Chapter 4 Quadratic Relations I

Chapter 4 Prerequisite Skills

Chapter 4 Prerequisite Skills Question 1 Page 166

a) 13.1

b) –1.7

c) –7.7

d) –17.1

Chapter 4 Prerequisite Skills Question 2 Page 166

a) –6.5

b) $17.5 - (-8.6)$
$= 17.5 + 8.6$
$= 26.1$

c) –10.5

d) $-10 - (-3.3)$
$= -10 + 3.3$
$= -6.7$

Chapter 4 Prerequisite Skills Question 3 Page 166

a) 41.4

b) –25.2

c) –130

d) 10.65

Chapter 4 Prerequisite Skills Question 4 Page 166

a) $3x + (-5x)$
$= 3x - 5x$
$= -2x$

b) $9x^2 - (-10) + 3x - 2x^2$
$= 9x^2 - 2x^2 + 3x + 10$
$= 7x^2 + 3x + 10$

c) $3x^2 - 4x + 2x + 5x^2$
$= 3x^2 + 5x^2 - 4x + 2x$
$= 8x^2 - 2x$

d) $-5x^2 + 2x - (3x^2 - 2)$
$= -5x^2 + 2x - 3x^2 + 2$
$= -8x^2 + 2x + 2$

Chapter 4 Prerequisite Skills Question 5 Page 166

a) $3(0)^2 = 0$

b) $-9(0)^2 + 6 = 6$

c) $6(0 + 4)^2$
$= 6(4)^2$
$= 96$

d) $-3(0 + 8)^2 - 10$
$= -3(8)^2 - 10$
$= -202$

Chapter 4 Prerequisite Skills Question 6 Page 166

a) $-2(3)^2 = -18$

b) $5(3)^2 + 2 = 47$

c) $-4(3 - 8)^2$
$= -4(25)$
$= -100$

d) $(3 + 9)^2 + 7$
$= (12)^2 + 7$
$= 151$

Chapter 4 Prerequisite Skills Question 7 Page 166

a) $12(-4)^2$
$= 12(16)$
$= 192$

b) $-11(-4)^2 - 7$
$= -11(16) - 7$
$= -176 - 7$
$= -183$

c) $-3(11)^2 = -363$

d) $9(-16)^2 - 20$
$= 9(256) - 20$
$= 2304 - 20$
$= 2284$

Chapter 4 Prerequisite Skills Question 8 Page 166

a)

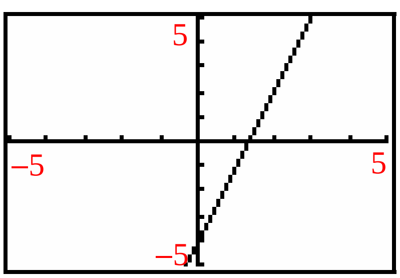
x	*y*
–2	–10
–1	–7
0	–4
1	–1
2	2

b)

x	*y*
8	17
12	15
16	13
20	11
24	9

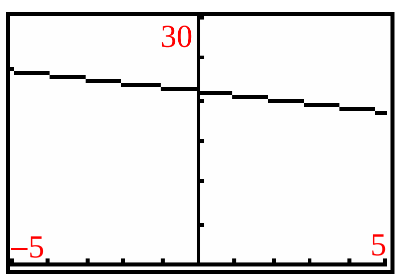
Chapter 4 Prerequisite Skills Question 9 Page 166

a)



slope: 3; *y*-intercept: –4

b)



slope: –0.5; *y*-intercept: 21

Chapter 4 Prerequisite Skills **Question 10 Page 166**

a)

x	*y*	First Differences
3	8	
		7
4	15	
		8
5	23	
		8
6	31	
		8
7	39	

b)

x	*y*	First Differences
13	0	
		1
14	1	
		3
15	4	
		5
16	9	
		7
17	16	

c)

x	*y*	First Differences
–5	3	
		0
–4	3	
		0
–3	3	
		0
–2	3	
		0
–1	3	

Chapter 4 Prerequisite Skills **Question 11 Page 167**

a) slope: $-\frac{1}{2}$; *y*-intercept: 3

b) slope: 5; *y*-intercept: 1

c) slope: 2; *y*-intercept: –4

d) slope: $\frac{2}{5}$; *y*-intercept: $-3\frac{4}{5}$

Chapter 4 Prerequisite Skills **Question 12 Page 167**

a) a translation of 4 units down

b) a translation of 4 units to the right and 3 units up

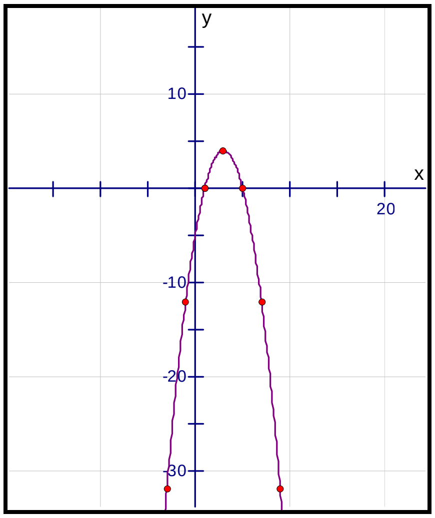
c) a reflection in the vertical line halfway between the pentagons;
or a rotation of 180º about a point halfway between the pentagons

d) a rotation of 180º about the point (–1.5, 0)

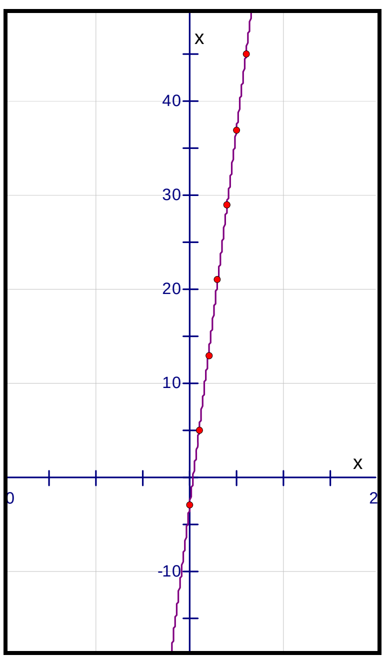
Chapter 4 Section 1 Modelling With Quadratic Equations

Chapter 4 Section 1 Question 1 Page 174

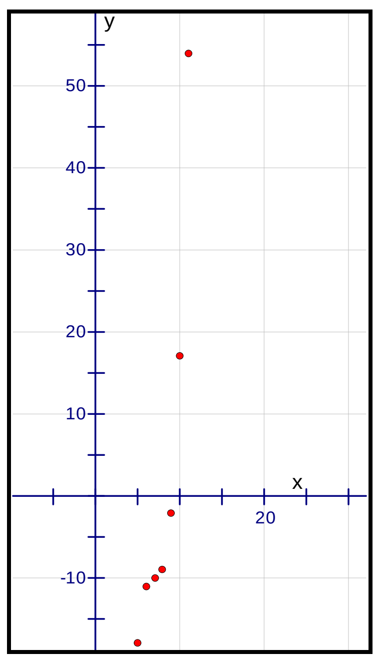
a) quadratic; graph should be similar to:



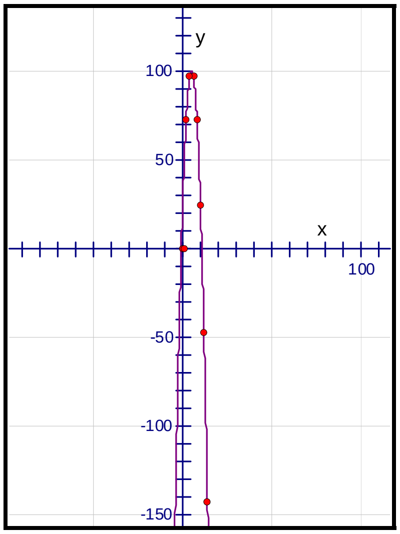
b) linear; graph should be similar to:



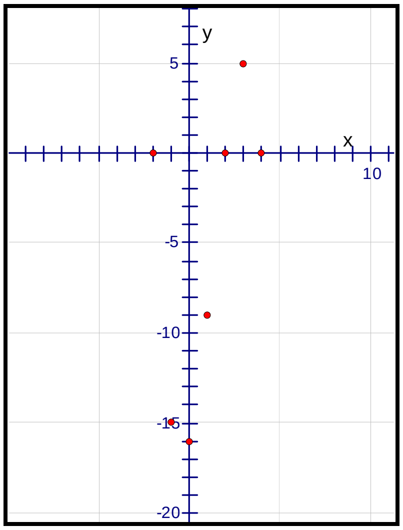
c) neither; graph should be similar to:



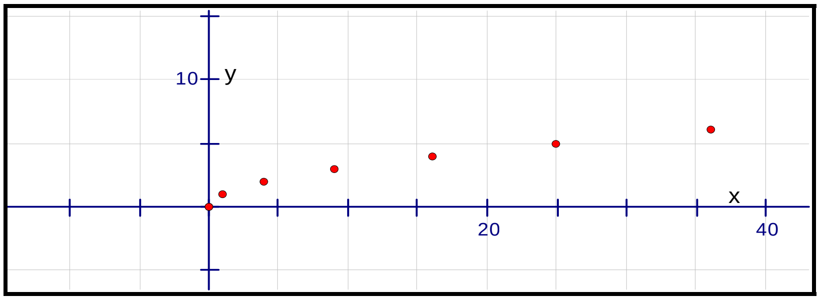
d) quadratic; graph should be similar to:



e) neither; graph should be similar to:



f) neither; graph should be similar to:



Chapter 4 Section 1 **Question 2 Page 174**

a) not quadratic; the first differences are always –9.

x	*y*	First Differences
–30	250	
–29	241	–9
–28	232	–9
–27	223	–9
–26	214	–9
–25	205	–9
–24	196	–9

b) not quadratic; the second differences are not constant.

x	*y*	First Differences	Second Differences
18	0		
20	3	3	
22	4	1	–2
24	4	0	–1
26	0	–4	–4
28	–5	–5	–1
30	–12	–7	–2

c) quadratic; the second differences are always 16.

x	*y*	First Differences	Second Differences
3	128		
6	200	72	
9	288	88	16
12	392	104	16
15	512	120	16
18	648	136	16
21	800	152	16

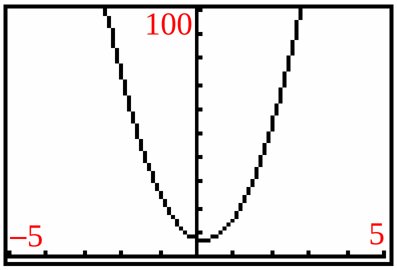
d) not quadratic; the second differences are not constant.

x	*y*	First Differences	Second Differences
1	2		
2	4	2	
3	8	4	2
4	16	8	4
5	32	16	8
6	64	32	16
7	128	64	32

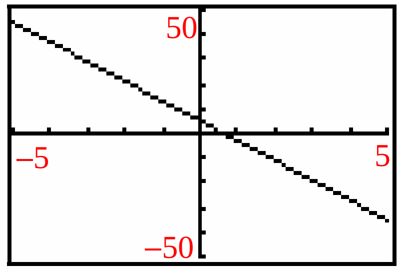
Chapter 4 Section 1 **Question 3 Page 175**

a) i) quadratic; the relation has an x^2-term.
ii) not quadratic; there is no x^2-term.
iii) quadratic; the relation has an x^2-term.
iv) not quadratic; there is no x^2-term.
v) quadratic; the relation has an x^2-term.
vi) not quadratic; there is no x^2-term.

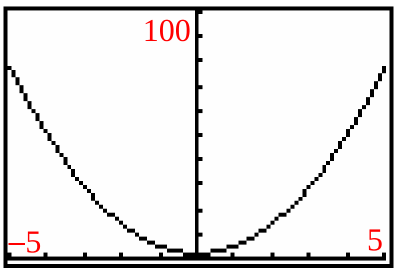
b) i)



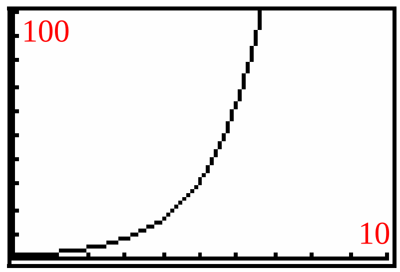
ii)



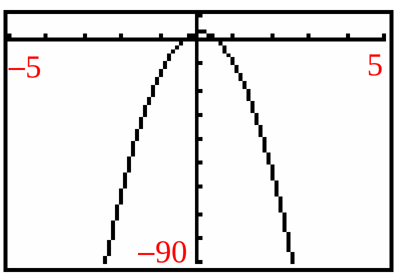
iii)



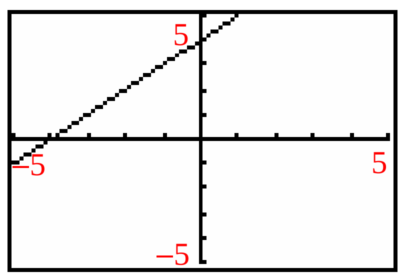
iv)



v)



vi)



Chapter 4 Section 1 **Question 4 Page 175**

Numerical answers may vary.

