Chapter 1 - Polynomial Functions

Power Functions

- A **polynomial expression** has the form:

$$a_nx^n+a_{n-1}x^{n-1}+a_{n-2}x^{n-1}+ \dots + a_3x^3+ a_2x^2+ a_1x+ a_0$$

- Where:
  - **Exponent** (n) is a whole number
  - x is the **variable really**
  - the **coefficients** $a_0,a_1,\dots,a_n$ are real numbers
  - the **degree** of the function is n, the exponent of the greatest power of x.
  - a_n, the coefficient of the greatest power of x, the **leading coefficient**
  - a_0, the term without a variable, the **constant term.**
  - A **polynomial function** has the form:

$$f(x) = a_nx^n+a_{n-1}x^{n-1}+a_{n-2}x^{n-1}+ \dots + a_3x^3+ a_2x^2+ a_1x+ a_0$$

- A **power function** is a polynomial of the form $y=ax^n$, where n is a whole number. They are often the base of transformations which build on top.
- **Even-degree** power functions have line symmetry in the y-axis.
- **Odd-degree** power functions have point symmetry about the origin.

Characteristics of a Polynomial Function

- **Odd Degree Functions**
  - Positive leading coefficient (+)
    - Graph end behavior extends from Q3 – Q1.
- Negative Leading coefficient (-)
  - Graph end behavior extends from Q2 – Q4.
- Odd degree polynomials have atleast 1 x-intercept, and up to a maximum of n x-intercepts, where n is the degree of the function.
- Domain of all odd-degree polynomial functions is {x E R}, and the range is {y E R}.
- Odd-Degree Functions have no max/min points.
- Odd-Degree functions may have point symmetry.
- **Even Degree Functions**
  - Positive leading coefficient (+)
    - Graph end behavior extends from Q2 – Q1.
    - Will have a minimum point.
    - Range is {y E R, y <= a}, where a is the minimum value of the function.
- Negative Leading coefficient (-)
  - Graph end behavior extends from Q3 – Q4
  - Will have a maximum point.
  - Range is {y E R, y >= a}, where a is the maximum value of the function
- Even-Degree polynomials may have zero to a maximum of n-intercepts, where n is the degree of the function.
- The domain of all even-degree polynomials is {x E R}.
- Even-Degree polynomials may have line symmetry.
- For any polynomial function of degree n, the nth differences
  - Are equal or constant

    - Have the same sign as the leading coefficient
    - Are equal to a[n!], where a is the leading coefficient.
    - Constant Difference = leading coefficient[degree or nth differences !]
        - D = a [n!]

Equations and graphs of a polynomial function

- The graph of a polynomial function can be sketched using the x-intercepts, the degree of the function, and the sign of the leading coefficient
- The x-intercepts of the graph of a polynomial function are the roots of the corresponding polynomial equation
- When a polynomial is in factored form, the zeros can be easily determined from the factors. When a factor is repeated n times, the corresponding zero has order n.
- The graph of a polynomial function changes sign only at x-intercepts that correspond to zeros of odd order. At x-intercepts that correspond to zeros of even order, the graph touches but does not cross the x-axis.
- An even function satisfies the property f(-x) = f(x) for all x in its domain and is symmetric about the y-axis. An even-degree polynomial function is an even function if the exponent of each term is even.
- An even function satisfies the property f(-x) = -f(x) for all x in its domain and is rotationally symmetric about the origin. An odd-degree polynomial function is an odd function if the exponent of each term is odd.

Determining Odd or Even Functions

Even Functions

- In even functions, f(x)=f(-x), by algebraically proving it.
- They are always symmetrical along the Y-axis
- Their exponents always add to an even number

Odd functions

- In odd functions, f(-x) = -f(x), by algebraically proving it.
- Always point symmetry at the origin

How to graph polynomial functions

1. Determine degree of function (This will allow you to know what type it is)
2. Determine and graph the X-Intercepts
3. Look at the sign of the coefficient and determine end behavior
4. Look at each intercept and determine if it’s a single, double, or triple root.
5. Graph the lines with everything above in mind

Transformations of a polynomial function

- Translations occur in the form:

$$y = a[k(x - d)]^n + c$$

- **a**
  - a<0, reflection along the x-axis. Graph opens downwards and creates a maximum point
  - a>1, vertical stretch by a factor of a.
  - 0<a<1, vertical compression by factor of a.
  - a>0, no reflection. Graph opens upwards and creates a minimum point.
- **k**
  - k<0, reflection along y-axis
  - 0<k<1, horizontal stretch by a factor of 1/k
  - k>1, horizontal compression by factor of 1/k
- **c**
  - C >0, translations C units up
  - C <0, translations |c| units down
- **d**
  - d>0, translations d units right
  - d<0, translations d units left
  - -d is shown as x+d in the equation
  - d is shown as x-d in the equation
  - Remember to factor out k, if necessary, to obtain the appropriate d value.

Slopes of Secants and average rate of change

- **Rate of change** is a measure of how quickly one quantity changes with respects to another quantity.
- Average rates of change:
  - Represent the rate of change over a specified interval
  - Correspond to the slope of a secant between two points $P_1(x_1,y_1)$ and $P_2(x_2,y_2)$ on a curve.
  - An average rate of change can be determined by calculating the slope between the two points given using the equation.

Slopes of tangents and instantaneous rate of change

- An **instantaneous rate of change** corresponds to the slope of a tangent to a point on a curve.
- An approximate value for an instantaneous rate of change at a point may be determined using:
  - A graph, either by estimating the slope of a secant passing through that point or by sketching the tangent and estimating the slope between the tangent point and a second point on the approximate tangent line.
  - A table of values by estimating the slope between a point and a nearby point in the table
  - An equation, by estimating the slope using very short intervals between the tangent point and a second point found using the equation.