5 Relationships

GETTING STARTED

Words You Need to Communicate Effectively

Match each term on the left with its example or definition on the right.

a) relation — iii

b) linear relation — v

c) table of values — i

d) scatter plot — iv

e) trend — ii

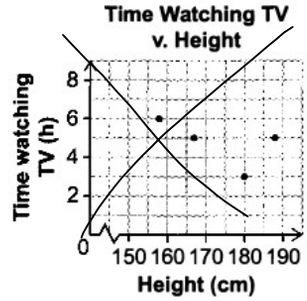
i)

Height (cm)	Time watching TV (h)
180	3
167	5
158	6
188	5

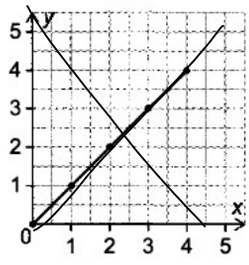
ii) a description of the general direction for data in a table or scatter plot

iii) a description of how two variables are connected

iv)



v)



Connections You Need for Success

PLOTTING POINTS

To plot a point, look at its coordinates.

The first, or x, coordinate tells how far to move left or right from the origin, (0, 0).

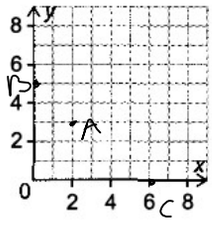
The second, or y, coordinate tells how far to move up or down from the origin.

Example

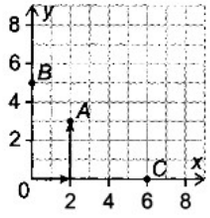
Plot and label each point.

- $A(2, 3)$
- $B(0, 5)$
- $C(6, 0)$

x y
A(2, 3)



Solution



I started at the origin for each point.

For point $A(2, 3)$, I moved 2 units to the right. Then I moved 3 units up.

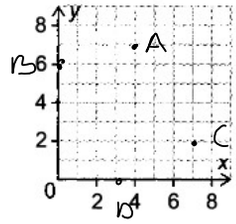
For point $B(0, 5)$, I moved 0 units to the right. Then I moved 5 units up. This point is on the y-axis.

For point $C(6, 0)$, I moved 6 units to the right. Then I moved 0 units up. This point is on the x-axis.

Your Turn

1. Plot and label each point.

- $A(4, 7)$
- $B(0, 6)$
- $C(7, 2)$
- $D(3, 0)$



CREATING A SCATTER PLOT

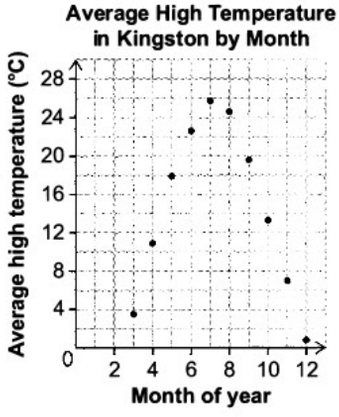
A scatter plot is a graphical method of showing the relationship between two variables in which points are plotted on a coordinate grid.

Example

Create a scatter plot for the average high temperature for Kingston versus the month, using the table of values.

Average Monthly High Temperature for Kingston		
Month	**Month number**	**Average high temperature (°C)**
March	3	3.5
April	4	10.9
May	5	17.9
June	6	22.6
July	7	25.7
August	8	24.6
September	9	19.9
October	10	13.3
November	11	7.0
December	12	0.8

Solution



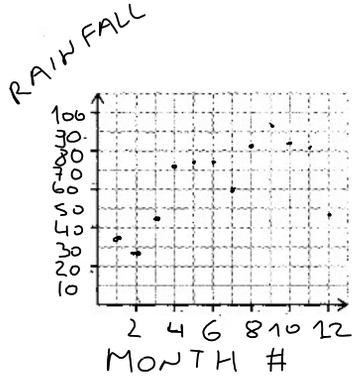
The temperature appears to depend on the month number, so I labelled the horizontal axis as the month number.

I chose a scale to use on each axis by looking at the range of numbers in the table. I used the data from month number and average high temperature in each row to create ordered pairs and plot points.

Your Turn

2. Create a scatter plot for the average monthly rainfall for Kingston versus the month, using the table of values.

Average Monthly Rainfall for Kingston		
Month	**Month number**	**Average monthly rainfall (mm)**
January	1	35
February	2	29
March	3	46
April	4	71
May	5	75
June	6	75
July	7	60
August	8	82
September	9	94
October	10	85
November	11	81
December	12	49



SUBTRACTING INTEGERS

One strategy to subtract integers is to add the opposite.

Example

Subtract.

a) $7 - (-2)$ **b)** $-10 - (+3)$

Solution

a) $7 - (-2) = 7 + (+2)$

$7 - (-2) = 9$

b) $-10 - (+3) = -10 + (-3)$

$-10 - (+3) = -13$

Your Turn

3. Subtract.

a) $5 - (-2) =$ ____

b) $7 - (-7) =$ ____

c) $-5 - (-4) =$ ____

d) $-7 - (-4) =$ ____

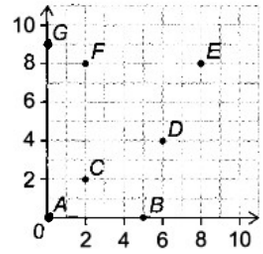
e) $-2 - (-2) =$ ____

f) $8 - 8 =$ ____

Practice

4. State the coordinates of each point.

A (0,0)
B (5,0)
C (2,2)
D (6,4)
E (8,8)
F (2,8)
G (0,9)

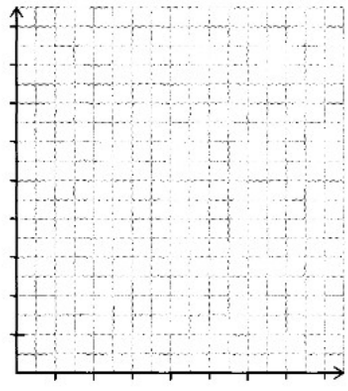


5. This table tracks the number of chin-ups Jim completed in 30 s.

Time (s)	0	5	10	15	20	25	30
Chin-ups	0	4	7	10	13	15	17

a) Create a scatter plot of this data on the grid provided.

b) Do you think a relationship exists between chin-ups and time? Explain.



6. Determine the difference.

a) $26 - (-10)$

b) $-15 - 8$

c) $19 - 12$

d) $-12 - (-12)$

5.1 Interpreting Graphs

KEEP IN MIND

- A scatter plot is a graph that shows relationships between two variables, when a relationship exists.
- When the points form a pattern, there is a relationship between the variables.
- If there is no relationship, the points will be randomly scattered.

