Chapter 2 Polynomial Equations and Inequalities

Chapter 2 Prerequisite Skills

Chapter 2 Prerequisite Skills Question 1 Page 82

a) $$\begin{array}{r}124\\28\overline{)3476}\\\underline{28}\\67\\\underline{56}\\116\\\underline{112}\\4\end{array}$$

124 R4

b) $$\begin{array}{r}161\\37\overline{)5973}\\\underline{37}\\227\\\underline{222}\\53\\\underline{37}\\16\end{array}$$

161 R16

c) $$\begin{array}{r}147\\17\overline{)2508}\\\underline{17}\\80\\\underline{68}\\128\\\underline{119}\\9\end{array}$$

147 R9

147 R9

d) $$\begin{array}{r}358\\19\overline{)6815}\\\underline{57}\\111\\\underline{95}\\165\\\underline{152}\\13\end{array}$$

358 R13

Chapter 2 Prerequisite Skills **Question 2 Page 82**

a) $P(-1) = (-1)^3 - 5(-1)^2 + 7(-1) - 9$

$= -1 - 5 - 7 - 9$

$= -22$

b) $P(3) = (3)^3 - 5(3)^2 + 7(3) - 9$

$= 27 - 45 + 21 - 9$

$= -6$

c) $P(-2) = (-2)^3 - 5(-2)^2 + 7(-2) - 9$

$= -8 - 20 - 14 - 9$

$= -51$

d) $P\left(-\frac{1}{2}\right) = \left(-\frac{1}{2}\right)^3 - 5\left(-\frac{1}{2}\right)^2 + 7\left(-\frac{1}{2}\right) - 9$

$= -\frac{1}{8} - \frac{5}{4} - \frac{7}{2} - 9$

$= -13.875$

e) $P\left(\frac{2}{3}\right) = \left(\frac{2}{3}\right)^3 - 5\left(\frac{2}{3}\right)^2 + 7\left(\frac{2}{3}\right) - 9$

$= \frac{8}{27} - \frac{20}{9} + \frac{14}{3} - 9$

$= -\frac{169}{27}$

Chapter 2 Prerequisite Skills Question 3 Page 82

a) $(x^3 + 3x^2 - x + 1)(x - 2) + 5 = x^4 - 2x^3 + 3x^3 - 6x^2 - x^2 + 2x + x - 2 + 5$
$= x^4 + x^3 - 7x^2 + 3x + 3$

b) $(2x^3 - 4x^2 + x - 3)(x + 4) - 7 = 2x^4 + 8x^3 - 4x^3 - 16x^2 + x^2 + 4x - 3x - 12 - 7$
$= 2x^4 + 4x^3 - 15x^2 + x - 19$

c) $(x^3 + 4x^2 - x + 8)(3x - 1) + 6 = 3x^4 - x^3 + 12x^3 - 4x^2 - 3x^2 + x + 24x - 8 + 6$
$= 3x^4 + 11x^3 - 7x^2 + 25x - 2$

d) $(x - \sqrt{2})(x + \sqrt{2}) = x^2 + \sqrt{2}x - \sqrt{2}x - 2$
$= x^2 - 2$

e) $(x - 3\sqrt{5})(x + 3\sqrt{5}) = x^2 + 3\sqrt{5}x - 3\sqrt{5}x - 45$
$= x^2 - 45$

f) $(x - 1 + \sqrt{3})(x - 1 - \sqrt{3}) = x^2 - x - \sqrt{3}x - x + 1 + \sqrt{3} + \sqrt{3}x - \sqrt{3} - 3$
$= x^2 - 2x - 2$

Chapter 2 Prerequisite Skills Question 4 Page 82

a) $(x - 2)(x + 2)$

b) $(5m - 7)(5m + 7)$

c) $(4y - 3)(4y + 3)$

d) $3(4c^2 - 9) = 3(2c - 3)(2c + 3)$

e) $2(x^4 - 16) = 2(x^2 - 4)(x^2 + 4)$
$= 2(x - 2)(x + 2)(x^2 + 4)$

f) $3(n^4 - 4) = 3(n^2 - 2)(n^2 + 2)$

Chapter 2 Prerequisite Skills Question 5 Page 82

a) $(x + 3)(x + 2)$

b) $(x - 4)(x - 5)$

c) $(b + 7)(b - 2)$

d) $(2x + 3)(x - 5)$

e) $(2x - 3)^2$

f) $(2a - 1)(3a - 2)$

g) $(3m - 4)^2$

h) $(m - 3)(3m - 1)$

Chapter 2 Prerequisite Skills **Question 6 Page 82**

a) $(x - 5)(x + 3) = 0$
$x = -3$ or $x = 5$

b) $(x + 1)(4x - 3) = 0$
$x = -1$ or $x = \frac{3}{4}$

c) $4(4x^2 - 9) = 0$
$4(2x + 3)(2x - 3) = 0$
$x = -\frac{3}{2}$ or $x = \frac{3}{2}$

d) $9x^2 - 48x + 15 = 0$
$3(3x^2 - 16x + 5) = 0$
$3(3x - 1)(x - 5) = 0$
$x = \frac{1}{3}$ or $x = 5$

e) $8x^2 + 12x - 20 = 0$
$4(2x^2 + 3x - 5) = 0$
$4(2x + 5)(x - 1) = 0$
$x = -\frac{5}{2}$ or $x = 1$

f) $21x^2 - 10x + 1 = 0$
$(7x - 1)(3x - 1) = 0$
$x = \frac{1}{7}$ or $x = \frac{1}{3}$

Chapter 2 Prerequisite Skills **Question 7 Page 82**

a) $$x = \frac{-6 \pm \sqrt{6^2 - 4(5)(-1)}}{2(5)}$$
$$= \frac{-6 \pm \sqrt{36+20}}{10}$$
$$= \frac{-6 \pm \sqrt{56}}{10}$$
$$= \frac{-3 \pm \sqrt{14}}{5}$$
$x \doteq -1.3$ or $x \doteq 0.1$

b) $$x = \frac{7 \pm \sqrt{(-7)^2 - 4(2)(4)}}{2(2)}$$
$$= \frac{7 \pm \sqrt{49-32}}{4}$$
$$= \frac{7 \pm \sqrt{17}}{4}$$
$x \doteq 0.7$ or $x \doteq 2.8$

c) $$x = \frac{-2 \pm \sqrt{2^2 - 4(4)(-3)}}{2(4)}$$
$$= \frac{-2 \pm \sqrt{4+48}}{8}$$
$$= \frac{-2 \pm \sqrt{52}}{8}$$
$$= \frac{-1 \pm \sqrt{13}}{4}$$
$x \doteq -1.2$ or $x \doteq 0.7$

d) $$x = \frac{7 \pm \sqrt{(-7)^2 - 4(6)(-20)}}{2(6)}$$
$$= \frac{7 \pm \sqrt{49+480}}{12}$$
$$= \frac{7 \pm \sqrt{529}}{12}$$
$$= \frac{7 \pm 23}{12}$$
$x \doteq -1.3$ or $x = 2.5$

a) $y = a(x + 4)(x - 1)$

$2 = a[(-1) + 4][(-1) - 1]$

$2 = -6a$

$a = -\frac{1}{3}$

$y = -\frac{1}{3}(x+4)(x-1)$

b) $y = ax(x - 3)$

$6 = a(2)(2 - 3)$

$6 = -2a$

$a = -3$

$y = -3x(x - 3)$

c) $y = a(x + 3)(x - 4)$

$24 = a(3 + 3)(3 - 4)$

$24 = -6a$

$a = -4$

$y = -4(x + 3)(x - 4)$

d) $y = a(x + 1)(x - 5)$

$-10 = a(4 + 1)(4 - 5)$

$-10 = -5a$

$a = 2$

$y = 2(x + 1)(x - 5)$

e) $y = a(2x + 1)(2x - 3)$

$9 = a(2(0) + 1)(2(0) - 3)$

$9 = -3a$

$a = -3$

$y = -3(2x + 1)(2x - 3)$

Chapter 2 Prerequisite Skills **Question 9 Page 83**

a) i) x-intercepts are –4 and 1

ii) above the x-axis: $x < -4$ and $x > 1$
below the x-axis: $-4 < x < 1$

b) i) x-intercepts are –1, 1 and 2

ii) above the x-axis: $-1 < x < 1$ and $x > 2$
below the x-axis: $x < -1$ and $1 < x < 2$

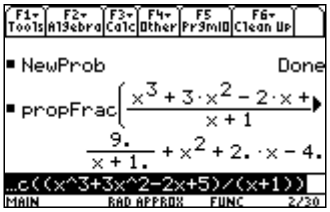
c) i) x-intercepts are –2, –1, 1, and 2

ii) above the x-axis: $-2 < x < -1$ and $1 < x < 2$
below the x-axis: $x < -2$ and $-1 < x < 1$ and $x > 2$

Chapter 2 Section 1 The Remainder Theorem

Chapter 2 Section 1 Question 1 Page 91

a)



```
■ NewProb                          Done
■ propFrac((x^3 + 3·x^2 - 2·x +▶)/(x + 1))
          9./(x + 1.) + x^2 + 2.·x - 4.
...c((x^3+3x^2-2x+5)/(x+1))
```

$$\frac{x^3+3x^2-2x+5}{x+1}=x^2+2x-4+\frac{9}{x+1}$$

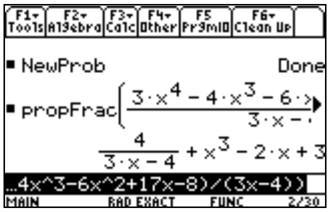
b) $x + 1 \neq 0$
$x \neq -1$

c) $x^3 + 3x^2 - 2x + 5 = (x + 1)(x^2 + 2x - 4) + 9$

d) $(x + 1)(x^2 + 2x - 4) + 9 = x^3 + 2x^2 - 4x + x^2 + 2x - 4 + 9$
$= x^3 + 3x^2 - 2x + 5$

Chapter 2 Section 1 Question 2 Page 91

a)



```
■ NewProb                          Done
■ propFrac((3·x^4 - 4·x^3 - 6·>▶)/(3·x - ·))
          4/(3·x - 4) + x^3 - 2·x + 3
...4x^3-6x^2+17x-8)/(3x-4))
```

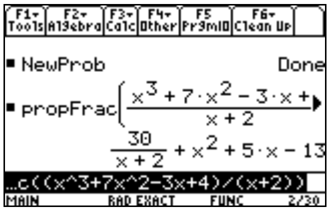
$$\frac{3x^4-4x^3-6x^2+17x-8}{3x-4}=x^3-2x+3+\frac{4}{3x-4}$$

b) $3x - 4 \neq 0$
$x \neq \frac{4}{3}$

c) $3x^4 - 4x^3 - 6x^2 + 17x - 8 = (3x - 4)(x^3 - 2x + 3) + 4$

d) $(3x - 4)(x^3 - 2x + 3) + 4 = 3x^4 - 6x^2 + 9x - 4x^3 + 8x - 12 + 4$
$= 3x^4 - 4x^3 - 6x^2 + 17x - 8$

a)



■ NewProb Done

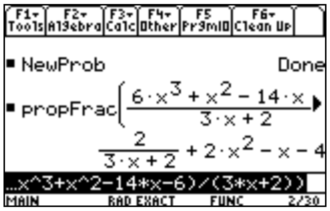
■ propFrac$\left(\frac{x^3+7\cdot x^2-3\cdot x+}{x+2}\right.$▸

$\frac{30}{x+2}+x^2+5\cdot x-13$

`...c((x^3+7x^2-3x+4)/(x+2))`

$$\frac{x^3+7x^2-3x+4}{x+2}=x^2+5x-13+\frac{30}{x+2},\ x\neq-2$$

b)



■ NewProb Done

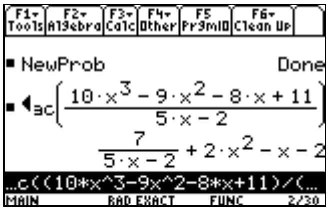
■ propFrac$\left(\frac{6\cdot x^3+x^2-14\cdot x}{3\cdot x+2}\right.$▸

$\frac{2}{3\cdot x+2}+2\cdot x^2-x-4$

`...x^3+x^2-14*x-6)/(3*x+2))`

$$\frac{6x^3+x^2-14x-6}{3x+2}=2x^2-x-4+\frac{2}{3x+2},\ x\neq-\frac{2}{3}$$

c)



■ NewProb Done

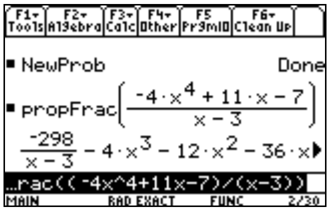
■ ◂ac$\left(\frac{10\cdot x^3-9\cdot x^2-8\cdot x+11}{5\cdot x-2}\right)$

$\frac{7}{5\cdot x-2}+2\cdot x^2-x-2$

`...c((10*x^3-9x^2-8*x+11)/(...`

$$\frac{10x^3-9x^2-8x+11}{5x-2}=2x^2-x-2+\frac{7}{5x-2},\ x\neq\frac{2}{5}$$

d)

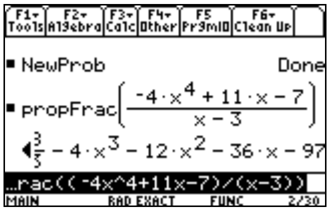


■ NewProb Done

■ propFrac$\left(\frac{^-4\cdot x^4+11\cdot x-7}{x-3}\right)$

$\frac{^-298}{x-3}-4\cdot x^3-12\cdot x^2-36\cdot x$▸

`...rac((-4x^4+11x-7)/(x-3))`



■ NewProb Done

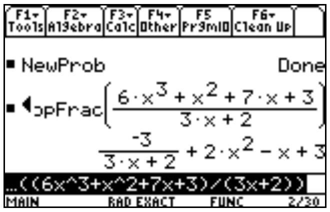
■ propFrac$\left(\frac{^-4\cdot x^4+11\cdot x-7}{x-3}\right)$

◂$\frac{3}{3}-4\cdot x^3-12\cdot x^2-36\cdot x-97$

`...rac((-4x^4+11x-7)/(x-3))`

$$\frac{-4x^4+11x-7}{x-3}=-4x^3-12x^2-36x-97-\frac{298}{x-3},\ x\neq3$$

e)



■ NewProb Done

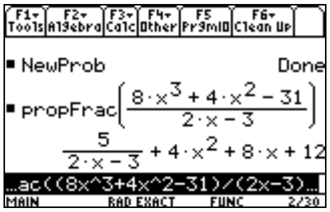
■ ◀pFrac$\left(\frac{6\cdot x^3+x^2+7\cdot x+3}{3\cdot x+2}\right)$

$\frac{-3}{3\cdot x+2}+2\cdot x^2-x+3$

```
...((6x^3+x^2+7x+3)/(3x+2))
MAIN    RAD EXACT    FUNC    2/30
```

$$\frac{6x^3+x^2+7x+3}{3x+2}= 2x^2-x+3-\frac{3}{3x+2},\ x\neq-\frac{2}{3}$$

f)



■ NewProb Done

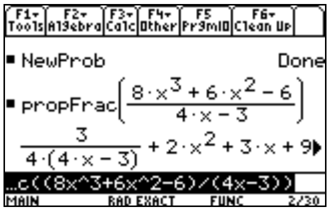
■ propFrac$\left(\frac{8\cdot x^3+4\cdot x^2-31}{2\cdot x-3}\right)$

$\frac{5}{2\cdot x-3}+4\cdot x^2+8\cdot x+12$

```
...ac((8x^3+4x^2-31)/(2x-3)...
MAIN    RAD EXACT    FUNC    2/30
```

$$\frac{8x^3+4x^2-31}{2x-3}= 4x^2+8x+12+\frac{5}{2x-3},\ x\neq\frac{3}{2}$$

g)

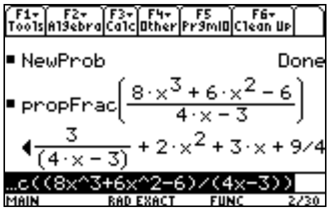


■ NewProb Done

■ propFrac$\left(\frac{8\cdot x^3+6\cdot x^2-6}{4\cdot x-3}\right)$

$\frac{3}{4\cdot(4\cdot x-3)}+2\cdot x^2+3\cdot x+9$▶

```
...c((8x^3+6x^2-6)/(4x-3))
MAIN    RAD EXACT    FUNC    2/30
```



■ NewProb Done

■ propFrac$\left(\frac{8\cdot x^3+6\cdot x^2-6}{4\cdot x-3}\right)$

◀$\frac{3}{(4\cdot x-3)}+2\cdot x^2+3\cdot x+9/4$

```
...c((8x^3+6x^2-6)/(4x-3))
MAIN    RAD EXACT    FUNC    2/30
```

$$\frac{8x^3+6x^2-6}{4x-3}= 2x^2+3x+\frac{9}{4}+\frac{3}{4(4x-3)},\ x\neq\frac{3}{4}$$

Chapter 2 Section 1 **Question 4 Page 91**

a) $(2x - 3)(3x + 4) + R = 6x^2 - x + 15$

$6x^2 + 8x - 9x - 12 + R = 6x^2 - x + 15$

$R = 6x^2 - 6x^2 - x - 8x + 9x + 15 + 12$

$R = 27$

b) $(x + 2)(x^2 - 3x + 4) + R = x^3 - x^2 - 2x - 1$

$x^3 - 3x^2 + 4x + 2x^2 - 6x + 8 + R = x^3 - x^2 - 2x - 1$

$R = x^3 - x^3 - x^2 + 3x^2 - 2x^2 - 2x - 4x + 6x - 1 - 8$

$R = -9$

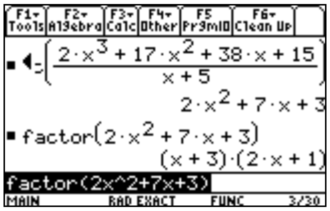
c) $(x - 4)(2x^2 + 3x - 1) + R = 2x^3 - 5x^2 - 13x + 2$

$2x^3 + 3x^2 - x - 8x^2 - 12x + 4 + R = 2x^3 - 5x^2 - 13x + 2$

$R = 2x^3 - 2x^3 - 5x^2 - 3x^2 + 8x^2 - 13x + x + 12x + 2 - 4$

$R = -2$

Chapter 2 Section 1 **Question 5 Page 91**

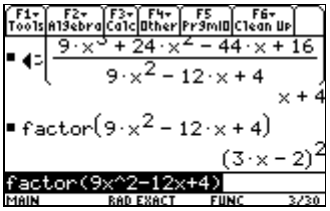


```
■ ◀:( 2·x^3 + 17·x^2 + 38·x + 15 )
      ( x + 5 )
                    2·x^2 + 7·x + 3
■ factor(2·x^2 + 7·x + 3)
                 (x + 3)·(2·x + 1)
factor(2x^2+7x+3)
```

$2x^3 + 17x^2 + 38x + 15 = (x + 5)(x + 3)(2x + 1)$

The possible dimensions of the box are $(x + 5)$ cm by $(x + 3)$ cm by $(2x + 1)$ cm.

Chapter 2 Section 1 **Question 6 Page 91**



```
■ ◀:( 9·x^3 + 24·x^2 - 44·x + 16 )
      ( 9·x^2 - 12·x + 4 )
                              x + 4
■ factor(9·x^2 - 12·x + 4)
                         (3·x - 2)^2
factor(9x^2-12x+4)
```

$9x^3 + 24x^2 - 44x + 16 = (x + 4)(3x - 2)^2$

The possible dimensions of the box are $(3x - 2)$ cm by $(3x - 2)$ cm by $(x + 4)$ cm.

Chapter 2 Section 1 **Question 7 Page 91**

a) $P(-1) = 2(-1)^3 + 7(-1)^2 - 8(-1) + 3$
$= -2 + 7 + 8 + 3$
$= 16$

b) $P(2) = 2(2)^3 + 7(2)^2 - 8(2) + 3$
$= 16 + 28 - 16 + 3$
$= 31$

c) $P(-3) = 2(-3)^3 + 7(-3)^2 - 8(-3) + 3$
$= -54 + 63 + 24 + 3$
$= 36$

d) $P(4) = 2(4)^3 + 7(4)^2 - 8(4) + 3$
$= 128 + 112 - 32 + 3$
$= 211$

e) $P(1) = 2(1)^3 + 7(1)^2 - 8(1) + 3$
$= 2 + 7 - 8 + 3$
$= 4$

Chapter 2 Section 1 **Question 8 Page 91**

a) $P(-2) = (-2)^3 + 3(-2)^2 - 5(-2) + 2$
$= -8 + 12 + 10 + 2$
$= 16$

b) $P(-2) = 2(-2)^3 - (-2)^2 - 3(-2) + 1$
$= -16 - 4 + 6 + 1$
$= -13$

c) $P(-2) = (-2)^4 + (-2)^3 - 5(-2)^2 + 2(-2) - 7$
$= 16 - 8 - 20 - 4 - 7$
$= -23$

Chapter 2 Section 1 Question 9 Page 91

a) $P(-3) = (-3)^3 + 2(-3)^2 - 3(-3) + 9$

$= -27 + 18 + 9 + 9$

$= 9$

b) $P(-2) = 2(-2)^3 + 7(-2)^2 - (-2) + 1$

$= -16 + 28 + 2 + 1$

$= 15$

c) $P(3) = (3)^3 + 2(3)^2 - 3(3) + 5$

$= 27 + 18 - 9 + 5$

$= 41$

d) $P(2) = (2)^4 - 3(2)^2 - 5(2) + 2$

$= 16 - 12 - 10 + 2$

$= -4$

Chapter 2 Section 1 Question 10 Page 92

a) $P(-1) = k(-1)^3 + 5(-1)^2 - 2(-1) + 3$

$7 = -k + 5 + 2 + 3$

$k = 10 - 7$

$k = 3$

b) $P(3) = 3(3)^3 + 5(3)^2 - 2(3) + 3$

$= 81 + 45 - 6 + 3$

$= 123$

Chapter 2 Section 1 Question 11 Page 92

a) $f(2) = (2)^4 - c(2)^3 + 7(2) - 6 = -8$

$-8 = 16 - 8c + 14 - 6$

$8c = 24 - 8$

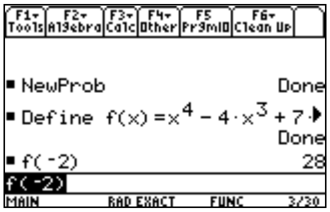
$c = 4$

b) $f(-2) = (-2)^4 - 4(-2)^3 + 7(-2) - 6$

$= 16 + 32 - 14 - 6$

$= 28$

c)



```
■ NewProb                  Done
■ Define f(x) = x⁴ - 4·x³ + 7·▶
                           Done
■ f(-2)                      28
f(-2)
```

Chapter 2 Section 1 **Question 12 Page 92**

$$\begin{aligned}P(2)&=-2(2)^3+b(2)^2-5(2)+2\\&=-16+4b-10+2\\&=4b-24\end{aligned}$$

$$\begin{aligned}P(-1)&=-2(-1)^3+b(-1)^2-5(-1)+2\\&=2+b+5+2\\&=b+9\end{aligned}$$

Since the remainders are equal,

$$\begin{aligned}4b-24&=b+9\\3b&=33\\b&=11\end{aligned}$$

Chapter 2 Section 1 **Question 13 Page 92**

$$\begin{aligned}f(1)&=(1)^3+6(1)^2+k(1)-4\\&=1+6+k-4\\&=k+3\end{aligned}$$

$$\begin{aligned}f(-2)&=(-2)^3+6(-2)^2+k(-2)-4\\&=-8+24-2k-4\\&=-2k+12\end{aligned}$$

Since the remainders are equal,

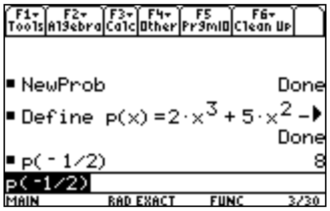
$$\begin{aligned}k+3&=-2k+12\\3k&=9\\k&=3\end{aligned}$$

Chapter 2 Section 1 **Question 14 Page 92**

a) $$\begin{aligned}P\left(-\frac{1}{2}\right)&=2\left(-\frac{1}{2}\right)^3+5\left(-\frac{1}{2}\right)^2-6\left(-\frac{1}{2}\right)+4\\&=-\frac{1}{4}+\frac{5}{4}+3+4\\&=8\end{aligned}$$

b) $$\begin{array}{r}x^2+2x-4\\2x+1\overline{\smash{\big)}\,2x^3+5x^2-6x+4}\\\underline{2x^3+x^2}\\4x^2-6x\\\underline{4x^2+2x}\\-8x+4\\\underline{-8x-4}\\8\end{array}$$

c)

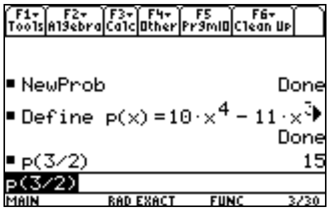


```
■ NewProb                 Done
■ Define p(x)=2·x^3+5·x^2-▸
                          Done
■ p(-1/2)                    8
p(-1/2)
MAIN      RAD EXACT    FUNC    3/30
```

Chapter 2 Section 1 Question 15 Page 92

a) $$P\left(\frac{3}{2}\right) = 10\left(\frac{3}{2}\right)^4 - 11\left(\frac{3}{2}\right)^3 - 8\left(\frac{3}{2}\right)^2 + 7\left(\frac{3}{2}\right) + 9$$
$$= \frac{405}{8} - \frac{297}{8} - 18 + \frac{21}{2} + 9$$
$$= 15$$

b)



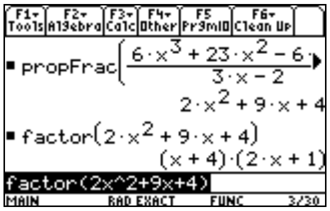
```
■ NewProb                  Done
■ Define p(x)=10·x^4 - 11·x^3▸
                           Done
■ p(3/2)                     15
p(3/2)
```

Chapter 2 Section 1 Question 16 Page 92

a) $$P\left(\frac{2}{3}\right) = 6\left(\frac{2}{3}\right)^3 + 23\left(\frac{2}{3}\right)^2 - 6\left(\frac{2}{3}\right) - 8$$
$$= \frac{16}{9} + \frac{92}{9} - 4 - 8$$
$$= 0$$

b) $(3x - 2)$ is a factor of $6x^3 + 23x^2 - 6x - 8$ since there is no remainder.

c)

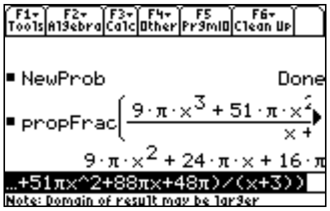


```
■ propFrac((6·x^3 + 23·x^2 - 6·▸)/(3·x - 2))
                     2·x^2 + 9·x + 4
■ factor(2·x^2 + 9·x + 4)
                 (x + 4)·(2·x + 1)
factor(2x^2+9x+4)
```

$(3x - 2)(x + 4)(2x + 1)$

Chapter 2 Section 1 Question 17 Page 92

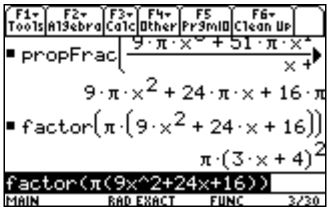
a)



```
■ NewProb                          Done
■ propFrac( 9·π·x^3 + 51·π·x^2 / x +
           9·π·x^2 + 24·π·x + 16·π
...+51πx^2+88πx+48π)/(x+3))
Note: Domain of result may be larger
```

$\pi(9x^2 + 14x + 16)$; this result represents the area of the base of the cylindrical container, i.e., the area of a circle.

b)



```
■ propFrac( 9·π·x^3 + 51·π·x^2 / x +
           9·π·x^2 + 24·π·x + 16·π
■ factor(π·(9·x^2 + 24·x + 16))
                       π·(3·x + 4)^2
factor(π(9x^2+24x+16))
MAIN        RAD EXACT      FUNC       3/30
```

$\pi(3x + 4)^2(x + 3)$

c) Volumes are given to the nearest cubic centimetre.

Value of x	Radius (cm)	Height (cm)	Volume (cm^3)
2	10	5	1 571
3	13	6	3 186
4	16	7	5 630
5	19	8	9 073
6	22	9	13 685
7	25	10	19 635
8	28	11	27 093

Chapter 2 Section 1 **Question 18 Page 92**

a) $-5t^2 + 15t + 1 = (t - b)(-5t - 5b + 15) - 5b^2 + 15b + 1$

b)
$$\begin{aligned}
Q(t) &= \frac{h(t) - h(b)}{t - b} \\
&= \frac{-5t^2 + 15t + 1 - \left[-5b^2 + 15b + 1\right]}{t - b} \\
&= \frac{-5t^2 + 15t + 1 + 5b^2 - 15b - 1}{t - b} \\
&= \frac{-5(t^2 - b^2) + 15(t - b)}{t - b} \\
&= \frac{-5(t - b)(t + b) + 15(t - b)}{t - b} \\
&= \frac{(t - b)\left[-5(t + b) + 15\right]}{t - b} \\
&= -5t - 5b + 15
\end{aligned}$$

Rearrange the division statement from part a).

$$\frac{-5t^2 + 15t + 1 - \left[-5b^2 + 15b + 1\right]}{t - b} = -5t - 5b + 15$$

c) The instantaneous rate of change at t for the function $h(t)$.
Diagrams may vary depending on choice of b. All should be linear graphs with a slope of -5 and a y-intercept of $15 - 5b$.

d) Answers may vary. A sample solution is shown.
At $t = b$ there is a hole in the graph; the graph is discontinuous at $t = b$.

e)
$$\begin{aligned}
h(3) &= -5(3)^2 + 15(3) + 1 \\
&= -45 + 45 + 1 \\
&= 1
\end{aligned}$$

At 3 s, the height of the javelin is 1 m.

Chapter 2 Section 1 — Question 19 Page 93

a) $$\begin{aligned} h(1.5) &= -5(1.5)^2 + 8.3(1.5) + 1.2 \\ &= -11.25 + 12.45 + 1.2 \\ &= 2.4 \end{aligned}$$

b) At 1.5 s the shot put is 2.4 m above the ground.

Chapter 2 Section 1 — Question 20 Page 93

$$\begin{aligned} m(-3)^3 - 3(-3)^2 + n(-3) + 2 &= -1 \\ -27m - 27 - 3n + 2 &= -1 \\ -27m - 3n &= 24 \\ 9m + n &= -8 \end{aligned}$$

$$\begin{aligned} m(2)^3 - 3(2)^2 + n(2) + 2 &= -4 \\ 8m - 12 + 2n + 2 &= -4 \\ 8m + 2n &= 6 \\ 4m + n &= 3 \end{aligned}$$

Subtract the two equations to solve for m.

$$\begin{aligned} 9m + n &= -8 \\ -4m + n &= 3 \\ 5m &= -11 \\ m &= -\frac{11}{5} \end{aligned}$$

Substitute m into $4m + n = 3$ to solve for n.

$$\begin{aligned} 4\left(-\frac{11}{5}\right) + n &= 3 \\ n &= 3 + \frac{44}{5} \\ n &= \frac{59}{5} \end{aligned}$$

Chapter 2 Section 1 **Question 21 Page 93**

$$\begin{aligned} 3(2)^3+a(2)^2+b(2)-9&=-5 \\ 24+4a+2b-9&=-5 \\ 4a+2b&=-20 \\ 2a+b&=-10 \end{aligned} \qquad \begin{aligned} 3(-1)^3+a(-1)^2+b(-1)-9&=-16 \\ -3+a-b-9&=-16 \\ a-b&=-4 \end{aligned}$$

Add the two equations to solve for a.

$$\begin{aligned} 2a+b&=-10 \\ a-b&=-4 \\ 3a&=-14 \\ a&=-\frac{14}{3} \end{aligned}$$

Substitute a into $a-b=-4$ to solve for b.

$$\begin{aligned} -\frac{14}{3}-b&=-4 \\ b&=-\frac{14}{3}+4 \\ b&=-\frac{2}{3} \end{aligned}$$

Chapter 2 Section 1 **Question 22 Page 93**

$$\begin{aligned} 3(-k)^2+10(-k)-3&=5 \\ 3k^2-10k-8&=0 \\ (3k+2)(k-4)&=0 \end{aligned}$$

$k=-\frac{2}{3}$ or $k=4$

Chapter 2 Section 1 **Question 23 Page 93**

$$\begin{array}{r} ? \\ 4\overline{)x} \\ \underline{x-3} \\ 3 \end{array} \qquad\qquad \begin{array}{r} 1 \\ x-3\overline{)x} \\ \underline{x-3} \\ 3 \end{array}$$

$x-3=4$

$x=7$

Substitute $x=7$ into $\frac{5x}{4}$.

$\frac{35}{4}=8\text{ R3}$

The remainder is 3.

Chapter 2 Section 1 **Question 24 Page 93**

$$\begin{aligned}
a &= \text{BC} \\
&= \sqrt{(3-4)^2+(2-8)^2} \\
&= \sqrt{1+36} \\
&= \sqrt{37} \\
b &= \text{AC} \\
&= \sqrt{(6-4)^2+(4-8)^2} \\
&= \sqrt{4+16} \\
&= \sqrt{20} \\
c &= \text{AB} \\
&= \sqrt{(6-3)^2+(4-2)^2} \\
&= \sqrt{9+4} \\
&= \sqrt{13} \\
s &= \frac{1}{2}\left(\sqrt{37}+\sqrt{20}+\sqrt{13}\right) \\
&\doteq 7.08 \\
A &\doteq \sqrt{7.08\left(7.08-\sqrt{37}\right)\left(7.08-\sqrt{20}\right)\left(7.08-\sqrt{13}\right)} \\
A &\doteq 8
\end{aligned}$$

Chapter 2 Section 1 **Question 25 Page 93**

If a right triangle is inscribed in a circle, then its hypoteneuse is a diameter of the circle. The median, MK, is the radius of the circle. HM is half the diameter which is the radius, therefore HM = MK.

Chapter 2 Section 2 **The Factor Theorem**

Chapter 2 Section 2 **Question 1 Page 102**

a) $x - 4$

b) $x + 3$

c) $3x - 2$

d) $4x + 1$

Chapter 2 Section 2 **Question 2 Page 102**

a) $P(-3) = (-3)^3 + (-3)^2 - (-3) + 6$
$= -27 + 9 + 3 + 6$
$= -9$
No.

b) $P(-3) = 2(-3)^3 + 9(-3)^2 + 10(-3) + 3$
$= -54 + 81 - 30 + 3$
$= 0$
Yes.

c) $P(-3) = (-3)^3 + 27$
$= -27 + 27$
$= 0$
Yes.

Chapter 2 Section 2 **Question 3 Page 102**

a) $P(-4) = (-4)^3 + 3(-4)^2 - 6(-4) - 8$
$= -64 + 48 + 24 - 8$
$= 0$
Since the remainder is zero, $P(x)$ is divisible by $(x + 4)$ and $(x + 4)$ is a factor of $P(x)$.
$P(-1) = (-1)^3 + 3(-1)^2 - 6(-1) - 8$
$= -1 + 3 + 6 - 8$
$= 0$
Since the remainder is zero, $P(x)$ is divisible by $(x + 1)$ and $(x + 1)$ is a factor of $P(x)$.
$P(2) = (2)^3 + 3(2)^2 - 6(2) - 8$
$= 8 + 12 - 12 - 8$
$= 0$
Since the remainder is zero, $P(x)$ is divisible by $(x - 2)$ and $(x - 2)$ is a factor of $P(x)$.

$P(x) = (x - 2)(x + 1)(x + 4)$

b) $$\begin{aligned} P(-6) &= (-6)^3 + 4(-6)^2 - 15(-6) - 18 \\ &= -216 + 144 + 90 - 18 \\ &= 0 \end{aligned}$$

Since the remainder is zero, $P(x)$ is divisible by $(x + 6)$ and $(x + 6)$ is a factor of $P(x)$.

$$\begin{aligned} P(-1) &= (-1)^3 + 4(-1)^2 - 15(-1) - 18 \\ &= -1 + 4 + 15 - 18 \\ &= 0 \end{aligned}$$

Since the remainder is zero, $P(x)$ is divisible by $(x + 1)$ and $(x + 1)$ is a factor of $P(x)$.

$$\begin{aligned} P(3) &= (3)^3 + 4(3)^2 - 15(3) - 18 \\ &= 27 + 36 - 45 - 18 \\ &= 0 \end{aligned}$$

Since the remainder is zero, $P(x)$ is divisible by $(x - 3)$ and $(x - 3)$ is a factor of $P(x)$.

$P(x) = (x - 3)(x + 1)(x + 6)$

c) $$\begin{aligned} P(-3) &= (-3)^3 - 3(-3)^2 - 10(-3) + 24 \\ &= -27 - 27 + 30 + 24 \\ &= 0 \end{aligned}$$

Since the remainder is zero, $P(x)$ is divisible by $(x + 3)$ and $(x + 3)$ is a factor of $P(x)$.

$$\begin{aligned} P(2) &= (2)^3 - 3(2)^2 - 10(2) + 24 \\ &= 8 - 12 - 20 + 24 \\ &= 0 \end{aligned}$$

Since the remainder is zero, $P(x)$ is divisible by $(x - 2)$ and $(x - 2)$ is a factor of $P(x)$.

$$\begin{aligned} P(4) &= (4)^3 - 3(4)^2 - 10(4) + 24 \\ &= 64 - 48 - 40 + 24 \\ &= 0 \end{aligned}$$

Since the remainder is zero, $P(x)$ is divisible by $(x - 4)$ and $(x - 4)$ is a factor of $P(x)$.

$P(x) = (x - 4)(x - 2)(x + 3)$

Chapter 2 Section 2 **Question 4 Page 102**

a $P(x) = x^3 + x^2 - 9x - 9$

Group the first two terms and factor out x^2.

Then, group the second two terms and factor out –9.

$P(x) = x^2(x + 1) - 9(x + 1)$

Factor out $x + 1$ and then factor the difference of squares

$$\begin{aligned}P(x) &= (x + 1)(x^2 - 9)\\ &= (x + 1)(x - 3)(x + 3)\end{aligned}$$

$P(x) = (x + 1)(x - 3)(x + 3)$

b) $P(x) = x^3 - x^2 - 16x + 16$

Group the first two terms and factor out x^2.

Then, group the second two terms and factor out –16.

$P(x) = x^2(x - 1) - 16(x - 1)$

Factor out $x - 1$ and then factor the difference of squares.

$$\begin{aligned}P(x) &= (x - 1)(x^2 - 16)\\ &= (x - 1)(x - 4)(x + 4)\end{aligned}$$

$P(x) = (x - 1)(x - 4)(x + 4)$

c) $P(x) = 2x^3 - x^2 - 72x + 36$

Group the first two terms and factor out x^2.

Then, group the second two terms and factor out –36.

