Chapter 8 **Compound Interest**

Chapter 8 Prerequisite Skills

Chapter 8 Prerequisite Skills **Question 1 Page 420**

a) $1.2 \times 4.3 = 5.16$

b) $2 \times 1.05 = 2.1$

c) $1000(0.04)(7) = 280$

d) $350(0.035)(2.5) = 30.625$

e) $500 + 500(0.09)(0.5) = 522.5$

f) $950 + 950(0.04)\left(\frac{7}{12}\right) = 972.1\overline{6}$

g) $675\left[1 + (0.025)(4)\right] = 742.5$

h) $1000\left[1 + (0.038)\left(\frac{1}{2}\right)\right] = 1019$

Chapter 8 Prerequisite Skills **Question 2 Page 420**

a) $0.04 \div 2 = 0.02$

b) $0.05 \div 2 = 0.025$

c) $0.064 \div 2 = 0.032$

d) $0.064 \div 4 = 0.016$

e) $0.06 \div 12 = 0.005$

f) $0.085 \div 4 = 0.021\ 25$

g) $0.03 \div 12 = 0.0025$

h) $0.095 \div 2 = 0.0475$

Chapter 8 Prerequisite Skills **Question 3 Page 420**

a) $6\% = \frac{6}{100} = 0.06$

b) $4\% = \frac{4}{100} = 0.04$

c) $2.5\% = \frac{2.5}{100} = 0.025$

d) $18\% = \frac{18}{100} = 0.18$

e) $18.5\% = \frac{18.5}{100} = 0.185$

f) $12.25\% = \frac{12.25}{100} = 0.1225$

g) $0.5\% = \frac{0.5}{100} = 0.005$

h) $2.33\% = \frac{2.33}{100} = 0.0233$

Chapter 8 Prerequisite Skills **Question 4 Page 420**

a) $0.05 \times \$400 = \$20$

b) $0.03 \times \$1000 = \$30$

c) $0.055 \times \$2000 = \$110$

d) $0.07 \times \$350 = \$24.50$

e) $0.06 \times \$10\,000 = \$600$

f) $0.045 \times \$2500 = \$112.50$

g) $0.011225 \times \$200\,000 = \$2245$

h) $0.0664 \times \$3500 = \$232.40$

Chapter 8 Prerequisite Skills **Question 5 Page 420**

a) $6\% \div 2 = 3\%$

$\frac{6}{100} \div 2 = 0.03$

b) $8.4\% \div 2 = 4.2\%$

$\frac{8.4}{100} \div 2 = 0.042$

c) $9.3\% \div 12 = 0.775\%$

$\frac{9.3}{100} \div 12 = 0.007\,75$

d) $5.2\% \div 4 = 1.3\%$

$\frac{5.2}{100} \div 4 = 0.013$

e) $16\% \div 4 = 4\%$

$\frac{16}{100} \div 4 = 0.04$

$21.6\% \div 12 = 1.8\%$

f) $\frac{21.6}{100} \div 12 = 0.018$

g) $7.5\% \div 4 = 1.875\%$

$\frac{7.5}{100} \div 4 = 0.018\,75$

h) $3.3\% \div 12 = 0.275\%$

$\frac{3.3}{100} \div 12 = 0.002\,75$

Chapter 8 Prerequisite Skills **Question 6 Page 420**

Estimates may vary. For example:

a) 4.1% of \$1000 is about \$40.
(Use 4% of \$1000.)

b) 9.9% of \$5000 is about \$500.
(Use 10% of \$5000.)

c) 3.8% of \$200 is about \$8.
(Use 4% of \$200.)

d) 5.1% of \$690 is about \$35.
(Use 5% of \$700.)

e) 4% of \$329.17 is about \$13.
(Use 4% of \$325.)

f) 5% of \$236 712 is about \$12 000.
(Use 5% of \$240 000.)

Chapter 8 Prerequisite Skills **Question 7 Page 420**

a) $1.03^2 = 1.06$

b) $1.06^8 = 1.59$

c) $200(1.03)^6 = 238.81$

d) $5000(1.0225)^{10} = 6246.02$

e) $2^{-1} = 0.5$

f) $5^{-2} = 0.04$

g) $1.03^{-6} = 0.84$

h) $1.005^{-12} = 0.94$

Chapter 8 Prerequisite Skills **Question 8 Page 421**

a) $I = Prt$
$= \$500(0.05)(2)$
$= \$50$

b) $I = Prt$
$= \$1200(0.08)(0.5)$
$= \$48$

c) $I = Prt$
$= \$4000(0.075)(0.75)$
$= \$225$

d) $I = Prt$
$= \$4000(0.075)\left(3 + \frac{8}{12}\right)$
$= \$1100$

Chapter 8 Prerequisite Skills **Question 9 Page 421**

a) $I = Prt$
$= \$1000(0.06)(3)$
$= \$180$

b) $I = Prt$
$= \$800(0.072)(1.5)$
$= \$86.40$

c) $I = Prt$
$= \$1200(0.048)\left(\frac{315}{365}\right)$
$= \$49.71$

d) $I = Prt$
$= \$4000(0.05)\left(2 + \frac{3}{12}\right)$
$= \$450$

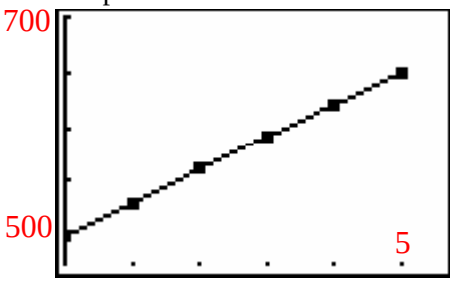
Chapter 8 Section 1 **Simple and Compound Interest**

Chapter 8 Section 1 **Question 1 Page 428**

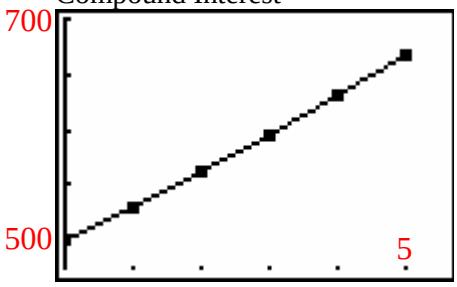
Year	Simple Interest ($)	Amount ($)
0		500.00
1	30	530.00
2	30	560.00
3	30	590.00
4	30	620.00
5	30	650.00

Year	$A = P(1.06)$	Amount ($)
0		500.00
1	500.00(1.06)	530.00
2	530.00(1.06)	561.80
3	561.80(1.06)	595.508
4	595.508(1.06)	631.23848
5	631.23848(1.06)	669.11279

Simple Interest



Compound Interest

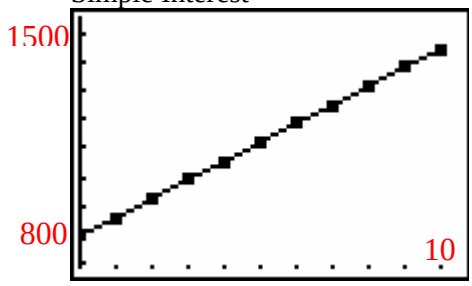


Chapter 8 Section 1 **Question 2 Page 428**

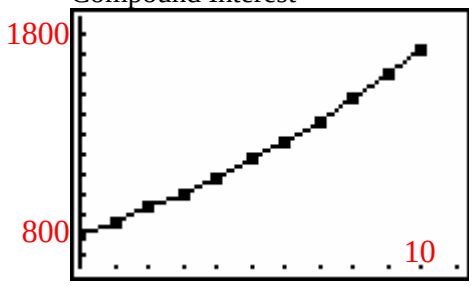
Year	Simple Interest ($)	Amount ($)
0		800.00
1	64	864.00
2	64	928.00
3	64	992.00
4	64	1056.00
5	64	1120.00
6	64	1184.00
7	64	1248.00
8	64	1312.00
9	64	1376.00
10	64	1440.00

Year	$A = P(1.08)$	Amount ($)
0		800.00
1	800.00(1.08)	864.00
2	864.00(1.08)	933.12
3	933.12(1.08)	1007.7696
4	1007.7696(1.08)	1088.39117
5	1088.39117(1.08)	1175.46246
6	1175.46246(1.08)	1269.49946
7	1269.49946(1.08)	1371.05942
8	1371.05942(1.08)	1480.74417
9	1480.74417(1.08)	1599.20370
10	1599.20370(1.08)	1727.14000

Simple Interest



Compound Interest



Chapter 8 Section 1 Question 3 Page 428

To calculate the amount of annual interest use $I = Prt$, with $P = \$750$, $r = 5\%$, and $t = 1$ year.

Year	Simple Interest ($)	Amount ($)
0		750.00
1	750(0.05) = 37.50	787.50
2	750(0.05) = 37.50	825.00
3	750(0.05) = 37.50	862.50
4	750(0.05) = 37.50	900.00
5	750(0.05) = 37.50	937.50

At the end of five years, Shu Ying's simple interest investment totals $937.50.

Now calculate the yearly compound interest using the same rate and principal amount.

Year	$A = P(1.05)$	Amount ($)
0		750.00
1	750(1.05)	787.50
2	787.50(1.05)	826.88
3	826.88(1.05)	868.22
4	868.22(1.05)	911.63
5	911.63(1.05)	957.21

By the end of year 2 notice the compound interest investment has a greater value.
The difference between the values increases with each year due to the effect of compounding.
At the end of five years Shu Jin's compound interest investment totals $957.21.
This is $957.21 – $937.50 = $19.71 more than Shu Ying's investment.

Chapter 8 Section 1 Question 4 Page 428

a) Use the simple interest equation $A = P(1 + rt)$, with $P = \$1000$, $r = 6.5\%$, and $t = 6$ years.

$$A = 1000\big(1 + (0.065)(6)\big)$$
$$= 1390$$

The value of the investment using simple interest is $1390.

b) If the $1000 dollars is compounded annually at 6.5%, the value at the end of each year for six years is:

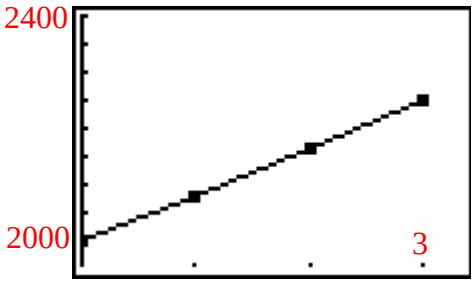
Year	$A = P(1.065)$	Amount ($)
0		1000.00
1	1000(1.065)	1065.00
2	1065(1.065)	1134.2225
3	1134.2225(1.065)	1207.950
4	1207.950(1.065)	1286.466
5	1286.466(1.065)	1370.087
6	1370.087(1.065)	1459.142

The value at the end of six years is $1459.14.

Chapter 8 Section 1 **Question 5 Page 428**

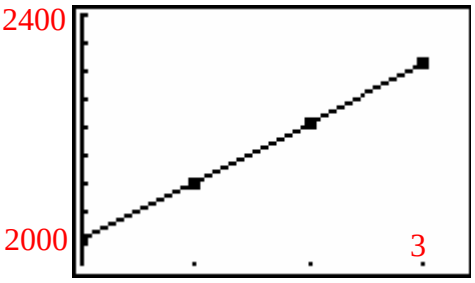
a) $2000 is invested at 4% per year for three years compounded annually.

Year	$A = P(1.04)$	Amount ($)
0		2000.00
1	2000(1.04)	2080.00
2	2080(1.04)	2163.20
3	2163.20(1.04)	2249.73



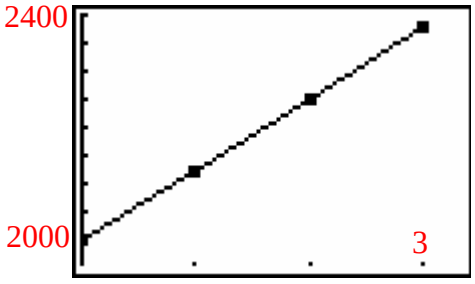
b) $2000 is invested at 5% per year for three years compounded annually.

