## Chapter 7 Exponents

### Chapter 7 Prerequisite Skills

### Chapter 7 Prerequisite Skills Question 1 Page 354

**a)** $6 \times 6 = 6^2$

**b)** $7 \times 7 \times 7 \times 7 = 7^4$

**c)** $(–2) \times (–2) \times (–2) = (–2)^3$

**d)** $(4)(4)(4)(4)(4)(4)(4)(4) = 4^8$

**e)** $\frac{1}{4} \times \frac{1}{4} \times \frac{1}{4} \times \frac{1}{4} \times \frac{1}{4} = \left(\frac{1}{4}\right)^5$

**f)** $\left(-\frac{4}{5}\right)\left(-\frac{4}{5}\right) = \left(-\frac{4}{5}\right)^2$

### Chapter 7 Prerequisite Skills Question 2 Page 354

**a)** $5^2 = 25$

**b)** $7^3 = 343$

**c)** $10^5 = 100\ 000$

**d)** $(–3)^2 = 9$

**e)** $–3^2 = –9$

**f)** $–12^2 = –144$

**g)** $\left(\frac{1}{2}\right)^2 = \frac{1}{4}$

**h)** $\left(-\frac{1}{3}\right)^4 = \frac{1}{81}$

**i)** $\left(-\frac{1}{5}\right)^3 = -\frac{1}{125}$

### Chapter 7 Prerequisite Skills Question 3 Page 354

**a)** $y = 2x + 5$
slope 2, $y$-intercept 5

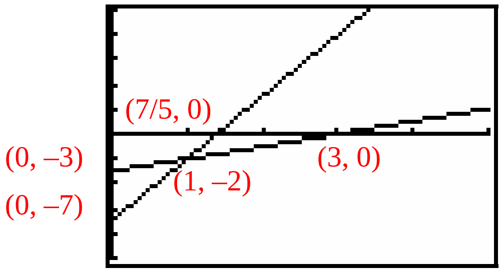
**b)** $y = 3x – 1$
slope 3, $y$-intercept –1

**c)** $y = –4x + 3$
slope –4, $y$-intercept 3

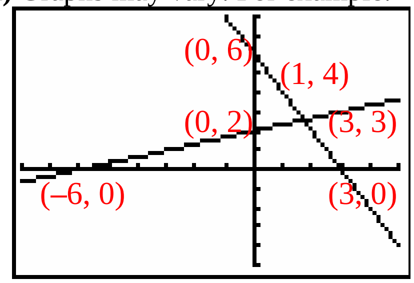
**d)** $y = -\frac{1}{2}x - \frac{2}{3}$.
slope $-\frac{1}{2}$, $y$-intercept $-\frac{2}{3}$

**Chapter 7 Prerequisite Skills** **Question 4 Page 354**

**a), b)** Graphs may vary. For example:




**c), d)** Graphs may vary. For example:




**Chapter 7 Prerequisite Skills** **Question 5 Page 354**

If $y = mx + b$ represents Ahmed's salary, then $m$ = \$2 is his rate per call and $b$ = \$40 is his salary per day.

**Chapter 7 Prerequisite Skills** **Question 6 Page 354**

**a)**
$$\begin{aligned} A &= \pi(\text{r})^2 \\ &= 25\pi \\ &\approx 78.5 \text{ cm}^2 \end{aligned}$$

**b)**
$$\begin{aligned} I &= Prt \\ &= \$200 \times 0.06 \times 2 \\ &= \$24 \end{aligned}$$

**c)**
$$\begin{aligned} V &= s^3 \\ &= 5^3 \\ &= 125 \text{ m}^3 \end{aligned}$$

**d)**
$$\begin{aligned} P &= 2\,(l + w) \\ &= 2(17) \\ &= 34 \text{ cm} \end{aligned}$$

**Chapter 7 Prerequisite Skills** **Question 7 Page 355**

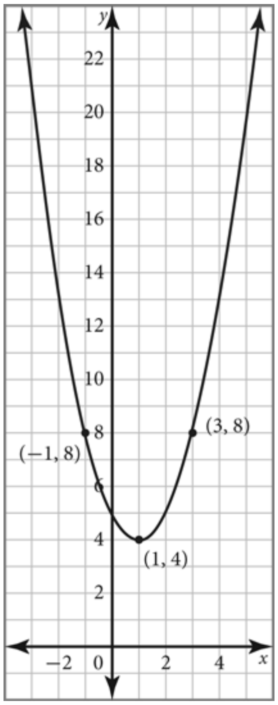
**a)** Answers may vary. For example:
The graph shown is the reflection of $y = x^2$ in the $x$-axis.

**b)** Answers may vary. For example:
The graph shown is that of $y = x^2$ moved 1 unit to the right and 2 units down.

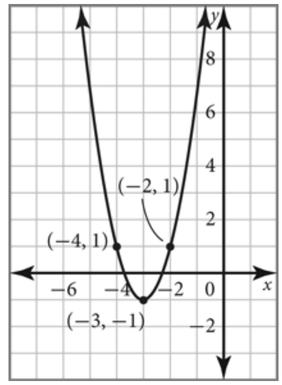
**c)** Answers may vary. For example:
The graph shown is that of $y = x^2$ moved 3 units to the left and 2 units up.

**Chapter 7 Prerequisite Skills** **Question 8 Page 355**

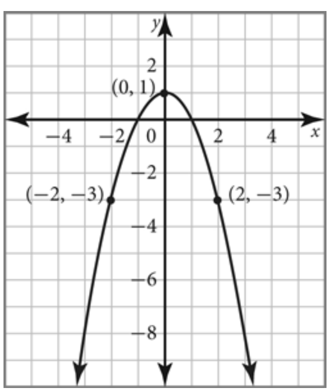
a)



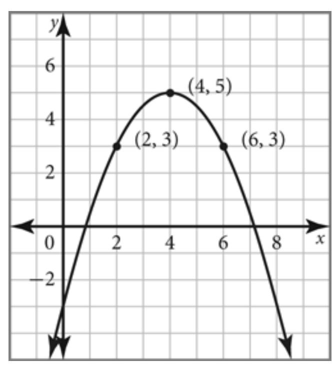

**b)**




**c)**




**d)**



## Chapter 7 Section 1 Exponent Rules

## Chapter 7 Section 1 Question 1 Page 360

**a)** $5^2 \times 5^2 = 5^{2+2}$
$= 5^4$
$= 625$

**b)** $2^4 \times 2^3 = 2^{4+3}$
$= 2^7$
$= 128$

**c)** $(-3)^2 \times (-3)^4 = (-3)^{2+4}$
$= (-3)^6$
$= 729$

**d)** $(-4)^3 \times (-4)^3 = (-4)^{3+3}$
$= (-4)^6$
$= 4096$

**e)** $\left(\frac{1}{4}\right)^2 \times \left(\frac{1}{4}\right)^3 = \left(\frac{1}{4}\right)^{2+3}$
$= \left(\frac{1}{4}\right)^5$
$= \frac{1}{1024}$

**f)** $\left(-\frac{1}{2}\right)^2 \times \left(-\frac{1}{2}\right) = \left(-\frac{1}{2}\right)^{2+1}$
$= \left(-\frac{1}{2}\right)^3$
$= -\frac{1}{8}$

## Chapter 7 Section 1 Question 2 Page 360

**a)** $6^5 \div 6^4 = 6^{5-4}$
$= 6^1$
$= 6$

**b)** $8^7 \div 8^5 = 8^{7-5}$
$= 8^2$
$= 64$

**c)** $12^8 \div 12^7 = 12^{8-7}$
$= 12^1$
$= 12$

**d)** $\frac{2^{10}}{2^6} = 2^{10-6}$
$= 2^4$
$= 16$

**e)** $\frac{(-2)^9}{(-2)^6} = (-2)^{9-6}$
$= (-2)^3$
$= -8$

**f)** $\frac{(-3)^6}{(-3)^4} = (-3)^{6-4}$
$= (-3)^2$
$= 9$

## Chapter 7 Section 1 Question 3 Page 361

**a)** $(5^2)^3 = 5^{2 \times 3}$
$= 5^6$
$= 15\ 625$

**b)** $(2^3)^3 = 2^{3 \times 3}$
$= 2^9$
$= 512$

**c)** $[(-4)^3]^2 = (-4)^{3 \times 2}$
$= (-4)^6$
$= 4096$

**d)** $\left(\frac{1}{7^2}\right)^2 = \frac{1^2}{7^{2 \times 2}}$
$= \frac{1^2}{7^4}$
$= \frac{1}{2401}$

**e)** $\left(\frac{1}{3^3}\right)^2 = \frac{1^2}{3^{3 \times 2}}$
$= \frac{1^2}{3^6}$
$= \frac{1}{729}$

**f)** $\left(-\frac{1}{10^2}\right)^4 = \frac{1^4}{10^{2 \times 4}}$
$= \frac{1^4}{10^8}$
$= \frac{1}{100\ 000\ 000}$

**Chapter 7 Section 1** **Question 4 Page 361**

**a)** $6^2 \times 6^3 = 6^5$
$= 7776$

$6^2 \times 6^3 = 36 \times 216$
$= 7776$

**b)** $7^4 \times 7^2 = 7^6$
$= 117\ 649$

$7^4 \times 7^2 = 2401 \times 49$
$= 117\ 649$

**c)** $9^5 \div 9^3 = 9^2$
$= 81$

$9^5 \div 9^3 = 59\ 049 \div 729$
$= 81$

**d)** $\frac{(-7)^4}{(-7)^3} = (-7)^1$
$= -7$

$\frac{(-7)^4}{(-7)^3} = \frac{2401}{-343}$
$= -7$

**e)** $(5^2)^3 = 5^6$
$= 15\ 625$

$(5^2)^3 = 25^3$
$= 15\ 625$

**f)** $(10^5)^2 = 10^{10}$
$= 10\ 000\ 000\ 000$

$(10^5)^2 = (100\ 000)^2$
$= 10\ 000\ 000\ 000$

**g)** $(-8)^3(-8) = (-8)^4$
$= 4096$

$(-8)^3(-8) = (-512)(-8)$
$= 4096$

**h)** $[(-1)^{11}]^9 = (-1)^{99}$
$= -1$

$[(-1)^{11}]^9 = (-1)^{99}$
$= -1$

**Chapter 7 Section 1** **Question 5 Page 361**

**a)** $9^4 \times 9^5 = 9^9$
$= 387\ 420\ 489$

**b)** $(7^2)^4 = 7^8$
$= 5\ 764\ 801$

**c)** $(-6) \times (-6)^5 = (-6)^6$
$= 46\ 656$

**d)** $24^6 \div 24^5 = 24^1$
$= 24$

**e)** $\frac{9^7}{9^5} = 9^2$
$= 81$

**f)** $\left(\frac{3}{4}\right)^5 \times \left(\frac{3}{4}\right)^2 = \left(\frac{3}{4}\right)^7$
$= \frac{2187}{16\ 384}$

**g)** $(4^3)^5 = 4^{15}$
$= 1\ 073\ 741\ 824$

**h)** $\frac{(-8)^9}{(-8)^6} = (-8)^3$
$= -512$

**i)** $\left(-\frac{5}{7}\right)^8 \div \left(-\frac{5}{7}\right)^4 = \left(-\frac{5}{7}\right)^4$
$= \frac{625}{2401}$

**Chapter 7 Section 1** **Question 6 Page 361**

An earthquake with magnitude 6 has an intensity of $10^6$ and one with magnitude 8 has an intensity of $10^8$.

$$\frac{10^8}{10^6} = 10^{8-6}$$
$$= 10^2$$
$$= 100$$

An earthquake with magnitude 8 is 100 times more powerful than one with a magnitude 6.

**Chapter 7 Section 1** **Question 7 Page 361**

The Indian Ocean earthquake was magnitude 9, while the Vancouver Island earthquake was magnitude 4.
The difference in intensity was $10^9 \div 10^4 = 10^5 = 100\ 000$.
The Indian Ocean earthquake was 100 000 times stronger.

**Chapter 7 Section 1** **Question 8 Page 361**

The 4.2 magnitude earthquake was stronger.
The difference in intensity was $10^{4.2} \div 10^{2.8} = 10^{1.4}$
$\approx 25$
The 4.2 magnitude earthquake was about 25 times stronger.

