

Unit 3

Exploring Relationships: Lines and Curves of Best Fit

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Scatter Plots

1. Graphing the Data

Given this data about studying and marks, answer the following questions.

a) Which of these statements do you think makes more sense?

Explain.

i) The number of minutes spent studying has an effect on the mark on the exam

or

ii) The mark on the exam has an effect on the number of minutes spent studying.

Time Spent Studying (min)	Mark on Exam
120	53
340	62
520	86
170	49
250	70
275	64
200	51
325	77
90	35
140	43
600	90
470	80
320	55
100	25

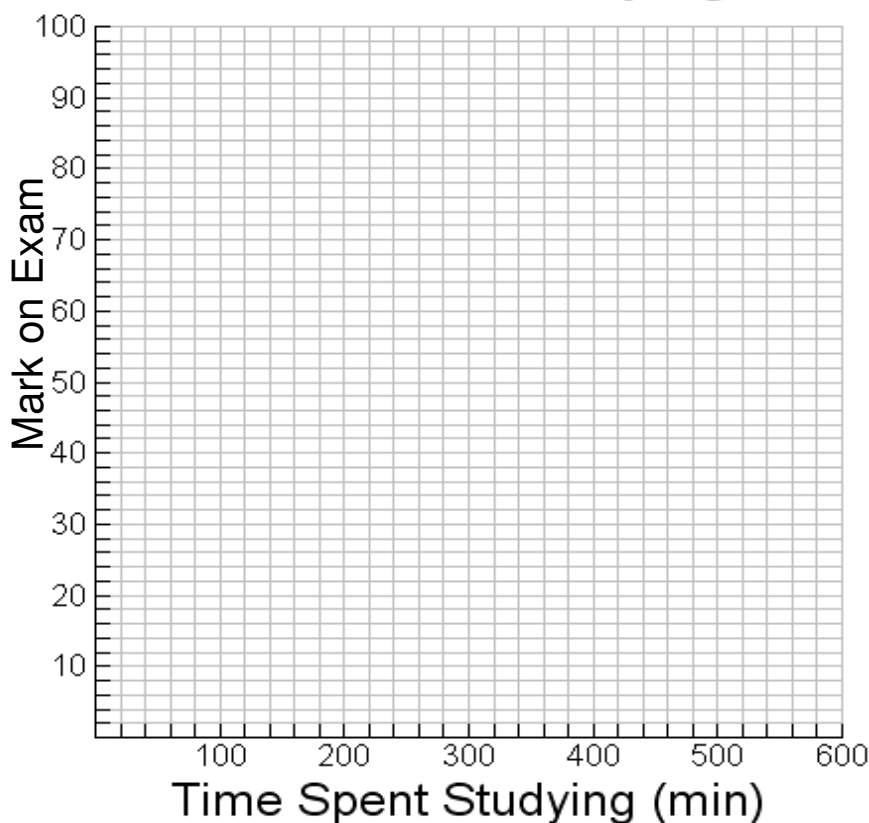
INDEPENDENT VARIABLE:

DEPENDENT VARIABLE:

b) Graph the data in the space provided

c) The **trend** of the data describes the direction of the points. How would you describe the trend for these points?

Marks and Studying



d) The **relationship** of the data describes how the two variables are connected. What relationship does the graph show between studying and the exam mark?

e) Describe the meaning of the point (520, 86).

2. Identifying Variables

When looking at relationships, the variable that is more likely to cause an effect on the other variable is called the **independent variable**. The one that is being affected is the **dependent variable**. When graphed, independent variables go on the horizontal (x) axis while the dependent variable goes on the vertical (y) axis. These variables become the axis labels.

a) This data set gives the Serving size and calories for 10 sandwiches at Arby's. For this relationship, identify the independent variable _____ and dependent variable _____

Sandwich	Serving size (grams)	Calories
Arby's Melt with Cheddar	148	368
Beef 'n Cheddar	189	487
Giant Roast Beef	228	555
Junior Roast Beef	126	324
Regular Roast Beef	154	388
Breaded Chicken Fillet	205	536
Chicken Cordon Bleu	240	623
Roast Chicken Santa Fe	182	436
French Dip	195	475
Ham 'n Cheese Melt	141	329

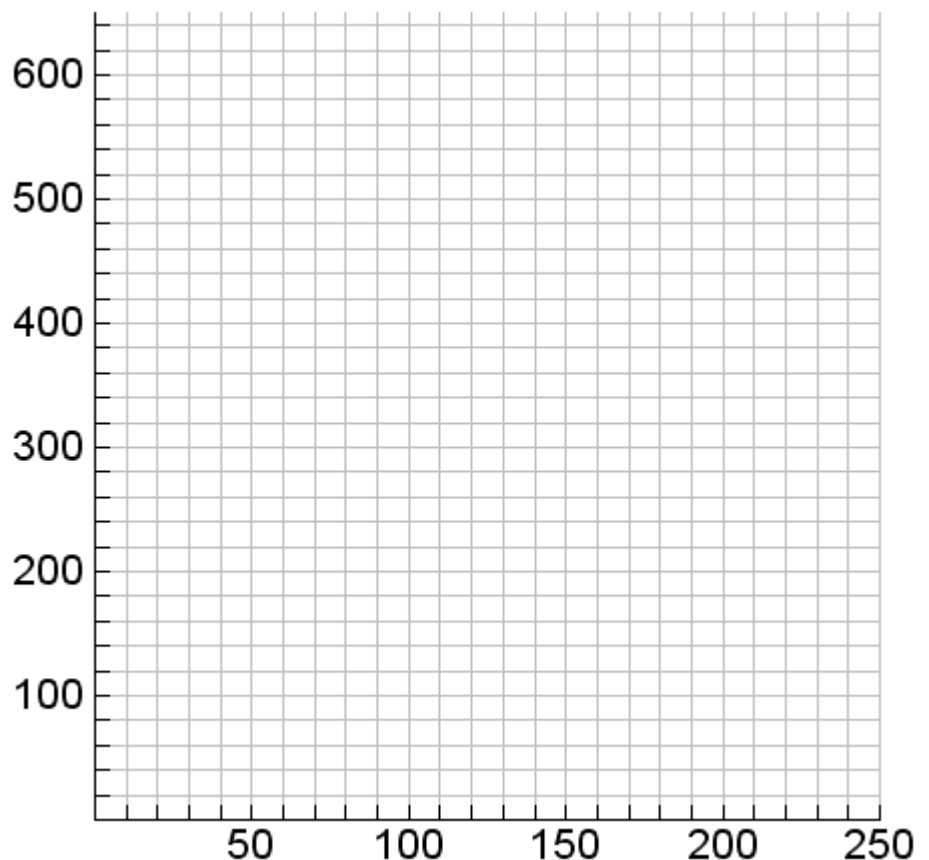
b) On a graph the independent variable goes on the horizontal (x) axis and the dependent variable goes on the vertical (y) axis. Use this information to label the graph below properly.

c) Graph the data

d) Describe the trend seen.

e) Describe the relationship seen.

f) Describe the meaning of the point (154, 388)



3. Creating the scale

a) Below is some data relating TV watching and reading. Identify the independent and dependent variables.

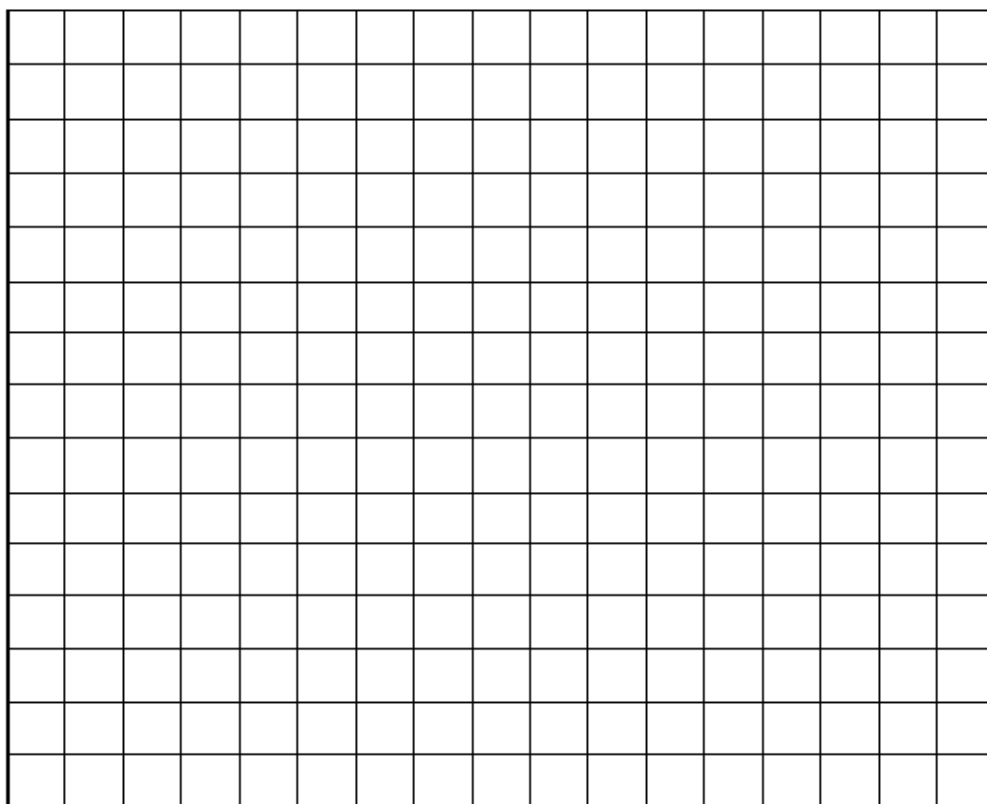
Independent _____

Dependent _____

Hrs of TV Watched per Week	6	20	30	12	15	21	26	25	24	23
Hrs Spent Reading per Week	12	6	2	9.5	8.5	5.5	3	3	4	4

b) Describe any problems in you had in answering part a)

c) Describe how you would make the scales for this data and graph the data.



c) Describe the meaning of the point (12, 9.5).

d) Describe the trend seen.

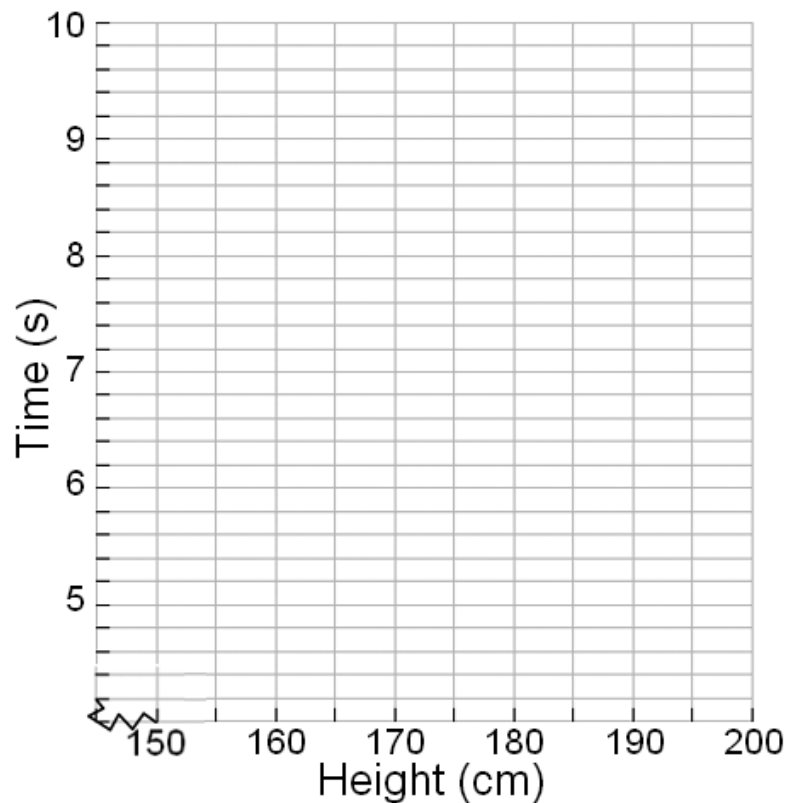
e) Describe the relationship seen.