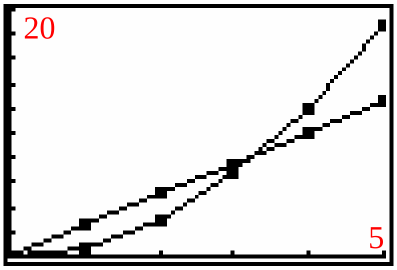
a) minimum: 400 m

b) maximum: 82 m

c) minimum: 0 m

Chapter 4 Section 1 **Question 5 Page 175**

a)



b) Carl's distance-time relationship is quadratic since his graph is a parabola.

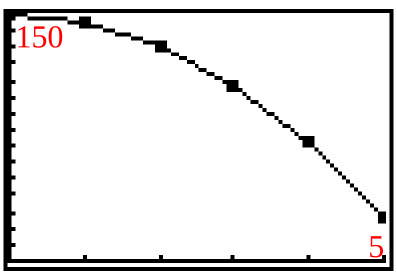
Chapter 4 Section 1 **Question 6 Page 176**

a)

Time (s)	Height (m)
0	150.0
1	145.1
2	130.4
3	105.9
4	71.6
5	27.5

b) Yes, it is quadratic because there is a t^2-term in the relation, and because the second differences in the heights are constant, –9.8.

c)



Chapter 4 Section 1 Question 7 Page 176

a) Use the Y= key to enter the two equations into a graphing calculator.
Then, look at TABLE to find the values for the table of values.

On Earth:

Time (s)	Height (m)
0	0
1	10.1
2	10.4
3	0.9

On the Moon:

Time (s)	Height (m)
0	0
2	26.8
4	47.2
6	61.2
8	68.8
10	70.0
12	64.8
14	53.2
16	35.2
18	10.8

b) Curve A models the motion of the ball on the moon since the ball goes higher and takes longer to land; Curve B models the motion of the ball on Earth.

Chapter 4 Section 1 Question 8 Page 176

a) "Quad–" is from the Latin root *quadri*, which means four.

b) A quadratic relation has a squared variable; the Latin root of the word "quadratic" is *quadratus*, which means made square; and squares have four sides; explanations may vary. See http://mathworld.wolfram.com/Quadratic.html for more information.

Chapter 4 Section 1 **Question 9 Page 177**

a)

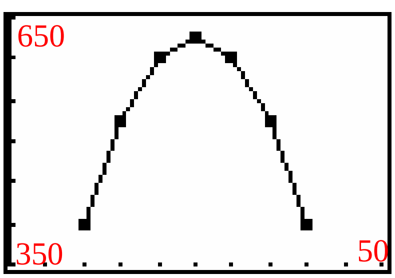
Length (m)	Width (m)	Perimeter (m)
40	10	2(40) + 2(10) = 100
35	15	2(35) + 2(15) = 100
30	20	2(30) + 2(20) = 100
25	25	2(25) + 2(25) = 100
20	30	2(20) + 2(30) = 100
15	35	2(15) + 2(35) = 100
10	40	2(10) + 2(40) = 100

The pen could have the following dimensions:
40 m by 10 m
35 m by 15 m
30 m by 20 m
25 m by 25 m
20 m by 30 m
15 m by 35 m
10 m by 40 m

b)

Length (m)	Width (m)	Perimeter (m)	Area (m^2)
40	10	2(40) + 2(10) = 100	400
35	15	2(35) + 2(15) = 100	525
30	20	2(30) + 2(20) = 100	600
25	25	2(25) + 2(25) = 100	625
20	30	2(20) + 2(30) = 100	600
15	35	2(15) + 2(35) = 100	525
10	40	2(10) + 2(40) = 100	400

c)



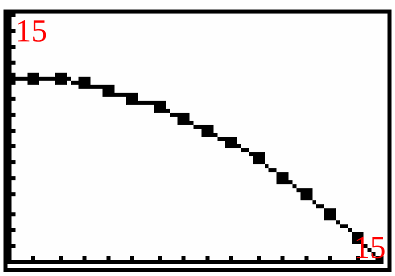
d) From the above graph, the pen with the largest possible area has dimensions 25 m by 25 m.

Chapter 4 Section 1 **Question 10 Page 177**

a) Use the Y= key to enter the equation $y = -0.05x^2 + 11.25$ into a graphing calculator. Then, look at TABLE to find the values for the table of values.

Horizontal Distance from Cliff (m)	Vertical Distance from Base of Cliff (m)
0	11.25
1	11.20
2	11.05
3	10.80
4	10.45
5	10.00
6	9.45
7	8.80
8	8.05
9	7.20
10	6.25
11	5.20
12	4.05
13	2.80
14	1.45
15	0

b)



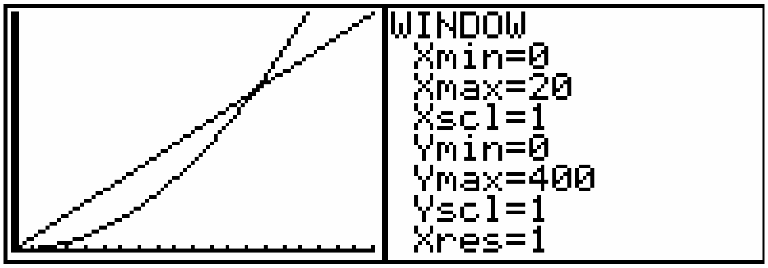
c) The snowboarder will land when the vertical distance *h* is equal to zero. From the table, this occurs when the horizontal distance *d* is 15 m.

Chapter 4 Section 1 **Question 11 Page 178**

Solutions for Achievement Checks are in the *Teacher Resource*.

Chapter 4 Section 1 **Question 12 Page 179**

Use a graphing calculator.
Use the `Y=` key to enter the equations $y = 20x$ and $y = 1.5x^2$.
The graph shown uses the chosen `WINDOW` settings.

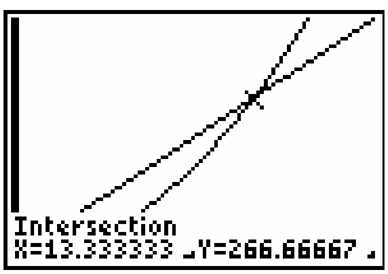


To find where and when the police officer and the speeder meet, examine the `TABLE` of values. The intersecting point lies between $x = 13$ and $x = 14$.

X	Y1	Y2
10	200	150
11	220	181.5
12	240	216
13	260	253.5
14	280	294
15	300	337.5
16	320	384

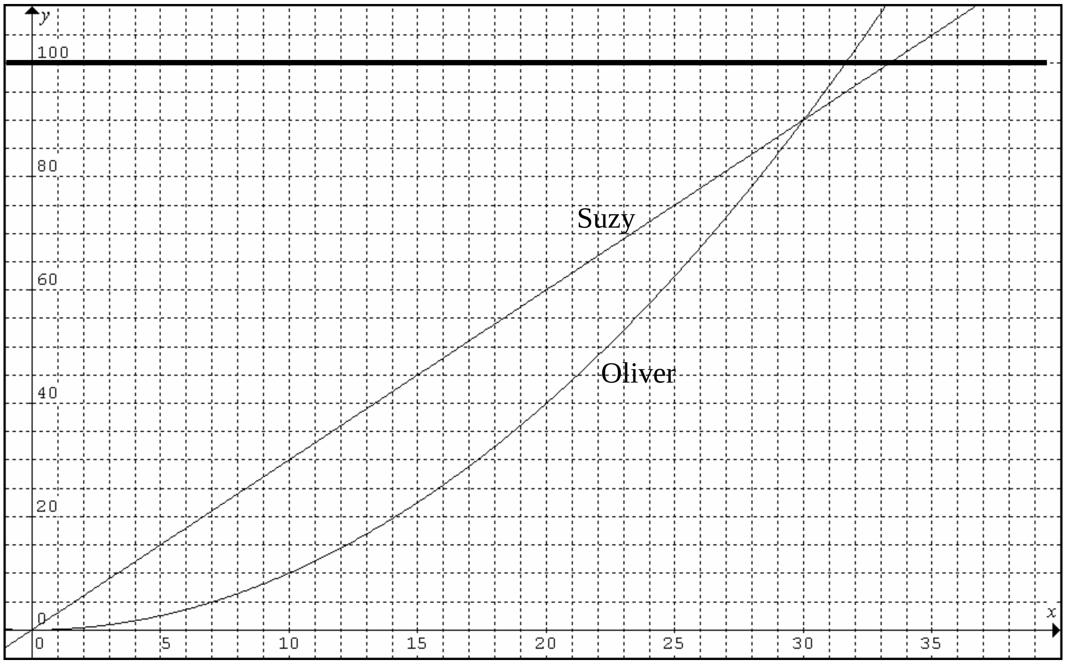
X=13

A more accurate intersecting point can be found using **`CALC`** $\rightarrow$ **`5: intersect`** on the calculator, as shown below. The officer will catch the speeder after 13.3 s, and both of them will be 266.7 m form the point where the officer started.



Chapter 4 Section 1 **Question 13 Page 179**

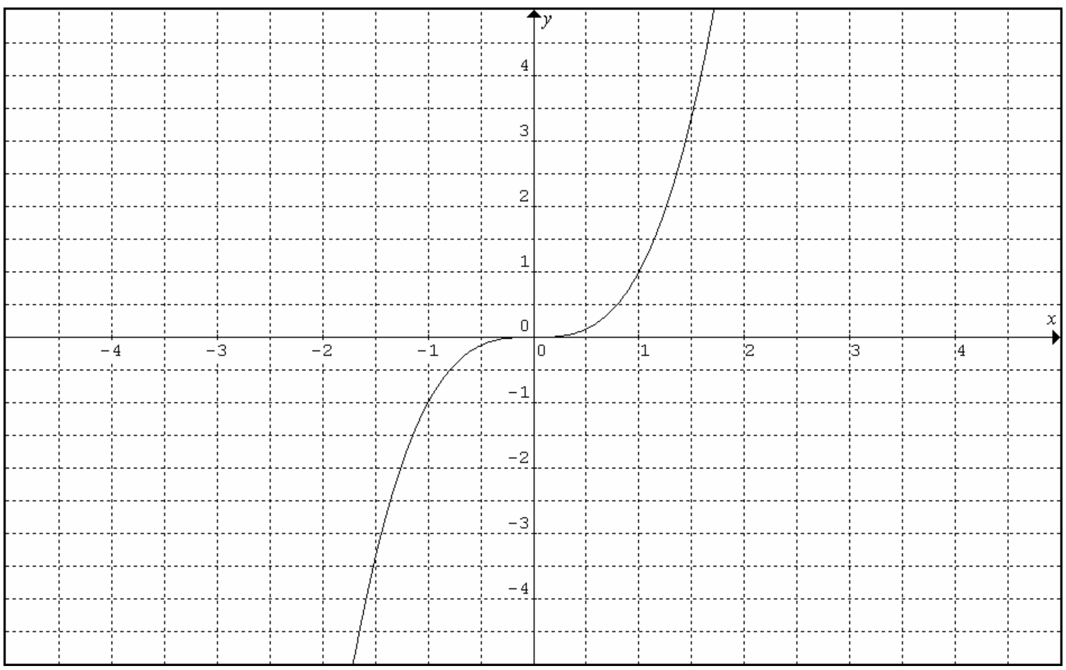
Graph the two relations on the same graph and draw a line for *d* = 100 as shown.
Oliver reaches this line in less time than Suzy; Oliver will win the race.