**Chapter 7 Section 1 Question 9 Page 362**

**a)** Answers may vary. For example:
$3^8 = 3^7 \times 3^1$
$3^8 = 3^5 \times 3^3$
$3^8 = 3^4 \times 3^4$

**b)** Answers may vary. For example:
$2^5 = 2^6 \div 2^1$
$2^5 = 2^7 \div 2^2$
$2^5 = 2^8 \div 2^3$

**c)** Answers may vary. For example:
$7^{12} = (7^6)^2$
$7^{12} = (7^3)^4$
$7^{12} = (7^2)^6$

**Chapter 7 Section 1 Question 10 Page 362**

**a)** Yes, they are equivalent.
$(64)^2 = (2^6)^2 = 2^{12}$
$(16)^3 = (2^4)^3 = 2^{12}$

**b)** Answers may vary. For example:
$(64)^2 = (8^2)^2 = 8^4$

**Chapter 7 Section 1 Question 11 Page 362**

**a)** The probability of rolling two 5s using two dice is:
$$\frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$$

**b)** The probability of rolling three 5s with three dice is:
$$\frac{1}{6} \times \frac{1}{6} \times \frac{1}{6} = \frac{1}{6^3}$$
$$= \frac{1}{216}$$

**Chapter 7 Section 1 Question 12 Page 362**

**a)** The area of a square of side $\frac{1}{2}$ in. is $\left(\frac{1}{2}\right)^2 = \frac{1}{4}$ in.$^2$.

**b)** The area of a square of side $\frac{1}{4}$ ft is $\left(\frac{1}{4}\right)^2 = \frac{1}{16}$ ft$^2$.

**Chapter 7 Section 1** **Question 13 Page 362**

**a)** If the side of the square is $\frac{1}{2} \div 12 = \frac{1}{24}$ ft, the area is $\left(\frac{1}{24}\right)^2 = \frac{1}{576}$ ft$^2$.

**b)** If the side of the square is $\frac{1}{4} \times 12 = 3$ in., the area is $3^2 = 9$ in.$^2$.

**Chapter 7 Section 1** **Question 14 Page 363**

| Measurement to be Calculated | Formula | Dimensions Given | Calculated Measurement |
|---|---|---|---|
| Area of a Circle | $A = \pi r^2$ | $r = \pi$ cm | 31 cm$^3$ |
| Volume of a Cube | $V = s^3$ | $s = \frac{1}{2}$ in. | $\frac{1}{8}$ in.$^3$ |
| Volume of a Sphere | $V = \frac{4}{3}\pi r^3$ | $r = \frac{1}{8}$ in. | 0.0082 in.$^3$ |
| Volume of a Cylinder | $V = \pi r^2 h$ | $r = h = 5$ cm | 392.7 cm$^3$ |

**Chapter 7 Section 1** **Question 15 Page 363**

$$\begin{aligned} V &= (l \times w)^3 \\ &= (3 \times 2)^3 \\ &= 6^3 \\ &= 216 \text{ cm}^3 \end{aligned}$$

The approximate volume of the Rubik's Cube® is 216 cm.

**Chapter 7 Section 1** **Question 16 Page 363**

**a)** $$\begin{aligned} 10^{3.1} \times 10^{4.2} &= 10^{7.3} \\ &= 19\ 952\ 623.15 \end{aligned}$$

**b)** $$\begin{aligned} \frac{10^{7.9}}{10^{3.1}} &= 10^{4.8} \\ &= 63\ 095.73 \end{aligned}$$

**c)** $$\begin{aligned} 2^{4.8} \times 2^{1.6} &= 2^{6.4} \\ &= 84.45 \end{aligned}$$

**d)** $$\begin{aligned} \left(\frac{1}{2}\right)^{7.8}\left(\frac{1}{2}\right)^{1.1} &= \left(\frac{1}{2}\right)^{8.9} \\ &= 0.0021 \end{aligned}$$

**Chapter 7 Section 1 Question 17 Page 363**

**a)** $(4x^3)(2x^4) = 8x^7$

**b)** $\dfrac{-12a^5b^3}{3a^2b} = -4a^3b^2$

**c)** $(m^2n^3)^5 = m^{10}n^{15}$

**d)** $$\begin{aligned}\left(\frac{k^5h^2}{k^2}\right)^3 &= (k^3h^2)^3 \\ &= k^9h^3\end{aligned}$$

**Chapter 7 Section 2** **Zero and Negative Exponents**

**Chapter 7 Section 2** **Question 1 Page 367**

**a)** $\frac{1}{9^5} = 9^{-5}$

**b)** $6^3 = \left(\frac{1}{6}\right)^{-3}$

**c)** $5^{-2} = \left(\frac{1}{5}\right)^2$

**d)** $\frac{1}{4^{-1}} = 4^1$
$= 4$

**Chapter 7 Section 2** **Question 2 Page 368**

**a)** $5^2 = 25$
$5^{-2} = \frac{1}{25}$

**b)** $2^1 = 2$
$2^{-1} = \frac{1}{2}$

**c)** $4^4 = 256$
$4^{-4} = \frac{1}{256}$

**d)** $10^3 = 1000$
$10^{-3} = \frac{1}{1000}$

**e)** $1^6 = 1$
$1^{-6} = 1$

**f)** $2^9 = 512$
$2^{-9} = \frac{1}{512}$

**g)** $\mathbf{(-3)^4 =} 81$
$(-3)^{-4} = \frac{1}{81}$

**h)** $\mathbf{(-8)^1 =} -8$
$(-8)^{-1} = -\frac{1}{8}$

**Chapter 7 Section 2** **Question 3 Page 368**

**a)** $12^0 = 1$

**b)** $8^{-1} = \frac{1}{8}$

**c)** $6^{-2} = \frac{1}{36}$

**d)** $100\,000^0 = 1$

**e)** $500^{-1} = \frac{1}{500}$

**f)** $5^{-3} = \frac{1}{125}$

**g)** $(-2)^{-8} = \frac{1}{256}$

**h)** $(-10)^{-3} = -\frac{1}{1000}$

**i)** $\left(\frac{1}{6}\right)^{-2} = 36$

**j)** $3^{-5} = \frac{1}{243}$

**k)** $\left(\frac{1}{3}\right)^{-3} = 27$

**l)** $(-7)^3 = -343$

**Chapter 7 Section 2 Question 4 Page 368**

Answers may vary. For example:
$4^3 \div 4^1 = 4^2 = 16$ (Divide each term by $4^1$ to get the next term.)
$4^2 \div 4^1 = 4^1 = 4$
$4^1 \div 4^1 = 4^0 = 1$
$4^0 \div 4^1 = 4^{-1} = \frac{1}{4}$
$4^{-1} \div 4^1 = 4^{-2} = \frac{1}{4^2}$
$4^{-2} \div 4^1 = 4^{-3} = \frac{1}{4^3}$

**Chapter 7 Section 2 Question 5 Page 368**

Answers may vary. For example:
In question 3, the answers were whole numbers or fractions. The calculator gives decimals instead of fractions. Fractions are accurate values, but the approximate decimal values are easier for comparison purposes.

**a)** 1

**b)** 0.125

**c)** 0.02778

**d)** 1

**e)** 0.002

**f)** 0.008

**g)** 0.0039

**h)** –0.001

**i)** 36

**j)** 0.0041

**k)** 27

**l)** –343

**Chapter 7 Section 2** **Question 6 Page 368**

**a)**

$$\frac{8^7}{8^5} = \frac{8 \times 8 \times \cancel{8} \times \cancel{8} \times \cancel{8} \times \cancel{8} \times \cancel{8}}{\cancel{8} \times \cancel{8} \times \cancel{8} \times \cancel{8} \times \cancel{8}}$$
$$= 8 \times 8$$
$$= 8^2$$

$$\frac{8^7}{8^5} = 8^2$$

**b)**

$$\frac{5^4}{5^9} = \frac{\cancel{5} \times \cancel{5} \times \cancel{5} \times \cancel{5}}{5 \times 5 \times 5 \times 5 \times 5 \times \cancel{5} \times \cancel{5} \times \cancel{5} \times \cancel{5}}$$
$$= \frac{1}{5 \times 5 \times 5 \times 5 \times 5}$$
$$= \frac{1}{5^5}$$
$$= 5^{-5}$$

$$\frac{5^4}{5^9} = 5^{-5}$$

**c)**

$$\frac{7}{7^3} = \frac{\cancel{7}}{7 \times 7 \times \cancel{7}}$$
$$= \frac{1}{7 \times 7}$$
$$= \frac{1}{7^2}$$
$$= 7^{-2}$$

$$\frac{7}{7^3} = 7^{-2}$$

**d)**

$$\frac{12^5}{12^8} = \frac{\cancel{12} \times \cancel{12} \times \cancel{12} \times \cancel{12} \times \cancel{12}}{12 \times 12 \times 12 \times \cancel{12} \times \cancel{12} \times \cancel{12} \times \cancel{12} \times \cancel{12}}$$
$$= \frac{1}{12 \times 12 \times 12}$$
$$= \frac{1}{12^3}$$
$$= 12^{-3}$$

$$\frac{12^5}{12^8} = 12^{-3}$$

**e)**

$$\frac{(-4)^7}{(-4)^8} = \frac{\cancel{(-4)} \times \cancel{(-4)} \times \cancel{(-4)} \times \cancel{(-4)} \times \cancel{(-4)} \times \cancel{(-4)} \times \cancel{(-4)}}{(-4) \times \cancel{(-4)} \times \cancel{(-4)} \times \cancel{(-4)} \times \cancel{(-4)} \times \cancel{(-4)} \times \cancel{(-4)} \times \cancel{(-4)}}$$
$$= \frac{1}{(-4)}$$
$$= (-4)^{-1}$$

$$\frac{(-4)^7}{(-4)^8} = (-4)^{-1}$$

**f)**

$$\frac{(-3)^2}{(-3)^7} = \frac{\cancel{(-3)} \times \cancel{(-3)}}{(-3) \times (-3) \times (-3) \times (-3) \times (-3) \times \cancel{(-3)} \times \cancel{(-3)}}$$
$$= \frac{1}{(-3) \times (-3) \times (-3) \times (-3) \times (-3)}$$
$$= \frac{1}{(-3)^5}$$
$$= (-3)^{-5}$$

$$\frac{(-3)^2}{(-3)^7} = (-3)^{-5}$$

**Chapter 7 Section 2** **Question 7 Page 368**

**a)**
$$\frac{6^5}{6^5} = \frac{\cancel{6} \times \cancel{6} \times \cancel{6} \times \cancel{6} \times \cancel{6}}{\cancel{6} \times \cancel{6} \times \cancel{6} \times \cancel{6} \times \cancel{6}}$$
$$= 1$$
$$\frac{6^5}{6^5} = 6^{5-5}$$
$$= 6^0$$
$$= 1$$

**b)**
$$\frac{8^4}{8^4} = \frac{\cancel{8} \times \cancel{8} \times \cancel{8} \times \cancel{8}}{\cancel{8} \times \cancel{8} \times \cancel{8} \times \cancel{8}}$$
$$= 1$$
$$\frac{8^4}{8^4} = 8^{4-4}$$
$$= 8^0$$
$$= 1$$

**c)**
$$\frac{16^6}{16^6} = \frac{\cancel{16} \times \cancel{16} \times \cancel{16} \times \cancel{16} \times \cancel{16} \times \cancel{16}}{\cancel{16} \times \cancel{16} \times \cancel{16} \times \cancel{16} \times \cancel{16} \times \cancel{16}}$$
$$= 1$$
$$\frac{16^6}{16^6} = 16^{6-6}$$
$$= 16^0$$
$$= 1$$

**d)**
$$\frac{2^7}{2^7} = \frac{\cancel{2} \times \cancel{2} \times \cancel{2} \times \cancel{2} \times \cancel{2} \times \cancel{2} \times \cancel{2}}{\cancel{2} \times \cancel{2} \times \cancel{2} \times \cancel{2} \times \cancel{2} \times \cancel{2} \times \cancel{2}}$$
$$= 1$$
$$\frac{2^7}{2^7} = 2^{7-7}$$
$$= 2^0$$
$$= 1$$

**e)**
$$\frac{(-9)^3}{(-9)^3} = \frac{\cancel{(-9)} \times \cancel{(-9)} \times \cancel{(-9)}}{\cancel{(-9)} \times \cancel{(-9)} \times \cancel{(-9)}}$$
$$= 1$$
$$\frac{(-9)^3}{(-9)^3} = (-9)^{3-3}$$
$$= (-9)^0$$
$$= 1$$

**f)**
$$\frac{(-7)^2}{(-7)^2} = \frac{\cancel{(-7)} \times \cancel{(-7)}}{\cancel{(-7)} \times \cancel{(-7)}}$$
$$= 1$$
$$\frac{(-7)^2}{(-7)^2} = (-7)^{2-2}$$
$$= (-7)^0$$
$$= 1$$

**Chapter 7 Section 2** **Question 8 Page 369**

**a)** $8^3 \times 8^{-1} = 8^2$
$= 64$

**b)** $\frac{4^2}{4^{-1}} = 4^{2-(-1)}$
$= 4^3$
$= 64$

**c)** $\frac{1}{(2^4)^3} = \frac{1}{2^{12}}$
$= \frac{1}{4096}$

**d)** $(-3)^3(-3)^{-1} = (-3)^2$
$= 9$

**e)** $(10^{-2})^3 = 10^{-6}$
$= \frac{1}{1\ 000\ 000}$

**f)** $\left(\frac{1}{2^4}\right)\left(\frac{1}{2^4}\right) = \left(\frac{1}{2^8}\right)$
$= \frac{1}{256}$

**g)** $6^2 \div 6^5 = 6^{-3}$
$= \frac{1}{216}$

**h)** $5^{-7} \times 5^4 = 5^{-3}$
$= \frac{1}{125}$

**i)** $(4^3)^{-2} = 4^{-6}$
$= \frac{1}{4096}$

**j)** $\left(\frac{1}{3}\right)^{-6} \times \left(\frac{1}{3}\right)^3 = \left(\frac{1}{3}\right)^{-3}$
$= 3^3$
$= 27$

**k)** $\left(\frac{1}{9}\right)^{-9} \times \left(\frac{1}{9}\right)^7 = \left(\frac{1}{9}\right)^{-2}$
$= 9^2$
$= 81$

**l)** $(5^{-2})^3 = 5^{-6}$
$= \frac{1}{15\ 625}$

**Chapter 7 Section 2** **Question 9 Page 369**

**a)** After 1600 years, $\frac{1}{2}$ of 16 g = 8 g of radium-226 remains.

**b)** 8000 years = 5 × 1600 years

The remaining mass of 16 g after 8000 years would be $\left(\frac{1}{2}\right)^5 \times 16 = 0.5$ g.