4. Is There Always a Relationship?

a) What kind of relationship do you think exists between the two variables seen below?

Height (cm)	166	173	171	175	155	183	175	170	170	163	155	163
Time to run 50m (sec)	7.0	8.9	7.5	8.0	7.1	7.9	8.2	7.8	8.2	8.4	8.5	9.5

b) Graph and label this data in the space provided below. Be sure to try to make the graph fill the graph paper as much as possible.



c) Describe the trend seen.

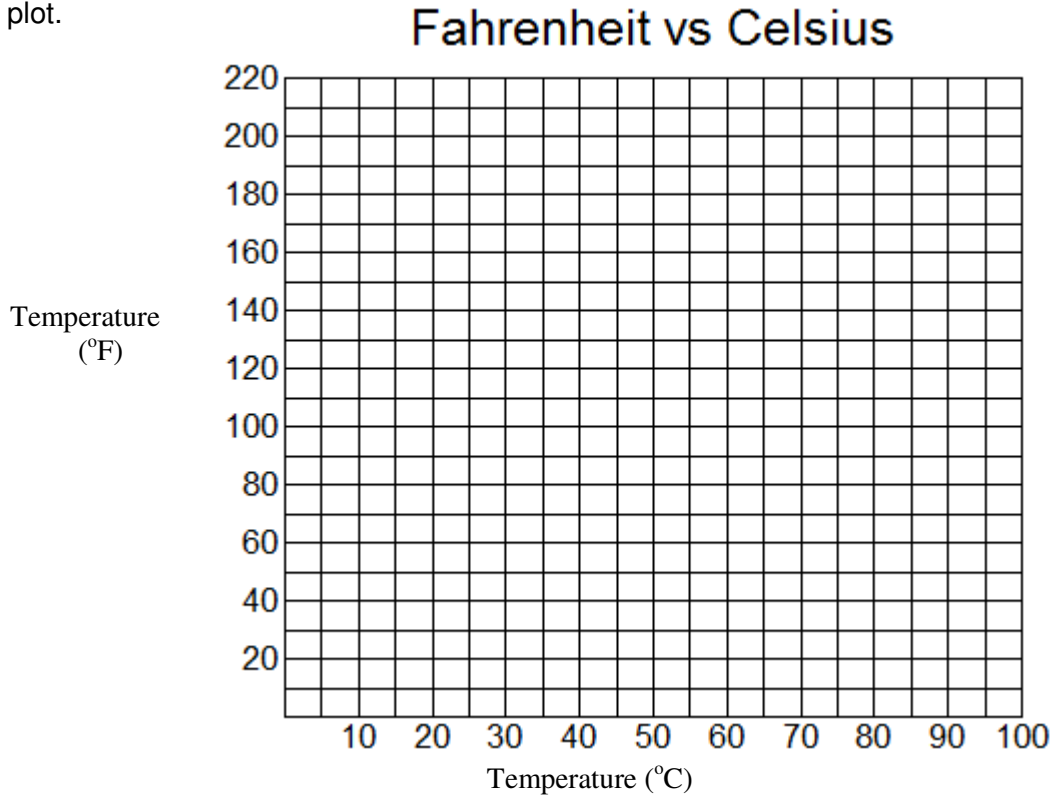
d) Describe the relationship seen

PRACTICE

1. The relationship between the Fahrenheit and Celsius scales can be seen in the table below.

Temperature (°C)	0	20	40	60	80	100
Temperature (°F)	32	68	104	140	176	212

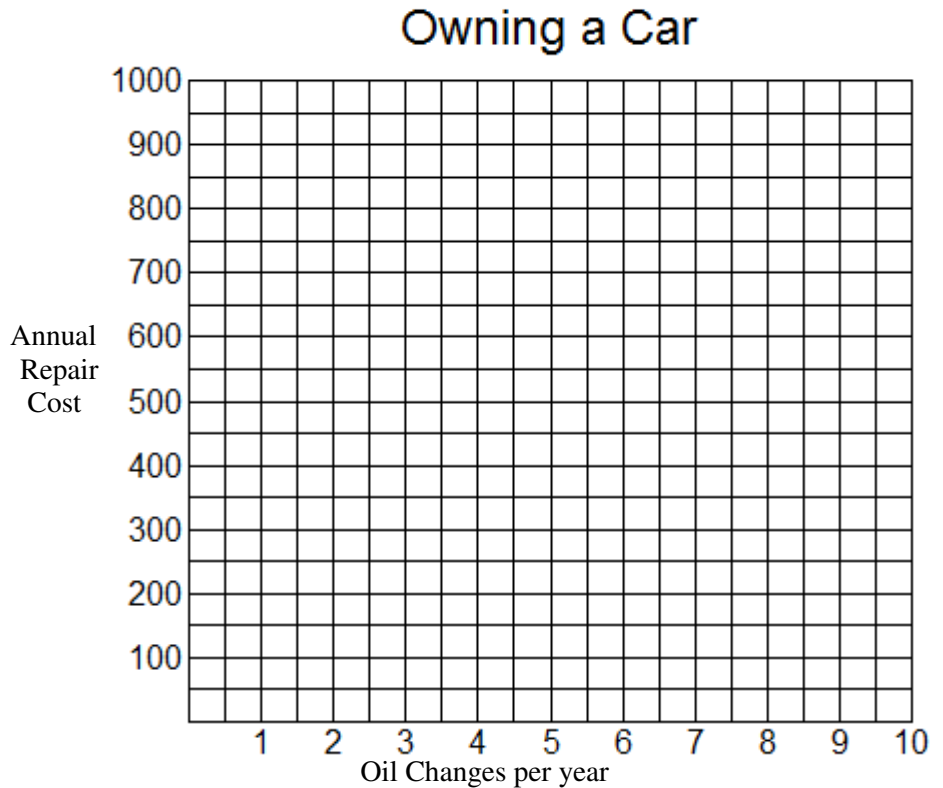
Create a scatter plot.



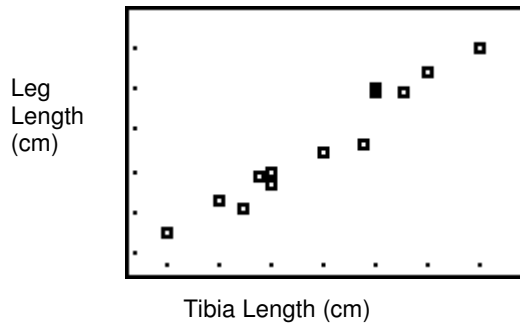
2. Below is a comparison of the number of oil changes in a year and the cost of auto repairs.

Create a scatter plot.

Oil Changes (per year)	Annual Repair Cost (\$)
0	850
5	310
5	270
7	125
5	400
8	110
6	150
8	95
5	300
1	560
10	0
3	400
2	650
1	750
3	450



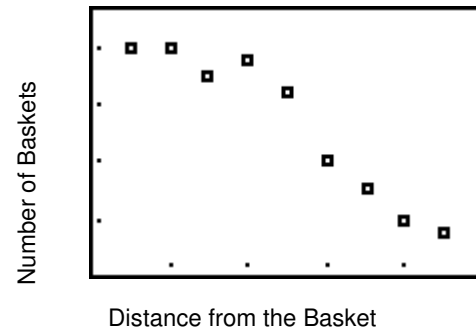
Plotted Points



1. Trend:

2. Relationship

3. The graph can be used to determine the length of a person's leg if you know the length of the tibia bone.
 - Agree
 - Disagree
 - Pass



1. Trend:

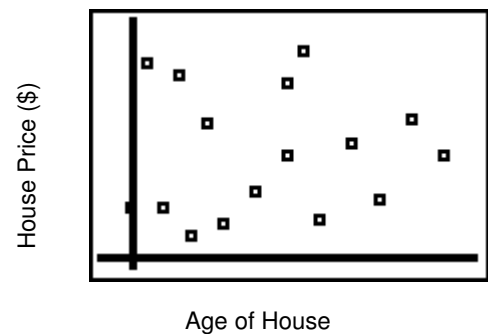
2. Relationship:

3. The graph can be used to determine the number of baskets you will make if you know the distance from the basket.
 - Agree
 - Disagree
 - Pass

1. Trend:

2. Relationship:


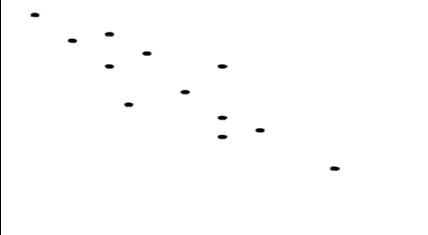
3. The graph can be used to determine the price of the house if you know how old it is.
 - Agree
 - Disagree
 - Pass




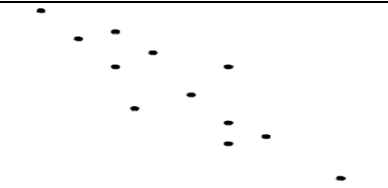
Scatter Plots - Types of Correlation

Correlation helps to describe the relationship between 2 quantities in a graph. Correlation can be described as positive or negative, strong or weak or none.

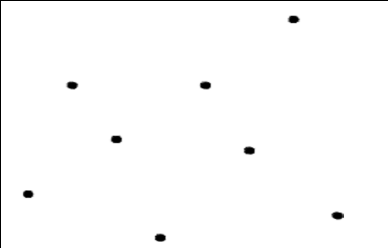
Positive or Negative Correlation

	<p>A scatter plot shows a _____ correlation when the pattern rises up to the right.</p> <p><i>This means that the two quantities increase together.</i></p>
	<p>A scatter plot shows a _____ correlation when the pattern falls down to the right.</p> <p><i>This means that as one quantity increases the other decreases.</i></p>

Strong or Weak Correlation

	<p>If the points nearly form a line, then the correlation is _____.</p> <p>To visualize this, enclose the plotted points in an oval. If the oval is <i>narrow</i>, then the correlation is <i>strong</i>.</p>
	<p>Here the points are still considered strong but not as strong as above. If the points are dispersed more widely, but still form a rough line, then the correlation can still be _____. But the <i>wider</i> the oval, the <i>weaker</i> the correlation is.</p>

No Correlation

	<p>A scatter plot shows _____ correlation when no pattern appears.</p> <p><i>Hint:</i> <i>If the points are roughly enclosed by a circle, then there is no correlation.</i></p>
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Journal Entry:

Jackie missed today's lesson. Send her a text message explaining the difference between positive, negative and no correlation.