$P(x) = x^2(2x - 1) - 36(2x - 1)$

Factor out $2x - 1$ and then factor the difference of squares.

$$\begin{aligned}P(x) &= (2x - 1)(x^2 - 36)\\ &= (2x - 1)(x - 6)(x + 6)\end{aligned}$$

$P(x) = (2x - 1)(x - 6)(x + 6)$

d) $P(x) = x^3 - 7x^2 - 4x + 28$

Group the first two terms and factor out x^2.

Then, group the second two terms and factor out –4.

$P(x) = x^2(x - 7) - 4(x - 7)$

Factor out $x - 7$ and then factor the difference of squares.

$$\begin{aligned}P(x) &= (x - 7)(x^2 - 4)\\ &= (x - 7)(x - 2)(x + 2)\end{aligned}$$

$P(x) = (x - 7)(x - 2)(x + 2)$

e) $P(x) = 3x^3 + 2x^2 – 75x – 50$

Group the first two terms and factor out x^2.

Then, group the second two terms and factor out –25.

$P(x) = x^2(3x + 2) – 25(3x + 2)$

Factor out $3x + 2$ and then factor the difference of squares.

$P(x) = (3x + 2)(x^2 – 25)$

$\quad = (3x + 2)(x – 5)(x + 5)$

$P(x) = (3x + 2)(x – 5)(x + 5)$

f) $P(x) = 2x^4 + 3x^3 – 32x^2 – 48x$

Group the first two terms and factor out x^3.

Then, group the second two terms and factor out $–16x$.

$P(x) = x^3(2x + 3) – 16x(2x + 3)$

Factor out $(2x + 3)$ and then factor $x^3 – 16x$.

$P(x) = (2x + 3)(x^3 – 16x)$

$\quad = x(2x + 3)(x – 4)(x + 4)$

$P(x) = x(2x + 3)(x – 4)(x + 4)$

Chapter 2 Section 2 **Question 5 Page 102**

a) $P(x) = 3x^3 + x^2 – 22x – 24$

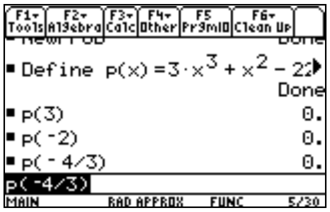
Let b represent the factors of the constant term –24, which are ±1, ±2, ±3, ±4, ±6, ±8, ±12, and ±24.

Let a represent the factors of the constant term 3, which are ±1 and ±3.

The possible values of $\frac{b}{a}$ are

$\pm\frac{1}{1}, \pm\frac{1}{3}, \pm\frac{2}{1}, \pm\frac{2}{3}, \pm\frac{3}{1}, \pm\frac{3}{3}, \pm\frac{4}{1}, \pm\frac{4}{3}, \pm\frac{6}{1}, \pm\frac{6}{3}, \pm\frac{8}{1}, \pm\frac{8}{3}, \pm\frac{12}{1}, \pm\frac{12}{3}, \pm\frac{24}{1}, \pm\frac{24}{3}$.

Test the values of $\frac{b}{a}$ for x to find the zeros using a graphing calculator.



```
■ Define p(x) = 3·x³ + x² – 22▸
                           Done
■ p(3)                       0.
■ p(⁻2)                      0.
■ p(⁻4/3)                    0.
p(⁻4/3)
MAIN      RAD APPROX   FUNC    5/30
```

The zeros are 3, –2, and $-\frac{4}{3}$.

The corresponding factors are $(x – 3)$, $(x + 2)$, and $(3x + 4)$.

$3x^3 + x^2 – 22x – 24 = (x – 3)(x + 2)(3x + 4)$

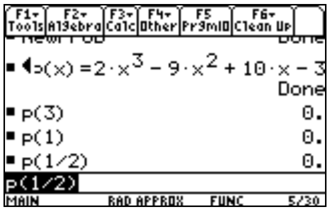
b) $P(x) = 2x^3 - 9x^2 + 10x - 3$

Let b represent the factors of the constant term –3, which are ±1 and ±3.

Let a represent the factors of the constant term 2, which are ±1 and ±2.

The possible values of $\frac{b}{a}$ are $\pm\frac{1}{1}, \pm\frac{1}{2}, \pm\frac{3}{1}, \pm\frac{3}{2}$.

Test the values of $\frac{b}{a}$ for x to find the zeros using a graphing calculator.



```
■ ◀p(x) =2·x^3 - 9·x^2 + 10·x - 3
                              Done
■ p(3)                          0.
■ p(1)                          0.
■ p(1/2)                        0.
p(1/2)
MAIN        RAD APPROX   FUNC   5/30
```

The zeros are 3, 1, and $\frac{1}{2}$.

The corresponding factors are $(x - 3)$, $(x - 1)$, and $(2x - 1)$.

$2x^3 - 9x^2 + 10x - 3 = (x - 3)(x - 1)(2x - 1)$

c) $P(x) = 6x^3 - 11x^2 - 26x + 15$

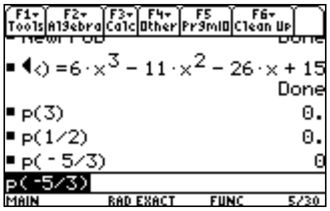
Let b represent the factors of the constant term 15, which are ±1, ±3, ±5, and ±15.

Let a represent the factors of the constant term 6, which are ±1, ±2, ±3, and ±6.

The possible values of $\frac{b}{a}$ are

$\pm\frac{1}{1}, \pm\frac{1}{2}, \pm\frac{1}{3}, \pm\frac{1}{6}, \pm\frac{3}{1}, \pm\frac{3}{2}, \pm\frac{3}{3}, \pm\frac{3}{6}, \pm\frac{5}{1}, \pm\frac{5}{2}, \pm\frac{5}{3}, \pm\frac{5}{6}, \pm\frac{15}{1}, \pm\frac{15}{2}, \pm\frac{15}{3}, \pm\frac{15}{6}$.

Test the values of $\frac{b}{a}$ for x to find the zeros using a graphing calculator.



```
■ ◀) =6·x^3 - 11·x^2 - 26·x + 15
                              Done
■ p(3)                          0.
■ p(1/2)                        0.
■ p(-5/3)                        0
p(-5/3)
MAIN        RAD EXACT    FUNC   5/30
```

The zeros are 3, $\frac{1}{2}$, and $-\frac{5}{3}$.

The corresponding factors are $(x - 3)$, $(2x - 1)$, and $(3x + 5)$.

$6x^3 - 11x^2 - 26x + 15 = (x - 3)(2x - 1)(3x + 5)$

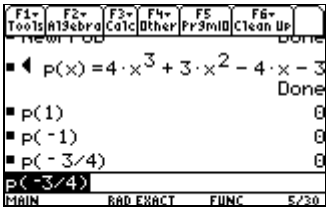
d) $P(x) = 4x^3 + 3x^2 - 4x - 3$

Let b represent the factors of the constant term –3, ±1, and ±3.

Let a represent the factors of the constant term 4, ±1, ±2, and ±4.

The possible values of $\frac{b}{a}$ are $\pm\frac{1}{1}, \pm\frac{1}{2}, \pm\frac{1}{4}, \pm\frac{3}{1}, \pm\frac{3}{2}, \pm\frac{3}{4}$.

Test the values of $\frac{b}{a}$ for x to find the zeros using a graphing calculator.



```
■ ◂ p(x) =4·x³ + 3·x² − 4·x − 3
                           Done
■ p(1)                        0
■ p(⁻1)                       0
■ p(⁻3/4)                     0
p(⁻3/4)
```

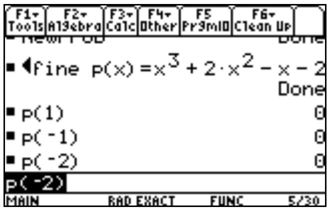
The zeros are 1, –1, and $-\frac{3}{4}$.

The corresponding factors are $(x - 1)$, $(x + 1)$, and $(4x + 3)$.

$4x^3 + 3x^2 - 4x - 3 = (x - 1)(x + 1)(4x + 3)$

Chapter 2 Section 2 **Question 6 Page 102**

a)

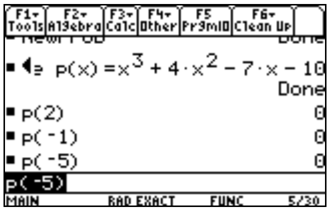


```
■ ◂fine p(x) =x³ + 2·x² − x − 2
                           Done
■ p(1)                        0
■ p(⁻1)                       0
■ p(⁻2)                       0
p(⁻2)
```

The zeros are 1, –1, and –2. The corresponding factors are $(x - 1)$, $(x + 1)$, and $(x + 2)$.

$x^3 + 2x^2 - x - 2 = (x - 1)(x + 1)(x + 2)$

b)

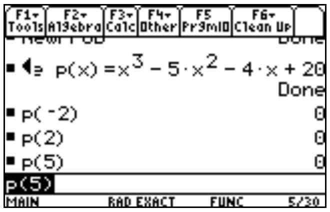


```
■ ◂e p(x) =x³ + 4·x² − 7·x − 10
                           Done
■ p(2)                        0
■ p(⁻1)                       0
■ p(⁻5)                       0
p(⁻5)
```

The zeros are –5, –1, and 2. The corresponding factors are $(x - 2)$, $(x + 1)$, and $(x + 5)$.

$x^3 + 4x^2 - 7x - 10 = (x - 2)(x + 1)(x + 5)$

c)

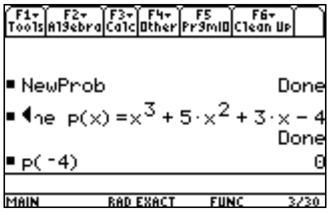


```
■ ◂e p(x) =x^3 − 5·x^2 − 4·x + 20
                              Done
■ p(−2)                          0
■ p(2)                           0
■ p(5)                           0
p(5)
MAIN       RAD EXACT   FUNC   5/30
```

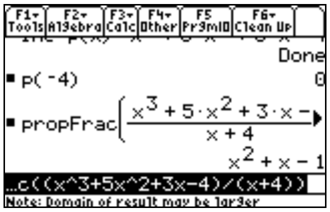
The zeros are –2, 2, and 5. The corresponding factors are (x – 5), (x – 2), and (x + 2).

$x^3 - 5x^2 - 4x + 20 = (x - 5)(x - 2)(x + 2)$

d)



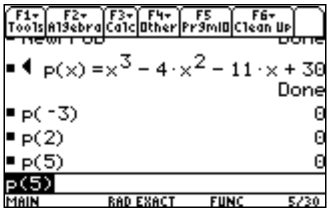
```
■ NewProb                     Done
■ ◂ne p(x) =x^3 + 5·x^2 + 3·x − 4
                              Done
■ p(−4)                          0

MAIN       RAD EXACT   FUNC   3/30
```



```
                              Done
■ p(−4)                          0
■ propFrac( x^3 + 5·x^2 + 3·x −▸
            x + 4
                        x^2 + x − 1
…c((x^3+5x^2+3x−4)/(x+4))
Note: Domain of result may be larger
```

The zero is –4. The corresponding factors are (x + 4) and ($x^2 + x - 1$).
$x^3 + 5x^2 + 3x - 4 = (x + 4)(x^2 + x - 1)$

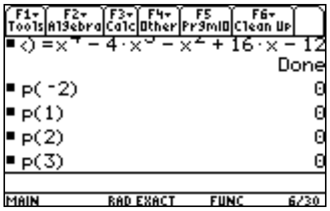
e)



```
■ ◂ p(x) =x^3 − 4·x^2 − 11·x + 30
                              Done
■ p(−3)                          0
■ p(2)                           0
■ p(5)                           0
p(5)
MAIN       RAD EXACT   FUNC   5/30
```

The zeros are –3, 2, and 5. The corresponding factors are (x – 5), (x – 2), and (x + 3).
$x^3 - 4x^2 - 11x + 30 = (x - 5)(x - 2)(x + 3)$.

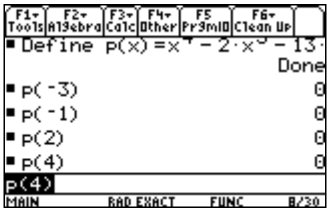
f)



```
■ <) =x^4 − 4·x^3 − x^2 + 16·x − 12
                              Done
■ p(−2)                          0
■ p(1)                           0
■ p(2)                           0
■ p(3)                           0

MAIN       RAD EXACT   FUNC   6/30
```

The zeros are –2, 1, 2, and 3.
The corresponding factors are (x – 3), (x + 2), (x – 1), and (x – 2).
$x^4 - 4x^3 - x^2 + 16x - 12 = (x - 3)(x + 2)(x - 1)(x - 2)$

g)

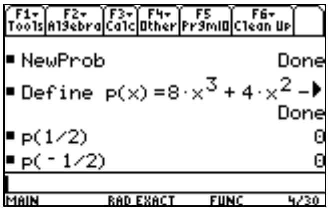


```
■ Define p(x)=x  - 2·x  - 13·
                        Done
■ p(-3)                    0
■ p(-1)                    0
■ p(2)                     0
■ p(4)                     0
p(4)
MAIN      RAD EXACT    FUNC     8/30
```

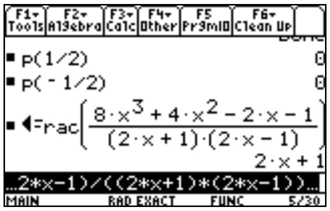
The zeros are –3, –1, 2, and 4.
The corresponding factors are (x – 4), (x – 2), (x + 1), and (x + 3).
$x^4 - 2x^3 - 13x^2 + 14x + 24 = (x - 4)(x - 2)(x + 1)(x + 3)$

Chapter 2 Section 2 **Question 7 Page 102**

a)



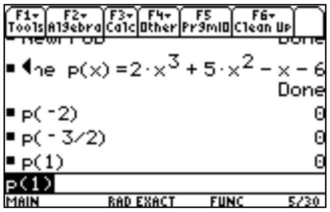
```
■ NewProb                 Done
■ Define p(x)=8·x³+4·x²-▸
                          Done
■ p(1/2)                     0
■ p(-1/2)                    0

MAIN      RAD EXACT    FUNC     4/30
```



```
■ p(1/2)                     0
■ p(-1/2)                    0
■ ◂Frac( 8·x³+4·x²-2·x-1       )
        ─────────────────
        (2·x+1)·(2·x-1)
                         2·x+1
…2*x-1)/((2*x+1)*(2*x-1))…
MAIN      RAD EXACT    FUNC     5/30
```

The zeros are $\frac{1}{2}$ and $-\frac{1}{2}$ (order 2). The corresponding factors are (2x – 1) and $(2x + 1)^2$.
$8x^3 + 4x^2 - 2x - 1 = (2x - 1)(2x + 1)^2$

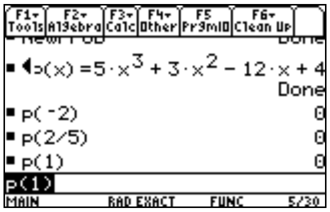
b)



```
■ ◂ne p(x)=2·x³+5·x²-x-6
                          Done
■ p(-2)                      0
■ p(-3/2)                    0
■ p(1)                       0
p(1)
MAIN      RAD EXACT    FUNC     5/30
```

The zeros are – 2, $-\frac{3}{2}$, and 1. The corresponding factors are (x – 1), (x + 2), and (2x + 3).
$2x^3 + 5x^2 - x - 6 = (x - 1)(x + 2)(2x + 3)$

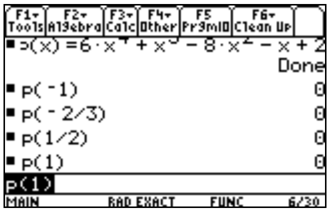
c)



```
■ ◂p(x)=5·x³+3·x²-12·x+4
                          Done
■ p(-2)                      0
■ p(2/5)                     0
■ p(1)                       0
p(1)
MAIN      RAD EXACT    FUNC     5/30
```

The zeros are – 2, $\frac{2}{5}$, and 1. The corresponding factors are (x – 1), (x + 2), and (5x – 2).
$5x^3 + 3x^2 - 12x + 4 = (x - 1)(x + 2)(5x - 2)$

d)



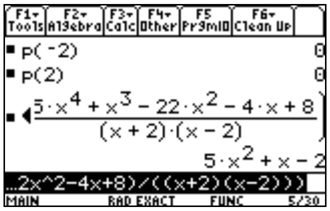
```
■ p(x) =6·x^4 + x^3 − 8·x^2 − x + 2
                                Done
■ p(-1)                            0
■ p(-2/3)                          0
■ p(1/2)                           0
■ p(1)                             0
p(1)
MAIN        RAD EXACT    FUNC   6/30
```

The zeros are – 1, $-\frac{2}{3}$, $\frac{1}{2}$, and 1.

The corresponding factors are $(x – 1)$, $(x + 1)$, $(2x – 1)$, and $(3x + 2)$.

$6x^4 + x^3 – 8x^2 – x + 2 = (x – 1)(x + 1)(2x – 1)(3x + 2)$

e)

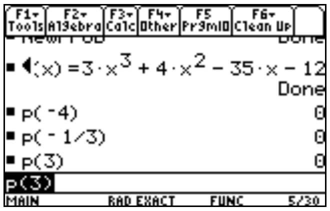


```
■ p(-2)                            0
■ p(2)                             0
■ (5·x^4 + x^3 − 22·x^2 − 4·x + 8)/((x + 2)·(x − 2))
                         5·x^2 + x − 2
...2x^2-4x+8)/((x+2)(x-2)))
MAIN        RAD EXACT    FUNC   5/30
```

The zeros are – 2 and 2. The corresponding factors are $(x – 2)$, $(x + 2)$, and $(5x^2 + x – 2)$.

$5x^4 + x^3 – 22x^2 – 4x + 8 = (x – 2)(x + 2)(5x^2 + x – 2)$

f)

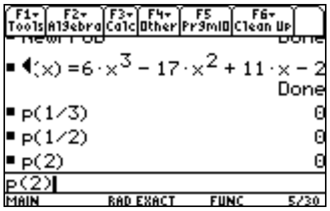


```
■ p(x) =3·x^3 + 4·x^2 − 35·x − 12
                                Done
■ p(-4)                            0
■ p(-1/3)                          0
■ p(3)                             0
p(3)
MAIN        RAD EXACT    FUNC   5/30
```

The zeros are – 4, $-\frac{1}{3}$, and 3. The corresponding factors are $(x – 3)$, $(x + 4)$, and $(3x + 1)$.

$3x^3 + 4x^2 – 35x – 12 = (x – 3)(x + 4)(3x + 1)$

g)

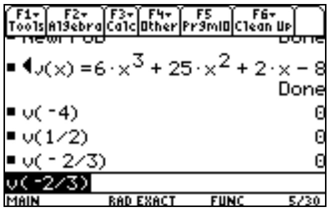


```
■ p(x) =6·x^3 − 17·x^2 + 11·x − 2
                                Done
■ p(1/3)                           0
■ p(1/2)                           0
■ p(2)                             0
p(2)
MAIN        RAD EXACT    FUNC   5/30
```

The zeros are $\frac{1}{3}$, $\frac{1}{2}$, and 2. The corresponding factors are $(x – 2)$, $(2x – 1)$, and $(3x – 1)$.

$6x^3 – 17x^2 + 11x – 2 = (x – 2)(2x – 1)(3x – 1)$

Chapter 2 Section 2 Question 8 Page 102



```
■ ◀v(x) =6·x^3 + 25·x^2 + 2·x − 8
                              Done
■ v(⁻4)                          0
■ v(1/2)                         0
■ v(⁻2/3)                        0
v(⁻2/3)
```

The zeros are $-4, -\frac{2}{3},$ and $\frac{1}{2}$.

The corresponding factors are (x + 4), (2x – 1), and (3x + 2).

$6x^3 + 25x^2 + 2x - 8 = (x + 4)(2x - 1)(3x + 2)$

Possible dimensions of the rectangular block of soapstone in cubic metres are (x + 4) by (2x – 1) by (3x + 2).

Chapter 2 Section 2 Question 9 Page 102

$$\begin{aligned} P(-2) &= (-2)^3 - 2k(-2)^2 + 6(-2) - 4 \\ 0 &= -8 - 8k - 12 - 4 \\ 8k &= -24 \\ k &= -3 \end{aligned}$$

Chapter 2 Section 2 Question 10 Page 102

$$\begin{aligned} P\left(\frac{2}{3}\right) &= 3\left(\frac{2}{3}\right)^3 - 5\left(\frac{2}{3}\right)^2 + k\left(\frac{2}{3}\right) + 2 \\ 0 &= \frac{8}{9} - \frac{20}{9} + \frac{2}{3}k + 2 \\ -\frac{8}{9} + \frac{20}{9} - \frac{18}{9} &= \frac{2}{3}k \\ -\frac{2}{3} &= \frac{2}{3}k \\ k &= -1 \end{aligned}$$

Chapter 2 Section 2 **Question 11 Page 102**

a) $$\begin{aligned} P(1) &= 2(1)^3+5(1)^2-1(1)-6 \\ &= 2+5-1-6 \\ &= 0 \end{aligned}$$

Since the remainder is zero, $P(x)$ is divisible by $(x - 1)$ and $(x - 1)$ is a factor of $P(x)$.

Use division to find the other factors.

$$\begin{array}{r} 2x^2+7x+6 \\ x-1\overline{)2x^3+5x^2-x-6} \\ \underline{2x^3-2x^2} \qquad\quad \\ 7x^2-x \quad\;\; \\ \underline{7x^2-7x} \quad\;\; \\ 6x-6 \\ \underline{6x-6} \\ 0 \end{array}$$

$$\begin{aligned} 2x^3+5x^2-x-6 &= (x-1)(2x^2+7x+6) \\ &= (x-1)(x+2)(2x+3) \end{aligned}$$

b) $$\begin{aligned} P(-1) &= 4(-1)^3-7(-1)-3 \\ &= -4+7-3 \\ &= 0 \end{aligned}$$

Since the remainder is zero, $P(x)$ is divisible by $(x + 1)$ and $(x + 1)$ is a factor of $P(x)$.

Use division to find the other factors.

$$\begin{array}{r} 4x^2-4x-3 \\ x+1\overline{)4x^3+0x^2-7x-3} \\ \underline{4x^3+4x^2} \qquad\quad \\ -4x^2-7x \quad\;\; \\ -\underline{4x^2-4x} \quad\;\; \\ -3x-3 \\ \underline{-3x-3} \\ 0 \end{array}$$

$$\begin{aligned} 4x^3-7x-3 &= (x+1)(4x^2-4x-3) \\ &= (x+1)(2x-3)(2x+1) \end{aligned}$$

c) $P(1) = 6(1)^3 + 5(1)^2 - 21(1) + 10$

$= 6 + 5 - 21 + 10$

$= 0$

Since the remainder is zero, $P(x)$ is divisible by $(x - 1)$ and $(x - 1)$ is a factor of $P(x)$.

Use division to find the other factors.

$$
\begin{array}{r}
6x^2+11x-10 \\
x-1\overline{)6x^3+5x^2-21x+10} \\
\underline{6x^3-6x^2} \\
11x^2-21x \\
\underline{11x^2-11x} \\
-10x+10 \\
\underline{-10x+10} \\
0
\end{array}
$$

$6x^3 + 5x^2 - 21x + 10 = (x - 1)(6x^2 + 11x - 10)$

$= (x - 1)(2x + 5)(3x - 2)$

d) $P(2) = 4(2)^3 - 8(2)^2 + 3(2) - 6$

$= 32 - 32 + 6 - 6$

$= 0$

Since the remainder is zero, $P(x)$ is divisible by $(x - 2)$ and $(x - 2)$ is a factor of $P(x)$.

Use division to find the other factors.

$$
\begin{array}{r}
4x^2+3 \\
x-2\overline{)4x^3-8x^2+3x-6} \\
\underline{4x^3-8x^2} \\
0x^2+3x-6 \\
\underline{3x-6} \\
0
\end{array}
$$

$4x^3 - 8x^2 + 3x - 6 = (x - 2)(4x^2 + 3)$

e) $P\left(\frac{1}{2}\right)=2\left(\frac{1}{2}\right)^3+\left(\frac{1}{2}\right)^2+\left(\frac{1}{2}\right)-1$

$$\begin{aligned}&=\frac{1}{4}+\frac{1}{4}+\frac{1}{2}-1\\&=0\end{aligned}$$

Since the remainder is zero, $P(x)$ is divisible by $(2x-1)$ and $(2x-1)$ is a factor of $P(x)$.

Use division to find the other factors.

$$\begin{array}{r} x^2+x+1 \\ 2x-1\overline{\smash{)}2x^3+x^2+x-1} \\ \underline{2x^3-x^2} \\ 2x^2+x \\ \underline{2x^2-x} \\ 2x-1 \\ \underline{2x-1} \\ 0 \end{array}$$

$2x^3+x^2+x-1=(2x-1)(x^2+x+1)$

f) $P(1) = (1)^4 - 15(1)^2 - 10(1) + 24$
$= 1 - 15 - 10 + 24$
$= 0$

Since the remainder is zero, $P(x)$ is divisible by $(x - 1)$ and $(x - 1)$ is a factor of $P(x)$.

Use division to find the other factors.

$$\begin{array}{r l}
 & \underline{x^3 + x^2 - 14x - 24} \\
x - 1 \big) & x^4 + 0x^3 - 15x^2 - 10x + 24 \\
 & \underline{x^4 - x^3} \\
 & \quad x^3 - 15x^2 \\
 & \quad \underline{x^3 - x^2} \\
 & \qquad -14x^2 - 10x \\
 & \qquad \underline{-14x^2 + 14x} \\
 & \qquad\qquad -24x + 24 \\
 & \qquad\qquad \underline{-24x + 24} \\
 & \qquad\qquad\qquad 0
\end{array}$$

$x^4 - 15x^3 - 10x + 24 = (x - 1)(x^3 + x^2 - 14x - 24)$

Factor $x^3 + x^2 - 14x - 24$:

$P(-2) = (-2)^3 + (-2)^2 - 14(-2) - 24$
$= -8 + 4 + 28 - 24$
$= 0$

Since the remainder is zero, $P(x)$ is divisible by $(x + 2)$ and $(x + 2)$ is a factor of $P(x)$.

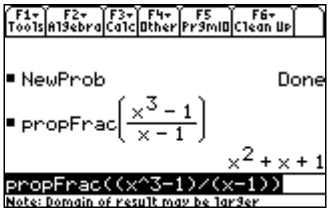
Use division to find the other factors.

$$\begin{array}{r l}
 & \underline{x^2 - x - 12} \\
x + 2 \big) & x^3 + x^2 - 14x - 24 \\
 & \underline{x^3 + 2x^2} \\
 & \quad -x^2 - 14x \\
 & \quad \underline{-x^2 - 2x} \\
 & \qquad -12x - 24 \\
 & \qquad \underline{-12x - 24} \\
 & \qquad\qquad 0
\end{array}$$

$x^4 - 15x^3 - 10x + 24 = (x - 1)(x + 2)(x^2 - x - 12)$
$= (x - 4)(x - 1)(x + 2)(x + 3)$

Chapter 2 Section 2 **Question 12 Page 103**

a) i) $P(1) = (1)^3 - 1$
$= 1 - 1$
$= 0$



■ NewProb Done

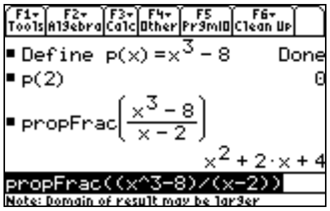
■ propFrac$\left(\dfrac{x^3-1}{x-1}\right)$ x^2+x+1

propFrac((x^3-1)/(x-1))

Note: Domain of result may be larger

$x^3 - 1 = (x - 1)(x^2 + x + 1)$

ii)



■ Define p(x) =$x^3 - 8$ Done

■ p(2) 0

■ propFrac$\left(\dfrac{x^3-8}{x-2}\right)$ $x^2+2\cdot x+4$

propFrac((x^3-8)/(x-2))

Note: Domain of result may be larger

$x^3 - 8 = (x - 2)(x^2 + 2x + 4)$

iii) $P(3) = (3)^3 - 27$
$= 27 - 27$
$= 0$

Since the remainder is zero, $P(x)$ is divisible by $(x - 3)$ and $(x - 3)$ is a factor of $P(x)$.

Use division to find the other factor.

$$\begin{array}{r|rrrr}
-3 & 1 & 0 & 0 & -27 \\
- & & -3 & -9 & -27 \\
\hline
\times & 1 & 3 & 9 & 0
\end{array}$$

$x^3 - 27 = (x - 3)(x^2 + 3x + 9)$

iv) $P(4) = (4)^3 - 64$
$= 64 - 64$
$= 0$

Since the remainder is zero, $P(x)$ is divisible by $(x - 4)$ and $(x - 4)$ is a factor of $P(x)$.

Use division to find the other factor.

$$\begin{array}{c|cccc} -4 & 1 & 0 & 0 & -64 \\ - & & -4 & -16 & -64 \\ \hline \times & 1 & 4 & 16 & 0 \end{array}$$

$x^3 - 64 = (x - 4)(x^2 + 4x + 16)$

b) $x^3 - a^3 = (x - a)(x^2 + ax + a^2)$

c) $(x - 5)(x^2 + 5x + 25)$

d) **i)** $(2x - 1)(4x^2 + 2x + 1)$

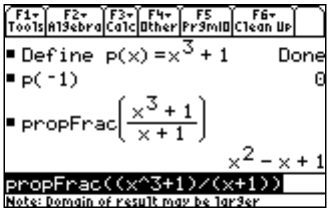
ii) $(5x^2 - 2)(25x^4 + 10x^2 + 4)$

iii) $(4x^4 - 3)(16x^8 + 12x^4 + 9)$

iv) $\left(\frac{2}{5}x - 4y^2\right)\left(\frac{4}{25}x^2 + \frac{8}{5}xy^2 + 16y^4\right)$

Chapter 2 Section 2 **Question 13 Page 103**

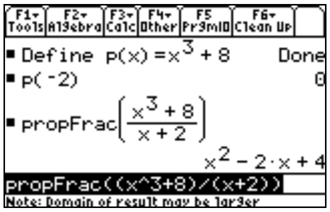
a) **i)**



```
■ Define p(x) = x^3 + 1        Done
■ p(-1)                           0
■ propFrac((x^3 + 1)/(x + 1))
                          x^2 - x + 1
propFrac((x^3+1)/(x+1))
Note: Domain of result may be larger
```

$x^3 + 1 = (x + 1)(x^2 - x + 1)$

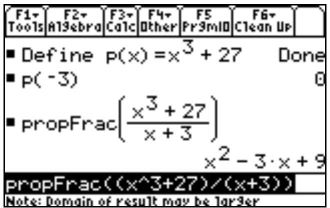
ii)



```
■ Define p(x) = x^3 + 8        Done
■ p(-2)                           0
■ propFrac((x^3 + 8)/(x + 2))
                        x^2 - 2·x + 4
propFrac((x^3+8)/(x+2))
Note: Domain of result may be larger
```

$x^3 + 8 = (x + 2)(x^2 - 2x + 4)$

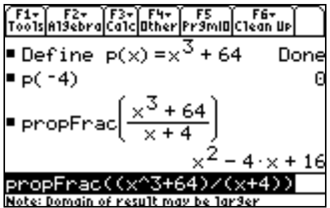
iii)



```
■ Define p(x) = x³ + 27      Done
■ p(-3)                         0
■ propFrac((x³ + 27)/(x + 3))
                     x² - 3·x + 9
propFrac((x^3+27)/(x+3))
Note: Domain of result may be larger
```

$x^3 + 27 = (x + 3)(x^2 - 3x + 9)$

iv)



```
■ Define p(x) = x³ + 64      Done
■ p(-4)                         0
■ propFrac((x³ + 64)/(x + 4))
                    x² - 4·x + 16
propFrac((x^3+64)/(x+4))
Note: Domain of result may be larger
```

$x^3 + 64 = (x + 4)(x^2 - 4x + 16)$

b) $x^3 + a^3 = (x + a)(x^2 - ax + a^2)$

c) $(x + 5)(x^2 - 5x + 25)$

d) **i)** $(2x + 1)(4x^2 - 2x + 1)$

ii) $(5x^2 + 2)(25x^4 - 10x^2 + 4)$

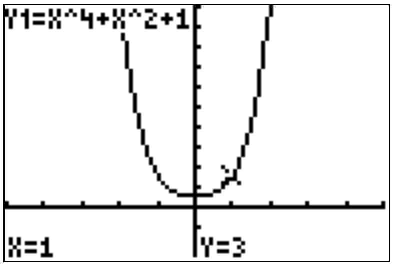
iii) $(4x^4 + 3)(16x^8 - 12x^4 + 9)$

iv) $\left(\frac{2}{5}x + 4y^2\right)\left(\frac{4}{25}x^2 - \frac{8}{5}xy^2 + 16y^4\right)$

Chapter 2 Section 2 Question 14 Page 103

$x^4 + x^2 + 1 = (x^2 + x + 1)(x^2 - x + 1)$
Neither factor has integer zeros so $x^4 + x^2 + 1$ is non-factorable over the integers.

From the graph, you can see there are no zeros.



Chapter 2 Section 2 Question 15 Page 103

a) let $m = x^2$

$$4x^4 - 37x^2 + 9 = 4m^2 - 37m + 9$$
$$= (m - 9)(4m - 1)$$
$$m = 9 \text{ or } m = \frac{1}{4}$$
$$x^2 = 9 \text{ or } x^2 = \frac{1}{4}$$
$$x = \pm 3 \text{ or } x = \pm\frac{1}{2}$$

$4x^4 - 37x^2 + 9 = (x - 3)(x + 3)(2x - 1)(2x + 1)$

b) let $m = x^2$

$$9x^4 - 148x^2 + 64 = 9m^2 - 148m + 64$$
$$= (m - 16)(9m - 4)$$
$$m = 16 \text{ or } m = \frac{4}{9}$$
$$x^2 = 16 \text{ or } x^2 = \frac{4}{9}$$
$$x = \pm 4 \text{ or } x = \pm\frac{2}{3}$$

$9x^3 - 148x^2 + 64 = (x - 4)(x + 4)(3x - 2)(3x + 2)$

Chapter 2 Section 2 **Question 16 Page 103**

Solutions to Achievement Check questions are provided in the Teacher's Resource.

Chapter 2 Section 2 **Question 17 Page 103**

a) The possible values of $\frac{b}{a}$ are $\pm\frac{1}{1}, \pm\frac{1}{2}, \pm\frac{2}{1}, \pm\frac{2}{2}, \pm\frac{3}{1}, \pm\frac{3}{2}, \pm\frac{4}{1}, \pm\frac{4}{2}, \pm\frac{6}{1}, \pm\frac{6}{2}, \pm\frac{12}{1}, \pm\frac{12}{2}$.

Test the values of $\frac{b}{a}$ for x to find the zeros.

$$\begin{aligned}P(2) &= 2(2)^5+3(2)^4-10(2)^3-15(2)^2+8(2)+12\\ &= 64+48-80-60+16+12\\ &= 0\end{aligned}$$

$$\begin{aligned}P(1) &= 2(1)^5+3(1)^4-10(1)^3-15(1)^2+8(1)+12\\ &= 2+3-10-15+8+12\\ &= 0\end{aligned}$$

$$\begin{aligned}P(-1) &= 2(-1)^5+3(-1)^4-10(-1)^3-15(-1)^2+8(-1)+12\\ &= -2+3+10-15-8+12\\ &= 0\end{aligned}$$

$$\begin{aligned}P(-2) &= 2(-2)^5+3(-2)^4-10(-2)^3-15(-2)^2+8(-2)+12\\ &= -64+48+80-60-16+12\\ &= 0\end{aligned}$$

$$\begin{aligned}P\left(-\frac{3}{2}\right) &= 2\left(-\frac{3}{2}\right)^5+3\left(-\frac{3}{2}\right)^4-10\left(-\frac{3}{2}\right)^3-15\left(-\frac{3}{2}\right)^2+8\left(-\frac{3}{2}\right)+12\\ &= -\frac{243}{16}+\frac{243}{16}+\frac{135}{4}-\frac{135}{4}-12+12\\ &= 0\end{aligned}$$

$$2x^5+3x^4-10x^3-15x^2+8x+12 = (x-2)(x-1)(x+1)(x+2)(2x+3)$$

b) The possible values of $\frac{b}{a}$ are $\pm\frac{1}{1}, \pm\frac{1}{2}, \pm\frac{1}{4}, \pm\frac{2}{1}, \pm\frac{2}{2}, \pm\frac{2}{4}, \pm\frac{4}{1}, \pm\frac{4}{2}, \pm\frac{4}{4}, \pm\frac{8}{1}, \pm\frac{8}{2}, \pm\frac{8}{4}$.