Example

Trevor and Tyra tracked the number of home runs her baseball team hit over the season.

Games played	0	2	6	7	10	14	18	21
Total home runs	0	3	5	9	9	14	22	25

a) Create a scatter plot to represent the data.

b) What does each point represent?

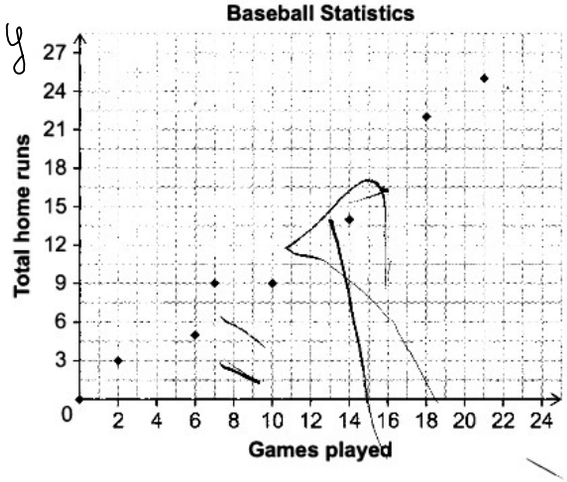
c) How many home runs were hit after 10 games were played?

d) Why might the total home runs have stayed the same from game 7 to game 10?

e) Describe any trends in the data. Justify your answer.

Solution

a)



I thought the number of home runs would depend on the number of games played. I labelled the horizontal axis "Games played."

There were 21 games, so I labelled the horizontal axis by 2s up to 24.

There were 25 home runs, so I labelled the vertical axis by 3s up to 27.

I plotted each point using the data from the table as ordered pairs (games played, total home runs).

I wrote a descriptive title.

b) Each point represents the number of home runs scored after a certain number of games have been played.

c) The coordinates of this point are (10, 9). After 10 games had been played, 9 home runs had been scored.

I located 10 on the horizontal axis. Then I went straight up to the point on the graph and looked at its home-run coordinate.

d) Perhaps the players who had hit home runs in earlier games were injured for games 7 to 10.

I think there could be several reasons why the number of home runs stayed the same, so I tried to think of one that made sense.

e) Generally, as the number of games increased, the total number of home runs also increased. This is an increasing trend.

I noticed that the points went upward to the right, although not in a perfectly straight line.

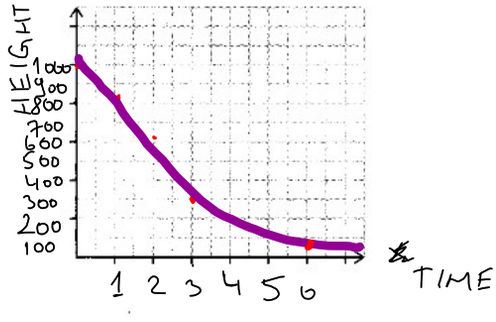
Practice

1. Taylor and her family took a hot air balloon ride. The table shows the time the balloon took to descend and its height from the ground.

Time (min)	0	1	2	3	4	5	6
Height above ground (ft)	1000	820	630	300	213	105	0

90 m

a) Create a scatter plot to display the data.



b) Does a relationship exist between height and time? Explain.

AS TIME PASSES, HEIGHT DECREASES.

c) Predict how high the balloon was from the ground 6 min after the descent began. Include this value on the scatter plot.

2. For each of the following statements, fill in the blanks to make the statement true.

Word bank:
increasing I
decreasing D
no NO

a) The age of a computer and its value would have a(n) D relationship.

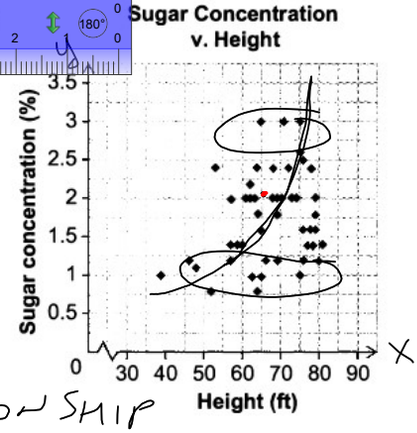
b) The amount of time you spend studying and the mark you get would have a(n) I relationship.

c) The number of cars you wash at a school car wash and the amount of money you make would have a(n) I relationship.

d) Your height and how much time you spend exercising would have NO relationship.

e) The distance you drive and the volume of gas in your gas tank would have a(n) D relationship.

3. This scatter plot shows the sugar concentration of trees according to their height. Use the plot to answer each question.



a) Describe the meaning of the point (65, 2.1).

HEIGHT OF THE TREE WITH SUGAR CONC. OF 2.1%

b) Describe any trends in the data. Justify your answer.

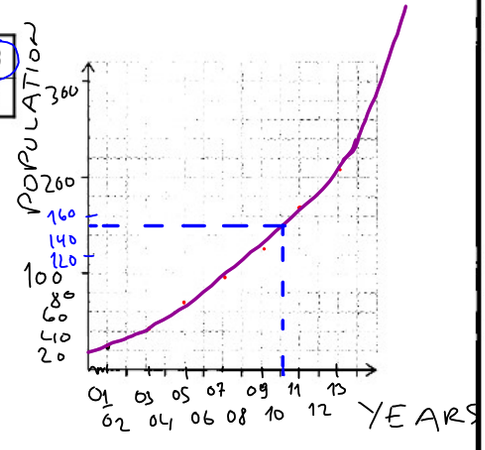
UP AND TO THE RIGHT

c) Does a relationship exist between sugar concentration and tree height? Explain.

NO STRONG RELATIONSHIP

4. A local park with a small lake attracts Canada geese. Officials have kept track of the population over time.

Year (X)	2001	2003	2005	2007	2009	2011	2013
Population (Y)	26	42	68	98	122	166	210



X	Y
2001	26
2003	42

a) Create a scatter plot of the data.

b) Does a relationship exist between the goose population and time? Explain.

AS YEARS PASS, POPULATION OF GEESE INCREASES

c) Estimate what the goose population was in 2010.

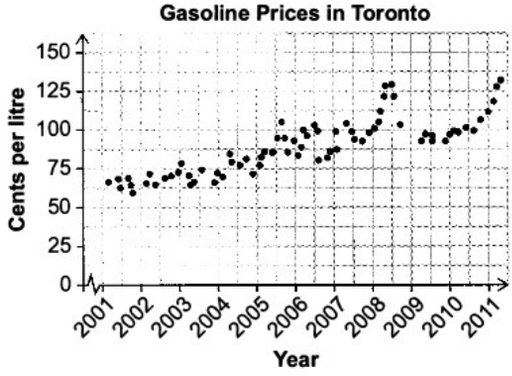
150

INTERPOLATION

EQAO Preparation

MULTIPLE CHOICE

5. This scatter plot shows gasoline prices from 2001 to 2011 at various filling stations in Toronto, Ontario.



Which statement about the trend is true?

a Gasoline prices increased, then decreased.

b Gasoline prices increased.

c Gasoline prices decreased.

d Gasoline prices did not change over the years.