**Chapter 7 Section 2** **Question 10 Page 369**

**a)** $1000 = 10^3$

**b)** $\frac{1}{1000} = \frac{1}{10^3}$
$= 10^{-3}$

**c)** One millionth is $10^{-6}$.

**d)** One billionth is $10^{-9}$.

**e)** One thousandth is $10^{-3}$. One billionth is $10^{-9}$.

The ratio is $\frac{10^{-3}}{10^{-9}} = 10^{-3-(-9)}$
$= 10^{-3+9}$
$= 10^6$

One thousandth is $10^6$ times larger.

**Chapter 7 Section 2** **Question 11 Page 369**

**a)** $2^{10} = 1024$ bytes

**b)** $2^{-10} = \frac{1}{1024}$ kilobytes

**c)** $(2^{10})^2 = 2^{20}$
$= 1\ 048\ 576$ bytes

**d)** $(2^{-10})^3 = 2^{-30}$
$= \frac{1}{1\ 073\ 741\ 824}$ gigabytes

**e)** $2^{-3} = \frac{1}{8}$ bytes

**f)** $2^{-40} \times 2^{-3} = 2^{-43}$
$= \frac{1}{8\ 796\ 093\ 022\ 208}$ terabytes

**Chapter 7 Section 2 Question 12 Page 370**

**a)** Normal conversation has an intensity level of 60dB and the threshold of hearing is 0 dB.

The ratio is: $\frac{10^6}{10^0} = 10^6$

$= 1\ 000\ 000$

Therefore, the sound of normal conversation is 1 000 000 times more intense than the threshold of hearing.

**b)** A front row seat at a rock concert has an intensity level of 110 dB and busy street traffic has an intensity level of 70 dB. The ratio is: $\frac{10^{11}}{10^7} = 10^4$

$= 10\ 000$

Therefore the rock concert sound in the front row is 10 000 times as intense as the sound of busy street traffic.

**c)** The human threshold of hearing is 0 dB and that of some dogs is –5 dB.

The ratio for humans to dogs is: $\frac{10^0}{10^{-0.5}} = 10^{0.5}$

$= 3.16$

Therefore the threshold of sound at which dogs can hear is 3.16 times less intense than that at which humans can hear.

**Chapter 7 Section 2 Question 13 Page 370**

**a)** The ratio for intensity of sound in question 12 is $\frac{10^a}{10^b}$.

Each increase in the intensity by a factor of 10 doubles the loudness, so $10^a$ has an increase in loudness of $2^a$.

$\frac{10^a}{10^b}$ has a loudness level of $\frac{2^a}{2^b}$.

**b)** $\frac{2^a}{2^b} = 2^{a-b}$

**c)** Answers may vary. For example:
It would take 10 vacuum cleaners, since it requires an increase by a factor of 10 to double the loudness.

**Chapter 7 Section 2** **Question 14 Page 371**

**a)** Use the formula for *C*. From 1920 to 2007 is 87 years, so $n = 87$; $T = \$150$.

$$\begin{aligned} C &= T(1.0323)^{-n} \\ &= \$150(1.0323)^{-87} \\ &= \$9.44 \end{aligned}$$

The coat would have cost \$9.44 in 1920.

**b)** Use the formula for *C*. From 1970 to 2007 is 37 years, so $n = 37$; $T = \$20\,000$.

$$\begin{aligned} C &= T(1.0323)^{-n} \\ &= \$20\,000(1.0323)^{-37} \\ &= \$6168.94 \end{aligned}$$

The car would have cost \$6168.94 in 1970.

**c)** Use the formula for *C*. From 1962 to 2007 is 45 years, so $n = 45$; $T = \$1.99$.

$$\begin{aligned} C &= T(1.0323)^{-n} \\ &= \$1.99(1.0323)^{-45} \\ &= \$0.48 \end{aligned}$$

The bag of dried fruit snacks would have cost \$0.48 in 1962.

**d)** Use the formula for *C*. From 1980 to 2007 is 27 years, so $n = 27$; $T = \$200\,000$.

$$\begin{aligned} C &= T(1.0323)^{-n} \\ &= \$200\,000(1.0323)^{-27} \\ &= \$84\,775.42 \end{aligned}$$

The condominium would have cost \$84 775.42 in 1980.

**Chapter 7 Section 2** **Question 15 Page 371**

**a)** After two years a 5 g amount of orange peel will reduce to:

$$\begin{aligned} 5 \times 10^{-1} \times 10^{-1} &= 5 \times 10^{-2} \\ &= 0.05 \text{ g} \end{aligned}$$

**b)** After 100 years the 16.5 g mass of aluminum will reduce to:

$$\begin{aligned} 16.5 \times 10^{-1} \times 10^{-1} &= 16.5 \times 10^{-2} \\ &= 0.165 \text{ g} \end{aligned}$$

**Chapter 7 Section 2 Question 16 Page 371**

**a)** $\dfrac{128}{1024} = \dfrac{2^7}{2^{10}}$
$= 2^{-3}$

**b)** $\dfrac{243}{6561} = \dfrac{3^5}{3^8}$
$= 3^{-3}$

**c)** $\dfrac{3125}{625} = \dfrac{5^5}{5^4}$
$= 5^1$

**d)** $\dfrac{49}{2401} = \dfrac{7^2}{7^4}$
$= 7^{-2}$

**e)** $\dfrac{11}{1331} = \dfrac{11^1}{11^3}$
$= 11^{-2}$

**f)** $\dfrac{512}{64} = \dfrac{2^9}{2^6}$
$= 2^3$

**g)** $\dfrac{1}{8} \times \dfrac{1}{16} = \dfrac{1}{2^3} \times \dfrac{1}{2^4}$
$= \dfrac{1}{2^7}$
$= 2^{-7}$

**h)** $\dfrac{1}{25} \times \dfrac{1}{125} = \dfrac{1}{5^2} \times \dfrac{1}{5^3}$
$= \dfrac{1}{5^5}$
$= 5^{-5}$

**Chapter 7 Section 2 Question 17 Page 371**

**a)** $y^{-1}z^2 = \dfrac{1}{4^2} \times (4^3)^2$
$= \dfrac{1}{4^2} \times 4^6$
$= 4^4$

**b)** $\dfrac{y}{z} = \dfrac{4^2}{4^3}$
$= 4^{-1}$

**c)** $\dfrac{z^{-4}}{y} = \dfrac{(4^3)^{-4}}{4^2}$
$= \dfrac{4^{-12}}{4^2}$
$= 4^{-14}$

**d)** $y^5z^{-2} = (4^2)^5(4^3)^{-2}$
$= 4^{10} \times 4^{-6}$
$= 4^4$

**e)** $\dfrac{y^{-2}z^{-4}}{y} = \dfrac{(4^2)^{-2}(4^3)^{-4}}{4^2}$
$= \dfrac{4^{-4} \times 4^{-14}}{4^2}$
$= 4^{-18}$

**f)** $\dfrac{yz}{y^2z^{-1}} = \dfrac{4^2 \times 4^3}{(4^2)^2 \times (4^3)^{-1}}$
$= \dfrac{4^7}{4^4 \times 4^{-3}}$
$= \dfrac{4^7}{4}$
$= 4^6$

**Chapter 7 Section 3 Investigate Exponential Relationships**

**Chapter 7 Section 3 Question 1 Page 377**

The ratio between successive terms is 2, which is a constant. Therefore the relation is exponential.

**Chapter 7 Section 3 Question 2 Page 377**

**a)** Answers may vary. For example:
This is a linear relation. The graphed points make a straight line.

**b)** Answers may vary. For example:
This could be an exponential relation. The curve is flat at the one end and increases rapidly.

**c)** Answers may vary. For example:
This is a quadratic relation. The graph is in the shape of a parabola that points downward.

**d)** Answers may vary. For example:
This could be an exponential relation. The graph is flat at one end and decreases rapidly.

**Chapter 7 Section 3 Question 3 Page 377**

**a)**

| Day | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| Number of Grains of Rice | 1 | 2 | 4 | 8 | 16 | 32 | 64 |

**b)** Answers may vary. For example:
Each day the number of grains of rice doubles (1, 2, 4, 8, …), so the ratio between successive terms is 2. Therefore the relation is exponential ($2^0$, $2^1$, $2^2$, $2^3$, …).

**c)** On the sixteenth day the farmer will receive $2^{15}$ = 32 768 grains of rice.

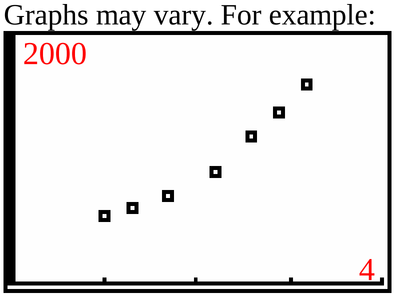
**d)** By the 31st day the king will have to give the farmer $2^{30}$ = 1 073 741 824 grains of rice. Therefore the king will run out of rice before the 64th day when he would need about $10^{300}$ grains of rice.

**Chapter 7 Section 3** **Question 4 Page 378**

**a)** Answers may vary. For example:

| Distance (cm) | AM Radio Frequency (kHz) |
|---|---|
| 1.0 | 540 |
| 1.3 | 600 |
| 1.7 | 700 |
| 2.2 | 900 |
| 2.6 | 1200 |
| 2.9 | 1400 |
| 3.2 | 1600 |

**b)** Graphs may vary. For example:




**c)** Answers may vary. For example:
The graph appears to be exponential.

**Chapter 7 Section 3** **Question 5 Page 378**

**a)** Answers may vary. For example:
The ratio for the first two prizes is \$200 ÷ \$100 = 2.
The ratio for the 2nd and 3rd prize is \$300 ÷ \$200 = 1.5.
The increase is not exponential because the ratios are different.

**b)** Answers may vary. For example:

| | | | | |
|---|---|---|---|---|
| \$62.50 | \$500 | \$4000 | \$32 000 | \$256 000 |
| \$125 | \$1000 | \$8000 | \$64 000 | \$512 000 |
| \$250 | \$2000 | \$16 000 | \$128 000 | \$1 024 000 |

**c)** Answers may vary. For example:
\$1, \$2, \$4, \$10, \$20

**Chapter 7 Section 3 Question 6 Page 378**

**a)** The graph slopes upward with an increasingly steep curve.

**b)** Using extrapolation from the graph, the number of bacteria present at the beginning of the test ($t$ = 0 h) was about 5000.

**c)** Using extrapolation, the estimated number of bacteria after 10 h is about 35 000.

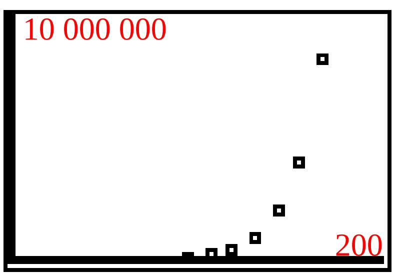
**d)** The trend in the bacterial growth is for a positive exponential increase.