Use the following set of variables to complete these correlations:

Age **Amount of studying** **Monthly spending**
Temperature
Take home pay **Hours of sleep per night** **Mark on a test**
Number of pets **Amount of water** **Amount of snow**
Number of hours worked **Hours of TV** **Length of hair** **Energy left**
Number of hours **watched at night** **Amount of books read** **in batteries**
listening to an ipod
Height of **Number of children** **Height of plants** **Age**
Snowman **in a family** **Snowboard Sales** **Final math mark**
Hours of TV watched per week **Amount in bank account**

1. List two variables that will show a positive correlation.

i. _____

ii. _____

2. List two variables that will show a negative correlation.

i. _____

ii. _____

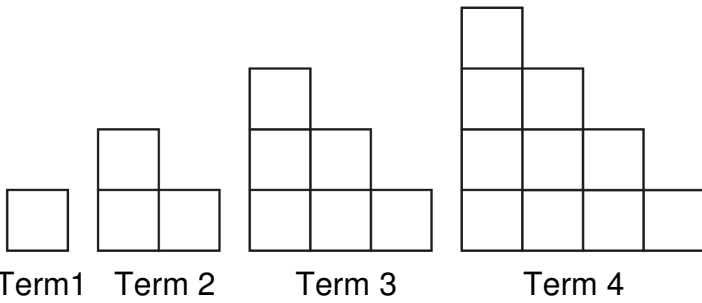
3. List two variables that will show no correlation.

i. _____

ii. _____

Curves of Best Fit

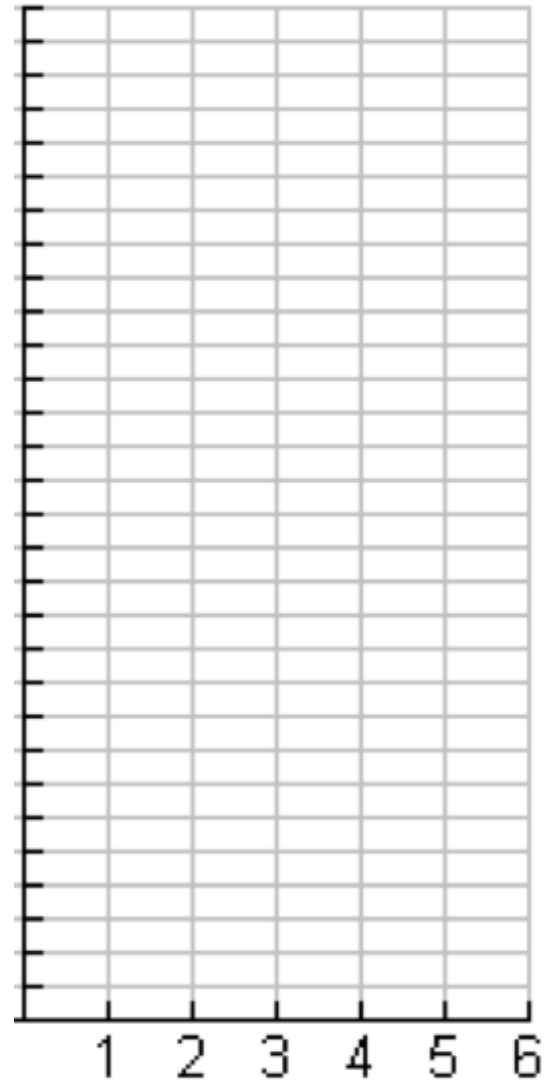
The first four terms of a pattern are built using cubes and are seen below.



Complete the following table according to the pattern.

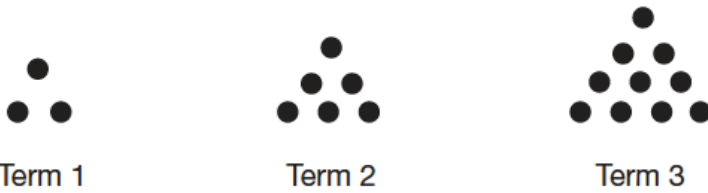
Term #, n	Total Number of Cubes
1	1
2	3
3	
4	
5	
6	

Graph the data from the table and draw a line or curve of best fit.



Stack It

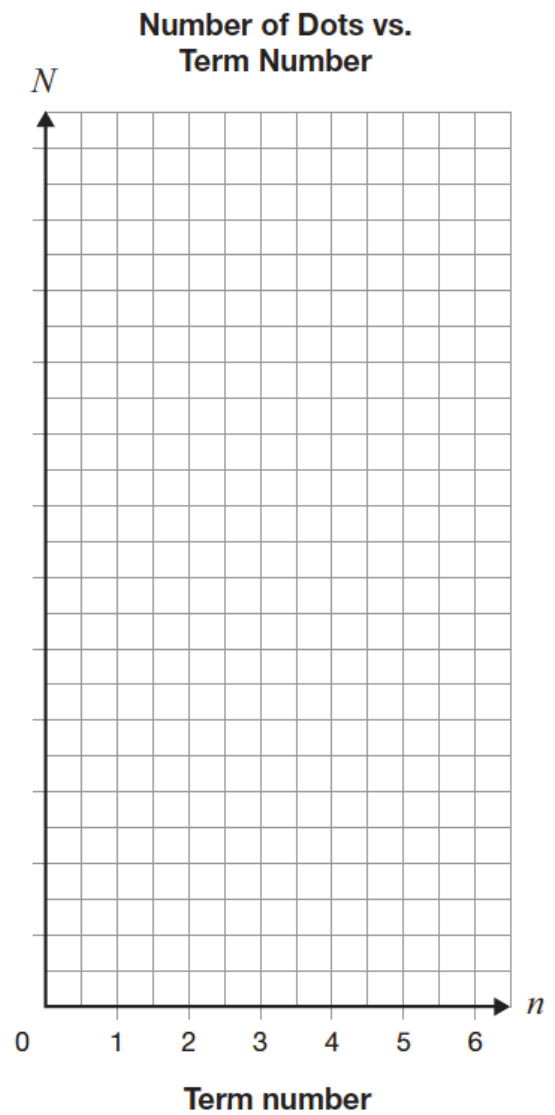
Juan draws the first three terms of a pattern as shown below.



The pattern continues to grow in the same way. Complete the following table according to the pattern.

Term number, n	Number of dots, N
1	3
2	6
3	
4	
5	
6	

Number of dots

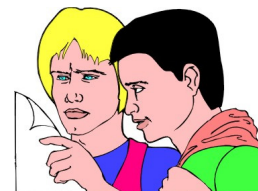


Graph the data from the table on the grid above. Add a scale for the N -axis.

Draw a line or curve of best fit for the data.

Relationships

Complete the following statements by yourself, then share your answers with your partner. Explain the reasons for your choice. Indicate if you and your partner agree or disagree.



Is There a Relationship?	My Partner and I:
As a person gets taller their arm span _____. <i>(gets wider, gets smaller, stays the same)</i>	___ agree ___ disagree
The longer a person's legs are _____ they run. <i>(the faster, the slower, will make no difference to how fast)</i>	___ agree ___ disagree
As a person's foot size increases, their walking stride _____. <i>(gets longer, gets shorter, stays the same)</i>	___ agree ___ disagree
As a person's forearm gets longer, their arm span _____. <i>(gets longer, gets shorter, stays the same length)</i>	___ agree ___ disagree
The longer a person's thumb is _____ their index finger. <i>(the longer, the shorter, will make no difference to the length of)</i>	___ agree ___ disagree
As a person gets taller, their foot size _____. <i>(gets longer, gets shorter, is not affected)</i>	___ agree ___ disagree

Data Collection – Is There a Relationship Here?

1) With a partner, measure and record each measurement to the nearest centimetre. Enter your data into the class data collection chart.

- a) total height _____ cm
- b) forearm _____ cm
- c) arm span from fingertips to fingertips _____ cm
- d) foot length _____ cm
- e) walking stride length _____ cm
- f) hand span _____ cm

2) Use Fathom or the graph paper provided to create scatter plots for the following for the following pairs of variables from your data set. Describe the trend, correlation, and relationships for each graph.

- a) Height vs Length of Forearm
- b) Height vs. Arm span
- c) Height vs. Hand span
- d) Height vs. Foot Length

3) Which model is the best predictor of the height? Give reasons for your answers.



4) Could I Be a Forensic Scientist?

Exploring the Problem

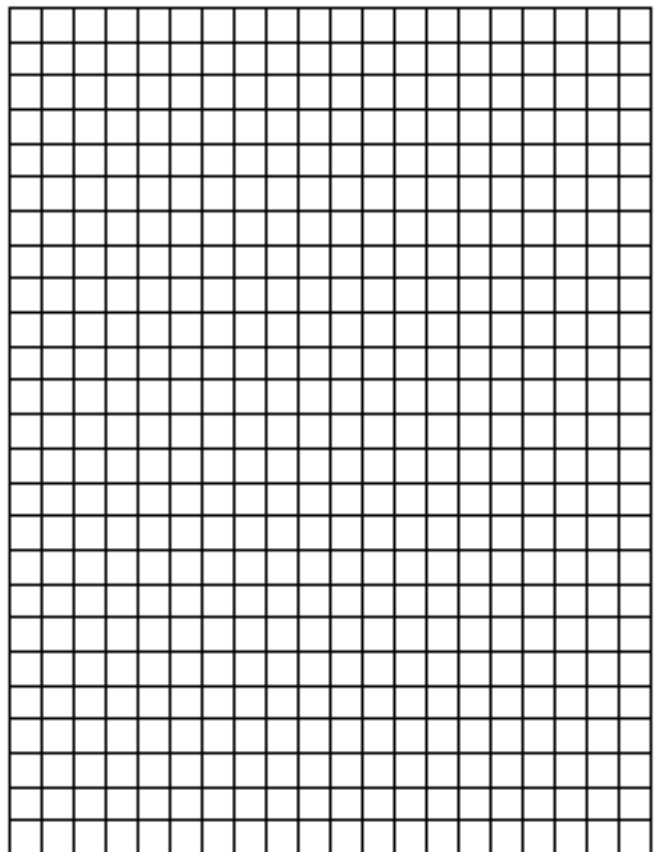
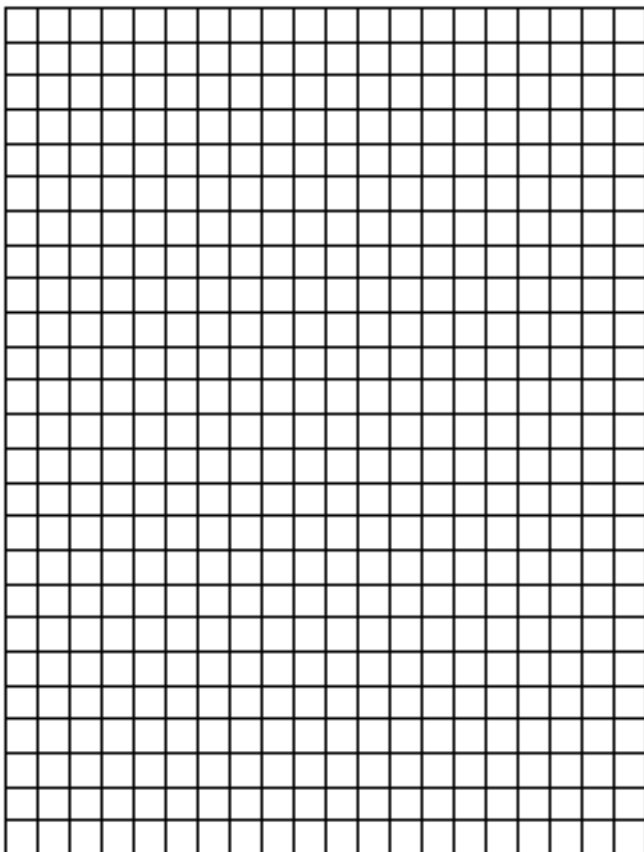
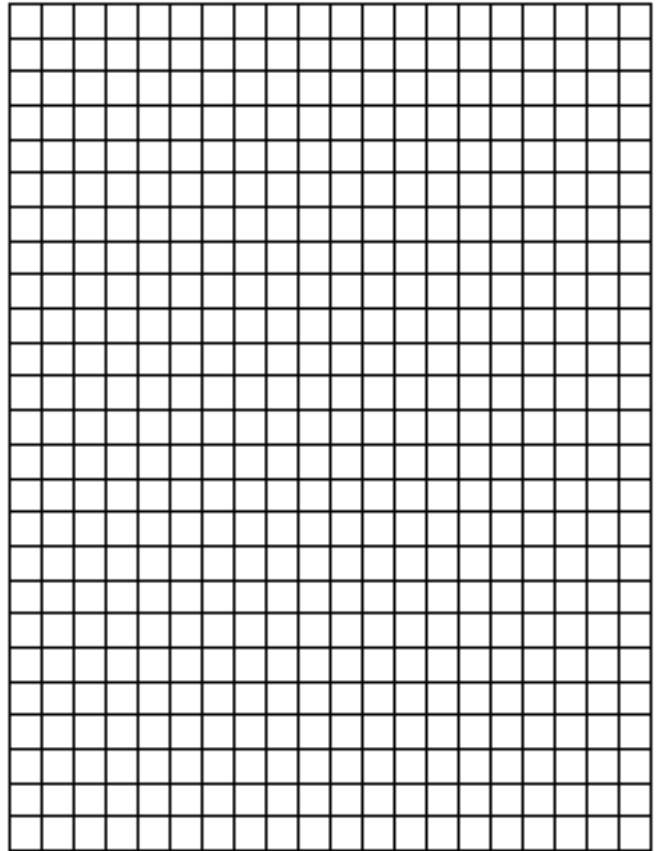
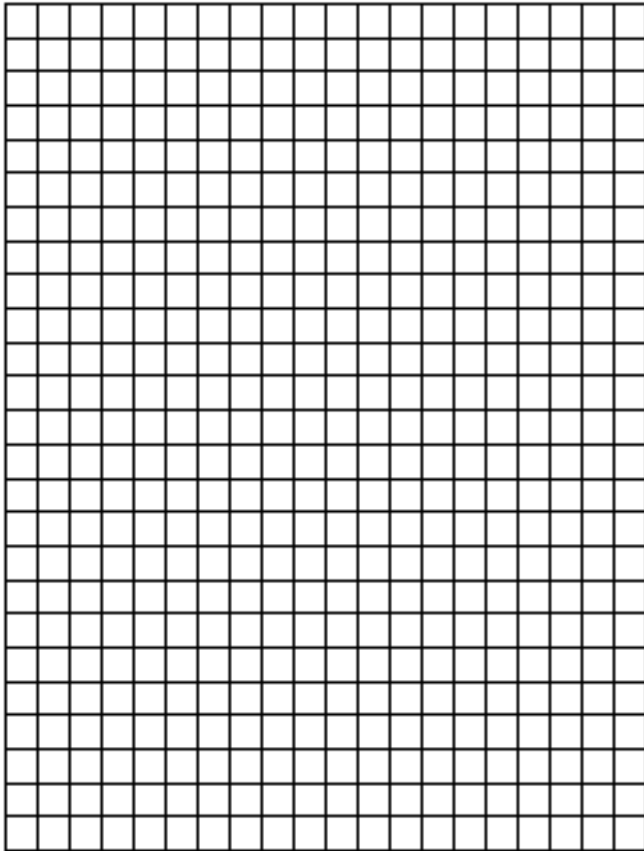
Remnants of a human skeleton were found at an archaeological dig that is thought to be the ruins of an ancient civilization. From the bones discovered, the scientists have determined the following:

- length of the forearm is 23 cm
- arm span is 185 cm
- hand span is 23 cm
- foot length is 24 cm

The scientists call you in as an expert in anthropology who is currently researching relationships between body measurements to help them determine an estimated height of the skeleton in question.

As the expert, your job will be to:

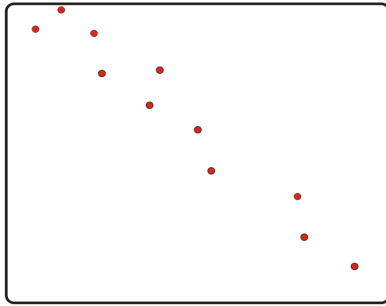
- estimate the height of the skeleton;
- explain the procedure you used to determine the height of the skeleton;



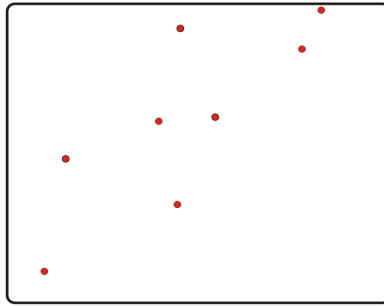
Scatter Plots, Correlation, and the Line of Best Fit

Using the graphs below, complete each of the following instructions.

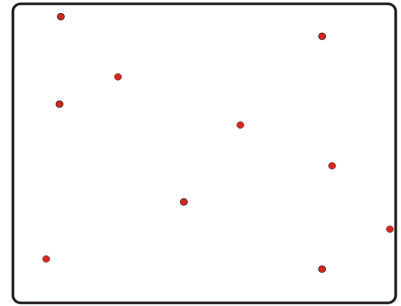
- 1) Draw a line of best fit if possible. If a line can be drawn, label the graph as *linear*. If a line cannot be drawn, label the graph as *non-linear*.
- 2) Label each graph as showing a *relationship* or *no relationship*.
- 3) The following instructions are for the **linear graphs** only.
 - a) Describe the correlation of each scatter plot as *positive* or *negative*.
 - b) Describe the correlation as *weak* or *strong*.



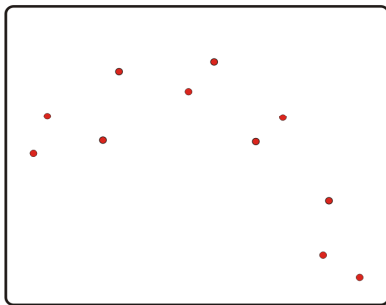
a) _____



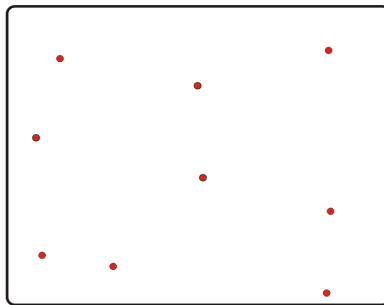
b) _____



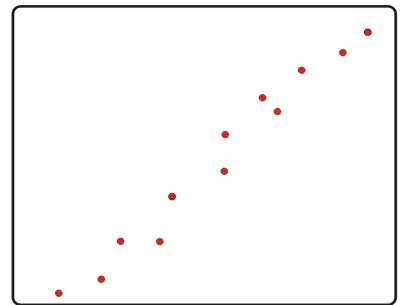
c) _____



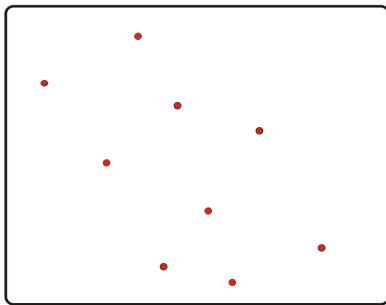
d) _____



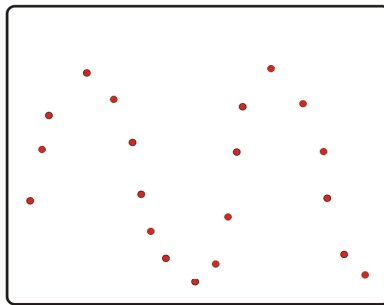
e) _____



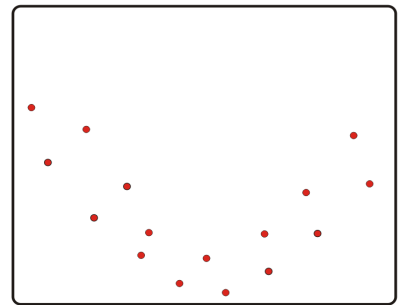
f) _____



g) _____



h) _____



i) _____

Practice Line of Best Fit

To be able to make predictions, we need to model the data with a line or a curve of best fit.

Rules for drawing a line of best fit:

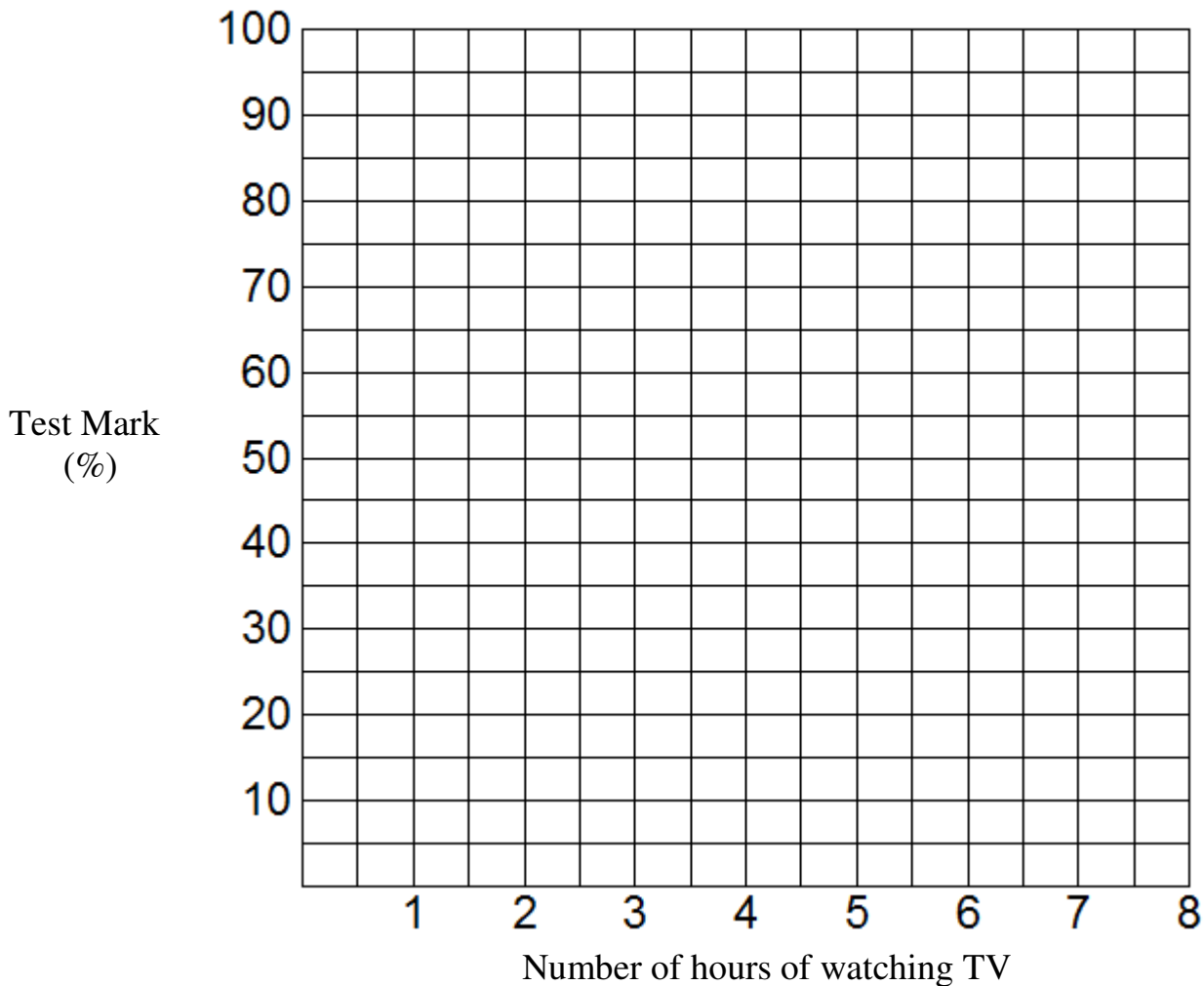
1. The line **must** follow the _____.
2. There should be _____ of points **distributed** above and below the line.

Use the information below to draw a scatter plot. Describe the correlation and draw the line of best fit.

The teachers at Holy Mary high school took a survey in their classes to determine if there is a relationship between the student's mark on a test and the number of hours watching T.V. the night before.

Mark %	75	70	68	73	59	57	80	65	63	55	85	70	55
Number of Hours	1	2	3	2	4	4.5	1	3	3.5	4	1	2.5	4

Hours of TV and Test Mark



Making Predictions Using the Lines of Best Fit

1. Explain what the point (4.5, 57) represents.
2. Predict how many hours of T.V. a student could watch to get a mark of 72.
3. Predict the mark a student would get if he watched 5 hours of T.V. the night before.
4. Predict the mark a student would get if he watched a half hour of T.V. the night before.
5. Predict how many hours of T.V, a student could watch to get a mark of 62.

Relationships Summary

A **scatter plot** is _____.

The points in a scatter plot often show a pattern, or _____.

From the pattern or trend you can describe the _____.

Example:

Julie gathered information about her age and height from the markings on the wall in her house.