6. Which situation can best be described by an increasing trend?

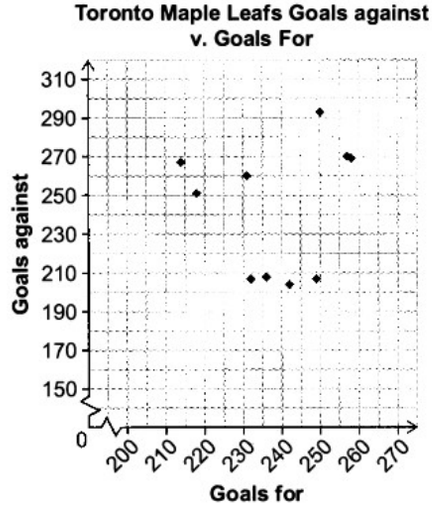
a the depth of snow as the temperature rises above 0 °C

b a person's height past the age of 30

c the hours he or she works and the amount a person earns

d the age of a refrigerator and its resale value

7. This scatter plot compares goals scored by the Toronto Maple Leafs to goals scored against them in the seasons from 2000–2001 to 2010–2011.



Which statement is true?

a There is no relationship between goals against and goals for.

b As the number of goals for decreases, the number of goals against increases.

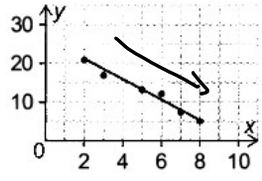
c The number of goals for stays the same, no matter how many goals against occurred.

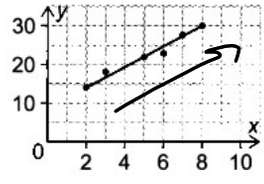
d As the number of goals for increases, the number of goals against increases.

5.2 Representing and Interpreting Linear Relations

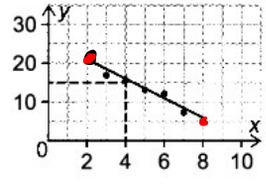
KEEP IN MIND

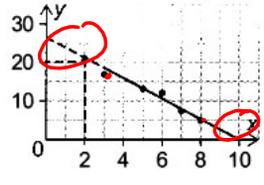
- A linear relation is a relationship in which the graph forms a straight line.
- When a scatter plot shows a linear trend, a line of best fit can be used as a model for the relationship.





- You can estimate a value between two known values of a linear relation by interpolating.
- You can predict a value before or after known values of a linear relation by extrapolating.





- To make predictions, you can model the data with a line of best fit.
  - The line should go through as many points as possible.
  - The line must follow the trend in the data.
  - There should be approximately the same number of points above and below the line.
  - You should not use only the beginning and end point to draw a line of best fit.
- Making a prediction **in**side the data is called **in**terpolation. This type of prediction is usually reliable.
- Making a prediction **ex**ternal to the data is called **ex**trapolation. When extrapolating, you extend the line of best fit beyond your data, so this type of prediction is usually less reliable.

Example

The operator of an ice-cream truck keeps track of how much ice cream is sold on days with different temperatures. She records the sales for the past 10 days.

x	Temperature (°C)	14.4	16.2	11.8	15.9	19.0	25.8	22.4	15.3	18.1	12.4
y	Sales ($)	215	325	185	332	406	698	421	356	401	190

T	S
14.4	215
16.2	325

a) Create a scatter plot and draw a line of best fit.

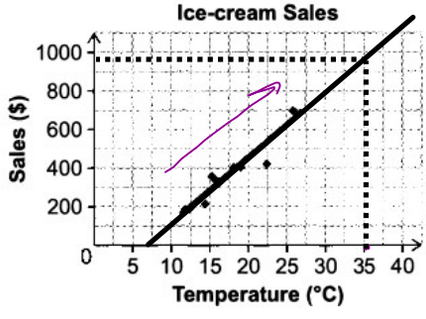
b) Determine if there is a trend in the data, and, if so, what kind. Explain.

c) The weather forecast says the temperature will be 35 °C today. Estimate what sales the operator will have today.

d) If the operator wants to make $500, what does the temperature have to be that day?

Solution

a)

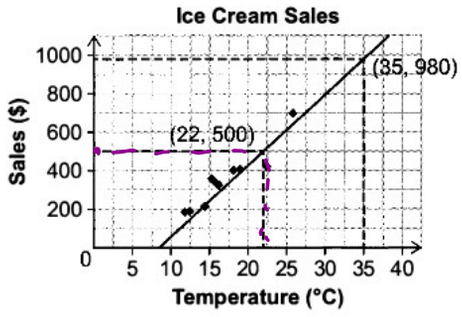


I created the scatter plot:
- I wrote a descriptive title.
- I wrote labels on the axes.
- I determined appropriate scales.
- I plotted the data points accurately.

I drew the line of best fit:
- I used a clear plastic ruler.
- I drew my line through as many points as possible.
- My line followed the direction of the data points, up and to the right.
- I tried to get about the same number of points above and below the line.
- I used more than just the beginning and end points to draw my line.

b) My line of best fit shows that there is a linear trend. Since the line rises from left to right, it is also an increasing trend. As the temperature increases her ice-cream sales also increase.

c),d)



When it is 35 °C, there should be about $980 in sales.

To achieve $500 in sales, the temperature should be about 22 °C.

I extended the line of best fit.

I extrapolated to estimate what the sales might be when it is 35 °C.

I went to 35 on the Temperature axis. Then I went straight up to the line of best fit.

From there, I went left to the Sales axis.

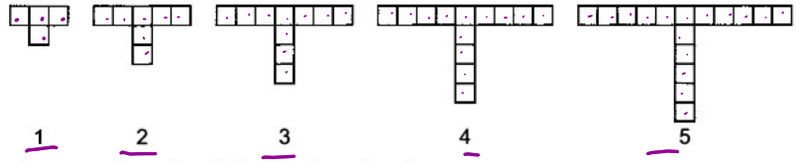
I interpolated to estimate what the temperature might be when the sales are $500.

I went to 500 on the Sales axis. Then I went straight right to the line of best fit.

From there, I went straight down to the Temperature axis.