Year	$A = P(1.05)$	Amount($)
0		2000.00
1	2000(1.05)	2100.00
2	2100(1.05)	2205.00
3	2205(1.05)	2315.25



c) $2000 is invested at 6% per year for three years compounded annually.

Year	$A = P(1.06)$	Amount($)
0		2000.00
1	2000(1.06)	2120.00
2	2120(1.06)	2247.20
3	2247.20(1.06)	2382.03

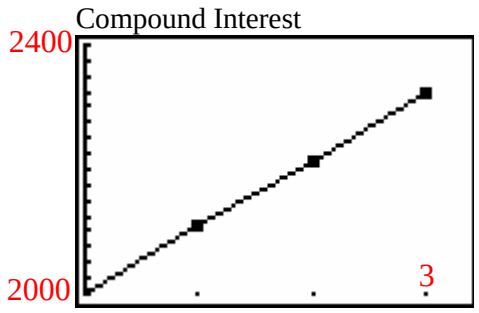


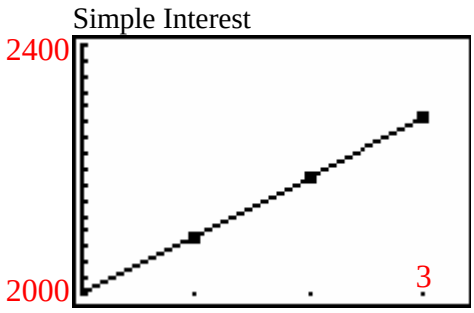
Chapter 8 Section 1 **Question 6 Page 428**

The difference in favour of compound interest is \$2341.81 – \$2324 = \$17.81, as shown by the following tables.

Year	Simple Interest (\$)	Amount (\$)
0		2000.00
1	108	2108.00
2	108	2216.00
3	108	2324.00

Year	$A = P(1.054)$	Amount (\$)
0		2000.00
1	2000(1.054)	2108.00
2	2108(1.054)	2221.832
3	2221.832(1.054)	2341.810 93





Chapter 8 Section 1 **Question 7 Page 428**

a) \$200 for one year at 3.8% simple interest: $200 \times 0.038 \times 1 = 7.60$
Each year the bank pays \$7.60 interest for the term of the investment.

b) Compound interest:

First year: 200(1.038) = 207.60
The interest is \$207.60 – \$200.00 = \$7.60.

Second year: 207.60(1.038) = 215.49
The interest is \$215.49 – \$207.60 = \$7.89.

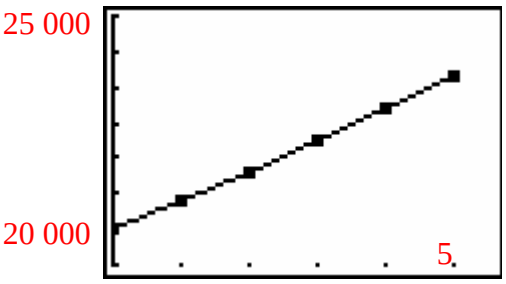
Third year: 215.49(1.038) = 223.69
The interest is \$223.68 – \$215.49 = \$8.19.

c) Simple interest is easier to calculate, since it is the same amount each year over the term of the investment.

Chapter 8 Section 1 **Question 8 Page 429**

a)

Year	$A = P(1.04)$	Amount ($)
0		20 000.00
1	20 000(1.04)	20 800.00
2	20 800(1.04)	21 632.00
3	21 632(1.04)	22 497.28
4	22 497.28(1.04)	23 397.1712
5	23 397.1712(1.04)	24 333.058



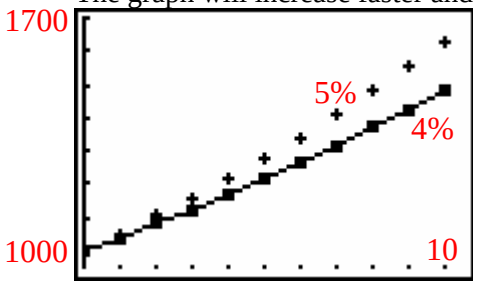
b) Investing $20 000 at 4% a year, simple interest, for five years would have given interest of $20 000(0.04)(5) = $4000.
Investing $20 000 at 4% per year, compounded annually, for five years gave interest of $20 000(1+ 0.04)^5 – $20 000 = $4333.058.
The compounding added $333.06 over the five-year period.

Chapter 8 Section 1 **Question 9 Page 429**

a) By interpolating from the graph, the $1000 investment is worth about $1125 after three years.

b) Since the investment of $1000 is worth about $1475 after ten years, it would be worth $1500 in about 10.4 years.

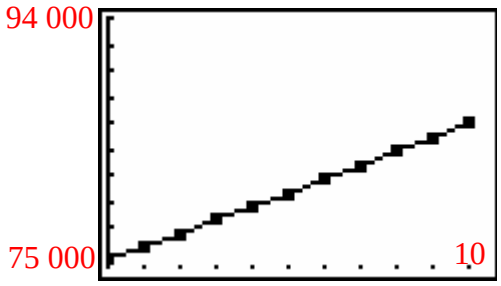
c) Answers may vary. For example:
The graph will increase faster and have a sharper curve.



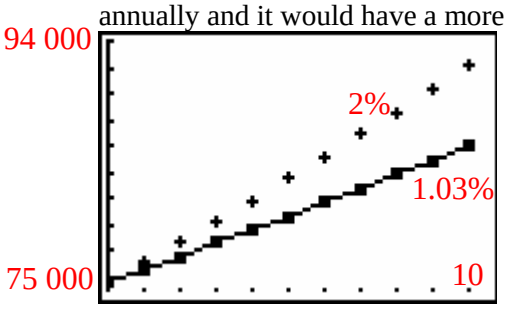
Chapter 8 Section 1 **Question 10 Page 429**

a)

Year	$A = P(1.013)$	Population
0		75 600.00
1	75 600(1.013)	76 582.80
2	76 582.8(1.013)	77 578.3764
3	77 578.3764(1.013)	78 586.8953
4	78 586.8953(1.013)	79 608.5249
5	79 608.5249(1.013)	80 643.4358
6	80 643.4358(1.013)	81 691.8004
7	81 691.8004(1.013)	82 753.7938
8	82 753.7938(1.013)	83 829.5931
9	83 829.5931(1.013)	84 919.3779
10	84 919.3779(1.013)	86 023.3298



b) Answers may vary. For example:
The graph would increase faster if the growth rate were 2% annually as opposed to 1.3% annually and it would have a more pronounced curve.



Chapter 8 Section 1 Question 11 Page 429

Marcy invested $200 at 4% simple annual interest for ten years.

$$A = P(1+rt)$$
$$=200(1+(0.04)(10))$$
$$= 280$$

At the end of ten years the investment was worth $280.

We wish to find what rate of interest compounded annually would give the same amount.
First we know that the interest rate would be less than 4%.

Use trial and error.
Try 3.75% ⇨ the value would be $289.01.
Try 3.5% ⇨ the value would be $282.12.
Try 3.4% ⇨ the value would be $279.41.
Try 3.42% ⇨ the value would be $279.95.
Try 3.43% ⇨ the value would be $280.22.

So the interest rate would be 3.42% per year, compounded annually.

Chapter 8 Section 2 **Compound Interest**

Chapter 8 Section 2 **Question 1 Page 432**

a) $500(1.02)^3 = 530.60$

b) $200(1.03)^7 = 245.97$

c) $1000(1.06)^4 = 1262.48$

d) $3500(1.0025)^8 = 3570.62$

e) $1350(1.0375)^{12} = 2099.86$

f) $12\ 500(1.041)^5 = 15\ 281.42$

Chapter 8 Section 2 **Question 2 Page 432**

a) $A = 2000(1 + 0.05)^3$

b) $A = 1000(1 + 0.04)^4$

c) $A = 50\ 000(1 + 0.03)^{20}$

d) $A = 750(1 + 0.005)^{12}$

Chapter 8 Section 2 **Question 3 Page 433**

a) $A = 1000(1 + 0.04)^5$
$= \$1216.65$
Interest = \$1216.65 – \$1000 = \$216.65

b) $A = 1000(1 + 0.04)^6$
$= \$1265.32$
Interest = \$1265.32 – \$1000 = \$265.32

c) $A = 1000(1 + 0.1625)^8$
$= \$1137.64$
Interest = \$1137.64 – \$1000 = \$137.64

d) $A = 1000(1 + 0.003)^{48}$
$= \$1154.64$
Interest = \$1154.64 – \$1000 = \$154.64

Chapter 8 Section 2 **Question 4 Page 433**

Ming Mei borrowed \$900 at 6% a year, compounded quarterly, for two years.

a) $A = 900(1 + 0.015)^2$
$= 1013.84$
Ming Mei must repay \$1013.84.

b) The amount of interest is \$1013.84 – \$900 = \$113.84.

Chapter 8 Section 2 Question 5 Page 433

Keisha plans to invest $5000 for five years at 6%.

a) Compounded annually: $A = 5000(1 + 0.06)^5$

$= 6691.13$

He would have $6691.13 after five years.

b) Compounded semi-annually: $A = 5000(1 + 0.03)^{10}$

$= 6719.58$

He would have $6719.58 after five years.

c) Compounded quarterly: $A = 5000(1 + 0.015)^{20}$

$= 6734.28$

He would have $6734.28 after five years.

d) Compounded monthly: $A = 5000(1 + 0.005)^{60}$

$= 6744.25$

He would have $6744.25 after five years.

e) Compounded daily: $A = 5000(1 + 0.0001643835616)^{1825}$

$= 6749.13$

He would have $6749.13 after five years.

Chapter 8 Section 2 Question 6 Page 433

$10 000 was invested at 5%, compounded semi-annually, for Tonya's education.

a) $A = 10\,000(1 + 0.025)^{24}$

$= 18\,087.26$

At the end of 12 years the investment was worth $18 087.26.

b) $A = 10\,000(1 + 0.025)^{36}$

$= 24\,325.35$

At the end of 18 years the investment was worth $24 235.35.

Chapter 8 Section 2 Question 7 Page 433

An investment of $2000 was made eight years ago at 13.6% per year, compounded annually.

$A = 2000(1 + 0.136)^8$

$= 5546.98$

At the end of eight years the investment is worth $5546.98.

Chapter 8 Section 2 Question 8 Page 433

P = \$5000, i = 4% a year, compounded quarterly, n = 10 years in total
Use $A = P(1 + i)^n$.

a) After one year: $A = 5000(1 + 0.01)^4$
$= \$5203.02$

After two years: $A = 5000(1 + 0.01)^8$
$= \$5414.28$

b) The interest earned in the second year is \$5414.28 – \$5203.02 = \$211.26.

c) After nine years: $A = 5000(1 + 0.01)^{36}$
$= \$7153.84$

After ten years: $A = 5000(1 + 0.01)^{40}$
$= \$7444.32$
The interest paid in the tenth year is \$7444.32 – \$7153.84 = \$290.48.

d) Answers may vary. For example:
The value of the investment is greater in the ninth year than in the first year. So the amount of interest earned by the investment in the tenth year is greater than the amount of interest earned in the second year even though the interest rate that is applied for both years is the same.

Chapter 8 Section 2 Question 9 Page 433

a) \$2000 is invested at 4.5% a year, compounded semi-annually, for three years.
$A = 2000(1 + 0.0225)^6$
$= 2285.65$
At the end of the three years, the investment is worth \$2285.65.
The amount of interest earned is \$2285.65 – \$2000.00 = \$285.65.