Chapter 4 Section 1   Question 14 Page 179

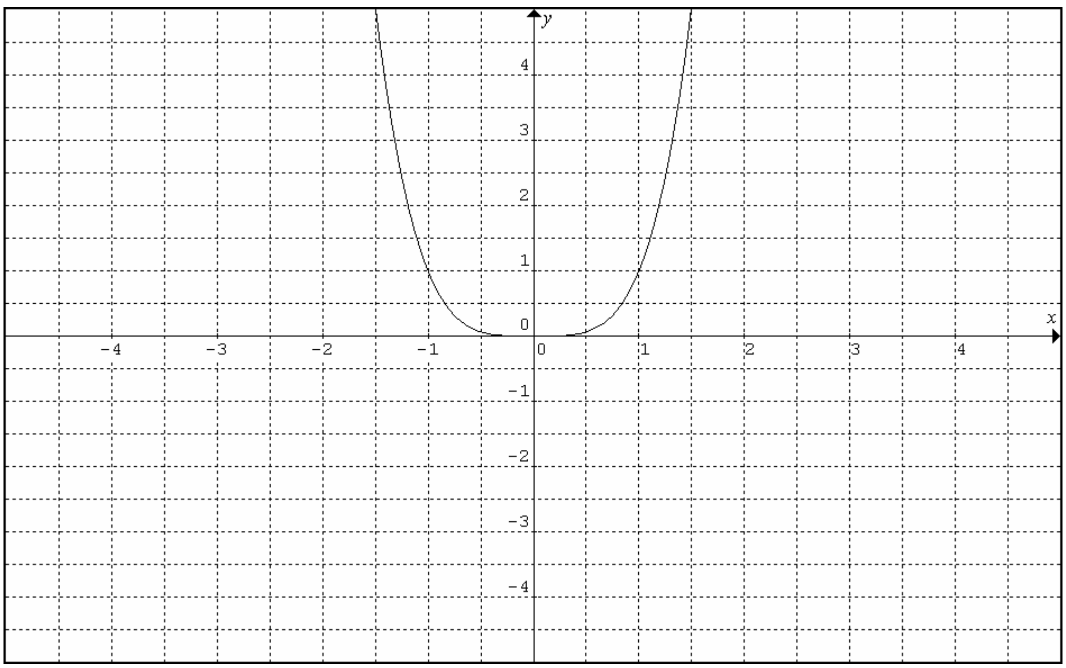
a) Explanations may vary. For example:
No, the relation is not quadratic; the second differences are not constant and the graph is not a parabola.

x	*y*	First Differences	Second Differences
0	0		
		1	
1	1		6
		7	
2	8		12
		19	
3	27		18
		37	
4	64		24
		61	
5	125		30
		91	
6	216		



b) Explanations may vary. For example:
No, the relation is not quadratic; the second differences are not constant and the graph is not a parabola.

x	*y*	First Differences	Second Differences
0	0		
		1	
1	1		14
		15	
2	16		50
		65	
3	81		110
		175	
4	256		194
		369	
5	625		302
		671	
6	1296		



Chapter 4 Section 1 Question 15 Page 179

Answers may vary.
A good place to start research is http://en.wikipedia.org/wiki/Catenary.
Other examples include suspension bridges, free hanging chains, and inverted catenary arches.

Chapter 4 Section 2 The Quadratic Relation $y = ax^2 + k$

Chapter 4 Section 2 Question 1 Page 190

a) $0 < a < 1$; the parabola opens upward and is vertically compressed.

b) $a < –1$; the parabola opens downward and is vertically stretched.

c) $–1 < a < 0$; the parabola opens downward and is vertically compressed.

d) $a > 1$; the parabola opens upward and is vertically stretched.

Chapter 4 Section 2 Question 2 Page 191

a) positive; the parabola is translated upward.

b) negative, the parabola is reflected in the x-axis and translated downward.

c) positive; the parabola is reflected in the x-axis and translated upward.

d) negative; the parabola is translated downward.

Chapter 4 Section 2 Question 3 Page 191

a) $k = 3$; vertex: (0, 3)

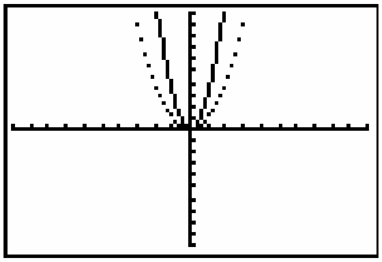
b) $k = –2$; vertex: (0, –2)

c) $k = 5$; vertex: (0, 5)

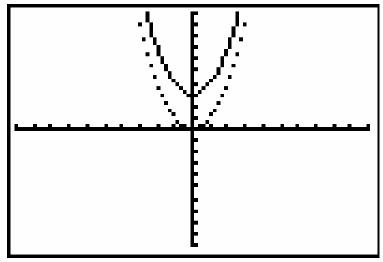
d) $k = –7$; vertex: (0, –7)

Chapter 4 Section 2 Question 4 Page 191

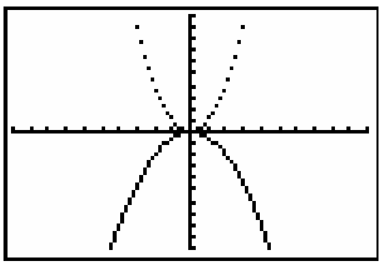
a) stretched vertically by a factor of 3



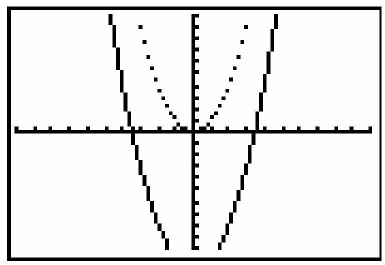
b) translated up by 3 units; has vertex (0, 3)



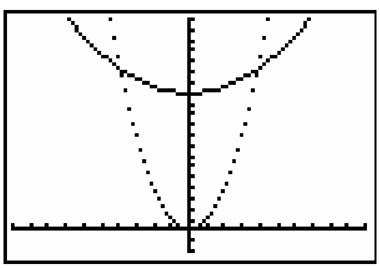
c) reflected in the x-axis; vertically compressed



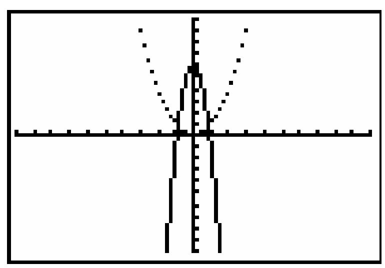
d) translated down by 12 units; has vertex (0, –12)



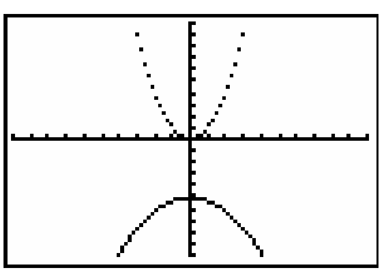
e) vertically compressed; translated up by 13 units; has vertex (0, 13)



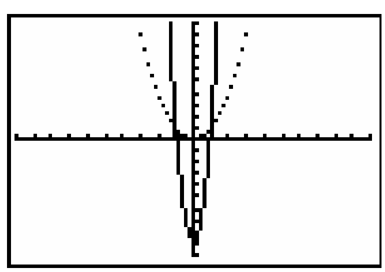
f) reflected in the x-axis, vertically stretched; translated up by 6 units; has vertex (0, 6)



g) reflected in the x-axis; vertically compressed; translated down by 5 units; has vertex (0, –5)



h) vertically stretched; translated down by 9 units; has vertex (0, –9)



Chapter 4 Section 2 Question 5 Page 191

a) Since the parabola is translated up, $k > 0$; since it is vertically compressed, $0 < a < 1$

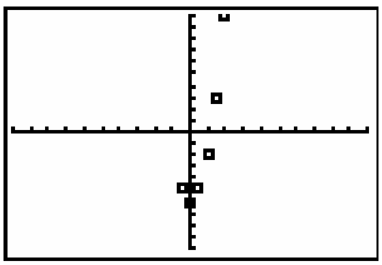
b) Since the parabola is translated down, $k < 0$; since it is vertically stretched, $a > 1$

c) Since the parabola is translated down, $k < 0$; since it is reflected in the x-axis and vertically stretched, $a < –1$

d) Since the parabola is translated up, $k < 0$; since it is reflected in the x-axis and vertically compressed, $–1 < a < 0$

Chapter 4 Section 2 Question 6 Page 192

a)



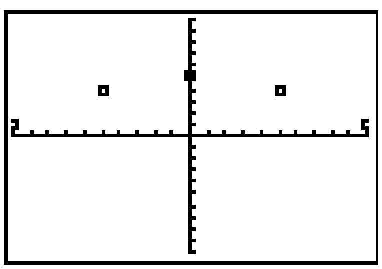
The graph appears to be a parabola, shifted down by 6 units, and with a > 0, since the curve is stretched vertically. The equation is of the form $y = ax^2 – 6$.
Use trial-and-error to find a. Use the point (1, –2) on the parabola.

Try $a = 3$: $3(1)^2 - 6 = -3 \neq -2$

Try $a = 4$: $4(1)^2 - 6 = -2$ [Guess is correct.]

The relation can be represented by the equation $y = 4x^2 - 6$.

b)



The graph appears to be a parabola, reflected in the x-axis, shifted up by 5 units, and with $–1 < a < 0$, since the curve is compressed vertically.
The equation is of the form $y = ax^2 + 5$.
Use trial-and-error to find a. Use the point (5, 4) on the parabola.

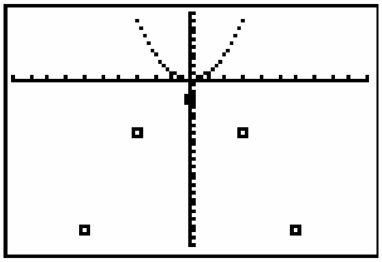
Try $a = –0.1$: $-0.1(5)^2 + 5 = 2.5 \neq 4$

Try $a = –0.01$: $-0.01(5)^2 + 5 = 4.75 \neq 4$

Try $a = –0.04$: $-0.04(5)^2 + 5 = 4$ [Guess is correct.]

The relation can be represented by the equation $y = -0.04x^2 + 5$.

c)



The graph appears to be a parabola, reflected in the x-axis, shifted down by 3 units, and with $-1 < a < 0$, since the curve is compressed vertically.
The equation is of the form $y = ax^2 - 3$.
Use trial and error to find a. Use the point (3, –8) on the parabola.
Try $a = -0.5$: $-0.5(3)^2 - 3 = -7.5 \neq -8$
Try $a = -0.6$: $-0.6(3)^2 - 3 = -8.4 \neq -8$
Try $a = -0.55$: $-0.55(3)^2 - 3 = -7.95 \neq -8$
Try $a = -\frac{5}{9}$: $-\frac{5}{9}(3)^2 - 3 = -8$ [A lucky (educated) guess is correct.]

The relation can be represented by the equation $y = -\frac{5}{9}x^2 - 3$.

Chapter 4 Section 2 Question 7 Page 192

a) widest: $y = 0.2x^2$ because $0.2 < 5$
vertex farthest from x-axis: $5x^2 + 6$ because $6 > 0$

b) widest: $y = -0.4x^2 - 8$ because $0.4 < 3$
vertex farthest from x-axis: $y = 3x^2 + 9$ because $9 > 8$

c) widest: $y = 2x^2 - 5$ because $2 < 5$
vertex farthest from x-axis: $y = 5x^2 + 7$ because $7 > 5$

d) widest: $y = 0.1x^2$ because $0.1 < 0.25$
vertex farthest from x-axis: $y = 0.25x^2 + 11$ because $11 > 0$

e) widest: $y = 0.03x^2 + 2$ because $0.03 < 0.2$
vertex farthest from x-axis: $y = 0.03x^2 + 2$ because $2 > 1$

f) widest: $y = 0.9x^2 + 6$ because $0.9 < 1$
vertex farthest from x-axis: they are both the same distance from the x-axis since $6 = 6$

Chapter 4 Section 2 Question 8 Page 192

a) When t = 0.5 s, $h = –0.85(0.5)^2 + 2 = 1.7875$.
The skateboarder is approximately 1.79 m off the ground.

When t = 1 s, $h = –0.85(1)^2 + 2 = 1.15$.
The skateboarder is 1.15 m off the ground.