Test the values of $\frac{b}{a}$ for x to find the zeros.

$$\begin{aligned}P(-2) &= 4(-2)^6 + 12(-2)^5 - 9(-2)^4 - 51(-2)^3 - 30(-2)^2 + 12(-2) + 8 \\ &= 256 - 384 - 144 + 408 - 120 - 24 + 8 \\ &= 0\end{aligned}$$

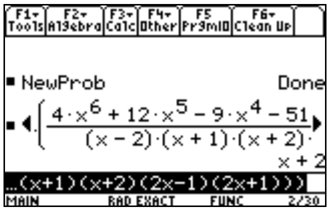
$$\begin{aligned}P(-1) &= 4(-1)^6 + 12(-1)^5 - 9(-1)^4 - 51(-1)^3 - 30(-1)^2 + 12(-1) + 8 \\ &= 4 - 12 - 9 + 51 - 30 - 12 + 8 \\ &= 0\end{aligned}$$

$$\begin{aligned}P\left(-\frac{1}{2}\right) &= 4\left(-\frac{1}{2}\right)^6 + 12\left(-\frac{1}{2}\right)^5 - 9\left(-\frac{1}{2}\right)^4 - 51\left(-\frac{1}{2}\right)^3 - 30\left(-\frac{1}{2}\right)^2 + 12\left(-\frac{1}{2}\right) + 8 \\ &= \frac{1}{16} - \frac{3}{8} - \frac{9}{16} + \frac{51}{8} - \frac{15}{2} - 6 + 8 \\ &= 0\end{aligned}$$

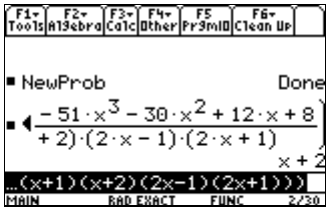
$$\begin{aligned}P\left(\frac{1}{2}\right) &= 4\left(\frac{1}{2}\right)^6 + 12\left(\frac{1}{2}\right)^5 - 9\left(\frac{1}{2}\right)^4 - 51\left(\frac{1}{2}\right)^3 - 30\left(\frac{1}{2}\right)^2 + 12\left(\frac{1}{2}\right) + 8 \\ &= \frac{1}{16} + \frac{3}{8} - \frac{9}{16} - \frac{51}{8} - \frac{15}{2} + 6 + 8 \\ &= 0\end{aligned}$$

$$\begin{aligned}P(2) &= 4(2)^6 + 12(2)^5 - 9(2)^4 - 51(2)^3 - 30(2)^2 + 12(2) + 8 \\ &= 256 + 384 - 144 - 408 - 120 + 24 + 8 \\ &= 0\end{aligned}$$

Only found 5 factors and the degree is 6, so one must have order 2.
Divide to determine the last factor.



```
■ NewProb                          Done
■ ◀( 4·x^6 + 12·x^5 − 9·x^4 − 51▶
     (x − 2)·(x + 1)·(x + 2)·
                                  x + 2
…(x+1)(x+2)(2x−1)(2x+1)))
```



```
■ NewProb                          Done
■ ◀ − 51·x^3 − 30·x^2 + 12·x + 8
    + 2)·(2·x − 1)·(2·x + 1)
                                  x + 2
…(x+1)(x+2)(2x−1)(2x+1)))
```

$4x^6 + 12x^5 - 9x^4 - 51x^3 - 30x^2 + 12x + 8 = (x - 2)(x + 1)(x + 2)^2(2x - 1)(2x + 1)$

Chapter 2 Section 2 **Question 18 Page 103**

$$\begin{aligned} P(2) &= 2(2)^3 + m(2)^2 + n(2) - 3 \\ 0 &= 16 + 4m + 2n - 3 \\ 4m + 2n &= -13 \end{aligned}$$

$$\begin{aligned} Q(2) &= (2)^3 - 3m(2)^2 + 2n(2) + 4 \\ 0 &= 8 - 12m + 4n + 4 \\ 12m - 4n &= 12 \\ 6m - 2n &= 6 \end{aligned}$$

Solve for *n* by adding *Q* and *P*.

$$\begin{aligned} 10m &= -7 \\ m &= -\frac{7}{10} \end{aligned}$$

Substitute *m* into *Q*.

$$\begin{aligned} 6\left(-\frac{7}{10}\right) - 2n &= 6 \\ -2n &= 6 + \frac{21}{5} \\ -2n &= \frac{51}{5} \\ n &= -\frac{51}{10} \end{aligned}$$

Chapter 2 Section 2 **Question 19 Page 103**

a) $$\begin{aligned} P(x) &= a(x+4)(4x+3)(2x-1) \\ P(-2) &= a(2)(-5)(-5) \\ 50 &= 50a \\ a &= 1 \end{aligned}$$

Therefore $P(x) = (x + 4)(4x + 3)(2x - 1)$.

b) $$\begin{aligned} P(x) &= a(x-3)(x+1)(3x-2)(2x+3) \\ P(1) &= a(-2)(2)(1)(5) \\ -18 &= -20a \\ a &= \frac{9}{10} \end{aligned}$$

Therefore $P(x) = \frac{9}{10}(x-3)(x+1)(3x-2)(2x+3)$.

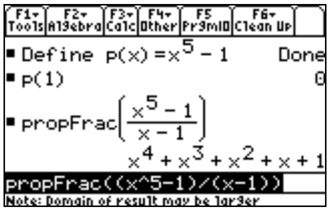
a) i) $(x - 1)(x + 1)(x^2 + 1)$

To help predict a pattern for b); $x^4 - 1$ partially factored is $(x - 1)(x^3 + x^2 + x + 1)$.

ii) $(x - 2)(x + 2)(x^2 + 4)$

To help predict a pattern for b); x^4 - 16 partially factored is $(x - 2)(x^3 + 2x^2 + 4x + 8)$.

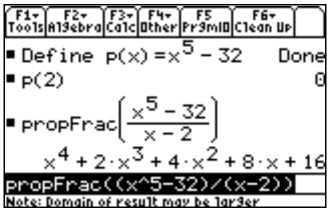
iii)



```
■ Define p(x) = x^5 - 1          Done
■ p(1)                              0
■ propFrac((x^5 - 1)/(x - 1))
                x^4 + x^3 + x^2 + x + 1
propFrac((x^5-1)/(x-1))
Note: Domain of result may be larger
```

$(x - 1)(x^4 + x^3 + x^2 + x + 1)$

iv)



```
■ Define p(x) = x^5 - 32         Done
■ p(2)                              0
■ propFrac((x^5 - 32)/(x - 2))
        x^4 + 2·x^3 + 4·x^2 + 8·x + 16
propFrac((x^5-32)/(x-2))
Note: Domain of result may be larger
```

$(x - 2)(x^4 + 2x^3 + 4x^2 + 8x + 16)$

b) $x^n - a^n = (x - a)(x^{n-1} + ax^{n-2} + a^2x^{n-3} + \ldots + a^{n-3}x^2 + a^{n-2}x + a^{n-1})$ where n is a positive integer.

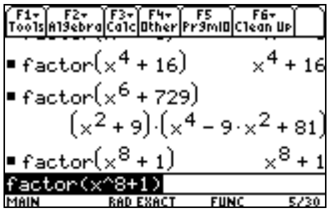
c) $(x - 1)(x^5 + x^4 + x^3 + x^2 + x + 1)$

d) i) $(x - 5)(x^2 + 25)$

ii) $(x - 3)(x^4 + 3x^3 + 9x^2 + 27x + 81)$

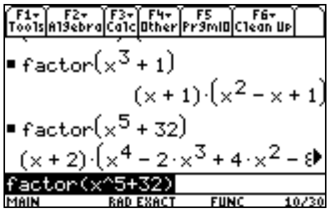
Chapter 2 Section 2 Question 21 Page 103

Yes, but only if n is odd.

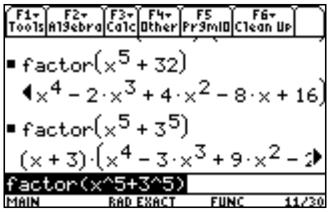


```
■ factor(x^4 + 16)          x^4 + 16
■ factor(x^6 + 729)
      (x^2 + 9)·(x^4 − 9·x^2 + 81)
■ factor(x^8 + 1)           x^8 + 1
factor(x^8+1)
MAIN       RAD EXACT    FUNC      5/30
```

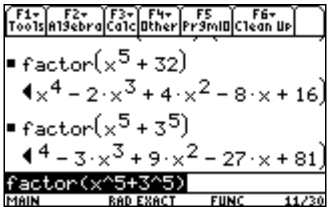
There is no pattern for $x^n + a^n$ when n is even.



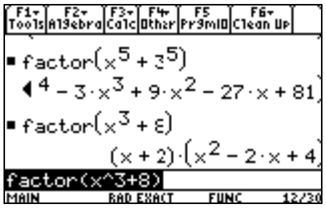
```
■ factor(x^3 + 1)
                 (x + 1)·(x^2 − x + 1)
■ factor(x^5 + 32)
  (x + 2)·(x^4 − 2·x^3 + 4·x^2 − 8▸
factor(x^5+32)
MAIN       RAD EXACT    FUNC     10/30
```



```
■ factor(x^5 + 32)
  ◂x^4 − 2·x^3 + 4·x^2 − 8·x + 16)
■ factor(x^5 + 3^5)
  (x + 3)·(x^4 − 3·x^3 + 9·x^2 − 2▸
factor(x^5+3^5)
MAIN       RAD EXACT    FUNC     11/30
```



```
■ factor(x^5 + 32)
  ◂x^4 − 2·x^3 + 4·x^2 − 8·x + 16)
■ factor(x^5 + 3^5)
  ◂4 − 3·x^3 + 9·x^2 − 27·x + 81)
factor(x^5+3^5)
MAIN       RAD EXACT    FUNC     11/30
```



```
■ factor(x^5 + 3^5)
  ◂4 − 3·x^3 + 9·x^2 − 27·x + 81)
■ factor(x^3 + 8)
                 (x + 2)·(x^2 − 2·x + 4)
factor(x^3+8)
MAIN       RAD EXACT    FUNC     12/30
```

Yes, but only if n is odd. Let $n = 2k + 1$. Then,
$x^{2k+1} + a^{2k+1} = (x + a)(x^{2k} - x^{2k-1}a + x^{2k-2}\,a^2 - x^{2k-3}a^3 + \dots - xa^{2k-1} + a^{2k})$.

Chapter 2 Section 2 Question 22 Page 103

$7x - 5$

Chapter 2 Section 3 **Polynomial Equations**

Chapter 2 Section 3 **Question 1 Page 110**

a) $x = 0$ or $x = -2$ or $x = 5$

b) $x = 1$ or $x = 4$ or $x = -3$

c) $x = -\frac{2}{3}$ or $x = -9$ or $x = 2$

d) $x = 7$ or $x = -\frac{2}{3}$ or $x = -1$

e) $x = \frac{1}{4}$ or $x = \frac{3}{2}$ or $x = -8$

f) $x = \frac{5}{2}$ or $x = -\frac{5}{2}$ or $x = 7$

g) $x = \frac{8}{5}$ or $x = -3$ or $x = \frac{1}{2}$

Chapter 2 Section 3 **Question 2 Page 110**

a) $x = -3$ or $x = -1$ or $x = 1$

b) $x = -1$ or $x = 3$ or $x = 4$

c) $x = -2$ or $x = -1$ or $x = 2$ or $x = 3$

d) $x = -5$ or $x = -2$ or $x = 1$

e) $x = -3$ or $x = -1$ or $x = 0$ or $x = 2$

Chapter 2 Section 3 **Question 3 Page 110**

a) $x = 4$

b) $(x - 1)(x + 1)(x^2 + 4) = 0$

$x = 1$ or $x = -1$

c) $(3x^2 + 27)(x - 4)(x + 4) = 0$

$x = 4$ or $x = -4$

d) $(x^2 - 1)(x^2 + 1)(x - 5)(x + 5) = 0$
$(x - 1)(x + 1)(x^2 + 1)(x - 5)(x + 5) = 0$

$x = -1$ or $x = 1$ or $x = 5$ or $x = -5$

e) $(2x - 3)(2x + 3)(x^2 + 16) = 0$
$x = \frac{3}{2}$ or $x = -\frac{3}{2}$

f) $(x + 4)(x + 3)(x - 7)(x + 7) = 0$

$x = 7$ or $x = -7$ or $x = -3$ or $x = -4$

g) $4(2x - 1)(x + 3)(x^2 - 25) = 0$
$4(2x - 1)(x + 3)(x - 5)(x + 5) = 0$
$x = -3$ or $x = \frac{1}{2}$ or $x = 5$ or $x = -5$

Chapter 2 Section 3 **Question 4 Page 110**

a) $y = x^3 - 4x^2 - 45x$
$0 = x(x^2 - 4x - 45)$
$0 = x(x - 9)(x + 5)$
$x = 0$ or $x = 9$ or $x = -5$

The x-intercepts are –5, 0, 9.

b) $f(x) = x^2(x^2 - 81)$
$0 = x^2(x - 9)(x + 9)$
$x = 0$ or $x = 9$ or $x = -9$

The x-intercepts are –9, 0, 9.

c) $P(x) = x(6x^2 - 5x - 4)$

$0 = x(3x - 4)(2x + 1)$

$x = 0$ or $x = \frac{4}{3}$ or $x = -\frac{1}{2}$

The x-intercepts are $-\frac{1}{2}$, 0, $\frac{4}{3}$.

d) $h(x) = x^2(x + 1) - 4(x + 1)$

$0 = (x^2 - 4)(x + 1)$

$0 = (x - 2)(x + 2)(x + 1)$

$x = 2$ or $x = -2$ or $x = -1$

The x-intercepts are –2, –1, 2.

e) $g(x) = (x^2 - 4)(x^2 + 4)$

$0 = (x - 2)(x + 2)(x^2 + 4)$

$x = 2$ or $x = -2$

The x-intercepts are –2, 2.

f) $k(x) = x^3(x - 2) - x(x - 2)$

$0 = (x^3 - x)(x - 2)$

$0 = x(x^2 - 1)(x - 2)$

$0 = x(x - 1)(x + 1)(x - 2)$

$x = 0$ or $x = 1$ or $x = -1$ or $x = 2$

The x-intercepts are –1, 0, 1, 2.

g) let $m = x^2$

$t(m) = m^2 - 29m + 100$

$0 = (m - 25)(m - 4)$

substitute x back in for m

$t(x) = (x^2 - 25)(x^2 - 4)$

$0 = (x - 5)(x + 5)(x - 2)(x + 2)$

$x = 5$ or $x = -5$ or $x = 2$ or $x = -2$

The x-intercepts are –5, –2, 2, 5.

Chapter 2 Section 3 **Question 5 Page 111**

Answers may vary. A sample solution is shown.

a) False. If the graph of a quartic function has four x-intercepts, then the corresponding quartic equation has four real roots.

b) True.

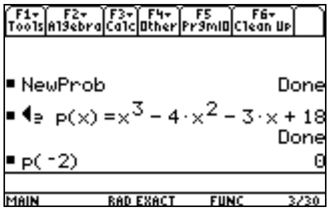
c) False. A polynomial equation of degree 3 has three or fewer real roots.

d) False. If a polynomial equation is not factorable, the roots can be determined by graphing.

e) True.

Chapter 2 Section 3 **Question 6 Page 111**

a) By the integral zero theorem test factors of 18, that is, ±1, ±2, ±3, ±6, ±9, ±18.



■ NewProb Done
■ Define p(x) = x^3 − 4·x^2 − 3·x + 18 Done
■ p(⁻2) 0

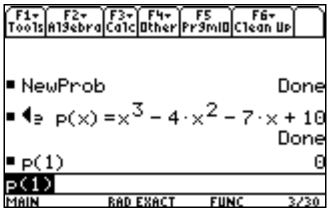
Since $x = -2$ is a zero of $P(x)$, $(x + 2)$ is a factor.

Use division to determine the other factor.

$$\begin{array}{c|cccc}
2 & 1 & -4 & -3 & 18 \\
- & & 2 & -12 & 18 \\
\hline
\times & 1 & -6 & 9 & 0
\end{array}$$

$$\begin{aligned}
P(x) &= x^3 - 4x^2 - 3x + 18 \\
0 &= (x+2)(x^2 - 6x + 9) \\
0 &= (x+2)(x-3)^2 \\
x &= -2 \text{ or } x = 3
\end{aligned}$$

b) By the integral zero theorem test factors of 10, that is, ±1, ±2, ±5, ±10.



```
■ NewProb                         Done
■ ◀e p(x) =x³ - 4·x² - 7·x + 10
                                  Done
■ p(1)                               0
p(1)
```

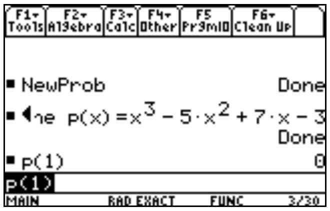
Since $x = 1$ is a zero of $P(x)$, $(x - 1)$ is a factor.

Use division to determine the other factor.

$$\begin{array}{c|cccc} -1 & 1 & -4 & -7 & 10 \\ - & & -1 & 3 & 10 \\ \hline \times & 1 & -3 & -10 & 0 \end{array}$$

$$\begin{aligned} P(x) &= x^3 - 4x^2 - 7x + 10 \\ 0 &= (x-1)(x^2 - 3x - 10) \\ 0 &= (x-1)(x-5)(x+2) \\ x &= 5 \text{ or } x = -2 \text{ or } x = 1 \end{aligned}$$

c) By the integral zero theorem test factors of –3, that is, ±1, ±3.



```
■ NewProb                         Done
■ ◀ne p(x) =x³ - 5·x² + 7·x - 3
                                  Done
■ p(1)                               0
p(1)
```

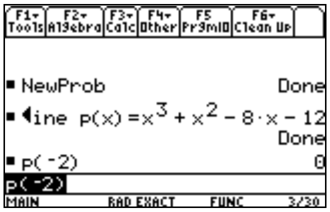
Since $x = 1$ is a zero of $P(x)$, $(x - 1)$ is a factor.

Use division to determine the other factor.

$$\begin{array}{c|cccc} -1 & 1 & -5 & 7 & -3 \\ - & & -1 & 4 & -3 \\ \hline \times & 1 & -4 & 3 & 0 \end{array}$$

$$\begin{aligned} P(x) &= x^3 - 5x^2 + 7x - 3 \\ 0 &= (x-1)(x^2 - 4x + 3) \\ 0 &= (x-1)(x-3)(x-1) \\ 0 &= (x-1)^2(x-3) \\ x &= 1 \text{ or } x = 3 \end{aligned}$$

d) By the integral zero theorem test factors of –12, that is, ±1, ±2, ±3, ±4, ±6, ±12.



```
■ NewProb                 Done
■ ◂ine p(x) = x^3 + x^2 - 8·x - 12
                          Done
■ p(-2)                      0
p(-2)
```

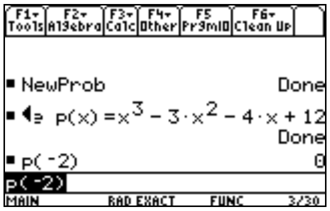
Since $x = -2$ is a zero of $P(x)$, $(x + 2)$ is a factor.

Use division to determine the other factor.

$$\begin{array}{r|rrrr} 2 & 1 & 1 & -8 & -12 \\ - & & 2 & -2 & -12 \\ \hline \times & 1 & -1 & -6 & 0 \end{array}$$

$$\begin{aligned} P(x) &= x^3 + x^2 - 8x - 12 \\ 0 &= (x+2)(x^2 - x - 6) \\ 0 &= (x+2)(x-3)(x+2) \\ 0 &= (x+2)^2(x-3) \\ x &= -2 \text{ or } x = 3 \end{aligned}$$

e) By the integral zero theorem test factors of 12, that is, ±1, ±2, ±3, ±4, ±6, ±12.



```
■ NewProb                 Done
■ ◂e p(x) = x^3 - 3·x^2 - 4·x + 12
                          Done
■ p(-2)                      0
p(-2)
```

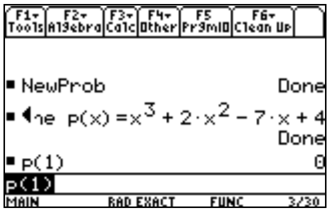
Since $x = -2$ is a zero of $P(x)$, $(x + 2)$ is a factor.

Use division to determine the other factor.

$$\begin{array}{r|rrrr} 2 & 1 & -3 & -4 & 12 \\ - & & 2 & -10 & 12 \\ \hline \times & 1 & -5 & 6 & 0 \end{array}$$

$$\begin{aligned} P(x) &= x^3 - 3x^2 - 4x + 12 \\ 0 &= (x+2)(x^2 - 5x + 6) \\ 0 &= (x+2)(x-2)(x-3) \\ x &= -2 \text{ or } x = 2 \text{ or } x = 3 \end{aligned}$$

f) By the integral zero theorem test factors of 4, that is, ±1, ±2, ±4.



```
■ NewProb                       Done
■ ◀ne p(x) =x^3 + 2·x^2 - 7·x + 4
                                Done
■ p(1)                             0
p(1)
MAIN      RAD EXACT    FUNC     3/30
```

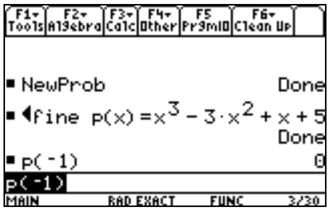
Since $x = 1$ is a zero of $P(x)$, $(x - 1)$ is a factor.

Use division to determine the other factor.

–1	1	2	–7	4
–		–1	–3	4
×	1	3	–4	0

$$\begin{aligned} P(x) &= x^3 + 2x^2 - 7x + 4 \\ 0 &= (x-1)(x^2 + 3x - 4) \\ 0 &= (x-1)(x+4)(x-1) \\ 0 &= (x-1)^2(x+4) \\ x &= -4 \text{ or } x = 1 \end{aligned}$$

g) By the integral zero theorem test factors of 5, that is, ±1, ±5.



```
■ NewProb                       Done
■ ◀fine p(x) =x^3 - 3·x^2 + x + 5
                                Done
■ p(-1)                            0
p(-1)
MAIN      RAD EXACT    FUNC     3/30
```

Since $x = -1$ is a zero of $P(x)$, $(x + 1)$ is a factor.

Use division to determine the other factor.

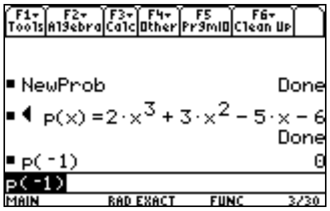
1	1	–3	1	5
–		1	–4	5
×	1	–4	5	0

$$\begin{aligned} P(x) &= x^3 - 3x^2 + x + 5 \\ 0 &= (x+1)(x^2 - 4x + 5) \\ x &= -1 \end{aligned}$$

Chapter 2 Section 3 **Question 7 Page 111**

a) Use the rational zero theorem to determine the values that should be tested.
Let b represent the factors of the constant term –6, which are ±1, ±2, ±3, ±6.
Let a represent the factors of the leading coefficient 2, which are ±1, ±2.
The possible values of $\frac{b}{a}$ are $\pm\frac{1}{1}, \pm\frac{1}{2}, \pm\frac{2}{1}, \pm\frac{2}{2}, \pm\frac{3}{1}, \pm\frac{3}{2}, \pm\frac{6}{1}, \pm\frac{6}{2}$.

Test the values of $\frac{b}{a}$ for x to find the zeros.



```
■ NewProb                         Done
■ p(x) = 2·x^3 + 3·x^2 − 5·x − 6
                                  Done
■ p(−1)                              0
p(−1)
```

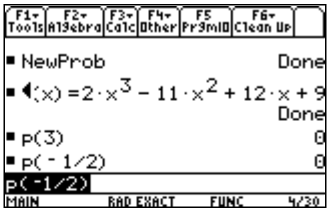
Since $x = -1$ is a zero of $P(x)$, $(x + 1)$ is a factor.

Use division to determine the other factor.

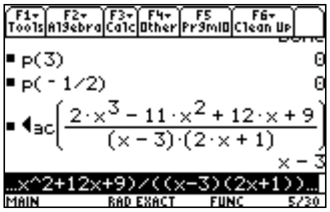
$$\begin{array}{c|cccc} 1 & 2 & 3 & -5 & -6 \\ - & & 2 & 1 & -6 \\ \hline \times & 2 & 1 & -6 & 0 \end{array}$$

$$\begin{aligned} P(x) &= 2x^3 + 3x^2 - 5x - 6 \\ 0 &= (x+1)(2x^2 + x - 6) \\ 0 &= (x+1)(2x-3)(x+2) \\ x &= -2 \text{ or } x = -1 \text{ or } x = \frac{3}{2} \end{aligned}$$

b) Use the rational zero theorem to determine the values that should be tested.
Let b represent the factors of the constant term 9, which are ±1, ±3, ±9.
Let a represent the factors of the leading coefficient 2, which are ±1, ±2.
The possible values of $\frac{b}{a}$ are $\pm\frac{1}{1}, \pm\frac{1}{2}, \pm\frac{3}{1}, \pm\frac{3}{2}, \pm\frac{9}{1}, \pm\frac{9}{2}$.



```
■ NewProb                          Done
■ ◂(x) =2·x^3 − 11·x^2 + 12·x + 9
                                   Done
■ p(3)                                0
■ p(− 1/2)                            0
p(-1/2)
```



```
■ p(3)                                0
■ p(− 1/2)                            0
■ ◂ac( (2·x^3 − 11·x^2 + 12·x + 9) / ((x − 3)·(2·x + 1)) )
                                  x − 3
...x^2+12x+9)/((x-3)(2x+1))...
```

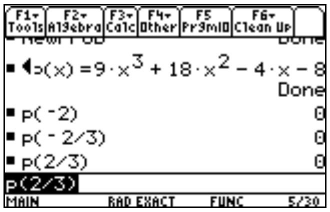
Since $x = 3$ is a zero of $P(x)$, $(x - 3)$ is a factor.

Since $x = -\frac{1}{2}$ is a zero of $P(x)$, $(2x + 1)$ is a factor.

Using division we discover that the factor $(x - 3)$ is of order 2.

$$P(x) = 2x^3 - 11x^2 + 12x + 9$$
$$0 = (x-3)(2x+1)(x-3)$$
$$0 = (2x+1)(x-3)^2$$
$$x = -\frac{1}{2} \text{ or } x = 3$$

c) Use the rational zero theorem to determine the values that should be tested.
Let b represent the factors of the constant term –8, which are ±1, ±2, ±4, ±8.
Let a represent the factors of the leading coefficient 9, which are ±1, ±3, ±9.
The possible values of $\frac{b}{a}$ are $\pm\frac{1}{1}, \pm\frac{1}{3}, \pm\frac{1}{9}, \pm\frac{2}{1}, \pm\frac{2}{3}, \pm\frac{2}{9}, \pm\frac{4}{1}, \pm\frac{4}{3}, \pm\frac{4}{9}, \pm\frac{8}{1}, \pm\frac{8}{3}, \pm\frac{8}{9}$.



```
■ ◀p(x) =9·x³ + 18·x² - 4·x - 8
                            Done
■ p(⁻2)                        0
■ p(⁻2/3)                      0
■ p(2/3)                       0
p(2/3)
```

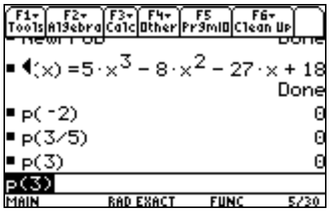
Since $x = –2$ is a zero of $P(x)$, $(x + 2)$ is a factor.

Since $x = -\frac{2}{3}$ is a zero of $P(x)$, $(3x + 2)$ is a factor.

Since $x = \frac{2}{3}$ is a zero of $P(x)$, $(3x – 2)$ is a factor.

$$0 = 9x^3 + 18x^2 - 4x - 8$$
$$0 = (x + 2)(3x + 2)(3x - 2)$$
$$0 = (x + 2)(3x + 2)(3x - 2)$$
$$x = -2 \text{ or } x = -\frac{2}{3} \text{ or } x = \frac{2}{3}$$

d) Use the rational zero theorem to determine the values that should be tested.
Let b represent the factors of the constant term 18, which are ±1, ±2, ±3, ±6, ±9, ±18.
Let a represent the factors of the leading coefficient 5, which are ±1, ±5.
The possible values of $\frac{b}{a}$ are $\pm\frac{1}{1}, \pm\frac{1}{5}, \pm\frac{2}{1}, \pm\frac{2}{5}, \pm\frac{3}{1}, \pm\frac{3}{5}, \pm\frac{6}{1}, \pm\frac{6}{5}, \pm\frac{9}{1}, \pm\frac{9}{5}, \pm\frac{18}{1}, \pm\frac{18}{5}$.



```
■ ◀(x) =5·x^3 - 8·x^2 - 27·x + 18
                               Done
■ p(-2)                           0
■ p(3/5)                          0
■ p(3)                            0
p(3)
MAIN       RAD EXACT    FUNC   5/30
```

Since $x = –2$ is a zero of $P(x)$, $(x + 2)$ is a factor.

Since $x = \frac{3}{5}$ is a zero of $P(x)$, $(5x – 3)$ is a factor.

Since $x = 3$ is a zero of $P(x)$, $(x – 3)$ is a factor.

$$5x^3 - 8x^2 - 27x + 18 = (x + 2)(5x - 3)(x - 3)$$
$$(x + 2)(5x - 3)(x - 3) = 0$$
$$x = –2 \text{ or } x = \frac{3}{5} \text{ or } x = 3$$

e) $8x^4 - 64x = 8x(x^3 - 8)$
$0 = 8x(x – 2)(x^2 + 2x + 4)$
$x = 0$ or $x = 2$

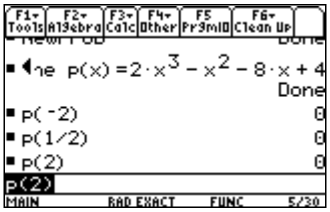
f) $4x^4 - 2x^3 - 16x^2 + 8x = 2x(2x^3 - x^2 - 8x + 4)$

Use the rational zero theorem to determine the values that should be tested.

Let b represent the factors of the constant term 4, which are ±1, ±2, ±4.

Let a represent the factors of the leading coefficient 2, which are ±1, ±2.

The possible values of $\frac{b}{a}$ are $\pm\frac{1}{1}, \pm\frac{1}{2}, \pm\frac{2}{1}, \pm\frac{2}{2}, \pm\frac{4}{1}, \pm\frac{4}{2}$.



```
■ ◂ne p(x) =2·x^3 - x^2 - 8·x + 4
                              Done
■ p(-2)                          0
■ p(1/2)                         0
■ p(2)                           0
p(2)
MAIN        RAD EXACT    FUNC  5/30
```

Since $x = -2$ is a zero of $P(x)$, $(x + 2)$ is a factor.

Since $x = \frac{1}{2}$ is a zero of $P(x)$, $(2x - 1)$ is a factor.

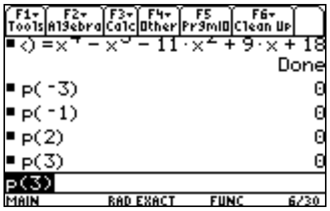
Since $x = 2$ is a zero of $P(x)$, $(x - 2)$ is a factor.

$$4x^4 - 2x^3 - 16x^2 + 8x = 2x(x+2)(2x-1)(x-2)$$

$$2x(x+2)(2x-1)(x-2) = 0$$

$x = -2$ or $x = 0$ or $x = \frac{1}{2}$ or $x = 2$

g) By the integral zero theorem test factors of 18, that is, ±1, ±2, ±3, ±6, ±9, ±18.



```
■ <) =x  - x  - 11·x  + 9·x + 18
                              Done
■ p(-3)                          0
■ p(-1)                          0
■ p(2)                           0
■ p(3)                           0
p(3)
MAIN        RAD EXACT    FUNC  6/30
```

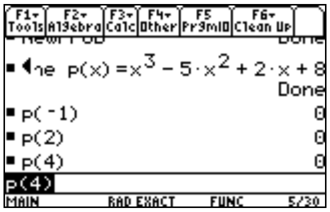
Since $x = -3$ is a zero of $P(x)$, $(x + 3)$ is a factor.
Since $x = -1$ is a zero of $P(x)$, $(x + 1)$ is a factor.
Since $x = 2$ is a zero of $P(x)$, $(x - 2)$ is a factor.
Since $x = 3$ is a zero of $P(x)$, $(x - 3)$ is a factor.

$$x^4 - x^3 - 11x^2 + 9x + 18 = (x+3)(x+1)(x-2)(x-3)$$

$$(x+3)(x+1)(x-2)(x-3) = 0$$

$x = -3$ or $x = -1$ or $x = 2$ or $x = 3$

a) By the integral zero theorem test factors of 8, that is, ±1, ±2, ±4, ±8.



```
■ ◂ne p(x) =x^3 - 5·x^2 + 2·x + 8
                             Done
■ p(-1)                         0
■ p(2)                          0
■ p(4)                          0
p(4)
MAIN       RAD EXACT    FUNC   5/30
```

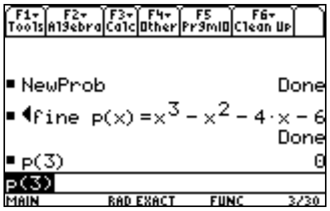
Since $x = -1$ is a zero of $P(x)$, $(x + 1)$ is a factor.
Since $x = 2$ is a zero of $P(x)$, $(x - 2)$ is a factor.
Since $x = 4$ is a zero of $P(x)$, $(x - 4)$ is a factor.

$$x^3 - 5x^2 + 2x + 8 = (x+1)(x-2)(x-4)$$

$$(x+1)(x-2)(x-4) = 0$$

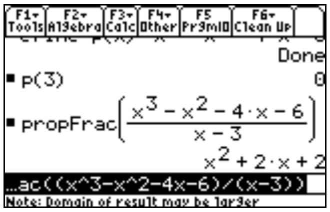
$x = -1$ or $x = 2$ or $x = 4$

b) By the integral zero theorem test factors of –6, that is, ±1, ±2, ±3, ±6.



```
■ NewProb                    Done
■ ◂fine p(x) =x^3 - x^2 - 4·x - 6
                             Done
■ p(3)                          0
p(3)
MAIN       RAD EXACT    FUNC   3/30
```

Divide to determine the other factor.



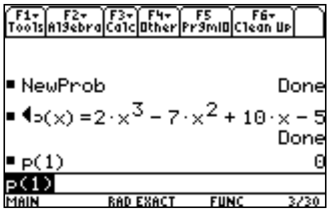
```
                             Done
■ p(3)                          0
■ propFrac((x^3 - x^2 - 4·x - 6)/(x - 3))
                        x^2 + 2·x + 2
...ac((x^3-x^2-4x-6)/(x-3))
Note: Domain of result may be larger
```

$$x^3 - x^2 - 4x - 6 = (x-3)(x^2 + 2x + 2)$$

$$(x-3)(x^2 + 2x + 2) = 0$$

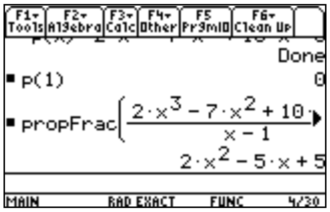
$$x = 3$$

c) Use the rational zero theorem to determine the values that should be tested.
Let b represent the factors of the constant term -5, which are ±1, ±5.
Let a represent the factors of the leading coefficient 2, which are ±1, ±2.
The possible values of $\frac{b}{a}$ are $\pm\frac{1}{1}, \pm\frac{1}{2}, \pm\frac{5}{1}, \pm\frac{5}{2}$.



```
■ NewProb                          Done
■ ◀ɔ(x) =2·x^3 − 7·x^2 + 10·x − 5
                                   Done
■ p(1)                                0
p(1)
```

Divide to determine the other factors.



```
                                   Done
■ p(1)                                0
■ propFrac( (2·x^3 − 7·x^2 + 10·▸ ) / (x − 1)
                        2·x^2 − 5·x + 5
```

$$2x^3 - 7x^2 + 10x - 5 = (x-1)(2x^2 - 5x + 5)$$
$$(x-1)(2x^2 - 5x + 5) = 0$$
$$x = 1$$

d) By the integral zero theorem test factors of –4, that is, ±1, ±2, ±4.

$$\begin{aligned} P(-1) &= (-1)^4 - (-1)^3 - 2(-1) - 4 \\ &= 1 + 1 + 2 - 4 \\ &= 0 \end{aligned}$$

Since $x = -1$ is a zero of $P(x)$, $(x + 1)$ is a factor.

Divide to determine the other factors.

$$\begin{array}{c|ccccc} 1 & 1 & -1 & 0 & -2 & -4 \\ - & & 1 & -2 & 2 & -4 \\ \hline \times & 1 & -2 & 2 & -4 & 0 \end{array}$$

$x^4 - x^3 - 2x - 4 = (x + 1)(x^3 - 2x^2 + 2x - 4)$

$$\begin{aligned} P(2) &= (2)^3 - 2(2)^2 + 2(2) - 4 \\ &= 8 - 8 + 4 - 4 \\ &= 0 \end{aligned}$$

Since $x = 2$ is a zero of $P(x)$, $(x - 2)$ is a factor.
Divide to determine the other factors.

$$\begin{array}{c|cccc} -2 & 1 & -2 & 2 & -4 \\ - & & -2 & 0 & -4 \\ \hline \times & 1 & 0 & 2 & 0 \end{array}$$

$x^4 - x^3 - 2x - 4 = (x + 1)(x - 2)(x^2 + 2)$

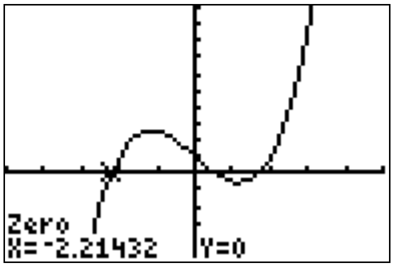
$(x + 1)(x - 2)(x^2 + 2) = 0$

$x = -1$ or $x = 2$

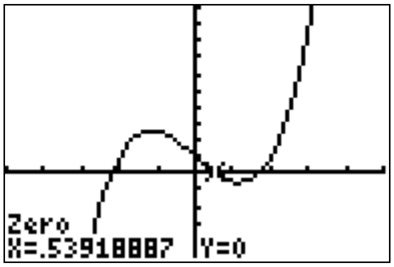
e) $x^4 + 13x^2 + 36 = 0$
$x^4 + 13x^2 = -36$
$x^4 + 13x^2$ cannot be negative.
$x^4 + 13x^2 + 36 = 0$ has no real roots since there are no real values of x that satisfy the equation.