\$2500 invested at 4.2% a year, compounded quarterly, for three years.
$A = 2500(1 + 0.0105)^{12}$
$= \$2833.84$
At the end of the three years, the investment is worth \$2833.84.
The amount of interest earned is \$2833.84 – \$2500.00 = \$333.84.

The \$2500 investment earned more interest.

b) Wayne's investments earned \$285.65 + \$333.84 = \$619.49 in total interest.

Chapter 8 Section 2 Question 10 Page 434

Sangar borrowed $8000 for one year at 4.8%, compounded monthly.

$A = 8000(1 + 0.004)^{12}$

$= \$8392.56$

Sanjiv borrowed $8000 for one year at 3.2%, compounded monthly.

$A = 8000(1 + 0.0026666667)^{12}$

$= \$8259.79$

Sangar paid $8392.56 – $8259.79 = $132.77 more interest than his brother.

Chapter 8 Section 2 Question 11 Page 434

a), b), c), d) Answers may vary.

Chapter 8 Section 2 Question 12 Page 434

$3000 borrowed for five years at 9%, compounded semi-annually:

$A = 3000(1 + 0.045)^{10}$

$= 4658.91$

The total amount of the loan is $4658.91.

$3000 borrowed for five years at 8.6%, compounded quarterly:

$A = 3000(1 + 0.0215)^{20}$

$= 4590.80$

The total amount of the loan is $4590.80.

Warren should take the loan at 8.6%, compounded quarterly, because he will pay $4590.80 – $4658.91 = $68.11 less interest.

Chapter 8 Section 2 Question 13 Page 434

a) For a bond of $3 million for ten years at 5%, compounded semi-annually:

$A = 3\,000\,000(1 + 0.025)^{20}$

$= 4\,915\,849.32$

At the end of the term, the town must pay $4 915 849.32.

b) The total interest paid is $4 915 849.32 – $3 000 000.00 = $1 915 849.32.

Chapter 8 Section 2 Question 14 Page 434

The projected population for Melville: $P = 102\,000(1.023)^{10}$
$= 128\,043$

The projected population for Markton: $P = 97\,000(1.037)^{10}$
$= 139\,495$

Markton will have the larger population by 139 495 – 128 043 = 11 452 people.

Chapter 8 Section 2 Question 15 Page 434

The amount borrowed for the stereo is \$1150 for one year at 12%, compounded monthly.
$A = 1150(1 + 0.01)^{12}$
$= 1295.85$
After one year Peter would have to pay \$1295.85 for the stereo.

Chapter 8 Section 2 Question 16 Page 435

Solutions for Achievement Checks are shown in the Teacher's Resource.

Chapter 8 Section 2 Question 17 Page 435

Sarah invested \$2000 for five years at 12.6%, compounded annually.
$A = 2000(1 + 0.126)^{5}$
$= 3620.11$
At the end of five years the value was \$3620.11.

She then invested the \$3620.11 for five years at 15.8% compounded semi-annually.
$A = 3620.11(1 + 0.079)^{10}$
$= 7743.48$
At the end of the five years the value was \$7743.48.

Chapter 8 Section 2 **Question 18 Page 435**

Find the value of the yearly interest i% in each case.

a) $512.50 = 500\left(1+\frac{i}{2}\right)^{1}$

$$\frac{512.50}{500} = 1+\frac{i}{2}$$
$$\frac{512.50}{500} - 1 = \frac{i}{2}$$
$$\frac{i}{2} = 1.025 - 1$$
$$\frac{i}{2} = 0.025$$
$$i = 0.05$$

The value of i is 5%.

b) $2020.05 = 2000\left(1+\frac{i}{12}\right)^{2}$

$$\sqrt{\frac{2020.05}{2000}} = 1+\frac{i}{12}$$
$$\frac{i}{12} = 1.005 - 1$$
$$\frac{i}{12} = 0.005$$
$$i = 12 \times 0.005$$
$$i = 0.06$$

The value of i is 6%.

c) $1025.16 = 1000\left(1+\frac{i}{4}\right)^{2}$

$$\frac{1025.16}{1000} = \left(1+\frac{i}{4}\right)^{2}$$
$$1.02516 = \left(1+\frac{i}{4}\right)^{2}$$
$$\sqrt{(1.02516)} = 1+\frac{i}{4}$$
$$\frac{i}{4} = 1.0125 - 1$$
$$i = 0.0125(4)$$
$$i = 0.05$$

The value of i is 5%.

Chapter 8 Section 2 Question 19 Page 435

a) A $500 Ontario Savings Bond is worth:

Year	Calculation	Amount ($)
0		500.00
1	500.00(1 + 0.037)	518.50
2	518.50(1 + 0.038)	538.203
3	538.203(1 + 0.039)	559.192917
4	559.192917(1 + 0.04)	581.5606337
5	581.5606337(1 + 0.0425)	606.2769606

At the end of five years the value of the bond is $606.28.

b) If Bryce tripled his investment to $1500, the investment value will also triple.
$606.2769606 \times 3 = 1818.830882$
The $1500 bond would be worth $1818.83 at the end of five years.

Chapter 8 Section 3 **Present Value**

Chapter 8 Section 3 **Question 1 Page 439**

a) $2000(1.04)^{-6} = 1580.63$

b) $50(1.005)^{-12} = 706.43$

c) $500(1.01)^{-10} = 452.64$

d) $10\,000(1.03)^{-8} = 7894.09$

e) $2450(1.0075)^{-18} = 2141.68$

f) $1500(1.1)^{-3} = 1126.97$

Chapter 8 Section 3 **Question 2 Page 439**

a) Use $P = A(1 + i)^{-n}$, with A = \$5000 and i = 6%, compounded annually, for four years.
$P = 5000(1 + 0.06)^{-4}$
$= 3960.47$
The present value is \$3960.47.

b) Use $P = A(1 + i)^{-n}$, with A = \$2000 and i = 4.5%, compounded semi-annually, for two years.
$P = 2000(1 + 0.02)^{-4}$
$= 1847.69$
The present value is \$1847.69.

c) Use $P = A(1 + i)^{-n}$, with A = \$1000 and i = 6%, compounded monthly, for three years.
$P = 1000(1 + 0.00375)^{-36}$
$= 873.94$
The present value is \$873.94.

d) Use $P = A(1 + i)^{-n}$, with A = \$10 000 and i = 8%, compounded quarterly, for five years.
$P = 10\,000(1 + 0.02)^{-20}$
$= 6729.71$
The present value is \$6729.71.

Chapter 8 Section 3 Question 3 Page 440

a) Use $P = A(1 + i)^{-n}$, with A = \$2000 and i = 6%, compounded semi-annually, for three years.
$P = 2000(1 + 0.03)^{-6}$
$= 1674.97$
The discounted value of the loan is \$1674.97.

b) Use $P = A(1 + i)^{-n}$, with A = \$5000 and i = 5% compounded quarterly, for four years.
$P = 5000(1 + 0.0125)^{-16}$
$= 4098.73$
The discounted value of the loan is \$4098.73.

c) Use $P = A(1 + i)^{-n}$, with A = \$100 000 and i = 7.5% compounded monthly, for five years.
$P = 100\,000(1 + 0.00625)^{-60}$
$= 68\,809.18$
The discounted value of the loan is \$68 809.18.

d) Use $P = A(1 + i)^{-n}$, with A = \$1000 and i = 5% compounded semi-annually, for three years.
$P = 1000(1 + 0.025)^{-6}$
$= 862.30$
The discounted value of the loan is \$862.30.

Chapter 8 Section 3 Question 4 Page 440

Use $P = A(1 + i)^{-n}$, with A = \$2604.52 and i = 9%, compounded semi-annually, for three years.
$P = 2604.52(1 + 0.045)^{-6}$
$= 2000$
Steve borrowed \$2000.

Chapter 8 Section 3 Question 5 Page 440

Use $P = A(1 + i)^{-n}$, with A= \$1000 and i = 4%, compounded quarterly, for two years.
$P = 1000(1 + 0.01)^{-8}$
$= 923.48$
The principal to be invested is \$923.48.

Chapter 8 Section 3 Question 6 Page 440

A = \$20 000 and i = 8%, compounded quarterly, for 18 years.
$P = 20\,000(1 + 0.02)^{-72}$
$= 4806.37$
The amount of interest that would be earned is \$20 000 – \$4806.37 = \$15 193.63.

Chapter 8 Section 3 Question 7 Page 440

Use $P = A(1 + i)^{-n}$, with A = \$50 000 and i = 6.3%, compounded monthly, for ten years,
$P = 50\,000(1 + 0.00525)^{-120}$
$= 26\,673.51$
The woman should invest \$26 673.51.

Chapter 8 Section 3 Question 8 Page 440

Use $P = A(1 + i)^{-n}$, with A = \$3000 and i = 5.7%, compounded semi-annually, for four years.
$P = 3000(1 + 0.0285)^{-8}$
$= 2396.00$
The creditor should be willing to accept \$2396.

Chapter 8 Section 3 Question 9 Page 440

Use $P = A(1 + i)^{-n}$, with A = \$50 000 and i = 3.5%, compounded annually, for three years.
$P = 50\,000(1 + 0.035)^{-3}$
$= 45\,097.14$
The equivalent value today is \$45 097.14.

Chapter 8 Section 3 Question 10 Page 440

Use $P = A(1 + i)^{-n}$, with A = \$2200 and i = 4.2%, compounded quarterly, for one year.
$P = 2200(1 + 0.00105)^{-4}$
$= 2109.98$
The present value of Option 2, the payment due in a year, is \$2109.98.

Option 1 means paying \$2399.99 now.
Option 2 is the equivalent of paying \$399.99 + \$2109.99 = \$2509.97 now.

So, Option 1 costs about \$290 less than Option 2 and is the better choice.

Chapter 8 Section 3 Question 11 Page 440

Use $P = A(1 + i)^{-n}$, with A = \$10 000 and i = 4.5%, compounded monthly, for two years and four months.
$P = 10\,000(1 + 0.00375)^{-28}$
$= 9005.01$
Jenay should invest \$9005.01.

Chapter 8 Section 3 Question 12 Page 441

A loan will be paid back in varying amounts over three years. Use $P = A(1 + i)^{-n}$ to find the combined value of the loan today.

Year 1: A = \$1000 and i = 7.5%, compounded semi-annually
$P = 1000(1 + 0.0375)^{-2}$
$= 929.02$

Year 2: A = \$2000 and i = 7.5%, compounded semi-annually
$P = 2000(1 + 0.0375)^{-4}$
$= 1726.15$

Year 3: A = \$3000 and i = 7.5% compounded semi-annually
$P = \$3000(1 + 0.0375)^{-6}$
$= \$2405.43$
Therefore, the combined value of the loan today is \$929.02 + \$1726.15 + \$2405.43 = \$5060.60.

Chapter 8 Section 3 Question 13 Page 441

a) Use $P = A(1 + i)^{-n}$, with $A = \$30\,000$ and $i = 8\%$, compounded monthly, for 6 months
$P = 30\,000[1 + (0.08 \div 12)]^{-6}$
$= 28\,827.51$
Andelko can borrow \$28 827.51.

b) The amount of interest will be \$30 000 – \$28 827.51 = \$1172.49.

Chapter 8 Section 3 Question 14 Page 441

Use $P = A(1 + i)^{-n}$.
Loan: $A = \$5000$, $i = 4.8\%$ compounded monthly, in 6 years
$P = 5000(1 + 0.004)^{-72}$
$= 3750.96$
The principal on the loan is \$3750.96.

Sold Loan: $A = \$5000$, $i = 4.2\%$ compounded quarterly, in 6 years
$P = 5000(1 + 0.0105)^{-24}$
$= 3891.33$
The discounted price of the loan is \$3891.33.

a) The new creditor will pay \$3891.33.
b) The original creditor will earn \$3891.33 – \$3750.96 = \$140.37.