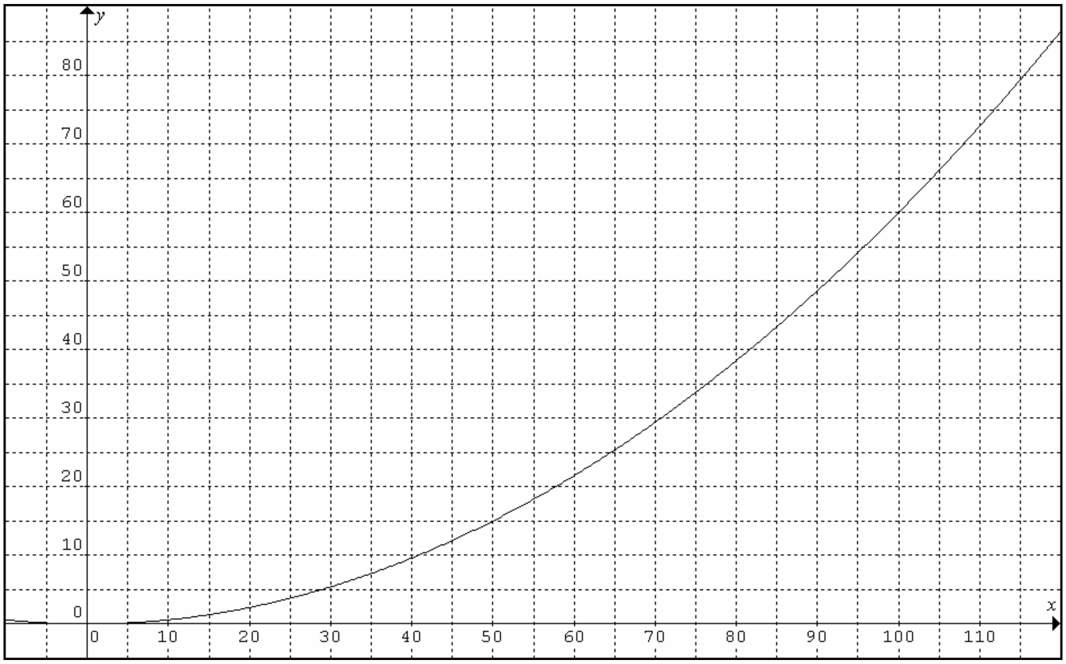
b) Solve for t when h = 0 using algebra, or else plot the relation and read the x-intercept off the graph.

$$\begin{aligned} -0.85t^2 + 2 &= 0 \\ -0.85t^2 &= -2 \\ t^2 &= \frac{-2}{0.85} \\ t^2 &= \sqrt{\frac{2}{0.85}} \\ t &\doteq 1.5 \end{aligned}$$

The skateboarder is in the air for about 1.5 s. (Note: this assumes that t > 0 for the jump.)

Chapter 4 Section 2 **Question 9 Page 192**

a) Choose an appropriate scale to graph the relation. There is no need to graph the negative values for *t*.



b) Values can be read off the graph or calculated using the equation for the relation.

i) 15 m

ii) 21.6 m

iii) 60 m

iv) 72.6 m

c) In the city, the extra stopping distance, in metres, is:
$0.006(60)^2 - 0.006\,(50)^2 = 6.6$

On the highway, the extra stopping distance, in metres, is:
$0.006(110)^2 - 0.006\,(10)^2 = 12.6$

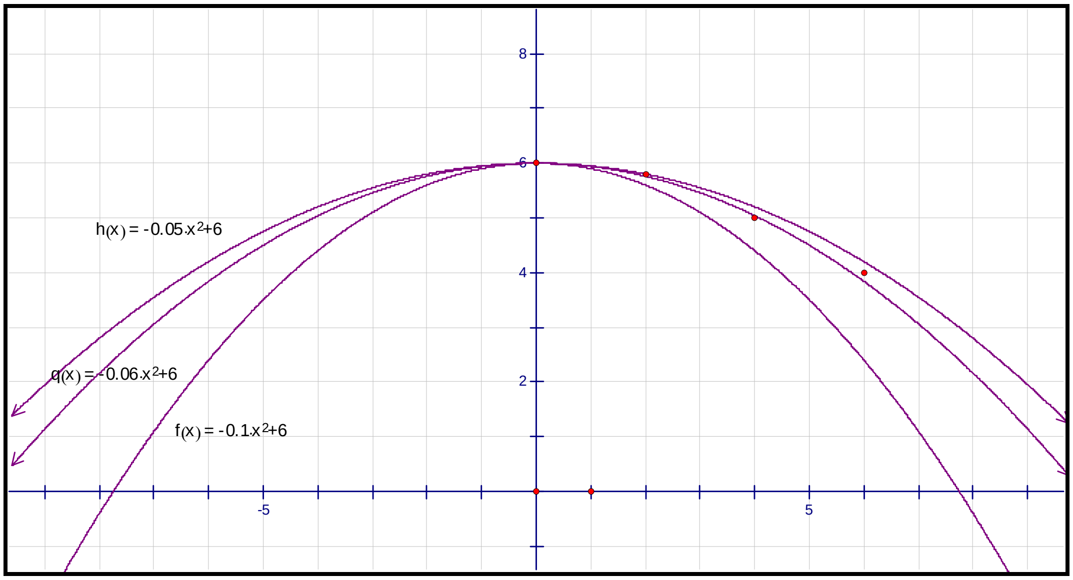
The stopping distance increases by about two times as much when going 10 km/h over the speed limit on the highway.

Chapter 4 Section 2 Question 10 Page 193

Answers may vary.

a) (0, 6)

b) The points (0, 6), (2, 5.8), (4, 5), and (6, 4) are on the curve.
Graph these points and fit them in an equation of the form $y = ax^2 + 6$.
Try values of a such as –0.1, –0.05, –0.06. The best fit seems to occur when a = –0.06.



c) The parabola can be modelled by the equation $y = -0.006x^2 + 6$.

Chapter 4 Section 2 Question 11 Page 193

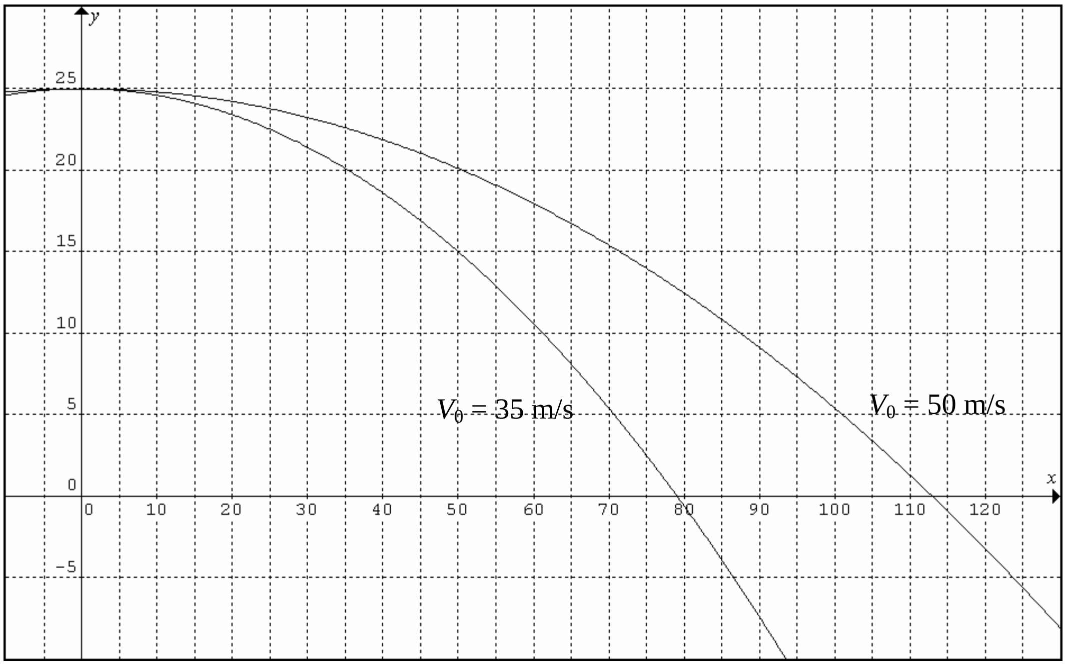
a) The first second of Jamie's jump occurs from t = 0 to t = 1.
Calculate the difference in his heights at these two times: $4.9(1)^2 - 4.9(0)^2 = 4.9$
The third second of Jamie's jump occurs from t = 2 to t = 3.
Calculate the difference in his heights at these two times: $4.9(3)^2 - 4.9(2)^2 = 24.5$
24.5 – 4.9 = 19.6

Jamie fell 19.6 m farther in the third second of his jump.

b) Using trail-and-error [$4.9(4)^2 = 78.4$], or graphing the relation and reading a value from the graph, it takes Jamie almost 4 s to make the 77-m jump.

Chapter 4 Section 2 Question 12 Page 193

Graph both relations on the same axes. Determine the *x*-intercepts by inspection.
They are about 79.05 and 112.94 (using the ZOOM feature of graphing software).
112.94 – 79.05 = 33.89
The difference in distance travelled is about 33.9 m.



Chapter 4 Section 3 **The Quadratic Relation $y = a(x - h)^2$**

Chapter 4 Section 3 **Question 1 Page 200**

a) $h = 5$; vertex is (5, 0)

b) $h = –3$; vertex is (–3, 0)

c) $h = –7$; vertex is (–7, 0)

d) $h = 2$; vertex is (2, 0)

Chapter 4 Section 3 **Question 2 Page 200**

a) $a = 1$, so the graph is neither stretched nor compressed;
$h = 7$, so the graph is translated 7 units to the right.

b) $a = –1$, so the graph is neither stretched nor compressed, but it is reflected in the x-axis;
$h = –3$, so the graph is translated 3 units to the left.

c) $a = 1.5$, so the graph is vertically stretched;
$h = –8$, so the graph is translated 8 units to the left.

d) $a = –0.8$, so the graph is vertically compressed and reflected in the x-axis;
$h = 2$, so the graph is translated 2 units to the right.

e) $a = 0.1$, so the graph is vertically compressed;
$h = 5$, so the graph is translated 5 units to the right.

f) $a = 2$, so the graph is vertically stretched;
$h = –1$, so the graph is translated 1 unit to the left.

g) $a = –2$, so the graph is vertically stretched and reflected in the x-axis;
$h = 8$, so the graph is translated 8 units to the right.

h) $a = 0.3$, so the graph is vertically compressed;
$h = –14$, so the graph is translated 14 units to the left.

Chapter 4 Section 3 **Question 3 Page 200**

a) The graph is vertically stretched and translated 7 units to the right, so $a > 1$ and $h = 7$.

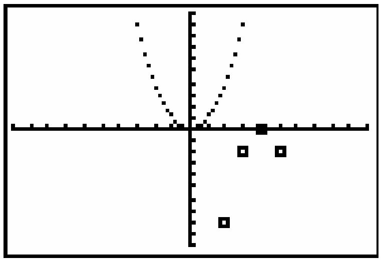
b) The graph is vertically compressed, reflected in the x-axis, and translated 5 units to the left, so $–1 < a < 0$ and $h = –5$.

c) The graph is neither vertically compressed nor stretched, but it is translated 8 units to the left, so $a = 1$ and $h = –8$.

d) The graph is vertically stretched, reflected in the x-axis and translated 2 units to the right, so $a < –1$ and $h = 2$.

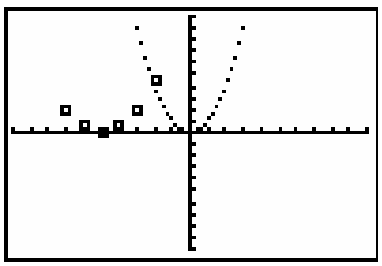
Chapter 4 Section 3 **Question 4 Page 201**

a)



The vertex is translated 4 units to the right, so $h = 4$; the graph is reflected in the x-axis and is vertically stretched, so $a < -1$. The equation for the relation is $y = -2(x-4)^2$.

b)



The vertex is translated 5 units to the left, so $h = -5$; the graph is vertically compressed, so $0 < a < 1$. The equation for the relation is $y = 0.5(x+5)^2$.

Chapter 4 Section 3 **Question 5 Page 201**

a) $y = 2(x + 3)^2$; because $3 > 1$

b) $y = -0.2(x - 8)^2$; because $8 > 3$

c) $y = 32(x - 10)^2$; because $10 > 3$

d) $y = 0.85(x + 9)^2$; because $9 > 2$

Chapter 4 Section 3 **Question 6 Page 201**

The general equation for a quadratic relation is written so that a positive h means a translation to the right and a negative h means a translation to the left.

