Section 2.1: Forces and Free-Body Diagrams
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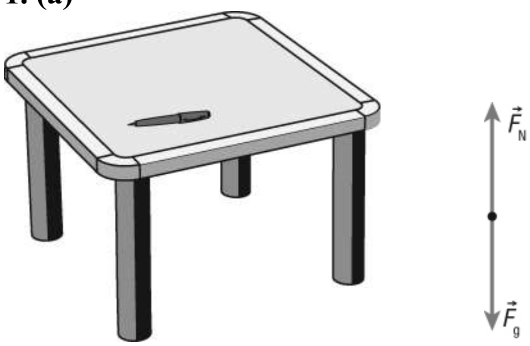
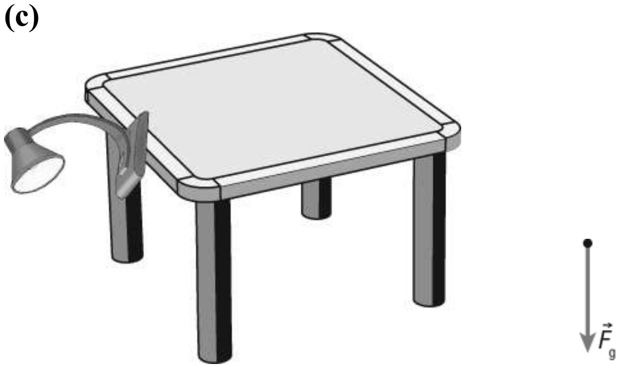
1. (a)



(b)

(c)



(d)

2. While the ball is in the air (from just after it leaves your hand, until just before it makes contact with the object that it will hit), it is only acted upon by one force—the force of gravity, $\vec{F}_g$. Therefore, for (a), (b), and (c), the FBD of the force acting on the ball is shown below.

3.

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1. (a) Given: $\vec{F}_a = 25$ N [forward 40.0° up]; $\vec{F}_g = 4.2$ N [down]

Required: $\Sigma\vec{F}$

Analysis: $\left|\Sigma\vec{F}\right| = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$; $\theta = \tan^{-1}\left(\dfrac{\Sigma F_y}{\Sigma F_x}\right)$. Choose forward and up as positive.

Solution: For the x-component of the force,

$$\begin{aligned}\vec{F}_{ax} &= \vec{F}\cos\theta \\ &= (25\text{ N})\cos 40.0° \\ \vec{F}_{ax} &= 19.15\text{ N (two extra digits carried)}\end{aligned}$$

$$\begin{aligned}\Sigma\vec{F}_x &= \vec{F}_{ax} + \vec{F}_{gx} \\ &= 19.15\text{ N} + 0.0\text{N} \\ \Sigma\vec{F}_x &= 19.15\text{ N (two extra digits carried)}\end{aligned}$$

For the y-component of the force,

$$\begin{aligned}\vec{F}_{ay} &= \vec{F}\sin\theta \\ &= (25\text{ N})\sin 40.0° \\ \vec{F}_{ay} &= 16.07\text{ N (two extra digits carried)}\end{aligned}$$

$$\Sigma\vec{F}_y = \vec{F}_{ay} + (-\vec{F}_{gy})$$
$$= 16.07 - 4.2 \text{ N}$$
$$\Sigma\vec{F}_y = 11.87 \text{ N (two extra digits carried)}$$

Construct $\Sigma\vec{F}$:

$$\left|\Sigma\vec{F}\right| = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$$
$$= \sqrt{(19.15 \text{ N})^2 + (11.87 \text{ N})^2}$$
$$= 22.53 \text{ N}$$
$$\left|\Sigma\vec{F}\right| = 23 \text{ N}$$

$$\theta = \tan^{-1}\left(\frac{\Sigma\vec{F}_y}{\Sigma\vec{F}_x}\right)$$
$$= \tan^{-1}\left(\frac{11.87 \cancel{\text{N}}}{19.15 \cancel{\text{N}}}\right)$$
$$\theta = 32°$$

Statement: The net force acting on the soccer ball is 23 N [32° above the horizontal].

(b) Given: $\vec{F}_1 = 15$ N [N 35° E]**;** $\vec{F}_2 = 25$ N [N 54° W]

Required: $\Sigma\vec{F}$

Analysis: $\left|\Sigma\vec{F}\right| = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$; $\theta = \tan^{-1}\left(\frac{\Sigma\vec{F}_y}{\Sigma\vec{F}_x}\right)$. Choose east and north as positive.

Solution: For the *x*-component of the force,

$$\vec{F}_{1x} = \vec{F}\cos\theta$$
$$= (15 \text{ N})\cos 35°$$
$$\vec{F}_{1x} = 12.29 \text{ N (two extra digits carried)}$$

$$\vec{F}_{2x} = \vec{F}\cos\theta$$
$$= (25 \text{ N})\cos 54°$$
$$\vec{F}_{2x} = 14.70 \text{ N (two extra digits carried)}$$

$$\Sigma\vec{F}_x = \vec{F}_{1x} + \vec{F}_{2x}$$
$$= 12.29 \text{ N} + 14.70 \text{ N}$$
$$\Sigma\vec{F}_x = 26.99 \text{ N (two extra digits carried)}$$

For the y-component of the force,

$$\vec{F}_{1y} = \vec{F}\sin\theta$$
$$= (15\text{ N})\sin 35°$$
$$\vec{F}_{1y} = 8.604\text{ N (two extra digits carried)}$$

$$\vec{F}_{2y} = \vec{F}\sin\theta$$
$$= (-25\text{ N})\sin 54°$$
$$\vec{F}_{2y} = -20.22\text{ N (two extra digits carried)}$$

$$\Sigma\vec{F}_y = \vec{F}_{1y} + \vec{F}_{2y}$$
$$= 8.604 - 20.22$$
$$\Sigma\vec{F}_y = -11.62\text{ N (two extra digits carried)}$$

Construct $\Sigma\vec{F}$:

$$\left|\Sigma\vec{F}\right| = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$$
$$= \sqrt{(26.99\text{ N})^2 + (-11.62\text{ N})^2}$$
$$= 29.39\text{ N}$$
$$\left|\Sigma\vec{F}\right| = 29\text{ N}$$

$$\theta = \tan^{-1}\left(\frac{\Sigma\vec{F}_y}{\Sigma\vec{F}_x}\right)$$
$$= \tan^{-1}\left(\frac{-11.62\not{\text{N}}}{26.99\not{\text{N}}}\right)$$
$$\theta = 23°$$

Statement: The net force acting on the sled is 29 N [N 23° W].

(c) Given: $\vec{F}_g = 4.4\times10^2$ N [down]; $\vec{F}_1 = 4.3\times10^2$ N [up 35° left]; $\vec{F}_2 = 2.8\times10^2$ N [up]

Required: $\Sigma\vec{F}$

Analysis: $\left|\Sigma\vec{F}\right| = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$; $\theta = \tan^{-1}\left(\frac{\Sigma\vec{F}_y}{\Sigma\vec{F}_x}\right)$. Choose right and up as positive.

Solution: For the x-component of the force,

$$\Sigma\vec{F}_x = \vec{F}_{gx} + \vec{F}_{1x} + \vec{F}_{2x}$$
$$= (0\text{ N}) + (-430\text{ N})\sin 35° + (0\text{ N})$$
$$\Sigma\vec{F}_x = -246.6\text{ N (two extra digits carried)}$$

For the *y*-component of the force,

$$\Sigma\vec{F}_y = \vec{F}_{gy} + \vec{F}_{1y} + \vec{F}_{2y}$$
$$= (-440\text{ N}) + (430\text{ N})\cos 35° + (280\text{ N})$$
$$= -440\text{ N} + 352.2\text{ N} + 280\text{ N}$$
$$\Sigma\vec{F}_y = 192.2\text{ N (two extra digits carried)}$$

Construct $\Sigma\vec{F}$:

$$\left|\Sigma\vec{F}\right| = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$$
$$= \sqrt{(246.6\text{ N})^2 + (192.2\text{ N})^2}$$
$$= 310\text{ N}$$
$$\left|\Sigma\vec{F}\right| = 3.1\times10^2\text{ N}$$

$$\theta = \tan^{-1}\left(\frac{\Sigma\vec{F}_y}{\Sigma\vec{F}_x}\right)$$
$$= \tan^{-1}\left(\frac{192.24\text{ N}}{246.64\text{ N}}\right)$$
$$\theta = 38°$$

Statement: The net force acting on the performer is 3.1×10^2 N [up 38° left] or [left 52° up].

2. (a) Given: $\vec{F}_1 = 1.2\times10^4$ N [E 12° N]; $\vec{F}_2 = 1.2\times10^4$ N [E 12° S]; $\Sigma\vec{F} = 0$ N

Required: $\vec{F}_f$

Analysis: $\Sigma\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_f$. Choose east and north as positive.

Solution: $\Sigma\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_f$

$$0 = \vec{F}_1 + \vec{F}_2 + \vec{F}_f$$
$$\vec{F}_f = -\vec{F}_1 - \vec{F}_2$$

$$\vec{F}_{fx} = -\vec{F}_{1x} - \vec{F}_{2x}$$
$$= -(1.2\times10^4\text{ N})\cos 12° - (1.2\times10^4\text{ N})\cos 12°$$
$$\vec{F}_{fx} = -2.3\times10^4\text{ N}$$

Statement: The force of friction on the rock is 2.3×10^4 N [W].

(b) Answers may vary. Sample answer: I notice that the forces of the tractors are equal in magnitude but act in directions symmetric about the east. So the net force of the tractors has to be due east, making the force of friction due west. Another way to say this is that the north component of one tractor force cancels the south component of the other.

3. Given: m_1 = 15.0 kg; m_2 = 7.0 kg; m_3 = 13.0 kg; g = 9.8 m/s^2

Required: $\vec{F}_{T1}$; $\vec{F}_{T2}$; $\vec{F}_{T3}$

Analysis: Draw an FBD for each mass. Choose up as positive.

Solution: The FBDs are shown below.

Mass 1 ($m_1 = 15.0$ kg)

$$\Sigma \vec{F}_y = \vec{F}_{T1y} + \vec{F}_{T2y} + \vec{F}_{gy}$$
$$0 \text{ N} = \vec{F}_{T1} - \vec{F}_{T2} - (15.0 \text{ kg})g$$
$$\vec{F}_{T1} - \vec{F}_{T2} = (15.0 \text{ kg})g \quad \text{(Equation 1)}$$

Mass 2 ($m_2 = 7.0$ kg)

$$\Sigma \vec{F}_y = \vec{F}_{T2y} + \vec{F}_{T3y} + \vec{F}_{gy}$$
$$0 \text{ N} = \vec{F}_{T2} - \vec{F}_{T3} - (7.0 \text{ kg})g$$
$$\vec{F}_{T2} - \vec{F}_{T3} = (7.0 \text{ kg})g \quad \text{(Equation 2)}$$

Mass 3 ($m_3 = 13.0$ kg)

$$\Sigma \vec{F}_y = \vec{F}_{3y} + \vec{F}_{gy}$$
$$0 \text{ N} = \vec{F}_{T3} - (13.0 \text{ kg})g$$
$$\vec{F}_{T3} = (13.0 \text{ kg})g \quad \text{(Equation 3)}$$

Solve for the tensions in the wires, working from equation (3) to equation (2) to equation (1):

$$\left|\Sigma\vec{F}\right| = \sqrt{(\Sigma\vec{F}_x)^2 + (\Sigma\vec{F}_y)^2}$$
$$= \sqrt{(0.3392\ \text{N})^2 + (1.2880\ \text{N})^2}$$
$$\left|\Sigma\vec{F}\right| = 1.3\ \text{N}$$

$$\vec{F}_{\text{T3}} = (13.0\ \text{kg})g \quad \text{(Equation 1)}$$
$$\vec{F}_{\text{T3}} = 1.3\times10^2\ \text{N}$$

$$\vec{F}_{\text{T2}} - \vec{F}_{\text{T3}} = (7.0\ \text{kg})g \quad \text{(Equation 2)}$$
$$\vec{F}_{\text{T2}} = \vec{F}_{\text{T3}} + (7.0\ \text{kg})g$$
$$= (13.0\ \text{kg})g + (7.0\ \text{kg})g$$
$$\vec{F}_2 = 2.0\times10^2\ \text{N}$$

$$\vec{F}_{\text{T1}} - \vec{F}_{\text{T2}} = (15.0\ \text{kg})g \quad \text{(Equation 3)}$$
$$\vec{F}_{\text{T1}} = \vec{F}_{\text{T2}} + (15.0\ \text{kg})g$$
$$= (13.0\ \text{kg})g + (7.0\ \text{kg})g + (15.0\ \text{kg})g$$
$$\vec{F}_{\text{T1}} = 3.4\times10^2\ \text{N}$$

Statement: The tension in the top wire is 3.4×10^2 N, in the middle wire it is 2.0×10^2 N, and in the bottom wire it is 1.3×10^2 N.

4. Given: $\vec{F}_{\text{air}} = 0.40$ N [32° above the horizontal]; $\vec{F}_{\text{g}} = 1.5$ N [down]

Required: $\Sigma\vec{F}$

Analysis: $\left|\Sigma\vec{F}\right| = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$; $\theta = \tan^{-1}\left(\frac{\Sigma\vec{F}_y}{\Sigma\vec{F}_x}\right)$; use forward and up as positive.

Solution: For the x-component of the force,

$$\Sigma\vec{F}_x = \vec{F}_{\text{air}x} + \vec{F}_{\text{g}x}$$
$$= (-0.40\ \text{N})\cos 32° + (0\ \text{N})$$
$$\Sigma\vec{F}_x = -0.3392\ \text{N (two extra digits carried)}$$

For the y-component of the force,

$$\Sigma\vec{F}_y = \vec{F}_{\text{air}y} + \vec{F}_{\text{g}y}$$
$$= (0.40\ \text{N})\sin 32° + (-1.5\ \text{N})$$
$$\Sigma\vec{F}_y = -1.288\ \text{N (two extra digits carried)}$$

Construct $\Sigma\vec{F}$:

$$\theta = \tan^{-1}\left(\frac{\Sigma\vec{F}_y}{\Sigma\vec{F}_x}\right)$$

$$= \tan^{-1}\left(\frac{1.288\ \cancel{\text{N}}}{0.3392\ \cancel{\text{N}}}\right) \text{ (two extra digits carried)}$$

$$\theta = 75°$$

Statement: The magnitude of the net force on the ball is 1.3 N [75° below the horizontal].

5. (a) Since the ball is at rest, the net force on it is 0 N.

(b) If I suddenly remove my hand, the only force acting on the ball is gravity. The net force is 16 N [down].

(c) Given: $\vec{F}_\text{g} = 16$ N [down]; $\vec{F}_\text{a} = 12$ N [right]

Required: $\Sigma\vec{F}$

Analysis: $\left|\Sigma\vec{F}\right| = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$; $\theta = \tan^{-1}\left(\frac{\Sigma\vec{F}_y}{\Sigma\vec{F}_x}\right)$. Choose forward and up as positive.