Practice

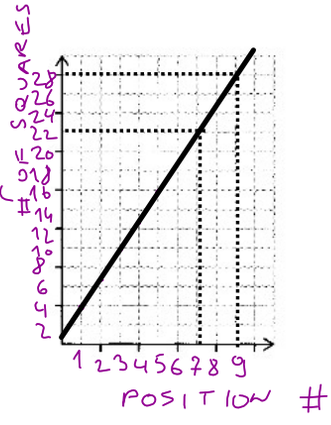
1. The first five positions in a pattern are shown.



a) Complete the table of values for the pattern.

Position number	1	2	3	4	5	6	7	8	9
Number of squares	4	7	10	13	16	19	22	25	28

b) Create a scatter plot and draw a line of best fit.



c) Determine if there is a trend in the data, and, if so, what kind. Explain.

UP AND TO THE RIGHT, AS THE POSITION # INCREASES, THE # OF SQUARES INCREASES AS WELL

d) Extrapolate to determine how many squares position 7 will have.

22 SQUARES

e) Extrapolate to determine which position has 28 squares.

POSITION 9

EQAO Preparation

MULTIPLE CHOICE

2. This table of values shows the wrist circumferences and heights of girls in a gym class.

Wrist circumference (mm)	Height (cm)
137.9	135.4
144.3	138.0
151.5	144.0
150.0	150.6
138.3	156.1
148.7	160.0
152.5	161.6
154.3	162.3
153.1	164.4
154.1	165.3
154.4	167.6
154.4	168.2

Which of these points was most likely determined by interpolation?

a (140, 140)

b (155, 171.2)

c (162.3, 154.3))

d (170, 143.2)

3. Use the data from question 2. Which of these points was most likely determined by extrapolation?

a (135, 137)

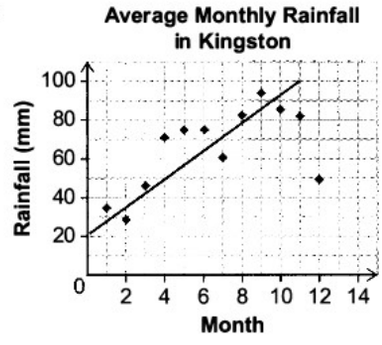
b (145, 150)

c (150, 156)

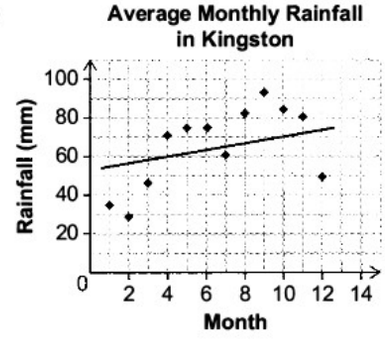
d (153, 160)

4. Which line of best fit is most accurate?

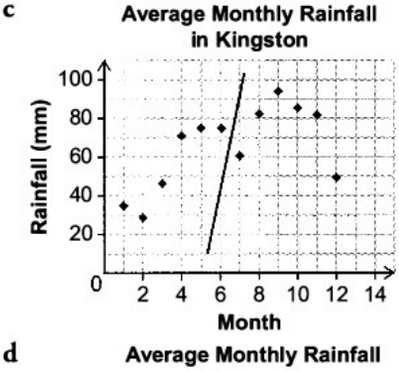
a



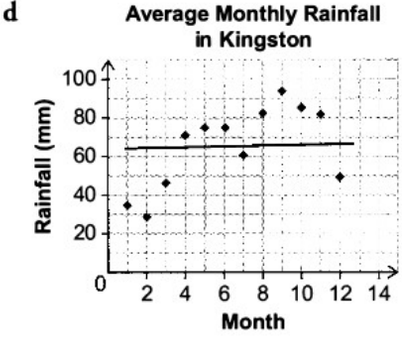
b



c



d



5.3 Representing and Interpreting Linear Relations with Technology

KEEP IN MIND

- For a set of data, you can use graphing technology to
  - create a scatter plot
  - plot the line of best fit, using a process called linear regression when there is a linear trend
- You can trace along a line of best fit to interpolate and extrapolate.
- Predictions made using graphs created with technology are more accurate than predictions made using hand-drawn graphs.

Example

Ashlynn's health class is studying healthy eating. She was asked to see if there is a relationship between the total fat and total calories in fast food sandwiches. She collected the following data:

Sandwich	Total fat (g)	Energy (calories)
hamburger	10	280
double hamburger	47	770
roast beef and Swiss cheese sandwich	42	810
double hamburger with cheese	49	800
veggie burger	16	380
six-inch sub with Italian cold cuts	25	480
ham sandwich	4	210
fried chicken sandwich	27	450
hamburger with cheese and bacon	29	580
children's size hamburger	9	270

a) Develop a hypothesis about a potential relationship.

b) Model the relationship using technology.

c) Interpolate or extrapolate to estimate the amount of calories in a sandwich with 20 g of fat.

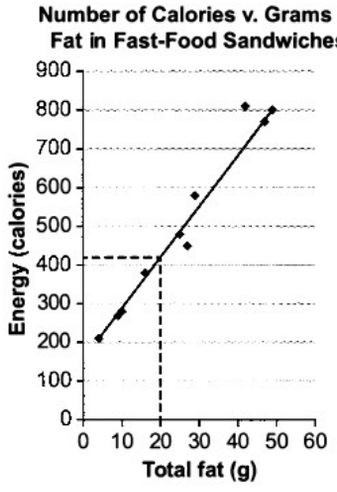
d) Conclude by stating whether your hypothesis was correct.

Solution

a) **Hypothesis:** There is a relationship between the total fat and the number of calories in a fast food sandwich.

I saw that, in the table, sandwiches high in fat are also high in calories. Sandwiches low in fat are also low in calories.

b),c)



I plotted the data using a spreadsheet. Then I used linear regression to draw the line of best fit. I could have used a graphing calculator instead.

I interpolated, because 20 g of fat lies within the range of known data.

A sandwich with 20 g of fat should have about 410 calories.

d) In conclusion, my hypothesis appears to be correct. There is an increasing linear relationship between total fat and total calories.

Practice

1. Fraser did push-ups every day for 2 wk. He kept a record of how many he could do.

Day	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Number of push-ups	3	3	5	7	7	9	12	15	19	21	24	28	29	32

a) Create a scatter plot of the data, using technology.

b) Develop a hypothesis about a potential relationship between the day and the number of push-ups.

c) Use linear regression to draw a line of best fit.

d) Estimate the number of push-ups he will be able to do on day 20.

2. Outline the strengths and weaknesses of using technology to represent and interpret linear relations.

3. Create a hypothesis for each topic, and state one way you could test it. Describe how you would display your results.

a) The school board wants to know whether a student's attendance is likely to affect the student's EQAO score.

b) A recording artist is deciding whether to release a new album on CD or not.

c) A company wants to know if customers are pleased with a new product.