Chapter 8 Section 3 Question 15 Page 441

Emilie borrowed $2700 at 8.6%, compounded quarterly.

a) Use $A = P(1 + i)^n$.
Year 1: P = \$2700, i = 8.6% compounded quarterly
$A = 2700(1 + 0.0215)^4$
$= 2939.80$
At the end of Year 1 Emilie owes \$2939.80.

Then Emilie pays \$1000, leaving \$1939.80 to pay in two years plus interest.
Years 2 and 3: P = \$1939.80 and i = 8.6%, compounded quarterly
$A = 1939.80(1.0215)^8$
$= 2299.66$
At the end of Year 3, Emilie will have to repay \$2299.66.

b) Suppose Emilie paid another \$1000 after two years.
From part a), after Year 1, \$1939.80 was left to pay.
Year 2: P = \$1939.80 and i = 8.6% compounded quarterly
$A = 1939.80(1.0215)^4$
$= 2112.08$
At the end of Year 2, Emilie would owe \$2112.08.

If Emilie pays \$1000, she has \$1112.08 to pay.
Year 3: P = \$1112.08 and i = 8.6% compounded quarterly.
$A = 1112.08(1.0215)^4$
$= 1210.84$
At the end of Year 3, Emilie has \$1210.84 left to pay.

c) From part a) if Emilie paid \$2000 after one year she has \$939.80 left to pay for 2 years at 8.6% compounded quarterly.
$A = 939.80(1.0215)^8$
$= 1114.15$
Emilie will have \$1114.15 to repay.

d) In part a) Emilie paid \$3299.66 in total.
In part b) she paid \$3210.84.
In part c) she paid \$3114.15.

Chapter 8 Section 3 Question 16 Page 441

a) \$1225.04 was repaid for a loan of \$1000 at 7% per year, compounded annually.
Use $P = A(1 + i)^{-n}$ to find n, the number of years it took to repay the loan.
$1000 = 1225.04(1 + 0.07)^{-n}$

Use guess and test.
For $n = 2$, the value of the right hand side (RHS) of the expression is \$1070.
For $n = 3$, RHS = \$1000
It took three years to repay the loan.

b) \$2979.69 was repaid for a loan of \$2000 at 8% per year, compounded monthly.
Use $P = A(1 + i)^{-n}$ to find n, the number of years it took to repay the loan.
$2000 = 2979.69(1 + (0.08 \div 12))^{-n}$

Use guess and test.
For $n = 12$, RHS = \$2751.33
For $n = 36$, RHS = \$2345.77
For $n = 60$, RHS = \$2000
It took $\frac{60}{12} = 5$ years to repay the loan.

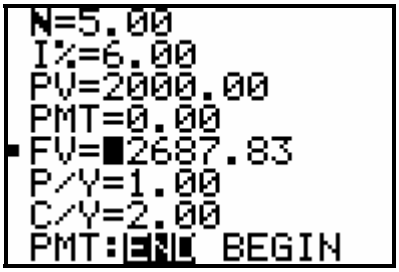
c) \$1097.84 was repaid for a loan of \$850 at 6.5% per year compounded semi-annually.
Use $P = A(1 + i)^{-n}$ to find n, the number of years it took to repay the loan.
$850 = 1097.84(1 + 0.0325)^{-n}$

Use guess and test.
For $n = 4$, RHS = \$966
For $n = 6$, RHS = \$906.18
For $n = 8$, RHS = \$850
So it will take $\frac{8}{2} = 4$ years to repay the loan.

Chapter 8 Section 4 The TVM Solver

Chapter 8 Section 4 Question 1 Page 444

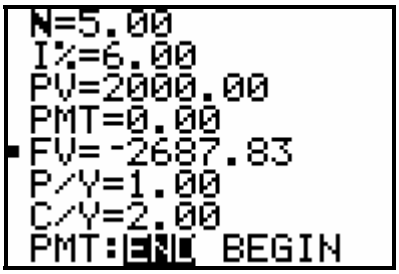
P = \$2000, i = 6%, compounded semi-annually, and N = 5 years
Use the TVM Solver to find the future value.



The value of the investment in five years is \$2687.83.

Chapter 8 Section 4 Question 2 Page 444

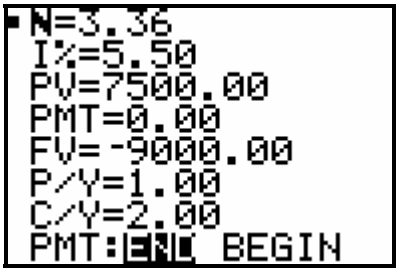
P = \$1000, i = 8.4%, compounded monthly, and N = 2 years
Use the TVM Solver to find the future value.



Ginny must repay \$1182.24.

Chapter 8 Section 4 Question 3 Page 444

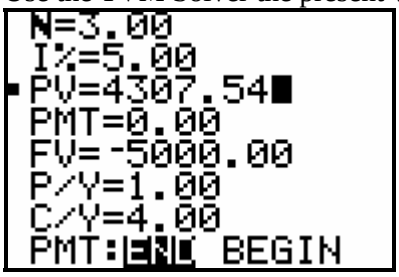
P = \$7500, FV = –\$9000, and i = 5.5%, compounded semi-annually
Use the TVM Solver to find the number of years.



It will take 3 years 6 months for Chin Lee to have enough to buy the motorcycle.

Chapter 8 Section 4 Question 4 Page 444

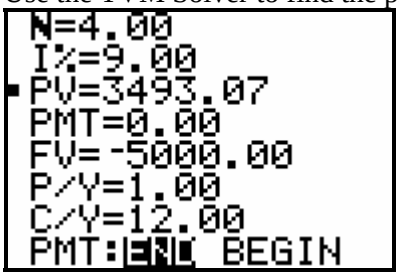
$FV = -\$5000$, $i = 5\%$, compounded quarterly, $N = 3$ years
Use the TVM Solver the present value of the investment.



Eduardo must invest $4307.54 today to have $5000 to buy the car in three years.

Chapter 8 Section 4 Question 5 Page 444

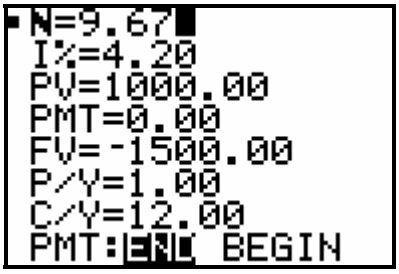
$FV = -\$5000$, $i = 9\%$, compounded monthly, $N = 4$ years
Use the TVM Solver to find the present value of the loan.



The creditor would pay $3493.07 for the loan due in four years.

Chapter 8 Section 4 Question 6 Page 444

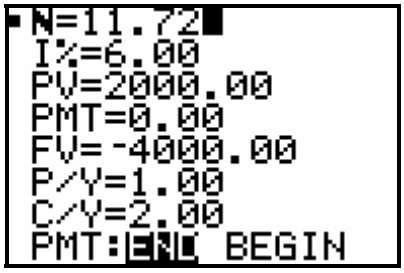
$P = \$1000$, $FV = -\$1500$, and $i = 4.2\%$, compounded monthly
Use the TVM Solver to find the number of years.



It will take about 9 years 9 months for Maria's investment to reach $1500.

Chapter 8 Section 4 Question 7 Page 444

a) P = \$2000, FV = –\$4000, i = 6%, compounded semi-annually
Use the TVM Solver to find the number of years.

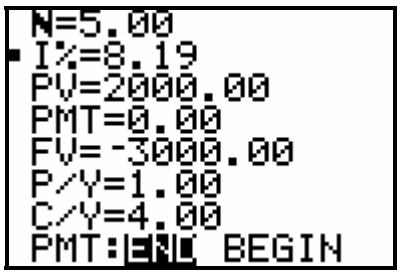


Since the compounding is semi-annual, it will take 12 years for Keenan's investment to double to \$4000.

b) Answers may vary. For example:
Yes. Any investment will double in this time.
In the formula $A = P(1 + i)^n$,
if $A = 2P$
then $2P = P(1 + i)^n$
$2 = (1 + i)^n$
Therefore, the time to double an investment depends on the value of i and the compounding period, and not on the actual amount invested.

Chapter 8 Section 4 Question 8 Page 444

a) P = \$2000, FV = –\$3000, N = 5 years compounded quarterly
Use the TVM Solver to find i.

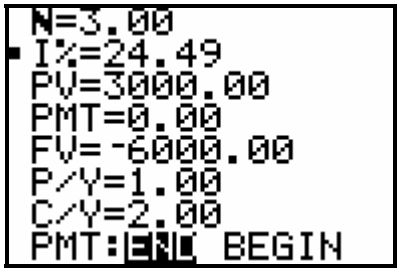


i = 8.2%

b) No. Since an interest rate of i = 8.2% did not double the \$2000 in five years, it will not double any amount invested for the same period.

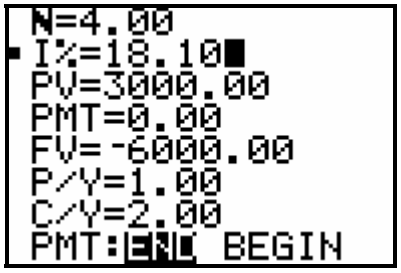
Chapter 8 Section 4 **Question 9 Page 444**

a) $P = \$3000$, $FV = -\$6000$, $N = 3$ years compounded semi-annually.
Use the TVM Solver to find i.



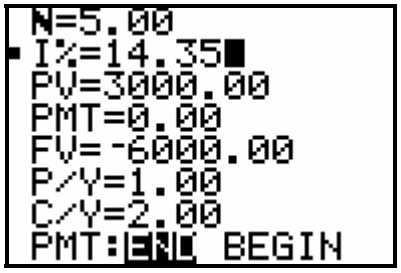
$i = 24.5\%$

b) $P = \$3000$, $FV = -\$6000$, $N = 4$ years compounded semi-annually
Use the TVM Solver to find i.



$i = 18.1\%$

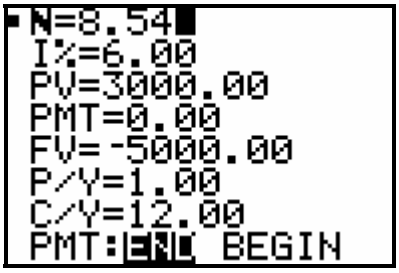
c) $P = \$3000$, $FV = -\$6000$, $N = 5$ years, compounded semi-annually
Use the TVM Solver to find i.



$i = 14.35\%$

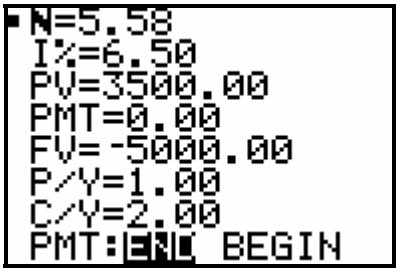
Chapter 8 Section 4 Question 10 Page 445

P = \$3000, FV = –\$5000, i = 6%, compounded monthly
Use the TVM Solver to find N.



N = 8.54 years

P = \$3500, FV = –\$5000, i = 6.5%, compounded semi-annually
Use the TVM Solver to find N.

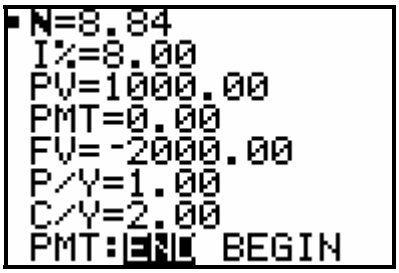


N = 5.58 years

So, \$3500 invested at 6.5% compounded semi-annually will reach \$5000 almost three years faster.