Solution: For the x-component of the force,

$$\Sigma\vec{F}_x = \vec{F}_{\text{a}x} + \vec{F}_{\text{g}x}$$

$$= 12\text{ N} + 0\text{ N}$$

$$\Sigma\vec{F}_x = 12\text{ N}$$

For the y-component of the force,

$$\Sigma\vec{F}_y = \vec{F}_{\text{a}y} + \vec{F}_{\text{g}y}$$

$$= (0\text{ N}) + (-16\text{ N})$$

$$\Sigma\vec{F}_y = -16\text{ N}$$

Construct $\Sigma\vec{F}$:

$$\left|\Sigma\vec{F}\right| = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$$

$$= \sqrt{(12\text{ N})^2 + (16\text{ N})^2}$$

$$\left|\Sigma\vec{F}\right| = 20\text{ N}$$

$$\theta = \tan^{-1}\left(\frac{\Sigma\vec{F}_y}{\Sigma\vec{F}_x}\right)$$

$$= \tan^{-1}\left(\frac{16\ \cancel{\text{N}}}{12\ \cancel{\text{N}}}\right)$$

$$\theta = 53°$$

Statement: The net force on the basketball is 20 N [right 53° down].

(d) Given: $\vec{F}_g = 16$ N [down]; $\vec{F}_a = 26$ N [up 45° right]

Required: $\Sigma\vec{F}$

Analysis: $\left|\Sigma\vec{F}\right| = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$; $\theta = \tan^{-1}\left(\dfrac{\Sigma\vec{F}_y}{\Sigma\vec{F}_x}\right)$. Choose forward and up as positive.

Solution: For the x-component of the force,

$$\begin{aligned}\Sigma\vec{F}_x &= \vec{F}_{ax} + \vec{F}_{gx} \\ &= (26\text{ N})\cos 45° + 0\text{ N} \\ \Sigma\vec{F}_x &= 18.38\text{ N (two extra digits carried)}\end{aligned}$$

For the y-component of the force,

$$\begin{aligned}\Sigma\vec{F}_y &= \vec{F}_{ay} + \vec{F}_{gy} \\ &= (26\text{ N})\sin 45° + (-16\text{ N}) \\ \Sigma\vec{F}_y &= -2.385\text{ N (two extra digits carried)}\end{aligned}$$

Construct $\Sigma\vec{F}$:

$$\begin{aligned}\left|\Sigma\vec{F}\right| &= \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2} \\ &= \sqrt{(18.38\text{ N})^2 + (2.385\text{ N})^2} \\ \left|\Sigma\vec{F}\right| &= 19\text{ N}\end{aligned}$$

$$\begin{aligned}\theta &= \tan^{-1}\left(\frac{\Sigma\vec{F}_y}{\Sigma\vec{F}_x}\right) \\ &= \tan^{-1}\left(\frac{2.385\text{ }\cancel{\text{N}}}{18.38\text{ }\cancel{\text{N}}}\right) \\ \theta &= 7.4°\end{aligned}$$

Statement: The net force on the basketball is 19 N [right 7.4° up].

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1. Table 1 Common Forces

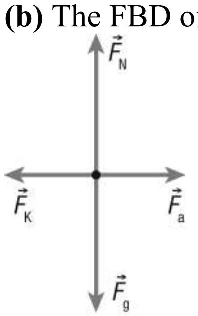
Name	Symbol	Contact/ non-contact	Direction	Example in daily life
force of gravity	$\vec{F}_g$	non-contact	down	A peanut butter sandwich falls to the floor because it is pulled by gravity.
normal force	$\vec{F}_N$	contact	perpendicular to surface	A tea cup sits on the surface of a table, held up by the normal force.
string tension	$\vec{F}_T$	contact	away from object	A child uses tension in a leash to pull her dog.
friction	$\vec{F}_f$	contact	opposite to direction of motion or tendency to motion	Friction in the car brake pads causes the car to slow down.
static friction	$\vec{F}_S$	contact	along the surface, opposite to sum of the other forces	Static friction between a sled and the snow has to be overcome before the sled will slide.
kinetic friction	$\vec{F}_K$	contact	along the surface, opposite to direction of motion	Kinetic friction between my skate blades and the ice causes me to slow down.
air resistance	$\vec{F}_{air}$	contact	opposite to direction of motion	A falling sheet of paper is subject to air resistance as well as the force of gravity.
applied force (push or pull)	$\vec{F}_a$	contact	any direction	My friends help me push my car out of the ditch.

2. A pulley is a device that changes the direction of string tension but does not change its magnitude. This means that the tension in the cord is 22 N throughout. Therefore, the student's statement is not valid.

3. Ropes can only pull and never push because the rope just sags when you push on it and, therefore, you cannot exert a force.

4. (a) The forces acting on the textbook are the force of gravity, the normal force, the applied force, and the force of kinetic friction.

(b) The FBD of the textbook is shown below.



5. Given: $\vec{F}_A = 2.3$ N [S 35° W]; $\vec{F}_B = 3.6$ N [N 14° W]; $\vec{F}_C = 4.2$ N [S 34° E]

(a) Required: $\vec{F}_A + \vec{F}_B + \vec{F}_C$

Analysis: $\vec{F}_A + \vec{F}_B + \vec{F}_C$. Choose north and east as positive.

Solution:

$$\begin{aligned}\Sigma\vec{F}_x &= \vec{F}_{Ax} + \vec{F}_{Bx} + \vec{F}_{Cx}\\ &= (-2.3\text{ N})\sin 35° + (-3.6\text{ N})\sin 14° + (4.2\text{ N})\sin 24°\\ &= -1.319\text{ N} - 0.8709\text{ N} + 1.708\text{ N}\\ \Sigma\vec{F}_x &= -0.4819\text{ N (two extra digits carried)}\end{aligned}$$

$$\Sigma\vec{F}_y = \vec{F}_{Ay} + \vec{F}_{By} + \vec{F}_{Cy}$$
$$= (-2.3 \text{ N})\cos 35° + (3.6 \text{ N})\cos 14° + (-4.2 \text{ N})\cos 24°$$
$$= -1.884 \text{ N} + 3.493 \text{ N} - 3.836 \text{ N}$$
$$\Sigma\vec{F}_y = -2.227 \text{ N (two extra digits carried)}$$

Construct $\Sigma\vec{F}$:

$$\left|\Sigma\vec{F}\right| = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$$
$$= \sqrt{(0.4819 \text{ N})^2 + (2.227 \text{ N})^2}$$
$$\left|\Sigma\vec{F}\right| = 2.3 \text{ N}$$

$$\theta = \tan^{-1}\left(\frac{\Sigma\vec{F}_y}{\Sigma\vec{F}_x}\right)$$
$$= \tan^{-1}\left(\frac{2.227 \cancel{\text{N}}}{0.4819 \cancel{\text{N}}}\right)$$
$$\theta = 78°$$

Statement: $\vec{F}_A + \vec{F}_B + \vec{F}_C = 2.3$ N [W 78° S] or [S 12° W]

(b) Required: $\vec{F}_B - \vec{F}_C$

Analysis: $\vec{F}_B - \vec{F}_C$. Choose north and east as positive.

Solution:

For the *x*-component of the force,

$$\left(\vec{F}_B - \vec{F}_C\right)_x = \vec{F}_{Bx} - \vec{F}_{Cx}$$
$$= (-3.6 \text{ N})\sin 14° - (4.2 \text{ N})\sin 24°$$
$$= -0.8709 \text{ N} - 1.708 \text{ N}$$
$$\left(\vec{F}_B - \vec{F}_C\right)_x = -2.579 \text{ N (two extra digits carried)}$$

For the *y*-component of the force,

$$\left(\vec{F}_B - \vec{F}_C\right)_y = \vec{F}_{By} - \vec{F}_{Cy}$$
$$= (3.6 \text{ N})\cos 14° - (-4.2 \text{ N})\cos 24°$$
$$= +3.493 \text{ N} + 3.836 \text{ N}$$
$$\left(\vec{F}_B - \vec{F}_C\right)_y = 7.329 \text{ N (two extra digits carried)}$$

Construct $\Sigma\vec{F}$:

$$\left|\vec{F}_B - \vec{F}_C\right| = \sqrt{\left(\vec{F}_B - \vec{F}_C\right)_x^2 + \left(\vec{F}_B - \vec{F}_C\right)_y^2}$$

$$= \sqrt{(2.579\text{ N})^2 + (7.329\text{ N})^2}$$

$$\left|\vec{F}_B - \vec{F}_C\right| = 7.8\text{ N}$$

$$\theta = \tan^{-1}\left(\frac{\left(\vec{F}_B - \vec{F}_C\right)_y}{\left(\vec{F}_B - \vec{F}_C\right)_x}\right)$$

$$= \tan^{-1}\left(\frac{7.329\ \cancel{\text{N}}}{2.579\ \cancel{\text{N}}}\right)$$

$$\theta = 71°$$

Statement: $\vec{F}_B - \vec{F}_C = 7.8$ N [W 71° N] or [N 19° W]

6. Given: $\vec{F}_A = 33$ N [E 22° N]; $\vec{F}_B = 42$ N [S 15° E]; $\Sigma\vec{F} = 0$ N

Required: $\vec{F}_C$

Analysis: $\vec{F}_A + \vec{F}_B + \vec{F}_C = 0$ N. Choose north and east as positive.

Solution: For the *x*-component of the force,

$$\vec{F}_{Cx} = -\vec{F}_{Ax} - \vec{F}_{Bx}$$

$$= -(33\text{ N})\cos 22° - (42\text{ N})\sin 15°$$

$$= -30.60\text{ N} - 10.87\text{ N}$$

$$\vec{F}_{Cx} = -41.47\text{ N (two extra digits carried)}$$

For the *y*-component of the force,

$$\vec{F}_{Cy} = -\vec{F}_{Ay} - \vec{F}_{By}$$

$$= -(33\text{ N})\sin 22° - (-42\text{ N})\cos 15°$$

$$= -12.36\text{ N} + 40.57\text{ N}$$

$$\vec{F}_{Cy} = 28.21\text{ N (two extra digits carried)}$$

Construct $\vec{F}_C$:

$$\left|\vec{F}_C\right| = \sqrt{(\vec{F}_{Cx})^2 + (\vec{F}_{Cy})^2}$$

$$= \sqrt{(41.47\text{ N})^2 + (28.21\text{ N})^2}\ \text{(two extra digits carried)}$$

$$\left|\vec{F}_C\right| = 50\text{ N}$$

$$\theta = \tan^{-1}\left(\frac{\vec{F}_{Cy}}{\vec{F}_{Cx}}\right)$$
$$= \tan^{-1}\left(\frac{28.21\ \cancel{N}}{41.47\ \cancel{N}}\right)$$
$$\theta = 34°$$

Statement: The force needed so that $\vec{F}_A + \vec{F}_B + \vec{F}_C = 0$ N is 50 N [W 34° N].

7. (a) Given: $\vec{F}_1 = 15$ N [N 24° E]; direction of $\vec{F}_2$ is [S]; direction of $\vec{F}_3$ is [W]; $\Sigma\vec{F} = 0$ N

Required: $|\vec{F}_2|$; $|\vec{F}_3|$

Analysis: $\vec{F}_1 = -\vec{F}_2 - \vec{F}_3$. Choose north and east as positive.

Solution: For the *x*-components of the force:

$$\vec{F}_{1x} = -\vec{F}_{2x} - \vec{F}_{3x}$$
$$(15\text{ N})\sin 24° = -(0\text{ N}) - \vec{F}_{3x}$$
$$\vec{F}_{3x} = -6.101\text{ N (two extra digits carried)}$$
$$\vec{F}_3 = 6.1\text{ N [W]}$$

For the *y*-components of the force:

$$\vec{F}_{1y} = -\vec{F}_{2y} - \vec{F}_{3y}$$
$$(15\text{ N})\cos 24° = -\vec{F}_{2y} - (0\text{ N})$$
$$\vec{F}_{2y} = -13.70\text{ N (two extra digits carried)}$$
$$\vec{F}_2 = 14\text{ N [S]}$$

Statement: The second child pulls with a force of 14 N [S] and the third child with a force of 6.1 N [W].

(b) Since the net force was zero when the second child lets go, the new net force has the same magnitude as $\vec{F}_2 = 14$ N but the opposite direction.

Therefore, the net force is 14 N [N].

(c) Given: $\vec{F}_1 = 15$ N [N 24° E]; $\Sigma\vec{F} = 0$ N

Analysis: $\Sigma\vec{F} = \vec{F}_1 + \vec{F}_3$. Choose north and east as positive.

Solution: $\Sigma\vec{F} = \vec{F}_1 + \vec{F}_3$

$$0 = 15\text{N} + \vec{F}_3$$
$$\vec{F}_3 = 0 - 15\text{ N}$$
$$\vec{F}_3 = -15\text{ N}$$

Statement: The third child must exert a force of 15 N [S 24° W] to cancel the force of the first child on her own.

8. Given: $\Sigma\vec{F} = 180$ N [E]; $\vec{F}_1 = 120$ N [E 14° S]

Required: $\vec{F}_2$

Analysis: $\Sigma\vec{F} = \vec{F}_1 + \vec{F}_2$. Choose north and east as positive.

Solution: For the x-components of the force:

$$\vec{F}_{2x} = \Sigma\vec{F}_x - \vec{F}_{1x}$$
$$= (180 \text{ N}) - (120 \text{ N})\cos 14°$$
$$\vec{F}_{2x} = 63.56 \text{ N (one extra digit carried)}$$

For the y-components of the force:

$$\vec{F}_{2\text{y}} = \Sigma\vec{F}_\text{y} - \vec{F}_{1\text{y}}$$
$$= (0 \text{ N}) - (120 \text{ N})\sin 14°$$
$$\vec{F}_{2\text{y}} = -29.03 \text{ N (one extra digit carried)}$$

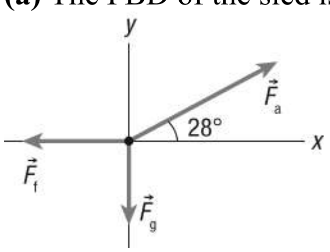
$$\left|\vec{F}_2\right| = \sqrt{(F_{2x})^2 + (F_{2y})^2}$$
$$= \sqrt{(63.56 \text{ N})^2 + (29.03 \text{ N})^2}$$
$$= 69.88 \text{ N (one extra digit carried)}$$
$$\left|\vec{F}_\text{C}\right| = 70 \text{ N}$$

$$\theta = \tan^{-1}\left(\frac{\vec{F}_{2y}}{\vec{F}_{2x}}\right)$$
$$= \tan^{-1}\left(\frac{29.03 \not{\text{N}}}{63.56 \not{\text{N}}}\right)$$
$$\theta = 25°$$

Statement: The second student exerts a force of 70 N [E 25° N].

9. Given: $\vec{F}_\text{a} = 55$ N [forward 28° up]; $\vec{F}_\text{g} = 120$ N; $\Sigma\vec{F} = 0$ N

(a) The FBD of the sled is shown below.



(b) Given: $\vec{F}_\text{a} = 55$ N [forward 28° up]; $\vec{F}_\text{g} = 120$ N; $\Sigma\vec{F} = 0$ N

Required: $\vec{F}_\text{N}$

Analysis: $\Sigma\vec{F}_y = \vec{F}_{\text{a}y} + \vec{F}_{\text{g}y} + \vec{F}_{\text{N}y}$. Choose forward and up as positive.