Chapter 2 Section 3 **Question 9 Page 111**

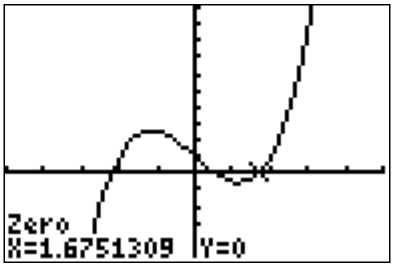
a) Set the mode to approximate.



```
Zero
X=-2.21432      Y=0
```



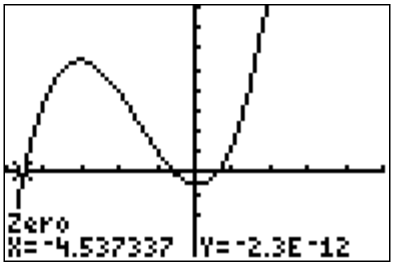
```
Zero
X=.53918887     Y=0
```



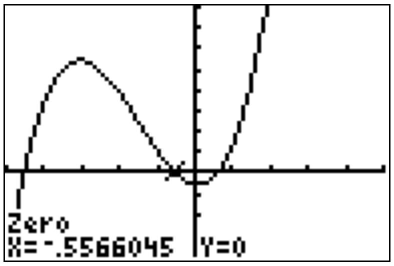
```
Zero
X=1.6751309     Y=0
```

$x \doteq -2.2$ or $x \doteq 0.5$ or $x \doteq 1.7$

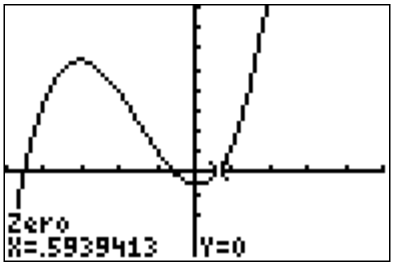
b)



```
Zero
X=-4.537337     Y=-2.3E-12
```



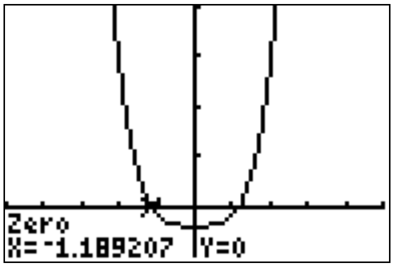
```
Zero
X=-.5566045     Y=0
```

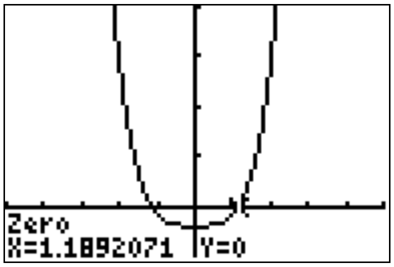


```
Zero
X=.5939413      Y=0
```

$x \doteq -4.5$ or $x \doteq -0.6$ or $x \doteq 0.6$

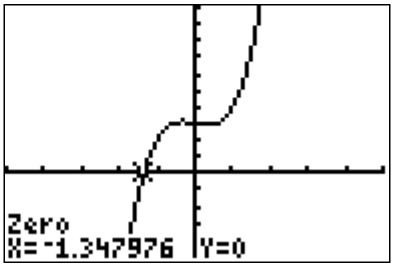
c)





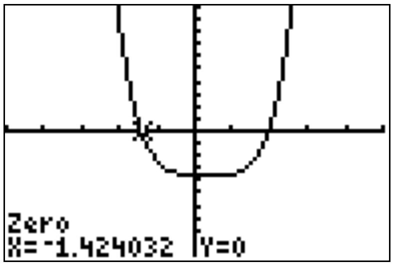
$x \doteq -1.2$ or $x \doteq 1.2$

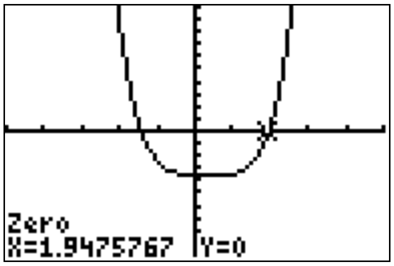
d)



$x \doteq -1.3$

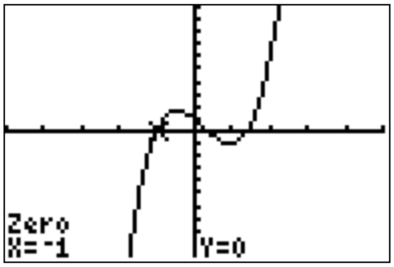
e)

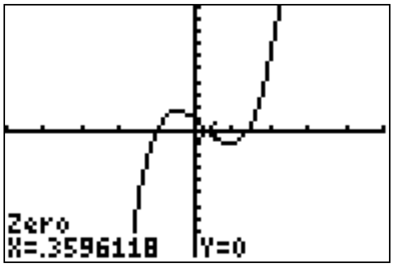


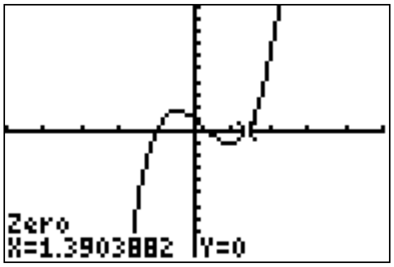


$x \doteq -1.4$ or $x \doteq 1.9$

f)

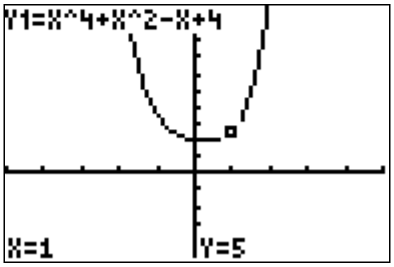






$x = -1$ or $x \doteq 0.4$ or $x = 1.4$

g)



There are no real roots.

Chapter 2 Section 3 **Question 10 Page 111**

Let x be the height of the tank.
width = $x - 3$

$$V(x) = w^2 \times h \quad \text{(square based)}$$

$$\begin{aligned} 20 &= (x-3)^2 x \\ 0 &= (x^2 - 6x + 9) - 20 \\ 0 &= x^3 - 6x^2 + 9x - 20 \end{aligned}$$

By the integral zero theorem test factors of 20, that is, ±1, ±2, ±4, ±5, ±20.

$$\begin{aligned} V(5) &= (5)^3 - 6(5)^2 + 9(5) - 20 \\ &= 125 - 150 + 45 - 20 \\ &= 0 \end{aligned}$$

Since $x = 5$ is a zero of $P(x)$, $(x - 5)$ is a factor.

Divide to determine the other factors.

$$\begin{array}{r|rrrr} -5 & 1 & -6 & 9 & -20 \\ - & & -5 & 5 & -20 \\ \hline \times & 1 & -1 & 4 & 0 \end{array}$$

$$\begin{aligned} V(x) &= (x-5)(x^2 - x + 4) \\ 0 &= (x-5)(x^2 - x + 4) \\ x &= x^2 - x + 4 \text{ or } x = 5 \end{aligned}$$

$$x = \frac{1 \pm \sqrt{1^2 - 4(1)(4)}}{2(1)}$$

$$x = \frac{1 \pm \sqrt{-15}}{2}$$

Since the only positive root is $x = 5$, the height of the tank is 5 m.
width = 2

The dimensions of the tank are 2 m by 2 m by 5 m.

Chapter 2 Section 3 **Question 11 Page 111**

$$\begin{aligned} V(x) &= (2x-7)(2x+3)(x-2) \\ 117 &= 4x^3-16x^2-5x+42 \\ 0 &= 4x^3-16x^2-5x-75 \end{aligned}$$

Use the rational zero theorem to determine the values that should be tested.

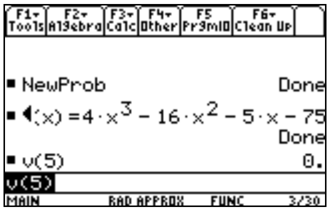
Let *b* represent the factors of the constant term –75, which are ±1, ±3, ±5, ±15, ±25, ±75.

Let *a* represent the factors of the leading coefficient 4, which are ±1, ±2, ±4.

The possible values of $\frac{b}{a}$ are

$$\pm\frac{1}{1}, \pm\frac{1}{2}, \pm\frac{1}{4}, \pm\frac{3}{1}, \pm\frac{3}{2}, \pm\frac{3}{4}, \pm\frac{5}{1}, \pm\frac{5}{2}, \pm\frac{5}{4}, \pm\frac{15}{1},$$

$$\pm\frac{15}{2}, \pm\frac{15}{4}, \pm\frac{25}{1}, \pm\frac{25}{2}, \pm\frac{25}{4}, \pm\frac{75}{1}, \pm\frac{75}{2}, \pm\frac{75}{4}.$$



```
■ NewProb                      Done
■ ◀(x) =4·x³ - 16·x² - 5·x - 75
                               Done
■ v(5)                           0.
v(5)
```

Since *x* = 5 is a zero of *V*(*x*), (*x* – 5) is a factor.

Divide to determine the other factors.

$$\begin{array}{r|rrrr} -5 & 4 & -16 & -5 & -75 \\ - & & -20 & -20 & -75 \\ \hline \times & 4 & 4 & 15 & 0 \end{array}$$

$$\begin{aligned} V(x) &= (x-5)(4x^2+4x+15) \\ 0 &= (x-5)(4x^2+4x+15) \\ x &= 5 \end{aligned}$$

Since the only positive real root is *x* = 5:
width: 2*x* – 7 = 3
length: 2*x* + 3 = 13
height: *x* – 2 = 3
The dimensions are 13 m by 3 m by 3 m.

Chapter 2 Section 3 **Question 12 Page 111**

Answers may vary. A sample solution is shown.
Yes, for example: $x^3 + 2 = 0$

$$x^3 = -2$$
$$x = \sqrt[3]{-2}$$
$$x \doteq -1.26$$

Chapter 2 Section 3 **Question 13 Page 111**

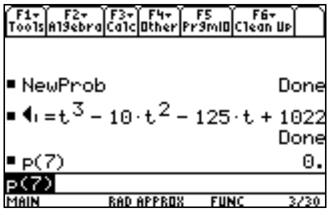
Answers may vary. A sample solution is shown.
No. If the radical part of the quadratic is negative, then two non-real roots occur.
Example:

$$x^3 - x^2 + 5x - 5 = 0$$
$$x^2(x-1) + 5(x-1) = 0$$
$$(x^2+5)(x-1) = 0$$
$$x^2 = -5 \text{ or } x = 1$$
$$x = \pm\sqrt{-5} \text{ or } x = 1$$

Chapter 2 Section 3 Question 14 Page 111

$$-4t^3+40t^2+500t=4088$$
$$-4t^3+40t^2+500t-4088=0$$
$$-4(t^3-10t^2-125t+1022)=0$$

By the integral zero theorem test factors of 1022, that is, ±1, ±2, ±7, ±146, ±511, ±1022.



```
■ NewProb                         Done
■ ◀ı =t³ - 10·t² - 125·t + 1022
                                  Done
■ p(7)                              0.
p(7)
MAIN      RAD APPROX   FUNC       3/30
```

Since $t = 7$ is a zero of $P(t)$, $(t - 7)$ is a factor.

Use division to find any other factors.

−7	1	−10	−125	1022
−		−7	21	1022
×	1	−3	−146	0

$$0=-4(t^3-10t^2-125t+1022)$$
$$0=-4(t-7)(t^2-3t-146)$$
$$t=7$$

or

$$t=\frac{3\pm\sqrt{(-3)^2-4(1)(-146)}}{2}$$
$$t=\frac{3\pm\sqrt{593}}{2}$$
$$t\doteq 13.7 \text{ or } t\doteq -10.7$$

Since time cannot be negative and $0 \le t \le 10$, $t = 7$ h.
It takes the plane 7 hours to fly 4088 km.

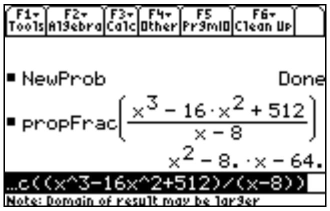
Chapter 2 Section 3 **Question 15 Page 111**

$$d(x) = 0.0005(x^4 - 16x^3 + 512x)$$
$$0 = 0.0005x(x^3 - 16x^2 + 512)$$

Let $P(x) = x^3 - 16x^2 + 512$

$$\begin{aligned} P(8) &= (8)^3 - 16(8)^2 + 512 \\ &= 512 - 1024 + 512 \\ &= 0 \end{aligned}$$

Since $x = 8$ is a zero of $P(x)$, $(x - 8)$ is a factor.



■ NewProb Done

■ propFrac$\left(\dfrac{x^3 - 16 \cdot x^2 + 512}{x - 8}\right)$ $x^2 - 8. \cdot x - 64.$

…c((x^3-16x^2+512)/(x-8))

Note: Domain of result may be larger

$$x^3 - 16x^2 + 512 = 0$$
$$(x - 8)(x^2 - 8x - 64) = 0$$
$$x = 8$$

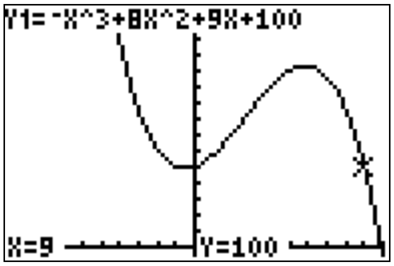
or

$$x = \frac{8 \pm \sqrt{(-8)^2 - 4(1)(-64)}}{2(1)}$$
$$x = \frac{8 \pm \sqrt{320}}{2}$$
$$x \doteq 12.9 \text{ or } x \doteq -4.9$$

The weight should be placed 0 m or 8 m or approximately 12.9 m from the end.

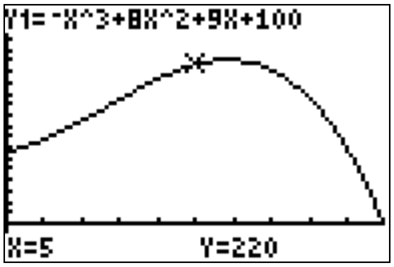
Chapter 2 Section 3 Question 16 Page 112

a)



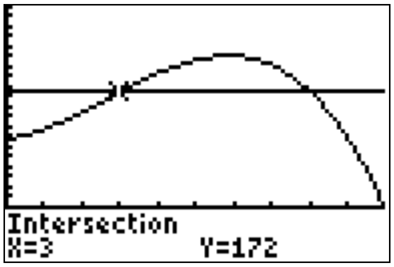
Domain: The price, x, of sunscreen cannot be negative and the number, D, of bottles sold cannot be negative. The domain is approximately $\{x \in \mathbb{R}, 0 \le x \le 9.923\}$.

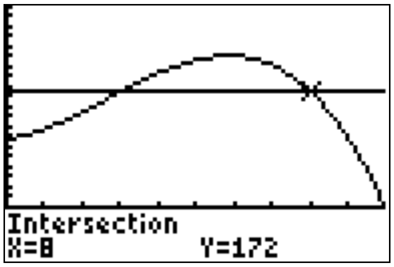
b)



22 000 bottles per month are sold when the price is \$5 per bottle.

c) On your graph, sketch the line $y = 172$ and find the points of intersection.





$x = 3$ or $x = 8$; If the selling price is \$3 per bottle or \$8 per bottle, then 17 200 bottles of sunscreen will be sold per month.

Chapter 2 Section 3 **Question 17 Page 112**

a)

$$\begin{aligned} 2(x-1)^3 &= 16 && \\ (x^2-2x+1)(x-1) &= 8 && \text{Divide both sides by 2.} \\ x^3-x^2-2x^2+2x+x-1 &= 8 && \text{Expand.} \\ x^3-3x^2+3x-9 &= 0 && \text{Collect like terms.} \\ x^2(x-3)+3(x-3) &= 0 && \text{Factor by grouping.} \\ (x^2+3)(x-3) &= 0 && \\ x &= 3 && \end{aligned}$$

Equation could also be solved by factoring difference of cubes.

$$\begin{aligned} 2(x-1)^3 &= 16 && \\ (x-1)^3-8 &= 0 && \text{Divide both sides by 2.} \\ \left[\left(x-1\right)-2\right]\left[(x-1)^2+2(x-1)+4\right] &= 0 && \text{Factor the difference of cubes.} \\ (x-3)(x^2-2x+1+2x-2+4) &= 0 && \text{Expand and add like terms.} \\ (x-3)(x^2+3) &= 0 && \\ x &= 3 && \end{aligned}$$

b) $2(x^2-4x)^2-5(x^2-4x)=3$

$2(x^2-4x)^2-5(x^2-4x)-3=0$

Let $m=x^2-4x$.

$2m^2-5m-3=0$

$(m-3)(2m+1)=0$

$m=3$ or $m=-\dfrac{1}{2}$

Substitute x^2-4x back in for *m*.

$x^2-4x=3$ or $x^2-4x=-\dfrac{1}{2}$ Multiply by 2.

$$\begin{aligned} &x^2-4x-3=0 && 2x^2-8x+1=0 \\ &x=\frac{4\pm\sqrt{(-4)^2-4(1)(-3)}}{2(1)} && x=\frac{8\pm\sqrt{(-8)^2-4(2)(1)}}{2(2)} \\ &x=\frac{4\pm\sqrt{28}}{2} && x=\frac{8\pm\sqrt{56}}{4} \end{aligned}$$

$x \doteq 4.6$ or $x \doteq -0.6$ or $x \doteq 3.9$ or $x \doteq 0.1$

Chapter 2 Section 3 **Question 18 Page 112**

a)

$$\begin{aligned} 2x^3+(k+1)x^2 &= 4-x^2 \\ 2x^3+(k+1)x^2-4+x^2 &= 0 \\ 2(-2)^3+(k+1)(-2)^2-4+(-2)^2 &= 0 \\ -16+4k+4-4+4 &= 0 \\ 4k &= 12 \\ k &= 3 \end{aligned}$$

b) $2x^3+(k+1)x^2-4+x^2=0$

$2x^3+(3+1)x^2-4+x^2=0$

$2x^3+5x^2-4=0$

Since –2 is a root of the equation, (x + 2) is a factor.
Divide to determine the other factors.

$$\begin{array}{r} 2x^2+x-2 \\ x+2\overline{)2x^3+5x^2+0x-4} \\ \underline{2x^3+4x^2} \qquad\quad \\ x^2+0x \qquad \\ \underline{x^2+2x} \qquad \\ -2x-4 \\ \underline{-2x-4} \\ 0 \end{array}$$

$(x+2)(2x^2+x-2)=0$

$x=-2$

or

$$x=\frac{-1\pm\sqrt{(1)^2-4(2)(-2)}}{2(2)}$$

$$x=\frac{-1\pm\sqrt{17}}{4}$$

$x \doteq 2$ or $x \doteq -1.3$ or $x \doteq 0.8$

Chapter 2 Section 3 **Question 19 Page 112**

length = $(32 - 2x)$
width = $(28 - 2x)$
height = x

$$V(x) = (32-2x)(28-2x)x$$
$$1920 = 4x^3 - 120x^2 + 896x$$
$$0 = 4x^3 - 120x^2 + 896x - 1920$$
$$0 = 4(x^3 - 30x^2 + 224x - 480)$$

$$\begin{aligned} V(4) &= (4)^3 - 30(4)^2 + 224(4) - 480 \\ &= 64 - 480 + 896 - 480 \\ &= 0 \end{aligned}$$

Since $x = 4$ is a zero of $V(x)$, $(x - 4)$ is a factor.

Divide to determine the other factors.

–4	1	–30	224	–480
–		–4	104	–480
×	1	–26	120	0

$$(x-4)(x^2 - 26x + 120) = 0$$
$$(x-4)(x-6)(x-20) = 0$$

If $x = 4$	If $x = 6$	If $x = 20$
length = 24	length = 20	length = –8; cannot have negative length
width = 20	width = 16	
height = 4	height = 6	

The dimensions of the boxes are 24 cm by 20 cm by 4 cm or 20 cm by 16 cm by 6 cm.

Chapter 2 Section 3 **Question 20 Page 112**

a) $(x-3)(x^2+3x+9)=0$

$x=3$

or

$$x=\frac{-3\pm\sqrt{(3)^2-4(1)(9)}}{2(1)}$$
$$x=\frac{-3\pm\sqrt{-27}}{2}$$
$$x=\frac{-3\pm\sqrt{(-1)3\times 9}}{2}$$
$$x=\frac{-3\pm\sqrt{-1}\times\sqrt{3}\times\sqrt{9}}{2}$$
$$x=\frac{-3\pm 3i\sqrt{3}}{2}$$
$$x=\frac{-3+3i\sqrt{3}}{2}\text{ or } x=\frac{-3-3i\sqrt{3}}{2}$$
$$x=3\text{ or } x=\frac{-3+3i\sqrt{3}}{2}\text{ or } x=\frac{-3-3i\sqrt{3}}{2}$$

b)
$$\begin{aligned}0&=\left[x-(3+i)\right]\left[x-(3-i)\right](x+4)\\&=\left[x^2-(3-i)x-(3+i)x+(3-i)(3+i)\right](x+4)\\&=\left[x^2-3x+i-3x-i+9+3i-3i-i^2\right](x+4)\\&=\left[x^2-6x+9-(-1)\right](x+4)\\&=(x^2-6x+10)(x+4)\\&=x^3+4x^2-6x^2-24x+10x+40\\&=x^3-2x^2-14x+40\end{aligned}$$

This equation is not unique since any multiple of it would have the same roots (e.g., $2x^3-4x^2-28x+80=0$).

Chapter 2 Section 3 Question 21 Page 112

$$V(x)=x(x+1)(x+2)$$

$$(x+1)(x+1+2)(x+2+3)=x(x+1)(x+2)+456$$

$$(x+1)(x+3)(x+5)=x(x^2+3x+2)+456$$

$$(x^2+4x+3)(x+5)=x^3+3x^2+2x+456$$

$$x^3+5x^2+4x^2+20x+3x+15=x^3+3x^2+2x+456$$

$$x^3-x^3+9x^2-3x^2+23x-2x+15-456=0$$

$$6x^2+21x-441=0$$

$$3(2x^2+7x-147)=0$$

$$3(2x+21)(x-7)=0$$

$$x\xcancel{=-\frac{21}{2}} \text{ or } x=7 \qquad \text{Reject the negative root.}$$

smaller box	larger box
height = x = 7	height = x + 1 = 8
width = x + 1 = 8	width = x + 3 = 10
length = x + 2 = 9	length = x + 5 = 12

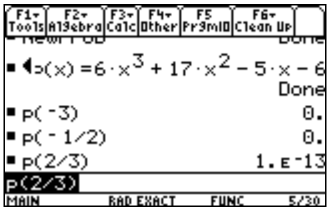
The dimensions of the smaller box are 9 cm by 8 cm by 7 cm. The dimensions of the larger box are 12 cm by 10 cm by 8 cm.

Chapter 2 Section 3 **Question 22 Page 112**

Use the rational zero theorem to determine the values that should be tested.
Let b represent the factors of the constant term –6, which are ±1, ±2, ±3, ±6.
Let a represent the factors of the leading coefficient 6, which are ±1, ±2, ±3, ±6.

The possible values of $\frac{b}{a}$ are

$$\pm\frac{1}{1},\ \pm\frac{1}{2},\ \pm\frac{1}{3},\ \pm\frac{1}{6},\ \pm\frac{2}{1},\ \pm\frac{2}{2},\ \pm\frac{2}{3},\ \pm\frac{2}{6},\ \pm\frac{3}{1},\ \pm\frac{3}{2},\ \pm\frac{3}{3},\ \pm\frac{3}{6},\ \pm\frac{6}{1},\ \pm\frac{6}{2},\ \pm\frac{6}{3},\ \pm\frac{6}{6}.$$



```
■ ◀p(x) =6·x^3 + 17·x^2 – 5·x – 6
                                Done
■ p(-3)                           0.
■ p(- 1/2)                        0.
■ p(2/3)                      1.E-13
p(2/3)
```

Since $x = –3$ is a zero of $V(x)$, $(x + 3)$ is a factor.

Since $x = -\frac{1}{2}$ is a zero of $V(x)$, $(2x + 1)$ is a factor.

Since $x = \frac{2}{3}$ is a zero of $V(x)$, $(3x – 2)$ is a factor.

$a = –3$; $b = -\frac{1}{2}$; $c = \frac{2}{3}$

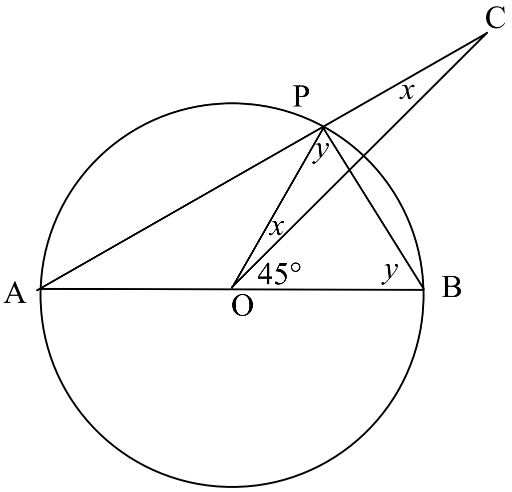
$a + b = –3 + \left(-\frac{1}{2}\right) = -\frac{7}{2}$; $(2x + 7)$ is a factor.

$\frac{a}{b} = \frac{-3}{-\frac{1}{2}} = 6$; $(x – 6)$ is a factor.

$ab = –3\left(-\frac{1}{2}\right) = \frac{3}{2}$; $(2x – 3)$ is a factor.

$$\begin{aligned}0 &= (2x+7)(x-6)(2x-3)\\ &= (2x^2-5x-42)(2x-3)\\ &= 4x^3-6x^2-10x^2+15x-84x+126\\ &= 4x^3-16x^2-69x+126\\ &\text{or}\\ &= x^3-4x^2-\frac{69}{4}x+\frac{63}{2}\end{aligned}$$

Chapter 2 Section 3 **Question 23 Page 112**



The diameter of a circle subtends a right triangle to any point on a circle.
Therefore, ∠APB = ∠CPB = 90°.
From ΔPOB:
$45 + x + 2y = 180$
$x + 2y = 135$ ①

From ΔPOC:
$y + 90 + 2x = 180$
$2x + y = 90$ ②

Solve for x.
① – 2②
$x + 2y - (4x + 2y) = 135 - 180$
$-3x = -45$
$x = 15$
∠POC = 15°

Chapter 2 Section 3 **Question 24 Page 112**

Try different values of k.

Through trial and error when $k = 5$ the equation has a double root.
$2x^3 - 9x^2 + 12x - 5 = 0$
$(x - 1)^2(2x - 5) = 0$
When $k = 4$ the equation has a double root.

$2x^3 - 9x^2 + 12x - 4$
$(x - 2)^2(2x - 1) = 0$
When $k = 5$ the equation has a double root.

$k = 4$ and $k = 5$
The product is 20.

Chapter 2 Section 4 **Families of Polynomial Functions**

Chapter 2 Section 4 **Question 1 Page 119**

a) The factor associated with –7 is $(x + 7)$ and the factor associated with –3 is $(x + 3)$. An equation for this family is $y = k(x + 7)(x + 3)$, where $k \in \mathbb{R}$, $k \neq 0$.

b) Answers may vary. A sample solution is shown.
$y = 2(x + 7)(x + 3)$, $y = -3(x + 7)(x + 3)$

c) Substitute $x = 2$ and $y = 18$ into the equation.

$$18 = k(2 + 7)(2 + 3)$$
$$18 = 45k$$
$$k = \frac{18}{45}$$
$$k = \frac{2}{5}$$
$$y = \frac{2}{5}(x + 7)(x + 3)$$

Chapter 2 Section 4 **Question 2 Page 119**

C (has different zeros)

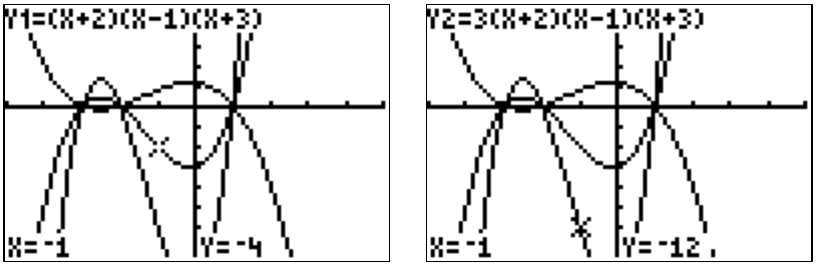
Chapter 2 Section 4 **Question 3 Page 119**

A, B, and D (same zeros)

Chapter 2 Section 4 **Question 4 Page 119**

A, C, E (zeros are –3, –2, 1)
B, D, F(zeros are –1, 2, 3)

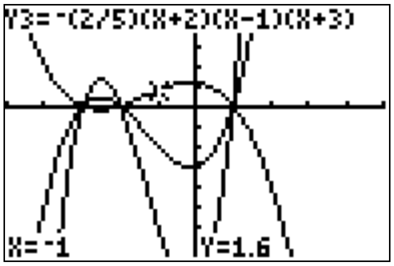
A



Y1=(X+2)(X-1)(X+3)
X=-1 Y=-4

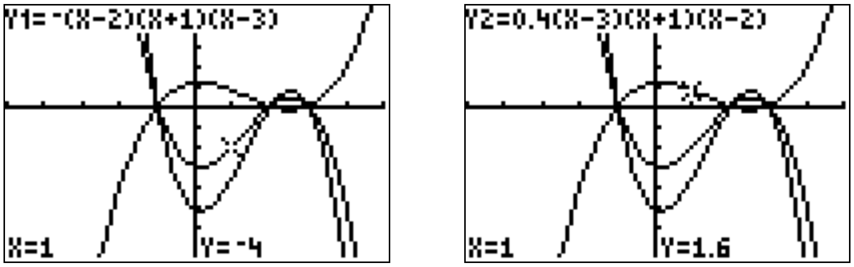
Y2=3(X+2)(X-1)(X+3)
X=-1 Y=-12

E



Y3=-(2/5)(X+2)(X-1)(X+3)
X=-1 Y=1.6

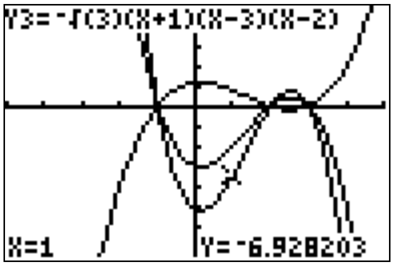
B



Y1=-(X-2)(X+1)(X-3)
X=1 Y=-4

Y2=0.4(X-3)(X+1)(X-2)
X=1 Y=1.6

F



Y3=-√(3)(X+1)(X-3)(X-2)
X=1 Y=-6.928203

Chapter 2 Section 4 **Question 5 Page 120**

a) $y = k(x + 5)(x - 2)(x - 3)$
b) $y = k(x - 1)(x - 6)(x + 3)$
c) $y = k(x + 4)(x + 1)(x - 9)$
d) $y = kx(x + 7)(x - 2)(x - 5)$

Chapter 2 Section 4 **Question 6 Page 120**

a) A From the graph, the x-intercepts are –2, 1, and 3. The corresponding factors are $(x + 2)$, $(x - 1)$, and $(x - 3)$.
An equation for the family of polynomial functions with these zeros is
$y = k(x + 2)(x - 1)(x - 3)$.
Select a point that the graph passes through, such as (0, 6).
Substitute $x = 0$ and $y = 6$ into the equation to solve for k.
$6 = (2)(-1)(-3)k$
$k = 1$
An equation is $y = (x + 2)(x - 1)(x - 3)$.

B From the graph, the x-intercepts are –2, 1, and 3. The corresponding factors are $(x + 2)$, $(x - 1)$, and $(x - 3)$.
An equation for the family of polynomial functions with these zeros is
$y = k(x + 2)(x - 1)(x - 3)$.
Select a point that the graph passes through, such as (0, –3).
Substitute $x = 0$ and $y = -3$ into the equation to solve for k.
$-3 = (2)(-1)(-3)k$
$6k = -3$
$k = -\frac{1}{2}$
An equation is $y = -\frac{1}{2}(x + 2)(x - 1)(x - 3)$.

C From the graph, the x-intercepts are –2, 2, and 3. The corresponding factors are $(x + 2)$, $(x - 2)$, and $(x - 3)$.
An equation for the family of polynomial functions with these zeros is
$y = k(x + 2)(x - 2)(x - 3)$.
Select a point that the graph passes through, such as (0, –6).
Substitute $x = 0$ and $y = -6$ into the equation to solve for k.
$-6 = (2)(-2)(-3)k$
$12k = -6$
$k = -\frac{1}{2}$
An equation is $y = -\frac{1}{2}(x + 2)(x - 2)(x - 3)$.

D From the graph, the x-intercepts are –2, 1, and 3. The corresponding factors are $(x + 2)$, $(x – 1)$, and $(x – 3)$.
An equation for the family of polynomial functions with these zeros is
$y = k(x + 2)(x – 1)(x – 3)$.
Select a point that the graph passes through, such as (0, 12).
Substitute $x = 0$ and $y = 12$ into the equation to solve for k.
$12 = (2)(–1)(–3)k$
$k = 2$
An equation is $y = 2(x + 2)(x – 1)(x – 3)$.

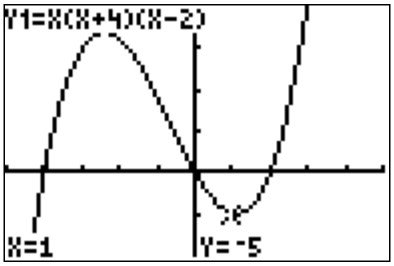
Chapter 2 Section 4 **Question 7 Page 120**

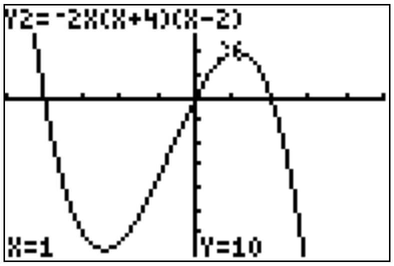
a) The corresponding factors are $(x + 4)$, $(x – 2)$, and x.
An equation for the family of polynomial functions with these zeros is
$y = kx(x + 4)(x – 2)$

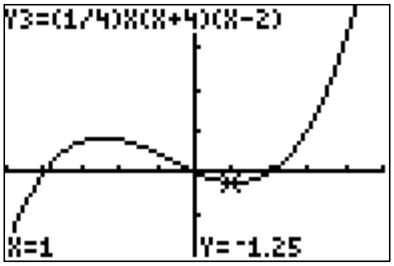
b) Answers may vary. A sample solution is shown.
$y = x(x + 4)(x – 2)$, $y = –2x(x + 4)(x – 2)$

c) Substitute $x = –2$ and $y = 4$ into the equation and solve for k.
$4 = k(–2)(–2 + 4)(–2 – 2)$
$4 = 16k$
$k = \frac{1}{4}$
An equation is $y = \frac{1}{4}x(x + 4)(x – 2)$.

d) Answers may vary. A sample solution is shown.







Chapter 2 Section 4 **Question 8 Page 120**

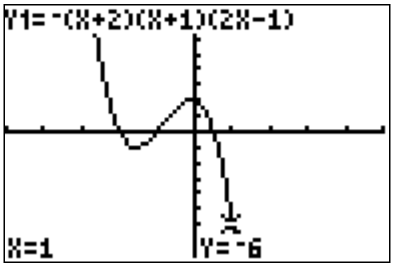
a) $y = k(x + 2)(x + 1)(2x - 1)$

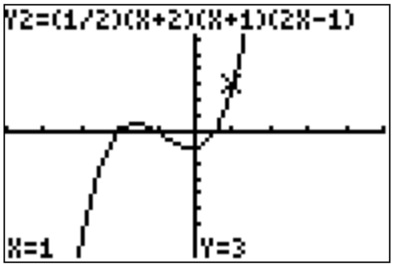
b) Answers may vary. A sample solution is shown.

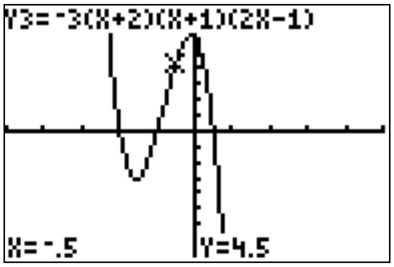
$y = -(x + 2)(x + 1)(2x - 1)$, $y = \frac{1}{2}(x + 2)(x + 1)(2x - 1)$

c) Substitute $x = 0$ and $y = 6$ and solve for k.
$6 = k(2)(1)(-1)$
$k = -3$
An equation is $y = -3(x + 2)(x + 1)(2x - 1)$.

d) Answers may vary. A sample solution is shown.







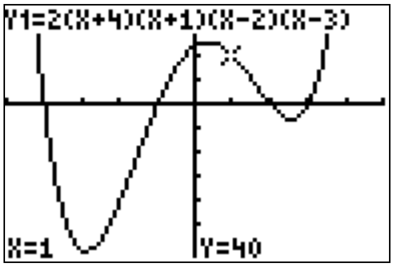
Chapter 2 Section 4 **Question 9 Page 120**

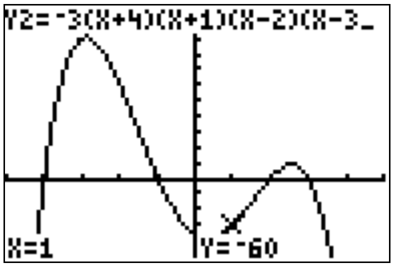
a) $y = k(x + 4)(x + 1)(x – 2)(x – 3)$

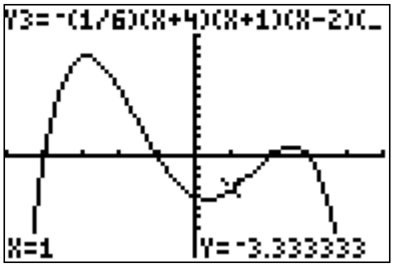
b) Answers may vary. A sample solution is shown.
$y = 2(x + 4)(x + 1)(x – 2)(x – 3)$, $y = –3(x + 4)(x + 1)(x – 2)(x – 3)$

c) Substitute $x = 0$ and $y = –4$ and solve for k.
$–4 = k(4)(1)(–2)(–3)$
$24k = –4$
$k = -\frac{1}{6}$
An equation is $y = -\frac{1}{6}(x + 4)(x + 1)(x – 2)(x – 3)$.

d) Answers may vary. A sample solution is shown.







Chapter 2 Section 4 **Question 10 Page 120**

a) $y = k(2x + 5)(x + 1)(2x – 7)(x – 3)$

b) Answers may vary. A sample solution is shown.
$y = -\frac{1}{2}(2x + 5)(x + 1)(2x – 7)(x – 3)$
$y = 2(2x + 5)(x + 1)(2x – 7)(x – 3)$

c) Substitute $x = -2$ and $y = 25$ and solve for k.

$25 = k[2(-2) + 5](-2 + 1)[2(-2) - 7](-2 - 3)$

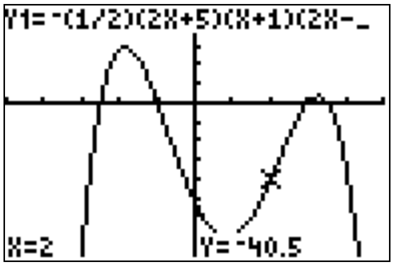
$25 = k(1)(-1)(-11)(-5)$

$25 = -55k$

$$k = -\frac{5}{11}$$

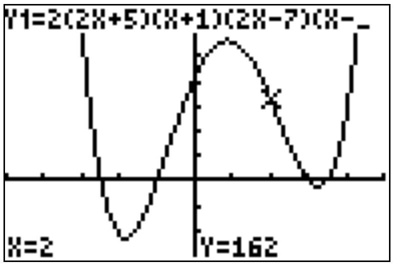
An equation is $y = -\frac{5}{11}(2x + 5)(x + 1)(2x - 7)(x - 3)$.

d) Answers may vary. A sample solution is shown.



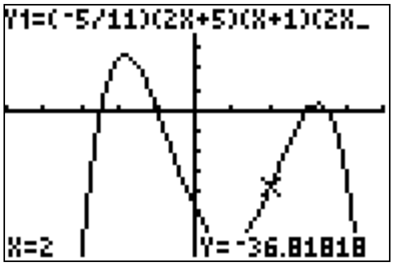
Y1=⁻(1/2)(2X+5)(X+1)(2X-_

X=2 Y=⁻40.5



Y1=2(2X+5)(X+1)(2X-7)(X-_

X=2 Y=162



Y1=(⁻5/11)(2X+5)(X+1)(2X_

X=2 Y=⁻36.81818

Chapter 2 Section 4 **Question 11 Page 120**

a) The factors are $(x-1+\sqrt{2}), (x-1-\sqrt{2})$ and $(2x+1)$.

$$\begin{aligned} y &= k(x-1+\sqrt{2})(x-1-\sqrt{2})(2x+1) \\ &= k(x^2-x-\sqrt{2}x-x+1+\sqrt{2}+\sqrt{2}x-\sqrt{2}-2)(2x+1) \\ &= k(x^2-2x-1)(2x+1) \\ &= k(2x^3+x^2-4x^2-2x-2x-1) \\ &= k(2x^3-3x^2-4x-1) \end{aligned}$$

b) Substitute $x = 3$ and $y = 35$ and solve for k.

$$\begin{aligned} 35 &= k\left[2(3)^3-3(3)^2-4(3)-1\right] \\ 35 &= k(54-27-12-1) \\ 35 &= 14k \\ k &= \frac{5}{2} \end{aligned}$$

An equation is $y = \frac{5}{2}(2x^3 - 3x^2 - 4x - 1)$.