4. Miguel did some research on the minimum wage for an adult in Ontario and found these facts.

Year	1986	1987	1988	1989	1990	1991	1992	1994
Minimum Wage ($)	4.35	4.55	4.75	5.00	5.40	6.00	6.35	6.70
Year	1995	2004	2005	2006	2007	2008	2009	2010
Minimum Wage ($)	6.85	7.14	7.45	7.75	8.00	8.75	9.50	10.25

a) Develop a hypothesis about whether there is a relationship between the data.

b) Make a scatter plot of the data using technology.

c) State whether your hypothesis was correct and explain your thinking.

5. Mei Lin measured her heart rate every school day for 2 wk after biking to school.

Day	1 (Mon.)	2 (Tue.)	3 (Wed.)	4 (Thu.)	5 (Fri.)
Heart rate (beats/min)	116	104	125	112	121
Day	6 (Mon.)	7 (Tue.)	8 (Wed.)	9 (Thu.)	10 (Fri.)
Heart rate (beats/min)	114	110	106	122	119

a) Make a scatter plot of the data using technology.

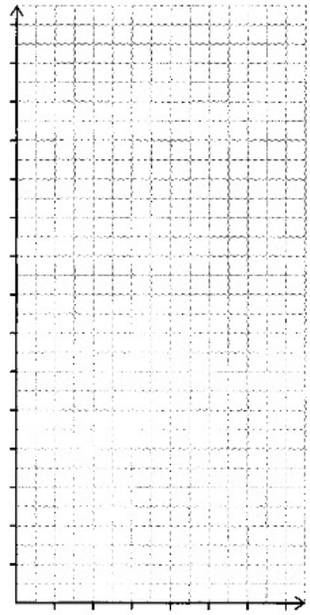
b) Use the scatter plot to describe the relationship, if any, between day and heart rate.

5 MID-CHAPTER REVIEW

5.1 Interpreting Graphs

1. This table of values shows the height of a plant on different days.

Time (days)	Height (mm)
1	5
3	13
5	20
7	33
9	51



a) Create a scatter plot to display the data.

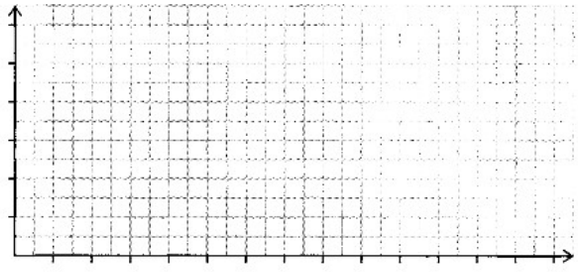
b) Describe the trend in the data.

c) Does a relationship exist?

2. This table shows Armour's scores after playing a video game.

Games played	2	5	8	9	13
Total score	200	950	1500	2300	5400

a) Create a scatter plot to display the data.



b) Describe the trend in the data.

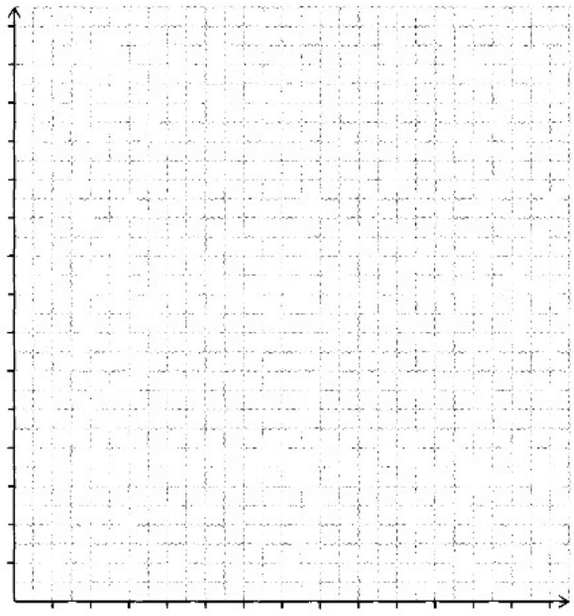
c) Does a relationship exist?

3. The manager of a convenience store records how many umbrellas she sells when it is raining.

Rainfall (mm)	0	2.5	4.0	9.8	11.0	12.0
Umbrellas sold	0	1	5	8	12	11

a) Determine if there is a trend in the data, and, if so, what kind. Explain.

b) Create a scatter plot and draw a line of best fit.



c) The weather forecast calls for 6.0 mm of rain today. Estimate using interpolation how many umbrellas the manager can expect to sell today.

d) Last Monday, the manager sold 10 umbrellas. Estimate the amount of rain that fell that day.

4. Cheryl measures the wrist circumference and foot length of some high school students.

Wrist circumference (mm)	150	160	146	170	180	185	150	150	160
Foot length (cm)	22	23	20	27	30	26	25	27	23
Wrist circumference (mm)	150	157	200	150	160	170	130	190	170
Foot length (cm)	25	22	23	24	24	27	23	28	26

a) Create a scatter plot of the data using technology.

b) Develop a hypothesis about a potential relationship between a student's wrist circumference and foot length.

c) Use linear regression to draw a line of best fit.

d) Estimate the foot length of a person whose wrist circumference measures 195 mm.

e) Estimate the wrist circumference of a person whose foot length measures 21 cm.

EQAO Preparation

MULTIPLE CHOICE

5. This scatter plot shows the prices of desktop computers in Ontario from 2002 to 2011.

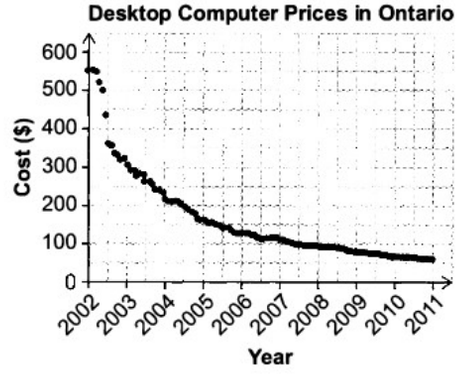
Which statement is true?

a Computer prices increased, then decreased.

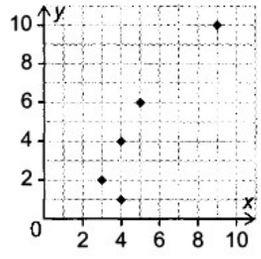
b Computer prices decreased in a linear fashion.

c Computer prices decreased in a non-linear fashion.

d There is no relationship between computer prices and time.