Solution: $\Sigma\vec{F}_y = \vec{F}_{ay} + \vec{F}_{gy} + \vec{F}_{Ny}$

$$0\ \text{N} = (55\ \text{N})\sin 28° + (-120\ \text{N}) + \vec{F}_\text{N}$$
$$\vec{F}_\text{N} = -25.82\ \text{N} + 120\text{N}$$
$$= 94.18\ \text{N}$$
$$\vec{F}_\text{N} = 94\ \text{N}$$

Statement: The normal force acting on the sled is 94 N [up]. The magnitude of the normal force is less than the magnitude of the force of gravity because the applied force has an upward component. In effect, the applied force lifts some of the weight from the surface, reducing the normal force.

(c) Given: $\vec{F}_\text{a} = 55\ \text{N}$ [forward 28° up]; $\Sigma\vec{F} = 0\ \text{N}$

Required: $\vec{F}_\text{S}$

Analysis: $\Sigma\vec{F}_x = \vec{F}_{ax} + \vec{F}_{Sx}$. Choose forward and up as positive.

Solution: $\Sigma\vec{F}_x = \vec{F}_{ax} + \vec{F}_{Sx}$

$$0\ \text{N} = (55\ \text{N})\cos 28° - \vec{F}_\text{S}$$
$$\vec{F}_\text{S} = 48.56\ \text{N}$$
$$\vec{F}_\text{S} = 49\ \text{N}$$

Statement: The force of static friction acting on the sled is 49 N [backward].

Section 2.2: Newton's Laws of Motion
Tutorial 1 Practice, pages 72–73

1. (a) Given: $m = 1.2\times10^2$ kg; $\vec{F}_1 = 1.5\times10^2$ N [N]; $\vec{F}_2 = 2.2\times10^2$ N [W]

Required: $\vec{a}$

Analysis: $\left|\Sigma\vec{F}\right| = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$; $\theta = \tan^{-1}\left(\dfrac{\Sigma F_y}{\Sigma F_x}\right)$; $\Sigma\vec{F} = m\vec{a}$. Choose east and north as positive.

Solution: For the *x*-components of the force:

$$\begin{aligned}\Sigma\vec{F}_x &= \vec{F}_{1x} + \vec{F}_{2x}\\ &= (0\text{ N}) + (-220\text{ N})\\ \Sigma\vec{F}_x &= -220\text{ N}\end{aligned}$$

For the *y*-components of the force:

$$\begin{aligned}\Sigma\vec{F}_y &= \vec{F}_{1y} + \vec{F}_{2y}\\ &= (150\text{ N}) + (0\text{ N})\\ \Sigma\vec{F}_y &= 150\text{ N}\end{aligned}$$

Construct $\Sigma\vec{F}$:

$$\begin{aligned}\left|\Sigma\vec{F}\right| &= \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}\\ &= \sqrt{(220\text{ N})^2 + (150\text{ N})^2}\\ \left|\Sigma\vec{F}\right| &= 266.3\text{ N (two extra digits carried)}\end{aligned}$$

$$\begin{aligned}\theta &= \tan^{-1}\left(\frac{\Sigma\vec{F}_y}{\Sigma\vec{F}_x}\right)\\ &= \tan^{-1}\left(\frac{150\text{ N}}{220\text{ N}}\right)\\ \theta &= 34^\circ\end{aligned}$$

Calculate $\vec{a}$:

$$\begin{aligned}\Sigma\vec{F} &= m\vec{a}\\ \vec{a} &= \frac{\Sigma\vec{F}}{m}\\ &= \frac{266.3\text{ N [W 34° N]}}{120\text{ kg}}\\ \vec{a} &= 2.2\text{ m/s}^2\text{ [W 34° N]}\end{aligned}$$

Statement: The acceleration of the mass is 2.2 m/s^2 [W 34° N] or 2.2 m/s^2 [N 56° W].

(b) Given: $m = 26$ kg; $\vec{F}_1 = 38$ N [N 24° E]; $\vec{F}_2 = 52$ N [N 36° E]

Required: $\vec{a}$

Analysis: $\left|\Sigma\vec{F}\right| = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$; $\theta = \tan^{-1}\left(\dfrac{\Sigma F_y}{\Sigma F_x}\right)$; $\Sigma\vec{F} = m\vec{a}$. Choose east and north as positive.

Solution: For the x-components of the force:

$$\begin{aligned}\Sigma\vec{F}_x &= \vec{F}_{1x} + \vec{F}_{2x} \\ &= (38\ N)\sin 24° + (52\ \text{N})\sin 36° \\ &= 15.46\ \text{N} + 30.57\ \text{N} \\ \Sigma\vec{F}_x &= 46.03\ \text{N (two extra digits carried)}\end{aligned}$$

For the y-components of the force:

$$\begin{aligned}\Sigma\vec{F}_y &= \vec{F}_{1y} + \vec{F}_{2y} \\ &= (38\ \text{N})\cos 24° + (52\ \text{N})\cos 36° \\ &= 34.72\ \text{N} + 42.07 \\ \Sigma\vec{F}_y &= 76.79\ \text{N (two extra digits carried)}\end{aligned}$$

Construct $\Sigma\vec{F}$:

$$\begin{aligned}\left|\Sigma\vec{F}\right| &= \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2} \\ &= \sqrt{(46.03\ \text{N})^2 + (76.79\ \text{N})^2} \\ \left|\Sigma\vec{F}\right| &= 89.53\ \text{N (two extra digits carried)}\end{aligned}$$

$$\begin{aligned}\theta &= \tan^{-1}\left(\frac{\Sigma\vec{F}_y}{\Sigma\vec{F}_x}\right) \\ &= \tan^{-1}\left(\frac{76.79\ \cancel{\text{N}}}{46.03\ \cancel{\text{N}}}\right) \\ \theta &= 59°\end{aligned}$$

Calculate $\vec{a}$:

$$\begin{aligned}\Sigma\vec{F} &= m\vec{a} \\ \vec{a} &= \frac{\Sigma\vec{F}}{m} \\ &= \frac{89.53\ \text{N [E 59° N]}}{26\ \text{kg}} \\ \vec{a} &= 3.4\ \text{m/s}^2\ \text{[E 59° N]}\end{aligned}$$

Statement: The acceleration of the mass is 3.4 m/s^2 [E 59° N] or 3.4 m/s^2 [N 31° E].

2. Given: $m = 65\text{ kg}$; $\vec{F}_1 = 2.2\times10^2\text{ N [E 42° N]}$; $\vec{F}_\text{f} = 1.9\times10^2\text{ N [W]}$; $\vec{a} = 2.0\text{ m/s}^2\text{ [E]}$

Required: $\vec{F}_2$

Analysis: $\Sigma\vec{F} = m\vec{a}$; $\Sigma\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_\text{f}$. Choose east and north as positive.

Solution:

Calculate the net force $\Sigma\vec{F}$:

$$\begin{aligned}\Sigma\vec{F} &= m\vec{a}\\ &= (65\text{ kg})(2.0\text{ m/s}^2)\\ \Sigma\vec{F} &= 130\text{ N [E]}\end{aligned}$$

Find $\vec{F}_2$ in terms of the other forces:

$$\begin{aligned}\Sigma\vec{F} &= \vec{F}_1 + \vec{F}_2 + \vec{F}_\text{f}\\ \vec{F}_2 &= \Sigma\vec{F} - \vec{F}_1 - \vec{F}_\text{f}\end{aligned}$$

Calculate the components of $\vec{F}_2$.

For the *x*-components of the force:

$$\begin{aligned}\vec{F}_{2x} &= \Sigma\vec{F}_x - \vec{F}_{1x} - \vec{F}_{\text{f}x}\\ &= (130\text{N}) - (220\text{ N})\cos 42° - (-190\text{ N})\\ &= 130\text{ N} - 163.5\text{ N} + 190\text{ N}\\ \vec{F}_{2x} &= 156.5\text{ N (two extra digits carried)}\end{aligned}$$

For the *y*-components of the force:

$$\begin{aligned}\vec{F}_{2y} &= \Sigma\vec{F}_y - \vec{F}_{1y} - \vec{F}_{\text{f}y}\\ &= (0\text{N}) - (-220\text{ N})\sin 42° - (0\text{ N})\\ \vec{F}_{2y} &= 147.2\text{ N (two extra digits carried)}\end{aligned}$$

Construct $\vec{F}_2$:

$$\begin{aligned}\left|\vec{F}_2\right| &= \sqrt{(F_{2x})^2 + (F_{2y})^2}\\ &= \sqrt{(156.5\text{ N})^2 + (147.2\text{ N})^2}\\ &= 214.9\text{ N}\\ \left|\vec{F}_2\right| &= 2.1\times10^2\text{ N}\end{aligned}$$

$$\begin{aligned}\theta &= \tan^{-1}\left(\frac{\vec{F}_{2y}}{\vec{F}_{2x}}\right)\\ &= \tan^{-1}\left(\frac{147.2\text{ N}}{156.5\text{ N}}\right)\\ \theta &= 43°\end{aligned}$$

Statement: The second student applies a force of 2.1×10^2 N [E 43° N] to the trunk.

3. Given: $m = 1.5 \times 10^2$ kg; $\vec{F}_g = 1.47 \times 10^3$ n [down]; $\vec{F}_1 = 1.8 \times 10^3$ N [up 30.0° left];

$\vec{F}_2 = 1.8 \times 10^3$ N [up 30.0° right]

Required: $\vec{a}$

Analysis: $\Sigma\vec{F} = m\vec{a}$. Choose right and up as positive.

Solution: For the *x*-components of the force:

$$\begin{aligned}\Sigma\vec{F}_x &= \vec{F}_{gx} + \vec{F}_{1x} + \vec{F}_{2x} \\ &= (0 \text{ N}) + (-1800 \text{ N})\sin 30.0° + (1800 \text{ N})\sin 30.0° \\ \Sigma\vec{F}_x &= 0 \text{ N}\end{aligned}$$

For the *y*-components of the force:

$$\begin{aligned}\Sigma\vec{F}_y &= \vec{F}_{gy} + \vec{F}_{1y} + \vec{F}_{2y} \\ &= (-1470 \text{ N}) + (1800 \text{ N})\cos 30.0° + (1800 \text{ N})\cos 30.0° \\ \Sigma\vec{F}_y &= 1648 \text{ N (two extra digits carried)}\end{aligned}$$

Since the *x*-component of $\Sigma\vec{F}$ is zero, $\Sigma\vec{F} = 1648$ N [up].

Calculate $\vec{a}$:

$$\begin{aligned}\Sigma\vec{F} &= m\vec{a} \\ \vec{a} &= \frac{\Sigma\vec{F}}{m} \\ &= \frac{1648 \text{ N [up]}}{150 \text{ kg}} \\ \vec{a} &= 11 \text{ m/s}^2 \text{ [up]}\end{aligned}$$

Statement: The acceleration of the beam is 11 m/s^2 [up].

Tutorial 2 Practice, page 74

1. Answers may vary. Sample answers:

(a) The rocket engine pushes down on the burning fuel ($\vec{F}_{\text{R on F}}$) while the burning fuel pushes up on the rocket ($\vec{F}_{\text{F on R}}$).

(b) The plane pushes back on the air passing through the jets ($\vec{F}_{\text{P on A}}$) while the air passing through the jets pushes the plane forward ($\vec{F}_{\text{A on P}}$).

(c) The runner pushes down on the ground ($\vec{F}_{\text{R on G}}$) while the ground pushes up on the runner's foot ($\vec{F}_{\text{G on R}}$). This last force takes the form of the normal force of the ground on the runner.

2. Given: $m = 56$ kg; $\Delta t_1 = 0.75$ s; $v_i = 0$ m/s; $\vec{v}_f = 75$ cm/s [W] $= 0.75$ m/s [W]

Required: $\vec{a}$

Analysis: $\vec{a} = \dfrac{\Delta\vec{v}}{\Delta t}$. Choose east as positive.

Solution: $\vec{a} = \dfrac{\Delta v}{\Delta t}$

$$= \frac{(-0.75 \text{ m/s}) - (0 \text{ m/s})}{0.75 \text{ s}}$$

$$= -1.0 \text{ m/s}^2$$

$$\vec{a} = 1.0 \text{ m/s}^2 \text{ [W]}$$

Statement: The magnitude of the (constant) acceleration is 1.0 m/s^2 [W].

(b) Given: $m = 56$ kg**;** $\vec{a} = 1.0 \text{ m/s}^2$ [W]

Required: $\vec{F}_{\text{S on W}}$

Analysis: $\vec{F}_{\text{S on W}} = m\vec{a}$

Solution: Force of the swimmer on the wall:

$$\vec{F}_{\text{S on W}} = m\vec{a}$$

$$= (56 \text{ kg})(1.0 \text{ m/s}^2)$$

$$\vec{F}_{\text{S on W}} = 56 \text{ N [E]}$$

Statement: The force exerted by the swimmer on the wall is 56 N [E].

(c) Given: $\vec{F}_{\text{S on W}} = 56$ N [E]

Required: $\vec{F}_{\text{W on S}}$

Analysis: $\vec{F}_{\text{W on S}} = m\vec{a}$. The force of the swimmer on the wall, $\vec{F}_{\text{S on W}}$ **,** is the action–reaction partner to the force of the wall on the swimmer, $\vec{F}_{\text{W on S}}$. It is the force of the wall on the swimmer that causes the swimmer's acceleration.

Solution: The force of the wall on the swimmer:

$$\vec{F}_{\text{W on S}} = -\vec{F}_{\text{S on W}}$$

$$= -56 \text{ N [E]}$$

$$\vec{F}_{\text{W on S}} = 56 \text{ N [W]}$$

Statement: The force exerted by the wall on the swimmer is 56 N [W].

(d) Given: $\vec{a} = 1.0 \text{ m/s}^2$ [W]; $\Delta t_2 = 1.50 \text{ s} - 0.75 \text{ s} = 0.75 \text{ s}$

Required: $\Delta\vec{d} = \Delta\vec{d}_1 + \Delta\vec{d}_2$

Analysis: $\Delta d_1 = v_{\text{i}}\Delta t + \dfrac{1}{2}a\left(\Delta t\right)^2$; $\Delta d_2 = v_{\text{f}}\Delta t_2$.

Solution: The first distance covered is Δd_1:

$$\Delta d_1 = v_{\text{i}}\Delta t + \frac{1}{2}\overset{\text{r}}{a}\left(\Delta t\right)^2$$

$$\Delta d_1 = (0 \text{ m/s})(0.75 \text{ s}) + \frac{1}{2}(1.0 \text{ m/s}^2)\left(0.75 \text{ s}\right)^2$$

$$\Delta d_1 = 0.2813 \text{ m (two extra digits carried)}$$

The second distance covered is Δd_2:

$$\Delta d_2 = v_f \Delta t_2$$
$$= (0.75 \text{ m/s})(0.75 \text{ s})$$
$$\Delta d_2 = 0.5625 \text{ m (two extra digits carried)}$$

The total distance covered is:

$$\Delta \vec{d} = \Delta \vec{d}_1 + \Delta \vec{d}_2$$
$$= 0.2813 \text{ m} + 0.5625 \text{ m}$$
$$\Delta \vec{d} = 0.84 \text{ m}$$

Statement: Both parts of the swimmer's motion were away from the wall. The total displacement is 84 cm [W].