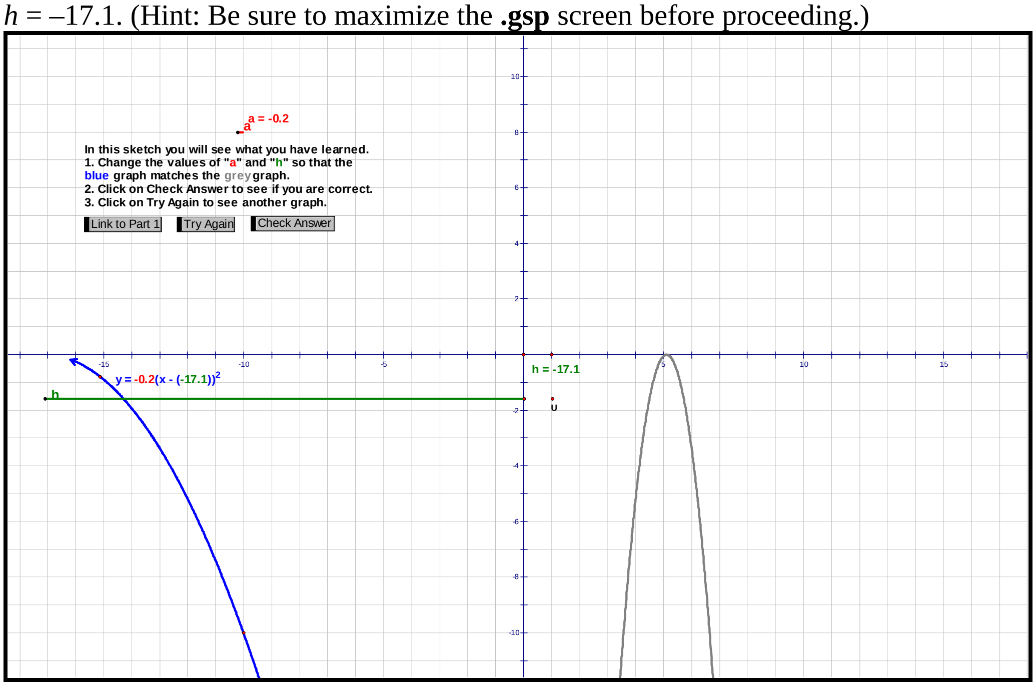
Chapter 4 Section 3 Question 7 Page 201

a)

b) From the table of values for the relation, $d = 4$ when w is about 0 and 14.
So, the width of the headlight from edge to edge is about 14 cm – 0 cm = 14 cm.

Chapter 4 Section 3 Question 8 Page 202

a) The relation will have an equation of the form $y = -\frac{5}{5^2}(x-h)^2$. This simplifies to $y = -0.2(x-h)^2$. The only known point on this curve is the one at the start of the ramp and its co-ordinates are (–10, –10). Using **4.3 Investigation.gsp** from text page 195 and plotting the point (–10, –10), the blue curve can be manipulated until it passes through the point, i.e., when $h = -17.1$. (Hint: Be sure to maximize the **.gsp** screen before proceeding.)



b) Decrease (e.g., a value below –17); if the mountain biker has a greater speed when travelling horizontally, he will have to jump sooner so that he does not overshoot the ramp.

c) The relation will have an equation of the form $y = -\frac{5}{10^2}(x-h)^2$. This simplifies to $y = -0.05(x-h)^2$. Again, the only known point on this curve is (–10, –10). Using trial-and-error with **4.3 Investigation.gsp**, $h = -26.1$ when $a = -0.04$ and $h = -22.4$ when $a = -0.06$. Both of these h-values are below –17.

Chapter 4 Section 3 Question 9 Page 203

a) The vertex of the parabola is at (282, 0), as shown in the diagram.

b) The parabola passes through (0, 70). The equation of the curve will be of the form $y = a(x - 282)^2$. Substitute (0, 60) into the equation:

$$70 = a\left(-282\right)^2$$

$$a = \frac{70}{79524}$$
$$= 0.000\ 88$$

c) The equation for quadratic relation that models the parabola is $y = 0.000\ 88(x - 282)^2$.

d) The cables should be attached at the point that is the *y*-intercept.
Check by substituting $x = 0$ into the equation for the parabola.

$$y = 0.000\ 88(0 - 282)^2$$
$$= 69.98$$

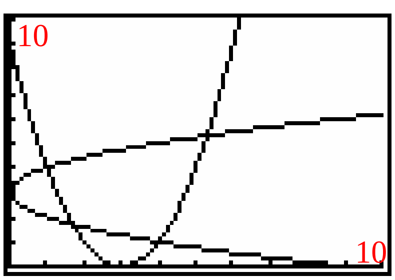
The cables are attached at a height of 70.0 m above the deck on the support tower.

Chapter 4 Section 3 Question 10 Page 203

a)

x	$y = (x - 3)^2$	$y = 3 + \sqrt{x}$	$y = 3 - \sqrt{x}$
0	9.00	3.00	3.00
1	4.00	4.00	2.00
2	1.00	4.41	1.59
3	0.00	4.73	1.27
4	1.00	5.00	1.00
5	4.00	5.24	0.76
6	9.00	5.45	0.55
7	16.00	5.65	0.35

b)



c) Answers may vary. For example:
They are similar in that the first parabola has the same shape as the other two curves combined; they are different in that the first parabola is a reflection of the other two through the line $y = x$.

Chapter 4 Section 4 The Quadratic Relation $y = a(x - h)^2 + k$

Chapter 4 Section 4 Question 1 Page 212

a) i) (5, 1) **ii)** positive

b) i) (–2, –7) **ii)** positive

c) i) (–4, –2) **ii)** negative

d) i) (2, 6) **ii)** negative

e) i) (–8, 4) **ii)** positive

f) i) (6, –2) **ii)** negative

g) i) (2, –5) **ii)** positive

h) i) (–8, 1) **ii)** negative

Chapter 4 Section 4 Question 2 Page 213

a) i) (3, 12) **ii)** upward **iii)** stretched

b) i) (10, –1) **ii)** downward **iii)** compressed

c) i) (–4, –8) **ii)** downward **iii)** stretched

d) i) (–20, –5) **ii)** downward **iii)** neither

e) i) (11, –3) **ii)** upward **iii)** compressed

f) i) (–2, 9) **ii)** upward **iii)** stretched

g) i) (–6, 7) **ii)** downward **iii)** compressed

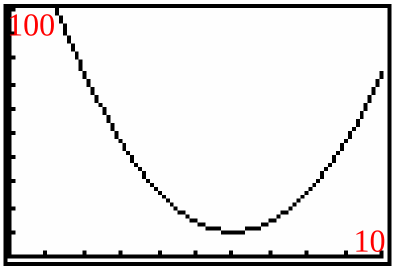
h) i) (8, 2) **ii)** upward **iii)** stretched

i) i) (–2, –1) **ii)** upward **iii)** stretched

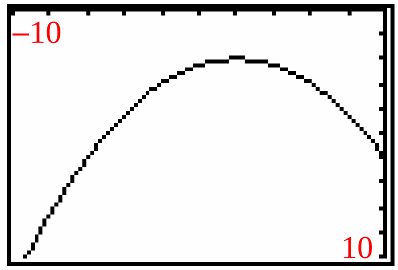
j) i) (4, 6) **ii)** downward **iii)** compressed

Chapter 4 Section 4 **Question 3 Page 213**

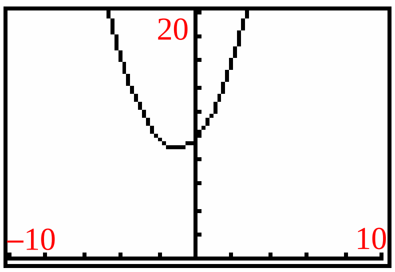
a)



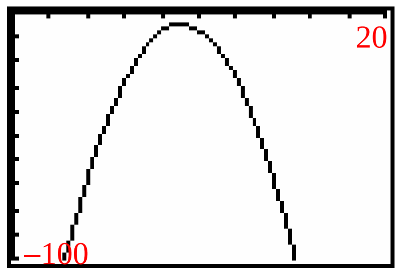
b)



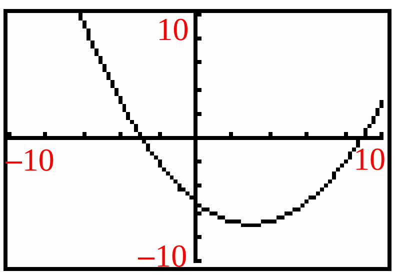
c)



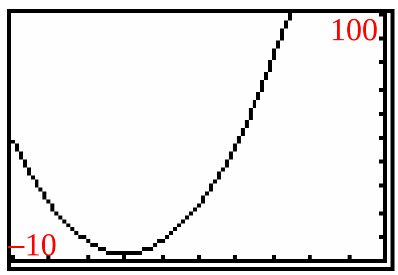
d)



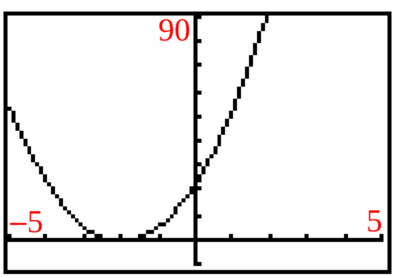
e)



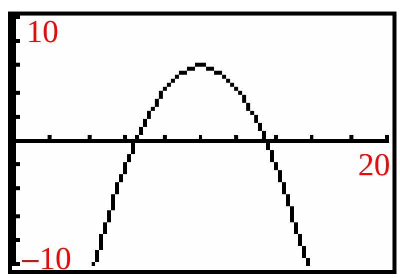
f)



g)



h)



Chapter 4 Section 4 Question 4 Page 213

a) The vertex is (0, 0).
The equation is of the form $y = a(x - h)^2 + k$.
This becomes $y = a(x - 0)^2 + 0$.
To solve for a, use the point (1, 3) on the parabola.

$$3 = a(1)^2 + 0$$
$$3 = a$$
$$a = 3$$

The equation is $y = 3x^2$.

b) The vertex is (7, 3).
The equation is of the form $y = a(x - h)^2 + k$.
This becomes $y = a(x - 7)^2 + 3$.
To solve for a, use the point (11, 9) on the parabola.

$$9 = a(11 - 7)^2 + 3$$
$$9 = 16a + 3$$
$$6 = 16a$$
$$a = \frac{3}{8}$$
$$a = 0.375$$

The equation is $y = 0.375(x - 7)^2 + 3$.

c) The vertex is (–4, 8).
The equation is of the form $y = a(x - h)^2 + k$.
This becomes $y = a(x + 4)^2 + 8$.
To solve for a, use the point (–3, 2) on the parabola.

$$2 = a(-3 + 4)^2 + 8$$
$$2 = 1a + 8$$
$$a = -6$$

The equation is $y = -6(x + 4)^2 + 8$.

d) The vertex is (–6, 1).
The equation is of the form $y = a(x - h)^2 + k$.
This becomes $y = a(x + 6)^2 + 1$.
To solve for a, use the point (–4, 3) on the parabola.

$$3 = a(-4 + 6)^2 + 1$$
$$3 = 4a + 1$$
$$2 = 4a$$
$$a = \frac{2}{4}$$
$$a = 0.5$$

The equation is $y = 05(x + 6)^2 + 1$.

Chapter 4 Section 4 Question 5 Page 214

a) The vertex is (–2, –3).
The equation is of the form $y = a(x - h)^2 + k$.
This becomes $y = a(x + 2)^2 - 3$.
To solve for a, use the point (–12, –2) on the parabola.

$$-2 = a(-12 + 2)^2 - 3$$
$$-2 = 100a - 3$$
$$1 = 100a$$
$$a = \frac{1}{100}$$
$$a = 0.01$$

The equation is $y = 0.01(x + 2)^2 - 3$.

b) The vertex is (8, 3).
The equation is of the form $y = a(x - h)^2 + k$.
This becomes $y = a(x - 8)^2 + 3$.
To solve for a, use the point (7, 1) on the parabola.

$$1 = a(7 - 8)^2 + 3$$
$$1 = 1a + 3$$
$$a = -2$$

The equation is $y = -2(x - 8)^2 + 3$.
The equation is of the form $y = a(x - h)^2 + k$.
This becomes $y = a(x - 5)^2 - 7$.
To solve for a, use the point (4, 3) on the parabola.

$$3 = a(4 - 5)^2 - 7$$
$$3 = 1a - 7$$
$$a = 10$$

The equation is $y = 10(x - 5)^2 - 7$.

d) The vertex is (–3, –1).
The equation is of the form $y = a(x - h)^2 + k$.
This becomes $y = a(x + 3)^2 - 1$.
To solve for a, use the point (–2, –5) on the parabola.

$$-5 = a(-2 + 3)^2 - 1$$
$$-5 = 1a - 1$$
$$a = -4$$

The equation is $y = -4(x + 3)^2 - 1$.

Chapter 4 Section 4 Question 6 Page 214

a) The vertex is (8.7, 7.6).

b) The variable *d* represents the horizontal distance from where the football was kicked. Therefore, $d = 0$ when the football is at its initial height.
Substitute $d = 0$ into the equation.

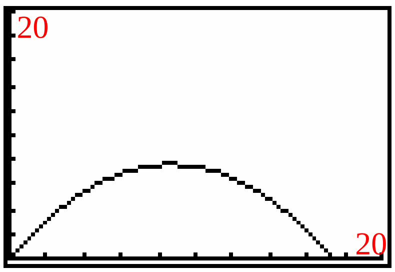
$h = -0.1(0 - 8.7)^2 + 7.6$

$h = -0.1 \times 75.69 + 7.6$

$h = 0.031$

The football's initial height is 0.031 m.

c)

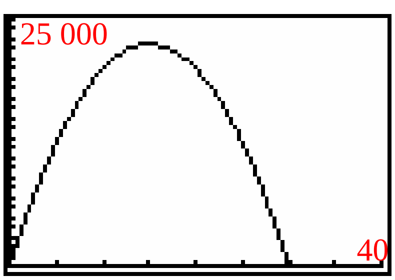


d) The vertex is (8.7, 7.6), with 8.7 m being the horizontal distance travelled when the football reaches its greatest height 7.6 m above ground.