**Chapter 7 Section 3 Question 7 Page 378**

**a)**

| Time (h) | Number of Bacteria (1000s) |
|---|---|
| 12 | 1 000 |
| 24 | 2 000 |
| 36 | 4 000 |
| 48 | 8 000 |
| 60 | 16 000 |
| 72 | 32 000 |
| 84 | 64 000 |
| 96 | 128 000 |
| 108 | 256 000 |
| 120 | 512 000 |
| 132 | 1 024 000 |
| 144 | 2 048 000 |
| 156 | 4 096 000 |
| 168 | 8 192 000 |

**b)**




**c)** From the table in part a) we can see that the number of bacteria will reach 1 024 000 in 132 h, or 5.5 days. This is just over one million.

**Chapter 7 Section 3** **Question 8 Page 380**

**a)** Answers may vary. For example:
The graph shows a steeply declining curve that then levels off.

**b)** The time for the liquid to cool to 30°C can be interpolated from the graph. It is approximately 16 min.

**c)** The graph levelled off at 20°C. This is room temperature.

**Chapter 7 Section 3** **Question 9 Page 380**

**a)** If an image is reduced 90% and that copy again reduced 90%, the size is:

$$\begin{aligned} 90\% \times 90\% &= \frac{9}{10} \times \frac{9}{10} \\ &= \frac{81}{100} \\ &= 81\% \end{aligned}$$

**b)** $(0.9)^2 = 0.81$
$(0.90)^3 = 0.729$
$(0.90)^4 = 0.6561$
$(0.90)^5 = 0.5904$
$(0.90)^6 = 0.5314$
$(0.90)^7 = 0.4783$

The image would have to be reduced 7 times for it to be less than 50% of the original size.

**Chapter 7 Section 3** **Question 10 Page 380**

**a)** Answers may vary. For example:
The curve of best fit has an exponential trend, but it is not a simple exponential curve because the ratios for successive years are not exactly equal. For example,

$$1996 - 1997 \approx \frac{110}{90} = 1.22$$

$$2001 - 2002 \approx \frac{730}{520} = 1.40$$

$$2002 - 2003 \approx \frac{1100}{730} = 1.5$$

**b)** Answers may vary. For example:
By extrapolating from the curve of best fit, the value of high-tech exports in 2004 would be approximately 1600 times US$1 000 000 000. This is US$160 000 000 000 or US$160 billion dollars. By 2010 the value of high-tech exports would be approximately US$1.4 trillion.

**c)** Answers may vary. For example:
It is unlikely that exports of high-tech equipment will continue to grow exponentially. They would probably start to level off. Political factors, tariffs, etc. can impact their value exports.

**Chapter 7 Section 3 Question 11 Page 381**

Solutions for Achievement Checks are shown in the Teacher's Resource.

**Chapter 7 Section 3 Question 12 Page 381**

**a)** Stage 1 consists of a vertical line 1 cm long.
In Stage 2, two 0.5 cm lines are added to the top end of the 1 cm line forming angles of 120°.
In Stage 3, two 0.25 cm lines are added to the ends of the 0.5 cm extensions forming 120° angles, etc.
The number of branches or extensions grows exponentially by doubling at each stage:
$1, 2, 2^2, 2^3, \ldots$

The branches decrease in size exponentially: $1, \frac{1}{2}, \left(\frac{1}{2}\right)^2, \left(\frac{1}{2}\right)^3, \ldots$

**b)** Answers may vary. For example:
Stage 1 consists of one white equilateral triangle.
Stage 2 consists of 4 equilateral triangles 3 of which are white and one quarter the size of the original one.
In Stage 3, each of the 3 white triangles from Stage 2 is broken up into 4 equilateral triangles, $\frac{1}{16}$ the size of the original, there are now 9 white triangles etc.
The number of white triangles triples at each stage so the growth is exponential:
$1, 3, 3^2, 3^3, \ldots$

Their size also decreases exponentially: $1, \frac{1}{4}, \left(\frac{1}{4}\right)^2, \left(\frac{1}{4}\right)^3, \ldots$

**Chapter 7 Section 4** **Exponential Relations**

**Chapter 7 Section 4** **Question 1 Page 390**

**a)** Answers may vary. For example:
The growth is exponential. The graph is like $y = (2.5)^x$ and the ratio of successive terms is about 1.67.

**b)** The growth is linear. This is the graph of $y = x + 1$.

**c)** Answers may vary. For example:
The growth is exponential. This is the graph of $y = –2^x$ and the ratio of successive terms is 2.

**d)** Answers may vary. For example:
The growth is quadratic. The relation is $y = (x + 2)^2 – 1$.

**e)** Answers may vary. For example:
The growth is linear. The relation is $y = –x$.

**Chapter 7 Section 4** **Question 2 Page 390**

**a)** Answers may vary. For example:
Graph C is the graph of $y = 2^x$.
The coordinates (0, 1), (1, 2), and (2, 4) lie on the graph.

**b)** Answers may vary. For example:
Graph A is the graph of $y = 10^x$.
The coordinates (0, 1) and (1, 10) lie on the graph.

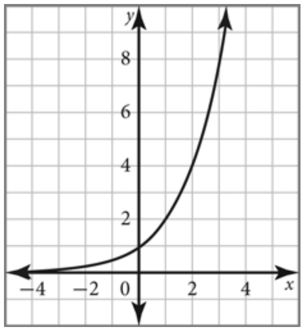
**c)** Answers may vary. For example:
Graph D is the graph of $y = \left(\frac{1}{2}\right)^x$.
The coordinates (0, 1) and (–1, 2) lie on the graph.

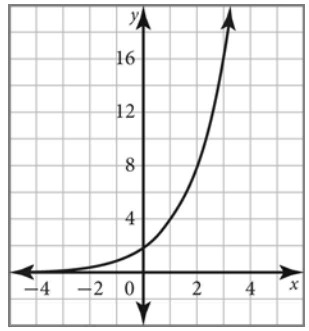
**d)** Answers may vary. For example:
Graph B is the graph of $y = (0.1)^x$.
The coordinates (0, 1) and (–1, 10) lie on the graph.

**Chapter 7 Section 4** **Question 3 Page 391**

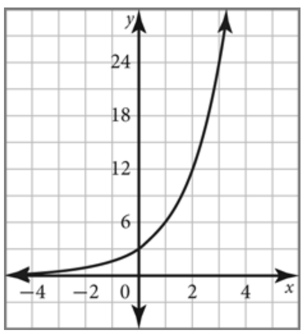
**a)** Graphs may vary. For example:

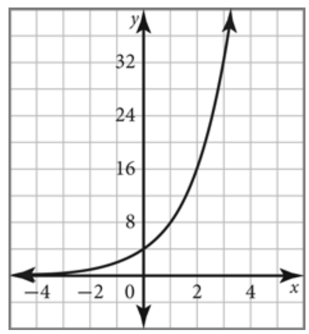
**i)** $y = 2^x$ **ii)** $y = 2(2^x)$




**iii)** $y = 3(2^x)$ **iv)** $y = 4(2^x)$



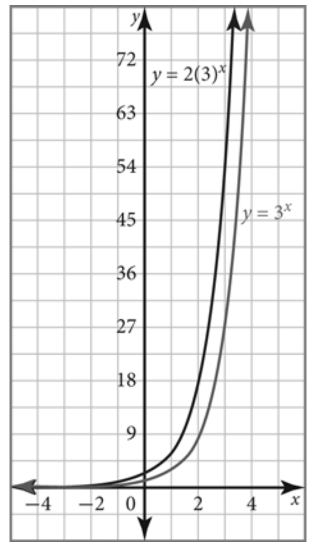




**b)** For $y = a(b^x)$, greater values of $a$ cause the graph to grow faster. If $a$ is positive the values of $y$ are positive; if $a$ is negative the values of $y$ are negative.

**Chapter 7 Section 4** **Question 4 Page 391**

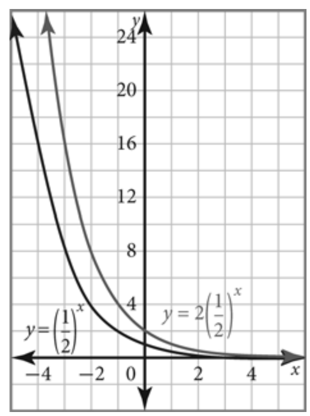
**a)** Answers may vary. For example:

| x | −1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|
| $y = 3^x$ | $\frac{1}{3}$ | 1 | 3 | 9 | 27 |
| $y = 2(3^x)$ | $\frac{2}{3}$ | 2 | 6 | 18 | 54 |



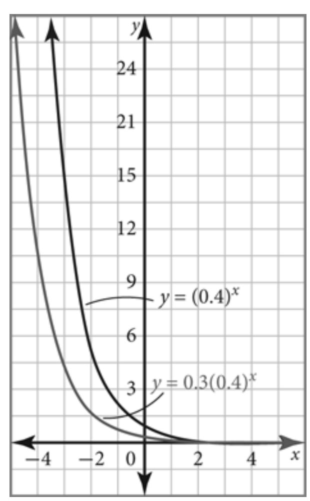

**b)** Answers may vary. For example:

| x | −1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|
| $y = \left(\frac{1}{2}\right)^x$ | 2 | 1 | $\frac{1}{2}$ | $\frac{1}{4}$ | $\frac{1}{8}$ |
| $y = 2\left(\frac{1}{2}\right)^x$ | 4 | 2 | 1 | $\frac{1}{2}$ | $\frac{1}{4}$ |




**c)** Answers may vary. For example:

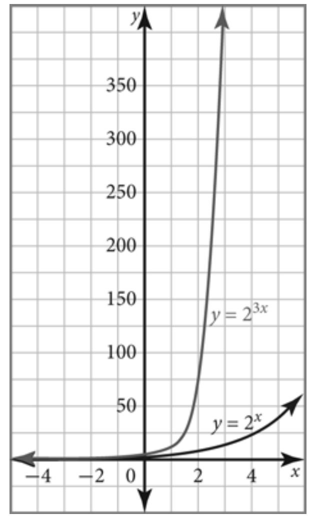
| x | −1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|
| $y = (0.4)^x$ | 2.5 | 1 | 0.4 | 0.16 | 0.064 |
| $y = 0.3(0.4)^x$ | 0.75 | 0.3 | 0.12 | 0.048 | 0.0192 |



**Chapter 7 Section 4 Question 5 Page 391**

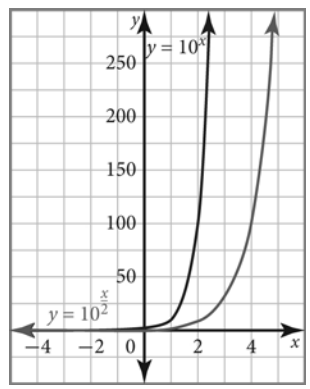
**a)** Answers may vary. For example:

| **x** | −1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|
| $y = 2^x$ | $\frac{1}{2}$ | 1 | 2 | 4 | 8 |
| $y = 2^{3x}$ | $\frac{1}{\ }$ | 1 | 8 | 64 | 512 |



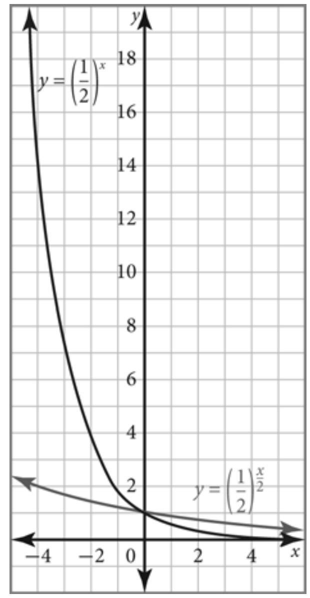

**b)** Answers may vary. For example:

| **x** | −1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|
| $y = 10^x$ | $\frac{1}{10}$ | 1 | 10 | 100 | 1000 |
| $y = 10^{\frac{x}{2}}$ | 0.316 | 1 | 3.16 | 10 | 31.6 |



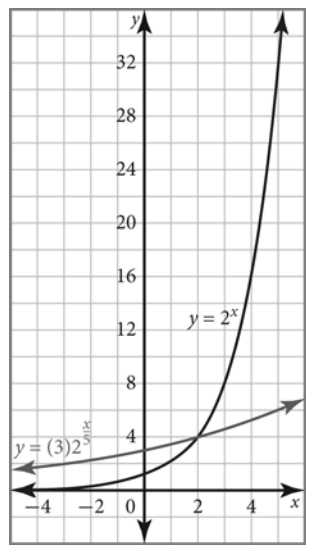
**c)** Answers may vary. For example:

| x | −1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|
| $y = \left(\frac{1}{2}\right)^x$ | 2 | 1 | $\frac{1}{2}$ | $\frac{1}{4}$ | $\frac{1}{8}$ |
| $y = \left(\frac{1}{2}\right)^{\frac{x}{4}}$ | 1.19 | 1 | 0.84 | 0.707 | 0.59 |



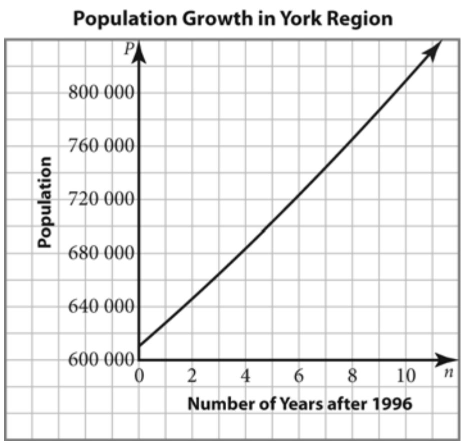

**d)** Answers may vary. For example:

| x | −1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|
| $y = 2^x$ | $\frac{1}{2}$ | 1 | 2 | 4 | 8 |
| $y = (3)2^{\frac{x}{5}}$ | 2.61 | 3 | 3.45 | 3.96 | 4.55 |



**Chapter 7 Section 4** **Question 6 Page 391**

**a)**




**b)** The *P*-intercept is the population of York Region in 1996, when $n = 0$.

$P = 610\ 000(1.029)^0$

$= 610\ 000 \times 1$

$= 610\ 000$

In 1996, the population of York Region was 610 000.