3. (a) Given: $m_{\text{boy}} = 32.5 \text{ kg}$; $m_{\text{mattress}} = 2.50 \text{ kg}$; $\Sigma\vec{F}_{\text{boy}} = 0 \text{ N}$; $\Sigma\vec{F}_{\text{mattress}} = 0 \text{ N}$

Required: $\vec{F}_{\text{W on M}}$

Analysis: $\vec{F}_{\text{B on M}} + \vec{F}_{\text{W on M}} + \vec{F}_g = 0 \text{ N}$

Solution: Equation for upward force of the water on the mattress:

$$\vec{F}_{\text{B on M}} + \vec{F}_{\text{W on M}} + \vec{F}_g = 0 \text{ N}$$
$$-\left|\vec{F}_{\text{B on M}}\right| + \left|\vec{F}_{\text{W on M}}\right| - \left|\vec{F}_g\right| = 0 \text{ N}$$
$$\left|\vec{F}_{\text{W on M}}\right| = \left|\vec{F}_{\text{B on M}}\right| + \left|\vec{F}_g\right|$$
$$= (32.5 \text{ kg})(9.8 \text{ m/s}^2) + (2.50 \text{ kg})(9.8 \text{ m/s}^2)$$
$$= 318.5 \text{ N} + 24.5 \text{ N}$$
$$\left|\vec{F}_{\text{W on M}}\right| = 3.4 \times 10^2 \text{ N}$$

Statement: The upward force of the water on the mattress is 3.4×10^2 N.

(b) Given: $m_{\text{boy}} = 32.5 \text{ kg}$; $m_{\text{mattress}} = 2.50 \text{ kg}$; $\Sigma\vec{F}_{\text{boy}} = 0 \text{ N}$; $\Sigma\vec{F}_{\text{mattress}} = 0 \text{ N}$

Required: $\vec{F}_{\text{B on M}}$

Analysis: $\vec{F}_{\text{B on M}} = -\vec{F}_{\text{M on B}}$. Choose up as positive.

Solution:

Since $\vec{F}_{\text{B on M}} = -\vec{F}_{\text{M on B}}$; solve by determining the upward force of the mattress on the boy, $\vec{F}_{\text{M on B}}$.

$$\vec{F}_{\text{M on B}} + \vec{F}_g = 0 \text{ N}$$
$$\left|\vec{F}_{\text{M on B}}\right| = \left|\vec{F}_g\right|$$
$$= (32.5 \text{ kg})(9.8 \text{ m/s}^2)$$
$$= 318.5 \text{ N (two extra digits carried)}$$
$$\left|\vec{F}_{\text{M on B}}\right| = 3.2 \times 10^2 \text{ N}$$

Statement: The force that the boy exerts on the mattress, $\vec{F}_{\text{B on M}}$, is 3.2×10^2 N.

(c) Since $\vec{F}_{\text{B on M}} = -\vec{F}_{\text{M on B}}$, the upward force of the mattress on the boy is 3.2×10^2 N.

4. Given: $m_\text{P} = 0.20\text{ kg}$; $\vec{a}_\text{P} = 25\text{ m/s}^2$ [forward]; $\vec{a}_\text{L} = 0.25\text{ m/s}^2$ [backward]

Required: m_L

Analysis: $\vec{F}_\text{P on L} = -\vec{F}_\text{L on P}$. Choose forward as positive.

Solution: $\vec{F}_\text{P on L} = -\vec{F}_\text{L on P}$

$$m_\text{L}\vec{a}_\text{L} = -m_\text{P}\vec{a}_\text{P}$$

$$m_\text{L} = \frac{-m_\text{P}\vec{a}_\text{P}}{\vec{a}_\text{L}}$$

$$m_\text{L} = \frac{-(0.20\text{ kg})(25\ \cancel{\text{m/s}^2})}{-0.25\ \cancel{\text{m/s}^2}}$$

$$m_\text{L} = 20\text{ kg}$$

Statement: The mass of the launcher is 20 kg.

Section 2.2 Questions, page 76

1. According to Newton's first law of motion, when the snowboarder suddenly encounters the rough patch on the hill, her body will continue in a forward motion, but her snowboard may stick instead of slide.

2. As you are sitting in the bus tossing the ball vertically, both you and the ball are acted upon by the forward motion of the bus. By Newton's first law of motion, both you and the ball continue in a forward direction. From your point of view, the ball remains directly in front of you and will not hit you in the face unless a horizontal force is exerted.

3. (a) Given: $v_\text{i} = 4.2\text{ m/s}$ [E]; $v_\text{f} = 0\text{ m/s}$; $m = 41\text{ kg}$; $\vec{F}_\text{f} = 25\text{ N}$ [W]

Required: $\vec{a}$

Analysis: $\Sigma\vec{F} = m\vec{a}$

Solution: $\Sigma\vec{F} = m\vec{a}$

$$\vec{a} = \frac{\Sigma\vec{F}}{m}$$

$$\vec{a} = \frac{25\text{ N [W]}}{41\text{ kg}}$$

$$= 0.6098\text{ m/s}^2\text{ [W] (two extra digits carried)}$$

$$\vec{a} = 0.61\text{ m/s}^2\text{ [W]}$$

Statement: The child's acceleration across the ice is 0.61 m/s^2 [W].

(b) Given: $v_\text{i} = 4.2\text{ m/s}$ [E]; $\vec{a} = 0.61\text{ m/s}^2$ [W]

Required: Δt

Analysis: $\vec{a} = \dfrac{\Delta\vec{v}}{\Delta t}$. Choose east as positive.

Solution: $a = \dfrac{\Delta v}{\Delta t}$

$$\Delta t = \frac{\Delta v}{a}$$

$$= \frac{0 \text{ m/s} - 4.2 \text{ m/s}}{-0.6098 \text{ m/s}^2}$$

$$\Delta t = 6.9 \text{ s}$$

Statement: It will take the child 6.9 s to stop.

4. Given: $|\vec{a}| = a = 12 \text{ m/s}^2$; $|\vec{F}| = F = 2.2 \times 10^2 \text{ N}$

Required: m

Analysis: $\vec{F} = m\vec{a}$

Solution: $\vec{F} = m\vec{a}$

$$m = \frac{\vec{F}}{\vec{a}}$$

$$= \frac{2.2 \times 10^2 \text{ N}}{12 \text{ m/s}^2}$$

$$m = 18 \text{ kg}$$

Statement: The mass of the object is 18 kg.

5. (a) Given: $v_i = 0$ m/s; $v_f = 2.5$ m/s [forward]; $\Delta t_1 = 1.0 \text{ min} = 60 \text{ s}$; $m = 1.2 \times 10^3$ kg

Required: $\vec{F}_N$

Analysis: $\vec{F} = m\vec{a}$. Choose forward as positive.

Solution: $\vec{a} = \dfrac{\Delta v}{\Delta t}$

$$= \frac{2.5 \text{ m/s} - 0 \text{ m/s}}{60 \text{ s}}$$

$$\vec{a} = 0.0417 \text{ m/s}^2 \text{ (two extra digits carried)}$$

$$\vec{F}_N = m\vec{a}$$

$$= (1.2 \times 10^3 \text{ kg})(0.0417 \text{ m/s})$$

$$\vec{F}_N = 50 \text{ N}$$

Statement: The normal force between the two bumpers is 50 N.

(b) Given: $v_2 = v_f = 2.5$ m/s; $\Delta d = 2.0 \text{ km} = 2000 \text{ m}$

Required: $\Delta t = \Delta t_1 + \Delta t_2$

Analysis: $\Delta d = \Delta d_1 + \Delta d_2$

Solution: $\Delta d_1 = \left(\dfrac{v_i + v_f}{2}\right)\Delta t$

$$= \left(\frac{0\text{ m/s} + 2.5\text{ m/s}}{2}\right)(60\text{ s})$$

$$= \frac{(2.5\text{ m/}\cancel{\text{s}})}{2}(60\cancel{\text{s}})$$

$$\Delta d_1 = 75\text{ m}$$

$$\Delta d = \Delta d_1 + \Delta d_2$$

$$\Delta d_2 = \Delta d - \Delta d_1$$

$$= 2000\text{ m} - 75\text{ m}$$

$$\Delta d_2 = 1925\text{ m}$$

The second time interval is:

$$v_2 = \frac{\Delta d_2}{\Delta t_2}$$

$$\Delta t_2 = \frac{\Delta d_2}{v_2}$$

$$= \frac{1925\ \cancel{\text{m}}}{2.5\ \cancel{\text{m}}\text{/s}}$$

$$= 770\text{ s}$$

$$\Delta t_2 = 7.7 \times 10^2\text{ s}$$

Statement: It takes 7.7×10^2 s to reach the repair shop.

6. Given: $m = 250$ kg; $\vec{F}_1 = 150$ N [E]; $\vec{F}_2 = 350$ N [S 45° W]

Required: $\vec{a}$

Analysis: $\Sigma\vec{F} = m\vec{a}$. Choose east and north as positive.

Solution: For the x-components of the force:

$$\Sigma\vec{F}_x = \vec{F}_{1x} + \vec{F}_{2x}$$

$$= (150\text{ N}) + (-350\text{ N})\sin 45°$$

$$= 150\text{ N} - 247.49\text{ N}$$

$$\Sigma\vec{F}_x = -97.49\text{ N (one extra digit carried)}$$

For the y-components of the force:

$$\Sigma\vec{F}_y = \vec{F}_{1y} + \vec{F}_{2y}$$

$$= (0\text{ N}) + (-350\text{ N})\cos 45°$$

$$\Sigma\vec{F}_y = -247.49\text{ N (two extra digits carried)}$$

Construct $\Sigma\vec{F}$:

$$\left|\Sigma\vec{F}\right| = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$$
$$= \sqrt{(97.49\text{ N})^2 + (247.49\text{ N})^2}$$
$$\left|\Sigma\vec{F}\right| = 266\text{ N}$$

$$\theta = \tan^{-1}\left(\frac{\Sigma F_y}{\Sigma F_x}\right)$$
$$= \tan^{-1}\left(\frac{247.49\ \cancel{\text{N}}}{97.49\ \cancel{\text{N}}}\right)$$
$$\theta = 68°$$

Calculate $\vec{a}$:

$$\Sigma\vec{F} = m\vec{a}$$
$$\vec{a} = \frac{\Sigma\vec{F}}{m}$$
$$= \frac{266\text{ N [W 68° S]}}{250\text{ kg}}$$
$$\vec{a} = 1.1\text{ m/s}^2\text{ [W 68° S]}$$

Statement: The acceleration of the mass is 1.1 m/s^2 [W 68° S] or 1.1 m/s^2 [S 22° W].

7. Answers may vary. Sample answers:

(a) When a tennis racquet hits a tennis ball, exerting a force on the ball, the tennis racquet pushes forward on the tennis ball and the tennis ball pushes back on the racquet.

(b) When a car is moving at high speed and runs into a tree, exerting a force on the tree, the car pushes forward on the tree and the tree pushes back on the car.

(c) When two cars are moving in opposite directions and collide head-on, the first car pushes the second car in the direction of the first car's initial velocity. The second car pushes the first car in the opposite direction.

(d) When a person leans on a wall, exerting a force on the wall, the person pushes forward on the wall and the wall pushes back on the person.

(e) When a mass hangs by a string attached to the ceiling, and the string exerts a force on the mass, the mass pulls down on the string and the string pulls up on the mass.

(f) When a bird sits on a telephone pole, exerting a force on the pole, the bird pushes down on the telephone pole and the telephone pole pushes up on the bird.

8. Given: $m_1 = m_2 = 5.2$ kg

Required: $\vec{F}_1$; $\vec{F}_2$

Analysis: $\Sigma\vec{F} = 0$ N . Choose right and up as positive.

Solution: $\Sigma\vec{F} = 0 \text{ N}$

$$\vec{F}_1 - m_1 g = 0 \text{ N}$$
$$\vec{F}_1 = (5.2 \text{ kg})(9.8 \text{ m/s}^2)$$
$$\vec{F}_1 = 51 \text{ N}$$

Equation for second mass:

$$\Sigma\vec{F} = 0 \text{ N}$$
$$\vec{F}_2 - m_2 g = 0 \text{ N}$$
$$\vec{F}_2 = (5.2 \text{ kg})(9.8 \text{ m/s}^2)$$
$$\vec{F}_2 = 51 \text{ N}$$

Statement: The tension is each string is 51 N.
(b) The spring scale reads 51 N, the string tension on the left side that is pulling the hook.
(c) The answers would remain the same if you removed one mass and held everything in place. A weight of 51 N would be exerted on the remaining mass, balanced by a string tension of 51 N. The spring scale is at rest so the other string also would also have a tension of 51 N. You have to pull down with a force of 51 N, effectively replacing the weight of the mass you removed.

9. Given: $m = 62 \text{ kg}$; $\vec{F}_{\text{ground}} = 1.1 \times 10^3 \text{ N [backward 55° up]}$

Required: $\vec{a}$

Analysis: $\Sigma\vec{F} = m\vec{a}$. Choose forward and up as positive.

Solution:

For the x-components of the force:

$$\begin{aligned} \Sigma\vec{F}_x &= \vec{F}_{\text{earth } x} + \vec{F}_{gx} \\ &= (-1100 \text{ N})\cos 55° + (0 \text{ N}) \\ &= 150 \text{ N} - 247.49 \text{ N} \\ \Sigma\vec{F}_x &= -630.9 \text{ N (two extra digits carried)} \end{aligned}$$

For the y-components of the force:

$$\begin{aligned} \Sigma\vec{F}_y &= \vec{F}_{\text{ground } y} + \vec{F}_{gy} \\ &= (1100)\sin 55° - (62 \text{ kg})(9.8 \text{ m/s}^2) \\ &= 901.1 \text{ N} - 607.6 \text{ N} \\ \Sigma\vec{F}_y &= 293.5 \text{ N (two extra digits carried)} \end{aligned}$$

$$\begin{aligned} \left|\Sigma\vec{F}\right| &= \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2} \\ &= \sqrt{(630.9 \text{ N})^2 + (293.5 \text{ N})^2} \\ \left|\Sigma\vec{F}\right| &= 695.8 \text{ N (two extra digits carried)} \end{aligned}$$

$$\theta = \tan^{-1}\left(\frac{\Sigma F_y}{\Sigma F_x}\right)$$
$$= \tan^{-1}\left(\frac{293.5\ \cancel{\text{N}}}{630.9\ \cancel{\text{N}}}\right)$$
$$\theta = 25°$$

$$\Sigma\vec{F} = m\vec{a}$$
$$\vec{a} = \frac{\Sigma\vec{F}}{m}$$
$$= \frac{695.8\ \text{N [backward 25° up]}}{62\ \text{kg}}$$
$$\vec{a} = 11\ \text{m/s}^2\ \text{[backward 25° up]}$$

Statement: The acceleration of the athlete is 11 m/s^2 [backward 25° up].

Section 2.3: Applying Newton's Laws of Motion
Tutorial 1 Practice, page 79

1. Given: $\overrightarrow{F}_{\text{gB}} = 2.8\text{ N}$; $\overrightarrow{F}_{\text{gA}} = 6.5\text{ N}$; $\overrightarrow{F}_{\text{f}} = 1.4\text{ N}$

(a) Required: F_1

Analysis: $\Sigma\vec{F} = 0\text{ N}$

Solution: The FBD for block B is shown below.