Chapter 2 Section 4 **Question 12 Page 120**

a)

$$\begin{aligned} y &= k(x-3)^2(x+4+\sqrt{3})(x+4-\sqrt{3}) \\ &= k(x^2-6x+9)(x^2+4x-\sqrt{3}x+4x+16-4\sqrt{3}+\sqrt{3}x+4\sqrt{3}-3) \\ &= k(x^2-6x+9)(x^2+8x+13) \\ &= k(x^4+8x^3+13x^2-6x^3-48x^2-78x+9x^2+72x+117) \\ &= k(x^4+2x^3-26x^2-6x+117) \end{aligned}$$

b) Substitute $x = 1$ and $y = –22$ and solve for k.

$$\begin{aligned} -22 &= k\left[1^4+2(1)^3-26(1)^2-6(1)+117\right] \\ -22 &= k(88) \\ k &= -\frac{1}{4} \end{aligned}$$

An equation is $y = -\frac{1}{4}(x^4 + 2x^3 - 26x^2 - 6x + 117)$.

Chapter 2 Section 4 **Question 13 Page 120**

a) $y = k(x+1-\sqrt{5})(x+1+\sqrt{5})(x-2+\sqrt{2})(x-2-\sqrt{2})$

$$= k(x^2 + x + \sqrt{5}x + x + 1 + \sqrt{5} - \sqrt{5}x - \sqrt{5} - 5)\times$$
$$(x^2 - 2x - \sqrt{2}x - 2x + 4 + 2\sqrt{2} + \sqrt{2}x - 2\sqrt{2} - 2)$$
$$= k(x^2+2x-4)(x^2-4x+2)$$
$$= k(x^4 - 4x^3 + 2x^2 + 2x^3 - 8x^2 + 4x - 4x^2 + 16x - 8)$$
$$= k(x^4 - 2x^3 - 10x^2 + 20x - 8)$$

b) Substitute $x = 0$ and $y = –32$ and solve for k.

$–32 = k(–8)$

$k = 4$

An equation is $y = 4(x^4 – 2x^3 – 10x^2 + 20x – 8)$.

Chapter 2 Section 4 **Question 14 Page 120**

From the graph, the x-intercepts are –2, 1, and 3. The corresponding factors are $(x + 2)$, $(x – 1)$, and $(x – 3)$.

An equation for the family of polynomial functions with these zeros is

$y = k(x + 2)(x – 1)(x – 3)$.

The y-intercept is –12.

Substitute $x = 0$ and $y = –12$ and solve for k.

$–12 = k(2)(–1)(–3)$

$k = –2$

An equation is $y = –2(x + 2)(x – 1)(x – 3)$.

Chapter 2 Section 4 **Question 15 Page 121**

From the graph, the x-intercepts are -3 (order 2), 1, and $\frac{3}{2}$.
The corresponding factors are $(x + 3)^2$, $(x - 1)$, and $(2x - 3)$.
An equation for the family of polynomial functions with these zeros is
$y = k(x + 3)^2(x - 1)(2x - 3)$.
The y-intercept is 27.
Substitute $x = 0$ and $y = 27$ and solve for k.
$27 = k(3)^2(-1)(-3)$
$27 = 27k$
$k = 1$
An equation is $y = (x + 3)^2(x - 1)(2x - 3)$.

Chapter 2 Section 4 **Question 16 Page 121**

From the graph, the x-intercepts are $-\frac{7}{2}$, -2, 0, and 1. The corresponding factors are x, $(2x + 7)$, $(x + 2)$, and $(x - 1)$.
An equation for the family of polynomial functions with these zeros is
$y = kx(2x + 7)(x + 2)(x - 1)$.
The graph passes through $\left(-\frac{3}{2}, -15\right)$.

Substitute $x = -\frac{3}{2}$ and $y = -15$ and solve for k.

$$-15 = k\left(-\frac{3}{2}\right)\left[2\left(-\frac{3}{2}\right)+7\right]\left(-\frac{3}{2}+2\right)\left(-\frac{3}{2}-1\right)$$

$$-15 = k\left(-\frac{3}{2}\right)(4)\left(\frac{1}{2}\right)\left(-\frac{5}{2}\right)$$

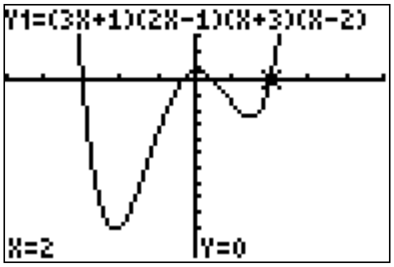
$$-15 = \frac{15}{2}k$$

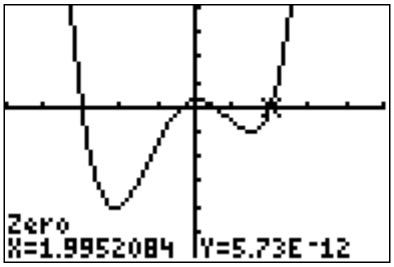
$k = -2$
An equation is $y = -2x(2x + 7)(x + 2)(x - 1)$.

Chapter 2 Section 4 **Question 17 Page 121**

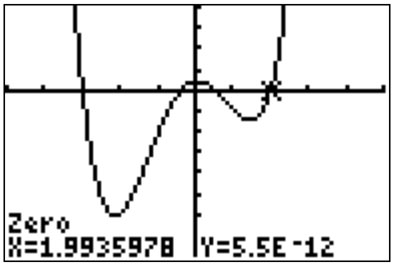
Set A: no; the zeros are different

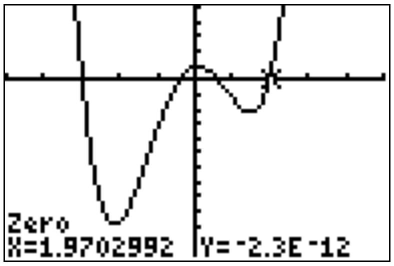
$y = (3x + 1)(2x - 1)(x + 3)(x - 2)$ $y = 2(3x + 1)(2x - 1)(x + 3)(x - 2) + 1$





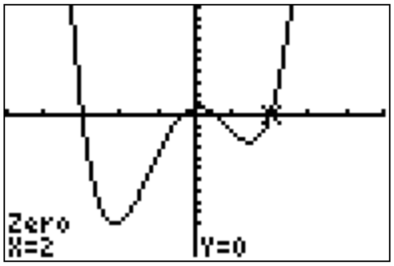
$y = 3(3x + 1)(2x - 1)(x + 3)(x - 2) + 2$ $y = 4(3x + 1)(2x - 1)(x + 3)(x - 2) + 3$

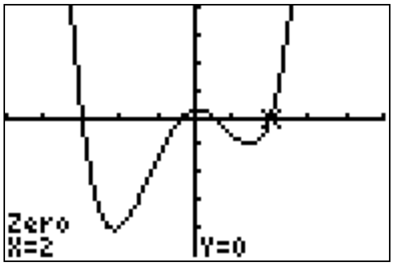




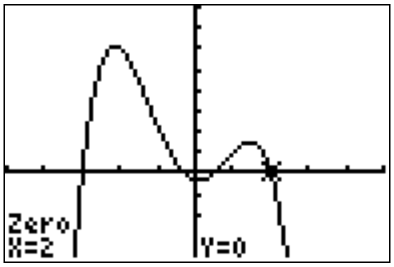
Set B: yes; the zeros are the same

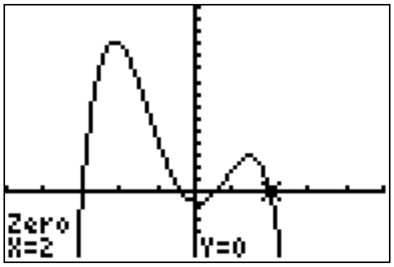
$y = (3x + 1)(2x - 1)(x + 3)(x - 2)$ $y = (3x + 1)(4x - 2)(x + 3)(x - 2)$





$y = 3(3x + 1)(1 - 2x)(x + 3)(x - 2)$ $y = 4(3x + 1)(2x - 1)(x + 3)(6 - 3x)$





Chapter 2 Section 4 **Question 18 Page 121**

a) height = x
width = $30 - x$
length = $48 - 2x$

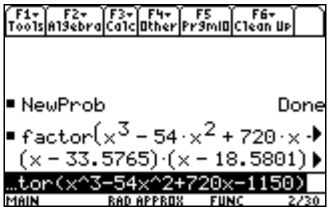
$V(x) = x(48 - 2x)(30 - x)$

b)

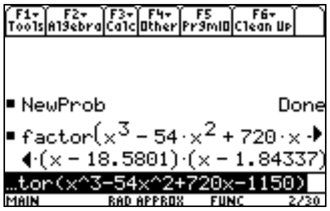
$$x(48-2x)(30-x)=2300$$
$$x(1440-48x-60x+2x^2)=2300$$
$$2x^3-108x^2+1440x-2300=0$$
$$2(x^3-54x^2+720x-1150)=0$$

Using the integral zero theorem test the values ±1, ±2, ±5, ±10, ±23, ±25, ±46, ±50, ±115, ±230, ±575, ±1150.
These values do not work.

Factor using CAS on approximate mode.



```
■ NewProb                    Done
■ factor(x^3 - 54·x^2 + 720·x ▸
  (x - 33.5765)·(x - 18.5801) ▸
...tor(x^3-54x^2+720x-1150)
MAIN      RAD APPROX    FUNC    2/30
```



```
■ NewProb                    Done
■ factor(x^3 - 54·x^2 + 720·x ▸
 ◂·(x - 18.5801)·(x - 1.84337)
...tor(x^3-54x^2+720x-1150)
MAIN      RAD APPROX    FUNC    2/30
```

$$2(x-33.5765)(x-18.5801)(x-1.84337)=0$$
$$x \doteq 33.6 \text{ or } x \doteq 18.6 \text{ or } x \doteq 1.84$$

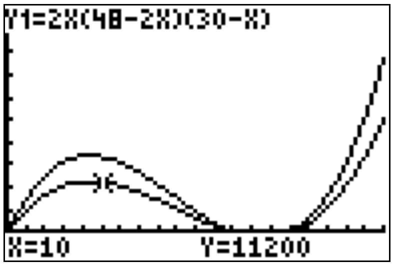
height = 33.6
width = –3.6
length = –19.2
Disregard the negative dimensions.

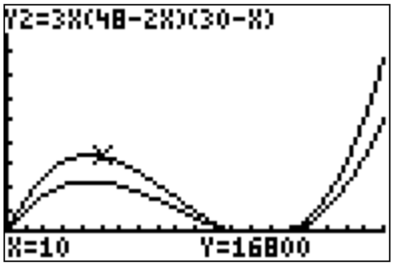
height $\doteq$ 18.6	height $\doteq$ 1.84
width $\doteq$ 11.4	width $\doteq$ 28.16
length $\doteq$ 10.8	length $\doteq$ 44.31

The possible dimensions of the box are approximately 44.31 cm by 28.16 cm by 1.84 cm or 18.6 cm by 11.4 cm by 10.8 cm.

c) volume doubles; volume triples; family of functions with zeros 24, 30, 0

d) Answers may vary. A sample solution is shown.





Chapter 2 Section 4 **Question 19 Page 121**

$y = kx(x - 30)(x - 60)(x - 90)(x - 120)(x - 150)$

Chapter 2 Section 4 **Question 20 Page 122**

a) height = x
width = $24 - 2x$
length = $36 - 2x$

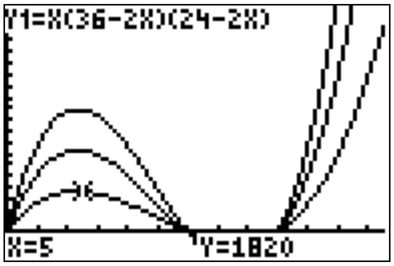
$V(x) = x(36 - 2x)(24 - 2x)$

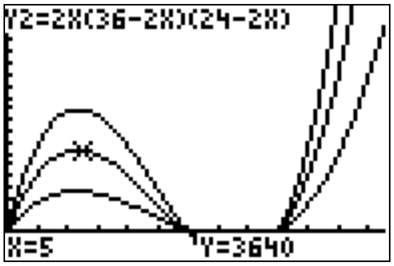
b) i) $V(x) = 2x(36 - 2x)(24 - 2x)$

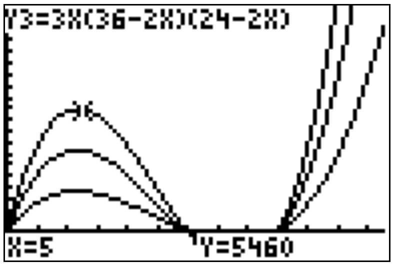
ii) $V(x) = 3x(36 - 2x)(24 - 2x)$

c) Family of functions with the same zeros: 0, 12, and 18.

d) Note that the domain and range are greater or equal to zero.







e) $x(36-2x)(24-2x)=1820$

$$x(864-120x+4x^2)-1820=0$$
$$4x^3-120x^2+864x-1820=0$$
$$4(x^3-30x^2+216x-455)=0$$
$$P(x)=x^3-30x^2+216x-455$$
$$P(5)=(5)^3-30(5)^2+216(5)-455$$
$$=125-750+1080-455$$
$$=0$$

Since 5 is a zero of the equation, $(x-5)$ is a factor.
Divide to determine the other factors.

$$\begin{array}{r|rrrr} -5 & 1 & -30 & 216 & -455 \\ & - & -5 & 125 & -455 \\ \hline \times & 1 & -25 & 91 & 0 \end{array}$$

$$(x-5)(x^2-25x+91)=0$$
$$x=5$$
or
$$x=\frac{25\pm\sqrt{(-25)^2-4(1)(91)}}{2(1)}$$
$$x=\frac{25\pm\sqrt{261}}{2}$$
$$x \doteq 20.58 \text{ or } x \doteq 4.42$$

height = 5	height ≐ 20.58	height ≐ 4.42
width = 14	width ≐ −17.16	width ≐ 15.16
length = 26	length ≐ −5.16	length ≐ 27.16

Disregard the negative dimensions.

The possible dimensions of the box are approximately 27.16 cm by 15.16 cm by 4.42 cm or 26 cm by 14 cm by 5 cm.

Chapter 2 Section 4 **Question 21 Page 122**

Solutions to Achievement Check questions are provided in the Teacher's Resource.

Chapter 2 Section 4 Question 22 Page 122

a) Answers may vary. A sample solution is shown.
$y = k(3x - 2)(x - 5)(x + 3)(x + 2)$

b) 4

c) Answers may vary. A sample solution is shown.
Substitute $x = -1$ and $y = -96$ and solve for k.

$$-96 = k[3(-1) - 2](-1 - 5)(-1 + 3)(-1 + 2)$$
$$-96 = k(-5)(-6)(2)(1)$$
$$-96 = 60k$$
$$k = -\frac{8}{5}$$

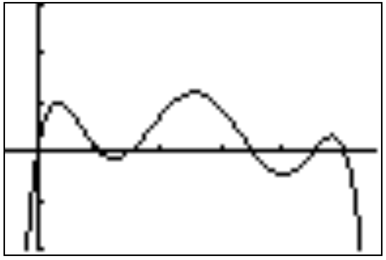
An equation is $y = -\frac{8}{5}(3x - 2)(x - 5)(x + 3)(x + 2)$.

d) Answers may vary. A sample solution is shown.

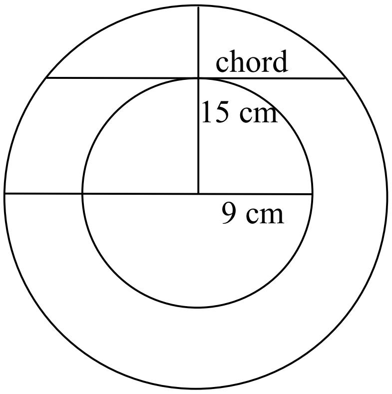
$$y = \frac{8}{5}(3x - 2)(x - 5)(x + 3)(x + 2)$$

Chapter 2 Section 4 Question 23 Page 122

Answers may vary. A sample solution is shown.
$y = x(x - 20)(x - 30)(x - 70)(x - 90)(x - 100) \div 100\ 000\ 000$



Chapter 2 Section 4 **Question 24 Page 122**



$$
\begin{aligned}
\text{chord} &= 2\sqrt{r^2 - d^2} \\
&= 2\sqrt{15^2 - 9^2} \\
&= 2\sqrt{144} \\
&= 2(12) \\
&= 24
\end{aligned}
$$

The length of the chord is 24 cm.

Chapter 2 Section 4 **Question 25 Page 122**

$$
\begin{aligned}
g(x^2+2) &= x^4 + 5x^2 + 3 \\
&= x^4 + 4x^2 + x^2 + 4 - 1 \\
&= (x^4 + 4x^2 + 4) + x^2 - 1 \\
&= (x^2+2)^2 + (x^2+2) - 3 \quad \text{Factor } x^4 + 4x^2 + 4.
\end{aligned}
$$

We have that $g(x) = x^2 + x - 3$.

$$
\begin{aligned}
g(x^2-1) &= (x^2-1)^2 + (x^2-1) - 3 \\
&= x^4 - 2x^2 + 1 + x^2 - 4 \\
&= x^4 - x^2 - 3
\end{aligned}
$$

$g(x^2 - 1) = x^4 - x^2 - 3$

Chapter 2 Section 5 Solve Inequalities Using Technology

Chapter 2 Section 5 Question 1 Page 129

a) $-7 < x \leq -1$

b) $-2 < x \leq 6$

c) $x < -3, x \geq 4$

d) $x \leq -1, x \geq 1$

Chapter 2 Section 5 Question 2 Page 129

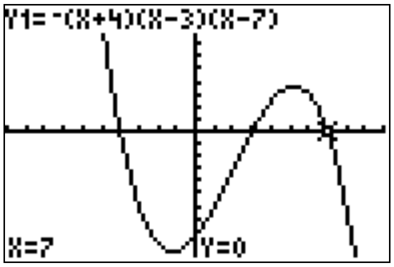
a) $x < -1, -1 < x < 5, x > 5$

b) $x < -7, -7 < x < 0, 0 < x < 2, x > 2$

c) $x < -6, -6 < x < 0, 0 < x < 1, x > 1$

d) $x < -4, -4 < x < -2, -2 < x < \frac{2}{5}, \frac{2}{5} < x < 4.3, x > 4.3$

Chapter 2 Section 5 Question 3 Page 129



Y1=⁻(X+4)(X−3)(X−7)

Chapter 2 Section 5 Question 4 Page 129

a) $f(x) > 0$ when $x < -2$ or $1 < x < 6$

b) $f(x) < 0$ when $-3.6 < x < 0$ or $x > 4.7$

Chapter 2 Section 5 Question 5 Page 130

a) i) The x-intercepts are –6 and 3.

ii) $f(x) > 0$ when $-6 < x < 3$

iii) $f(x) < 0$ when $x < -6$, $x > 3$

b) i) The x-intercepts are –2 and 5.

ii) $f(x) > 0$ when $x < -2$, $x > 5$

iii) $f(x) < 0$ when $-2 < x < 5$

c) i) The x-intercepts are –4, 3, and 5.

ii) $f(x) > 0$ when $-4 < x < 3$, $x > 5$

iii) $f(x) < 0$ when $x < -4$, $3 < x < 5$

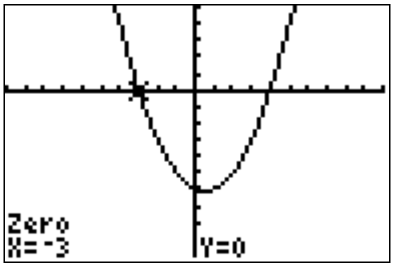
d) i) The x-intercepts are –4 and 1.

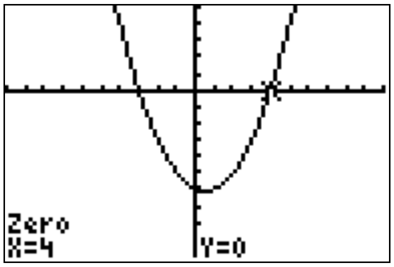
ii) $f(x) > 0$ when $x < -4$

iii) $f(x) < 0$ when $-4 < x < 1$, $x > 1$

Chapter 2 Section 5 Question 6 Page 130

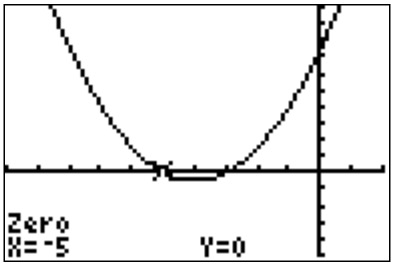
a)

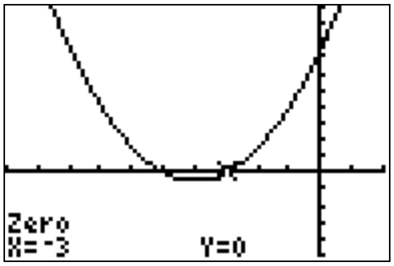




The values that satisfy the inequality $x^2 - x - 12 < 0$ are the values of x for which the graph is negative (below the x-axis). From the graph, this occurs when $-3 < x < 4$.

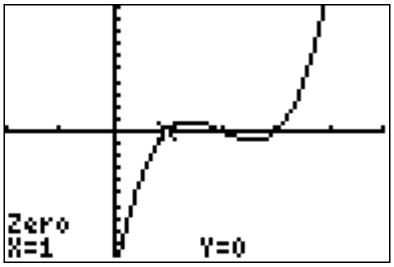
b)

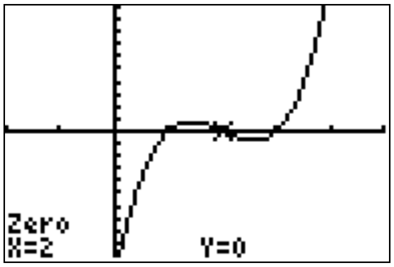


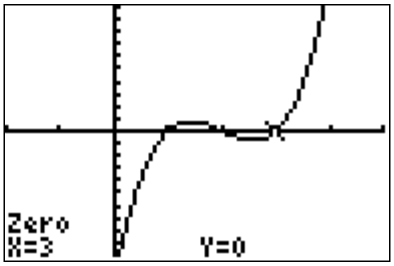


The values that satisfy the inequality $x^2 + 8x + 15 \le 0$ are the values of x for which the graph is zero or negative (on or below the x-axis). From the graph, this occurs when $-5 \le x \le -3$.

c)

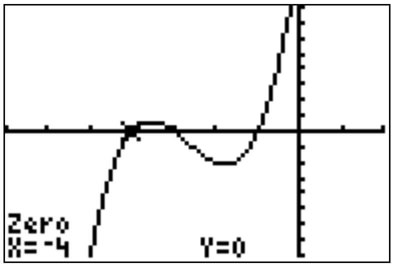


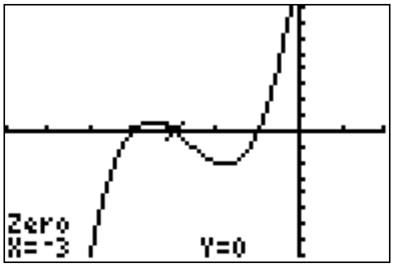


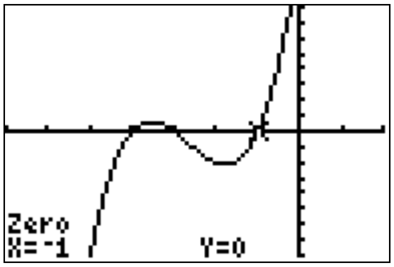


The values that satisfy the inequality $x^3 - 6x^2 + 11x - 6 > 0$ are the values of x for which the graph is positive (above the x-axis). From the graph, this occurs when
$1 < x < 2$, $x > 3$.

d)

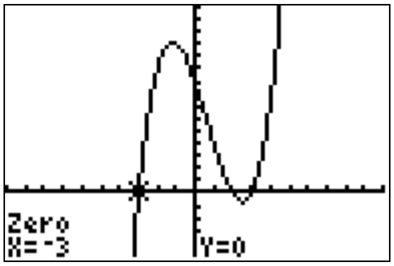


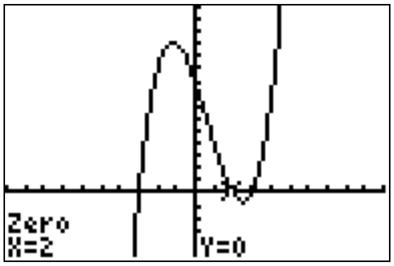


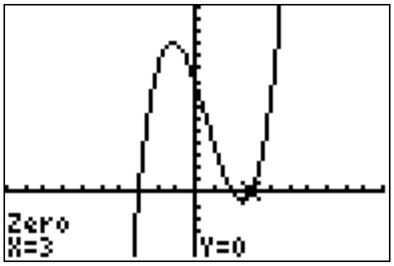


The values that satisfy the inequality $x^3 + 8x^2 + 19x + 12 \geq 0$ are the values of x for which the graph is zero or positive (on or above the x-axis). From the graph, this occurs when
$-4 \leq x \leq -3, x \geq -1$

e)

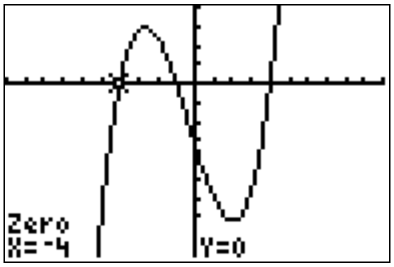


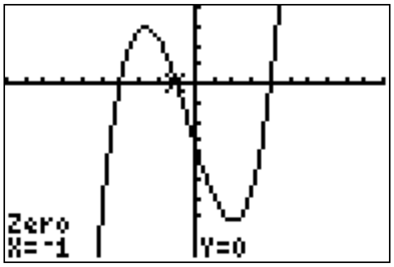


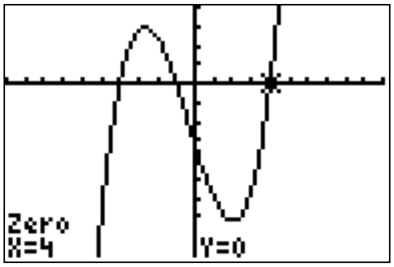


The values that satisfy the inequality $x^3 - 2x^2 - 9x + 18 < 0$ are the values of x for which the graph is negative (below the x-axis). From the graph, this occurs when
$x < -3, 2 < x < 3$.

f)



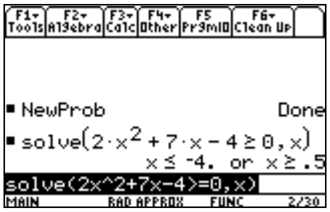




The values that satisfy the inequality $x^3 + x^2 - 16x - 16 \leq 0$ are the values of x for which the graph is zero or negative (on or below the x-axis). From the graph, this occurs when $x \leq -4$, $-1 \leq x \leq 4$.

Chapter 2 Section 5 **Question 7 Page 130**

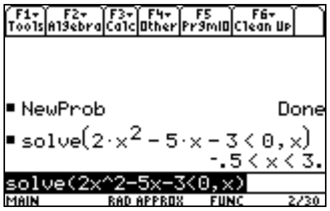
a)



```
■ NewProb                     Done
■ solve(2·x² + 7·x - 4 ≥ 0, x)
                 x ≤ -4. or x ≥ .5
solve(2x^2+7x-4)>=0,x)
```

$x \leq -4$ or $x \geq 0.5$

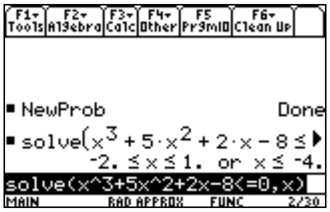
b)



```
■ NewProb                     Done
■ solve(2·x² - 5·x - 3 < 0, x)
                      -.5 < x < 3.
solve(2x^2-5x-3<0,x)
```

$-0.5 < x < 3$

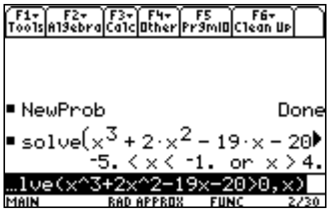
c)



```
■ NewProb                     Done
■ solve(x³ + 5·x² + 2·x − 8 ≤ ▸
       -2. ≤ x ≤ 1.  or  x ≤ -4.
solve(x^3+5x^2+2x-8<=0,x)
```

$x \le -4$ or $-2 \le x \le 1$

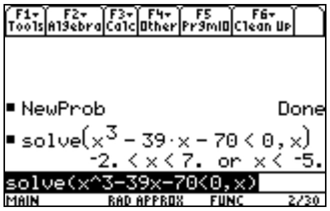
d)



```
■ NewProb                     Done
■ solve(x³ + 2·x² − 19·x − 20▸
       -5. < x < -1.  or  x > 4.
…lve(x^3+2x^2-19x-20>0,x)
```

$-5 < x < -1$ or $x > 4$

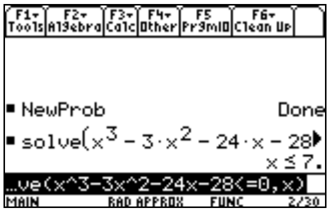
e)



```
■ NewProb                     Done
■ solve(x³ − 39·x − 70 < 0, x)
       -2. < x < 7.  or  x < -5.
solve(x^3-39x-70<0,x)
```

$x < -5$ or $-2 < x < 7$

f)

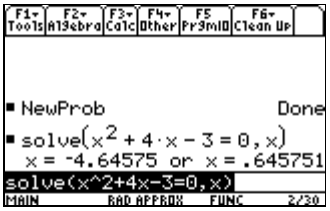


```
■ NewProb                     Done
■ solve(x³ − 3·x² − 24·x − 28▸
                          x ≤ 7.
…ve(x^3-3x^2-24x-28<=0,x)
```

$x \le 7$

Chapter 2 Section 5 **Question 8 Page 130**

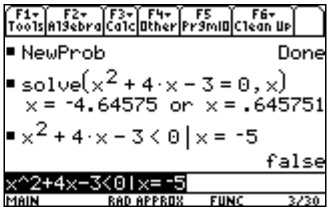
a)



```
■ NewProb                       Done
■ solve(x^2 + 4·x - 3 = 0, x)
  x = -4.64575 or x = .645751
solve(x^2+4x-3=0,x)
```

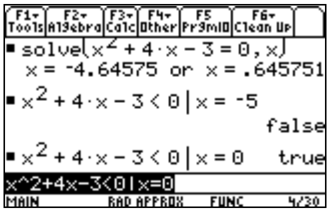
The roots are approximately –4.65 and 0.65.
The intervals are $x < -4.65$, $-4.65 < x < 0.65$, and $x > 0.65$.

For $x < -4.65$, test $x = -5$.



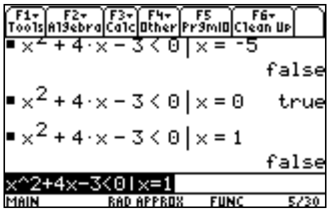
```
■ NewProb                       Done
■ solve(x^2 + 4·x - 3 = 0, x)
  x = -4.64575 or x = .645751
■ x^2 + 4·x - 3 < 0 | x = -5
                               false
x^2+4x-3<0|x=-5
```

For $-4.65 < x < 0.65$, test $x = 0$.



```
■ solve(x^2 + 4·x - 3 = 0, x)
  x = -4.64575 or x = .645751
■ x^2 + 4·x - 3 < 0 | x = -5
                               false
■ x^2 + 4·x - 3 < 0 | x = 0     true
x^2+4x-3<0|x=0
```

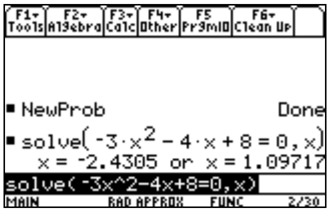
For $x > 0.65$, test 1.



```
■ x^2 + 4·x - 3 < 0 | x = -5
                               false
■ x^2 + 4·x - 3 < 0 | x = 0     true
■ x^2 + 4·x - 3 < 0 | x = 1
                               false
x^2+4x-3<0|x=1
```

The solution is $-4.65 < x < 0.65$ since the inequality is true for the values tested in this interval.

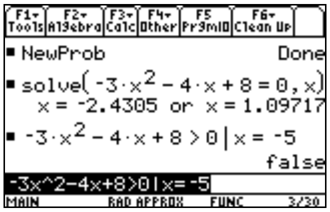
b)



```
■ NewProb                    Done
■ solve(⁻3·x²-4·x+8=0,x)
    x = ⁻2.4305 or x = 1.09717
solve(⁻3x^2-4x+8=0,x)
MAIN      RAD APPROX   FUNC   2/30
```

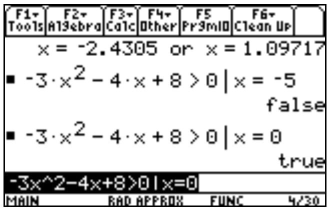
The roots are –2.43 and 1.10.
The intervals are $x < -2.43$, $-2.43 < x < 1.10$, and $x > 1.10$.

For $x < -2.43$, test $x = -5$.



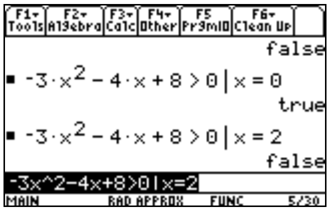
```
■ NewProb                    Done
■ solve(⁻3·x²-4·x+8=0,x)
    x = ⁻2.4305 or x = 1.09717
■ ⁻3·x²-4·x+8>0|x = ⁻5
                            false
⁻3x^2-4x+8>0|x=⁻5
MAIN      RAD APPROX   FUNC   3/30
```

For $-2.43 < x < 1.10$, test 0.



```
    x = ⁻2.4305 or x = 1.09717
■ ⁻3·x²-4·x+8>0|x = ⁻5
                            false
■ ⁻3·x²-4·x+8>0|x = 0
                             true
⁻3x^2-4x+8>0|x=0
MAIN      RAD APPROX   FUNC   4/30
```

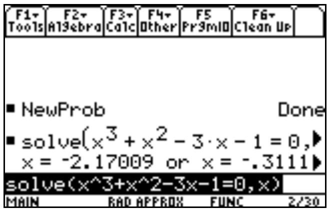
For $x > 1.10$, test $x = 2$.



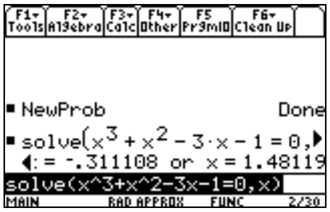
```
                            false
■ ⁻3·x²-4·x+8>0|x = 0
                             true
■ ⁻3·x²-4·x+8>0|x = 2
                            false
⁻3x^2-4x+8>0|x=2
MAIN      RAD APPROX   FUNC   5/30
```

The solution is $-2.43 < x < 1.10$ since the inequality is true for the value tested in this interval.

c)



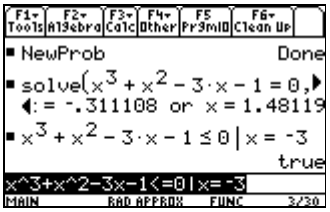
```
■ NewProb                      Done
■ solve(x^3 + x^2 - 3·x - 1 = 0,▸
  x = ⁻2.17009 or x = ⁻.3111▸
solve(x^3+x^2-3x-1=0,x)
MAIN      RAD APPROX   FUNC    2/30
```



```
■ NewProb                      Done
■ solve(x^3 + x^2 - 3·x - 1 = 0,▸
 ◂: = ⁻.311108 or x = 1.48119
solve(x^3+x^2-3x-1=0,x)
MAIN      RAD APPROX   FUNC    2/30
```

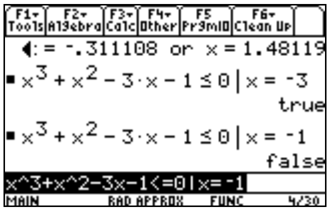
The roots are approximately –2.17, –0.31, and 1.48.
The intervals are $x < –2.17$, $–2.17 < x < –0.31$, $–0.31< x < 1.48$, and $x > 1.48$.

For $x < –2.17$, test $x = –3$.



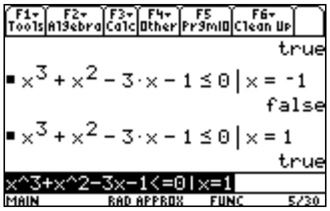
```
■ NewProb                      Done
■ solve(x^3 + x^2 - 3·x - 1 = 0,▸
 ◂: = ⁻.311108 or x = 1.48119
■ x^3 + x^2 - 3·x - 1 ≤ 0 | x = ⁻3
                               true
x^3+x^2-3x-1<=0|x=-3
MAIN      RAD APPROX   FUNC    3/30
```

For $–2.17 < x < –0.31$, test $x = –1$.



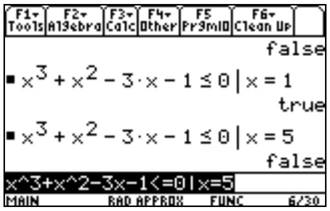
```
 ◂: = ⁻.311108 or x = 1.48119
■ x^3 + x^2 - 3·x - 1 ≤ 0 | x = ⁻3
                               true
■ x^3 + x^2 - 3·x - 1 ≤ 0 | x = ⁻1
                              false
x^3+x^2-3x-1<=0|x=-1
MAIN      RAD APPROX   FUNC    4/30
```

For $–0.31< x < 1.48$, test $x = 1$.



```
                               true
■ x^3 + x^2 - 3·x - 1 ≤ 0 | x = ⁻1
                              false
■ x^3 + x^2 - 3·x - 1 ≤ 0 | x = 1
                               true
x^3+x^2-3x-1<=0|x=1
MAIN      RAD APPROX   FUNC    5/30
```

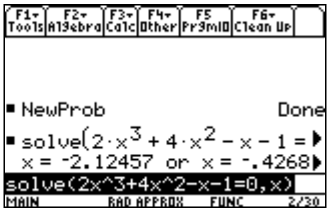
For $x > 1.48$, test $x = 5$.