6. Consider this scatter plot.



Which table of values was used to create it?

a

x	y
2	3
1	4
4	4
6	5
10	9

b

x	y
3	2
4	1
4	4
5	6
9	10

c

x	y
1	2
2	4
3	6
4	8
5	10

d

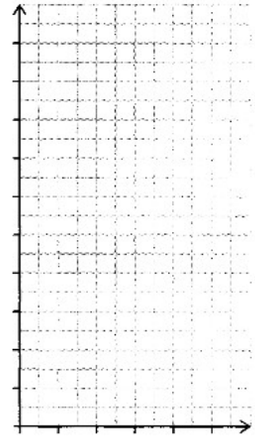
x	y
3	2
1	4
4	4
6	5
10	5

OPEN RESPONSE: MATH MARKS

7. Consider this data for a sample of Grade 9 students.

Science mark (%)	63	68	78	82	84	86	94
Math mark (%)	65	66	74	79	88	82	89

a) Do you think there is a relationship between math and science marks? Explain.



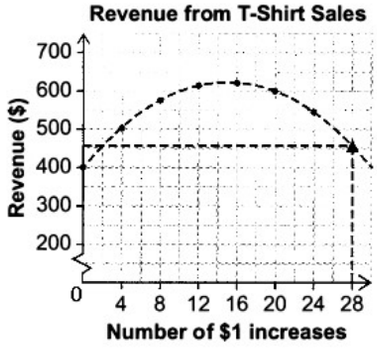
b) Create a scatter plot and draw a line of best fit.

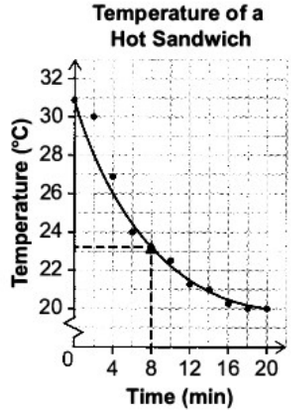
c) Estimate what mark a Grade 9 student might get in science if her math mark is 70%.

5.4 Representing and Identifying Non-linear Relations

KEEP IN MIND

- A non-linear relation is a relation in which the trend in the graph does not form a straight line.
- When a scatter plot shows a non-linear trend, you can use a curve of best fit as a model of the relationship to help you visualize the relationship between the variables.
- A curve of best fit should pass through as many points as possible, with the remaining points distributed equally on either side of the curve.
- You can estimate a value between two known values of a non-linear relation by interpolating.
- You can predict a value before or after known values of a non-linear relation by extrapolating.





Example

a) Graph the data in the table of values.

b) Is there a relationship between time and the height of the ball? Explain.

c) Is this relationship linear or non-linear? Explain.

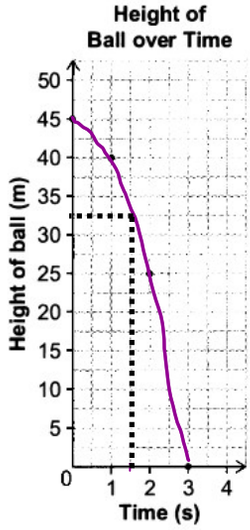
d) Is this relationship increasing or decreasing? Explain.

e) Talia thinks that the height of the ball will be 42 m at 1.5 s. Do you agree with her? Why or why not?

Time (s)	Height of ball (m)
0	45
1	40
2	25
3	0

Solution

a)



I decided that the horizontal axis should be time and the vertical axis should be height, because I think the height is dependent on time.

b) There is a relationship.

It seems that as the time increases, the height of the ball decreases.

c) The relationship is non-linear.

It appears that the values for height do not decrease by the same amount each time. The points appear to lie on a curve and not a straight line.

d) The relation is decreasing.

The values in the second column of the table get smaller as you move down the table.

e) Talia is wrong. The point (1.5, 42) does not follow the trend on the scatter plot.

It doesn't make sense that the ball would decrease in height after 1 s and then increase again after 1.5 s if the ball is falling.

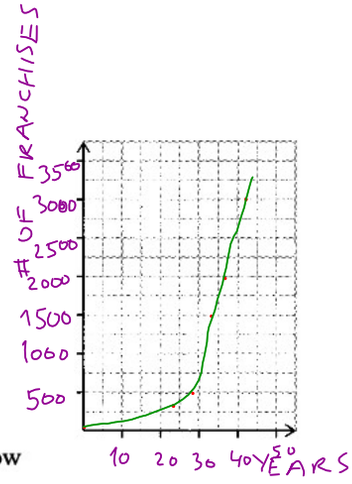
Practice

1. The total number of Tim Hortons franchises since 1964 is shown.

Years since 1964	Total number of franchises
0	1
23	300
27	500
33	1500
36	2000
42	3000

a) Create a scatter plot. Is there a relationship between the total number of franchises and the number of years since 1964?

AS THE YEARS INCREASE, # OF FRAN. INCREASES



b) Is this relationship linear or non-linear? Explain.

B/C WE USED A CURVE OF BEST FIT

2. The data below shows the winning distance of the women's discus throw at the Olympics since 1928.

Years since 1928	0	4	8	20	24	28	32
Winning distance (m)	39.62	40.58	47.63	41.92	51.42	53.69	55.10
Years since 1928	36	40	44	48	52	56	60
Winning distance (m)	57.27	58.28	66.62	69.00	69.96	65.36	72.30
Years since 1928	64	68	72	76	80	84	
Winning distance (m)	70.06	69.66	68.40	67.02	64.74	69.11	

a) Make a conjecture about the relationship between the winning distance and the years since 1928.

b) Create a scatter plot of the data.

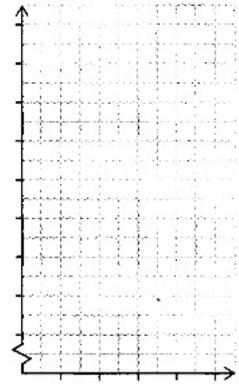
c) Based on your graph, was your conjecture correct? Explain.

YES, THERE IS AN INCREASE IN DISTANCE OVER YEARS

d) Draw a line of best fit or a curve of best fit. ✓

e) Predict the distance of the winning throw in 2024.