Equation for block B:

$$\begin{aligned}\Sigma\vec{F} &= 0\text{ N}\\ \vec{F}_1 - \vec{F}_{\text{gB}} &= 0\text{ N}\\ \vec{F}_1 &= \vec{F}_{\text{gB}}\\ \vec{F}_1 &= 2.8\text{ N}\end{aligned}$$

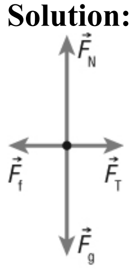
Statement: The tension in the vertical rope is 2.8 N.

(b) Given: $\vec{F}_{\text{gA}} = 6.5\text{ N}$; $\vec{F}_{\text{f}} = 1.4\text{ N}$

Required: $\vec{F}_2$; $\vec{F}_\text{N}$

Analysis: $\Sigma\vec{F} = 0\text{ N}$

Solution: The FBD for block A is shown below.



Equations for block A:

$$\begin{aligned}\Sigma\vec{F}_x &= 0\text{ N}\\ \vec{F}_2 - \vec{F}_\text{f} &= 0\text{ N}\\ \vec{F}_2 &= \vec{F}_\text{f}\\ \vec{F}_2 &= 1.4\text{ N}\end{aligned}$$

$$\begin{aligned}\Sigma\vec{F} &= 0\text{ N}\\ \vec{F}_\text{N} - \vec{F}_{\text{gA}} &= 0\text{ N}\\ \vec{F}_\text{N} &= \vec{F}_{\text{gA}}\\ \vec{F}_\text{N} &= 6.5\text{ N}\end{aligned}$$

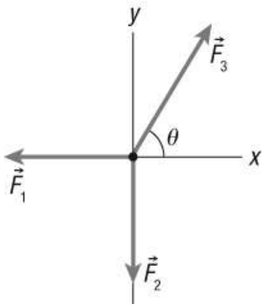
Statement: The tension in the horizontal rope is 1.4 N. The normal force acting on block A is 6.5 N.

(c) Given: $\vec{F}_2 = 1.4$ N

Required: $\vec{F}_3$

Analysis: $\Sigma\vec{F} = 0$ N

Solution: The FBD for point P is shown below.



Equations for point P.

For the *x*-components of the force:

$$\begin{aligned}\Sigma\vec{F}_x &= 0 \text{ N}\\ \vec{F}_{3x} - \vec{F}_2 &= 0 \text{ N}\\ \vec{F}_{3x} &= \vec{F}_2\\ \vec{F}_{3x} &= 1.4 \text{ N}\end{aligned}$$

For the *y*-components of the force:

$$\begin{aligned}\Sigma\vec{F} &= 0 \text{ N}\\ \vec{F}_{3y} - \vec{F}_1 &= 0 \text{ N}\\ \vec{F}_{3y} &= \vec{F}_1\\ \vec{F}_{3y} &= 2.8 \text{ N}\end{aligned}$$

Construct the vector $\vec{F}_3$ from its components:

$$\begin{aligned}\left|\vec{F}_3\right| &= \sqrt{(\vec{F}_{3x})^2 + (\vec{F}_{3y})^2}\\ &= \sqrt{(1.4 \text{ N})^2 + (2.8 \text{ N})^2}\\ \left|\vec{F}_3\right| &= 3.1 \text{ N}\end{aligned}$$

$$\begin{aligned}\theta &= \tan^{-1}\left(\frac{\vec{F}_{3y}}{\vec{F}_{3x}}\right)\\ &= \tan^{-1}\left(\frac{2.8 \cancel{\text{N}}}{1.4 \cancel{\text{N}}}\right)\\ \theta &= 63°\end{aligned}$$

Statement: The tension in the third rope is 3.1 N [right 63° up].

2. Given: $m = 62\text{ kg}$; $\vec{F}_\text{T} = 7.1\times10^2\text{ N [right 32° up]}$

Required: $\vec{F}_\text{W}$

Analysis: $\Sigma\vec{F} = 0\text{ N}$

Balance the *x*-components of the forces:

$$\begin{aligned}
\Sigma\vec{F}_x &= 0\text{ N}\\
\vec{F}_{\text{W}x} + \vec{F}_\text{T}\cos\theta &= 0\text{ N}\\
\vec{F}_{\text{W}x} &= -\vec{F}_\text{T}\cos\theta\\
&= -(710\text{ N})\cos 32°\\
\vec{F}_{\text{W}x} &= -602.1\text{ N (two extra digits carried)}
\end{aligned}$$

Balance the *y*-components of the forces:

$$\begin{aligned}
\Sigma\vec{F}_y &= 0\text{ N}\\
\vec{F}_{\text{W}y} + \vec{F}_{\text{T}y} - \vec{F}_\text{g} &= 0\text{ N}\\
\vec{F}_{\text{W}y} &= -\vec{F}_\text{T}\sin\theta + mg\\
\vec{F}_{\text{W}y} &= -(710\text{ N})\sin 32° + (62\text{ kg})(9.8\text{m/s}^2)\\
&= -376.2\text{ N} + 607.6\text{ N}\\
\vec{F}_{\text{W}y} &= 231.4\text{ N (two extra digits carried)}
\end{aligned}$$

Construct the vector $\vec{F}_\text{W}$ from its components:

$$\begin{aligned}
\left|\vec{F}_\text{W}\right| &= \sqrt{(\vec{F}_{\text{W}x})^2 + (\vec{F}_{\text{W}y})^2}\\
&= \sqrt{(602.1\text{ N})^2 + (231.4\text{ N})^2}\text{ (two extra digits carried)}\\
\left|\vec{F}_\text{W}\right| &= 6.5\times10^2\text{ N}
\end{aligned}$$

$$\begin{aligned}
\theta &= \tan^{-1}\left(\frac{F_{\text{W}y}}{F_{\text{W}x}}\right)\\
&= \tan^{-1}\left(\frac{231.4\ \cancel{\text{N}}}{602.1\ \cancel{\text{N}}}\right)\\
\theta &= 21°
\end{aligned}$$

Statement: The force exerted by the wall on the climber's feet is 6.5×10^2 N [left 21° up].

3. Given: $\vec{F}_1 = 60.0\text{ N [E 30.0° S]}$; $\vec{F}_2 = 50.0\text{ N [E 60.0° N]}$; $\Sigma\vec{F} = 0\text{ N}$

Required: $\vec{F}_3$

Analysis: $\Sigma\vec{F} = 0\text{ N}$; $\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 0\text{ N}$. Choose east and north as positive.

Solution: For the *x*-components of the force:

$$\vec{F}_{1x} + \vec{F}_{2x} + \vec{F}_{3x} = 0\text{N}$$

$$\begin{aligned}\vec{F}_{3x} &= -(60.0\text{ N})\cos 30.0° - (50.0\text{ N})\cos 60.0° \\ &= -51.96\text{ N} - 25.0\text{ N} \\ \vec{F}_{3x} &= -76.96\text{ N (two extra digits carried)}\end{aligned}$$

For the *y*-components of the force:

$$\vec{F}_{1y} + \vec{F}_{2y} + \vec{F}_{3y} = 0\text{ N}$$

$$\begin{aligned}\vec{F}_{3y} &= -(-60.0\text{ N})\sin 30.0° - (50.0\text{ N})\sin 60.0° \\ &= 30.0\text{ N} - 43.30\text{ N} \\ \vec{F}_{3y} &= -13.30\text{ N (two extra digits carried)}\end{aligned}$$

Construct the vector $\vec{F}_3$ from its components:

$$\begin{aligned}\left|\vec{F}_3\right| &= \sqrt{(\vec{F}_{3x})^2 + (\vec{F}_{3y})^2} \\ &= \sqrt{(76.96\text{ N})^2 + (13.30\text{ N})^2} \\ \left|\vec{F}_3\right| &= 78\text{ N}\end{aligned}$$

$$\begin{aligned}\theta_3 &= \tan^{-1}\left(\frac{\vec{F}_{3y}}{\vec{F}_{3x}}\right) \\ &= \tan^{-1}\left(\frac{13.30\ \cancel{\text{N}}}{76.96\ \cancel{\text{N}}}\right) \\ \theta_3 &= 9.8°\end{aligned}$$

Statement: The magnitude of the force is 78 N, at an angle [W 9.8° S].

Tutorial 2 Practice, pages 81–82

1. (a) Solutions may vary. Sample solution:

Given: $m_1 = 1.2\text{ kg}; m_2 = 1.8\text{ kg};\ \vec{a} = 1.2\text{ m/s}^2\text{ [up]}$

Required: $\vec{F}_1$; $\vec{F}_2$

Analysis: $\Sigma\vec{F} = m\vec{a}$

Solution: Equation for top block (mass m_1):

$$\Sigma\vec{F} = m_1\vec{a}$$
$$\vec{F}_1 - \vec{F}_2 - m_1 g = m_1\vec{a}$$
$$\vec{F}_1 = \vec{F}_2 + m_1 g + m_1\vec{a}$$
$$\vec{F}_1 = \vec{F}_2 + m_1(g + \vec{a}) \text{ (Equation 1)}$$

Equation for bottom block (mass m_2):

$$\Sigma\vec{F} = m_2\vec{a}$$
$$\vec{F}_2 - m_2 g = m_2\vec{a}$$
$$\vec{F}_2 = m_2 g + m_2\vec{a}$$
$$= m_2(g + \vec{a}) \text{ (Equation 2)}$$
$$= (1.8 \text{ kg})(9.8 \text{ m/s}^2 + 1.2 \text{ m/s}^2)$$
$$\vec{F}_2 = 20 \text{ N}$$

To calculate $\vec{F}_1$, substitute Equation 2 into Equation 1:

$$\vec{F}_1 = \vec{F}_2 + m_1(g + \vec{a})$$
$$\vec{F}_1 = m_2(g + \vec{a}) + m_1(g + \vec{a})$$
$$= (m_2 + m_1)(g + \vec{a}) \text{ (Equation 3)}$$
$$= (3.0 \text{ kg})(11.0 \text{ m/s}^2)$$
$$\vec{F}_1 = 33 \text{ N}$$

Statement: The tension in the top string is 33 N, and the tension in the bottom string is 20 N.

(b) Given: $m_1 = 1.2$ kg; $m_2 = 1.8$ kg; maximum string tension is 38 N

Required: maximum $\vec{a}$ that will not break the string

Analysis: $(m_2 + m_1)(g + a) \le 38$ N

Solution: $(m_2 + m_1)(g + \vec{a}) \le 38$ N

$$g + \vec{a} \le \frac{38 \text{ N}}{m_2 + m_1}$$
$$\vec{a} \le \frac{38 \text{ N}}{3.0 \text{ kg}} - g$$
$$\vec{a} \le 12.67 \text{ m/s}^2 - 9.8 \text{ m/s}^2$$
$$\vec{a} \le 2.9 \text{ m/s}^2$$

Statement: The maximum acceleration of the elevator that will not break the strings is 2.9 m/s^2 [up].

2. (a) Given: $m = 63\text{ kg}$; $\vec{F}_\text{f} = 0\text{ N}$; $\theta = 14°$ [above the horizontal]

Required: $\vec{F}_\text{N}$

Analysis: $\Sigma\vec{F}_y = 0\text{ N}$

Solution:

$$\begin{aligned}
\Sigma\vec{F}_y &= 0\text{ N} \\
\vec{F}_\text{N} - \vec{F}_{\text{g}y} &= 0\text{ N} \\
\vec{F}_\text{N} &= mg\cos\theta \\
&= (63\text{ kg})(9.8\text{ m/s}^2)\cos 14° \\
\vec{F}_\text{N} &= 6.0\times10^2\text{ N}
\end{aligned}$$

Statement: The magnitude of the normal force on the skier is $6.0\times10^2\text{ N}$.

(b) Given: θ=14° [above the horizontal]; g = 9.8 m/s^2

Required: $\vec{a} = |\vec{a}|$

Analysis: $\Sigma\vec{F}_x = m\vec{a}$. Choose +$x$-direction as the direction of acceleration, parallel to the hillside.

Solution:

$$\begin{aligned}
\Sigma\vec{F}_x &= m\vec{a} \\
\vec{F}_{\text{g}x} &= m\vec{a} \\
\vec{a} &= \frac{\vec{F}_{\text{g}x}}{m} \\
&= \frac{\cancel{m}g\sin\theta}{\cancel{m}} \\
&= (9.8\text{ m/s}^2)\sin 14° \\
\vec{a} &= 2.4\text{ m/s}^2
\end{aligned}$$

Statement: The magnitude of the skier's acceleration is 2.4 m/s^2.

3. Given: $\vec{a} = 1.9\text{ m/s}^2$ [down hill]; $\vec{F}_\text{f} = 0\text{ N}$

Required: θ

Analysis: $\vec{a} = g\sin\theta$

Solution:

$$\begin{aligned}
\vec{a} &= g\sin\theta \\
\theta &= \sin^{-1}\left(\frac{\vec{a}}{g}\right) \\
&= \sin^{-1}\left(\frac{1.9\ \cancel{\text{m/s}^2}}{9.8\ \cancel{\text{m/s}^2}}\right) \\
\theta &= 11°
\end{aligned}$$

Statement: The angle between the hill and the horizontal is 11°.

4. (a) Given: $\vec{F}_{\text{a}} = 82$ N [right 17° up]; $\vec{F}_{\text{N}} = 213$ N; $\vec{a} = 0.15$ m/s^2 [right]

Required: m

Analysis: $\Sigma\vec{F}_y = 0$ N. Choose right and up as positive.

Solution: For the y-components of the forces:

$$\Sigma\vec{F}_{\text{y}} = 0 \text{ N}$$

$$\vec{F}_{\text{N}} + \vec{F}_{\text{a}}\sin\theta - mg = 0 \text{ N}$$

$$m = \frac{\vec{F}_{\text{N}} + \vec{F}_{\text{a}}\sin\theta}{g}$$

$$= \frac{213 \text{ N} + (82 \text{ N})\sin 17°}{9.8 \text{ m/s}^2}$$

$$m = 24.18 \text{ kg} \text{ (two extra digits carried)}$$

Statement: The mass of the desk is 24 kg.

(b) Given: $\vec{F}_{\text{a}} = 82$ N [right 17° up]; $\vec{F}_{\text{N}} = 213$ N; $\vec{a} = 0.15$ m/s^2 [right]

Required: $\vec{F}_{\text{f}}$

Analysis: $\Sigma\vec{F}_x = m\vec{a}$

Solution:

$$\Sigma\vec{F}_x = m\vec{a}$$

$$\vec{F}_{\text{a}x} - \vec{F}_{\text{f}} = m\vec{a}$$

$$\vec{F}_{\text{f}} = \vec{F}_{\text{a}x} - m\vec{a}$$

$$= (82 \text{ N})\cos 17° - (24.18 \text{ kg})(0.15 \text{ m/s}^2)$$

$$\vec{F}_{\text{f}} = 75 \text{ N}$$

Statement: The magnitude of the friction force on the desk is 75 N.