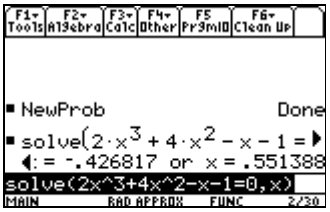
```
                              false
■ x^3 + x^2 - 3·x - 1 ≤ 0 | x = 1
                               true
■ x^3 + x^2 - 3·x - 1 ≤ 0 | x = 5
                              false
x^3+x^2-3x-1<=0|x=5
MAIN      RAD APPROX   FUNC    6/30
```

The solution is $x \le –2.17$ or $–0.31 \le x \le 1.48$, since the inequality is true for the values tested in these intervals.
$x \le –2.17$ or $–0.31 \le x \le 1.48$

d)



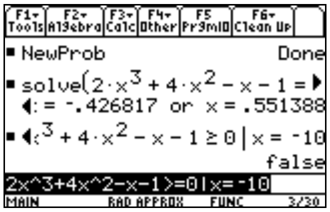
```
■ NewProb                    Done
■ solve(2·x³ + 4·x² − x − 1 = ▸
  x = -2.12457 or x = -.4268▸
solve(2x^3+4x^2-x-1=0,x)
```



```
■ NewProb                    Done
■ solve(2·x³ + 4·x² − x − 1 = ▸
  ◂: = -.426817 or x = .551388
solve(2x^3+4x^2-x-1=0,x)
```

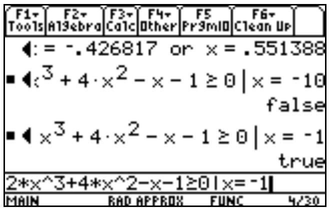
The roots are –2.12, –0.43, and 0.55.
The intervals are $x < -2.12$, $-2.12 < x < -0.43$, $-0.43 < x < 0.55$, and $x > 0.55$.

For $x < -2.12$, test $x = -10$.



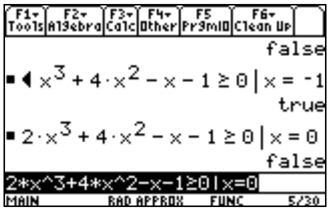
```
■ NewProb                    Done
■ solve(2·x³ + 4·x² − x − 1 = ▸
  ◂: = -.426817 or x = .551388
■ ◂³ + 4·x² − x − 1 ≥ 0 | x = -10
                            false
2x^3+4x^2-x-1>=0|x=-10
```

For $-2.12 < x < -0.43$, test $x = -1$.



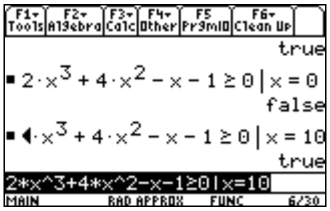
```
  ◂: = -.426817 or x = .551388
■ ◂³ + 4·x² − x − 1 ≥ 0 | x = -10
                            false
■ ◂ x³ + 4·x² − x − 1 ≥ 0 | x = -1
                             true
2*x^3+4*x^2-x-1≥0|x=-1
```

$-0.43 < x < 0.55$, test $x = 0$.



```
                            false
■ ◂ x³ + 4·x² − x − 1 ≥ 0 | x = -1
                             true
■ 2·x³ + 4·x² − x − 1 ≥ 0 | x = 0
                            false
2*x^3+4*x^2-x-1≥0|x=0
```

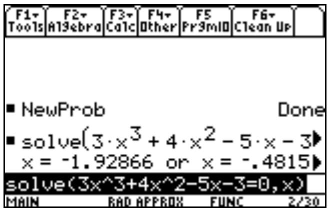
$x > 0.55$, test $x = 10$.



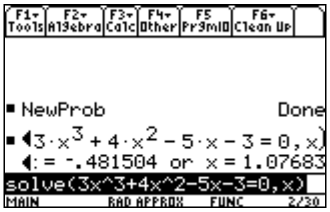
```
                             true
■ 2·x³ + 4·x² − x − 1 ≥ 0 | x = 0
                            false
■ ◂·x³ + 4·x² − x − 1 ≥ 0 | x = 10
                             true
2*x^3+4*x^2-x-1≥0|x=10
```

The solution is $-2.12 \leq x \leq -0.43$ or $x \geq 0.55$, since the inequality is true for the values tested in these intervals.

e)



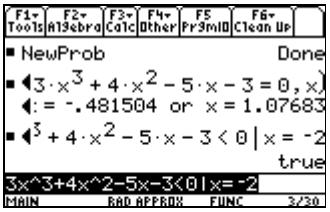
```
■ NewProb                   Done
■ solve(3·x^3 + 4·x^2 - 5·x - 3▸
  x = ⁻1.92866 or x = ⁻.4815▸
solve(3x^3+4x^2-5x-3=0,x)
```



```
■ NewProb                   Done
■ ◂3·x^3 + 4·x^2 - 5·x - 3 = 0, x)
  ◂: = ⁻.481504 or x = 1.07683
solve(3x^3+4x^2-5x-3=0,x)
```

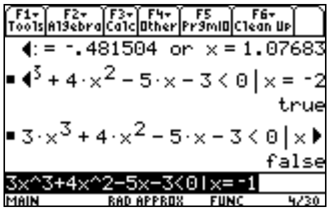
The roots are –1.93, –0.48, and 1.08.
The intervals are $x < -1.93$, $-1.93 < x < -0.48$, $-0.48 < x < 1.08$, and $x > 1.08$.

$x < -1.93$, test $x = -2$.



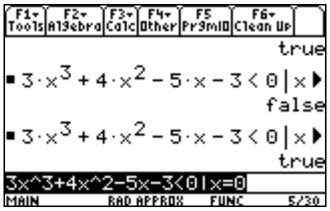
```
■ NewProb                   Done
■ ◂3·x^3 + 4·x^2 - 5·x - 3 = 0, x)
  ◂: = ⁻.481504 or x = 1.07683
■ ◂3 + 4·x^2 - 5·x - 3 < 0 | x = ⁻2
                            true
3x^3+4x^2-5x-3<0|x=-2
```

$-1.93 < x < -0.48$, test $x = -1$.



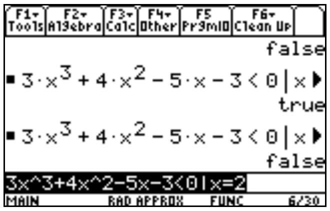
```
  ◂: = ⁻.481504 or x = 1.07683
■ ◂3 + 4·x^2 - 5·x - 3 < 0 | x = ⁻2
                            true
■ 3·x^3 + 4·x^2 - 5·x - 3 < 0 | x▸
                           false
3x^3+4x^2-5x-3<0|x=-1
```

$-0.48 < x < 1.08$, test $x = 0$.



```
                            true
■ 3·x^3 + 4·x^2 - 5·x - 3 < 0 | x▸
                           false
■ 3·x^3 + 4·x^2 - 5·x - 3 < 0 | x▸
                            true
3x^3+4x^2-5x-3<0|x=0
```

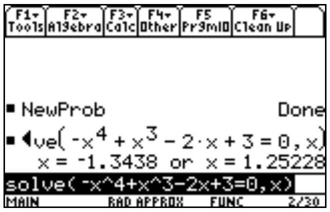
$x > 1.08$, test $x = 2$.



```
                           false
■ 3·x^3 + 4·x^2 - 5·x - 3 < 0 | x▸
                            true
■ 3·x^3 + 4·x^2 - 5·x - 3 < 0 | x▸
                           false
3x^3+4x^2-5x-3<0|x=2
```

The solution is $x < -1.93$ or $-0.48 < x < 1.08$, since the inequality is true for the values tested in these intervals.

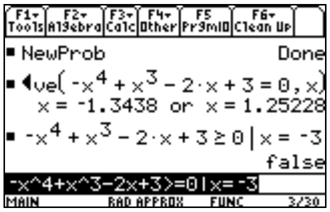
f)



```
■ NewProb                       Done
■ ◂ve( -x^4 + x^3 - 2·x + 3 = 0, x)
   x = -1.3438 or x = 1.25228
solve(-x^4+x^3-2x+3=0,x)
```

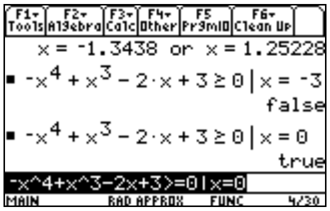
The roots are –1.34 and 1.25.
The intervals are $x < -1.34$, $-1.34 < x < 1.25$, and $x > 1.25$.

$x < -1.34$, test $x = -3$



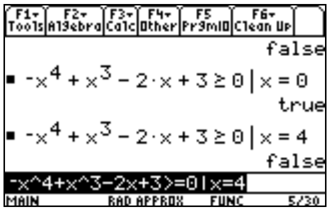
```
■ NewProb                       Done
■ ◂ve( -x^4 + x^3 - 2·x + 3 = 0, x)
   x = -1.3438 or x = 1.25228
■ -x^4 + x^3 - 2·x + 3 ≥ 0 | x = -3
                               false
-x^4+x^3-2x+3>=0|x=-3
```

$-1.34 < x < 1.25$, test $x = 0$.



```
   x = -1.3438 or x = 1.25228
■ -x^4 + x^3 - 2·x + 3 ≥ 0 | x = -3
                               false
■ -x^4 + x^3 - 2·x + 3 ≥ 0 | x = 0
                                true
-x^4+x^3-2x+3>=0|x=0
```

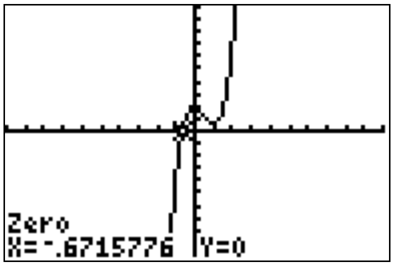
$x > 1.25$, test $x = 4$.



```
                               false
■ -x^4 + x^3 - 2·x + 3 ≥ 0 | x = 0
                                true
■ -x^4 + x^3 - 2·x + 3 ≥ 0 | x = 4
                               false
-x^4+x^3-2x+3>=0|x=4
```

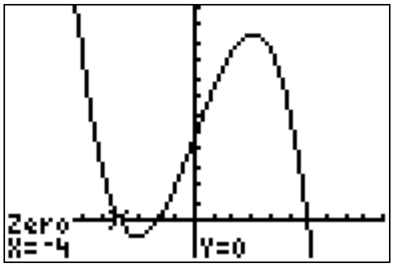
The solution is $-1.34 \le x \le 1.25$, since the inequality is true for the value tested in this interval.

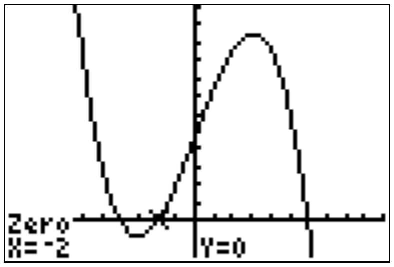
a)

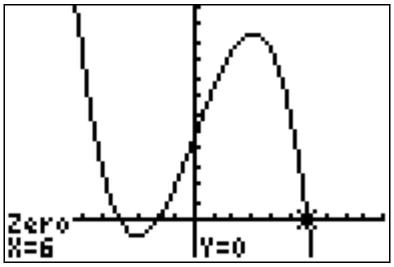


The values that satisfy the inequality $5x^3 - 7x^2 - x + 4 > 0$ are the values of x for which the graph is positive (above the x-axis). From the graph, this occurs approximately when $x > -0.67$.

b)

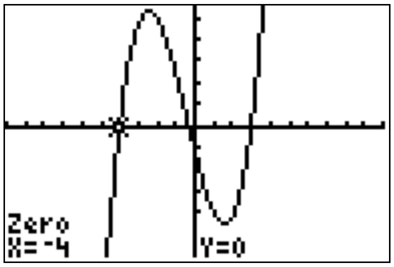


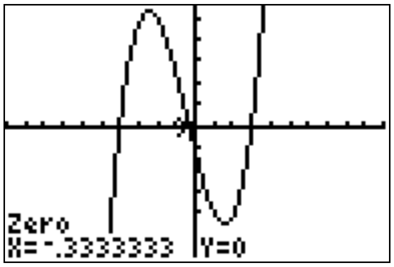


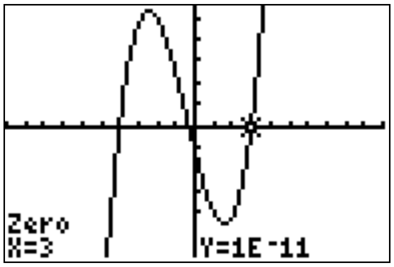


The values that satisfy the inequality $-x^3 + 28x + 48 \geq 0$ are the values of x for which the graph is zero or positive (on or above the x-axis). From the graph, this occurs when $x \leq -4$ or $-2 \leq x \leq 6$.

c)

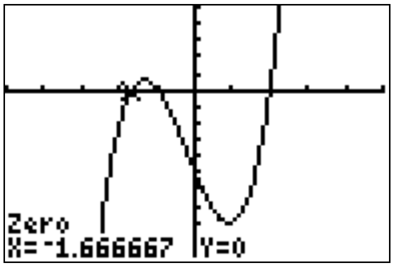


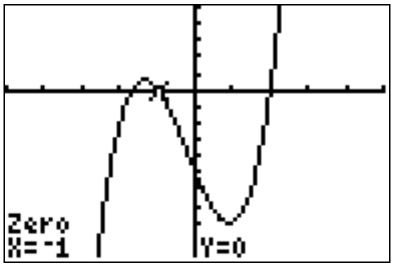


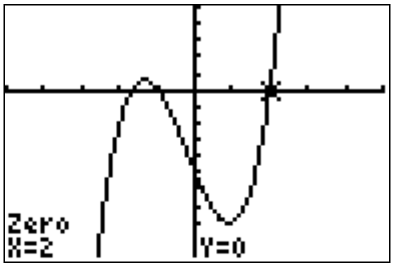


The values that satisfy the inequality $3x^3 + 4x^2 - 35x - 12 \le 0$ are the values of x for which the graph is zero or negative (on or below the x-axis). From the graph, this occurs when $x \le -4$ or $-\frac{1}{3} \le x \le 3$.

d)



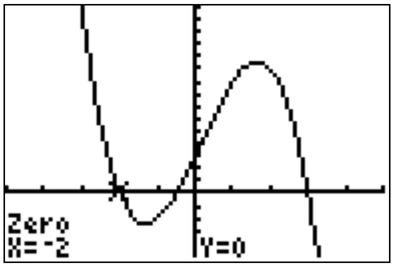


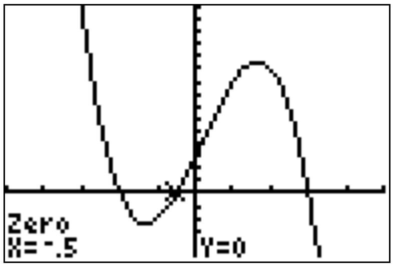


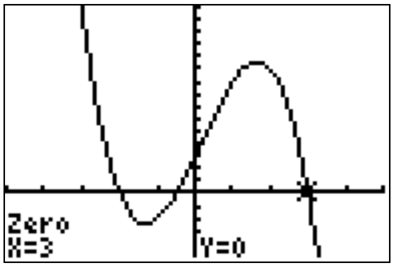
The values that satisfy the inequality $3x^3 + 2x^2 - 11x - 10 < 0$ are the values of x for which the graph is negative (below the x-axis). From the graph, this occurs when

$x < -\frac{5}{3}$ or $-1 < x < 2$.

e)

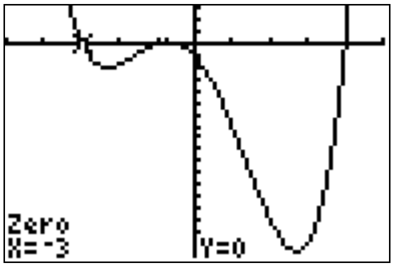


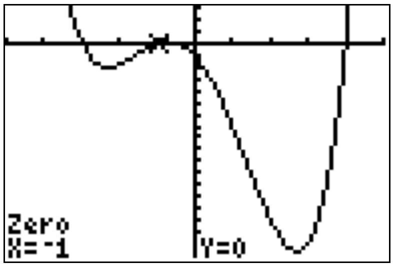


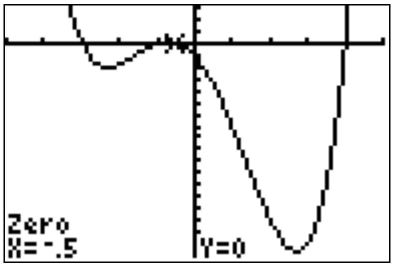


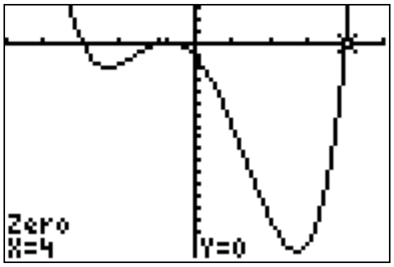
The values that satisfy the inequality $-2x^3 + x^2 + 13x + 6 > 0$ are the values of x for which the graph is positive (above the x-axis). From the graph, this occurs when $x < -2$ or $-\frac{1}{2} < x < 3$.

f)





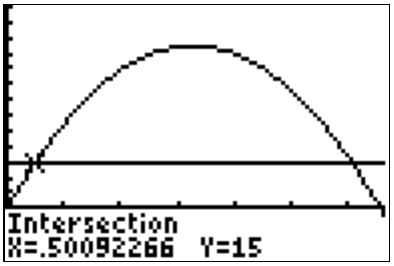




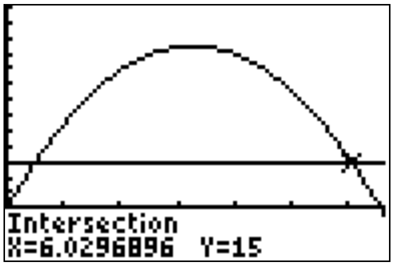
The values that satisfy the inequality $2x^4 + x^3 - 26x2 - 37x - 12 > 0$ are the values of x for which the graph is positive (above the x-axis). From the graph, this occurs when

$x < -3$ or $-1 < x < -\frac{1}{2}$ or $x > 4$.

Chapter 2 Section 5 **Question 10 Page 131**



```
Intersection
X=.50092266  Y=15
```

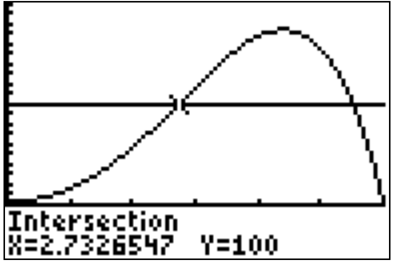


```
Intersection
X=6.0296896  Y=15
```

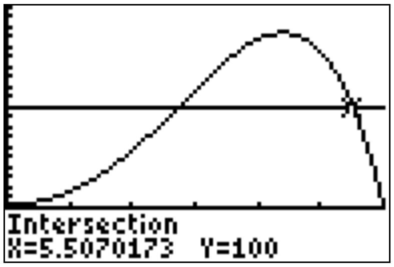
The height of the ball is greater than 15 m approximately when $0.50 < t < 6.03$, or between about 0.5 s and 6.03 s.

Chapter 2 Section 5 **Question 11 Page 131**

a)



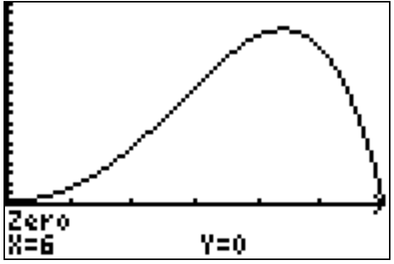
```
Intersection
X=2.7326547  Y=100
```



```
Intersection
X=5.5070173  Y=100
```

The tent caterpillar population was greater than 10 000 approximately when $2.73 < t < 5.51$, or between later in the second week and halfway through the fifth week.

b)



```
Zero
X=6          Y=0
```

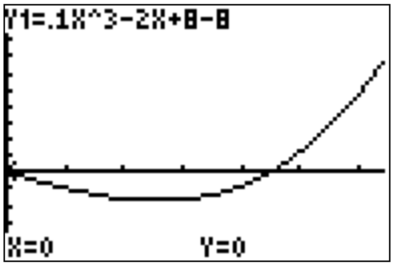
There are no tent caterpillars left.

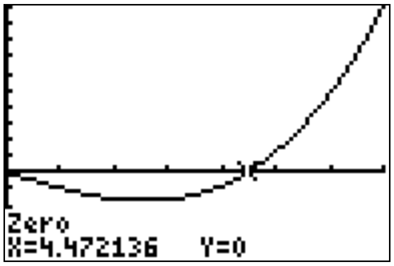
a) Write the inequality as $0.1t^3 - 2t + 8 < 8$

$$0.1t^3 - 2t + 8 - 8 < 0$$

$$0.1t^3 - 2t < 0$$

Graph the function $y = 0.1t^3 - 2t$





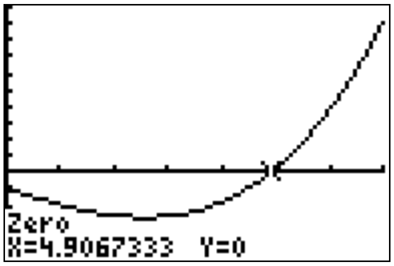
There are fewer than 8000 on-line customers between 0 and approximately 4.47 years.

b) Write the inequality as $0.1t^3 - 2t + 8 > 10$

$$0.1t^3 - 2t + 8 - 10 > 0$$

$$0.1t^3 - 2t - 2 > 0$$

Graph the function $y = 0.1t^3 - 2t - 2$



The number of on-line customers exceeds 10 000 after approximately 4.91 years.

Chapter 2 Section 5 Question 13 Page 131

Answers may vary. A sample solution is shown.

a) i) $(x - 1)(x^2 + 1) > 0$ or $x^3 - x^2 + x - 1$
ii) $x(x - 1)^2 > 0$ or $x^3 - 2x^2 + x$
iii) $x(x - 1)^2 > 0$ or $x^3 - 2x^2 + x$

b) i) $x > 1$
ii) $0 < x < 1, x > 1$
iii) $0 < x < 1, x > 1$

Chapter 2 Section 5 Question 14 Page 131

Answers may vary. A sample solution is shown.

a) i) $(x^2 + 1)(x^2 + 3) > 0$ or $x^4 + 4x^2 + 3$
ii) $x^2(x - 1)^2 > 0$ or $x^4 - 2x^3 + x^2$
iii) $x^2(x - 1)(x + 1) > 0$ or $x^4 - x^2$
iv) $x^2(x - 1)^2 > 0$ or $x^4 - 2x^3 + x^2$

b) i) $x \in \mathbb{R}$
ii) $x < 0, 0 < x < 1, x > 1$
iii) $x < 0, x > 1$
iv) $x < 0, 0 < x < 1, x > 1$

Chapter 2 Section 5 Question 15 Page 131

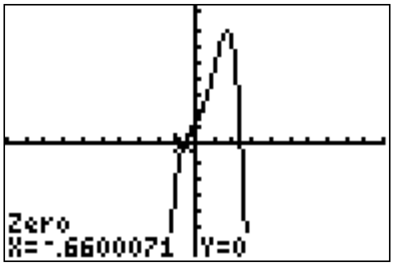
Answers may vary. A sample solution is shown.

a) $(3x + 2)(5x - 4)(2x - 7) > 0, -30x^3 - 109x^2 - 2x - 56 < 0$

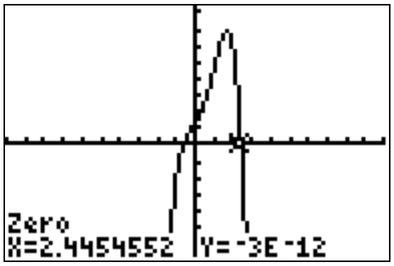
b) $x^3 - 2x^2 - 10x + 8 > 0, -x^3 + 2x^2 + 10x - 8 < 0$

Chapter 2 Section 5 Question 16 Page 131

$3x^4 - 6x^4 + 5x^3 + 2x^2 + x^2 - 4x + 9x + 6 - 2 \geq 0$
$-3x^4 + 5x^3 + 3x^2 + 5x + 4 \geq 0$



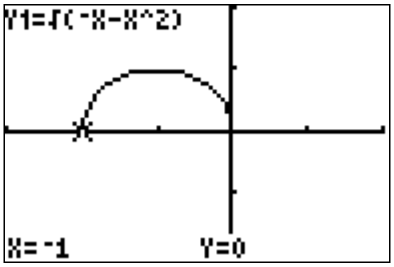
Zero
X=⁻.6600071 Y=0

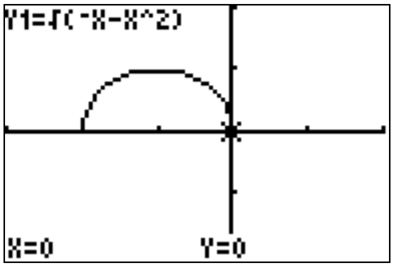


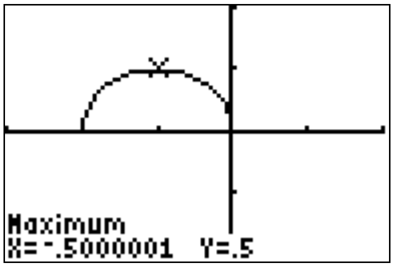
Zero
X=2.4454552 Y=⁻3E⁻12

The equality is satisfied for approximately $-0.66 \leq x \leq 2.45$.

a) Graph the function.

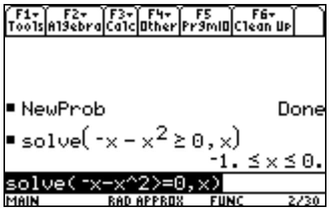






or

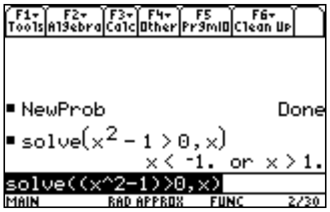
Domain



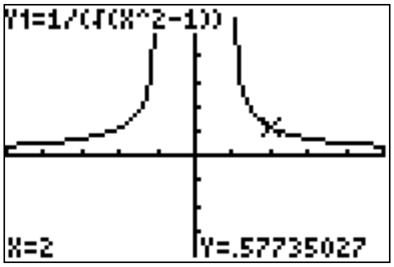
```
■ NewProb                 Done
■ solve(-x - x² ≥ 0, x)
                   -1. ≤ x ≤ 0.
solve(-x-x^2)>=0,x)
MAIN      RAD APPROX   FUNC   2/30
```

$\left\{x \in \mathbb{R}, -1 \le x \le 0\right\}$, $\left\{y \in \mathbb{R}, 0 \le y \le \frac{1}{2}\right\}$

b) Domain Range



```
■ NewProb                 Done
■ solve(x² - 1 > 0, x)
              x < -1. or x > 1.
solve((x^2-1)>0,x)
MAIN      RAD APPROX   FUNC   2/30
```



$\left\{x \in \mathbb{R}, x < -1, x > 1\right\}$, $\left\{y \in \mathbb{R}, y > 0\right\}$

Chapter 2 Section 5 **Question 18 Page 131**

PQ = PR (both tangents to the circle from the same point)
QO = RO (radius of the circle)
PO = PO (common)
Therefore, ΔPQO $\cong$ ΔPRO and $\angle$POQ = $\angle$POR

Chapter 2 Section 5 **Question 19 Page 131**

$$\begin{aligned} f\left(\frac{5}{3}\right) &= k\left(\frac{5}{3}\right)^2 - b\left(\frac{5}{3}\right) + k \\ 0 &= \frac{25}{9}k - \frac{5}{3}b + k \\ 0 &= \left(\frac{25}{9} + 1\right)k - \frac{5}{3}b \\ \frac{34}{9}k &= \frac{15}{9}b \\ 34k &= 15b \\ \frac{k}{b} &= \frac{15}{34} \\ k : b &= 15 : 34 \end{aligned}$$

Chapter 2 Section 5 **Question 20 Page 131**

$$(\text{PR})^2 = 4^2 + (\text{RS})^2$$
$$(\text{QR})^2 = 6^2 + (\text{RS})^2$$

$$\begin{aligned} 10^2 &= (\text{PR})^2 + (\text{QR})^2 \\ 10^2 &= \left(4^2 + (\text{RS})^2\right) + \left(6^2 + (\text{RS})^2\right) \\ 100 &= 16 + (\text{RS})^2 + 36 + (\text{RS})^2 \\ 48 &= 2(\text{RS})^2 \\ 24 &= (\text{RS})^2 \\ \sqrt{24} &= \text{RS} \\ 2\sqrt{6} &= \text{RS} \end{aligned}$$

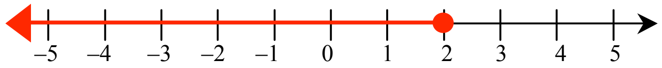
The exact length of RS is $2\sqrt{6}$.

Chapter 2 Section 6 Solve Factorable Polynomial Inequalities Algebraically

Chapter 2 Section 6 Question 1 Page 138

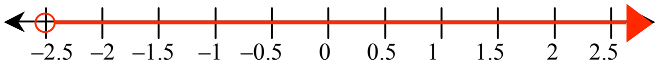
a) $x \le 5 - 3$

$x \le 2$



b) $2x > -4 - 1$

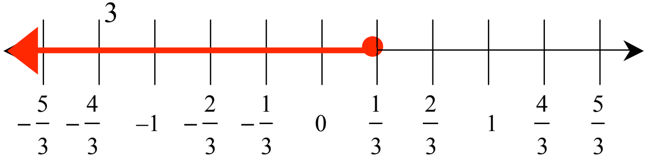
$x > -\frac{5}{2}$



c) $-3x \ge 6 - 5$

$-3x \ge 1$

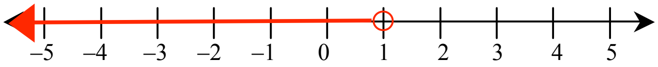
$x \le \frac{1}{3}$



d) $7x - 3x < 4$

$4x < 4$

$x < 1$

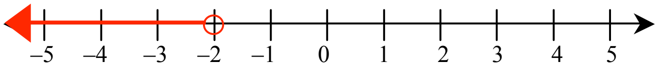


e) $2 - 20 > 5x + 4x$

$-18 > 9x$

$-2 > x$

$x < -2$



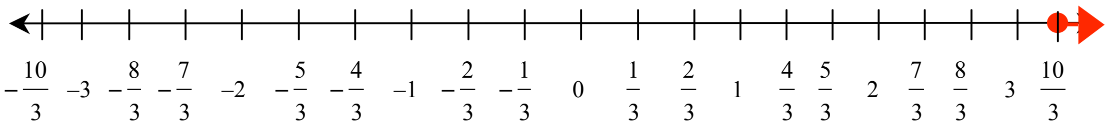
f) $2 - 2x \le x - 8$

$2 + 8 \le x + 2x$

$10 \le 3x$

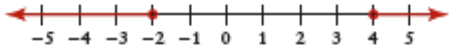
$\frac{10}{3} \le x$

$x \ge \frac{10}{3}$

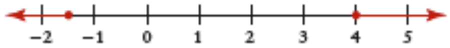


Chapter 2 Section 6 Question 2 Page 138

a) $x < -2$ or $x > 4$

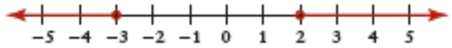


b) $x \le -\frac{3}{2}$ or $x \ge 4$

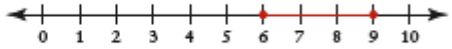


Chapter 2 Section 6 Question 3 Page 138

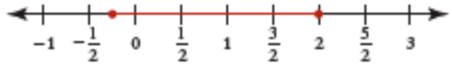
a) $x < -3$ or $x > 2$



b) $6 \le x \le 9$



c) $-\frac{1}{4} \le x \le 2$



Chapter 2 Section 6 Question 4 Page 138

a) Consider all cases.

Case 1
$x + 2 > 0 \quad 3 - x > 0 \quad x + 1 < 0$
$x > -2 \quad x < 3 \quad x < -1$
$-2 < x < -1$ is a solution.

Case 2
$x + 2 > 0 \quad 3 - x < 0 \quad x + 1 > 0$
$x > -2 \quad x > 3 \quad x > -1$
$x > 3$ is a solution.

Case 3
$x + 2 < 0 \quad 3 - x > 0 \quad x + 1 > 0$
$x < -2 \quad x < 3 \quad x > -1$
No solution since no x-values common to all three inequalities.

Case 4
$x + 2 < 0 \quad 3 - x < 0 \quad x + 1 < 0$
$x < -2 \quad x > 3 \quad x < -1$
No solution since no x-values common to all three inequalities.

Combining the results of all the cases, the solution is $-2 < x < -1$ or $x > 3$.

b) Consider all cases.

Case 1

$-x + 1 \geq 0 \quad 3x - 1 \geq 0 \quad x + 7 \geq 0$

$1 \geq x \quad 3x \geq 1 \quad x \geq -7$

$x \leq 1 \quad x \geq \frac{1}{3}$

$\frac{1}{3} \leq x \leq 1$ is a solution.

Case 2

$-x + 1 \geq 0 \quad 3x - 1 \leq 0 \quad x + 7 \leq 0$

$x \leq 1 \quad x \leq \frac{1}{3} \quad x \leq -7$

$x \leq -7$ is a solution.

Case 3

$-x + 1 \leq 0 \quad 3x - 1 \geq 0 \quad x + 7 \leq 0$

$x \geq 1 \quad x \geq \frac{1}{3} \quad x \leq -7$

No solution since no x-values common to all three inequalities.

Case 4

$-x + 1 \leq 0 \quad 3x - 1 \leq 0 \quad x + 7 \geq 0$

$x \geq 1 \quad x \leq \frac{1}{3} \quad x \geq -7$

No solution since no x-values common to all three inequalities.

Combining the results of all the cases, the solution is $x \leq -7$ or $\frac{1}{3} \leq x \leq 1$.

c) Consider all cases.

Case 1

$7x + 2 > 0 \quad 1 - x > 0 \quad 2x + 5 > 0$

$7x > -2 \quad 1 > x \quad 2x > -5$

$x > -\frac{2}{7} \quad x < 1 \quad x > -\frac{5}{2}$

$-\frac{2}{7} < x < 1$ is a solution.

Case 2

$7x + 2 > 0 \quad 1 - x < 0 \quad 2x + 5 < 0$

$x > -\frac{2}{7} \quad x > 1 \quad x < -\frac{5}{2}$

No solution since no x-values common to all three inequalities.

Case 3

$7x + 2 < 0 \quad 1 - x < 0 \quad 2x + 5 > 0$

$x < -\frac{2}{7} \quad x > 1 \quad x > -\frac{5}{2}$

No solution since no x-values common to all three inequalities.

Case 4

$7x + 2 < 0 \quad 1 - x > 0 \quad 2x + 5 < 0$

$x < -\frac{2}{7} \quad x < 1 \quad x < -\frac{5}{2}$

$x < -\frac{5}{2}$ is a solution.

Combining the results of all the cases, the solution is $x < -\frac{5}{2}$ or $-\frac{2}{7} < x < 1$.

d) Consider all cases.

Case 1

$x + 4 \le 0 \qquad -3x + 1 \le 0 \qquad x + 2 \le 0$

$x \le -4 \qquad -3x \le -1 \qquad x \le -2$

$x \ge \frac{1}{3}$

No solution since no x-values common to all three inequalities.

Case 2

$x + 4 \ge 0 \qquad -3x + 1 \ge 0 \qquad x + 2 \le 0$

$x \ge -4 \qquad x \le \frac{1}{3} \qquad x \le -2$

$-4 \le x \le -2$ is a solution.

Case 3

$x + 4 \ge 0 \qquad -3x + 1 \le 0 \qquad x + 2 \ge 0$

$x \ge -4 \qquad x \ge \frac{1}{3} \qquad x \ge -2$

$x \ge \frac{1}{3}$ is a solution.

Case 4

$x + 4 \le 0 \qquad -3x + 1 \ge 0 \qquad x + 2 \ge 0$

$x \le -4 \qquad x \le \frac{1}{3} \qquad x \ge -2$

No solution since no x-values common to all three inequalities.

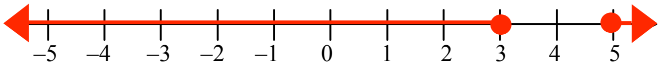
Combining the results of all the cases, the solution is $-4 \le x \le -2$ or $x \ge \frac{1}{3}$.

a) $(x - 3)(x - 5) \geq 0$
Consider all cases.

Case 1
$x - 3 \geq 0 \qquad x - 5 \geq 0$
$x \geq 3 \qquad x \geq 5$
Solution is $x \geq 5$.

Case 2
$x - 3 \leq 0 \qquad x - 5 \leq 0$
$x \leq 3 \qquad x \leq 5$
Solution is $x \leq 3$.

Combining the results of all the cases, the solution is $x \leq 3$ or $x \geq 5$.

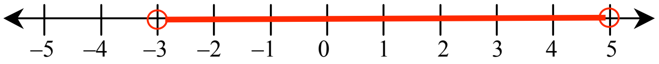


b) $(x - 5)(x + 3) < 0$
Consider all cases.

Case 1
$x - 5 < 0 \qquad x + 3 > 0$
$x < 5 \qquad x > -3$
Solution is $-3 < x < 5$.

Case 2
$x - 5 > 0 \qquad x + 3 < 0$
$x > 5 \qquad x < -3$
No solution since no x-values common to both inequalities.

Combining the results of all the cases, the solution is $-3 < x < 5$.



c) $(3x - 4)(5x + 2) \leq 0$
Consider all cases.

Case 1

$3x - 4 \leq 0 \qquad 5x + 2 \geq 0$

$3x \leq 4 \qquad 5x \geq -2$

$x \leq \frac{4}{3} \qquad x \geq -\frac{2}{5}$

Solution is $-\frac{2}{5} \leq x \leq \frac{4}{3}$.

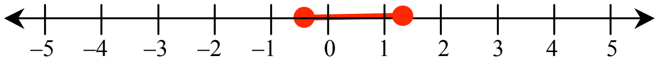
Case 2

$3x - 4 \geq 0 \qquad 5x + 2 \leq 0$

$x \geq \frac{4}{3} \qquad x \leq -\frac{2}{5}$

No solution since no x-values common to both inequalities.

Combining the results of all the cases, the solution is $-\frac{2}{5} \leq x \leq \frac{4}{3}$.



d) Factor using the factor theorem.
$(x - 3)(x - 1)(x + 2) < 0$
Consider all cases.

Case 1
$x - 3 < 0$ $x - 1 < 0$ $x + 2 < 0$
$x < 3$ $x < 1$ $x < -2$

$x < -2$ is a solution.

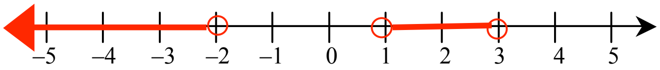
Case 2
$x - 3 < 0$ $x - 1 > 0$ $x + 2 > 0$
$x < 3$ $x > 1$ $x > -2$

$1 < x < 3$ is a solution.

Case 3
$x - 3 > 0$ $x - 1 < 0$ $x + 2 > 0$
$x > 3$ $x < 1$ $x > -2$
No solution since no x-values common to all three inequalities.

Case 4
$x - 3 > 0$ $x - 1 > 0$ $x + 2 < 0$
$x > 3$ $x > 1$ $x < -2$
No solution since no x-values common to all three inequalities.

Combining the results of all the cases, the solution is $x < -2$ or $1 < x < 3$.



e) $(x-1)(x+1)(2x+3) \geq 0$
Consider all cases.