**c)** **i)** For 2015, $n = 19$ (2015 – 1996):

$P = 610\ 000(1.029)^{19}$

$= 610\ 000 \times 1.721440913$

$= 1\ 050\ 079$

The projected population for 2015 is 1 050 079.

**ii)** For 2031, $n = 35$ (2031 – 1996):

$P = 610\ 000(1.029)^{35}$

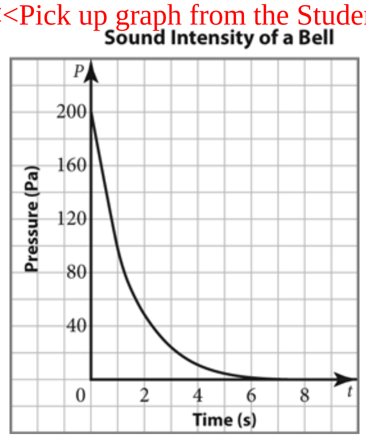
$= 610\ 000 \times 2.719807185$

$= 1\ 659\ 082$

The projected population for 2031 is 1 659 082.

**Chapter 7 Section 4** **Question 7 Page 391**

**a)** <<Pick up graph from the Student Edition Answers No 7 a) P.575>>




**b)** The $P$-intercept, the value of the sound pressure, is 200 Pa at the instant $t = 0$, when the bell started to ring.

**c)** **i)** The sound pressure after 1 s is:

$P = 200\ (0.5)^1$

$= 200 \times (0.5)$

$= 100$ Pa

**ii)** The sound pressure after 2 s is:

$P = 200\ (0.5)^2$

$= 200 \times (0.25)$

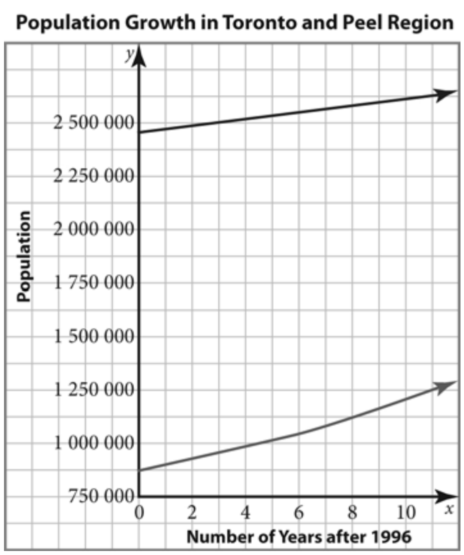
$= 50$ Pa

**Chapter 7 Section 4 Question 8 Page 392**

**a)**

| Number of years after 1996, *n* | Population of Toronto, *P* |
|---|---|
| 1 | 2 473 966 |
| 2 | 2 488 315 |
| 3 | 2 502 747 |
| 4 | 2 517 263 |
| 5 | 2 531 863 |
| 6 | 2 546 548 |
| 7 | 2 561 318 |
| 8 | 2 576 174 |
| 9 | 2 591 116 |
| 10 | 2 606 144 |

| Number of years after 1996, *n* | Population of Peel, *P* |
|---|---|
| 1 | 907 800 |
| 2 | 937 758 |
| 3 | 968 704 |
| 4 | 1 000 671 |
| 5 | 1 033 693 |
| 6 | 1 067 805 |
| 7 | 1 103 043 |
| 8 | 1 139 443 |
| 9 | 1 177 045 |
| 10 | 1 215 887 |




**b)** The growth rate for Toronto is 1.0058 and that for Peel Region is 1.033. Since the Peel rate is higher, the population will increase faster. This is shown by the steeper slope of the graph representing the Peel data.

**Chapter 7 Section 4 Question 9 Page 392**

**a)** This is a case for an exponential decay model. The rate of decrease is $\frac{1}{4}$ the speed for each elapsed second.

**b)** This is a case for a quadratic model. The stone accelerates under gravity, which is a quadratic expression.

**c)** This is a linear model since the increase each second, 4 km/h, is constant.

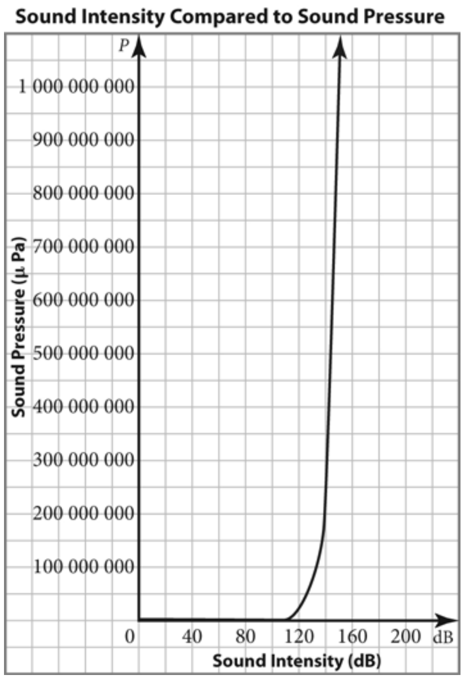
**d)** This is an exponential growth model since the growth of the bacteria doubles every 3 h**.** Therefore the ratio between successive 3 h time periods is 2.

**e)** This is a quadratic model since the ball moves under the force of gravity, which gives a quadratic expression.

**f)** This is an exponential decay model since the height the ball bounces decreases by the same percent for each bounce.

**Chapter 7 Section 4** **Question 10 Page 392**

**a)**




**b)** Normal conversation sound measures 60dB.

This gives $P = 20 \times 10^{\frac{60}{20}}$

$= 20 \times 10^3$

$= 20\ 000$ µPa

A rock concert sound measures 120dB.

This gives $P = 20 \times 10^{\frac{120}{20}}$

$= 20 \times 10^6$

$= 20\ 000\ 000$ µPa

The sound pressure for the rock concert is 1000 times greater.

**c)** Ear drums perforate at 160 dB.

This gives $P = 20 \times 10^{\frac{160}{20}}$

$= 20 \times 10^8$

$= 2\ 000\ 000\ 000$ µPa

**Chapter 7 Section 4 Question 11 Page 392**

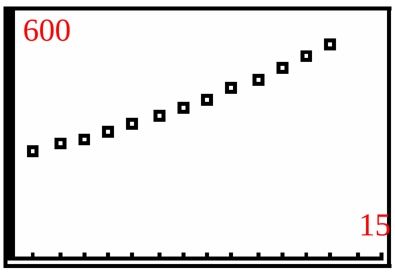
**a)** Answers may vary. For example:

The ratio of increase between C and C sharp is $\frac{277.2}{261.6} = 1.06$.

The ratio for C sharp and D is $\frac{293.7}{277.2} = 1.06$.

The ratio for the other successive notes is also 1.06.
Therefore the relationship can be modelled with exponential growth.
In Example 4, the frequencies of "A" notes, which are an octave apart, were graphed.
This was also modelled with exponential growth.

**b)**




The graph of the relation increases faster than a linear relation.

**Chapter 7 Section 4 Question 12 Page 393**

**a)** Answers may vary. For example:
The population of Canada in 1861 was about 3 million and the population in 1871 was about 3.2 million, so the annual rate change was $\frac{3.2}{3.0} = 1.067$.

**b)** Answers may vary. For example:
The population of Canada in 1951 was about 14.5 million and the population in 1961 was about 19 million so the annual rate change was $\frac{19}{14.5} = 1.310$.

**c)** Answers may vary. For example:
The population in 1991 was about 29.5 million and the population in 2001 was about 32.6 million so the annual rate change was $\frac{32.6}{29.5} = 1.105$.

**d)** Answers may vary. For example:
The growth is approximately exponential as the growth rate varies between 1.067 and 1.310.

**e)** Answers may vary. For example:
The doubling time was from 1861 until about 1906, a period of about 45 years.

**f)** Answers may vary. For example:
Between 1861 and 1906 (i.e., 45 years), the population doubled from about 3.5 to 7 million, and from 1956 and 2001 (i.e., 45 years) the population just about doubled from about 16–17 million to about 33 million. So for the population to almost double again it should take about another 45 years, that is, until 2046.

**Chapter 7 Section 4 Question 13 Page 393**

**a)** Start with a square and add one to each new diagram. This is a linear pattern.

Diagram 4

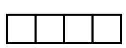
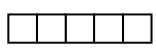


Diagram 5



**b)** Start two vertical squares and add a pair to each new diagram. This is a linear pattern.

Diagram 4

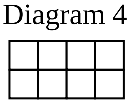
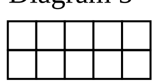


Diagram 5



**c)** To get the next diagram, add a row to one side of the triangle. This is a quadratic relation.

Diagram 4

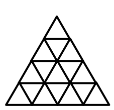
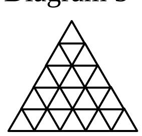


Diagram 5



**Chapter 7 Section 4 Question 14 Page 394**

**a)** The decrease in temperature in degrees Celsius from 0 m to 1000 m is 5.16 – 15.00 = –9.84.
This decrease is the same for each 1000 m increase in height.
The pressure at 0 m is 10.13 Pa and at 1000 m it is 8.99 Pa.

$$\frac{8.99}{10.13} = 0.8875 \times 100\%$$
$$= 88.75\%$$

This means that the pressure has declined by about 11.25%.

Between 1000 m and 2000 m the ratio is $\frac{7.95}{8.99} = 0.8843 \times 100\%$
$$= 88.43\%$$

So the pressure decrease is about 11.57%.
For each of the successive 1000 m increases in height, the ratios are:
0.8818, 0.8787, 0.8766, 0.8722, 0.8726, 0.8637

Therefore all the decreases are between about 12% and 13%.

**b)** For temperature the pattern is linear since the temperature decreases by the same amount, 9.84°C for each 1000 m.
For pressure, the decrease is almost exponential since the ratios of successive pressures are about 0.88.

**c)** As the height above the earth increases, the air becomes thinner, so there are fewer molecules per cubic metre. This means that the air is under less pressure and, consequently, cooler.

**Chapter 7 Section 4 Question 15 Page 394**

**a)**
$$y = 52(0.5)(0.5)^{x-1}$$
$$= 52 \times (0.5)^1 \times (0.5)^{x-1}$$
$$= 52 \times (0.5)^{1+(x-1)}$$
$$= 52(0.5)^x$$

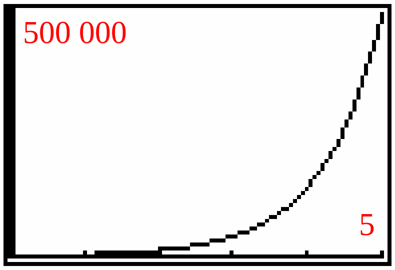
Write $y = 52(0.5)^x$ as $y = 52(0.5)(0.5)^{x-1}$.