5. (a) Given: $m_1 = 9.1$ kg; $m_2 = 12$ kg; $m_3 = 8.7$ kg; $\vec{F}_3 = 29$ N [right 23° up]

Required: $\vec{a}$

Analysis: $\Sigma F_x = ma$. Choose right and up as positive.

Solution: For the x-components of the force:

$$\Sigma\vec{F}_x = m_{\text{T}}\vec{a}$$

$$\vec{F}_{3x} = m_{\text{T}}\vec{a}$$

$$\vec{a} = \frac{\vec{F}_3\cos\theta}{m_{\text{T}}}$$

$$= \frac{(29 \text{ N})\cos 23°}{29.8 \text{ kg}}$$

$$\vec{a} = 0.8958 \text{ m/s}^2 \text{ (two extra digits carried)}$$

Statement: The carts accelerate at 0.90 m/s^2 to the right.

(b) Given: $m_3 = 8.7\text{ kg}$; $\vec{a} = 0.8958\text{ m/s}^2$

Required: $\vec{F}_1$

Analysis: $\Sigma\vec{F}_x = m\vec{a}$

Solution: For the *x*-components of the force:

$$\Sigma\vec{F}_x = m_3\vec{a}$$
$$\vec{F}_1 = m_3\vec{a}$$
$$= (8.7\text{ kg})(0.8958\text{ m/s}^2)$$
$$= 7.793\text{ N (two extra digits carried)}$$
$$\vec{F}_1 = 7.8\text{ N}$$

Statement: The tension in the cord between m_3 and m_2 is 7.8 N.

(c) Given: $\vec{a} = 0.90\text{ m/s}^2$; $\vec{F}_1 = 7.793\text{ N}$

Required: $\vec{F}_2$

Analysis: $\Sigma\vec{F}_x = m\vec{a}$

Solution: Using the *x*-components of the force:

$$\Sigma\vec{F}_x = m_2\vec{a}$$
$$-\vec{F}_1 + \vec{F}_2 = m_2\vec{a}$$
$$\vec{F}_2 = \vec{F}_1 + (12\text{ kg})(0.8958\text{ m/s}^2)$$
$$= 7.793\text{ N} + 10.75\text{ N}$$
$$\vec{F}_2 = 19\text{ N}$$

Statement: The tension in the cord between m_2 and m_1 is 19 N.

6. (a) Given: $m_\text{A} = 4.2\text{ kg}; m_\text{B} = 1.8\text{ kg}; \theta = 32°$

Required: $\vec{a}$

Analysis: $\Sigma\vec{F} = m\vec{a}$

Solution: Equation for block A:

$$\Sigma\vec{F}_y = m_\text{A}\vec{a}_y$$
$$\vec{F}_\text{gA} - \vec{F}_\text{T} = m_\text{A}\vec{a}$$
$$m_\text{A}g - \vec{F}_\text{T} = m_\text{A}\vec{a} \quad \text{(Equation 1)}$$

Equation for block B:

$$\Sigma\vec{F}_x = m_\text{B}\vec{a}_x$$
$$\vec{F}_\text{T} + \vec{F}_{\text{gB}x} = m_\text{B}\vec{a}$$
$$\vec{F}_\text{T} - m_\text{B}g\sin\theta = m_\text{A}\vec{a} \quad \text{(Equation 2)}$$

Solve for acceleration by adding equations (1) and (2):

$$(m_A g - \vec{F}_T) + (\vec{F}_T - m_B g \sin\theta) = m_A \vec{a} + m_B \vec{a}$$

$$(m_A - m_B \sin\theta) g = (m_A + m_B)\vec{a}$$

$$\vec{a} = \frac{(m_A - m_B \sin\theta) g}{m_A + m_B}$$

$$= \frac{(4.2 \text{ kg} - (1.8 \text{ kg})\sin 32°)(9.8 \text{ m/s}^2)}{4.2 \text{ kg} + 1.8 \text{ kg}}$$

$$= 5.302 \text{ m/s}^2 \text{ (two extra digits carried)}$$

$$\vec{a} = 5.3 \text{ m/s}^2$$

Statement: The blocks accelerate at 5.3 m/s^2.

(b) Given: $m_A = 4.2$ kg; $m_B = 1.8$ kg; $\theta = 32°$; $\vec{a} = 5.302$ m/s^2

Required: tension in the string, F_T

Analysis: We can substitute the value of acceleration into either of the equations from part (a) to solve for F_T. We will use Equation (1) because it is a bit simpler.

Solution: $m_A g - \vec{F}_T = m_A \vec{a}$

$$\vec{F}_T = m_A g + m_A \vec{a}$$

$$= m_A (g + \vec{a})$$

$$= (4.2 \text{ kg})(9.8 \text{ m/s}^2 + 5.302 \text{ m/s}^2)$$

$$\vec{F}_T = 19 \text{ N}$$

Statement: The tension in the string is 19 N.

Section 2.3 Questions, page 83

1. Given: $\vec{F}_1 = 30$ N [E 30° N]; $\vec{F}_2 = 40$ N [E 50° S]

Required: $\Sigma\vec{F}$

Analysis: $\left|\Sigma\vec{F}\right| = \sqrt{(\Sigma\vec{F}_x)^2 + (\Sigma\vec{F}_y)^2}$; $\theta = \tan^{-1}\left(\frac{\Sigma\vec{F}_y}{\Sigma\vec{F}_x}\right)$. Choose east and north as positive.

Solution: For the x-components of the force:

$$\Sigma\vec{F}_x = \vec{F}_{1x} + \vec{F}_{2x}$$

$$= (30 \text{ N})\cos 30° + (40 \text{ N})\cos 50°$$

$$= 25.98 \text{ N} + 25.71 \text{ N}$$

$$\Sigma\vec{F}_x = 51.69 \text{ N (two extra digits carried)}$$

For the *y*-components of the force:

$$\begin{aligned}\Sigma\vec{F}_y &= \vec{F}_{1y} + \vec{F}_{2y} \\ &= (30\ \text{N})\sin 30° + (-40\ \text{N})\sin 50° \\ &= 15\ \text{N} - 30.64\ \text{N} \\ \Sigma\vec{F}_y &= -15.64\ \text{N (two extra digits carried)}\end{aligned}$$

Construct $\Sigma\vec{F}$:

$$\begin{aligned}\left|\Sigma\vec{F}\right| &= \sqrt{(\Sigma\vec{F}_x)^2 + (\Sigma\vec{F}_y)^2} \\ &= \sqrt{(51.69\ \text{N})^2 + (15.64\ \text{N})^2} \\ &= 54.00\ \text{N} \\ \left|\Sigma\vec{F}\right| &= 54\ \text{N}\end{aligned}$$

$$\begin{aligned}\theta &= \tan^{-1}\left(\frac{\Sigma\vec{F}_y}{\Sigma\vec{F}_x}\right) \\ &= \tan^{-1}\left(\frac{15.64\ \cancel{\text{N}}}{51.69\ \cancel{\text{N}}}\right) \\ \theta &= 17°\end{aligned}$$

Statement: The total force exerted by the ropes on the skater is 54 N [E 17° S].

2. Given: $m = 45\ \text{kg}$

Required: $\vec{F}_{\text{T1}}$; $\vec{F}_{\text{T2}}$; $\vec{F}_{\text{T3}}$

Analysis: $\Sigma\vec{F} = 0\ \text{N}$

Solution: For the forces on the mass,

$$\begin{aligned}\vec{F}_{\text{T2}} - mg &= 0\ \text{N} \\ \vec{F}_{\text{T2}} &= mg\end{aligned}$$

For the *y*-components of the force:

$$\begin{aligned}\vec{F}_{\text{T2}y} + \vec{F}_{\text{T3}y} &= 0\ \text{N} \\ -\vec{F}_{\text{T2}} + \vec{F}_{\text{T3}}\sin\theta &= 0\ \text{N} \\ \vec{F}_{\text{T3}} &= \frac{\vec{F}_{\text{T2}}}{\sin\theta} \\ \vec{F}_{\text{T3}} &= \frac{mg}{\sin\theta}\end{aligned}$$

For the *x*-components of the force:

$$\vec{F}_{\text{T1}x} + \vec{F}_{\text{T3}x} = 0 \text{ N}$$
$$-\vec{F}_{\text{T1}} + \vec{F}_{\text{T3}}\cos\theta = 0 \text{ N}$$
$$\vec{F}_{\text{T1}} = \vec{F}_{\text{T3}}\cos\theta$$
$$= \left(\frac{mg}{\sin\theta}\right)\cos\theta$$
$$\vec{F}_{\text{T1}} = \frac{mg}{\tan\theta}$$

Calculate the tensions in the three cables.

$$\vec{F}_{\text{T1}} = \frac{mg}{\tan\theta}$$
$$= \frac{45 \text{ kg } (9.8 \text{ m/s}^2)}{\tan 60.0°}$$
$$\vec{F}_{\text{T1}} = 250 \text{ N}$$

$$\vec{F}_{\text{T2}} = mg$$
$$= 45 \text{ kg } (9.8 \text{ m/s}^2)$$
$$\vec{F}_{\text{T2}} = 440 \text{ N}$$

$$\vec{F}_{\text{T3}} = \frac{mg}{\sin\theta}$$
$$= \frac{45 \text{ kg } (9.8 \text{ m/s}^2)}{\sin 60.0°}$$
$$\vec{F}_{\text{T3}} = 510 \text{ N}$$

Statement: $\vec{F}_{\text{T1}}$ is 250 N, $\vec{F}_{\text{T2}}$ is 440 N, and $\vec{F}_{\text{T3}}$ is 510 N.

3. **Given:** $m = 2.5 \text{ kg}$; $\vec{F}_{\text{air}} = 12 \text{ N [right]}$; $\Sigma\vec{F} = 0 \text{ N}$

Required: θ

Analysis: $\Sigma\vec{F} = 0 \text{ N}$; $\left|\vec{F}_{\text{T}}\right| = \sqrt{(\vec{F}_{\text{T}x})^2 + (\vec{F}_{\text{T}y})^2}$; $\theta = \tan^{-1}\left(\frac{\vec{F}_{\text{T}y}}{\vec{F}_{\text{T}x}}\right)$. Choose right and up as positive.

Solution: For the *x*-components of the force:

$$\Sigma\vec{F}_x = 0 \text{ N}$$
$$\vec{F}_{\text{T}x} + \vec{F}_{\text{air}} = 0 \text{ N}$$
$$\vec{F}_{\text{T}x} = -\vec{F}_{\text{air}}$$
$$\vec{F}_{\text{T}x} = -12 \text{ N}$$

For the y-components of the force:

$$\Sigma\vec{F}_y = 0\text{ N}$$

$$\vec{F}_{\text{T}y} - mg = 0\text{ N}$$

$$\vec{F}_{\text{T}y} = mg$$

$$= (2.5\text{ kg})(9.8\text{ m/s}^2)$$

$$\vec{F}_{\text{T}x} = 24.5\text{ N (one extra digit carried)}$$

Construct $\vec{F}_\text{T}$ from its components:

$$\left|\vec{F}_\text{T}\right| = \sqrt{(\vec{F}_{\text{T}x})^2 + (\vec{F}_{\text{T}y})^2}$$

$$= \sqrt{(12\text{ N})^2 + (24.5\text{ N})^2}$$

$$\left|\vec{F}_\text{T}\right| = 27\text{ N}$$

$$\theta = \tan^{-1}\left(\frac{\vec{F}_{\text{T}y}}{\vec{F}_{\text{T}x}}\right)$$

$$= \tan^{-1}\left(\frac{24.5\ \cancel{\text{N}}}{12\ \cancel{\text{N}}}\right)$$

$$\theta = 64°$$

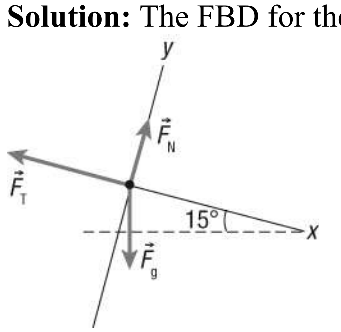
Statement: The tension is the rope is 27 N. The rope makes an angle of 64° with the horizontal.

4. (a) Given: $\theta = 15°$; $m = 1.41\times10^3\text{ kg}$; $\Sigma\vec{F} = 0\text{ N}$

Required: FBD showing the forces on the car

Analysis: Choose [down the hill] as the positive x-direction and [up perpendicular to hill] as the positive y-direction.

Solution: The FBD for the car is shown below.



(b) Given: $\Sigma\vec{F} = 0$ N
Required: equations for the conditions for static equilibrium along horizontal and vertical directions
Analysis: $\Sigma\vec{F}_x = 0$ N; $\Sigma\vec{F}_y = 0$ N
Solution: For the *x*-components of the force (horizontal):

$$\Sigma\vec{F}_x = 0 \text{ N}$$
$$\vec{F}_{gx} - \vec{F}_{\text{T}} = 0 \text{ N}$$
$$mg\sin\theta - \vec{F}_{\text{T}} = 0 \text{ N}$$

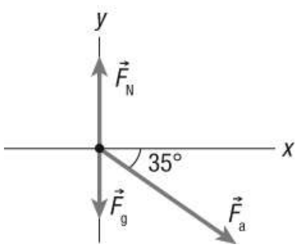
For the *y*-components of the force (vertical):

$$\Sigma\vec{F}_y = 0 \text{ N}$$
$$\vec{F}_{\text{N}} + \vec{F}_{gy} = 0 \text{ N}$$
$$\vec{F}_{\text{N}} - mg\cos\theta = 0 \text{ N}$$

(c) Given: $\theta = 15°$; $m = 1.41\times10^3$ kg; $\Sigma\vec{F} = 0$ N
Required: F_{T}
Analysis: $mg\sin\theta - \vec{F}_{\text{T}} = 0$ N
Solution: $mg\sin\theta - \vec{F}_{\text{T}} = 0$ N

$$\vec{F}_{\text{T}} = mg\sin\theta$$
$$= (1410 \text{ kg})(9.8 \text{ m/s}^2)\sin 15°$$
$$\vec{F}_{\text{T}} = 3.6\times10^3 \text{ N}$$

Statement: The tension in the cable is 3.6×10^3 N.
5. (a) Given: $\vec{F}_{\text{a}} = 42$ N [right 35° down]; $m = 18$ kg; $\Delta d = 5.0$ m
Required: FBD for the mower
Analysis: Choose forward and up as positive
Solution: The FBD for the lawn mower is shown below.



(b) Given: $\vec{F}_{\text{a}} = 42$ N [right 35° down]; $m = 18$ kg; $\Delta d = 5.0$ m
Required: a
Analysis: $\Sigma\vec{F}_x = m\vec{a}$

Solution: $\Sigma\vec{F}_x = m\vec{a}$

$$\vec{F}_{ax} = m\vec{a}$$

$$\vec{a} = \frac{\vec{F}_{ax}}{m}$$

$$= \frac{(42\ \text{N})\cos 35°}{18\ \text{kg}}$$

$$= 1.911\ \text{m/s}^2 \text{(two extra digits carried)}$$

$$\vec{a} = 1.9\ \text{m/s}^2$$

Statement: The acceleration of the mower is 1.9 m/s^2 [forward].