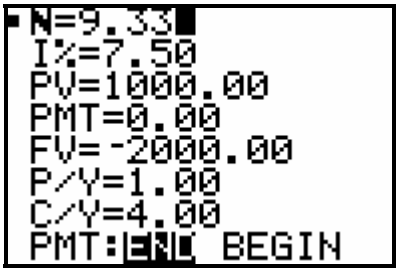
Chapter 8 Section 4 Question 11 Page 445

Find the shortest doubling time.
Case 1: P = \$1000, FV = –\$2000, i = 8%, compounded semi-annually
Use the TVM Solver to find the doubling time.



N = 8.84 years

Case 2: P = \$1000, FV = –\$2000, i = 7.5%, compounded quarterly
Use the TVM Solver to find the doubling time.

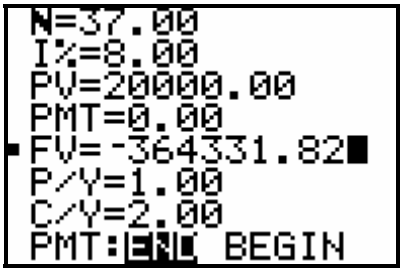


N = 9.33 years

Any amount invested at 8%, compounded semi-annually, will double in 8.84 years.
As in question 7b): in $A = P(1 + i)^n$ if $A = 2P$, then $2P = P(1 + i)^n$.
So $2 = (1 + i)^n$ and doubling is only dependent on i and n (i.e., rate and compounding periods).

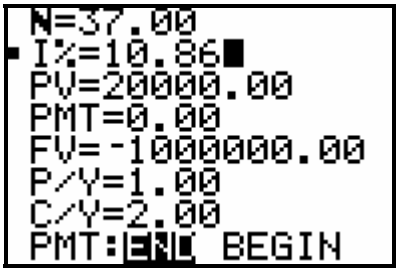
Chapter 8 Section 4 Question 12 Page 445

P = \$20 000, N = 37 years, i = 8%, compounded semi-annually
Use the TVM Solver to find FV.



FV = –\$364 331.82
A higher interest rate will be needed for the future value to be \$1 000 000.

P = \$20 000, FV = –\$1 000 000, and N = 37 years
Use the TVM Solver to find i.



i = 10.9%
The rate must be 10.9% for the investment to grow to \$1 000 000.

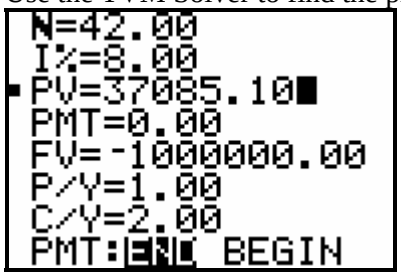
Chapter 8 Section 4 Question 13 Page 445

How to be worth one million dollars:

i) in 60 – 18 = 42 years

FV = –\$1 000 000, i = 8, and N = 42 years

Use the TVM Solver to find the present value of the investment.



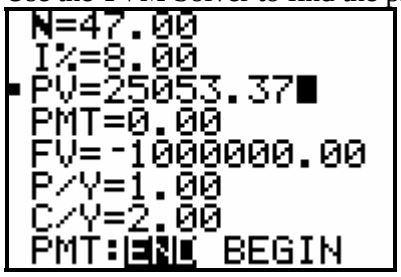
P = \$37 085.10

For an investment period of 42 years you need to invest \$37 085.10 today.

ii) in 65 – 18 = 47 years

FV = –\$1 000 000, i = 8, and N = 47 years

Use the TVM Solver to find the present value of the investment.

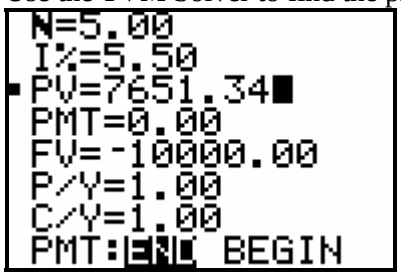


P = \$25 053.37

For an investment period of 47 years you need to invest \$25 053.37 today.

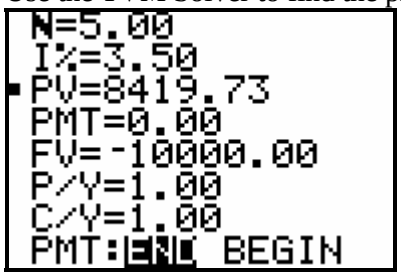
Chapter 8 Section 4 **Question 14 Page 445**

a) $FV = -\$10\,000$, $i = 5.5\%$, compounded annually, and $N = 5$ years
Use the TVM Solver to find the present value of the bond.



$P = \$7651.34$
The minimum fair discount price for the bond is \$7651.34.

b) $FV = -\$10\,000$, $i = 3.5\%$, compounded annually, and $N = 5$ years
Use the TVM Solver to find the present value of the bond.

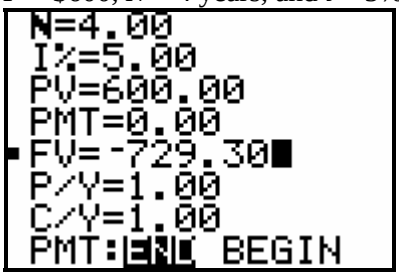


$PV = \$8419.73$
The maximum fair discount price for the bond is \$8419.73.

Chapter 8 Section 4 Question 15 Page 445

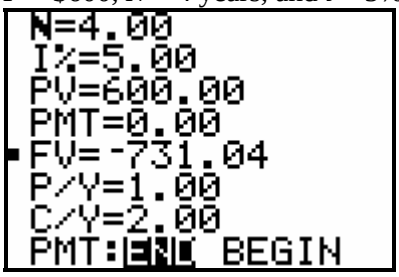
Answers may vary. For example:
In the following we will use the TVM Solver find *FV* to compare the interest rates for $N = 4$ years, $i = 5\%$, and various compounding periods. Then for $N = 4$ years, and $i = 5\%$, 6%, and 7%, all compounded semi-annually.

$P = \$600$, $N = 4$ years, and $i = 5\%$, compounded annually



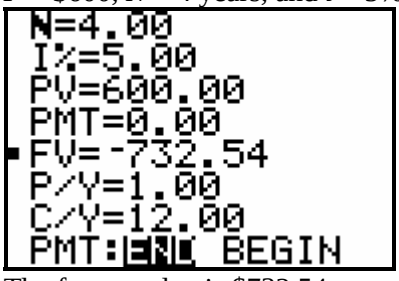
The future value is \$729.30.

$P = \$600$, $N = 4$ years, and $i = 5\%$, compounded semi-annually



The future value is \$731.04.

$P = \$600$, $N = 4$ years, and $i = 5\%$, compounded monthly



The future value is \$732.54.

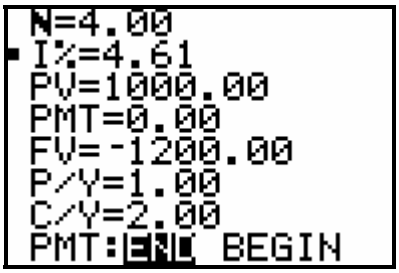
So the amounts of interest gradually increase as the number of compounding periods increase; i.e., \$129.30, \$131.04, \$132.54.

Similarly, if $i = 6\%$, compounded semi-annually, $FV = \$760.06$,
and if $i = 7\%$, compounded semi-annually, $FV = \$790.09$.

So the amounts of interest increase more quickly as the interest rate increases and the compounding period remains the same; i.e., \$131..04, \$160.06, \$190.09.

Chapter 8 Section 4 **Question 16 Page 445**

P = \$1000, FV = –\$1200, N = 4 years, interest is compounded semi-annually.
Use the TVM Solver to find i.



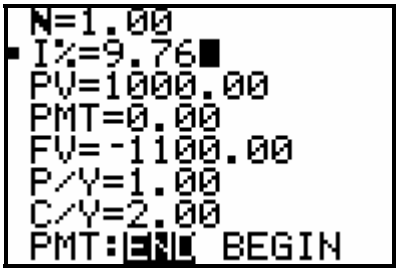
Naomi is offering to pay 4.61%, compounded semi-annually, on Reed's bond.

Chapter 8 Section 4 Question 17 Page 445

Simple interest: If \$1000 is invested for one year at 10% simple interest, the interest is $1000 \times \frac{10}{100} \times 1$ = \$100 and the value after one year is \$1100.

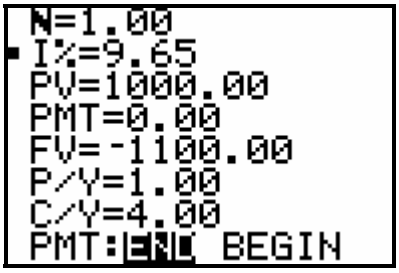
We can use the TVM Solver to find the interest rate that gives a value of \$1100 for a principal of \$1000 for one year, compounded semi-annually, quarterly, and monthly.

Compounding semi-annually: P = \$1000, FV = –\$1100, N = 1, and C/Y = 2
Use the TVM Solver to find i.



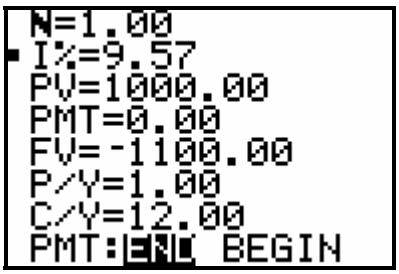
The interest rate, using semi-annual compounding, is 9.76%.

Compounding quarterly: P = \$1000, FV = –\$1100, N = 1, C/Y = 4
Use the TVM Solver to find i.



The interest rate, using quarterly compounding, is 9.65%.

Compounding monthly: P = \$1000, FV = –\$1100, N = 1, and C/Y = 12
Use the TVM Solver to find i.

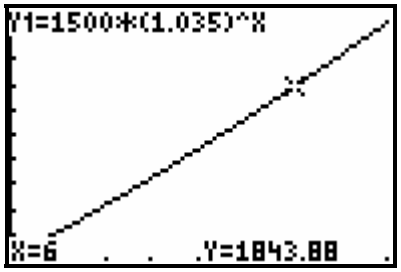


The interest rate, using monthly compounding, is 9.57%.

Chapter 8 Section 5 **Effects of Changing the Conditions on Investments and Loans**

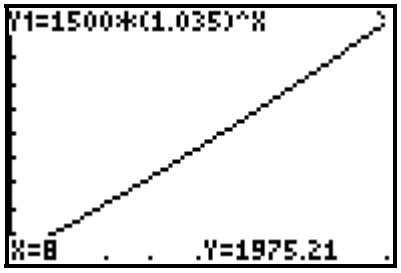
Chapter 8 Section 5 **Question 1 Page 450**

a)



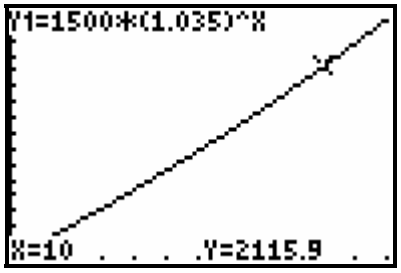
Amount: $1843.88; Interest: $1843.88 – $1500 = $343.88

b)



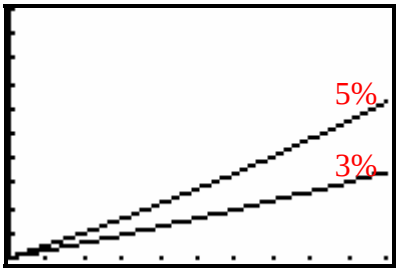
Amount: $1975.21; Interest: $1975.21 – $1500 = $475.21

c)



Amount: $2115.90; Interest: $2115.90 – $1500 = $615.90

Chapter 8 Section 5 **Question 2 Page 450**



Answers may vary. For example:
The graph representing 5% interest increases faster and has a more pronounced curve than the graph for 3% interest. A rate of 5% gives more interest than a rate of 3%.