**b)** Rewrite $y = 52(0.5)(0.5)^{x-1}$ as
$$y = (52 \times 0.5)(0.5)^{x-1}$$
$$= 26(0.5)^{x-1}$$

**Chapter 7 Section 5 Modelling Exponential Growth and Decay**

**Chapter 7 Section 5 Question 1 Page 401**

**a)**



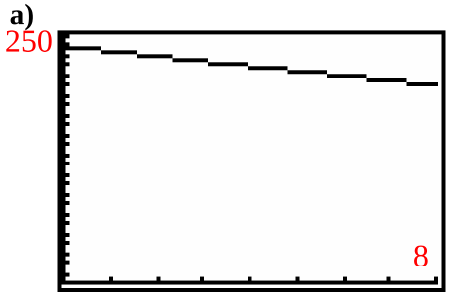

**b)** At the beginning of the growth $d = 0$, so there are $N = 1000(3.45)^0$
$= 1000(1)$
$= 1000$ cells.

**c)** After one day there will be $N = 1000(3.45)^1$
$= 1000(3.45)$
$= 3450$ cells.

**d)** After 5 days there will be $N = 1000(3.45)^5$
$= 1000 \times 488.75980$
$= 488\ 760$ cells.

**Chapter 7 Section 5 Question 2 Page 401**

**a)**




**b)** The current deer population is $P = 240(0.978)^0$
$= 240 \times 1$
$= 240$ deer.

**c)** The deer population after 8 years is expected to be $P = 240(0.978)^8$
$= 240 \times 0.83697$
$= 201$ deer.

**Chapter 7 Section 5** **Question 3 Page 402**

**a)** The amount of caffeine remaining in the person's body after 1 h is $\frac{113.1}{130} \times 100\% = 87\%$.

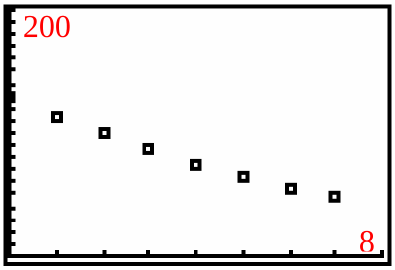
So 13% has been eliminated.

At the end of the second hour, we have $\frac{98.4}{113.1} \times 100\% = 87\%$.

So 13% has been eliminated from the person's body in the second hour.

Similarly for each additional hour 13% of the caffeine will be eliminated.

**b)**




**c)** After 18 h, there is $130 \times (0.87)^{18} = 10.6$ mg of caffeine remaining in the person's body.
After 19 h, there is 9.2 mg remaining.
It takes 19 h for there to be less than 10 mg.

**d)** Answers may vary. For example:
After 50 h there is $130 \times (0.875)^{8} = 0.1$ mg of caffeine remaining in the person's body.
So, it takes about 50 h for all the caffeine to be eliminated.

**Chapter 7 Section 5** **Question 4 Page 402**

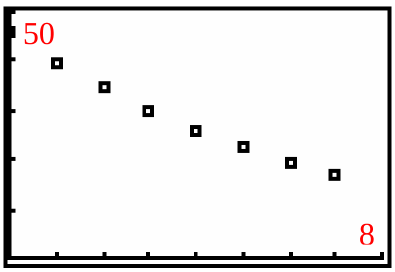
**a) i)**

| Time (h) | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| Mass of Caffeine (mg) | 45.6 | 39.7 | 34.5 | 30.0 | 26.1 | 22.7 | 19.8 | 17.2 |

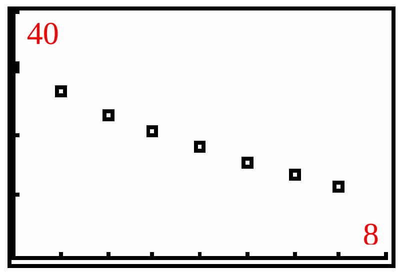
**ii)**

| Time (h) | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| Mass of Caffeine (mg) | 31.0 | 27.0 | 23.5 | 20.4 | 17.8 | 15.5 | 13.4 | 11.7 |

**b) i)**




**ii)**




**c)** Can of Cola: After 10 h the amount of caffeine will be reduced to
$45.6 \times (0.87)^{10} = 11.4$ mg.
Similarly, after 11 h the amount will be reduced to 9.9 mg.
It will take 11 h for the amount of caffeine in the person's body to be less than 10 mg.

Chocolate Bar: After 8 h the amount of caffeine will be reduced to
$31 \times (0.87)^{8} = 10.2$ mg.
Similarly, after 9 h the amount will be reduced to 8.9 mg.
It will take 9 h for the amount of caffeine in the person's body to be less than 10 mg.

**d)** Answers may vary. For example:
If Sandhya eats a chocolate bar, how long will it take for the caffeine to leave her body?
(Answer: about 41 h; $31 \times (0.87)^{41} = 0.1$ mg)

**Chapter 7 Section 5 Question 5 Page 402**

**a)** By interpolating from the graph, the temperature fell to 18°C after approximately 1.25 min.

**b)** Answers may vary. For example:
By interpolating from the graph, the temperature fell to 16°C after approximately 1.9 min.

**c)** Answers may vary. For example:
Yes, it is possible to extrapolate past 5 min. The cooling will continue gradually but the curve will tend to level off.

**d)** Answers may vary. For example:
Use the graph to estimate the temperature of the water after 6 min.
(Answer: about 8°C)

**Chapter 7 Section 5 Question 6 Page 403**

**a)** Yes. The results are close to demonstrating exponential decay. The ratio for successive 30 s periods ranges from 0.76–0.81.

**b)** Answers may vary. For example:
Since the ratios between successive amplitudes average about 0.78, the approximate rate of decay is 0.78 or 78%. That is, the amplitude reduces by 22% for each 30 s time period.

**c)** After 330 s, the amplitude would be about $0.7 \times 0.78 = 0.546$.
After 360 s, it would be $0.546 \times 0.78 = 0.426$.
The amplitude will become indiscernible between 330 s and 360 s.

**Chapter 7 Section 5** **Question 7 Page 403**

**a)** The level of $CO_2$ in the atmosphere is gradually increasing over time. From 1959 to 2002, it has increased by $\frac{(374.86 - 317.71)}{317.71} \times 100\% = 18\%$ .

**b)** The total increase from 1961 to 1962 was 320.58 – 319.48 = 1.1 ppm.
Percent increase: $\frac{1.1}{319.48} \times 100\% = 0.34\%$

The total increase from 1981 to 1982 was 343.56 – 342.51 = 1.05 ppm.
Percent increase: $\frac{1.05}{342.51} \times 100\% = 0.31\%$

The total increase from 2001 to 2002 was 374.86 – 372.87 = 1.99 ppm.
Percent increase: $\frac{1.99}{372.87} \times 100\% = 0.53\%$

**c)** 1960s:
Total increase: 326.66 – 319.03 = 7.63 ppm
Percent increase: $\frac{7.63}{319.03} \times 100\% = 2.39\%$

1970s:
Total increase: 338.89 – 328.13 = 10.76 ppm
Percent increase: $\frac{10.76}{328.13} \times 100\% = 3.28\%$

1980s:
Total increase: 355.42– 340.77 = 14.65 ppm
Percent increase: $\frac{14.65}{340.77} \times 100\% = 4.30\%$

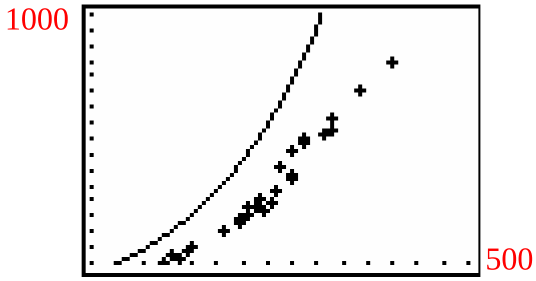
1990s:
Total increase: 371.14– 356.20 = 14.94 ppm
Percent Increase: $\frac{14.94}{356.20} \times 100\% = 4.19\%$

**d)** Answers may vary. For example:
The growth by decade gives a better sense of the exponential growth because of the longer time frame. In some cases annual results may decrease or stay the same, while the increase over the decade may be exponential.

**e)** Answers may vary. For example:
By what percent did the level of $CO_2$ increase between 1959 and 2002?
(Answer: 18%)

**Chapter 7 Section 5** **Question 8 Page 404**

**a)**




**b)** Answers may vary. For example:
No. The exponential curve of best fit does not closely match the data points.

**c)** Since the largest fish recorded is 438 mm long and has a mass of 840 g, we would expect a fish of 450 mm to have a mass of about 1000 g or 1 kg.
Using the **ExpReg** feature gives a value of $23.39 \times (1.008)^{450} = 1055$ g.

**d)** Answers may vary. For example:
Researchers could possibly compare fish sizes in different rivers for information on pollution and the health of the rivers.

**Chapter 7 Section 5** **Question 9 Page 404**

**a), b), c), d)** Answers may vary.

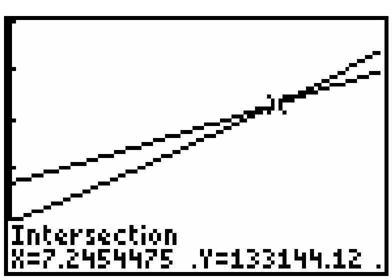
**Chapter 7 Section 5** **Question 10 Page 405**

Solutions for Achievement Checks are shown in the Teacher's Resource.

**Chapter 7 Section 5** **Question 11 Page 405**

After 7 years the population of Metropolis will be about $117\,000(1.018)^7 = 132\,562$.
After 7 years the population of Gotham will be about $109\,000(1.028)^7 = 132\,245$.
The populations will be the same in just over 7 years.

At 7.25 years the population of Metropolis will be $171\,000(1.018)^{7.25} = 133\,155$.
The population of Gotham will be $109\,000(1.028)^{7.25} = 133\,161$.



**Chapter 7 Section 5 Question 12 Page 405**

**a)** An exponential relation that models the world's population is:
$P = 4\ 500\ 000\ 000(1.02)^t$
Where $P$ is the population and $t$ is the time in years after 1980.
Since the rate increases at 2% a year the growth is 1.02 annually.

**b)** The world's population in 2015 is estimated as:
$P = 4\ 500\ 000\ 000(1.2)^{35}$
$\approx$ 9 000 000 000, or 9 billion

**c)** Assuming that the increase from 1970 to 1980 was 2% annual growth, the population in 1970 would have been:
$P = 4\ 500\ 000\ 000(0.98)^{10}$
= 3 676 827 631, or about 3.7 billion

## Chapter 7 Section 6 Solve Problems Involving Exponential Growth and Decay

### Chapter 7 Section 6 Question 1 Page 410

**a)** Assume there are $N_0$ *E. coli* bacteria to begin with.
When you double the number of bacteria, $N = 2N_0$.

$$2N_0 = N_0 \times 2^{\frac{t}{20}}$$
$$2 = 2^{\frac{t}{20}}$$
$$2^1 = 2^{\frac{t}{20}}$$
$$1 = \frac{t}{20}$$
$$t = 20$$

The doubling time for *E. coli* bacteria is 20 min.

**b)**
$$\begin{aligned} N &= 5000 \times 2^{\frac{60}{20}} \\ &= 5000 \times 2^3 \\ &= 5000 \times 8 \\ &= 40\ 000 \end{aligned}$$

After 1 h there are 40 000 *E. coli* bacteria.

**c)** 1 day = 24 h = 24 × 60 min

$$\begin{aligned} N &= 1000 \times 2^{\frac{24 \times 60}{20}} \\ &= 1000 \times 2^{72} \\ &= 1000 \times 4.7 \times 10^{21} \\ &= 4.7 \times 10^{24} \end{aligned}$$

After one day there are $4.7 \times 10^{24}$ *E. coli* bacteria.

### Chapter 7 Section 6 Question 2 Page 410

**a)** The intensity of light with 0 gels is $I = 1200\left(\frac{4}{5}\right)^0 = 1200$ W/cm$^2$.

**b)** The intensity of light with 1 gel is $I = 1200\left(\frac{4}{5}\right) = 960$ W/cm$^2$.

**c)** The intensity of light with 3 gels is $I = 1200\left(\frac{4}{5}\right)^3 = 614.4$ W/cm$^2$.

**d)** The intensity of light with 5 gels is $I = 1200\left(\frac{4}{5}\right)^5 = 393.2$ W/cm$^2$.

**Chapter 7 Section 6 Question 3 Page 411**

**a)** If $t$ is 1 day, then $W = W_0\left(\frac{1}{2}\right)^{0.36t}$

$$= 25\left(\frac{1}{2}\right)^{0.36(1)}$$
$$= 25 \times 0.779$$
$$= 19.5$$

After 1 day, the area of the wound will be 19.5 $mm^2$.

**b)** If $t$ is 4 days, then $W = W_0\left(\frac{1}{2}\right)^{0.36t}$

$$= 25\left(\frac{1}{2}\right)^{0.36(4)}$$
$$= 25 \times 0.369$$
$$= 9.2$$

After 4 days, the area of the wound will be 9.2 $mm^2$.