THE PREDICTED DISTANCE IN 2024 SHOULD BE ~ 66 m



EQAO Preparation

MULTIPLE CHOICE

3. Which table represents a non-linear relation?

a

n	C
0	2
1	6
2	18
3	54

c

n	C
0	1
1	4
2	7
3	10

b

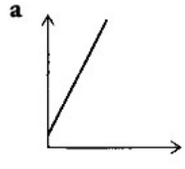
n	C
0	10
2	20
4	30
6	40

d

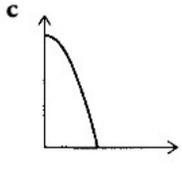
n	C
0	16
2	14
4	12
6	10

4. Which graph represents an increasing non-linear relation?

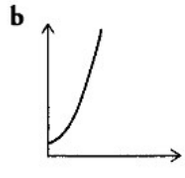
a



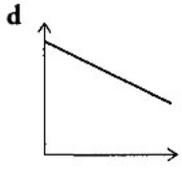
c



b



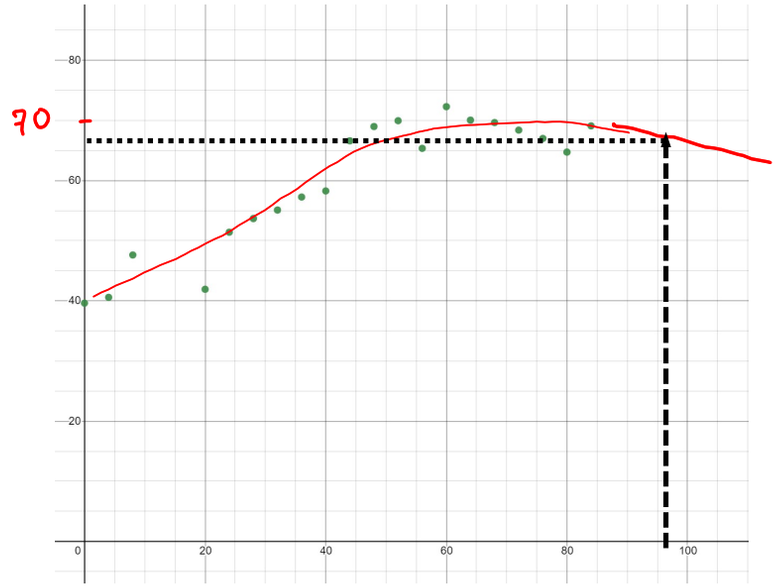
d



OPEN RESPONSE: SCIENCE EXPERIMENT

5. Determine whether the relationship between the number of bacteria and time is linear or non-linear. Explain.

Time (min)	Number of bacteria
5	8
10	15
15	30
20	61
25	120


70
80
60
40
20
0
20
40
60
80
100

5.5 Identifying and Modelling with Relations

KEEP IN MIND

- The first differences in a table of values are the differences between values of the dependent variable. You can use them to decide if a relation is linear or non-linear.
- When the values of the independent variable change by a constant amount and the values of the dependent variable also change by a constant amount (a constant rate of change), the relation is linear.
- If the values of the dependent variable do not change by a constant amount, the relation is non-linear, or there is no relationship.
- Creating a graphical model of a relation is useful when solving problems. You can determine additional information about the relation by interpolating and extrapolating.

x	y	First differences
0	8	
2	16	8
4	24	8
6	32	8

linear

x	y	First differences
0	5	
1	6	1
2	9	3
3	14	5

non-linear

Example

Each table of values represents a relationship. Use first differences to decide if the relationship is represented better by a line of best fit or by a curve of best fit.

a)

x	y
0	1
1	3
2	8
3	26
4	80
5	244

b)

x	y
0	58
1	55
2	51
3	48
4	45
5	41

Solution

a)

x	y	First differences
0	1	
1	3	3 − 1 = 2
2	8	8 − 3 = 5
3	26	26 − 8 = 18
4	80	80 − 26 = 54
5	244	244 − 80 = 164

I calculated the first differences. I saw that they are not the same, or even close.

This table of values is better represented with a curve of best fit.

I know that if the first differences are not the same, then the relationship is not linear.

b)

x	y	First differences
0	58	
1	55	55 − 58 = −3
2	51	51 − 55 = −4
3	48	48 − 51 = −3
4	45	45 − 48 = −3
5	41	41 − 45 = −4

I calculated the first differences.
I saw that they are close to the same.

This table of values is better represented with a line of best fit.

I know that if the first differences are the same, then the relationship is linear.

Practice

1. Each table of values represents a relationship. Use first differences to decide if the relation is represented better by a line of best fit or a curve of best fit.

a)

x	y	First differences
1	5	
2	11	11 − 5 = 6
3	17	17 − 11 = 6
4	23	23 − 17 = 6
5	29	29 − 23 = 6

LINEAR, USE LINE OF BEST FIT

b)

x	y	First differences
1	80	
2	40	40 − 80 = −40
3	20	20 − 40 = −20
4	10	10 − 20 = −10
5	5	5 − 10 = −5

NON-LINEAR

2. Sandra has been measuring the height of a flower for 5 wk. Her data is shown. Is the growth of the flower linear or non-linear? Use first differences to explain.

Week	Height (cm)	First differences
1	8.0	
2	10.1	10.1 − 8.0 = 2.1
3	11.9	11.9 − 10.1 = 1.8
4	14.0	14.0 − 11.9 = 2.1
5	15.8	15.8 − 14.0 = 1.8

3. The table of values shows the value of a car over several years.

Age of car (years)	Value of car ($)	First differences
0	20 000	
1	17 000	-3000
2	14 000	-3000
3	11 000	-3000
4	8 000	-3000

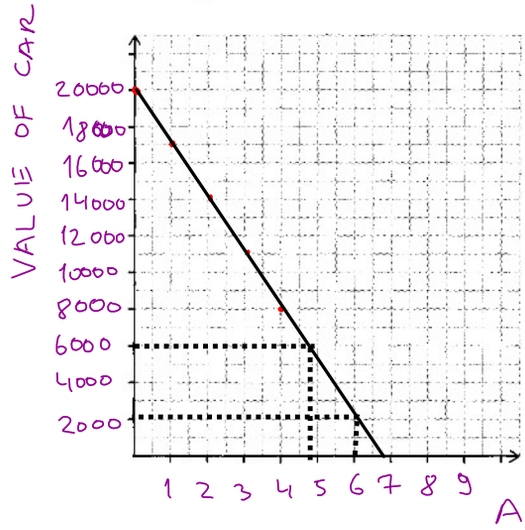
a) Does there appear to be a relationship between the car's value and its age? Explain.

AS THE CAR GETS OLDER, THE VALUE OF THE CAR GOES DOWN.

b) Is this relationship linear or non-linear? Explain.

B/C FIRST DIFFERENCES ARE CONSTANT (THE SAME)

c) Create a scatter plot of the data.



d) Draw a line of best fit or a curve of best fit for the relation.

e) Estimate the value of the car when it is 6 years old.

VALUE OF THE CAR IS $2000 AFTER 6 YRS

f) Estimate the age of the car if its value is $6000.