Age (years)	1	2	3	4	5	6	7	8
Height (cm)	70	82	93	98	106	118	127	135

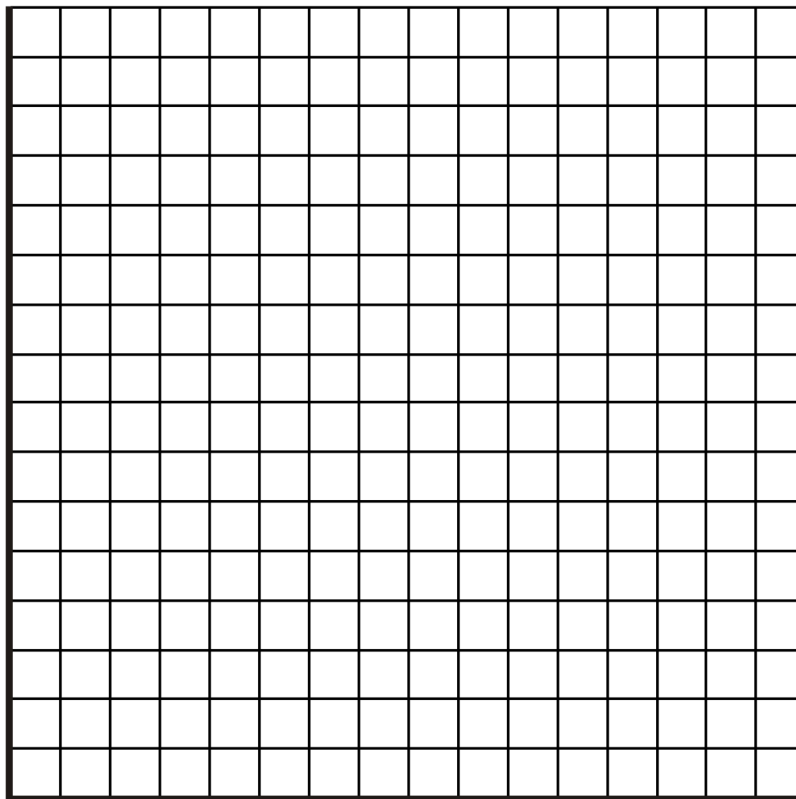
The **independent variable** is located on the _____ axis.
(These are the values you **choose**.)

The **dependent variable** is located on the _____ axis.
(These are the values you **calculate**.)

Independent Variable: _____

Dependent Variable: _____

- a) Construct a scatter plot.
- b) Describe the trend in the data.
- c) Describe the relationship.
- d) Describe the correlation.
- e) What does the point (4, 98) represent?
- f) Construct a line of best fit.



Making Predictions

Interpolate

When you interpolate, you are making a prediction _____ the data.

These predictions are usually _____.

Hint:
You are interpolating when the value you are finding is somewhere between the first point and the last point.

Extrapolate

When you extrapolate, you are making a prediction _____ the data.

It often requires you to _____ the line.

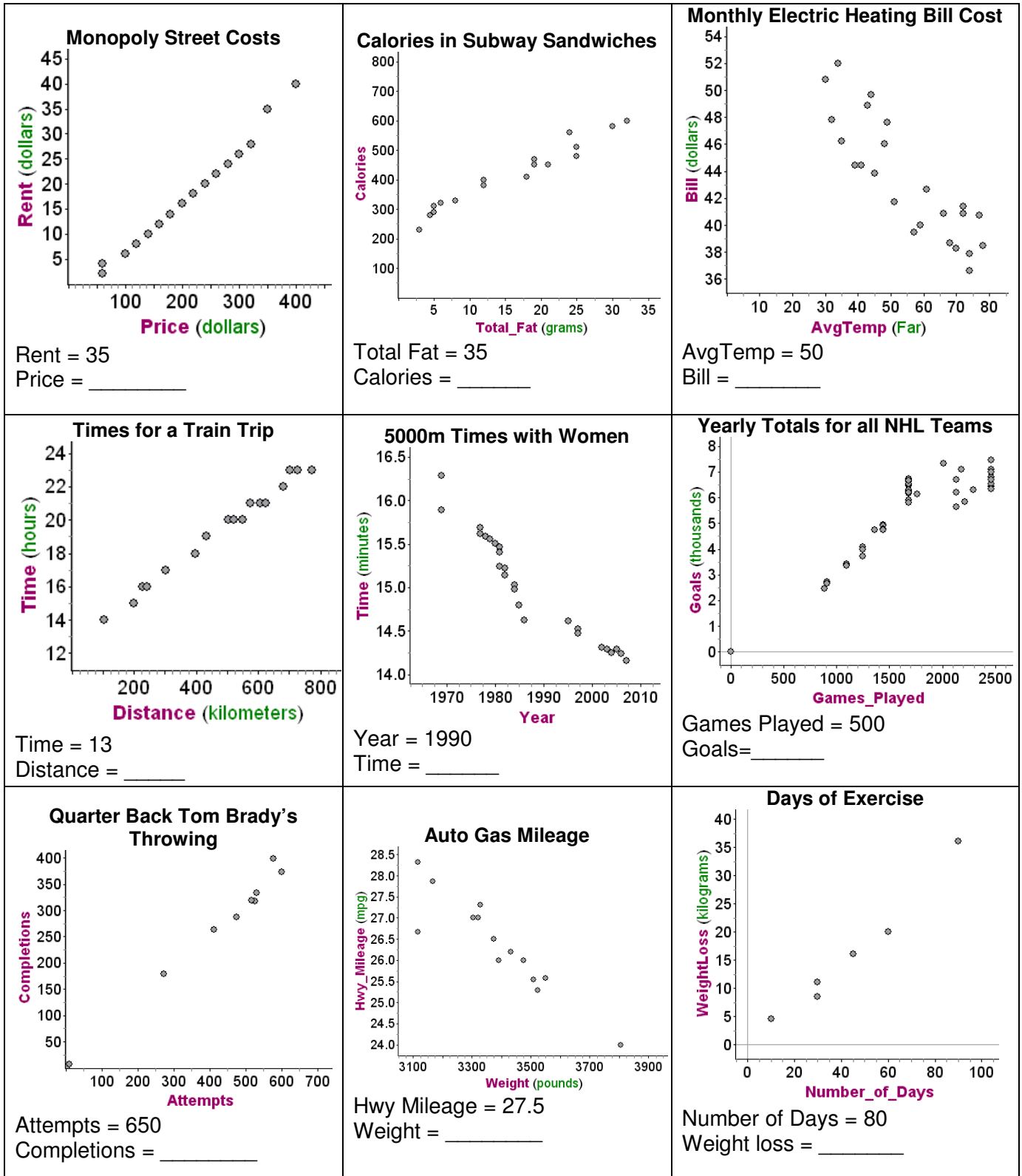
These predictions are less reliable.

You are extrapolating when the value you are finding is before the first point or after the last point. This means you may need to extend the line.

Use your line of best fit to estimate the following:

Question	Answer	Method of Prediction
How tall was Julie when she was 5 years old?		
How tall will Julie be when she is 9 years old?		
How old was Julie at 100 cm tall?		
How tall was Julie when she was born?		

- In each of the graphs below
- draw the line of best fit
 - for the given variable, estimate the other variable
 - indicate whether you interpolated or extrapolated for part b)



Forensic Analysis

Anthropologists and forensic scientists use data to determine information about people. Scientists can make predictions about the height, age, and sex of the person they are examining by looking for relationships in large amounts of data.

1. Construct a graph of the length of the humerus bone vs. the length of the radius.

(Avoid a large empty space in one corner of the graph by using the symbol to signal a “break” in the axis.)

2. Circle the point on the graph that represents the data for a radius that is 21.9 cm long.

How long is the humerus? _____

3. What does the point (21.4, 24) represent?

4. Describe the trend.

5. Describe the relationship.

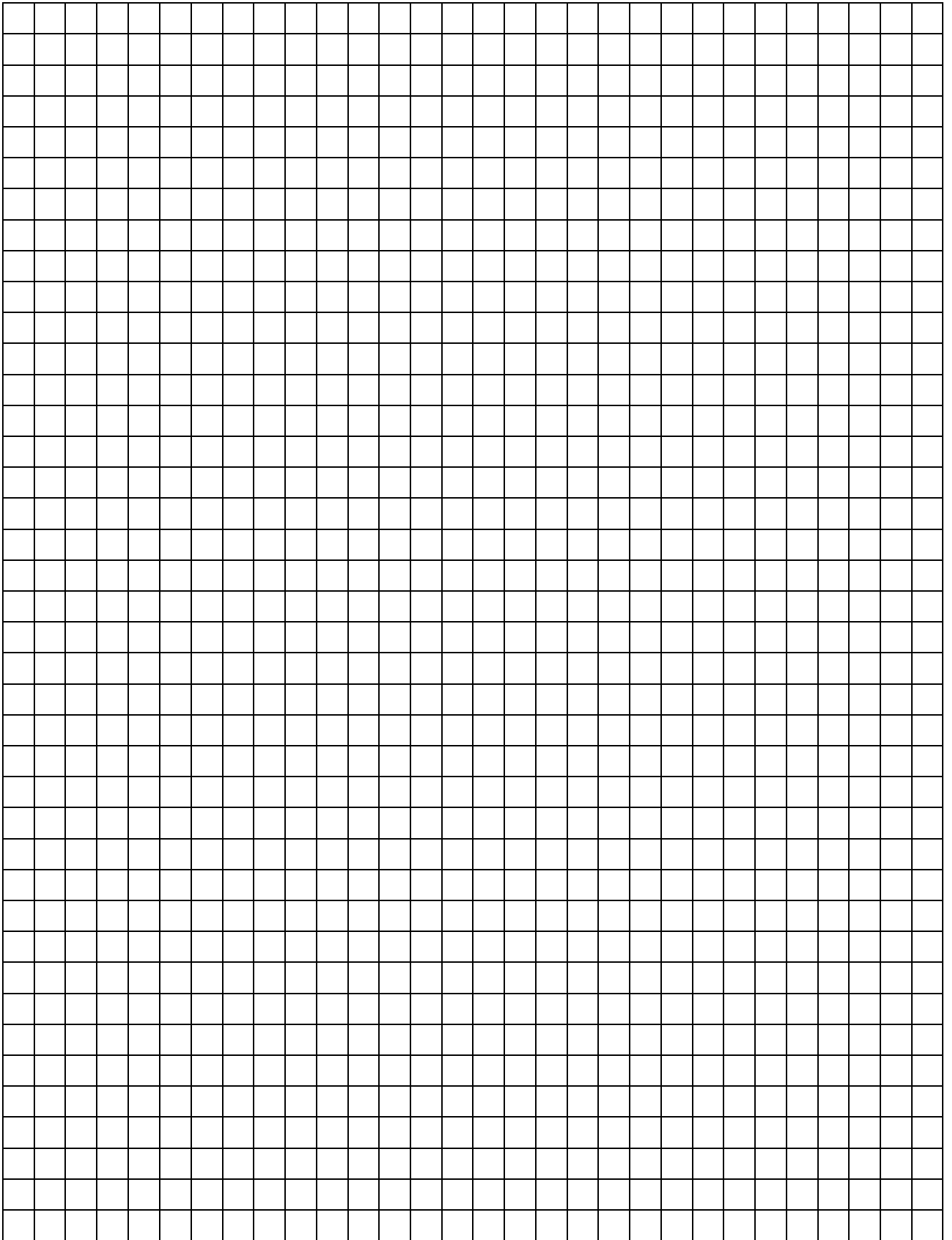
6. Describe the correlation.

- 7a. Draw a line of best fit.

- 7b. Use the line of best fit to predict the length of the humerus, if the radius is 24.5 cm long. Did you interpolate or extrapolate?

- 7c. Use the line of best fit to predict the length of the radius, if the humerus is 25 cm long. Did you interpolate or extrapolate?

Length of Radius (cm)	Length of Humerus (cm)
25	29.7
22	26.5
23.5	27.1
22.5	26
23	28
22.6	25.2
21.4	24
21.9	23.8
23.5	26.7
24.3	29
24	27



Linear and Non-Linear Relations

A. Jody works at a factory that produces square tiles for bathrooms and kitchens. She helps determine shipping costs by calculating the perimeter of each tile.

i) **Calculate** the perimeter and record your answers in the Perimeter column of the table.