**Chapter 7 Section 6 Question 4 Page 411**

**a)** If $n$ = 11 460 years, then $C = \left(\frac{1}{2}\right)^{\frac{n}{5730}}$

$$= \left(\frac{1}{2}\right)^{\frac{11\ 460}{5730}}$$
$$= 0.25 \text{ ppt}$$

An 11 460-year-old animal bone contains 0.25 parts per trillion of carbon-14.

**b)** If $n$ = 5000 years, then $C = \left(\frac{1}{2}\right)^{\frac{n}{5730}}$

$$= \left(\frac{1}{2}\right)^{\frac{5000}{5730}}$$
$$= 0.55 \text{ ppt}$$

A 5000-year-old map contains 0.55 parts per trillion of carbon-14.

**c)** If $n$ = 25 000 years, then $C = \left(\frac{1}{2}\right)^{\frac{n}{5730}}$

$$= \left(\frac{1}{2}\right)^{\frac{25\ 000}{5730}}$$
$$= 0.05 \text{ ppt}$$

A 25 000-year-old fossil contains 0.05 parts per trillion of carbon-14.

**Chapter 7 Section 6 Question 5 Page 411**

If the map was made in 1427, $n$ = (2007 – 1427) = 580 years.

$$C = \left(\frac{1}{2}\right)^{\frac{n}{5730}}$$

$$= \left(\frac{1}{2}\right)^{\frac{580}{5730}}$$

$$= 0.93 \text{ ppt}$$

The map contains 0.93 parts per trillion of carbon-14.

**Chapter 7 Section 6 Question 6 Page 411**

**a)** If $t = 0$, then $T = 190\left(\frac{1}{2}\right)^{0} = 190$

To find the freshness half-life of milk, let $T = \frac{190}{2}$.

$$\frac{190}{2} = 190\left(\frac{1}{2}\right)^{\frac{t}{10}}$$

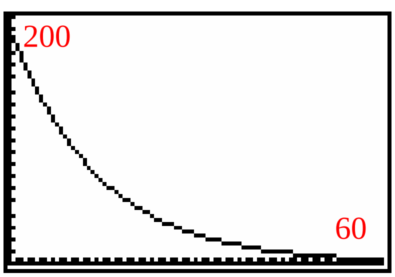
$$190\left(\frac{1}{2}\right)^{1} = 190\left(\frac{1}{2}\right)^{\frac{t}{10}}$$

$$\frac{t}{10} = 1$$

$$t = 10 \text{ h}$$

The freshness half-life of milk is 10 h.

**b)**



**c)** If $T$ = 22°C, then $22 = 190\left(\frac{1}{2}\right)^{\frac{t}{10}}$

So $\frac{22}{190} = \left(\frac{1}{2}\right)^{\frac{t}{10}} = 0.116$

$\left(\frac{1}{2}\right)^{\frac{30}{10}} = \left(\frac{1}{2}\right)^{3} = 0.125$

$\left(\frac{1}{2}\right)^{\frac{31}{10}} = 0.117$

So $t = 31$ and the milk at 22°C will be fresh for 31 h.

If $T$ = 4°C, then $4 = 190\left(\frac{1}{2}\right)^{\frac{t}{10}}$

$\frac{4}{190} = \left(\frac{1}{2}\right)^{\frac{t}{10}} = 0.0211$

Use trial and error.

$\left(\frac{1}{2}\right)^{\frac{50}{10}} = \left(\frac{1}{2}\right)^{5} = 0.03125$

$\left(\frac{1}{2}\right)^{\frac{55}{10}} = \left(\frac{1}{2}\right)^{5.5} = 0.022$

$\left(\frac{1}{2}\right)^{\frac{56}{10}} = 0.0206$

So $t = 56$ and the milk at 4°C will remain fresh for 56 h.

**Chapter 7 Section 6** **Question 7 Page 411**

**a)** The initial concentration is $C = C_0\left(\frac{1}{2}\right)^{\frac{t}{4}}$.

To find the half-life:

$C = \frac{C_0}{2}$, so substituting into the equation above,

$$\frac{C_0}{2} = C_0\left(\frac{1}{2}\right)^{\frac{t}{4}}$$

$$\frac{1}{2} = \left(\frac{1}{2}\right)^{\frac{t}{4}}$$

$$\frac{t}{4} = 1$$

$$t = 4$$

The half-life of the drug is 4 h.

**b)** **i)** If $C_0$ = 40 mg/ml and $t$ is 5,

$$C = 40\left(\frac{1}{2}\right)^{\frac{5}{4}} = 16.8$$

After 5 h, the drug's concentration is 16.8 mg/mL.

**ii)** If $t$ is 11.75 h,

$$C = 40\left(\frac{1}{2}\right)^{\frac{11.75}{4}}$$

$$= 5.2$$

After 11.75 h, the drug's concentration is 5.2 mg/mL.

**c)** After 20 h, the drug's concentration will be $C = 40\left(\frac{1}{2}\right)^{\frac{20}{4}}$ = 1.25 mg/ml.

After 25 h, the drug's concentration will be $C = 40\left(\frac{1}{2}\right)^{\frac{25}{4}}$ = 0.53 mg/ml.

So the second dose of the drug will need to be administered after about 25 h.

**Chapter 7 Section 6 Question 8 Page 412**

**a)** Answers may vary. For example:
The half-life is the time taken for the concentration of the drug in the system to reach half the initial level. The statement "every six hours" refers to level of concentration reached after 6 h when another oral dose is needed.

**b)** Answers may vary. For example:
If a person takes a second dose of the drug before the concentration in the bloodstream reaches the point when the next dose is needed, there is a severe danger that the concentration in the bloodstream will be greater than the intended maximum, which is an overdose.

**Chapter 7 Section 6 Question 9 Page 412**

| Sound Source | Intensity Level (dB) | Relative Intensity |
|---|---|---|
| Mosquito buzzing | 40 | $10^{-12} \times 10^{\frac{40}{10}} = 10^{-8}$ <br> $= 0.000\,000\,01$ |
| Rainfall | 50 | $10^{-12} \times 10^{\frac{50}{10}} = 10^{-7}$ <br> $= 0.000\,000\,1$ |
| Quiet alarm clock | 65 | $10^{-12} \times 10^{\frac{65}{10}} = 10^{-5.5}$ <br> $\doteq 0.000\,003$ |
| Loud alarm clock | 80 | $10^{-12} \times 10^{\frac{80}{10}} = 10^{-4}$ <br> $= 0.0001$ |
| Average factory | 90 | $10^{-12} \times 10^{\frac{90}{10}} = 10^{-3}$ <br> $= 0.001$ |
| Large orchestra | 98 | $10^{-12} \times 10^{\frac{98}{10}} = 10^{-2.2}$ <br> $\doteq 0.006$ |
| Car stereo | 125 | $10^{-12} \times 10^{\frac{125}{10}} = 10^{0.5}$ <br> $\doteq 3.16$ |

**Chapter 7 Section 6 Question 10 Page 412**

**a)** Answers may vary. For example:
Collisions per million vehicle kilometres: the number of collisions is divided by the total number of kilometres driven by all vehicles on the road.

**b)** For $s$ = 90 km/h:
$R = 0.534(1.03)^{90}$
$= 7.6$

For $s$ = 120 km/h:
$R = 0.534(1.03)^{120}$
$= 18.5$

**Chapter 7 Section 6** **Question 11 Page 413**

**a), b), c)** Answers may vary.

**Chapter 7 Section 6** **Question 12 Page 413**

**a)** Graphs may vary.

**b), c)** Answers may vary.

**Chapter 7 Section 6** **Question 13 Page 413**

The formula $A = A_0 \times \left(\frac{1}{2}\right)^{\frac{t}{h}}$ models the level of radioactivity, where $A_0$ is the initial level, $h$ is the half-life of the radioactive material, and $t$ is time.

**a)** For iodine-131, $h$ = 8.065 days and $A_0$ = 370 MBq.

After 3 days (March 4 – March 7),

$$A = (370)\left(\frac{1}{2}\right)^{\frac{3}{8.065}}$$
$$= 286 \text{ MBq}$$

After 12 days (March 4 – March 12),

$$A = (370)\left(\frac{1}{2}\right)^{\frac{12}{8.065}}$$
$$= 132 \text{ MBq}$$

**b)** For technetium-99, $h$ = 6.007 h and $A_0$ = 284 MBq.

After 13.17 h,

$$A = (284)\left(\frac{1}{2}\right)^{\frac{13.17}{6.007}}$$
$$= 62 \text{ MBq}$$

**Chapter 7 Section 6** **Question 14 Page 413**

We can use the exponential expression $P = a(b)^t$, where *a* and *b* are constants and *t* is in years, to model the growth. *P* is the population of foxes.

$325 = ab^0$
$a = 325$
$650 = 325(b^{15})$

$$b = \left(\frac{650}{325}\right)^{\frac{1}{15}}$$

$$b = 2^{\frac{1}{15}}$$

The relation is $P = 325\left(2^{\frac{1}{15}}\right)^t$

$$= 325(2)^{\frac{t}{15}}$$

20 years from now, *t* = 35 years

$P = 325(1.043)^{35}$
$= 1638$

## Chapter 7 Review

**Chapter 7 Review Question 1 Page 414**

**a)** $6^2 \times 6^3 = 6^5$
$= 7776$

**b)** $(-2)^2 \times (-2)^4 = (-2)^6$
$= 64$

**c)** $\frac{5^{10}}{5^7} = 5^3$
$= 125$

**d)** $\left(\frac{1}{3}\right)^3 \times \left(\frac{1}{3}\right)^3 = \left(\frac{1}{3}\right)^6$
$= \frac{1}{729}$

**e)** $(10^4)^2 = 10^8$
$= 100\ 000\ 000$

**f)** $[(-7)^2]^2 = (-7)^4$
$= 2401$

**g)** $\frac{3^8}{3^5} = 3^3$
$= 27$

**h)** $\left(-\frac{1}{2}\right)^2 \times \left(-\frac{1}{2}\right)^3 = \left(-\frac{1}{2}\right)^5$
$= -\frac{1}{32}$

**Chapter 7 Review Question 2 Page 414**

$A = s^2$
$= \left(\frac{3}{8}\right)^2$
$= \frac{9}{64}$ in.$^2$

**a)** $7^0 = 1$

**b)** $5^{-1} = \frac{1}{5}$

**c)** $8^{-3} = \frac{1}{8^3}$

$= \frac{1}{512}$

**d)** $\left(\frac{1}{50}\right)^0 = 1$

**e)** $\left(\frac{2}{3}\right)^{-2} = \frac{1}{\left(\frac{2}{3}\right)^2}$

$= \frac{1}{\left(\frac{4}{9}\right)}$

$= \frac{9}{4}$

**f)** $4^{-2} \times 4^5 = 4^3$

$= 64$

**g)** $\frac{7^2}{7^3} = 7^{-1}$

$= \frac{1}{7}$

**h)** $[(-3)^2]^{-1} = (9)^{-1}$

$= \frac{1}{9}$

**i)** $\frac{1}{2^{-3}} = (2^{-3})^{-1}$

$= 2^3$

$= 8$

**j)** $\frac{1}{5^{-1}} = (5^{-1})^{-1}$

$= 5$

**Chapter 7 Review Question 4 Page 414**

Graph a) represents an exponential relationship of the form $y = a(b)^{-x}$, where $a < 0$ (approximately –3) and $b > 1$.

Graph b) represents a quadratic relationship, i.e., $y = a(x - 0)^2 + 2.5$, with vertex at (0, 2.5).
Since the point (2.5, 7.5) is on the graph:

$$7.5 = a(2.5)^2 + 2.5$$
$$5 = a(6.25)$$
$$a = \frac{5}{6.25}$$
$$a = 0.8$$

Graph c) represents a linear relation of the form $y = mx + b$, passing through (0, –2.5) and (2, 1.5).
Since $b = -2.5$,

$$1.5 = m(2) - 2.5$$
$$2m = 4$$
$$m = 2$$

**Chapter 7 Review Question 5 Page 414**

**a)** After 8 h, there will be 2 × 30 000 = 60 000 bacteria.

**b)** After 16 h, there will be 4 × 30 000 = 120 000 bacteria.

**c)** After 4 days or 96 h, there will be $2^{12}$ × 30 000 = 122 880 000 bacteria.

**Chapter 7 Review Question 6 Page 414**

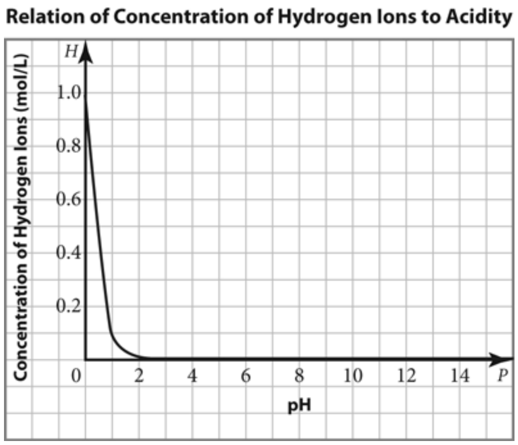
| $x$ | –4 | –3 | –2 | –1 | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|---|---|---|---|
| $y = 3x$ | –12 | –9 | –6 | –3 | 0 | 3 | 6 | 9 | 12 |
| $y = 3x^2$ | 48 | 27 | 12 | 3 | 0 | 3 | 12 | 27 | 48 |
| $y = 3^x$ | $3^{-4}$ | $3^{-3}$ | $3^{-2}$ | $3^{-1}$ | 1 | 3 | $3^2$ | $3^3$ | $3^4$ |

Answers may vary. For example:
The graphs are **similar** in that all three pass through the point (1, 3).