Case 1
$x-1 \geq 0$ $\quad x+1 \geq 0$ $\quad 2x+3 \geq 0$
$x \geq 1$ $\quad x \geq -1$ $\quad 2x \geq -3$
$x \geq -\frac{3}{2}$

$x \geq 1$ is a solution.

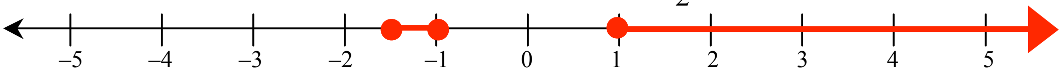
Case 2
$x-1 \leq 0$ $\quad x+1 \leq 0$ $\quad 2x+3 \geq 0$
$x \leq 1$ $\quad x \leq -1$ $\quad x \geq -\frac{3}{2}$

$-\frac{3}{2} \leq x \leq -1$ is a solution.

Case 3
$x-1 \leq 0$ $\quad x+1 \geq 0$ $\quad 2x+3 \leq 0$
$x \leq 1$ $\quad x \geq -1$ $\quad x \leq -\frac{3}{2}$
No solution since no x-values common to all three inequalities.

Case 4
$x-1 \geq 0$ $\quad x+1 \leq 0$ $\quad 2x+3 \leq 0$
$x \geq 1$ $\quad x \leq -1$ $\quad x \leq -\frac{3}{2}$
No solution since no x-values common to all three inequalities.

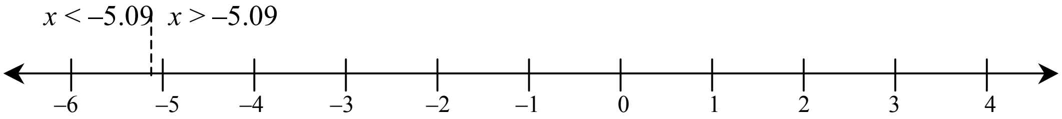
Combining the results of all the cases, the solution is $-\frac{3}{2} \leq x \leq -1$ or $x \geq 1$.



Chapter 2 Section 6 **Question 6 Page 139**

a) $(x + 5.09)(x^2 + 0.91x + 2.36) \geq 0$

Use the root –5.09 to break the number line into two intervals.



Test arbitrary values of x for each interval.

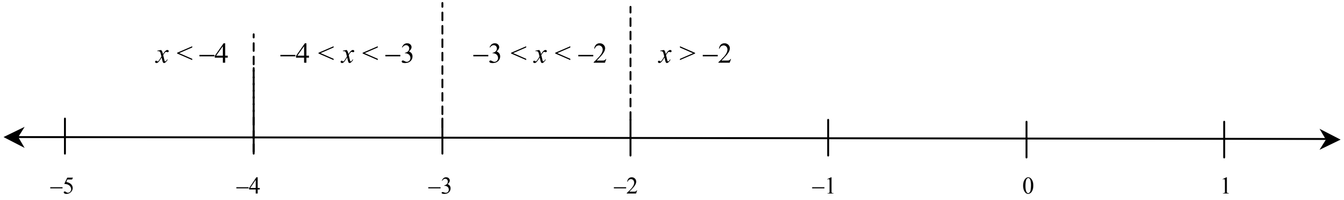
For $x < -5.09$, test $x = -6$.
$(-6)^3 + 6(-6)^3 + 6(-6)^2 + 7(-6) + 12 = -30$
Since $-30 < 0$, $x < -5.09$ is not a solution.

For $x > -5.09$, test $x = 0$.
$(0)^3 + 6(0)^2 + 7(0) + 12 = 12$
Since $12 > 0$, $x > -5.09$ is a solution.

The solution is approximately $x \geq -5.09$.

b) $(x + 2)(x + 3)(x + 4) < 0$

Use the roots –2, –3, and –4 to break the number line into four intervals.



For $x < -4$, test $x = -5$.
$(-5 + 2)(-5 + 3)(-5 + 4) = -6$
$-6 < 0$, $x < -4$ is a solution.

For $-4 < x < -3$, test $x = -3.5$.
$(-3.5 + 2)(-3.5 + 3)(-3.5 + 4) = 0.375$
$0.375 > 0$, $-4 < x < -3$ is not a solution.

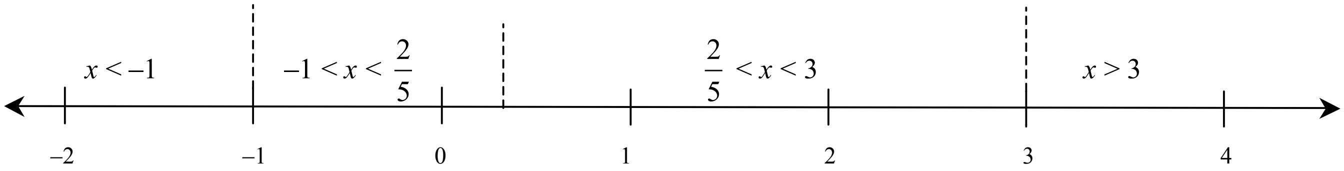
For $-3 < x < -2$, test $x = -2.5$.
$(-2.5 + 2)(-2.5 + 3)(-2.5 + 4) = -0.375$
$-0.375 < 0$, $-3 < x < -2$ is a solution.

For $x > -2$, test $x = 0$.
$(0 + 2)(0 + 3)(0 + 4) = 24$
$24 > 0$, $x > -2$ is not a solution.

The solution is $x < -4$ or $-3 \leq x \leq -2$.

c) $(x - 3)(x + 1)(5x - 2) \le 0$

Use the roots 3, −1, and $\frac{2}{5}$ to break the number line into four intervals.



For $x < -1$, test $x = -2$.
$(-2 - 3)(-2 + 1)[5(-2) - 2] = -60$
$-60 < 0$, $x < -1$ is a solution.

For $-1 < x < \frac{2}{5}$, test $x = 0$.
$(0 - 3)(0 + 1)[5(0) - 2] = 6$
$6 > 0$, $-1 < x < \frac{2}{5}$ is not a solution.

For $\frac{2}{5} < x < 3$, test $x = 1$.
$(1 - 3)(1 + 1)[5(1) - 2] = -12$
$-12 < 0$, $\frac{2}{5} < x < 3$ is a solution.

For $x > 3$, test $x = 4$.
$(4 - 3)(4 + 1)[5(4) - 2] = 90$
$90 > 0$, $x > 3$ is not a solution.

The solution is $x \le -1$ or $\frac{2}{5} \le x \le 3$.

d) Using CAS to factor.

$$6(x^2 - 2.64x + 2.40)(x^2 + 1.48x + 0.83) > 0$$

$$x^2 - 2.64x + 2.40 = 0$$

$$x = \frac{2.64 \pm \sqrt{(-2.64)^2 - 4(1)(2.40)}}{2(1)}$$

$$x = \frac{2.64 \pm \sqrt{-2.63}}{2}$$

There are no real roots.

The function is above the x-axis so it is positive for all values of x.
$x^2 + 1.48x + 0.83 > 0$ is true for all values of x.

Chapter 2 Section 6 **Question 7 Page 139**

a) $(x + 5)(x - 1) \le 0$
The roots are $x = -5$ and $x = 1$.
Consider all cases.

Case 1
$x < -5$ $x > 1$
No solution since no x-values common to both inequalities.

Case 2
$x > -5$ $x < 1$
Solution is $-5 < x < 1$.

Combining the results of all the cases, the solution is $-5 \le x \le 1$.

b) $(3 - x)(x + 2)(2x + 1) < 0$

The roots are $x = 3$, $x = -2$, and $x = -\frac{1}{2}$.

Consider all cases.

Case 1
$3 - x < 0 \quad x + 2 < 0 \quad 2x + 1 < 0$
$3 < x \quad x < -2 \quad 2x < -1$
$x > 3 \quad x < -\frac{1}{2}$

No solution since no x-values common to all three inequalities.

Case 2
$3 - x < 0 \quad x + 2 > 0 \quad 2x + 1 > 0$
$x > 3 \quad x > -2 \quad x > -\frac{1}{2}$

The solution is $x > 3$.

Case 3
$3 - x > 0 \quad x + 2 < 0 \quad 2x + 1 > 0$
$x < 3 \quad x < -2 \quad x > -\frac{1}{2}$

No solution since no x-values common to all three inequalities.

Case 4
$3 - x > 0 \quad x + 2 > 0 \quad 2x + 1 < 0$
$x < 3 \quad x > -2 \quad x < -\frac{1}{2}$

The solution is $-2 < x < -\frac{1}{2}$.

Combining the results of all the cases, the solution is $-2 < x < -\frac{1}{2}$ or $x > 3$.

c) $(x-1)(x+1)(2x+1) > 0$
Consider all cases.

Case 1
$x-1>0 \qquad x+1>0 \qquad 2x+1>0$
$x>1 \qquad x>-1 \qquad 2x>-1$
$x>-\frac{1}{2}$
$x>1$ is a solution.

Case 2
$x-1<0 \qquad x+1<0 \qquad 2x+1>0$
$x<1 \qquad x<-1 \qquad x>-\frac{1}{2}$
No solution since no x-values common to all three inequalities.

Case 3
$x-1>0 \qquad x+1<0 \qquad 2x+1<0$
$x>1 \qquad x<-1 \qquad x<-\frac{1}{2}$
No solution since no x-values common to all three inequalities.

Case 4
$x-1<0 \qquad x+1>0 \qquad 2x+1<0$
$x<1 \qquad x>1 \qquad x<-\frac{1}{2}$
$-1<x<-\frac{1}{2}$ is a solution.

Combining the results of all the cases, the solution is $-1<x<-\frac{1}{2}$ or $x>1$.

d) Factor first.

$(x-1)(x^2+x-4)=0$

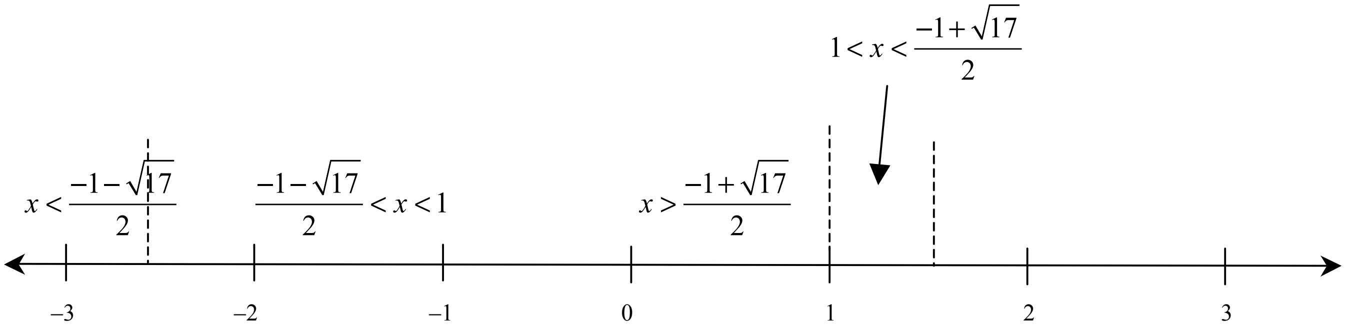
$x=1$

or

$x=\dfrac{-1\pm\sqrt{1^2-4(1)(-4)}}{2(1)}$

$x=\dfrac{-1+\sqrt{17}}{2}$ or $x=\dfrac{-1-\sqrt{17}}{2}$

Use the roots to break the number line into intervals.



For $x<\dfrac{-1-\sqrt{17}}{2}$, test $x = -3$.

$(-3)^3 - 5(-3) + 4 = -8$

$-8 < 0$, $x<\dfrac{-1-\sqrt{17}}{2}$ is not a solution.

For $\dfrac{-1-\sqrt{17}}{2}<x<1$, test $x = 0$.

$(0)^3 - 5(0) + 4 = 4$

$4 > 0$, $\dfrac{-1-\sqrt{17}}{2}<x<1$ is a solution.

For $1<x<\dfrac{-1+\sqrt{17}}{2}$, test $x = 1.5$.

$(1.5)^3 - 5(1.5) + 4 = -0.125$

$-0.125 < 0$, $1<x<\dfrac{-1+\sqrt{17}}{2}$ is not a solution.

For $x > \frac{-1+\sqrt{17}}{2}$, test $x = 3$.

$(3)^3 - 5(3) + 4 = 16$

$16 > 0$, $x > \frac{-1+\sqrt{17}}{2}$ is a solution.

The solution is $\frac{-1-\sqrt{17}}{2} \le x \le 1$ or $x \ge \frac{-1+\sqrt{17}}{2}$.

Chapter 2 Section 6 **Question 8 Page 139**

$$
\begin{aligned}
(6+x)(18+x)(20+x) &\ge 5280 \\
(x^2+24x+108)(20+x)-5280 &\ge 0 \\
x^3+44x^2+588x+2160-5280 &\ge 0 \\
x^3+44x^2+588x-3120 &\ge 0 \\
(x-4)(x^2+48x+780) &\ge 0
\end{aligned}
$$

The root is $x = 4$, $x^2 + 48x + 780$ is positive for all values of x.

$x > 4$, test $x = 5$.

$(5-4)(5^2+48(5)+780)=1045$

$1045 > 0$, $x > 4$

$x \ge 4$ is the solution.

$6 + 4 = 10$

$18 + 4 = 22$

$20 + 4 = 24$

22 cm by 24 cm by 10 cm are the minimum dimensions

Chapter 2 Section 6 **Question 9 Page 139**

$$
\begin{aligned}
0.5t^3 - 5.5t^2 + 14t &> 90 \\
0.5t^3 - 5.5t^2 + 14t - 90 &> 0 \\
0.5(t^3 - 11t^2 + 28t - 180) &> 0 \\
0.5(x - 10)(x^2 - x + 18) &> 0
\end{aligned}
$$
$x > 10$

The price of stock will be above $90 after 10 years (in 2009).

Chapter 2 Section 6 **Question 10 Page 139**

Solutions to Achievement Check questions are provided in the Teacher's Resource.

Chapter 2 Section 6 **Question 11 Page 139**

a) 8 cases
$x + 4$ negative, the rest positive
$x – 2$ negative, the rest positive
$x + 1$ negative, the rest positive
$x – 1$ negative, the rest positive
$x + 4$ positive, the rest negative
$x – 2$ positive, the rest negative
$x + 1$ positive, the rest negative
$x – 1$ positive, the rest negative

b) Answers may vary. A sample solution is shown.
Probably, because there are fewer intervals to look at than cases above:
$x < –4$, $–4 < x < –1$, $–1 < x$ 1, $1 < x < 2$, $x > 2$

Chapter 2 Section 6 **Question 12 Page 139**

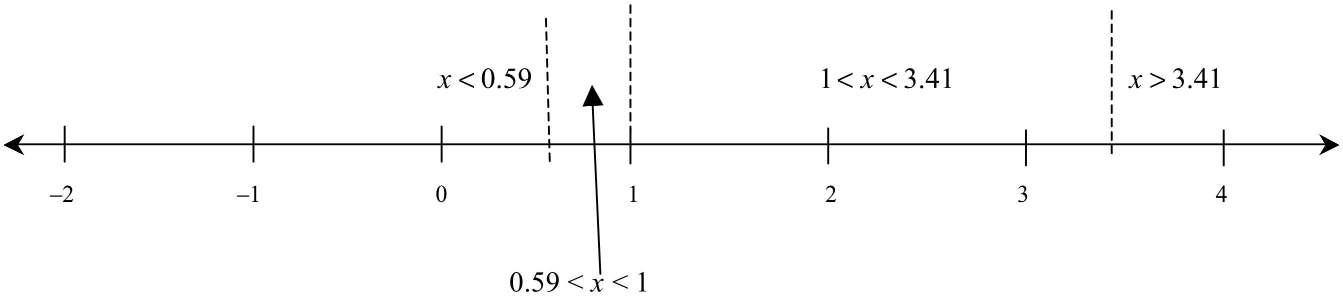
$$x^5-5x^4+7x^3-7x^2+6x-2=0$$
$$(x-1)(x^2+1)(x^2-4x+2)=0$$
$x=1$

or

$$x=\frac{4\pm\sqrt{(-4)^2-4(1)(2)}}{2(1)}$$
$$x=\frac{4\pm\sqrt{8}}{2}=\frac{4\pm2\sqrt{2}}{2}$$
$x=2+\sqrt{2}, x\doteq 3.41$

or $x=2-\sqrt{2}, x\doteq 0.59$

Use the roots to break the number line into three intervals.



For $x < 0.59$, test $x = -1$.
$(-1)^5 - 5(-1)^4 + 7(-1)^3 - 7(-1)^2 + 6(-1) - 2 = -28$
$-28 < 0$, $x < 0.59$ is a solution.

For $0.59 < x < 1$, test $x = 0.8$.
$(0.8)^5 - 5(0.8)^4 + 7(0.8)^3 - 7(0.8)^2 + 6(0.8) - 2 = 0.18$
$0.18 > 0$, $0.59 < x < 1$ is not a solution.

For $1 < x < 3.41$, test $x = 2$.
$(2)^5 - 5(2)^4 + 7(2)^3 - 7(2)^2 + 6(2) - 2 = -10$
$-10 < 0$, $1 < x < 3.41$ is a solution.

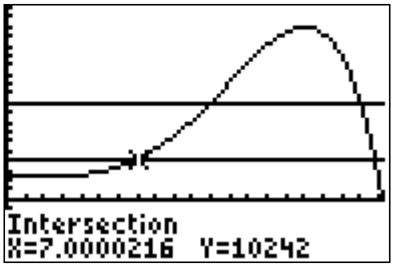
For $x > 3.41$, test $x = 5$.
$(5)^5 - 5(5)^4 + 7(5)^3 - 7(5)^2 + 6(5) - 2 = 728$
$728 > 0$, $x > 3.41$ is not a solution.

The solution is approximately $x < 0.59$ or $1 < x < 3.41$.

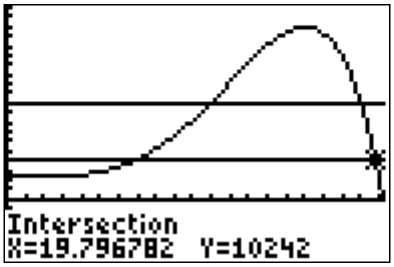
Chapter 2 Section 6 Question 13 Page 139

a) $10\ 242 < -0.15n^5 + 3n^4 + 5560 < 25\ 325$

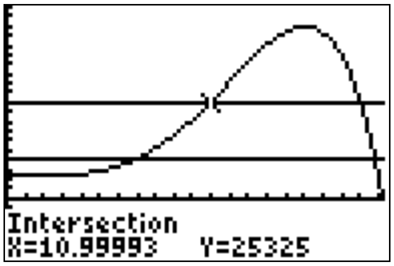
Graph $P(n)$ and the lines $P(n) = 10\ 242$ and $P(n) = 25\ 325$ and find the points of intersection.



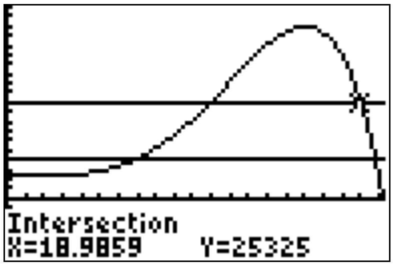
```
Intersection
X=7.0000216  Y=10242
```



```
Intersection
X=19.796782  Y=10242
```



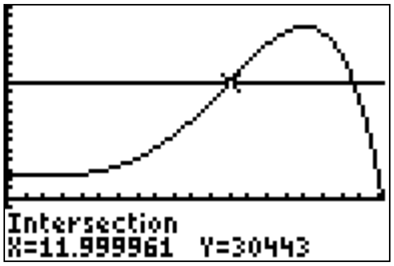
```
Intersection
X=10.99993    Y=25325
```



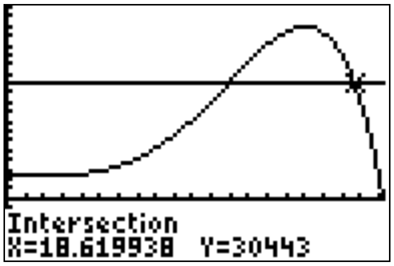
```
Intersection
X=18.9859     Y=25325
```

The population of the town will be between 10 242 and 25 325 at approximately $7 < n < 11$ or $19 < n < 20$, or between 7 and 11 years from today and between 19 and 20 years from today.

b)



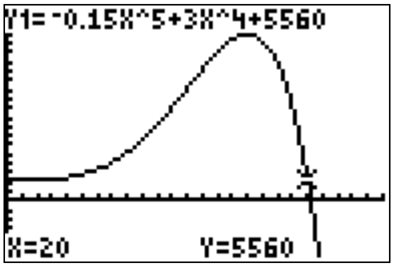
```
Intersection
X=11.999961  Y=30443
```



```
Intersection
X=18.619938  Y=30443
```

The population of the town is more than 30 443 at approximately $12 < n < 18.6$, or between 12 and 19 years from today.

c)



Not valid beyond 20 years. 20 years from today the population will have fallen to 5560, and in the next year it would fall below 0, which is not possible.

Chapter 2 Section 6 **Question 14 Page 139**

$x^4 - 76x^2 + 1156 \le 0$, $x^4 + 76x^2 - 1156 \ge 0$

Chapter 2 Section 6 **Question 15 Page 139**

Method 1:
Add line segments to make ΔPQA and ΔPBQ. Both triangles share ∠P and because PQ is tangent to the circle, ∠PQB = ∠QAB. Therefore, ΔPQA is similar to ΔPBQ.
So, write a ratio that can be used to determine the length of PQ:

$$\frac{\text{PQ}}{\text{AP}} = \frac{\text{BP}}{\text{PQ}}$$

$$\text{PQ}^2 = \text{AP} \times \text{BP}$$

$$\text{PQ}^2 = 22 \times 13$$

$$\text{PQ} = \sqrt{286}$$

Method 2:
From the tangent-secant theorem that states that if a tangent from an external point P meets the circle at Q and a secant from the same point P meets the circle at B and A, then

$$\text{PQ}^2 = \text{PA} \times \text{PB}$$

$$\text{PQ}^2 = 22 \times 13$$

$$\text{PQ} = \sqrt{286}$$

Chapter 2 Section 6 **Question 16 Page 139**

Instantaneous rate of change (slope) at the point (4, −3) on the circle is $\frac{4}{3}$.

Substitute $x = 4$ and $y = –3$ into $y = \frac{4}{3}x + b$.

$$
\begin{aligned}
-3 &= \frac{4}{3}(4) + b \\
-3 &= \frac{16}{3} + b \\
-\frac{9}{3} - \frac{16}{3} &= b \\
-\frac{25}{3} &= b
\end{aligned}
$$

$$y = \frac{4}{3}x - \frac{25}{3}$$

Chapter 2 Review

Chapter 2 Review **Question 1 Page 140**

a) i) $P(2) = (2)^3 + 9(2)^2 - 5(2) + 3 = 37$

ii)

$$\begin{array}{r|rrrr} -2 & 1 & 9 & -5 & 3 \\ - & & -2 & -22 & -34 \\ \hline \times & 1 & 11 & 17 & 37 \end{array}$$

$$\frac{x^3+9x^2-5x+3}{x-2} = x^2+11x+17+\frac{37}{x-2},\ x \neq 2$$

b) i) $P\left(-\frac{1}{3}\right) = 12\left(-\frac{1}{3}\right)^3 - 2\left(-\frac{1}{3}\right)^2 + \left(-\frac{1}{3}\right) - 11 = -12$

ii)

$$\begin{array}{r} 4x^2-2x+1 \\ 3x+1\overline{)12x^3-2x^2+x-11} \\ \underline{12x^3+4x^2} \\ -6x^2+x \\ \underline{-6x^2-2x} \\ 3x-11 \\ \underline{3x+1} \\ -12 \end{array}$$

$$\frac{12x^3-2x^2+x-11}{3x+1} = 4x^2-2x+1-\frac{12}{3x+1},\ x \neq -\frac{1}{3}$$

c) i) $P\left(\frac{1}{2}\right) = -8\left(\frac{1}{2}\right)^4 - 4\left(\frac{1}{2}\right) + 10\left(\frac{1}{2}\right)^3 - \left(\frac{1}{2}\right)^2 + 15 = \frac{27}{2}$

ii)

$$
\begin{array}{r}
-4x^3+3x^2+x-\dfrac{3}{2} \\
2x-1\overline{\big)-8x^4+10x^3-x^2-4x+15} \\
\underline{-8x^4+4x^3} \qquad\qquad\qquad\quad \\
6x^3-x^2 \qquad\qquad\quad \\
\underline{6x^3-3x^2} \qquad\qquad\quad \\
2x^2-4x \qquad\quad \\
\underline{2x^2-x} \qquad\quad \\
-3x+15 \\
\underline{-3x+\dfrac{3}{2}} \\
\dfrac{27}{2}
\end{array}
$$

$$\frac{-8x^4-4x+10x^3-x^2+15}{2x-1}=-4x^3+3x^2+x-\frac{3}{2}+\frac{27}{2(2x-1)},\ x\neq\frac{1}{2}$$

Chapter 2 Review **Question 2 Page 140**

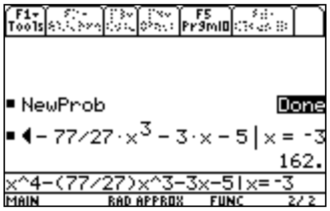
a) $f(3)=3^4+k(3)^3-3(3)-5=-10$

$$
\begin{aligned}
81+27k-9-5&=-10\\
27k&=-10-67\\
27k&=-77\\
k&=-\frac{77}{27}
\end{aligned}
$$

b) $f(-3)=(-3)^4-\dfrac{77}{27}(-3)^3-3(-3)-5$

$$=162$$

c)



```
■ NewProb                    Done
■ ◀- 77/27·x³ - 3·x - 5 | x = -3
                             162.
x^4-(77/27)x^3-3x-5|x=-3
MAIN        RAD APPROX    FUNC    2/2
```

Chapter 2 Review **Question 3 Page 140**

$$\begin{aligned} P(1) &= 4-3+b+6 \\ &= 7+b \\ P(-3) &= 4(-3)^3-3(-3)^2-3b+6 \\ &= -108-27-3b+6 \\ &= -129-3b \end{aligned}$$

$$\begin{aligned} 7+b &= -129-3b \\ b+3b &= -129-7 \\ 4b &= -136 \\ b &= -34 \end{aligned}$$

Chapter 2 Review **Question 4 Page 140**

a) $P(-1) = -1 - 4 - 1 + 6$
$= 0$
$(x + 1)$ is a factor.

Divide to determine the other factors.

$$\begin{array}{r|rrrr} 1 & 1 & -4 & 1 & 6 \\ - & & 1 & -5 & 6 \\ \hline \times & 1 & -5 & 6 & 0 \end{array}$$

$$\begin{aligned} x^3 - 4x^2 + x + 6 &= (x + 1)(x^2 - 5x + 6) \\ &= (x + 1)(x - 3)(x - 2) \end{aligned}$$

b) $P(-2) = 3(-2)^3 - 5(-2)^2 - 26(-2) - 8$
$= 0$
$(x + 2)$ is a factor.

$$\begin{array}{r|rrrr} 2 & 3 & -5 & -26 & -8 \\ - & & 6 & -22 & -8 \\ \hline \times & 3 & -11 & -4 & 0 \end{array}$$

$$\begin{aligned} 3x^3 - 5x^2 - 26x - 8 &= (x + 2)(3x^2 - 11x - 4) \\ &= (x + 2)(3x + 1)(x - 4) \end{aligned}$$

c) $P(1) = 5 + 12 - 101 + 48 + 36$
$= 0$

$(x – 1)$ is a factor.

$$\begin{array}{r|rrrrr} -1 & 5 & 12 & -101 & 48 & 36 \\ - & & -5 & -17 & 84 & 36 \\ \hline \times & 5 & 17 & -84 & -36 & 0 \end{array}$$

$$5x^4 + 12x^3 - 101x^2 + 48x + 36 = (x - 1)(5x^3 + 17x^2 - 84x - 36)$$

$P(3) = 5(3)^3 + 17(3)^2 - 84(3) - 36$
$= 0$

$(x – 3)$ is a factor.

$$\begin{array}{r|rrrr} -3 & 5 & 17 & -84 & -36 \\ - & & -15 & -96 & -36 \\ \hline \times & 5 & 32 & 12 & 0 \end{array}$$

$$\begin{aligned} 5x^4 + 12x^3 - 101x^2 + 48x + 36 &= (x - 1)(x - 3)(5x^2 + 32x + 12) \\ &= (x - 1)(x - 3)(x + 6)(5x + 2) \end{aligned}$$

Chapter 2 Review **Question 5 Page 140**

a) $$\begin{aligned} -4x^3 - 4x^2 + 16x + 16 &= -4(x^3 + x^2 - 4x - 4) \\ &= -4\left[x^2(x+1) - 4(x+1)\right] \\ &= -4(x+1)(x^2 - 4) \\ &= -4(x+1)(x+2)(x-2) \end{aligned}$$

b) $$\begin{aligned} 25x^3 - 50x^2 - 9x + 18 &= 25x^2(x-2) - 9(x-2) \\ &= (x-2)(25x^2 - 9) \\ &= (x-2)(5x-3)(5x+3) \end{aligned}$$

c) $$\begin{aligned} 2x^4 + 5x^3 - 8x^2 - 20x &= x(2x^3 + 5x^2 - 8x - 20) \\ &= x\left[2x(x^2 - 4) + 5(x^2 - 4)\right] \\ &= x(2x+5)(x^2 - 4) \\ &= x(2x+5)(x+2)(x-2) \end{aligned}$$

Chapter 2 Review — Question 6 Page 140

a) $V(-1) = -2 + 7 - 2 - 3$

$= 0$

$(x + 1)$ is a factor

$$\begin{array}{r|rrrr} 1 & 2 & 7 & 2 & -3 \\ - & & 2 & 5 & -3 \\ \hline \times & 2 & 5 & -3 & 0 \end{array}$$

$V(x) = (x + 1)(2x^2 + 5x - 3)$

$= (x + 1)(x + 3)(2x - 1)$

The dimensions are $(x + 1)$ m by $(x + 3)$ m by $(2x - 1)$ m.

b) (1 + 1) m by (1 + 3) m by (2 – 1) m, or 4 m by 2 m by 1 m

Chapter 2 Review — Question 7 Page 140

$$\begin{aligned} P(-3) &= (-3)^3 + 4(-3)^2 - 2(-3)k + 3 \\ 0 &= -27 + 36 + 6k + 3 \\ 6k &= -12 \\ k &= -2 \end{aligned}$$

Chapter 2 Review — Question 8 Page 140

$x = -4$ or $x = -2$ or $x = 3$

Chapter 2 Review — Question 9 Page 140

a) $5(x^2 + 4)3(x^2 - 16) = 0$

$15(x^2 + 4)(x + 4)(x - 4) = 0$

$x = -4$ or $x = 4$

b) $x = \dfrac{1 \pm \sqrt{(-1)^2 - 4(2)(-13)}}{2(2)}$

$x = \dfrac{1 - \sqrt{105}}{4}$ or $x = \dfrac{1 + \sqrt{105}}{4}$

Chapter 2 Review **Question 10 Page 140**

a) $P(-1) = -7 + 5 + 5 - 3$

$\quad = 0$

$(x + 1)$ is a factor.

$$\begin{array}{r|rrrr} 1 & 7 & 5 & -5 & -3 \\ - & & 7 & -2 & -3 \\ \hline \times & 7 & -2 & -3 & 0 \end{array}$$

$7x^3 + 5x^2 - 5x - 3 = (x+1)(7x^2 - 2x - 3)$

$x = -1$

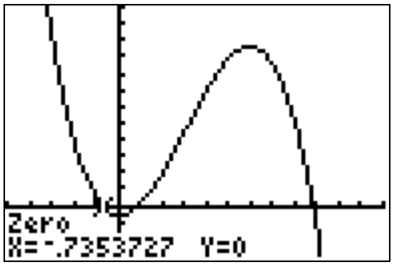
or

$$x = \frac{2 \pm \sqrt{(-2)^2 - 4(7)(-3)}}{2(7)}$$

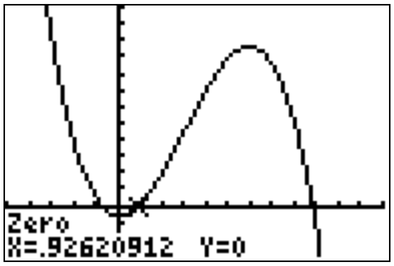
$$x = \frac{2 \pm \sqrt{88}}{14}$$

$x \doteq -0.5$ or $x \doteq 0.8$

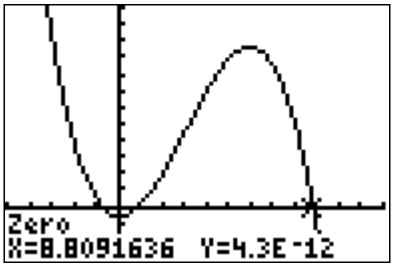
b) $-x^3 + 9x^2 - x - 6 = 0$



```
Zero
X=-.7353727  Y=0
```



```
Zero
X=.92620912  Y=0
```

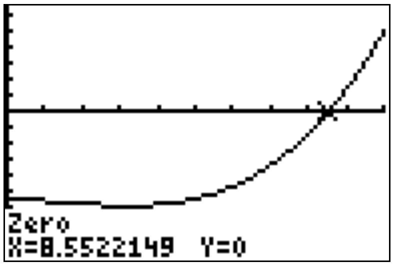


```
Zero
X=8.8091636  Y=4.3E-12
```

$x \doteq -0.7$ or $x \doteq 0.9$ or $x \doteq 8.8$

Chapter 2 Review **Question 11 Page 140**

$$
\begin{aligned}
V(x) &= l(l-5)(2l+1)\\
550 &= l(2l^2-9l-5)\\
0 &= 2l^3-9l^2-5l-550
\end{aligned}
$$



```
Zero
X=8.5522149  Y=0
```

$$
\begin{aligned}
l &\doteq 8.55\\
w &\doteq 8.55-5\\
&\doteq 3.55\\
h &\doteq 2(8.55)+1\\
&\doteq 18.1
\end{aligned}
$$

The possible dimensions of the box are approximately 8.55 cm by 3.55 cm by 18.10 cm

Chapter 2 Review **Question 12 Page 140**

B since the zeros are different.

Chapter 2 Review **Question 13 Page 140**

a)
$$
\begin{aligned}
y &= kx(x-2+\sqrt{5})(x-2-\sqrt{5})\\
y &= kx(x^2-2x-\sqrt{5}x-2x+4+2\sqrt{5}+\sqrt{5}x-2\sqrt{5}-5)\\
y &= kx(x^2-4x-1)\\
y &= k(x^3-4x^2-x)
\end{aligned}
$$

b) let $x = 2$ and $y = 20$
$$
\begin{aligned}
20 &= k(2^3-4(2)^2-2)\\
20 &= -10k\\
k &= -2\\
y &= -2(x^3-4x^2-x)
\end{aligned}
$$

Chapter 2 Review **Question 14 Page 141**

The zeros are –2 (order 2) and 1.

$y = k(x+2)^2(x-1)$

Using the point (−1, 6) from the graph, substitute $x = -1$ and $y = 6$.

$6 = k(-1+2)^2(-1-1)$

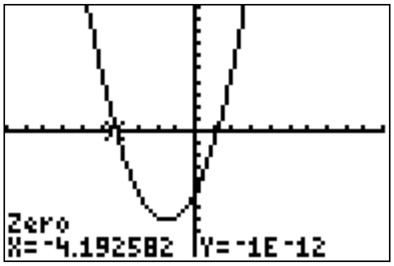
$6 = -2k$

$k = -3$

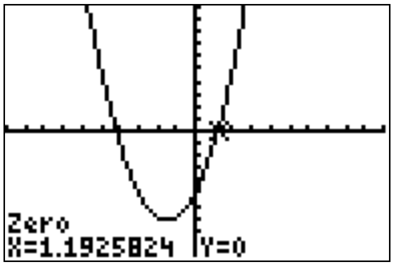
$y = -3(x+2)^2(x-1)$

Chapter 2 Review **Question 15 Page 141**

a)



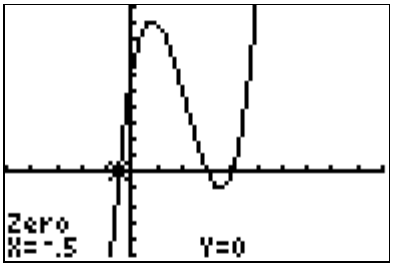
```
Zero
X=-4.192582  Y=-1E-12
```

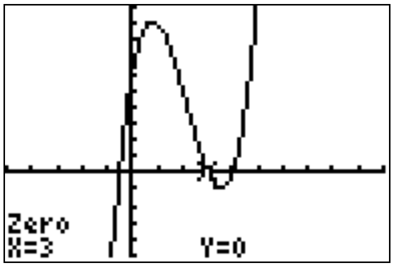


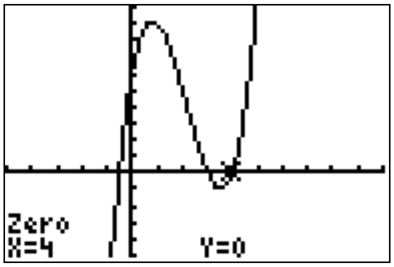
```
Zero
X=1.1925824  Y=0
```

The values that satisfy the inequality $x^2 + 3x - 5 \geq 0$ are the values of x for which the graph is zero or positive (on or above the x-axis). From the graph, this occurs approximately when $x \leq -4.2$ or $x \geq 1.2$.

b)



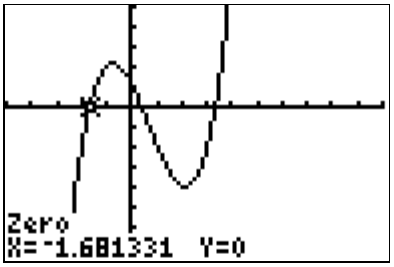


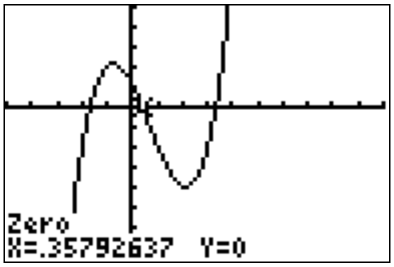


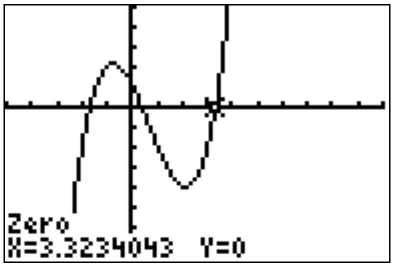
The values that satisfy the inequality $2x^3 - 13x^2 + 17x + 12 > 0$ are the values of x for which the graph is positive (above the x-axis). From the graph, this occurs when

$-\frac{1}{2} < x < 3$ or $x > 4$.

c)

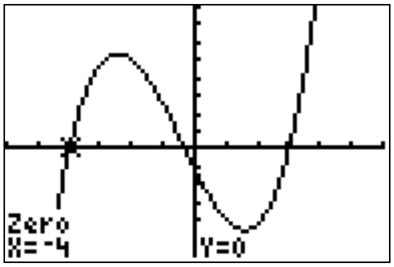


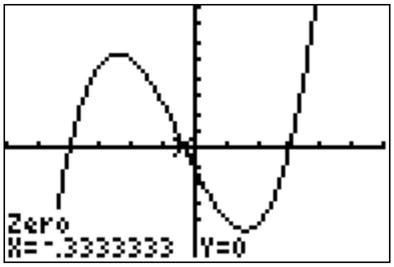


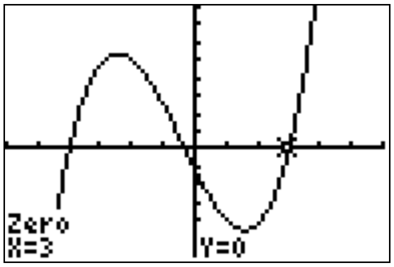


The values that satisfy the inequality $x^3 - 2x^2 - 5x + 2 < 0$ are the values of x for which the graph is negative (below the x-axis). From the graph, this occurs when approximately $x < -1.7$ or $0.4 < x < 3.3$.

d)

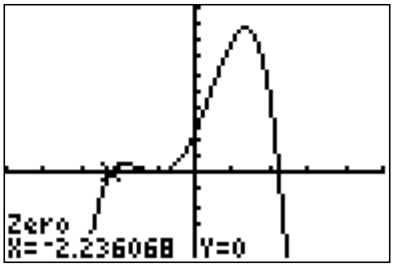


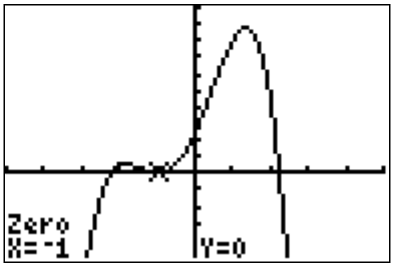


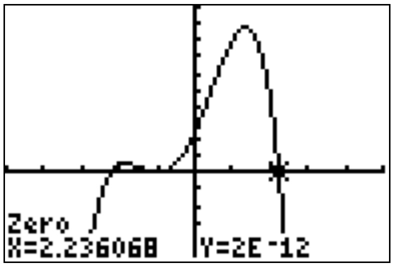


The values that satisfy the inequality $3x^3 + 4x^2 - 35x - 12 \leq 0$ are the values of x for which the graph is zero and negative (on or below the x-axis). From the graph, this occurs when $x \leq -4$ or $-\frac{1}{3} \leq x \leq 3$.

e)



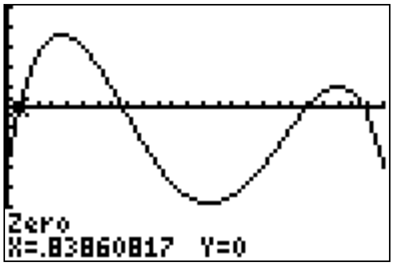




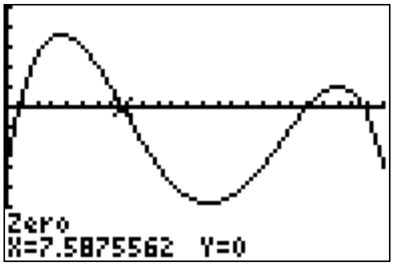
The values that satisfy the inequality $-x^4 - 2x^3 + 4x^2 + 10x + 5 < 0$ are the values of x for which the graph is negative (below the x-axis). From the graph, this occurs approximately when $x < -2.2$ or $x > 2.2$.