Side Length (cm)	Perimeter (cm)	First Differences
1		
2		
3		
4		
5		

ii) **Describe** what happens to the perimeter of each tile when the side length increases by one centimetre. _____

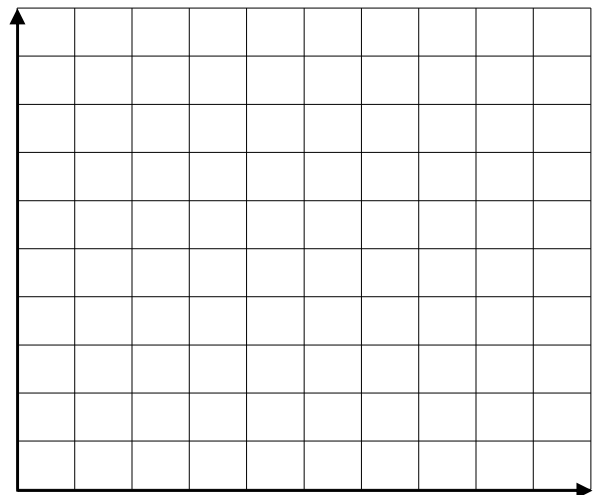
iii) **Construct** a graph of the perimeter vs. the side length. Include labels and titles.

a) Which variable is the independent variable?

b) Which variable is the dependent variable?

c) Use the graph to describe the relationship between the perimeter and side length of a tile.

d) Describe the shape of the graph.



iv) **Calculate** the first differences in the First Differences column of the table. What do you notice about the first differences?

v) **Summarize** your observations.

a) When the side length increases by one centimetre, the perimeter increases by _____.

b) The plotted points suggest a _____.

c) The first differences are _____.

B.Raj, another employee at the factory, also works with the tiles. He helps to determine the shipping costs by calculating the area of each tile.

i) **Calculate** the area and record your answers in the Area column of the table.

Length of sides (cm)	Area (cm ²)	First Differences
1		
2		
3		
4		
5		

ii) **Describe** what happens to the area of each tile when the side length of a tile increases by one centimetre. _____

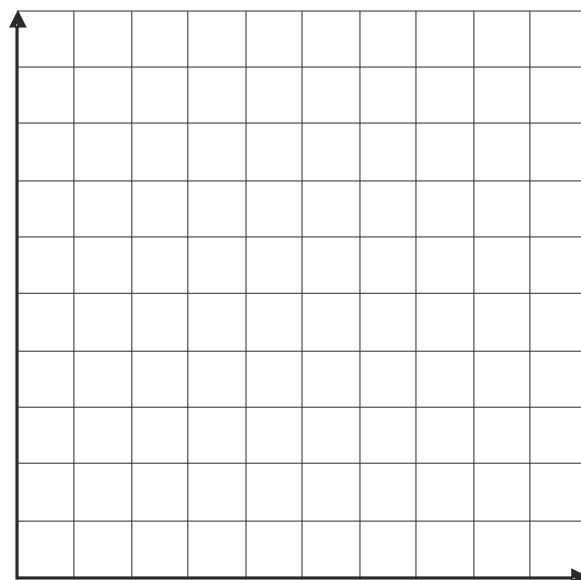
iii) **Construct** a graph of the area vs. the length of the sides of the tiles. Include labels and titles.

a) Which variable is the independent variable?

b) Which variable is the dependent variable?

c) Use the graph to describe the relationship between the area and the side length of the tile.

d) Describe the shape of the graph.



iv) **Calculate** the first differences in the First Differences column of the table. What do you notice about the first differences?

v) **Summarize** your observations.

a) When the side length increases by one centimetre, the area increases by _____.

b) The plotted points suggest a _____.

c) The first differences are _____.

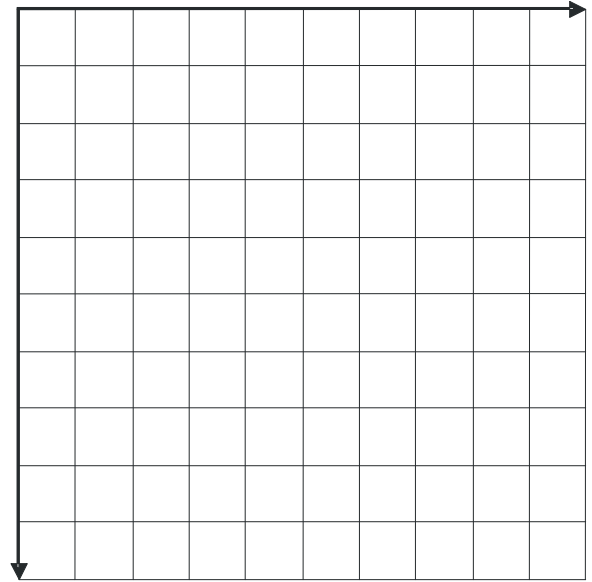
What You Have Discovered

Deep Sea Divers

The table below shows data collected as divers descend below sea level. Calculate the first differences. Use the first differences to determine if the relationship is linear or non-linear. Check your solution by graphing. Include labels and titles.

Time (min)	Depth (m)	First Differences
0	-2	
1	-4	
2	-6	
3	-8	
4	-10	

The relationship is:

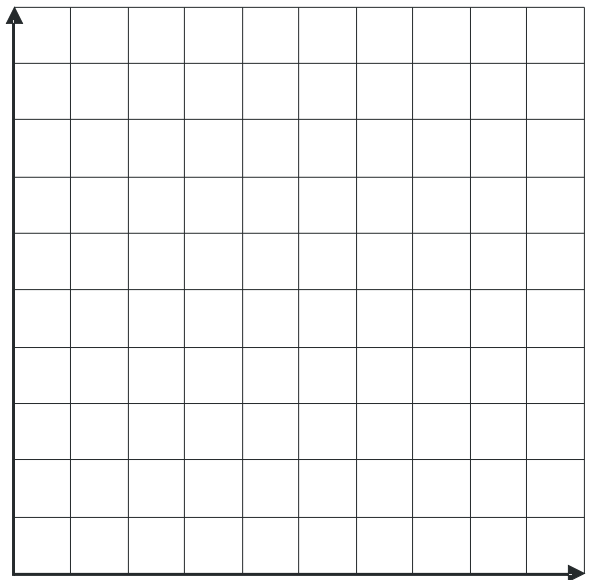


Hot Air Ballooning

The table shows data collected as a hot air balloon leaves the ground. Calculate the first differences. Use the first differences to determine if the relationship is linear or non-linear. Check your solution by graphing. Include labels and titles.

Time (sec)	Height (m)	First Differences
0	0	
1	1	
2	4	
3	9	
4	16	

The relationship is:



Practice

1. Calculate the first difference and classify the relation as linear or non-linear?

a)

x	y
1	0
2	4
3	8
4	12
5	16

b)

x	y
0	0
2	4
4	16
6	36
8	64

c)

x	y
1	2
2	4
3	7
4	11
5	16

d)

x	y
-1	1
0	-2
1	-5
2	-8
3	-11

2. Classify the relation as linear or non-linear.

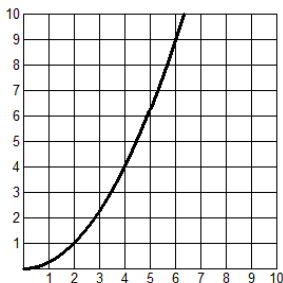
a)

Time (s)	Speed (m/s)
0	0
0.5	4.9
1.0	9.8
1.5	14.7
2.0	24.5

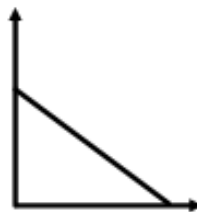
b)

Number of people	Cost (\$)
0	150
2	240
4	330
6	420
8	510

c)



d)



e)

Time (s)	Distance (m)
0	0
30	60
60	130
90	210
120	300

Unit 1 Practice

1. David went for a bike ride. The table shows his distance from home at different times.

a) Graph the data.

b) Describe the trend.

c) Describe the correlation.

d) Describe the relationship.

e) Draw a line of best fit.

f) How far from home was David after each time?

i) 15 min _____

ii) 55 min _____

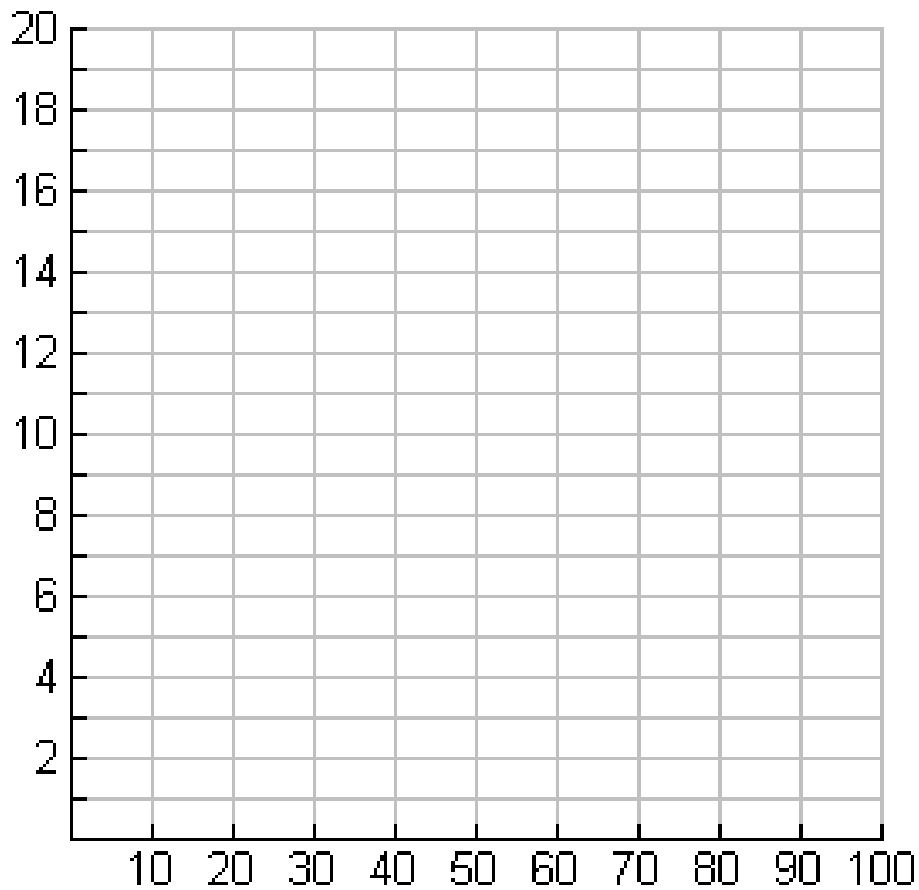
g) After how many minutes was David each distance from home?

i) 4 km _____

ii) 13 km _____

David's Distances from Home

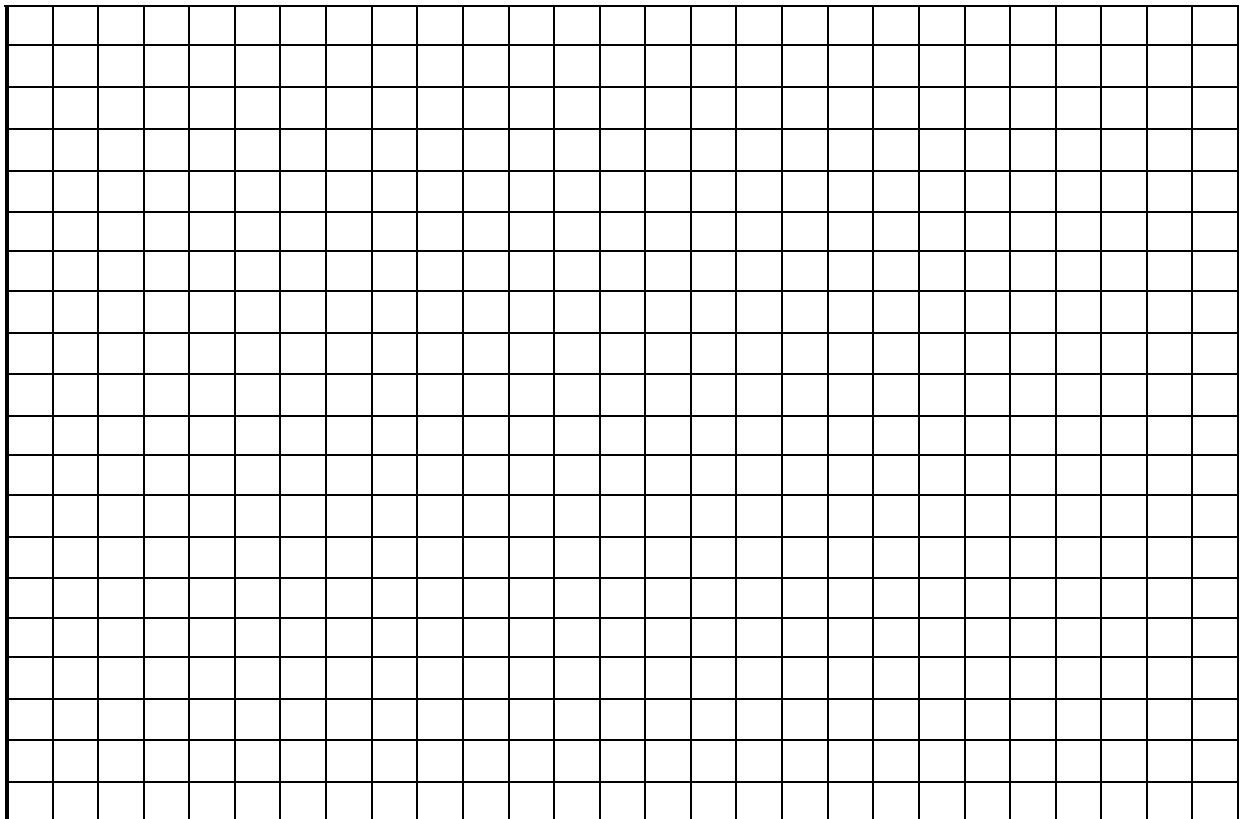
Time (min)	Distance (km)
0	0
10	2.5
20	5.0
30	7.5
40	10.0
50	12.5
60	15.0
70	17.5



2. The table shows, for 10 students, the number of hours their spent studying for their final exam and the mark they received on the exam.