(c) Given: $\vec{F}_a = 42\ \text{N [right 35° down]}$; $m = 18\ \text{kg}$

Required: $\vec{F}_N$

Analysis: Use the FBD to identify the forces with vertical components. Use $\Sigma\vec{F}_y = 0\ \text{N}$ to solve for the normal force.

Solution: $\Sigma\vec{F}_y = 0\ \text{N}$

$$\vec{F}_N + \vec{F}_{ay} - mg = 0\ \text{N}$$

$$\vec{F}_N = mg - \vec{F}_{ay}$$

$$= (18\ \text{kg})(9.8\ \text{m/s}^2) - (-42\ \text{N})\sin 35°$$

$$= 176.4\ \text{N} + 24.09\ \text{N}$$

$$\vec{F}_N = 200\ \text{N}$$

Statement: The normal force is 2.0×10^2 N [up].

(d) Given: $\Delta d = 5.0\ \text{m}$; $\vec{a} = 1.911\ \text{m/s}^2$

Required: $\vec{v}_f$

Analysis: $v_f^2 = v_i^2 + 2\vec{a}\Delta d$

Solution: $v_f^2 = v_i^2 + 2\vec{a}\Delta d$

$$v_f^2 = 2(1.911\ \text{m/s}^2)(5.0\ \text{m})$$

$$v_f = 4.4\ \text{m/s}$$

Statement: The velocity of the mower when it reaches the lawn is 4.4 m/s [forward].

6. (a) Given: $m = 1.3\ \text{kg}$; $\theta = 25°$; $\Sigma\vec{F} = 0\ \text{N}$

Required: $\vec{F}_a$

Analysis: $\Sigma\vec{F}_x = 0\ \text{N}$

Solution: $\Sigma\vec{F}_x = 0\text{ N}$

$$\vec{F}_\text{a} - mg\sin\theta = 0\text{ N}$$

$$\vec{F}_\text{a} = mg\sin\theta$$

$$= (1.3\text{ kg})(9.8\text{ m/s}^2)\sin 25°$$

$$\vec{F}_\text{a} = 5.4\text{ N}$$

Statement: A force of 5.4 N is required to pull the cart up the ramp at a constant velocity.

(b) Given: $m = 1.3\text{ kg}$; $\theta = 25°$; $\vec{a} = 2.2\text{ m/s}^2$ [up the ramp]

Required: $\vec{F}_\text{a}$

Analysis: $\Sigma\vec{F}_x = m\vec{a}_x$

Solution: $\Sigma\vec{F}_x = m\vec{a}$

$$\vec{F}_\text{a} - mg\sin\theta = m\vec{a}$$

$$\vec{F}_\text{a} = m\vec{a} + mg\sin\theta$$

$$= (1.3\text{ kg})(2.2\text{ m/s}^2) + (1.3\text{ kg})(9.8\text{ m/s}^2)\sin 25°$$

$$= 2.86\text{ N} + 5.384\text{ N}$$

$$\vec{F}_\text{a} = 8.2\text{ N}$$

Statement: A force of 8.2 N is required to pull the cart up the ramp at an acceleration of 2.2 m/s.

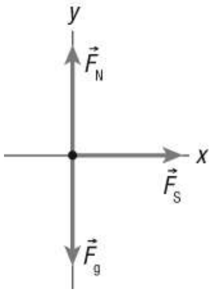
Section 2.4: Forces of Friction

Mini Investigation: Light from Friction, page 86

A. When we crushed the mints, the candy briefly gave off light.
B. The crystals of sugar in the mints rubbed together to create the friction that produces the light.

Tutorial 1 Practice, page 89

1. (a) An FBD of the top book during its acceleration is shown below.



(b) The force of static friction causes the top book to accelerate horizontally.

2. Given: $\underline{a} = 2.7\ \text{m/s}^2$

Required: μ_S

Analysis: $\vec{F}_S = \mu_S \vec{F}_N$; $\Sigma\vec{F} = m\vec{a}$

Solution: Equation for *y*-components of the force:

$$\begin{aligned} \Sigma\vec{F}_y &= 0\ \text{N} \\ \vec{F}_N - mg &= 0\ \text{N} \\ \vec{F}_N &= mg \end{aligned}$$

Equation for *x*-components of the force:

$$\begin{aligned} \Sigma\vec{F}_x &= m\vec{a} \\ \vec{F}_S &= m\vec{a} \\ \mu_S \vec{F}_N &= m\vec{a} \\ \mu_S \cancel{m} g &= \cancel{m}\vec{a} \\ \mu_S &= \frac{\vec{a}}{g} \\ &= \frac{2.7\ \cancel{\text{m/s}^2}}{9.8\ \cancel{\text{m/s}^2}} \\ \mu_S &= 0.28 \end{aligned}$$

Statement: The smallest coefficient of static friction between dinner plates that will prevent slippage is 0.28.

3. Given: $\vec{F}_T = 28\ \text{N [forward 29° up]}$; $\mu_S = 0.45$; $\mu_K = 0.45$; $\Sigma\vec{F} = 0\ \text{N}$

Required: *m*

Analysis: $\Sigma\vec{F} = m\vec{a}$

Solution: Equation for *y*-components of the force:

$$\Sigma\vec{F}_y = 0\text{ N}$$

$$\vec{F}_\text{N} + \vec{F}_\text{T}\sin\theta - mg = 0\text{ N}$$

$$\vec{F}_\text{N} = mg - \vec{F}_\text{T}\sin\theta \quad \text{(Equation 1)}$$

Equation for *x*-components of the force:

$$\Sigma\vec{F}_x = m\vec{a}$$

$$\vec{F}_\text{T}\cos\theta - \vec{F}_\text{S} = m\vec{a}$$

$$\vec{F}_\text{T}\cos\theta - \mu_\text{S}\vec{F}_\text{N} = m\vec{a} \quad \text{(Equation 2)}$$

Substitute Equation (1) into Equation (2) and solve for *m*:

$$\vec{F}_\text{T}\cos\theta - \mu_\text{S}\vec{F}_\text{N} = 0\text{ N}$$

$$\vec{F}_\text{T}\cos\theta - \mu_\text{S}\left(mg - \vec{F}_\text{T}\sin\theta\right) = 0\text{ N}$$

$$\vec{F}_\text{T}(\cos\theta + \mu_\text{S}\sin\theta) = \mu_\text{S}mg$$

$$m = \frac{\vec{F}_\text{T}(\cos\theta + \mu_\text{S}\sin\theta)}{\mu_\text{S}g}$$

$$= \frac{(28\text{ N})(\cos 29° + 0.45\sin 29°)}{0.45(9.8\text{ m/s}^2)}$$

$$m = 6.9\text{ kg}$$

Statement: The smallest possible mass for the box is 6.9 kg.

4. Given: $\theta = 6.0°$; $\vec{v}_\text{i} = 12$ m/s [down slope]; $v_\text{f} = 0$ m/s; $\mu_\text{K} = 0.14$

Required: Δd

Analysis: $\Sigma\vec{F} = m\vec{a}$; $v_\text{f}^2 = v_\text{i}^2 + 2a\Delta d$

Solution: Equation for *y*-components of the force:

$$\Sigma\vec{F}_y = 0\text{ N}$$

$$\vec{F}_\text{N} - mg\cos\theta = 0\text{ N}$$

$$\vec{F}_\text{N} = mg\cos\theta$$

Equation for *x*-components of the force:

$$\Sigma\vec{F}_x = m\vec{a}_x$$

$$mg\sin\theta - \vec{F}_\text{K} = m\vec{a}_x$$

$$\cancel{m}g\sin\theta - \mu_\text{K}\cancel{m}g\cos\theta = \cancel{m}\vec{a}_x$$

$$\vec{a}_x = g(\sin\theta - \mu_\text{K}\cos\theta)$$

$$= (9.8\text{ m/s}^2)(\sin 6.0° - 0.14\cos 6.0°)$$

$$\vec{a}_x = -0.3401\text{ m/s}^2 \text{ (two extra digits carried)}$$

Solve for the distance travelled:

$$v_f^2 = v_i^2 + 2\vec{a}_x \Delta d$$

$$\Delta d = \frac{v_f^2 - v_i^2}{2\vec{a}_x}$$

$$= \frac{(0 \text{ m/s})^2 - (12 \text{ m/s})^2}{2(-0.3401 \text{ m/s}^2)}$$

$$\Delta d = 2.1 \times 10^2 \text{ m}$$

Statement: The sled will slide for 2.1×10^2 m before coming to rest.

5. Given: $m = 39$ kg; direction of rope [forward 21° up]; $\mu_K = 0.23$; $\Sigma\vec{F} = 0$ N

Required: $\vec{F}_T$

Analysis: $\Sigma\vec{F} = 0$ N

Solution: Equation for *y*-components of the force:

$$\Sigma\vec{F}_y = 0 \text{ N}$$

$$\vec{F}_N + \vec{F}_{Ty} - mg = 0 \text{ N}$$

$$\vec{F}_N = mg - \vec{F}_T \sin\theta$$

Equation for *x*-components of the force:

$$\Sigma\vec{F}_x = 0 \text{ N}$$

$$\vec{F}_{Tx} - \vec{F}_K = 0 \text{ N}$$

$$\vec{F}_T \cos\theta - \mu_K \vec{F}_N = 0 \text{ N}$$

$$\vec{F}_T \cos\theta - \mu_K (mg - \vec{F}_T \sin\theta) = 0 \text{ N}$$

$$\vec{F}_T (\cos\theta + \mu_K \sin\theta) = \mu_K mg$$

$$\vec{F}_T = \frac{\mu_K mg}{\cos\theta + \mu_K \sin\theta}$$

$$= \frac{(0.23)(39 \text{ kg})(9.8 \text{ m/s}^2)}{\cos 21° + 0.23 \sin 21°}$$

$$F_T = 87 \text{ N}$$

Statement: A tension of 87 N in the rope is needed to keep the box moving at a constant velocity.

6. (a) Given: $m_1 = 24$ kg; $m_2 = 14$ kg; $\mu_K = 0.32$; $\vec{F}_a = 1.8 \times 10^2$ N [forward 25° up]

Required: $\vec{a}$

Analysis: $\Sigma\vec{F}_y = 0$ N ; $\Sigma\vec{F}_x = m_1\vec{a}$

Solution: Equation for *y*-components of the force:

$$\Sigma \vec{F}_y = 0 \text{ N}$$

$$\vec{F}_{\text{N1}} + \vec{F}_{\text{a}y} - m_1 g = 0 \text{ N}$$

$$\begin{aligned}\vec{F}_{\text{N1}} &= m_1 g - \vec{F}_{\text{a}} \sin\theta \\ &= (24 \text{ kg})(9.8 \text{ m/s}^2) - (180 \text{ N})\sin 25° \\ \vec{F}_{\text{N1}} &= 159.1 \text{ N (two extra digits carried)}\end{aligned}$$

$$\begin{aligned}\vec{F}_{\text{K1}} &= \mu_{\text{K}} \vec{F}_{\text{N1}} \\ &= 0.25(159.1 \text{ N}) \\ \vec{F}_{\text{K1}} &= 39.78 \text{ N (two extra digits carried)}\end{aligned}$$

Equation for *x*-components of the force:

$$\Sigma \vec{F}_x = m_1 \vec{a}$$

$$\vec{F}_{\text{a}x} - \vec{F}_{\text{K1}} - \vec{F}_{\text{T}} = m_1 \vec{a} \quad \text{(Equation 1)}$$

Equation for *y*-components of the force:

$$\Sigma \vec{F}_y = 0 \text{ N}$$

$$\vec{F}_{\text{N2}} - m_2 g = 0 \text{ N}$$

$$\begin{aligned}\vec{F}_{\text{N2}} &= m_2 g \\ &= (14 \text{ kg})(9.8 \text{ m/s}^2) \\ \vec{F}_{\text{N2}} &= 137.2 \text{ N (two extra digits carried)} \\ \vec{F}_{\text{K2}} &= \mu_{\text{K}} \vec{F}_{\text{N2}} \\ &= 0.25(137.2 \text{ N}) \\ \vec{F}_{\text{K2}} &= 34.3 \text{ N}\end{aligned}$$

Equation for *x*-components of the force:

$$\Sigma \vec{F}_x = m_2 \vec{a}$$

$$\vec{F}_{\text{T}} - \vec{F}_{\text{K2}} = m_2 \vec{a} \quad \text{(Equation 2)}$$

Add equations (1) and (2) to eliminate the tension of the rope. Solve for a.

$$\left(F_{\text{T}} - F_{\text{K2}}\right) + \left(F_{\text{a}x} - F_{\text{K1}} - F_{\text{T}}\right) = m_1 a + m_2 a$$

$$F_{\text{a}x} - F_{\text{K1}} - F_{\text{K2}} = (m_1 + m_2)a$$

$$a = \frac{F_{\text{a}x} - F_{\text{K1}} - F_{\text{K2}}}{m_1 + m_2}$$

$$= \frac{(180\ \text{N})\cos 25° - 39.78\ \text{N} - 34.3\ \text{N}}{38\ \text{kg}}$$

$$= 1.797\ \text{m/s}^2 \text{(two extra digits carried)}$$

$$a = 1.8\ \text{m/s}^2$$

Statement: The acceleration of the boxes is 1.8 m/s^2.

(b) Given: $m_2 = 14$ kg**;**; $\mu_{\text{K}} = 0.32$; $\vec{F}_{\text{a}} = 1.8 \times 10^2$ N [forward 25° up]

Required: $\vec{F}_{\text{T}}$

Analysis: $\vec{F}_{\text{T}} - \vec{F}_{\text{K2}} = m_2\vec{a}$

Solution: $\vec{F}_{\text{T}} - \vec{F}_{\text{K2}} = m_2\vec{a}$

$$\vec{F}_{\text{T}} = \mu_{\text{K}}\vec{F}_{\text{N2}} + m_2\vec{a}$$

$$= 0.25(137.2) + (14\ \text{kg})(1.797\ \text{m/s}^2)$$

$$\vec{F}_{\text{T}} = 59\ \text{N}$$

Statement: The tension in the rope is 59 N.

7. Given: $\mu_{\text{K}} = 0.20$; $\mu_{\text{S}} = 0.25$; $m = 100.0$ kg

Required: a

Analysis: $\Sigma\vec{F} = 0$ N ; $\Sigma\vec{F}_x = m\vec{a}_x$

Solution: For the y-components of the force:

$$\Sigma\vec{F}_y = 0\ \text{N}$$

$$\vec{F}_{\text{N}} - mg = 0\ \text{N}$$

$$\vec{F}_{\text{N}} = mg$$

$$= (100.0\ \text{kg})(9.8\ \text{m/s}^2)$$

$$\vec{F}_{\text{N}} = 980\ \text{N}$$

For the x-components of the force:

$$\Sigma\vec{F}_x = 0\ \text{N}$$

$$\vec{F}_{\text{a}} - \vec{F}_{\text{S}} = 0\ \text{N}$$

$$\vec{F}_{\text{a}} = \vec{F}_{\text{S}}$$

$$= \mu_{\text{s}}\vec{F}_{\text{N}}$$

$$= 0.25(980\ \text{N})$$

$$\vec{F}_{\text{a}} = 245\ \text{N}$$

Once the refrigerator is moving, an applied for 245 N produces acceleration, *a*:

$$\Sigma \vec{F}_x = m\vec{a}$$

$$\vec{F}_\text{a} - \vec{F}_\text{K} = m\vec{a}$$

$$\vec{a} = \frac{\vec{F}_\text{a} - \mu_\text{K}\vec{F}_\text{N}}{m}$$

$$= \frac{245\text{ N} - 0.20(980\text{ N})}{100.0\text{ kg}}$$

$$\vec{a} = 0.49\text{ m/s}^2$$

Statement: The acceleration when you apply minimum force needed to move the refrigerator is 0.49 m/s^2.