Chapter 8 Section 5 Question 3 Page 450

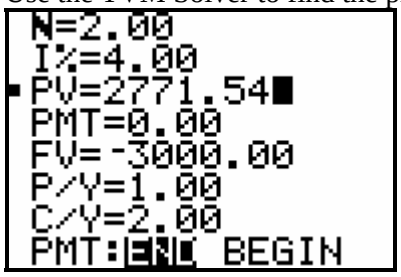
P = \$675, i = 3.4% per year, compounded semi-annually, and n = 5 years
Use $A = P(1 + i)^n$.
$A = 675(1.017)^{10}$
$= 798.94$
The amount of interest earned is \$798.94 – \$675 = \$123.94.

a) If the interest rate is doubled to 6.8% and all else remains the same,
$A = 675(1.034)^{10}$
$= 942.99$
The amount of interest earned is \$942.99 – \$675 = \$267.99.
The amount increases from \$798.94 to \$942.99 and the interest increases from \$123.94 to \$267.99.

b) If the time is doubled to ten years**,**
$A = 675(1.017)^{20}$
$= 945.63$
The amount of interest earned is \$945.63 – \$675 = \$270.63.
Therefore the amount increases from \$798.94 to \$945.63 and the interest increases from \$123.94 to \$270.63.

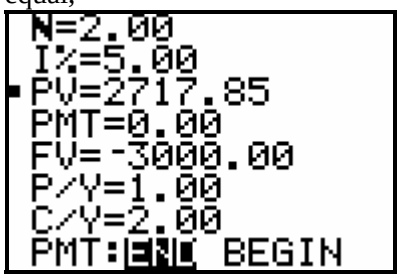
Chapter 8 Section 5 Question 4 Page 450

a) i = 4% per year, FV = –\$3000, N = 2, and C/Y = 2
Use the TVM Solver to find the present value.



P = \$2771.54
Nobuko needs to invest \$2771.54 to reach her goal.

b) Use the TVM Solver. If i = 5% per year, compounded semi-annually, and all else remains equal,



P = \$2717.85
At the higher rate of interest, Nobuko needs to invest \$2717.85 to reach her goal.

Chapter 8 Section 5 Question 5 Page 450

Answers may vary. For example:
Compare the results for compounding annually, semi-annually, quarterly and monthly for $P = \$3400$, $n = 3$ years, and $I = 6\%$ per year.
Annually: $A = 3400(1.06)^3 = 4049.45$
Semi-annually: $A = 3400(1.03)^6 = 4059.78$
Quarterly: $A = 3400(1.015)^{12} = 4065.10$
Monthly: $A = 3400(1.005)^{36} = 4068.71$

For more frequent compounding periods, the value (and the amount of interest) increase. Another way to look at this is to say that a smaller investment would have the same value with more frequent compounding periods.

Chapter 8 Section 5 Question 6 Page 450

Terry wants to invest \$6000 for 1 year at 5% per year.

a) Annual interest compared with interest compounded semi-annually:
Annual: $A = 6000(1.05) = 6300$
Semi-annual: $A = 6000(1.025)^2 = 6303.75$
Semi-annual compounding earns \$6303.75 – \$6300 = \$3.75 more interest than annual compounding.

b) Annual interest compared with interest compounded quarterly.
Quarterly: $A = 6000(1.0125)^4 = 6305.67$.
Quarterly compounding earns \$6305.67 – \$6300 = \$5.67 more interest than annual compounding.

c) Annual interest compared with interest compounded monthly:
Monthly: $A = 6000(1 + (1.05 \div 12))^{12} = 6306.97$
Monthly compounding earns \$6306.97 – 6300 = \$6.97 more interest than annual compounding.

Chapter 8 Section 5 Question 7 Page 451

Raheela hopes to have $18 000 in three years to buy a car.

a) Find P if A = \$18 000 and n = 3 years

i) at i = 4.5% per year, compounded monthly
$P = 18\,000(1.00375)^{-36}$
$= 15\,730.86$
Raheela needs to invest \$15 730.86 today at 4.5% per year, compounded monthly.

ii) at i = 4.5% per year, compounded semi-annually
$P = 18\,000(1.00225)^{-6}$
$= 15\,750.44$
Raheela needs to invest \$15 750.44 today at 4.5% per year, compounded semi-annually.

b) Answers may vary. For example:
The principal for compounding monthly is smaller. This is because more frequent compounding increases the amount of interest earned.

Chapter 8 Section 5 Question 8 Page 451

Barb has these choices for an investment.

Option A. P = \$10 000, i = 6.8% per year, simple interest, and n = 2 years

$$A = P(1 + i(n))$$
$$= 10\,000(1 + 0.068(2))$$
$$= 11\,360$$

Under Option A, the investment is worth \$11 360 at the end of two years.

Option B. P = \$10 000, i = 6.2% per year, compounded semi-annually, and n = 2 years

$$A = P(1 + i)^n$$
$$= 10\,000(1.031)^4$$
$$= 11\,298.86$$

Under Option B, the investment is worth \$11 298.86 at the end of two years.

Option C. 6.0% per year compounded quarterly

$$A = P(1 + i)^n$$
$$= 10\,000(1.015)^8$$
$$= 11\,264.93$$

Under Option C, the investment is worth \$11 264.93 at the end of two years.

Barb should choose Option A since it gains the most interest.

Chapter 8 Section 5 Question 9 Page 451

Answers may vary. For example:

a) If Jayeed can leave his money invested for four years:

Option A. P = \$4000, i = 3.25% per year, simple interest cashable any time, n = 4 years

$$\begin{aligned} A &= P(1+i(n)) \\ &= 4000(1+0.0325(4)) \\ &= 4520.00 \end{aligned}$$

Under Option A, the investment will be worth \$4520 in four years.

Option B. 3% per year compounded monthly cashable after 2 years

$$\begin{aligned} A &= P(1+i)^n \\ &= 4000(1.0025)^{24} \\ &= 4247.03 \end{aligned}$$

This amount can then be invested for an additional 2 years, when it is worth:

$$\begin{aligned} A &= 4247.03(1.0025)^{24} \\ &= 4509.31 \end{aligned}$$

Under Option B, the investment will be worth \$4509.31 in four years.

Option C. 3.5% per year compounded semi-annually cashable after four years

$$\begin{aligned} A &= P(1+i)^n \\ &= 4000(1.0175)^8 \\ &= 4595.53 \end{aligned}$$

Under Option C, the investment will be worth \$4595.53 in four years.

Option C earns the most interest, but it may not be the best option since it ties up Jayeed's \$4000 for four years.

b) If Jayeed wants his invested money in 2.5 years:

Option A. P = \$4000, i = 3.25% per year, simple interest cashable any time, n = 2.5 years

$$\begin{aligned} A &= P(1+i(n)) \\ &= 4000(1+0.0325(2.5)) \\ &= 4325.00 \end{aligned}$$

Under Option A, the investment will be worth \$4325 in 2.5 years.

Option B. Here the time is 2 years since the investment is cashable at that time.

$$\begin{aligned} A &= P(1+i)^n \\ &= 4000(1.0025)^{30} \\ &= 4311.13 \end{aligned}$$

Under Option B, the investment will be worth \$4247.03 in 2.5 years.

Option C. This plan cannot be used since it is not cashable for four years.

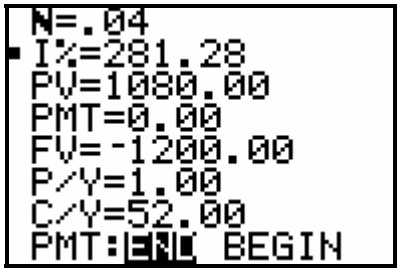
Option A is best for Jayeed since it earns more interest than Option B.

Chapter 8 Section 5 Question 10 Page 451

a) Jaspar will receive 90% of his \$1200 cheque, i.e., \$1200 × 0.9 = \$1080.

b) FV = –\$1200, P = \$1080, $N = \frac{1}{26}$, and C/Y = 52

Use the TVM Solver to find i.

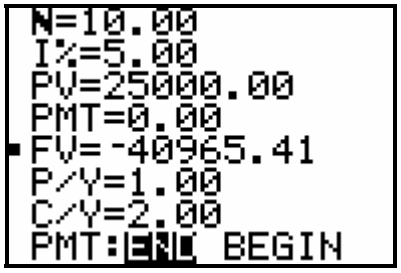


i = 281.28%

Chapter 8 Section 5 Question 11 Page 452

a) **Investment A:** P = \$25 000, N = 10, i = 5%, compounded semi-annually, and C/Y = 2

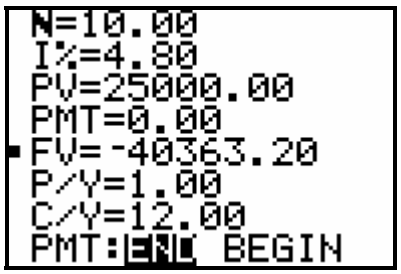
Use the TVM Solver to find FV.



FV = \$40 965.41

Investment B: P = \$25 000, N = 10, i = 4.8%, compounded monthly, and C/Y = 12

Use the TVM Solver to find FV.



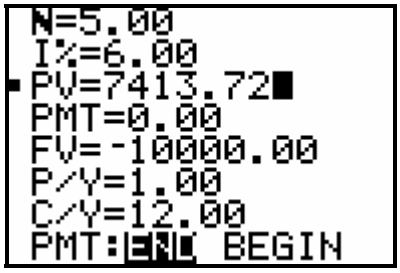
FV = \$40 363.19

The difference in interest is \$40 965.41 – \$40 363.19 = \$602.22.

b) Answers may vary.

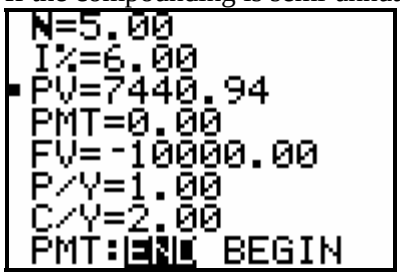
Chapter 8 Section 5 Question 12 Page 452

a) i) $FV = -\$10\,000$, $N = 5$, $i = 6\%$, compounded monthly, and C/Y = 12
Use the TVM Solver to find P.



$P = \$7413.72$

ii) If the compounding is semi-annual, C/Y = 2.



$P = \$7440.94$

b) The second principal is greater because there are only two, compared to 12, compounding periods.

Chapter 8 Section 5 Question 13 Page 452

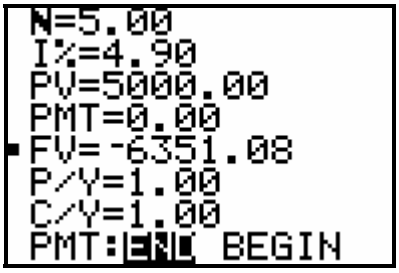
Answers may vary.

Chapter 8 Section 5 Question 14 Page 452

Solutions for Achievement Checks are shown in the Teacher's Resource.

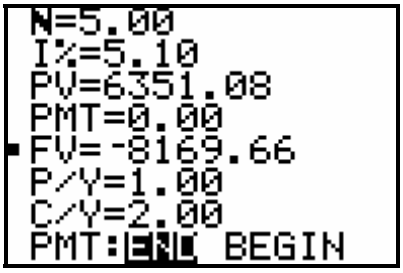
Chapter 8 Section 5 Question 15 Page 453

First five years: P = \$5000, N = 5 years, and i = 4.9% per year, compounded annually
Use the TVM Solver to find FV.



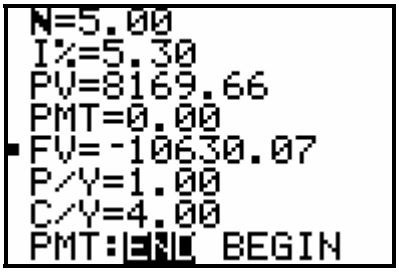
The investment is worth \$6351.08 in five years.