Chapter 4 Section 4 Question 7 Page 215

a)

Ticket Price, *P* ($)	Total Revenue, *R* ($)
0	0
5	12 500
10	20 000
15	22 500
20	20 000
25	12 500
30	0



b) Comparing $R = -100(P - 15)^2 + 22\,500$ to $y = a(x - h)^2 + k$, the vertex is (15, 22 500).

The *y*-coordinate of the vertex is the greatest total revenue, and the *x*-coordinate is the amount that, when used as the ticket price, results in this total revenue.

Chapter 4 Section 4 Question 8 Page 215

a) The vertex is (30, 9800).
The equation is of the form $y = a(x - h)^2 + k$.
This becomes $y = a(x - 30)^2 + 9800$.
To solve for a, use the point (0, 7300) on the parabola.

$$7300 = a(0 - 30)^2 + 9800$$
$$7300 = 900a + 9800$$
$$-2500 = 900a$$
$$a = \frac{-2500}{900}$$
$$a = -2.78$$

The equation is $y = -2.78(x - 30)^2 + 9800$.

b) Find y when $x = 20$.

$$y = -2.78(20 - 30)^2 + 9800$$
$$y = -2.78 \times 100 + 9800$$
$$y = 9522$$

The jet's altitude is 9522 m.

Chapter 4 Section 4 Question 9 Page 216

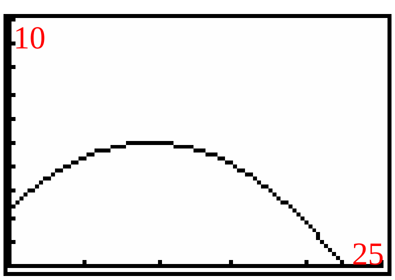
a) Substitute $d = 0$ into the equation $h = -0.03(d - 9.5)^2 + 5$.

$$h = -0.03(0 - 9.5)^2 + 5$$
$$h = -0.03 \times 90.25 + 5$$
$$h \doteq 2.3$$

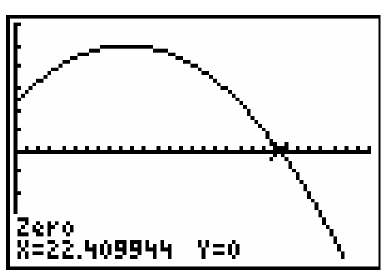
The initial height is 2.3 m.

b) The vertex is (9.5, 5). This indicates that when the biker is at a horizontal distance of 9.5 m from the end of the ramp, the biker is 5 m above ground.

c)



d)



The biker will land 22.4 m from the end of the ramp.

Chapter 4 Section 4 Question 10 Page 216

Solutions for Achievement Checks are in the *Teacher Resource*.

Chapter 4 Section 4 Question 11 Page 217

A number of strategies will work for matching the two graphs. For example:
1. Change a until the shape of the two parabolas match. Don't worry about position.
2. Change c until the y-intercepts of the two parabolas match.
3. Change b until the two curves coincide.

The form $y = a(x - h)^2 + k$ seems to be more useful for modelling quadratic relations, especially when the vertex of the parabola is already known.

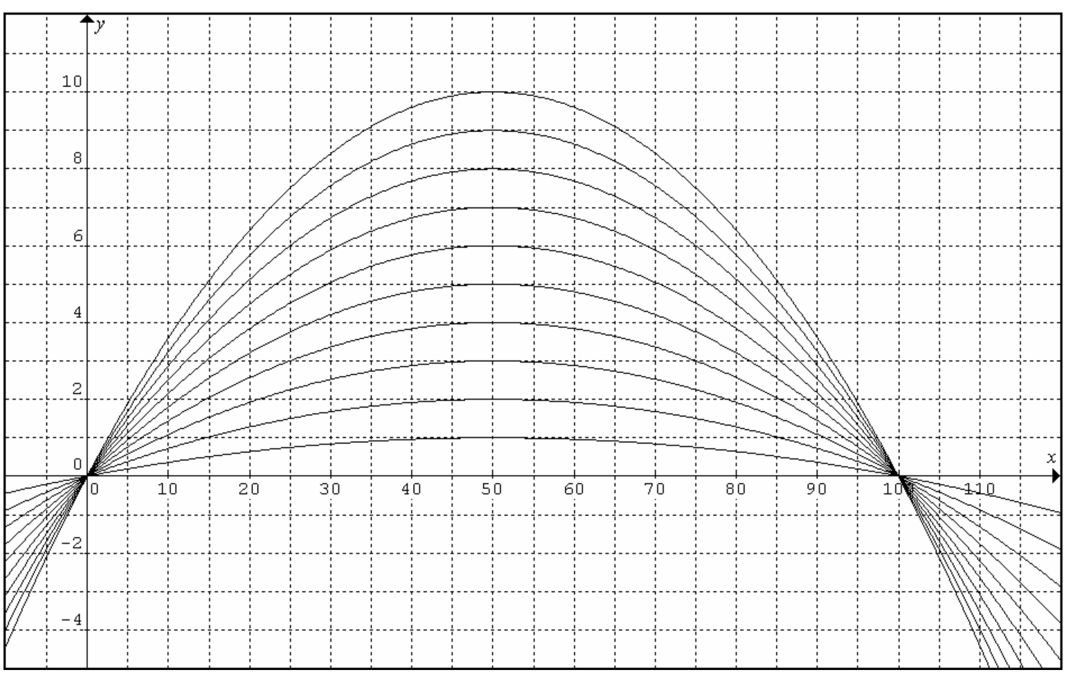
Chapter 4 Section 4 Question 12 Page 217

The required parabola needs to open downward and have x-intercepts at $x = 0$ and $x = 100$.
By symmetry, the vertex is located at $(50, k)$.
The equation will be of the form $y = a(x - 50)^2 + k$, where $a < 0$ and $k > 0$.
Since (100, 0) must be on the curve, substitute these coordinates into the equation.

$$0 = a\left(100 - 50\right)^2 + k$$
$$-k = 2500a$$
$$a = -\frac{k}{2500}$$

The equation for the relation is a family of equations of the form $y = -\frac{k}{2500}\left(x - 50\right)^2 + k$.

The graph below shows this family as $k = 1, 2, 3, \ldots 9, 10$.

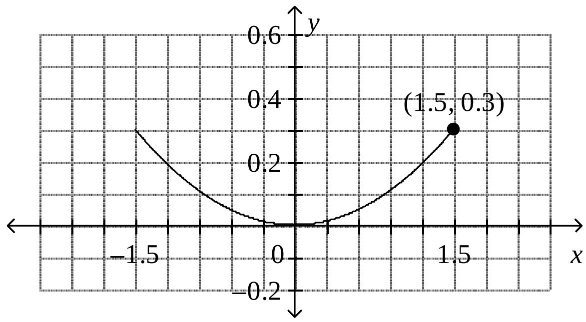


There can be more than one possible path. Explanations may vary.

Chapter 4 Section 4 Question 13 Page 217

a) Sketch a parabola for the dish with the vertex at (0, 0) to simplify the equation.

The equation is of the form $y = \frac{1}{4p}(x-0)^2$ or $y = \frac{1}{4p}x^2$.



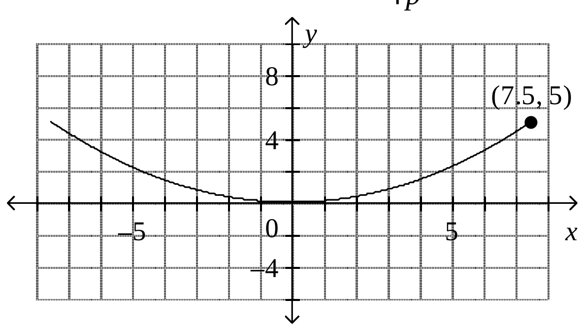
To solve for p, use the point (1.5, 0.3) on the parabola.

$$
\begin{aligned}
0.3 &= \frac{1}{4p}(1.5)^2 \\
0.3 \times 4p &= 2.25 \\
1.2p &= 2.25 \\
p &= \frac{2.25}{1.2} \\
p &\doteq 1.9
\end{aligned}
$$

The focus is approximately 1.9 m from the vertex.

b) The solution is similar to **a)** above.
Sketch a parabola for the dish with the vertex at (0, 0) to simplify the equation.

The equation is of the form $y = \frac{1}{4p}(x-0)^2$ or $y = \frac{1}{4p}x^2$.



To solve for p, use the point (7.5, 5) on the parabola.

$$
\begin{aligned}
5 &= \frac{1}{4p}(7.5)^2 \\
5 \times 4p &= 56.25 \\
20p &= 56.25 \\
p &= \frac{56.25}{20} \\
p &\doteq 2.8
\end{aligned}
$$

The focus is about 2.8 cm from the vertex.

Chapter 4 Section 5 Interpret Graphs of Quadratic Relations

Chapter 4 Section 5 Question 1 Page 222

a) Let $x = 0$. $y = -15(0)^2 + 25(0) - 7 = -7$

b) Let $x = 0$. $y = 0.45(0)^2 - 0.17(0) + 20 = 20$

c) Let $x = 0$. $y = 20(0 - 12)^2 + 15 = 2895$

d) Let $x = 0$. $y = -0.5(0 + 1.5)^2 + 4.5 = 3.375$

e) Let $x = 0$. $y = 10(0)^2 + 8(0) - 3 = -3$

f) Let $x = 0$. $y = 0.2(0 - 3.4)^2 + 1 = 3.312$

g) Let $x = 0$. $y = -0.1(0)^2 - 0.4(0) - 1.8 = -1.8$

h) Let $x = 0$. $y = -3(0 + 2)^2 - 9 = -21$

Chapter 4 Section 5 Question 2 Page 223

a) *x*-intercepts: –10, 2; *y*-intercept: –5; minimum: –9; vertex: (–4, –9)

b) *x*-intercepts: –2, –6; *y*-intercept: –6; maximum: 2; vertex: (–4, 2)

c) *x*-intercepts: 2, –20; *y*-intercept: 4; maximum: 12; vertex: (–9, 12)

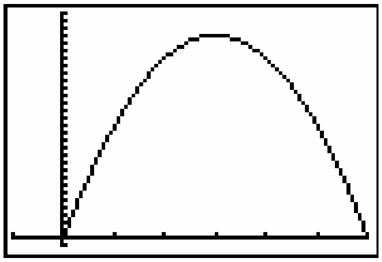
d) *x*-intercepts: –3, 1; *y*-intercept: –6; minimum: –8; vertex: (–1, –8)

e) *x*-intercepts: none; *y*-intercept: 12; minimum: 3; vertex: (6, 3)

f) *x*-intercepts: 2, –10; *y*-intercept: 10; maximum: 18; vertex: (–4, 18)

Chapter 4 Section 5 **Question 3 Page 224**

a)

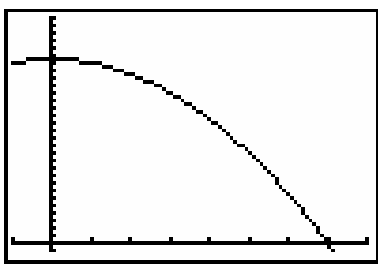


The vertex is (3, 27) and (0, 0) is a point on the parabola.

$$y = a(x-3)^2 + 27$$
$$0 = a(0-3)^2 + 27$$
$$-27 = 9a$$
$$a = -3$$

The equation is $y = -3(x-3)^2 + 27$.

b)

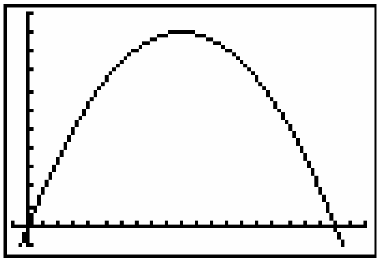


The vertex is (0, 24.5)
The equation is of the form $y = ax^2 + 24.5$
The point (7, 0) is on the parabola. Substitute these coordinates into the equation to find a.

$$0 = a(7)^2 + 24.5$$
$$-24.5 = 49a$$
$$a = \frac{-24.5}{49}$$
$$a = -0.5$$

The equation is $y = -0.5x^2 + 24.5$.

c)

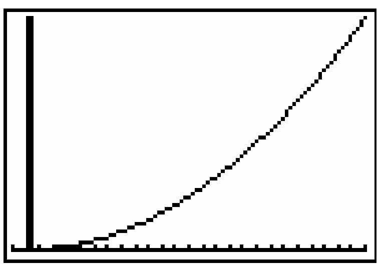


The vertex is (10, 10) and the point (20, 0) is a point on the parabola.

$$y = a(x-10)^2 + 10$$
$$0 = a(20-10)^2 + 10$$
$$-10 = 100a$$
$$a = \frac{-10}{100}$$
$$a = -0.1$$

The equation is $y = -0.1(x-10)^2 + 10$.

d)



The vertex is (0, 0).
The equation is of the form $y = ax^2$.
The point (25, 9375) is on the parabola.
Substitute these coordinates into the equation to find a.

$$9375 = a(25)^2$$
$$9375 = 625a$$
$$a = \frac{9375}{625}$$
$$a = 15$$

The equation is $y = 15x^2$.