Chapter 2 Review **Question 16 Page 141**

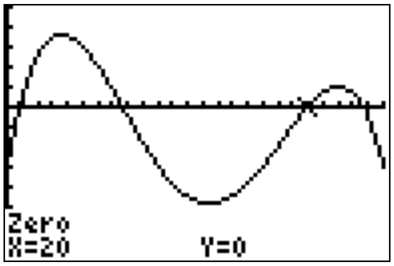
$$h(t) = -0.002t^4 + 0.104t^3 - 1.69t^2 + 8.5t + 9 > 15$$
$$-0.002t^4 + 0.104t^3 - 1.69t^2 + 8.5t - 6 > 0$$



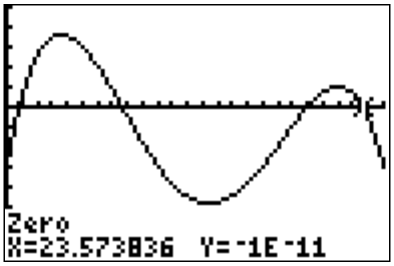
```
Zero
X=.83860817  Y=0
```



```
Zero
X=7.5875562  Y=0
```



```
Zero
X=20         Y=0
```



```
Zero
X=23.573836  Y=⁻1E⁻11
```

The values that satisfy the inequality $-0.002t^4 + 0.104t^3 - 1.69t^2 + 8.5t - 6 > 0$ are the values of x for which the graph is positive (above the x-axis). From the graph, this occurs approximately when approximately between 0.8 s and 7.6 s and between 20 s and 23.6 s.

Chapter 2 Review **Question 17 Page 141**

a) Consider all cases.

Case 1

$5x + 4 < 0$ $\qquad$ $x - 4 > 0$

$5x < -4$ $\qquad$ $x > 4$

$x < -\frac{4}{5}$

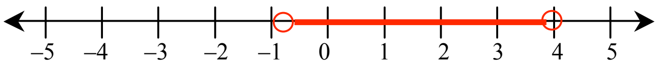
No solution since no x-values common to both inequalities.

Case 2

$5x + 4 > 0$ $\qquad$ $x - 4 < 0$

$x > -\frac{4}{5}$ $\qquad$ $x < 4$

$-\frac{4}{5} < x < 4$ is a solution.



Combining the results of all the cases, the solution is $-\frac{4}{5} < x < 4$.

b) $(2x + 3)(x - 1)(3x - 2) \geq 0$
Consider all cases.

Case 1

$$\begin{array}{lll} 2x + 3 > 0 & x - 1 > 0 & 3x - 2 > 0 \\ 2x > -3 & x > 1 & 3x > 2 \\ x > -\frac{3}{2} & & x > \frac{2}{3} \end{array}$$

$x > 1$ is a solution.

Case 2

$$\begin{array}{lll} 2x + 3 > 0 & x - 1 < 0 & 3x - 2 < 0 \\ x > -\frac{3}{2} & x < 1 & x < \frac{2}{3} \end{array}$$

$-\frac{3}{2} < x < \frac{2}{3}$ is a solution.

Case 3

$$\begin{array}{lll} 2x + 3 < 0 & x - 1 < 0 & 3x - 2 > 0 \\ x < -\frac{3}{2} & x < 1 & x > \frac{2}{3} \end{array}$$

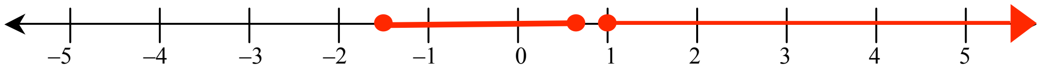
No solution since no x-values common to all three inequalities.

Case 4

$$\begin{array}{lll} 2x + 3 < 0 & x - 1 > 0 & 3x - 2 < 0 \\ x < -\frac{3}{2} & x > 1 & x < \frac{2}{3} \end{array}$$

No solution since no x-values common to all three inequalities.

Combining the results of all the cases, the solution is $-\frac{3}{2} \leq x \leq \frac{2}{3}$ or $x \geq 1$.

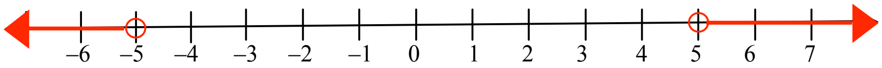


c) $(x^2 + 4x + 4)(x + 5)(x - 5) > 0$
Consider all cases.

Case 1
$x + 5 > 0 \qquad x - 5 > 0$
$\quad x > -5 \qquad\quad x > 5$
$x > 5$ is a solution.

Case 2
$x + 5 < 0 \qquad x - 5 < 0$
$\quad x < -5 \qquad\quad x < 5$
$x < -5$ is a solution.

Combining the results of all the cases, the solution is $x < -5$ or $x > 5$.



Chapter 2 Review **Question 18 Page 141**

a) $12x^2 + 25x - 7 = (3x + 7)(4x - 1)$
$(3x + 7)(4x - 1) \geq 0$
Consider all cases.

Case 1
$3x + 7 > 0 \qquad 4x - 1 > 0$
$\quad 3x > -7 \qquad\quad 4x > 1$
$\quad x > -\frac{7}{3} \qquad\quad x > \frac{1}{4}$
$x > \frac{1}{4}$ is a solution.

Case 2
$3x + 7 < 0 \qquad 4x - 1 < 0$
$\quad x < -\frac{7}{3} \qquad\quad x < \frac{1}{4}$
$x < -\frac{7}{3}$ is a solution.

Combining the results of all the cases, the solution is $x \leq -\frac{7}{3}$ or $x \geq \frac{1}{4}$.

b) $(x+4)(2x-3)(3x-1) \le 0$

Consider all cases.

Case 1

$x + 4 < 0 \qquad 2x - 3 < 0 \qquad 3x - 1 < 0$

$x < -4 \qquad 2x < 3 \qquad 3x < 1$

$x < \frac{3}{2} \qquad x < \frac{1}{3}$

$x < -4$ is a solution.

Case 2

$x + 4 < 0 \qquad 2x - 3 > 0 \qquad 3x - 1 > 0$

$x < -4 \qquad x > \frac{3}{2} \qquad x > \frac{1}{3}$

No solution since no x-values common to all three inequalities.

Case 3

$x + 4 > 0 \qquad 2x - 3 < 0 \qquad 3x - 1 > 0$

$x > -4 \qquad x < \frac{3}{2} \qquad x > \frac{1}{3}$

$\frac{1}{3} < x < \frac{3}{2}$ is a solution.

Case 4

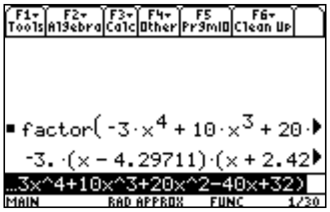
$x + 4 > 0 \qquad 2x - 3 > 0 \qquad 3x - 1 < 0$

$x > -4 \qquad x > \frac{3}{2} \qquad x < \frac{1}{3}$

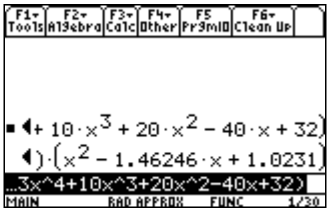
No solution since no x-values common to all three inequalities.

Combining the results of all the cases, the solution is $x \le -4$ or $\frac{1}{3} \le x \le \frac{3}{2}$.

c)



```
■ factor( -3·x^4 + 10·x^3 + 20·▸
  -3.·(x - 4.29711)·(x + 2.42▸
...3x^4+10x^3+20x^2-40x+32)
```



```
■ ◂+ 10·x^3 + 20·x^2 - 40·x + 32)
  ◂)·(x^2 - 1.46246·x + 1.0231)
...3x^4+10x^3+20x^2-40x+32)
```

$-3(x-4.3)(x+2.4)(x^2-1.5x+1.0)<0$

Consider all cases.

Case 1

$x - 4.3 > 0 \qquad x + 2.4 > 0$

$x > 4.3 \qquad x > -2.4$

$x > 4.3$ is a solution.

Case 2

$x - 4.3 < 0 \qquad x + 2.4 < 0$

$x < 4.3 \qquad x < -2.4$

$x < -2.4$ is a solution.

The solution is approximately $x < -2.4$ or $x > 4.3$.

Chapter Problem

Solutions for the Chapter Problem Wrap up are provided in the Teacher's Resource.

Chapter 2 Practice Test

Chapter 2 Practice Test Question 1 Page 142

The correct solution is **C**.

$$\begin{aligned} P(-2) &= 5(-2)^3 + 4(-2)^2 - 3(-2) + 2 \\ &= -16 \end{aligned}$$

Chapter 2 Practice Test Question 2 Page 142

The correct solution is **C**.

$$P(2) = 2(2)^3 - 5(2)^2 - 9(2) + 18 \neq 0$$

Chapter 2 Practice Test Question 3 Page 142

The correct solution is **D**.

The only set to include ±1 and $\pm\frac{1}{4}$

Chapter 2 Practice Test Question 4 Page 142

a)

$$\begin{array}{r|rrrr} 3 & 1 & -4 & 3 & -7 \\ - & & 3 & -21 & 72 \\ \hline \times & 1 & -7 & 24 & -79 \end{array}$$

$$\frac{x^3 - 4x^2 + 3x - 7}{x + 3} = x^2 - 7x + 24 - \frac{79}{x + 3}$$

b) $x \neq -3$

c) $(x + 3)(x^2 - 7x + 24) - 79$

d) $$\begin{aligned} (x + 3)(x^2 - 7x + 24) - 79 &= x^3 - 7x^2 + 24x + 3x^2 - 21x + 72 - 79 \\ &= x^3 - 4x^2 + 3x - 7 \end{aligned}$$

Chapter 2 Practice Test **Question 5 Page 142**

a)
$$\begin{aligned}
f(-2) &= (-2)^4 + (-2)^3k - 2(-2)^2 + 1 \\
5 &= 16 - 8k - 8 + 1 \\
8k &= 9 - 5 \\
8k &= 4 \\
k &= \frac{1}{2}
\end{aligned}$$

b)
$$\begin{aligned}
f(-4) &= (-4)^4 + (-4)^3\left(\frac{1}{2}\right) - 2(-4)^2 + 1 \\
&= 256 - 32 - 32 + 1 \\
&= 193
\end{aligned}$$

c)
$$\begin{array}{r l}
 & x^3 - \frac{7}{2}x^2 + 12x - 48 \\ \cline{2-2}
x+4 \big) & x^4 + \frac{1}{2}x^3 - 2x^2 + 0x + 1 \\
 & \underline{x^4 + 4x^3} \\
 & \qquad -\frac{7}{2}x^3 - 2x^2 \\
 & \qquad \underline{-\frac{7}{2}x^3 - 14x^2} \\
 & \qquad\qquad\qquad 12x^2 + 0x \\
 & \qquad\qquad\qquad \underline{12x^2 + 48x} \\
 & \qquad\qquad\qquad\qquad -48x + 1 \\
 & \qquad\qquad\qquad\qquad \underline{-48x - 192} \\
 & \qquad\qquad\qquad\qquad\qquad\quad 193
\end{array}$$

Chapter 2 Practice Test Question 6 Page 142

a) $P(-1) = -1 - 5 - 2 + 8$
$\quad = 0$

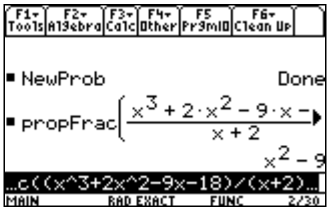
$(x + 1)$ is a factor.

$$\begin{array}{r|rrrr} 1 & 1 & -5 & 2 & 8 \\ - & & 1 & -6 & 8 \\ \hline \times & 1 & -6 & 8 & 0 \end{array}$$

$$\begin{aligned} x^3 - 5x^2 + 2x + 8 &= (x+1)(x^2 - 6x + 8) \\ &= (x+1)(x-4)(x-2) \end{aligned}$$

b) $P(-2) = (-2)^3 + 2(-2)^2 - 9(-2) - 18$
$\quad = 0$

$(x + 2)$ is a factor.



■ NewProb Done

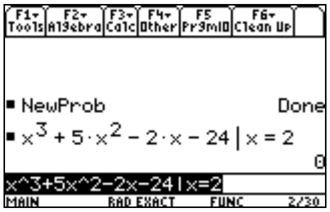
■ propFrac$\left(\dfrac{x^3 + 2\cdot x^2 - 9\cdot x -}{x+2}\right.$▶

$x^2 - 9$

...c((x^3+2x^2-9x-18)/(x+2)...

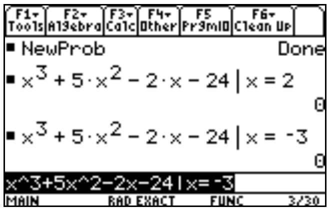
$$\begin{aligned} x^3 + 2x^2 - 9x - 18 &= (x + 2)(x^2 - 9) \\ &= (x + 2)(x + 3)(x - 3) \end{aligned}$$

c)



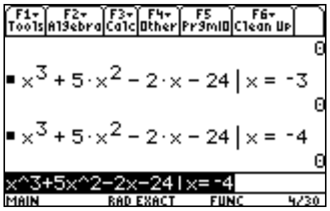
```
■ NewProb                      Done
■ x^3 + 5·x^2 − 2·x − 24 | x = 2
                                  0
x^3+5x^2-2x-24|x=2
MAIN      RAD EXACT    FUNC    2/30
```

(*x* – 2) is a factor.



```
■ NewProb                      Done
■ x^3 + 5·x^2 − 2·x − 24 | x = 2
                                  0
■ x^3 + 5·x^2 − 2·x − 24 | x = -3
                                  0
x^3+5x^2-2x-24|x= -3
MAIN      RAD EXACT    FUNC    3/30
```

(*x* + 3) is a factor.

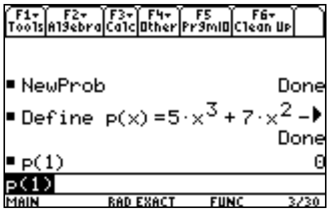


```
                                  0
■ x^3 + 5·x^2 − 2·x − 24 | x = -3
                                  0
■ x^3 + 5·x^2 − 2·x − 24 | x = -4
                                  0
x^3+5x^2-2x-24|x= -4
MAIN      RAD EXACT    FUNC    4/30
```

(*x* + 4) is a factor.

$x^3 + 5x^2 - 2x - 24 = (x - 2)(x + 3)(x + 4)$

d)



```
■ NewProb                    Done
■ Define p(x)=5·x³+7·x²-▸
                             Done
■ p(1)                          0
p(1)
MAIN      RAD EXACT   FUNC   3/30
```

$(x-1)$ is a factor.

$$\begin{array}{r@{}l}
 & \underline{5x^2+12x+4} \\
x-1 & \big)\,5x^3+7x^2-8x-4 \\
 & \underline{5x^3-5x^2} \\
 & \phantom{5x^3-{}}12x^2-8x \\
 & \phantom{5x^3-{}}\underline{12x^2-12x} \\
 & 4x-4 \\
 & \underline{4x-4} \\
 & 0
\end{array}$$

$$\begin{aligned}5x^3+7x^2-8x-4 &= (x-1)(5x^2+12x+4)\\ &= (x-1)(x+2)(5x+2)\end{aligned}$$

e) $P(-2)=(-2)^3+9(-2)^2+26(-2)+24$

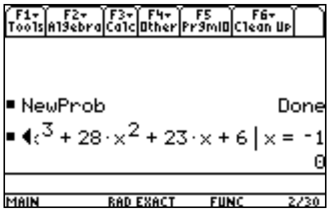
$=0$

$(x+2)$ is a factor.

$$\begin{array}{c|cccc}
2 & 1 & 9 & 26 & 24 \\
- & & 2 & 14 & 24 \\
\hline
\times & 1 & 7 & 12 & 0
\end{array}$$

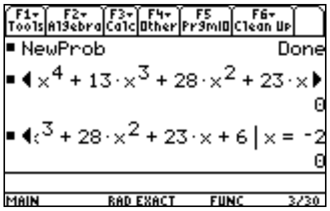
$$\begin{aligned}x^3+9x^2+26x+24 &= (x+2)(x^2+7x+12)\\ &= (x+2)(x+3)(x+4)\end{aligned}$$

f)



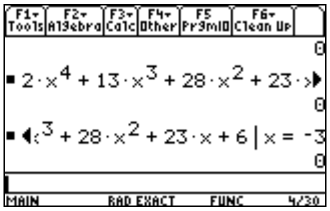
```
■ NewProb                    Done
■ ◀:3 + 28·x2 + 23·x + 6 | x = -1
                                0
```

($x + 1$) is a factor.



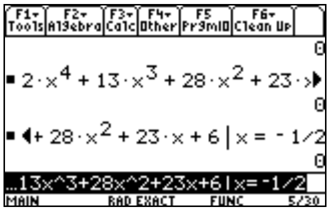
```
■ NewProb                    Done
■ ◀x4 + 13·x3 + 28·x2 + 23·x▶
                                0
■ ◀:3 + 28·x2 + 23·x + 6 | x = -2
                                0
```

($x + 2$) is a factor.



```
                                0
■ 2·x4 + 13·x3 + 28·x2 + 23·>▶
                                0
■ ◀:3 + 28·x2 + 23·x + 6 | x = -3
                                0
```

($x + 3$) is a factor.



```
                                0
■ 2·x4 + 13·x3 + 28·x2 + 23·>▶
                                0
■ ◀+ 28·x2 + 23·x + 6 | x = - 1/2
                                0
…13x^3+28x^2+23x+6|x=-1/2
```

($2x + 1$) is a factor.

$2x^4 + 13x^3 + 28x^2 + 23x + 6 = (x + 1)(x + 2)(x + 3)(2x + 1)$

Chapter 2 Practice Test **Question 7 Page 142**

$x = –5$ or $x = 3$ or $x = −2$

Chapter 2 Practice Test **Question 8 Page 142**

a) $x = 2$

b) $(x + 11)(x - 11)(x^2 + 16) = 0$
$x = -11$ or $x = 11$

c) $2(x^2 - 2x + 3)(x^2 - 25) = 0$
$2(x^2 - 2x + 3)(x - 5)(x + 5) = 0$
$x = 5$ or $x = -5$

d) $3(x^2 - 9)(x - 5)(x + 2) = 0$
$3(x - 3)(x + 3)(x - 5)(x + 2) = 0$
$x = 3$ or $x = -3$ or $x = 5$ or $x = -2$

Chapter 2 Practice Test **Question 9 Page 142**

a) $(x + 1)^2(x + 2) = 0$
$x = -1$ or $x = -2$

b) $(x - 3)(x - 1)(x + 4) = 0$
$x = 3$ or $x = 1$ or $x = -4$

c) $(2x - 3)(4x - 7)(4x + 7) = 0$
$x = \frac{3}{2}$ or $x = \frac{7}{4}$ or $x = -\frac{7}{4}$
$x = 1.5$ or $x = 1.75$ or $x = -1.75$

d) $x(3x - 2)(3x + 2)(5x - 3) = 0$
$x = 0$ or $x = \frac{2}{3}$ or $x = -\frac{2}{3}$ or $x = \frac{3}{5}$

Chapter 2 Practice Test **Question 10 Page 142**

Answers may vary. A sample solution is shown.

a) All involve polynomials; the equation is a statement about two equivalent expressions (e.g., $x^2 - x = x^7 + 8$), the inequality is a statement about two unequivalent expressions (e.g., $x^2 - x < x^7 + 8$), and the function is a relationship giving each element in the domain one corresponding value in the range (e.g., $y = x^7 + 8$).

b) When an polynomial equation such as $x^2 - x$ is equal to zero, the roots of the equation are the same as the zeros of the function $y = x^2 - x$ and the x-intercepts of the graph of $x^2 - x$.

Chapter 2 Practice Test **Question 11 Page 143**

a) $y = kx(x+3)(2x+3)(x-2)$

Using the point (−2, 4), substitute $x = -2$ and $y = 4$ and solve for k.

$$4 = k(-2)(-2+3)(2(-2)+3)(-2-2)$$
$$4 = -8k$$
$$k = -\frac{1}{2}$$
$$y = -\frac{1}{2}x(x+3)(2x+3)(x-2)$$

b) $x < -3, -\frac{3}{2} < x < 0, x > 2$

Chapter 2 Practice Test **Question 12 Page 143**

a) $y = k(x-5)^2(x+2+\sqrt{6})(x+2-\sqrt{6})$

$$y = k(x^2-10x+25)(x^2+2x-\sqrt{6}x+2x+4-2\sqrt{6}+\sqrt{6}x+2\sqrt{6}-6)$$
$$y = k(x^2-10x+25)(x^2+4x-2)$$
$$y = k(x^4+4x^3-2x^2-10x^3-40x^2+20x+25x^2+100x-50)$$
$$y = k(x^4-6x^3-17x^2+120x-50)$$

b) Substitute $x = 0$ and $y = 20$ and solve for k.

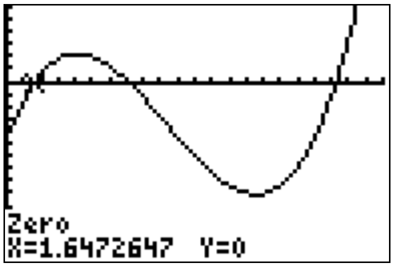
$$20 = k\left(0^4-6(0)^3-17(0)^2+120(0)-50\right)$$
$$20 = -50k$$
$$k = -\frac{2}{5}$$
$$y = -\frac{2}{5}(x^4-6x^3-17x^2+120x-50)$$

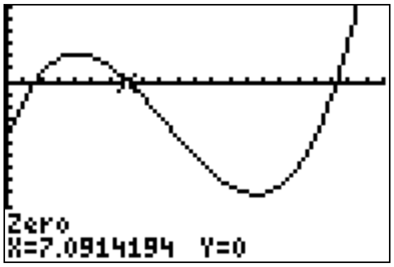
Chapter 2 Practice Test **Question 13 Page 143**

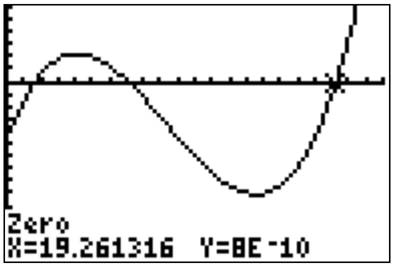
a) height = x
width = $(20 - 2x)$
length = $18 - x$
$V(x) = x(20 - 2x)(18 - x)$

b) $V(x) = x(20-2x)(18-x)$

$$450 = x(360-56x+2x^2)$$
$$0 = x(360-56x+2x^2)-450$$
$$0 = 2x^3-56x^2+360x-450$$







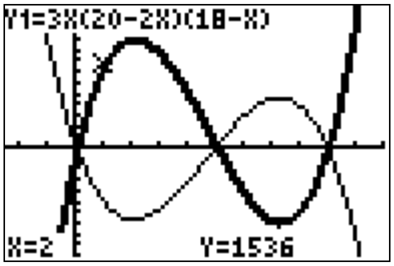
height ≐ 1.6	height ≐ 7.1	height ≐ 19.3
width ≐ 16.7	width ≐ 5.8	width ≐ –18.5
length ≐ 16.4	length ≐ 10.9	length ≐ –1.3

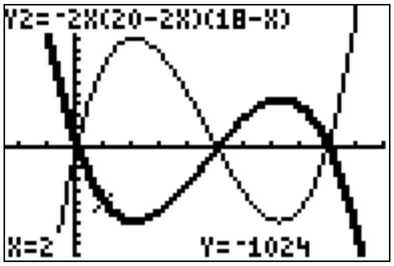
Disregard negative dimensions.

The possible dimensions of the box are approximately 16.7 cm by 16.4 cm by 1.6 cm or 5.8 cm by 10.9 cm by 7.1 cm

c) $V(x) = kx(20 - 2x)(18 - x)$

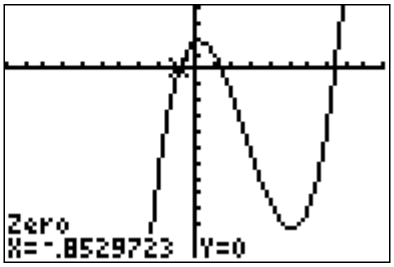
d) Answers may vary. A sample solution is shown.

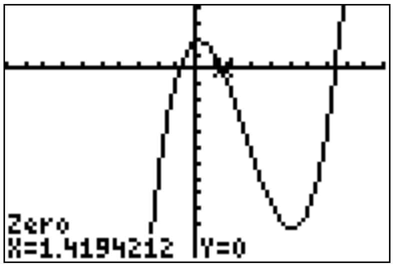


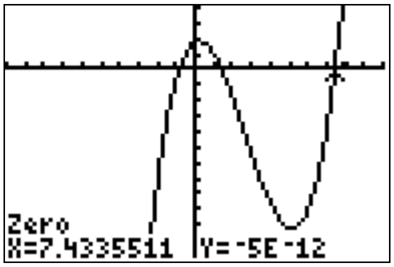


Chapter 2 Practice Test **Question 14 Page 143**

a) $x^3 - 8x^2 + 3x + 9 \leq 0$

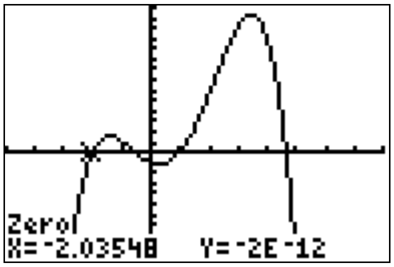


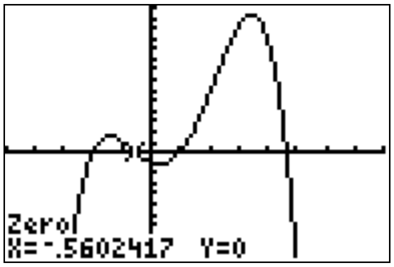


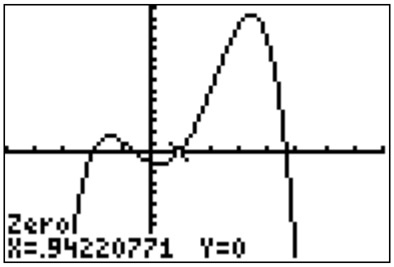


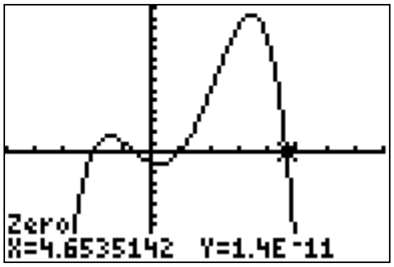
Approximately $x \leq -0.9$ or $1.4 \leq x \leq 7.4$.

b) $-x^4 + 3x^3 + 9x^2 - 5x - 5 > 0$





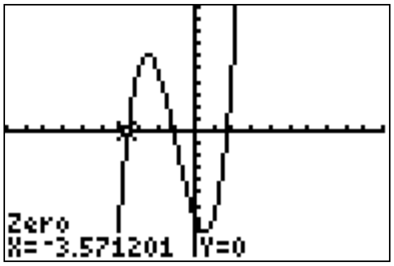


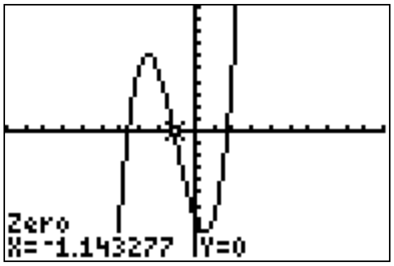


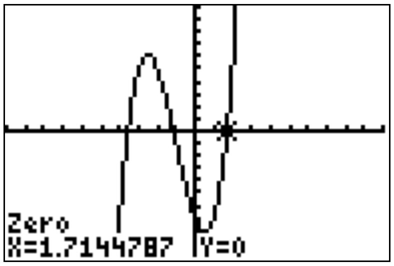
Approximately $-2.0 < x < -0.6$ or $0.9 < x < 4.7$.

Chapter 2 Practice Test Question 15 Page 143

a)

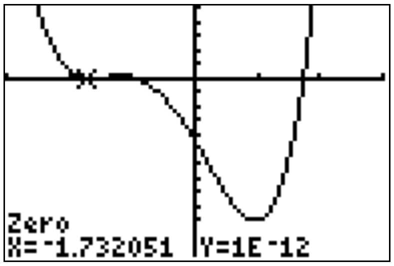

Zero
X=-3.571201 Y=0

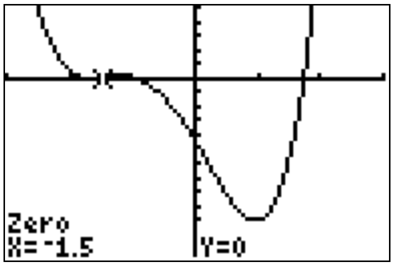

Zero
X=-1.143277 Y=0

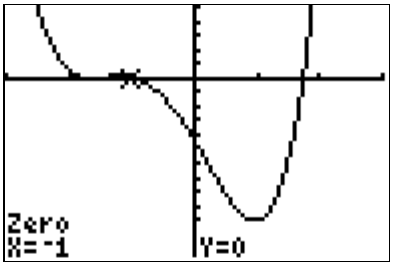

Zero
X=1.7144787 Y=0

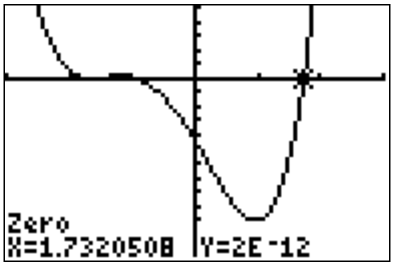
Approximately $x < -3.6$ or $-1.1 < x < 1.7$.

b)


Zero
X=-1.732051 Y=1E-12


Zero
X=-1.5 Y=0


Zero
X=-1 Y=0


Zero
X=1.7320508 Y=2E-12

$-1.5 \le x \le -1$ or approximately $x \le -1.7$ or $x \ge 1.7$.

Chapter 2 Practice Test **Question 16 Page 143**

a) $(3x - 4)(3x + 4) < 0$
Consider all cases.

Case 1
$3x - 4 > 0 \quad 3x + 4 < 0$
$3x > 4 \quad 3x < -4$
$x > \frac{4}{3} \quad x < -\frac{4}{3}$
No solution since no x-values common to both inequalities.

Case 2
$3x - 4 < 0 \quad 3x + 4 > 0$
$x < \frac{4}{3} \quad x > -\frac{4}{3}$
$-\frac{4}{3} < x < \frac{4}{3}$ is a solution.

Combining the results of all the cases, the solution is $-\frac{4}{3} < x < \frac{4}{3}$.

b)
$$-x(x^2 - 6x + 9) > 0$$
$$-x(x-3)^2 > 0$$
$$x(x-3)^2 < 0$$

$x < 0 \qquad x < 3$
The solution is $x < 0$.

c) $2x(x^2-9)+5(x^2-9)\le 0$

$(2x+5)(x-3)(x+3)\le 0$

Consider all cases.

Case 1

$2x+5<0$ $\quad x-3<0$ $\quad x+3<0$

$x<-\frac{5}{2}$ $\quad x<3$ $\quad x<-3$

$x<3$ is a solution.

Case 2

$2x+5<0$ $\quad x-3>0$ $\quad x+3>0$

$x<-\frac{5}{2}$ $\quad x>3$ $\quad x>-3$

No solution since no x-values common to all three inequalities.

Case 3

$2x+5>0$ $\quad x-3>0$ $\quad x+3<0$

$x>-\frac{5}{2}$ $\quad x>3$ $\quad x<-3$

No solution since no x-values common to all three inequalities.

Case 4

$2x+5>0$ $\quad x-3<0$ $\quad x+3>0$

$x>-\frac{5}{2}$ $\quad x<3$ $\quad x>-3$

$-\frac{5}{2}<x<3$ is a solution.

Combining the results of all the cases, the solution is $x\le -3$ or $-\frac{5}{2}\le x\le 3$.

d) $(x - 2)(2x + 1)(x + 1)(x + 3) \ge 0$
Consider all cases.

Case 1

$x > 2 \qquad x > -\frac{1}{2} \qquad x > -1 \qquad x > -3$
$x > 2$ is a solution.

Case 2

$x < 2 \qquad x < -\frac{1}{2} \qquad x > -1 \qquad x > -3$

$-1 < x < -\frac{1}{2}$ is a solution.

Case 3

$x > 2 \qquad x > -\frac{1}{2} \qquad x < -1 \qquad x < -3$
No solution since no x-values common to all four inequalities.

Case 4

$x < 2 \qquad x > -\frac{1}{2} \qquad x < -1 \qquad x > -3$
No solution since no x-values common to all four inequalities.

Case 5

$x < 2 \qquad x > -\frac{1}{2} \qquad x > -1 \qquad x < -3$
No solution since no x-values common to all four inequalities.

Case 6

$x > 2 \qquad x < -\frac{1}{2} \qquad x < -1 \qquad x > -3$
No solution since no x-values common to all four inequalities.

Case 7

$x > 2 \qquad x < -\frac{1}{2} \qquad x > -1 \qquad x < -3$
No solution since no x-values common to all four inequalities.

Case 8

$x < 2 \qquad x < -\frac{1}{2} \qquad x < -1 \qquad x < -3$
$x < -3$ is a solution.

Combining the results of all the cases, the solution is $x \le -3$ or $-1 \le x \le -\frac{1}{2}$ or $x \ge 2$.

Chapter 2 Practice Test **Question 17 Page 143**

a) $V(x) = x(32 - 2x)(40 - 2x)$

b) i) $V(x) = 2x(32 - 2x)(40 - 2x)$

ii) $V(x) = \frac{1}{2}x(32 - 2x)(40 - 2x)$

c) family of functions

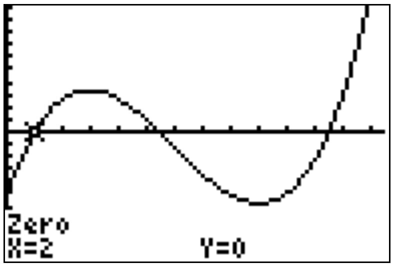
d) $V(x) = x(32 - 2x)(40 - 2x)$

$$V(x) > 2016$$

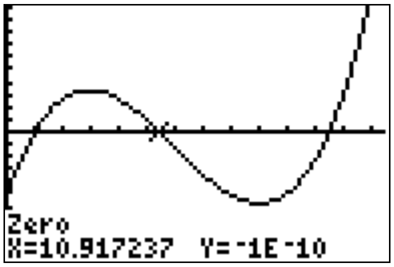
$$x(1280 - 144x + 4x^2) - 2016 > 0$$

$$4x^3 - 144x^2 + 1280x - 2016 > 0$$

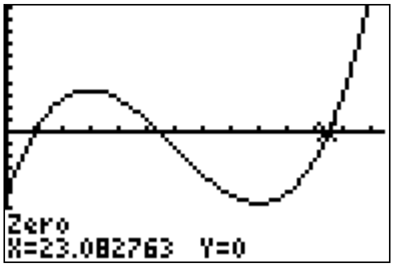
$$4(x^3 - 36x^2 + 320x - 504) > 0$$



```
Zero
X=2          Y=0
```



```
Zero
X=10.917237  Y=⁻1E⁻10
```



```
Zero
X=23.082763  Y=0
```

The values of x that will result in boxes of a volume greater than 2016 are approximately $2 < x < 10.9$ or $x > 23.1$.