Second five years: P = \$6351.08, N = 5 years, and i = 5.1% per year, compounded semi-annually
Use the TVM Solver to find FV.



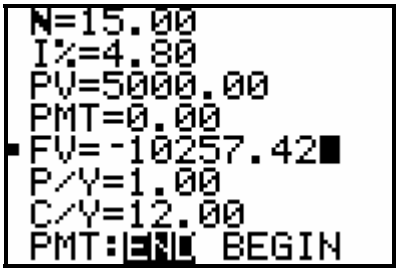
After the next five years the investment is worth \$8169.66.

Third five years: P = \$8169.66, N = 5 years, and I = 5.3% per year, compounded quarterly
Use the TVM Solver to find FV.



After the final five years the investment is worth \$10 630.07.

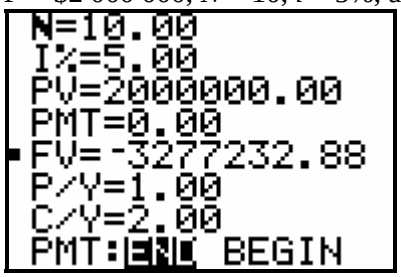
One investment for 15 years: P = \$5000, N = 15 years, I = 4.8% per year, compounded monthly
Use the TVM Solver to find FV.



After 15 years the single investment is worth \$10 257.42.
The three separate five-year investments earned \$10 630.07 – \$10 257.42 = \$372.65 more interest.

Chapter 8 Section 5 Question 16 Page 453

a) Use the TVM Solver to find *FV*.
P = \$2 000 000, *N* = 10, *i* = 5%, and C/Y = 2



After ten years, the \$2 million investment is worth \$3 277 232.88.

The cost of purchasing the aircraft would be the \$2 million plus the interest lost, which is equal to \$3 277 232.88.

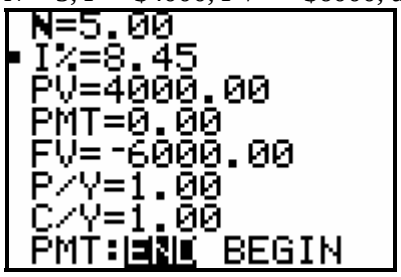
The aircraft can be resold for \$1 500 000, for a difference in cost of \$1 777 232.88 over the ten-year period.

b) At \$200 000 a year to lease, the total leasing cost is 10 × \$200 000 for ten years, which is \$2 000 000.

c) Answers may vary. For example:
It is cheaper to buy the plane when the resale is taken into account.

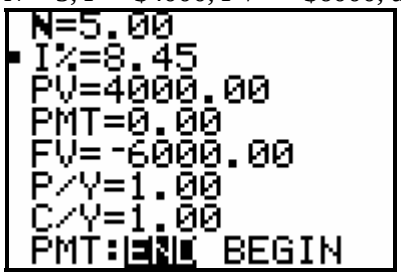
Chapter 8 Section 5 Question 17 Page 453

a) compounded annually: Use the TVM Solver to find *i*.
N = 5, *P* = $4000, *FV* = –$6000, and C/Y = 1



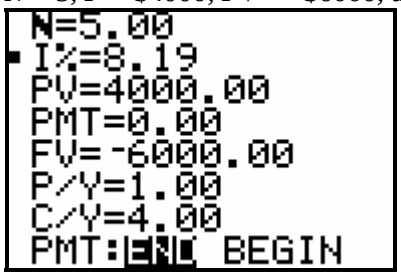
Jessica needs to earn 8.45%, compounded annually, to reach her goal.

b) compounded semi-annually: Use the TVM Solver to find *i*.
N = 5, *P* = $4000, *FV* = –$6000, and C/Y = 2



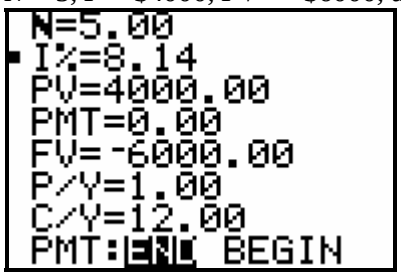
Jessica needs to earn 8.28%, compounded semi-annually, to reach her goal.

c) compounded quarterly: Use the TVM Solver to find *i*.
N = 5, *P* = $4000, *FV* = –$6000, and C/Y = 4



Jessica needs to earn 8.19%, compounded quarterly, to reach her goal.

d) compounded monthly: Use the TVM Solver to find *i*.
N = 5, *P* = $4000, *FV* = –$6000, and C/Y = 12

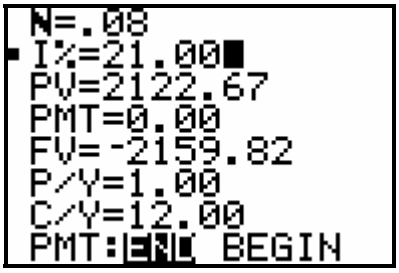


Jessica needs to earn 8.14%, compounded monthly, to reach her goal.

Chapter 8 Section 5 Question 18 Page 453

P = \$2122.67, FV = –\$2159.82, $N = \frac{1}{12}$, C/Y = 12

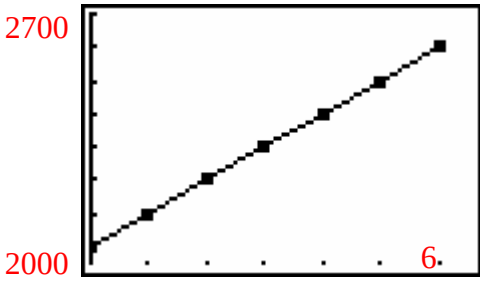
Use the TVM Solver to find i.



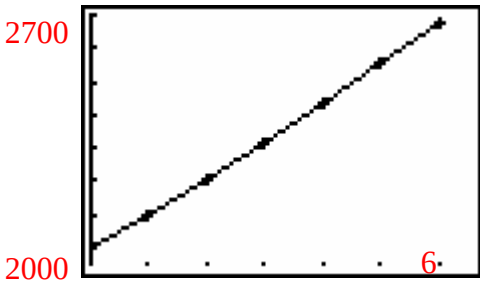
The effective annual interest rate on the credit card is 21.00%.

Chapter 8 Review **Question 1 Page 454**

Year	Simple Interest ($)	Amount ($)
0		2000
1	100	2100
2	100	2200
3	100	2300
4	100	2400
5	100	2500
6	100	2600



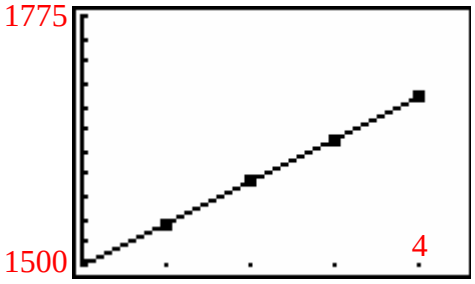
Year	$A = P(1.05)$	Amount ($)
0		2000.00
1	2000(1.05)	2100.00
2	2100(1.05)	2205.00
3	2205(1.05)	2315.25
4	2315.25(1.05)	2431.0125
5	2431.0125(1.05)	2552.56313
6	2552.56313(1.05)	2680.19128



Chapter 8 Review **Question 2 Page 454**

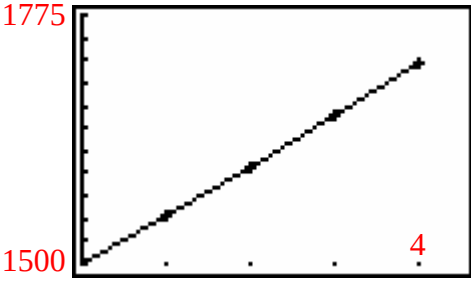
a)

Year	$A = P(1.03)$	Amount ($)
0		1500.00
1	1500(1.03)	1545.00
2	1545(1.03)	1591.35
3	1591.35(1.03)	1639.0905
4	1639.0905(1.03)	1688.26322



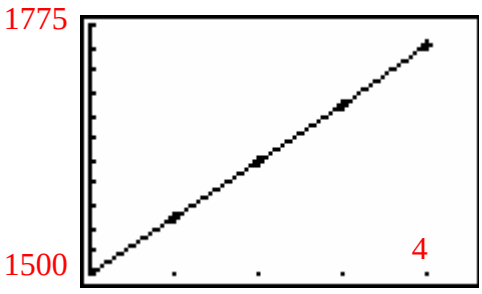
b)

Year	$A = P(1.035)$	Amount ($)
0		1500.00
1	1500(1.035)	1552.50
2	1552.5(1.035)	1606.8375
3	1606.8375(1.035)	1663.07681
4	1663.07681(1.035)	1721.28450



c)

Year	$A = P(1.04)$	Amount ($)
0		1500.00
1	1500(1.04)	1560.00
2	1560(1.04)	1622.40
3	1622.4(1.04)	1687.296
4	1687.296(1.04)	1754.78784



Chapter 8 Review **Question 3 Page 454**

a) By interpolating from the graph, the investment is worth about $2700 after five years.

b) By extrapolating from the graph, it would take the investment about 12 years to double to $4000.

c) If the interest rate was 4% instead of 6%, the graph would be less steep and have a lesser curve because of a lower interest rate.

Chapter 8 Review **Question 4 Page 454**

a) $A = 600(1 + 0.035)^6$

b) $A = 4000(1 + 0.0225)^{20}$

c) $A = 6000(1 + 0.007)^{36}$

d) $A = 1200(1 + 0.0225)^4$

Chapter 8 Review **Question 5 Page 454**

Use $A = P(1 + i)^n$ to evaluate the two investments.

a) **Plan A:** $A = 5000(1.01375)^{16}$
$= 6221.05$
Plan B: $A = 2500(1.029)^8$
$= 3142.41$
Plan A earned more money for Bill.

b) Interest earned on Plan A: $6221.05 – $5000 = $1221.05
Interest earned on Plan B: $3142.41 – $2500 = $642.41
Total interest: $1221.05 + $642.41 = $1863.46

Chapter 8 Review Question 6 Page 454

a) Use $A = P(1 + i)^n$. P = \$2300, i = 10% per year, compounded quarterly, and n = 5 years.

$A = 2300(1.025)^{20}$

$= 3768.82$

Barbara must repay \$3768.82 after five years.

b) The amount of interest paid is \$3768.82 – \$2300 = \$1468.82.

Chapter 8 Review Question 7 Page 454

How much should Neaz invest?

Use $P = A(1 + i)^{-n}$. A = \$4800, i = 5.7% per year, compounded monthly, and n = 5 years

$P = 4800(1.00475)^{-60}$

$= 3612.10$

Neaz should invest \$3612.10 to have \$4800 in five years.

Chapter 8 Review Question 8 Page 455

Use $P = A(1 + i)^{-n}$. A = \$10 000, i = 9.6% per year, compounded monthly, and n = 6 years

$P = 10\,000(1.008)^{-72}$

$= 5634.32$

The creditor should be willing to accept \$5634.32 to pay off the loan today.

Chapter 8 Review Question 9 Page 455

Compare the following three plans to finance buying a car.

Plan A: \$16 250 cash now

Plan B: \$1000 down and \$15 500 to be paid in one year

Use $P = A(1 + i)^{-n}$ to calculate the present value.

A = \$15 500, i = 5% per year, compounded semi-annually, and n = 1 year

$P = 15\,500(1.025)^{-2}$

$= 14\,753.12$

The total present cost of Plan B is \$1000 + \$14 753.12 = \$15 753.12.

Plan C: \$500 and \$16 000 to be paid in one year.

Use $P = A(1 + i)^{-n}$ to calculate the present value.

A = \$16 000, i = 5% per year, compounded semi-annually, and n = 1 year

$P = 16\,000(1.025)^{-2}$

$= 15\,229.03$

The total present cost of Plan C is \$500 + \$15 229.03 = \$15 729.03.

