

3.4 Solving Rational Equations and Inequalities

Solve: "Find the value of x that makes the statement true"

Solving Rational Equations:

Example:

a) $(3x-5) \cdot \frac{4}{3x-5} = 4(3x-5)$

$$\frac{4}{4} = 4(3x-5)$$

$$1 = 3x - 5$$

$$1 + 5 = 3x$$

$$6 = 3x$$

$$\boxed{x = 2} \quad x \neq \frac{5}{3}$$

b) $\frac{x-5}{x^2-3x-4} = \frac{3x+2}{x^2-1}$

$$x \neq 4, \pm 1$$

$$\frac{x-5}{(x-4)(x+1)} = \frac{3x+2}{(x-1)(x+1)}$$

$$\frac{x-5}{x-4} = \frac{3x+2}{x-1}$$

$$(x-1)(x-5) = (3x+2)(x-4)$$

$$x^2 - 5x - 1x + 5 = 3x^2 - 12x + 2x - 8$$

$$x^2 - 6x + 5 = 3x^2 - 10x - 8$$

$$x^2 - 3x^2 - 6x + 10x + 5 + 8 = 0$$

$$-2x^2 + 4x + 13 = 0$$

$$2x^2 - 4x - 13 = 0$$

$$x = \frac{4 \pm \sqrt{120}}{4}$$

$$= \frac{4 \pm 2\sqrt{30}}{4} = \frac{2 \pm \sqrt{30}}{2}$$

$$\frac{\sqrt{30 \times 4}}{\sqrt{30} \times \sqrt{4}} = \frac{\sqrt{30}}{2}$$

Practice:

a) $\frac{1}{x^2 - 2x - 7} = 1$

b) $\frac{2}{x-1} = \frac{5}{x+3}$

c) $\frac{x-3}{x-4} = \frac{x+2}{x+6}$

d) $\frac{1}{x} = \frac{x-34}{2x^2}$

e) $x - \frac{5}{x} = 4$

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2g)

$$\frac{x}{x-2} + \frac{1}{x+4} = \frac{2}{x^2-6x+8}$$

$$\frac{x(x+4) + (x-2)}{(x-2)(x+4)} = \frac{2}{(x-4)(x-2)}$$

$$x \neq 2, \pm 4$$

$$\frac{x^2 + 4x + x - 2}{x^2 + 4x - 2x - 8} = \frac{2}{x^2 - 6x + 8}$$

$$\frac{x^2 + 5x - 2}{x^2 + 2x - 8} = \frac{2}{x^2 - 6x + 8}$$

$$(x^2 + 5x - 2)(x^2 - 6x + 8) = 2(x^2 + 2x - 8)$$

$$x^4 - 6x^3 + 8x^2 + 5x^3 - 30x^2 + 40x - 2x^2 + 12x - 16 = 2x^2 + 4x - 16$$

$$x^4 - x^3 - 26x^2 + 48x = 0$$

$$x(x^3 - x^2 - 26x + 48) = 0$$

$$x(x-2)(x^2 + x - 24) = 0$$

$$\begin{array}{r|rrrr} 2 & 2 & -1 & -26 & 48 \\ & & 2 & 2 & -48 \\ \hline & 1 & 1 & -24 & 0 \end{array}$$

$$x = 0, \cancel{2}, \frac{-1 \pm \sqrt{97}}{2}$$

$$2g) \frac{x}{x-2} + \frac{1}{x+4} = \frac{2}{x^2-6x+8}$$

$$\frac{x(x+4) + (x-2)}{\cancel{(x-2)}(x+4)} = \frac{2}{(x-4)\cancel{(x-2)}}$$

$$\frac{x^2+4x+x-2}{x+4} = \frac{2}{x-4}$$

$$\frac{x^2+5x-2}{x+4} = \frac{2}{x-4}$$

$$(x-4)(x^2+5x-2) = 2(x+4)$$

$$x^3 + 5x^2 - 2x - 4x^2 - 20x + 8 = 2x + 8$$

$$x^3 + \underbrace{5x^2 - 4x^2}_{+x^2} - \underbrace{2x - 20x - 2x}_{-24x} + \underbrace{8 - 8}_0 = 0$$

$$x^3 + x^2 - 24x = 0$$

$$x(x^2 + x - 24) = 0$$

↓

$$x=0$$

↓ QUADRATIC FORMULA

$$x = \frac{-1 \pm \sqrt{97}}{2}$$

$$8f) \quad \frac{7x}{3x+3} - \frac{5}{4x-4} = \frac{3x}{2x+2} \quad x \neq \pm 1$$

$$\frac{7x(4x-4) - 5(3x+3)}{(3x+3)(4x-4)} = \frac{3x}{2x+2}$$

$$\frac{28x^2 - 28x - 15x - 15}{12x^2 - 12} = \frac{3x}{2x+2}$$

$$\frac{28x^2 - 43x - 15}{12x^2 - 12} = \frac{3x}{2x+2}$$

$$3x(12x^2 - 12) = (2x+2)(28x^2 - 43x - 15)$$

$$36x^3 - 36x = 56x^3 - 86x^2 - 30x + 56x^2 - 86x - 30$$

$$0 = 56x^3 - 36x^3 - 86x^2 + 56x^2 - 30x - 86x + 36x - 30$$

$$0 = 20x^3 - 30x^2 - 80x - 30$$

$$0 = 2x^3 - 3x^2 - 8x - 3$$

$$0 = (x+1)(2x^2 - 5x - 3)$$

$$0 = (x+1)(x-3)(2x+1)$$

$$x = \cancel{-1}, 3, -\frac{1}{2}$$

$$\begin{array}{r|rrrr} -1 & 2 & -3 & -8 & -3 \\ & & -2 & 5 & 3 \\ \hline & 2 & -5 & -3 & 0 \end{array}$$