Chapter 4 Section 5 **Question 4 Page 224**

a) The rocket reaches its maximum height at the vertex.
Comparing the equation to $y = a(x-h)^2 + k$, the vertex is (2, 169.6).

The maximum height is 169.6 m and this occurs after 2 s of flight.

b) To find the height after 5 s, substitute t = 5 into the equation.

$h = -4.9(5-2)^2 + 169.6$

$h = -4.9(9) + 169.6$

$h = 125.5$

The height of the rocket after 5 s will be 125.5 m.

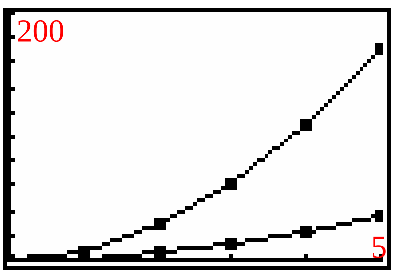
Chapter 4 Section 5 **Question 5 Page 224**

a) Smart Car Fortwo:

Time, *t* (s)	Distance, *d* (m)
0	0
1	1.4
2	5.6
3	12.6
4	22.4
5	35.0

Tesla Roadster:

Time, *t* (s)	Distance, *d* (m)
0	0
1	6.9
2	27.6
3	62.1
4	110.4
5	172.5



b) From the tables of values, the difference is: 172.5 m – 35.0 m = 137.5 m
These values are related to the coefficients of t^2 in the two relations.

c) The first second is from $t = 0$ to $t = 1$ and the fourth second is from $t = 3$ to $t = 4$.
In the first second, the Smart Car Fortwo travels: 1.4 m – 0 m = 1.4 m
In the fourth second, the Smart Car Fortwo travels: 22.4 m – 12.6 m = 9.8 m
In the first second, the Tesla Roadster travels: 6.9 m – 0 m = 6.9 m
In the first second, the Tesla Roadster travels: 110.4 m – 62.1 m = 48.3 m

The results indicate that the speed is increasing for each time period.

Chapter 4 Section 5 Question 6 Page 225

Answers may vary. For example:
Assume the *x*-axis is located across the top of the dish and that the *y*-axis is located on the left side of the diagram touching the dish.
The vertex of the parabola will be (20, –5).
The equation will be of the form $y = a(x - 20)^2 - 5$.
To find *a*, use one of these two points, (0, 0) or (40, 0), on the parabola.
Substitute the coordinates into the equation.

$$\begin{aligned} 0 &= a(0-20)^2 - 5 \\ 5 &= 400a \\ a &= \frac{5}{400} \\ a &= 0.0125 \end{aligned}$$

The shape of dish can be modelled by the relation $y = 0.0125(x - 20)^2 - 5$.

Chapter 4 Section 5 Question 7 Page 225

a) From previous work, the motion of a projectile can be modelled by a quadratic relation of the form $y = a(x - h)^2 + k$.

The vertex of this parabola is (4.5, 101.5). The equation is $y = a(x - 4.5)^2 + 101.25$.

The points (0, 0) and (9, 0) are on the parabola. Substitute the coordinates of either point on the parabola into the equation to solve for *a*.

$$\begin{aligned} 0 &= a(0-4.5)^2 + 101.25 \\ -101.25 &= 20.25a \\ a &= \frac{-101.25}{20.25} \\ a &= -5 \end{aligned}$$

The motion can be modelled by the relation $h = -5(t - 4.5)^2 + 101.25$.

b) Substitute $t = 3$ into the equation.

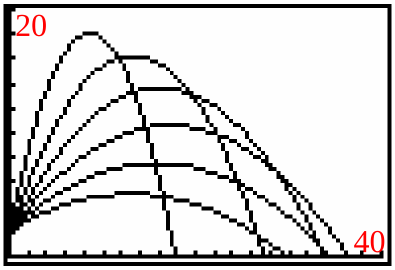
$$\begin{aligned} h &= -5(3-4.5)^2 + 101.25 \\ h &= -5(2.25) + 101.25 \\ h &= 90 \end{aligned}$$

The height of the projectile is 90 m after 3 s.

By examining the equation, $t = 6$ also gives the same height, since $(3 - 4.5)^2 = (6 - 4.5)^2$.
The first time is when the projectile is rising and the second time is when it is falling.
Explanations may vary.

Chapter 4 Section 5 Question 8 Page 225

a)



b) 45° (by examining where the graphs intersect the x-axis)

c) 75° (by examining which graph has the highest maximum)

d) This is the height of the ski ramp.

Chapter 4 Section 5 Question 9 Page 225

After 5 s, the Smart Car Fortwo has travelled 35.0 m.
Find how long it will take the Tesla Roadster to travel 35.0 m.
This can be done by trial-and-error, substituting in the equation $d = 6.9t^2$,
or using the algebraic solution below:

$$\begin{aligned} 35 &= 6.9t^2 \\ t^2 &= \frac{35}{6.9} \\ t &= \sqrt{\frac{35}{6.9}} \\ t &\doteq 2.3 \end{aligned}$$

The Tesla Roadster needs approximately 2.3 s to reach the distance 35.0 m.
It should start after 5 s – 2.3 s = 2.7 s.

Chapter 4 Review

Chapter 4 Review **Question 1 Page 226**

a) No; the greatest exponent is 1.

b) Yes; the greatest exponent is 2.

c) No; the second differences are not constant.

x	y	First Differences	Second Differences
–5	1		
		3	
0	4		9
		12	
5	16		36
		48	
10	64		144
		192	
15	256		

Chapter 4 Review **Question 2 Page 226**

a)

Time (s)	Height (m)
0.0	0
0.5	8.3
1.0	12.1
1.5	13.5
2.0	12.4
2.5	8.9
3.0	2.9

b) about 1.5 s

c) about 3.1 s

d) Four possible reasons:
- The second differences are constant.
- When graphed, the points in the table form a parabola.
- There is a t^2-term in the equation.
- The motion of a projectile under gravity is modelled by a quadratic relation.

Chapter 4 Review **Question 3 Page 226**

a) same shape; translated 3.4 units down

b) reflected in the x-axis; vertically compressed; has not been translated

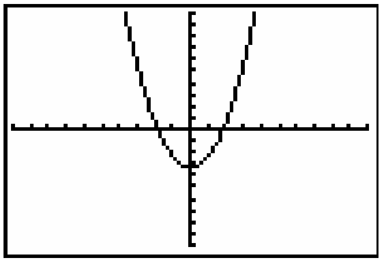
c) vertically compressed; translated 15 units up

d) vertically stretched; translated 3.4 units down

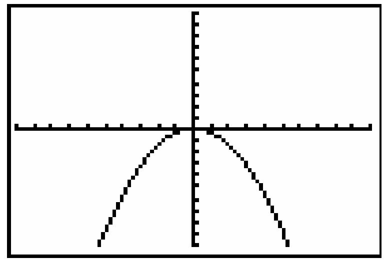
Chapter 4 Review **Question 4 Page 226**

Sketches may vary slightly.

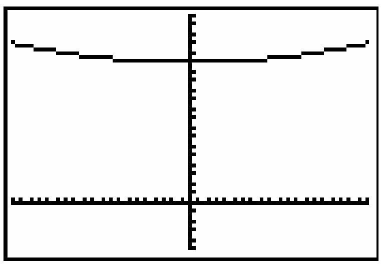
a) $y = x^2 - 3.4$



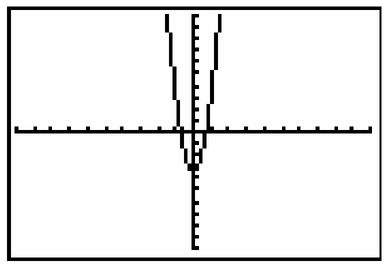
b) $y = -0.35x^2$



c) $y = -0.005x^2 + 15$

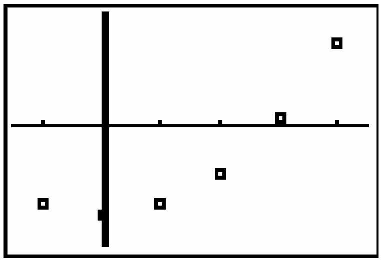
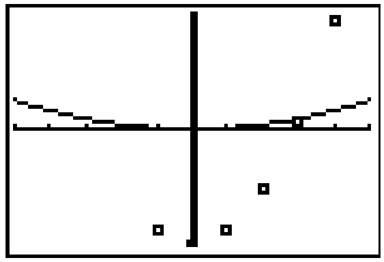


d) $y = 6.5x^2 - 3.4$



Chapter 4 Review **Question 5 Page 226**

a)

The graph appears to be a parabola shifted down by 100 units and with $a > 1$, since the curve is stretched vertically (graph on the right shows both the $y = x^2$ curve and the scatter plot). The equation is of the form $y = ax^2 - 100$.

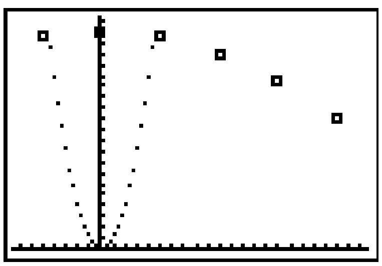
Use trial-and-error to find a. Use the point (1, –88) on the parabola.

Try $a=10$: $y = 10(1)^2 - 100 = -90 \neq -88$

Try $a=12$: $y = 12(1)^2 - 100 = -88$ [Guess is correct.]

The relation can be modelled by the equation $y = 12x^2 - 100$.

b)



The graph appears to be a parabola, reflected in the x-axis, shifted up by 20 units, and with $-1 < a < 0$, since the curve is compressed vertically. The equation is of the form $y = ax^2 + 20$.

Use trial-and-error to find a. Use the point (10, 18) on the parabola.

Try $a = -0.1$: $-0.1(10)^2 + 20 = 10 \neq 18$

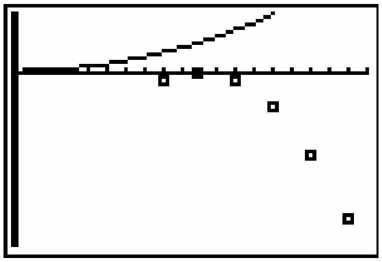
Try $a = -0.01$: $-0.01(10)^2 + 20 = 19 \neq 18$

Try $a = -0.02$: $-0.02(10)^2 + 20 = 18$ [Guess is correct.]

The relation can be modelled by the equation $y = -0.02x^2 + 20$.

Chapter 4 Review **Question 6 Page 226**

a)

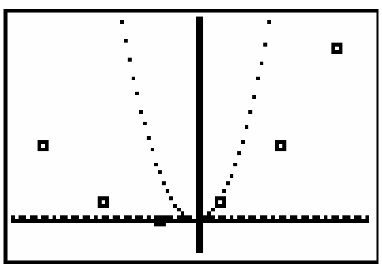


The graph appears to be a parabola with vertex at (10, 0).
The equation is of the form $y = a(x - 10)^2$.
The parabola is reflected in the x-axis and is stretched vertically, so $a < -1$.
The point (8, –32) is on the parabola. Substitute these coordinates into the equation.

$$-32 = a(8 - 10)^2$$
$$-32 = 4a$$
$$a = -8$$

The relation can be modelled by the equation $y = -8(x - 10)^2$.

b)



The graph appears to be a parabola with vertex at (–6, 0).
The equation is of the form $y = a(x + 6)^2$.
The parabola is compressed vertically, so $0 < a < 1$.
The point (4, 15) is on the parabola. Substitute these coordinates into the equation.

$$15 = a(4 + 6)^2$$
$$15 = 100a$$
$$a = 0.15$$

The relation can be modelled by the equation $y = 0.15(x + 6)^2$.