The graphs have **differences:**
$y = 3x$ is linear and has negative values.
$y = 3x^2$ is quadratic and symmetrical about the $y$-axis.
$y = 3^x$ is exponential and increases more rapidly than the other two graphs.

**Chapter 7 Review** **Question 7 Page 414**

**a)**




**b)** If the pH level is 7, then

$$H = \left(\frac{1}{10}\right)^7$$
$$= 10^{-7}$$
$$= 0.000\ 000\ 01$$

**c)** If the pH level is 7.6, then

$$H = \left(\frac{1}{10}\right)^{7.6}$$
$$= 0.000\ 000\ 025$$

**d)** The pH level of rain water is 5.6, so

$$H = \left(\frac{1}{10}\right)^{5.6}$$
$$= 10^{-5.6}$$
$$= 0.000\ 0025$$

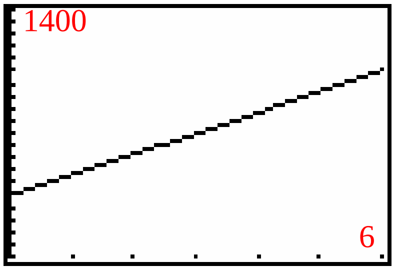
Acid rain has a pH level of 5.0,

$$H = \left(\frac{1}{10}\right)^{5.6}$$
$$= 10^{-5.6}$$
$$= 0.000\ 01$$

**Chapter 7 Review** **Question 8 Page 415**

The formula $P = 1250(1.013)^n$ models the population growth of raccoons, where *n* is in years.

**a)**




**b)** To find the current raccoon population, let *n* = 0. Then,

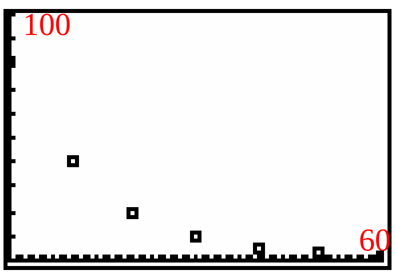
$$\begin{aligned} P &= 1250(1.013)^0 \\ &= 1250 \text{ raccoons} \end{aligned}$$

**c)** The expected population in 5 years is,

$$\begin{aligned} P &= 1250(1.013)^5 \\ &= 1333 \text{ raccoons} \end{aligned}$$

**Chapter 7 Review** **Question 9 Page 415**

**a)**




**b)** Answers may vary. For example:
The experiment demonstrates exponential decay since the amplitude is divided by for each successive 10 s period.

**c)** The rate of decay is $\frac{1}{2}$ (i.e., the half-life of the amplitude is 10 s).

**d)** For the amplitude of the swings to become unnoticeable, we have $80 \times \left(\frac{1}{2}\right)^n$, where *n* is in 10 s periods.

$80 \times \left(\frac{1}{2}\right)^8 = 0.3125$, and $80 \times \left(\frac{1}{2}\right)^9 = 0.15625$

It will take about 90 s for amplitude to be unnoticeable.

**Chapter 7 Review** **Question 10 Page 415**

**a)** The half-life of the drug is the time taken for $M$ to reach half the initial mass of the drug, i.e., 250 mg. Therefore, when $M$ = 250 mg,

$$250 = 500\left(\frac{1}{2}\right)^{\frac{t}{2}}$$

$$\frac{1}{2} = \left(\frac{1}{2}\right)^{\frac{t}{2}}$$

$$\frac{t}{2} = 1$$

$$t = 2$$

Therefore the half-life is 2 h.

**b)** The dosage of the drug was the initial mass of the drug in the system when $t$ = 0.

$$\begin{aligned} M &= 500\left(\frac{1}{2}\right)^{0} \\ &= 500 \times 1 \\ &= 500 \end{aligned}$$

The dosage of the drug was 500 mg.

**c)** **i)**

$$\begin{aligned} M &= 500\left(\frac{1}{2}\right)^{\frac{2}{2}} \\ &= 500 \times \frac{1}{2} \\ &= 250 \end{aligned}$$

After 2 h, $M$ = 250 mg.

**ii)**

$$\begin{aligned} M &= 500\left(\frac{1}{2}\right)^{\frac{6}{2}} \\ &= 500 \times \left(\frac{1}{2}\right)^{3} \\ &= 62.5 \end{aligned}$$

After 6h, $M$ = 62.5 mg.

**Chapter 7 Review** **Question 11 Page 415**

**a)** If $I_0$ = $34 000 in 1994 and $n$ is 10 years,

$$\begin{aligned} I &= \$34\ 000(1.041)^{10} \\ &= \$50\ 814 \end{aligned}$$

**b)** If $I_0$ = $50 000, then after 7 years

$$\begin{aligned} I &= \$50\ 000(1.041)^{7} \\ &= \$66\ 241 \end{aligned}$$

**c)** Since the growth factor is 1.041, the average yearly rate of increase of income is 4.1%.

**Chapter 7 Practice Test**

**Chapter 7 Practice Test** **Question 1 Page 416**

**a)** true

**b)** false

**c)** false

**d)** true

**Chapter 7 Practice Test** **Question 2 Page 416**

**a)** $3^3 \times 3^2 = 3^5$
$= 243$

**b)** $\frac{9^7}{9^5} = 9^2$
$= 81$

**c)** $(2^3)^2 = 2^6$
$= 64$

**d)** $6^0 = 1$

**e)** $7^{-2} = \frac{1}{7^2}$
$= \frac{1}{49}$

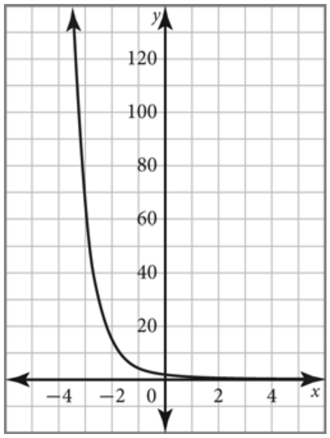
**f)** $\left(\frac{1}{5}\right)^{-3} = (5^{-1})^{-3}$
$= 5^3$
$= 125$

**g)** $4^{12} \times 4^{-3} \times 4^{-9} = 4^{12 + (-3) + (-9)}$
$= 4^0$
$= 1$

**h)** $\left(\frac{1}{3}\right)^2\left(\frac{1}{3}\right)^{-1} = \left(\frac{1}{3}\right)^{2-1}$
$= \frac{1}{3}$

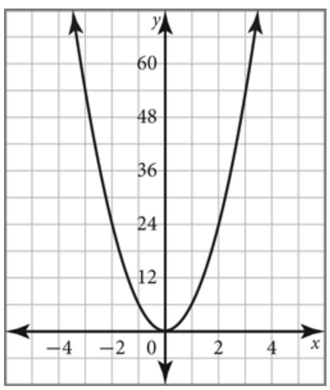
**Chapter 7 Practice Test** **Question 3 Page 416**

**a)** **i)**



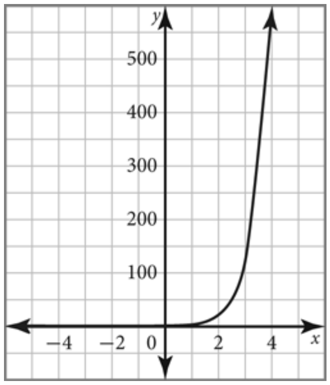

**ii)** The graph shows exponential decay with the relation divided by 4 for each successive *x*-value.

**b)** **i)**



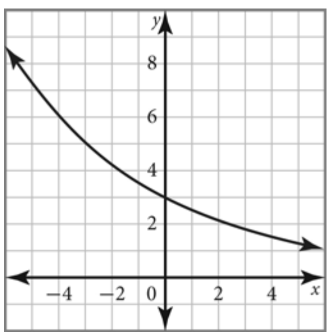

**ii)** The graph is a quadratic relation.
It is not exponential.

**c) i)**




**ii)** The graph shows exponential growth with the relation multiplied by 5 for successive *x*-values.

**d) i)**




**ii)** The graph shows exponential decay with the relation divided by 2 for each successive set of four *x*-values.

**Chapter 7 Practice Test** **Question 4 Page 416**

**a)** There are $3 \times 3 \times 3 = 3^3$ ft$^3$ in a yd$^3$.

**b)**
$$9 \text{ yd}^3 = 3^2 \text{ yd}^3$$
$$= (3^2 \times 3^3) \text{ ft}^3$$
$$= 3^5 \text{ ft}^2$$

**c)**
$$9 \text{ yd}^3 = 3^5 \text{ ft}^3$$
$$= 243 \text{ ft}^3$$

A total of $\frac{243}{3} = 81$ trips will need to be made.

**Chapter 7 Practice Test** **Question 5 Page 416**

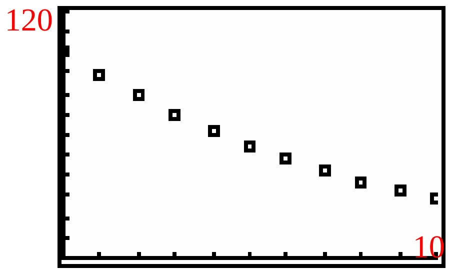
If $b > 0$ then $b^x > 0$ for any value of $x$.
Therefore $y = b^x$ will always be above the $x$-axis.

**Chapter 7 Practice Test** **Question 6 Page 416**

**a)** Answers may vary. For example:
All sketched graphs should approximately pass through the points (0, 2), representing the initial introduction of 2 moose in 1878 and (129, 150 000), representing the current population in 2007.

**b)** The situation is best matched by an exponential growth model (e.g., $y = 2(1.09)^x$).

**a)**




**b)** The equation that models the atmospheric pressure data is $P$ (101.3)(0.833)$^h$, where $P$ is in kiloPascals and $h$ is in kilometres.

**c)** At sea level, $h$ = 0 km,

$$\begin{aligned}(101.3)(0.833)^0 &= 101.3\times 1\\ &= 101.3 \text{ kPa}\end{aligned}$$

**d)** The equation $P = 101.3(0.833)^h$ does not match individual data points very accurately.

For $P$ = 89 kPa, $89 = 101.3(0.833)^h$, which gives $h \approx 0.9$ km.

The data give $h$ = 1.0 km for $P$ = 89.4 kPa.

That is, the data point is above the exponential curve of best fit.

Use ratios to get an approximate value of $h$ for $P$ = 89 kPa.

For $h$ = 1 km, $P$ = 89.4 kPa

For $h$ = 2 km, $P$ = 78.9 kPa

Therefore for the 1 km difference in $h$ there is a 10.5 kPa difference in $P$.

If $x$ is the height difference for 0.4 kPa, then

$$\frac{1.0}{x} = \frac{10.5}{0.4}$$

$$x = \frac{1.0 \times 0.4}{10.5}$$

$$x \approx 0.04$$

Therefore $h \approx$ 1.04 km = 1040 m.

**e)** Similarly, for $h$ = 4250 m the exponential equation is not a close enough match for the data points (4, 61.5) and (5, 54.3). The exponential equation gives the ordered pair (4.25, 46.6).

As in part d), we can use ratio to find an approximate value for $P$.

Let $x$ be the atmospheric pressure difference going from 4000 m to 4250 m, which is 0.25 km.

$$\frac{1.0}{0.25} = \frac{(61.5 - 54.3)}{x}$$

$$\frac{1.0}{0.25} = \frac{7.2}{x}$$

$$\begin{aligned}x &= 7.2 \times 0.25\\ &= 1.8\end{aligned}$$

Therefore for $h = 4250$, $P \approx (61.5 - 1.8)$ kPa = 59.7 kPa