8. Given: $\mu_\text{S} = 0.25$; $m = 110$ kg

Required: $\vec{F}_\text{a}$

Analysis: $\vec{F}_\text{S} = \mu_\text{S}\vec{F}_\text{N}$; $\Sigma\vec{F} = 0$ N

Solution: For the *y*-components of the force:

$$\Sigma \vec{F}_y = 0\text{ N}$$

$$\vec{F}_\text{N} - mg = 0\text{ N}$$

$$\vec{F}_\text{N} = mg$$

$$= (110\text{ kg})(9.8\text{ m/s}^2)$$

$$\vec{F}_\text{N} = 1078\text{ N (two extra digits carried)}$$

For the *x*-components of the force:

$$\Sigma \vec{F}_x = 0\text{ N}$$

$$\vec{F}_\text{a} - \vec{F}_\text{S} = 0\text{ N}$$

$$\vec{F}_\text{a} = \vec{F}_\text{S}$$

$$= \mu_\text{s}\vec{F}_\text{N}$$

$$= 0.25(1078\text{ N})$$

$$\vec{F}_\text{a} = 2.7 \times 10^2\text{ N}$$

Statement: The minimum force required to just set the stage prop into motion is 2.7×10^2 N.

Section 2.4 Questions, page 90

1. Given: $v_i = 20$ m/s; $v_f = 0$ m/s; $\Delta d = 40$ m

Required: coefficient of friction between the tires and the road, μ_K

Analysis: $v_f^2 = v_i^2 + 2a\Delta d$; $\Sigma\vec{F}_x = m\vec{a}_x$. Choose forward as positive.

Solution: Find the acceleration $\vec{a}$:

$$v_f^2 = v_i^2 + 2\vec{a}\Delta d$$

$$\vec{a} = -\frac{v_i^2}{2\Delta d}$$

$$= -\frac{(20 \text{ m/s})^2}{2(40 \text{ m})}$$

$$\vec{a} = -5.0 \text{ m/s}^2$$

Calculate μ_K:

$$\Sigma\vec{F}_x = m\vec{a}$$

$$-\vec{F}_K = m\vec{a}$$

$$-\mu_K \cancel{m} g = \cancel{m}\vec{a}$$

$$\mu_K = -\frac{\vec{a}}{g}$$

$$= \frac{5.0 \cancel{\text{m/s}^2}}{9.8 \cancel{\text{m/s}^2}}$$

$$\mu_K = 0.5$$

Statement: The coefficient of friction between the tires and the road is 0.5.

2. Given: $v_i = 50.0$ m/s; $\Delta t = 10.0$ s; $\mu_K = 0.030$

Required: v_f

Analysis: $\Sigma\vec{F}_x = m\vec{a}$. Choose forward as positive.

Solution: Find the acceleration *a*:

$$\Sigma\vec{F}_x = m\vec{a}$$

$$-\vec{F}_K = m\vec{a}$$

$$-\mu_K \cancel{m} g = \cancel{m}\vec{a}$$

$$\vec{a} = -\mu_K g$$

$$= -0.030(9.8 \text{ m/s}^2)$$

$$\vec{a} = -0.294 \text{ m/s}^2 \text{ (one extra digit carried)}$$

Calculate v_f:

$$v_f = v_i + a\Delta t$$
$$= 50.0 \text{ m/s} + (-0.294 \text{ m/s}^2)(10.0 \text{ s})$$
$$v_f = 47 \text{ m/s}$$

Statement: The speed of the puck after 10.0 s is 47 m/s.

3. (a) Given: $m = 2.0\times10^2$ kg; $\vec{F}_a = 3.5\times10^2$ N

Required: μ_S

Analysis: $\Sigma\vec{F} = 0$ N ; $\vec{F}_S = \mu_S\vec{F}_N$

Solution: For the *y*-components of the force:

$$\Sigma\vec{F}_y = 0 \text{ N}$$
$$\vec{F}_N - mg = 0 \text{ N}$$
$$\vec{F}_N = mg$$

For the *x*-components of the force:

$$\Sigma\vec{F}_x = 0 \text{ N}$$
$$\vec{F}_a - \vec{F}_S = 0 \text{ N}$$
$$\vec{F}_S = \vec{F}_a$$
$$\mu_S\vec{F}_N = \vec{F}_a$$
$$\mu_S = \frac{\vec{F}_a}{mg}$$
$$= \frac{350 \cancel{\text{N}}}{(200 \cancel{\text{kg}})(9.8 \cancel{\text{m/s}^2})}$$
$$\mu_S = 0.18$$

Statement: The coefficient of static friction between the floor and the sofa is 0.18.

(b) Given: $m = 2.0\times10^2$ kg; $\vec{F}_a = 3.5\times10^2$ N ; $v_i = 0$ m/s ; $v_f = 2.0$ m/s; $\Delta t = 5.0$ s

Required: μ_K

Analysis: $\Sigma\vec{F} = m\vec{a}$

Solution: Find the acceleration, $\vec{a}$:

$$\vec{a} = \frac{\Delta v}{\Delta t}$$
$$= \frac{2.0 \text{ m/s}}{5.0 \text{ s}}$$
$$\vec{a} = 0.40 \text{ m/s}^2$$

Using the x-components of the force:

$$\Sigma \vec{F}_x = m\vec{a}$$

$$\vec{F}_a - \vec{F}_K = m\vec{a}$$

$$\vec{F}_K = \vec{F}_a - m\vec{a}$$

$$\mu_K \vec{F}_N = \vec{F}_a - m\vec{a}$$

$$\mu_K = \frac{\vec{F}_a - m\vec{a}}{mg}$$

$$= \frac{350\text{ N} - (200\ \cancel{\text{kg}})(0.40\ \cancel{\text{m/s}^2})}{(200\ \cancel{\text{kg}})(9.8\ \cancel{\text{m/s}^2})}$$

$$\mu_K = 0.14$$

Statement: The coefficient of kinetic friction between the sofa and the floor is 0.14.

4. (a) Given: $\mu_S = 0.29$

Required: θ

Analysis: $\Sigma \vec{F} = 0\text{ N}$

Solution: For the y-components of the force:

$$\Sigma \vec{F}_y = 0\text{ N}$$

$$\vec{F}_N - mg\cos\theta = 0\text{ N}$$

$$\vec{F}_N = mg\cos\theta$$

For the x-components of the force:

$$\Sigma \vec{F}_x = 0\text{ N}$$

$$mg\sin\theta - \vec{F}_S = 0\text{ N}$$

$$mg\sin\theta = \mu_S \vec{F}_N$$

$$\cancel{mg}\sin\theta = \mu_S \cancel{mg}\cos\theta$$

$$\tan\theta = \mu_S$$

$$\theta = \tan^{-1}(0.29)$$

$$\theta = 16^\circ$$

Statement: The crate just begins to slip when the angle of inclination, θ, is 16°.

(b) Given: $\mu_K = 0.26$; $\theta = 16.17^\circ$

Required: $\vec{a}$

Analysis: $\Sigma \vec{F}_x = m\vec{a}$

Solution:
$$\Sigma\vec{F}_x = m\vec{a}$$
$$mg\sin\theta - \vec{F}_K = m\vec{a}$$
$$\cancel{m}g\sin\theta - \mu_K \cancel{m}g\cos\theta = \cancel{m}\vec{a}$$
$$\vec{a} = g(\sin\theta - \mu_K\cos\theta)$$
$$= (9.8 \text{ m/s}^2)(\sin 16.17° - 0.26\cos 16.17°)$$
$$\vec{a} = 0.28 \text{ m/s}^2$$
Statement: The crate accelerates at 0.28 m/s^2 [down the incline] when the coefficient of kinetic friction is 0.26.

5. (a) Two situations in which friction is helpful for an object moving on a horizontal surface are when running, so you can push yourself forward, and when walking on a snowy field so you can use traction from the snow to move yourself forward.

(b) Two situations in which it would be ideal if there were no friction when an object moves across a horizontal surface are: shooting the puck across the ice when playing hockey; and when trying to move a sled over a snowy sidewalk.

6. (a) Given: $m_1 = 45$ kg; $m_2 = 12$ kg; $\mu_S = 0.45$; $\mu_K = 0.35$

Required: $\vec{F}_{g2} \le \mu\vec{F}_N$

Analysis: To determine if this system is in static equilibrium, you need to determine the magnitude of the static friction, F_S, for mass m_1 and compare it to the tension in the string, $\vec{F}_{g2}$, for mass m_2.

Find F_S for mass m_1.
$$\vec{F}_S = \mu_S\vec{F}_N$$
$$= \mu_S m_1 g$$
$$= 0.45(45 \text{ kg})(9.8 \text{ m/s}^2)$$
$$\vec{F}_S = 200 \text{ N}$$
Find $\vec{F}_{g2}$ for mass m_2.
$$\vec{F}_{g2} = m_2 g$$
$$= (12 \text{ kg})(9.8 \text{ m/s}^2)$$
$$\vec{F}_{g2} = 120 \text{ N}$$
Statement: Yes, the system is in static equilibrium. The tension in the string for mass m_2 is not sufficient to break the force of static friction for mass m_1, so there is no acceleration.

(b) Given: $\vec{F}_{g2} = 120$ N

Required: $\vec{F}_T$

Analysis: As long as the two masses remain at rest, the tension in the string is equal to the force of gravity on mass m_2, $\vec{F}_T = \vec{F}_{g2} = m_2 g$.

Solution: $\vec{F}_T = \vec{F}_{g2}$

$$\vec{F}_T = 120 \text{ N}$$

Statement: The tension in the string is 120 N.

(c) Given: $m_1 = 45$ kg; $m_2 = 32$ kg; $\mu_K = 0.35$

Required: $\vec{a}$

Analysis: $\Sigma\vec{F} = m\vec{a}$

Solution: Find $\vec{F}_T$ for mass m_1.

$$\Sigma\vec{F}_x = m_1\vec{a}$$

$$\vec{F}_T - \vec{F}_K = m_1\vec{a}$$

$$\vec{F}_T - \mu_K m_1 g = m_1\vec{a}$$

Find $\vec{F}_T$ for mass m_2.

$$\Sigma\vec{F}_y = m_2\vec{a}$$

$$m_2 g - \vec{F}_T = m_2\vec{a}$$

Solve for $\vec{a}$:

$$\left(\vec{F}_T - \mu_K m_1 g\right) + \left(m_2 g - \vec{F}_T\right) = m_1\vec{a} + m_2\vec{a}$$

$$(m_2 - \mu_K m_1)g = (m_1 + m_2)\vec{a}$$

$$\vec{a} = \frac{(m_2 - \mu_K m_1)g}{m_1 + m_2}$$

$$= \frac{(32 \text{ kg} - 0.35(45 \text{ kg}))(9.8 \text{ m/s}^2)}{77 \text{ kg}}$$

$$\vec{a} = 2.1 \text{ m/s}^2$$

Statement: The acceleration of the system is 2.1 m/s^2, eliminating the string tension.

7. (a) Given: $\theta = 42°$

Required: μ_S

Analysis: $\Sigma\vec{F} = 0$ N

Solution: For the y-components of the force:

$$\Sigma\vec{F}_y = 0 \text{ N}$$

$$\vec{F}_N - mg\cos\theta = 0 \text{ N}$$

$$\vec{F}_N = mg\cos\theta$$

For the x-components of the force:

$$\Sigma\vec{F}_x = 0\text{ N}$$
$$mg\sin\theta - \vec{F}_S = 0\text{ N}$$
$$mg\sin\theta = \mu_S\vec{F}_N$$
$$\cancel{mg}\sin\theta = \mu_S\cancel{mg}\cos\theta$$
$$\tan\theta = \mu_S$$
$$\mu_S = \tan 42°$$
$$\mu_S = 0.90$$

Statement: The coefficient of static friction is 0.90.

(b) Given: $\theta = 35°$

Required: μ_K

Analysis: $\Sigma\vec{F}_x = 0\text{ N}$

Solution:

$$\Sigma\vec{F}_x = 0\text{ N}$$
$$mg\sin\theta - \vec{F}_K = 0\text{ N}$$
$$\cancel{mg}\sin\theta = \mu_K\cancel{mg}\cos\theta$$
$$\mu_K = \tan\theta$$
$$= \tan 35°$$
$$\mu_K = 0.70$$

Statement: The coefficient of kinetic friction is 0.70.

8. Given: $m_1 = 8.0\text{ kg}; m_2 = 12\text{ kg}; \theta_1 = 26°; \theta_2 = 39°; \mu_K = 0.21$

Required: $\vec{a}$

Analysis: $\Sigma\vec{F} = m\vec{a}$

Solution:

For the y-components of the force (mass m_1):

$$\Sigma\vec{F}_y = 0\text{ N}$$
$$\vec{F}_{N1} - m_1 g\cos\theta_1 = 0\text{ N}$$
$$\vec{F}_{N1} = m_1 g\cos\theta_1$$

For the y-components of the force (mass m_2):

$$\Sigma\vec{F}_y = 0\text{ N}$$
$$\vec{F}_{N2} - m_2 g\cos\theta_2 = 0\text{ N}$$
$$\vec{F}_{N2} = m_2 g\cos\theta_2$$

For the x-components of the force (mass m_1):

$$\Sigma\vec{F}_x = m_1\vec{a}$$

$$\vec{F}_\text{T} - m_1 g\sin\theta_1 - \vec{F}_\text{K1} = m_1\vec{a}$$

$$\vec{F}_\text{T} - m_1 g\sin\theta_1 - \mu_\text{K} m_1 g\cos\theta_1 = m_1\vec{a}$$

$$\vec{F}_\text{T} - 49.17\text{ N} = m_1\vec{a} \text{ (two extra digits carried)}$$

For the x-components of the force (mass m_2):

$$\Sigma\vec{F}_x = m_2\vec{a}$$

$$m_2 g\sin\theta_2 - \vec{F}_\text{K2} - \vec{F}_\text{T} = m_2\vec{a}$$

$$m_2 g\sin\theta_2 - \mu_\text{K} m_2 g\cos\theta_2 - \vec{F}_\text{T} = m_2\vec{a}$$

$$54.82\text{ N} - \vec{F}_\text{T} = m_2\vec{a} \text{ (two extra digits carried)}$$

Add the final equations for mass m_1 and mass m_2 to eliminate the string tension.

$$(\vec{F}_\text{T} - 49.17\text{ N}) + (54.82\text{ N} - \vec{F}_\text{T}) = (m_1 + m_2)\vec{a}$$

$$\vec{a} = \frac{54.82\text{ N} - 49.17\text{ N}}{20\text{ kg}}$$

$$\vec{a} = 0.28\text{ m/s}^2$$

Statement: The acceleration of the two masses (as a system) is 0.28 m/s^2 [clockwise].