Chapter 4 Review Question 7 Page 227

a) reflected in the *x*-axis; vertically compressed; translated 18 units to the right and 15 units up

b) vertically stretched; translated 1 unit to the left and 2 units down

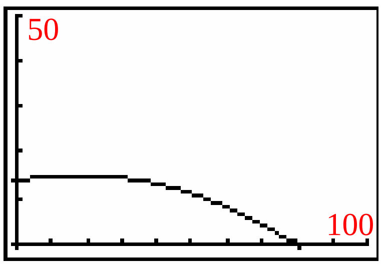
c) reflected in the *x*-axis; vertically stretched; translated 9 units to the left and 10.8 units up

d) vertically compressed; translated 40 units to the right

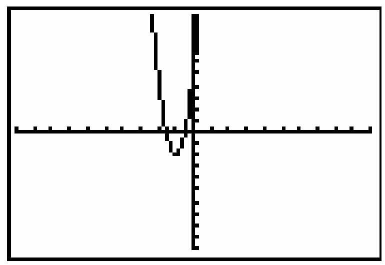
Chapter 4 Review Question 8 Page 227

Sketches may vary slightly.

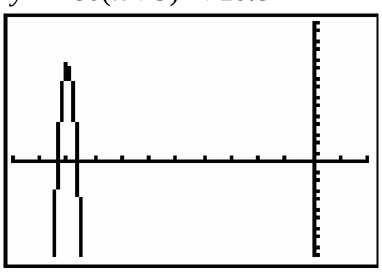
a) $y = -0.004(x - 18)^2 + 15$



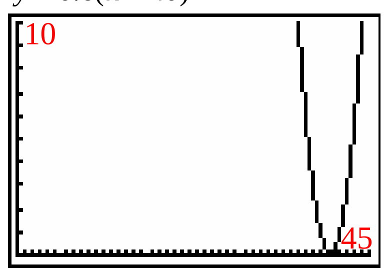
b) $y = 7(x + 1)^2 + 15$



c) $y = -80(x + 9)^2 + 10.8$



d) $y = 0.6(x - 40)^2$

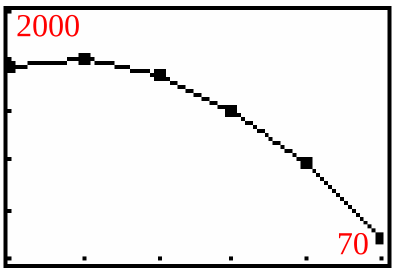


Chapter 4 Review **Question 9 Page 227**

a)

Hourly Rate ($)	Expected Number of Hours per Week	Weekly Revenue ($)
45	42	$1890
50	38	$1900
55	34	$1870
60	30	$1800
65	26	$1690
70	22	$1540

b)



c) Let the hourly rate be x and the weekly revenue be y.

The equation is of the form $y = a(x - h)^2 + k$.

It is not possible to pick the vertex from the table of values. The best choice of vertex is (50, 1900). (Advanced regression techniques show that the vertex for the parabola modelled by these data to be (48.75, 1901.25).)

Assume that the equation is $y = a(x - 50)^2 + 1900$.

To find a, substitute the coordinates of the point (60, 1800) into the equation.

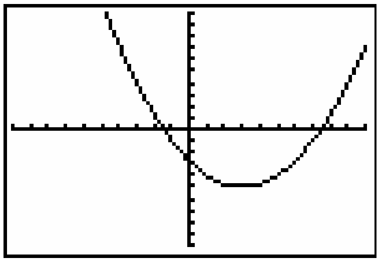
$$1800 = a(60 - 50)^2 + 1900$$
$$-100 = 100a$$
$$a = -1$$

The equation for the relation is $y = -(x - 50)^2 + 1900$.

d) The hourly rate that produces the maximum weekly revenue with the chosen model is $50/h. (The more accurate regression model produces a value of $48.75, but this is not a permitted value, since the wage must increase by increments of $5.)

Chapter 4 Review **Question 10 Page 227**

a)



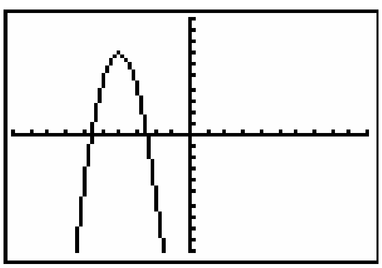
The vertex is (3, –5). The equation is of the form $y = a(x-3)^2 - 5$.

To find *a*, substitute the coordinates of the point (13, 20) into the equation.

$$20 = a(13-3)^2 - 5$$
$$25 = 100a$$
$$a = \frac{25}{100}$$
$$a = 0.25$$

The equation for the parabola is $y = 0.25(x-3)^2 - 5$.

b)



The vertex is (–4, 7). The equation is of the form $y = a(x+4)^2 + 7$.

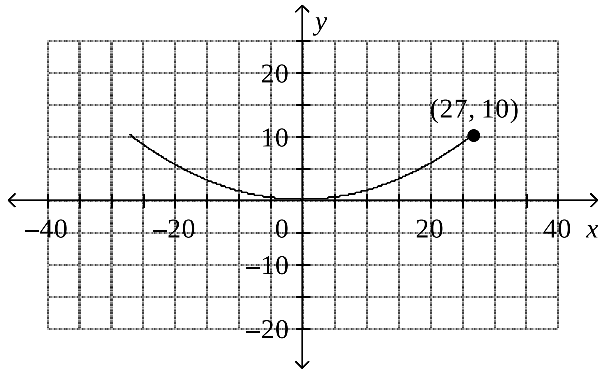
To find *a*, substitute the coordinates of the point (0, –39) into the equation.

$$-39 = a(0+4)^2 + 7$$
$$-46 = 16a$$
$$a = \frac{-46}{16}$$
$$a = -2.875$$

The equation for the parabola is $y = -2.875(x+4)^2 + 7$.

Chapter 4 Review **Question 11 Page 227**

Answers may vary. For example:
Sketch a parabola for the dish with the vertex at (0, 0) to simplify the equation.



The equation is of the form $y = ax^2$.
To solve for a, use the point (27, 10) on the parabola.
Substitute these coordinates into the equation

$$10 = a(27)^2$$
$$10 = 729a$$
$$a = 0.014$$

A relation that models this parabolic mirror is $y = 0.014x^2$.

Chapter 4 Review **Question 12 Page 227**

a)

d	0.0	0.5	1.0	1.5	2.0	2.5	3.0
h	2.15	5.4	7.35	8	7.35	5.4	2.15

b) 2.15 m (First y-value in table.)

c) By symmetry, the vertex is (1.5, 8).
The equation is of the form $y = a(x-1.5)^2 + 8$
To find a, substitute the coordinates of a point, such as (1, 7.35), into the equation.

$$7.35 = a(1-1.5)^2 + 8$$
$$-0.65 = 0.25a$$
$$a = \frac{-0.65}{0.25}$$
$$a = -2.6$$

The equation is $y = -2.6(x-1.5)^2 + 8$.

Chapter 4 Practice Test

Chapter 4 Practice Test **Question 1 Page 228**

B (The highest power of x is x^2.)

Chapter 4 Practice Test **Question 2 Page 228**

C [Speed is the first difference for distance. It is constant so speed is linear and not quadratic.]

Chapter 4 Practice Test **Question 3 Page 228**

C (Vertex is at (15, 3).)

Chapter 4 Practice Test **Question 4 Page 228**

C [(15 is the greatest distance.)

Chapter 4 Practice Test **Question 5 Page 228**

B (5 is the greatest factor.)

Chapter 4 Practice Test **Question 6 Page 228**

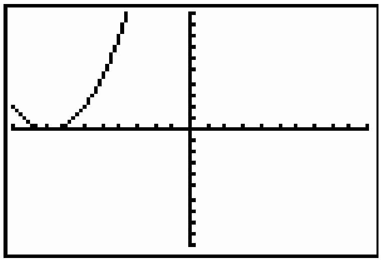
B

Chapter 4 Practice Test **Question 7 Page 228**

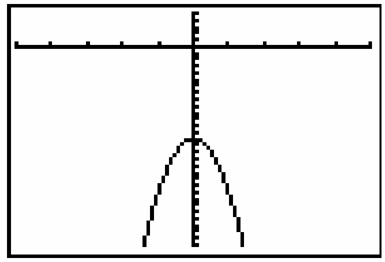
The first differences increase by a constant amount and the second differences are a constant.

Chapter 4 Practice Test **Question 8 Page 228**

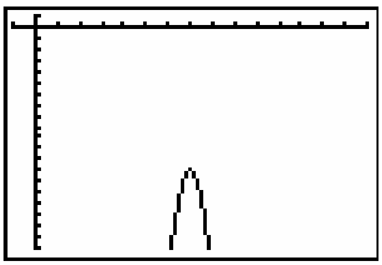
a) vertically compressed; translated 8 units to the left



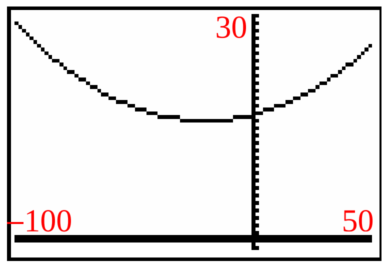
b) reflected in the x-axis; vertically stretched; translated 14 units down



c) reflected in the x-axis; vertically stretched; translated 7 units to the right and 13 units down



d) vertically compressed, translated 20 units to the left and 16 units up



Chapter 4 Practice Test **Question 9 Page 228**

a) The vertex is (35, 25). The equation is of the form $y = a\left(x - 35\right)^2 + 25$.

To find a, substitute the coordinates of the point (0, 0) or (70, 0) into the equation.

$$\begin{aligned} 0 &= a(0-35)^2 + 25 \\ -25 &= 1225a \\ a &= \frac{-25}{1225} \\ a &= -\frac{1}{49} \end{aligned}$$

The equation is $y = -\frac{1}{49}(x-35)^2 + 25$.

b) Substitute $x = 50$ into the equation.

$$\begin{aligned} y &= -\frac{1}{49}(50-35)^2 + 25 \\ y &= \frac{-225}{49} + 25 \\ y &\doteq 20 \end{aligned}$$

$$y = -\frac{1}{49}(x-35)^2 + 25$$

The soccer ball's height is approximately 20 m.

Chapter 4 Practice Test **Question 10 Page 229**

a)

Number of Layers	Total Number of Pennies
1	1
2	3
3	6
4	10
5	15
6	21
7	28
8	36
9	45
10	55

b) Answers may vary. For example:
The number of pennies equals the number of layers squared plus the number of layers, all divided by 2.

c) If you continue the table, the values are:

Number of Layers	Total Number of Pennies
11	66
12	78
13	91
14	105

A triangle of 14 layers requires 105 pennies.

d) ***Method 1***

The table could be extended for another 36 rows, but this method is tedious.

Method 2

This relation is quadratic, since the second differences are a constant, namely 1.
Try to find an equation to model this relation.
A sketch of the data suggests the vertex may be (0, 0).
The equation would be of the form $y = ax^2$
The point (1, 1) is on the curve, so a = 1. This gives the equation $y = x^2$.
Since no other points satisfy the equation, this is not the correct relation.
A careful sketch suggests the vertex to be $\left(-\frac{1}{2}, -\frac{1}{8}\right)$.
The equation would be of the form $y = a\left(x + \frac{1}{2}\right)^2 - \frac{1}{8}$
The point (1, 1) is on the curve, so $a = \frac{1}{2}$.
This gives the equation $y = \frac{1}{2}\left(x + \frac{1}{2}\right)^2 - \frac{1}{8}$.
All other points satisfy this equation.
Substitute x = 50 into the equation.

$$y = \frac{1}{2}\left(x + \frac{1}{2}\right) - \frac{1}{8}$$
$$y = \frac{1}{2}\left(50 + \frac{1}{2}\right) - \frac{1}{8}$$
$$y = 1275$$

The triangle with 50 layers has 1275 pennies.

Method 3

Using the rule found in part b), $y = \frac{x^2 + x}{2}$.
When x = 50, y = 1275.

The triangle with 50 layers has 1275 pennies.

Chapter 4 Practice Test **Question 11 Page 229**

a) Substitute d = 0 into the equation to find h.

$h = -0.2(0 - 2.5)^2 + 4.25$

$h = -0.2(6.25) + 4.25$

$h = 3$

The initial height is 3 m above the ground.

b) The greatest height is at the vertex, which is (2.5, 4.25).

The greatest height above the grounds is 4.25 m and this occurs when the horizontal distance is 2.5 m.

c) Check if the point (6, 3) is on the parabola.
Substitute d = 6 into the equation to find h.

$$\begin{aligned} h &= -0.2(6 - 2.5)^2 + 4.25 \\ &= -0.2(12.25) + 4.25 \\ &= 1.8 \end{aligned}$$

When d = 6, the height is 1.8 m, not 3 m.
The basketball will not go through the net.