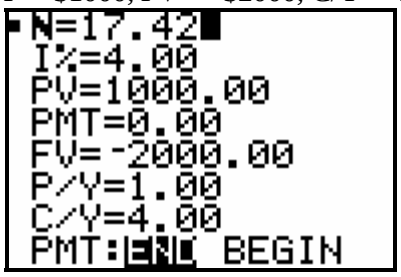
Therefore Plan C is the best deal.

Chapter 8 Review **Question 10 Page 455**

Present Value ($)	Future Value ($)	Term (years)	Compounding Period	Annual Interest Rate (%)
8000	12 000	5	monthly	8.14
6000	13 000	10	semi-annually	7.88
1340	2000	6.75	quarterly	6
100 000	1 000 000	29.5	semi-annually	8
4000	4376.21	3	monthly	3
16 149.25	25 000	8	quarterly	5.5

Chapter 8 Review **Question 11 Page 455**

a) Use the TVM Solver to find N, the time it takes an investment to double.
P = \$1000, FV = –\$2000, C/Y = 4, and i = 4%, compounded quarterly

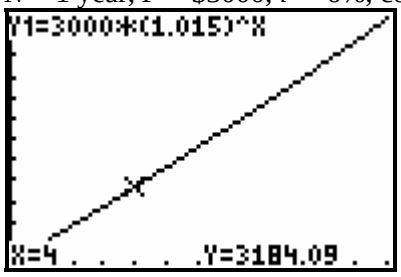


N = 17.42 years
It will take 17 years and 6 months for the investment to double.

b) Yes. Doubling depends on the rate and the compounding periods, not on the amount invested. Any amount would double in this length of time.

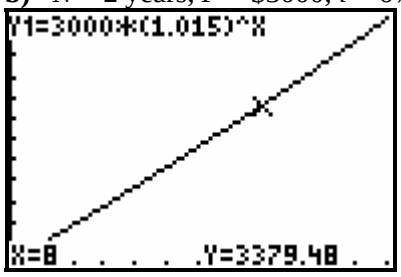
Chapter 8 Review **Question 12 Page 455**

a) N = 1 year, P = \$3000, i = 6%, compounded quarterly



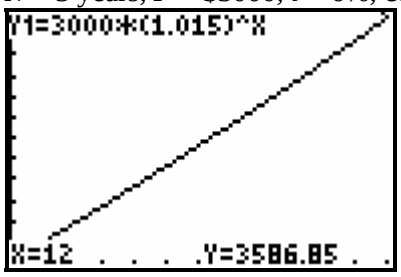
\$3184.09

b) N = 2 years, P = \$3000, i = 6%, compounded quarterly



\$3379.48

c) N = 3 years, P = \$3000, i = 6%, compounded quarterly



\$3586.85

Chapter 8 Review Question 13 Page 455

Compare returns using different rates of compounding.
Use $A = P(1 + i)^n$, with $P = \$2000$, $i = 7\%$, and $n = 2$ years

a) Compounded annually: $A = 2000(1.07)^2$

$= 2289.80$

Interest is \$2289.80 – \$2000 = \$289.80.

b) Compounded semi-annually: $A = 2000(1.035)^4$

$= 2295.05$

Interest is \$2295.05 – \$2000 = \$295.05.

c) Compounded quarterly: $A = 2000(1.0175)^8$

$= 2297.76$

Interest is \$2297.76 – \$2000 = \$297.76.

d) Compounded monthly: $A = 2000\left(1 + (0.07 \div 12)\right)^{24}$

$= 2299.61$

Interest is \$2299.61 – \$2000 = \$299.61.

Chapter 8 Review Question 14 Page 455

Marlon wants to compare purchasing a car for \$30 000 to leasing it for \$4000 a year.

a) Use $A = P(1 + i)^n$, with $P = \$30\ 000$, $i = 6\%$, and $n = 5$ years

$A = 30\ 000(1.015)^{20}$

$= 40\ 405.65$

If the car is sold after five years for \$12 000, Marlon's cost has been
\$40 405.65 – \$12 000 = \$ 28 405.65.

b) The cost of leasing the car for five years at \$4000 per year is \$20 000.

c) The better plan is for Marlon to lease the car since he will save \$8405.65 over the five-year period.

Chapter 8 Practice Test

Chapter 8 Practice Test **Question 1 Page 456**
B

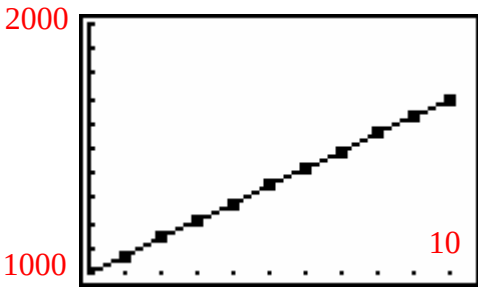
Chapter 8 Practice Test **Question 2 Page 456**
C

Chapter 8 Practice Test **Question 3 Page 456**
A

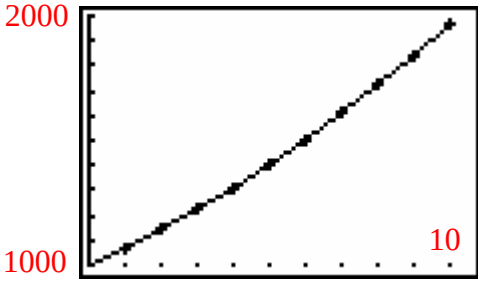
Chapter 8 Practice Test **Question 4 Page 456**
A and D

Chapter 8 Practice Test **Question 5 Page 456**

Year	Simple Interest ($)	Amount ($)
0		1000.00
1	70	1070.00
2	70	1140.00
3	70	1210.00
4	70	1280.00
5	70	1350.00
6	70	1420.00
7	70	1490.00
8	70	1560.00
9	70	1630.00
10	70	1700.00



Year	$A = P(1.07)$	Amount ($)
0		1000.00
1	1000.00(1.07)	1070.00
2	1070.00(1.07)	1144.90
3	1144.90 (1.07)	1225.043
4	1225.043 (1.07)	1310.79601
5	1310.79601 (1.07)	1402.55173
6	1402.55173 (1.07)	1500.73035
7	1500.73035 (1.07)	1605.78148
8	1605.78148 (1.07)	1718.18618
9	1718.18618 (1.07)	1838.45921
10	1838.45921 (1.07)	1967.15136



Chapter 8 Practice Test **Question 6 Page 456**

To calculate the amount Andrea will owe, use $A = P(1 + i)^n$.

a) P = \$768.42, i = 18.5%, and n = 1 month

$A = 768.42(1.0154167)^1$

$= 780.27$

After one month Andrea will owe \$780.27.

b) P = \$768.42, i = 18.5%, and n = 3 months

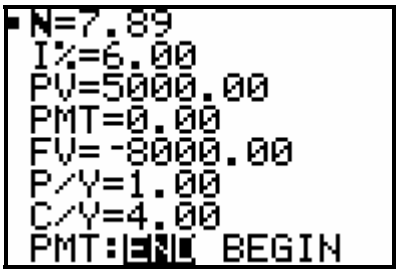
$A = 768.42(1.0154167)^3$

$= 804.51$

After three months Andrea will owe \$804.51.

Chapter 8 Practice Test **Question 7 Page 456**

Use the TVM Solver to find N. P = \$5000, FV = –\$8000, C/Y = 1, i = 6%, and C/Y = 4



It takes 8 years for \$5000 to grow to \$8000 at 6% per year, compounded quarterly.

Chapter 8 Practice Test **Question 8 Page 456**

Erik has two options for a loan. To evaluate the options, use $A = P(1 + i)^n$.

Loan A: P = \$2000, n = 3 years, and i = 10% per year, compounded semi-annually

$A = 2000(1.05)^6$

$= 2680.19$

The total cost of loan A is \$2680.19.

Loan B: P = \$2000, n = 3 years, and i = 9.2% per year, compounded quarterly

$A = 2000(1.023)^{12}$

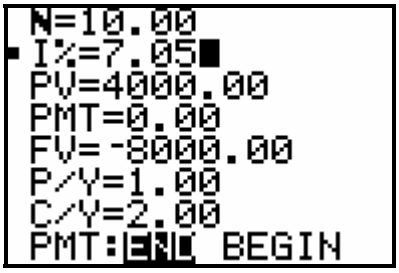
$= 2627.47$

The total cost of loan B is \$2627.47.

Erik should choose loan B since he will have to pay \$2680.19 – \$2627.47 = \$52.72 less interest.

Chapter 8 Practice Test **Question 9 Page 456**

What interest rate would double the investment?
Use the TVM Solver to find *i*. $N = 10$, $P = \$4000$, $FV = -\$8000$, and C/Y = 2



An interest rate of 7.05%, compounded semi-annually, will double the investment in ten years.

Chapter 8 Practice Test **Question 10 Page 457**

Use $P = A(1 + i)^{-n}$ to calculate the present value.
$A = \$15\,000$, $i = 6.6\%$ per year, compounded monthly, and $n = 8$ years

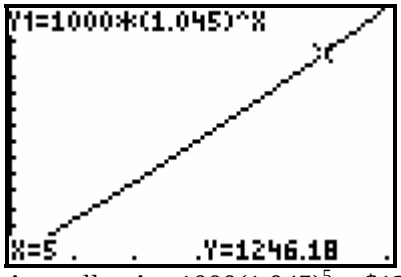
$$P = 15\,000(1.0055)^{-96}$$
$$= 8859.56$$

Jeeva's parents need to invest \$8859.56.

Chapter 8 Practice Test **Question 11 Page 457**

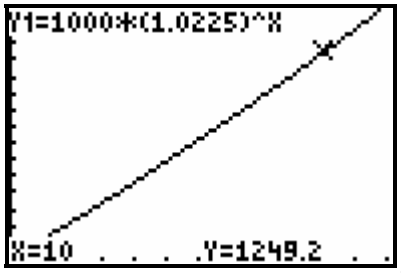
$1000 is invested for five years at 4.5% per year, compounded

a)



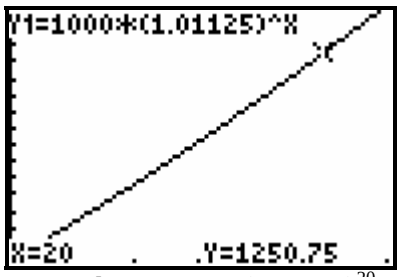
Annually: $A = 1000(1.045)^5 = \$1246.18$

b)



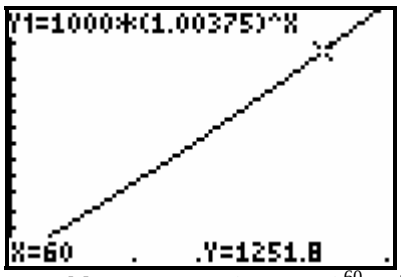
Semi-annually: $A = 1000(1.0225)^{10} = \$1249.20$

c)



Quarterly: $A = 1000(1.01125)^{20} = \$1250.75$

d)



Monthly: $A = 1000(1.00375)^{60} = \$1251.80$

As the number of compounding periods increases, the amount of interest increases.

Chapter 8 Practice Test Question 12 Page 457

a)

Annual Interest Rate (%)	Compounding Period	Scholarship Amount ($)
8.0	semi-annually	$4080.00
7.5	quarterly	$3856.79
5.5	semi-annually	$2787.81
7.0	semi-annually	$3561.25
9.0	annually	$4500.00

b) For P = \$50 000, what annual rate, compounded annually, is needed to give an amount of \$55 000?

$$55\,000 = 50\,000(1 + i)$$

$$\frac{55\,000}{50\,000} = 1 + i$$

$$1.1 - 1 = i$$

$$i = 0.1$$

So i = 10%

Check: 110% of \$50 000 is \$50 000 × 1.1 = \$55 000.