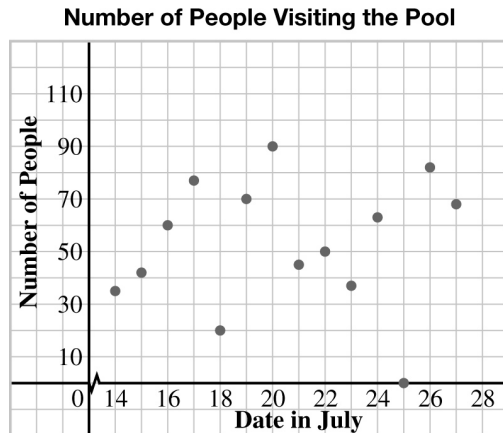
Study Time (hours)	10	0	1	3	5	4	7	9	8	5
Exam Mark (%)	100	50	60	70	65	80	95	80	90	55

- a) Create a scatter plot for this data.
- b) Explain what the point (4, 80) represents.
- c) Draw a line of best fit.
- d) Use your line of best fit to answer the questions:
 - i. How many hours would someone have to study to get a mark of 70%? _____
 - ii. What mark would someone get if they studied for 6 hours? _____



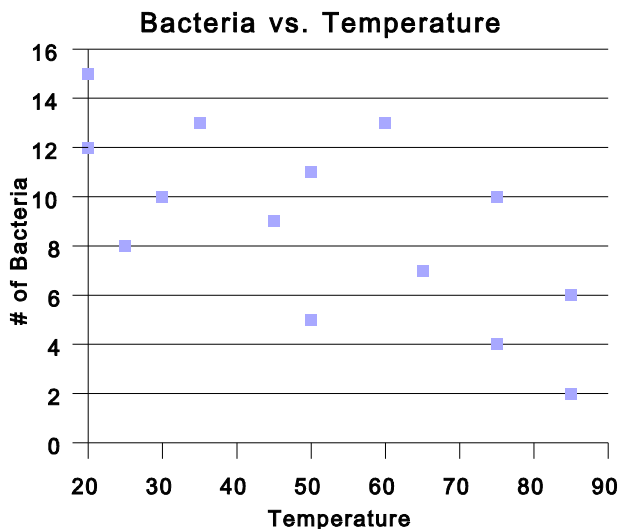
3. The graph shows the number of people that visited the Long Island Aquatic Club outdoor pool each day from July 14th to July 27th.

- a) How many people were at the pool each day?
 July 17th _____ July 23rd _____
- b) On what day(s) were each number of people at the pool?
 20 _____ 45 _____
- c) Which day had the highest attendance? _____
- d) Which day had the lowest attendance? _____



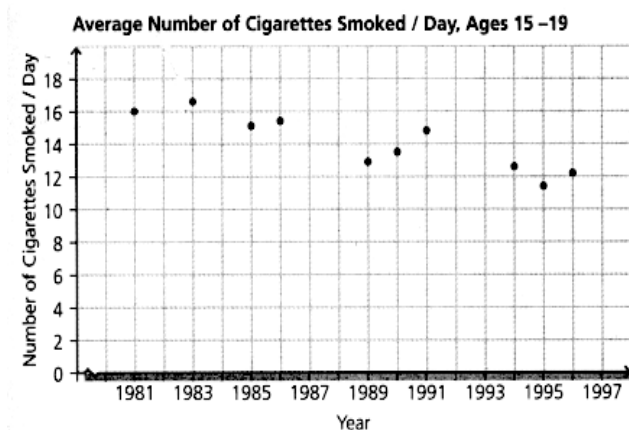
4. The following scatter plot shows the number of bacteria living in a culture at various temperatures.

- a) Draw a line of best fit.
- b) How many bacteria will live in a temperature of:
 40 degrees Celsius? _____
 80 degrees Celsius? _____
- c) Predict the temperature if:
 8 bacteria are in the culture _____
 14 bacteria are in the culture _____



5. The following graph represents the long term trends in smoking by teenagers aged 15 – 19.

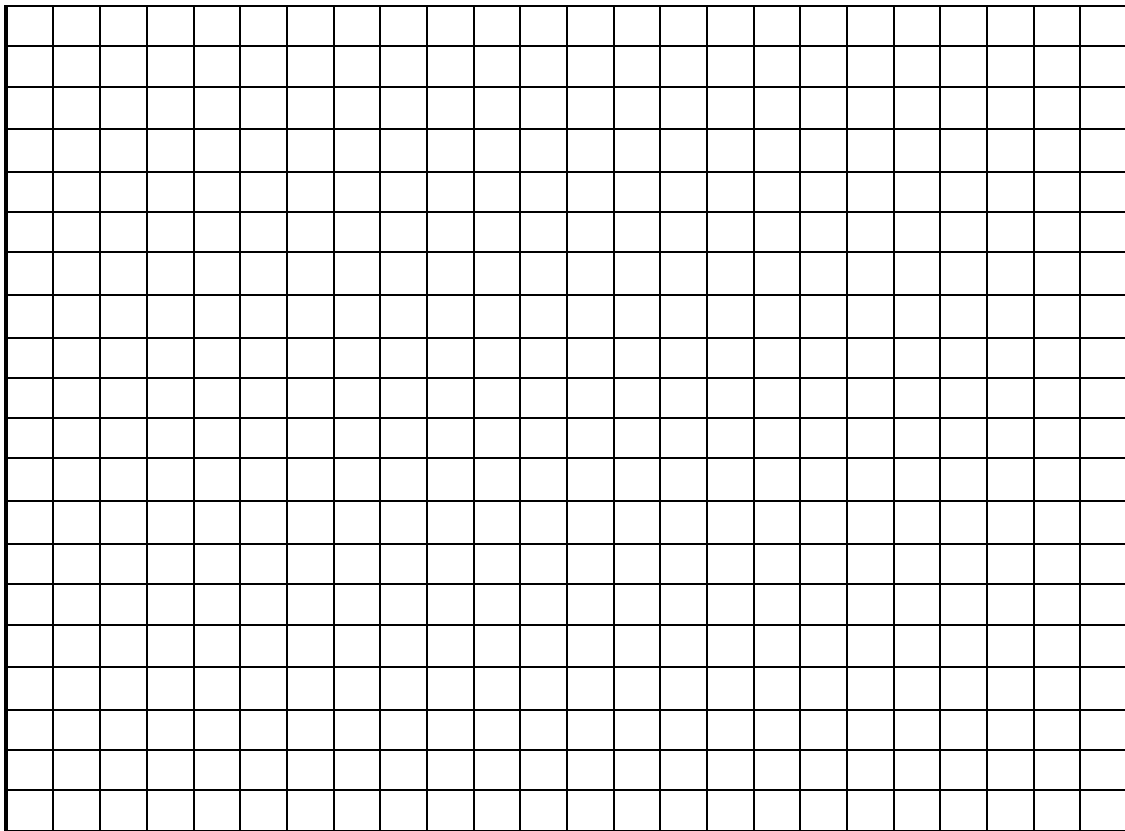
- a) Draw the line of best fit.
- b) What is the average number of cigarettes smoked per day by 15-19 year old in 1989?
- c) In what year were 15-19 year olds smoking an average of 14 cigarettes/day?



6. The table below shows the height of a certain type of tree over several years.

Age (years)	4	10	6	12	11	8	13	13	10	8	3	6
Height (metres)	2	6	4	5	6	3	5	8	4	5	3	3

- a) Create a scatter plot.
- b) Draw a line of best fit.
- c) What is the height of this tree after:
 5 years? _____ 9 years? _____
- d) Predict the age of a tree that is:
 1 m tall. _____ 7 m tall. _____



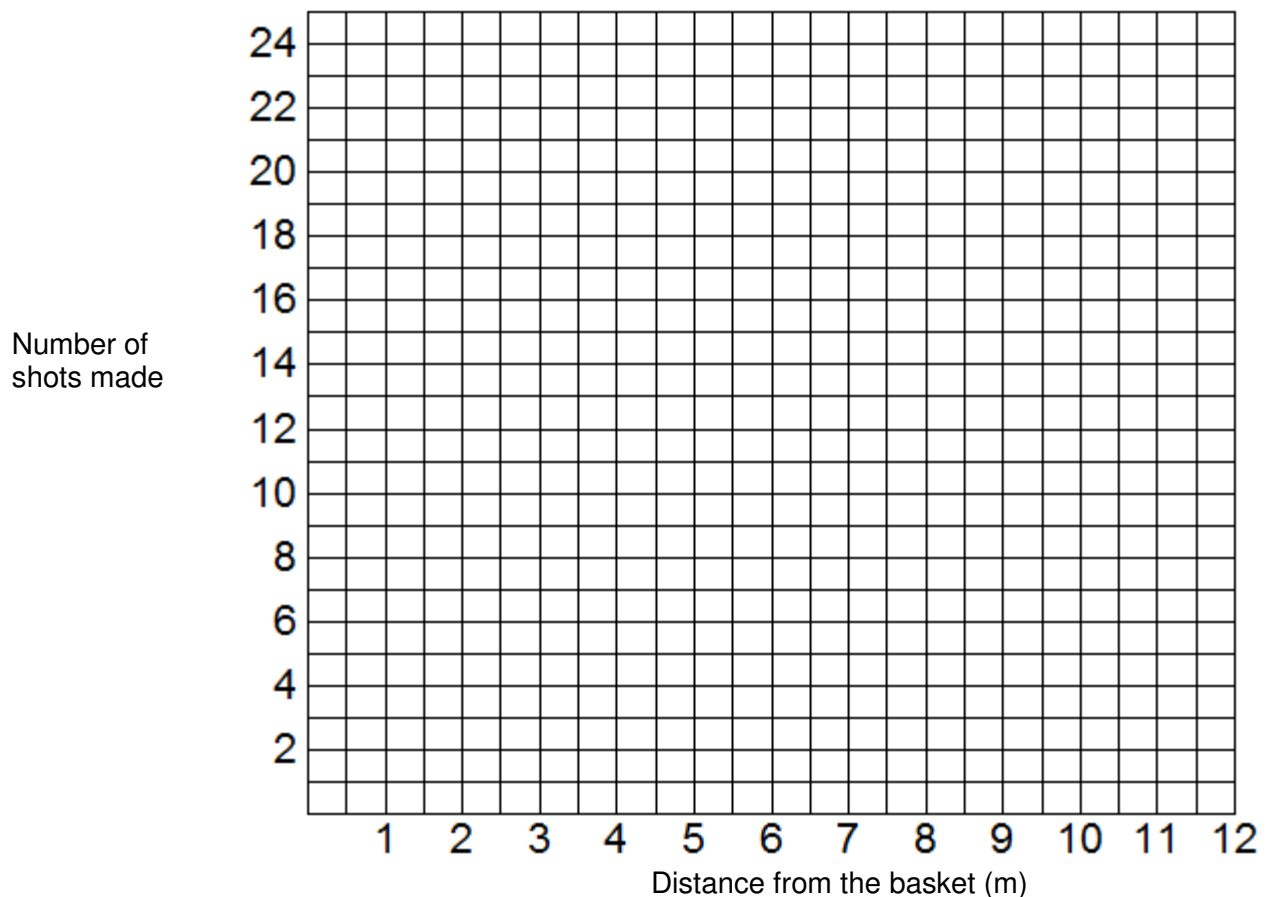
7. The table shows the number of successful shots Alex made a various distances from the basket.