AGE OF THE CAR IS 4.8 YRS

EQAO Preparation

MULTIPLE CHOICE

4. Which table represents a linear relation?

a

n	C
0	8
1	14
2	6
3	35
4	9

c

n	C
0	55
1	50
2	45
3	40
4	35

b

n	C
0	100
2	50
4	25
6	13
8	7

d

n	C
0	15
2	10
4	14
6	11
8	13

5. Adam works as a nurse. This table of values shows his annual salary in the last 5 years.

Year	Annual salary ($)
1	30 000
2	32 100
3	34 200
4	36 300
5	38 400

If the trend continues, what will Adam's annual salary be in year 7?

a $44 700 **c** $46 800

b $42 600 **d** $40 500

OPEN RESPONSE: SCIENCE EXPERIMENT

6. Rachel collects data for a science experiment in which she is cooling a liquid. Her data is shown below.

Time (s)	Temperature (°C)	First differences
0	50	
30	45	
60	40	
90	35	
120	30	
150	25	

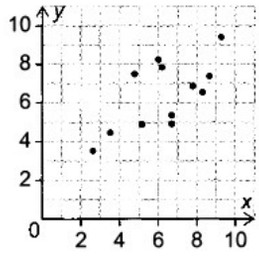
a) Determine whether the relationship between temperature and time is linear or non-linear. Provide your reasoning.

b) Predict the temperature of the liquid after 180 s.

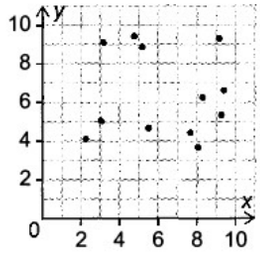
5 CHAPTER REVIEW

5.1 Interpreting Graphs

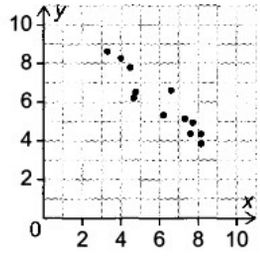
1. For each scatter plot, write the letter of the statement that best describes the relationship shown.



Letter: ____



Letter: ____



Letter: ____

A The relationship is linear and decreasing.

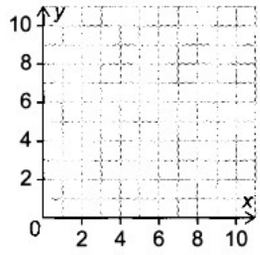
B The relationship is non-linear and increasing.

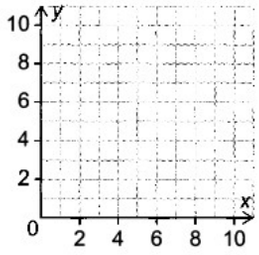
C The relationship is linear and increasing.

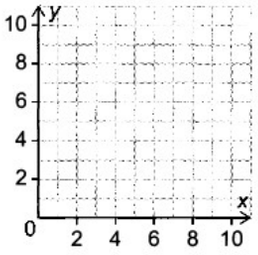
D There is no relationship.

2. Sketch an example of a scatter plot with each type of relationship.

a) non-linear and increasing **b)** linear and decreasing **c)** no relationship

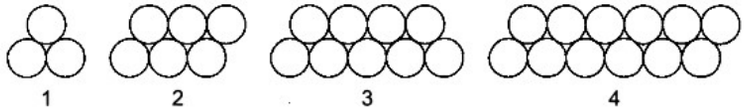






5.2 Representing and Interpreting Linear Relations

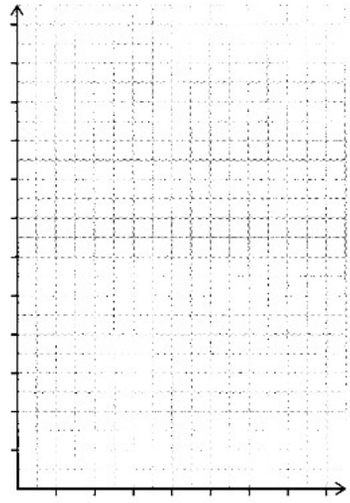
3. The first four positions in a pattern are shown.



a) Complete the table of values for the pattern.

Position number	1	2	3	4
Number of circles				

b) Determine if there is a trend in the data, and, if so, what kind. Explain.



c) Create a scatter plot and draw a line of best fit.

d) Extrapolate to determine how many circles position 7 will have.

e) Extrapolate to determine which position has 15 circles.

5.3 Representing and Interpreting Linear Relations with Technology

4. Beth has started to do an abdominal exercise called the "plank," where you try to hold the position shown for as long as possible. Beth records the time she can hold the plank each day.

Day	1	3	5	7	9	11	13
Time (s)	20	25	35	45	50	60	65

a) Make a scatter plot of the data using technology.

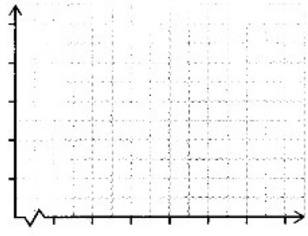
b) Develop a hypothesis about a potential relationship between the day and the length of time Beth can hold the plank.

5.4 Representing and Identifying Non-linear Relations

5. The table of values shows how long some students take to travel to school and the corresponding height of each student.

Travel time (min)	10	15	7	15	40	2	30	5	3	10
Height (cm)	159	168	150	167	201	185	179	172	182	155

a) Create a scatter plot of the data.



b) Is there a relationship between travel time and the height of the student? Explain.

c) Does it make sense to use interpolation to determine the travel time of a student whose height is 155 cm? Explain.

5.5 Identifying and Modelling with Relations

6. The table shows a tire's air pressure at specific altitudes.

Altitude (ft)	Air pressure (psi)	First differences
4000	13.3	
5000	12.8	
6000	12.3	
7000	11.8	

a) Is there a relationship between the altitude and air pressure? Explain.

b) Is this relationship linear or non-linear? Explain.

EQAO Preparation

MULTIPLE CHOICE

7. A ball is dropped from a building. The table of values shows the ball's height in metres.

Time (s)	Height (m)
0	250
1	245
2	230
3	205

Which choice best describes the situation?

a There is a non-linear, decreasing relationship between time and height, and at a time of 6 s, the ball will be 70 m above the ground.

b There is a linear, increasing relationship between time and height, and at a time of 6 s, the ball will be 100 m above the ground.

c There is no relationship between the ball's height and time, and it is not possible to estimate when the ball will be 70 m above the ground.

d There is a non-linear, increasing relationship between time and height, and at a time of 2.5 s, the ball will be about 218 m above the ground.

OPEN RESPONSE: FIRST DIFFERENCES

8. Christine was asked to determine the first differences for this set of data. Based on her answers, she says there is a non-linear relationship. Do you agree? Explain.

x	y	First differences
0	12	
4	0	−12
2	6	6
3	3	−3
1	9	6