Distance from the basket (m)	3	5	7	8	9	10
Number of shots made	22	17	16	10	8	3

- a) Create a scatter plot.
- b) Draw a line of best fit.
- c) Use your line of best fit to estimate the following:

Question	Answer	Method of Prediction interpolation or extrapolation
Number of shots Alex will make from 4.5 m?		
How far is Alex from the basket if he makes 18 shots?		
Number of shots Alex will make from 1 m?		
How far is Alex from the basket if he makes 2 shots?		

Hitting the Basket



EQAO Type Questions

1.

Which table shows a linear relationship between x and y ?

A

x	y
1	1
2	4
3	9
4	16

B

x	y
1	2
2	4
3	8
4	16

C

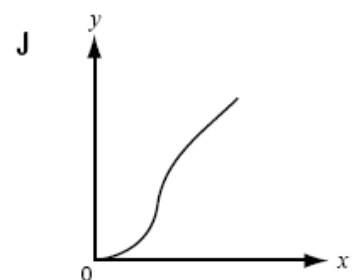
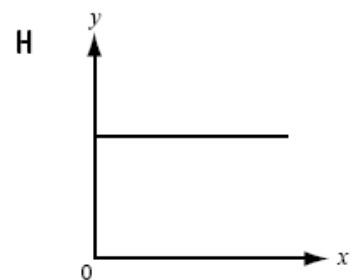
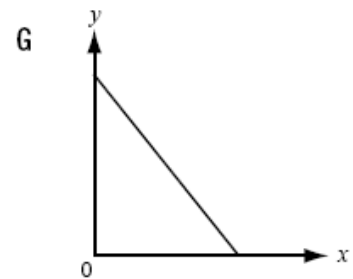
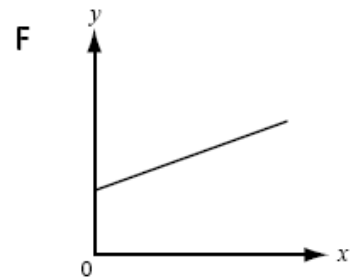
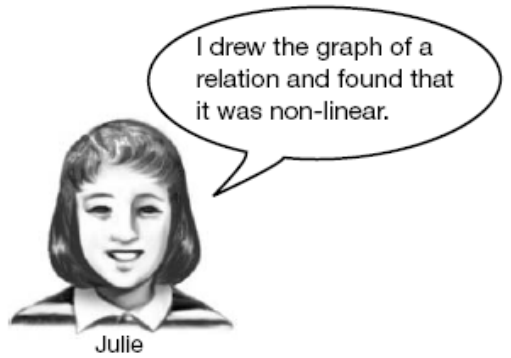
x	y
1	10
2	20
3	30
4	40

D

x	y
1	10
2	100
3	1000
4	10000

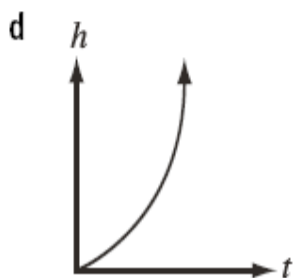
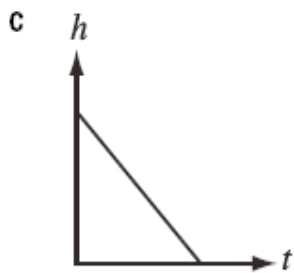
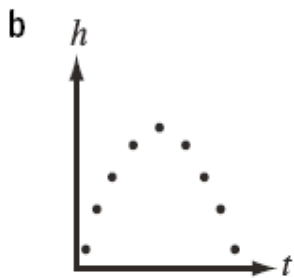
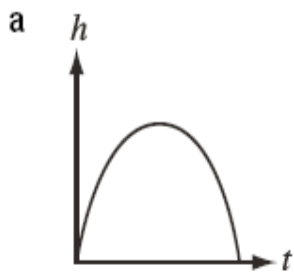
2.

Which graph does Julie draw?



3.

Which of the following graphs represents a linear relation?



4.

Simon records the height of a plant each day for five days.

Plant Growth Over Five Days

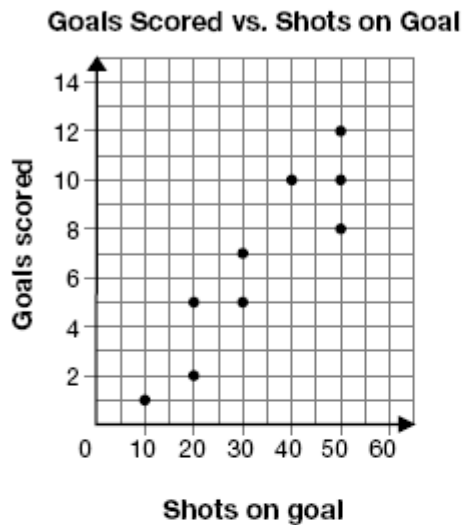
Day	Height (cm)
0	4
1	5
2	7
3	10
4	14

His chart shows that the relation between height and day

- a is a linear relation.
- b is a non-linear relation.
- c has a constant rate of change.
- d has a decreasing rate of change.

5.

The scatter plot below shows the number of goals scored versus shots on goal by different members of a soccer team during the course of a season.



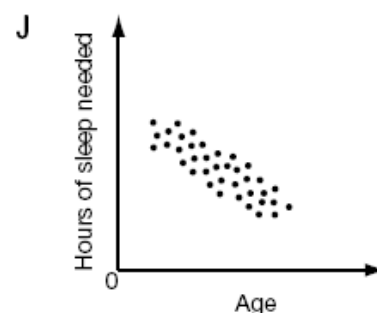
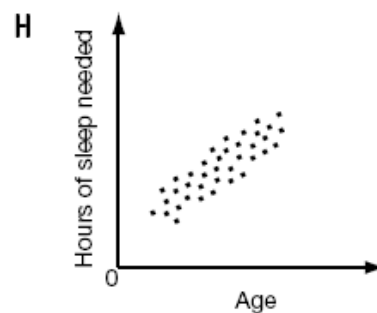
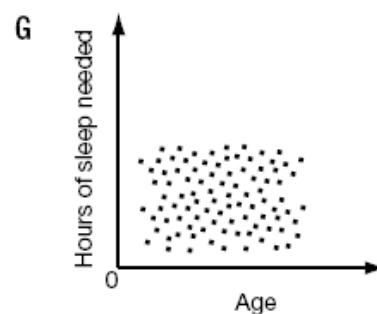
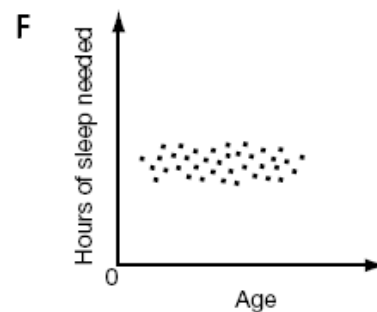
What relationship is shown in the scatter plot?

- A There is no relationship between shots on goal and the number of goals scored.
- B The number of goals scored decreases as the number of shots on goal increases.
- C The number of goals scored increases as the number of shots on goal increases.
- D The number of goals scored stays the same no matter how many shots on goal are taken.

6.

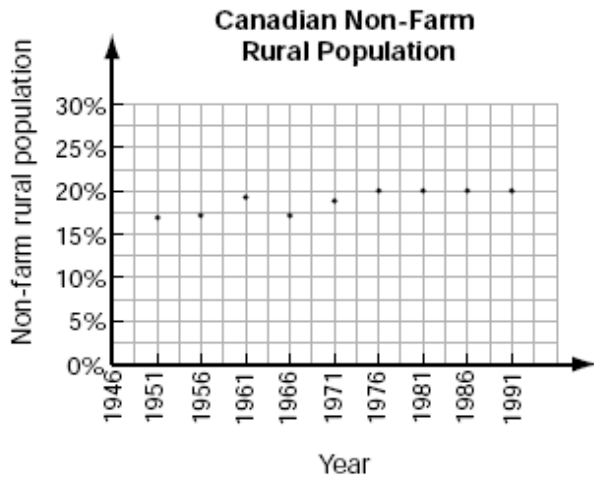
Sunil surveys 200 students to find out how much sleep they need. He finds that students need less sleep as they get older.

Which scatter plot most likely represents the survey results?



7.

Study the graph below, which shows how the percent of the Canadian population that is non-farm rural has varied over time.



Which statement about the percent is true?

- a It stayed constant from 1951 to 1991.
- b It increased from 1966 to 1976.
- c It decreased at a constant rate from 1961 to 1981.
- d It increased at a constant rate from 1951 to 1971.

8.

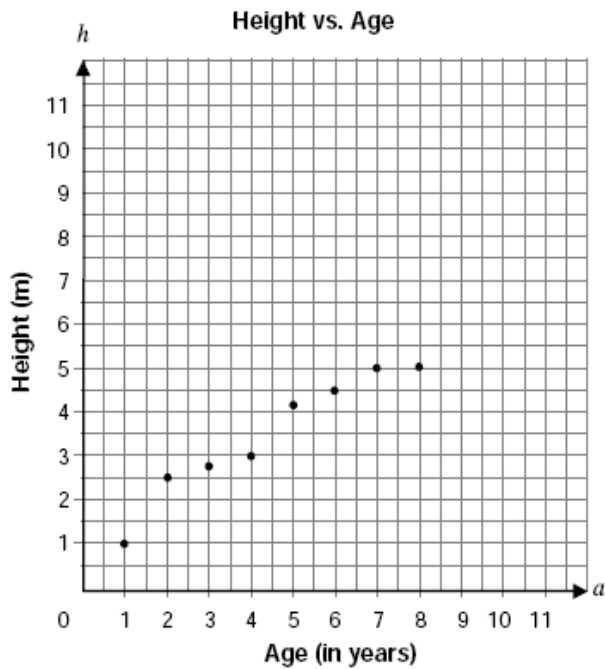
Karen collects data on the heights, in centimetres, and the ages, in years, of seven students in the school cafeteria. She enters the data into lists, using a graphing calculator.

L1	L2	L3	2
13	147		
14	165		
15	172		
16	175		
16	183		
18	195		
18	179		
L2(1)=148			

Which of the following scatter plots best matches the data points?

- a
- b
- c
- d

9. The graph below represents the relationship between the height, h , in metres, and the age, a , in years, of a tree.



What is the approximate **height** of the tree if it is **10 years old**?

- a 10.5 m
- b 8.5 m
- c 6.5 m
- d 4.5 m

10. The following tables express distance, in metres, as a function of time, in seconds:

Which table represents a **linear** relation?

a

Time (s)	Distance (m)
0	236
1	231
2	216
3	191

b

Time (s)	Distance (m)
0	1
1	2
2	4
3	8

c

Time (s)	Distance (m)
0	28
1	46
2	50
3	64

d

Time (s)	Distance (m)
0	16
1	12
2	8
3	4

1. Thrill Rides

Susanna travels to different amusement parks to ride 15 roller coasters and collect data about each ride.



She constructs a scatter plot to show the relationship between the **total length** of the ride, L , in metres, and the **maximum height** of its peaks, h , in metres.

Suzanna rides another roller coaster. The length of the ride on this roller coaster is **500m**. Determine its **maximum height**. Justify your answer.

