

## Unit 5

### Plane Geometry

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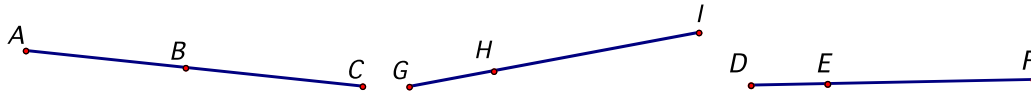
## Intro Problem: Drawing Parallels

1. Draw two parallel lines:
2. Use your own words to describe what it means for two lines to be parallel.
3. In the space below create a geometric design using only straight lines. Your diagram must contain at least three sets of parallel lines and any other lines needed to complete your design. When you are finished indicate any angles that are equal.

## Supplementary Angles (Review)

### Discovery

Each of the angles  $\angle ABC$ ,  $\angle DEF$  &  $\angle GHI$  are considered **straight angles**.



Describe what a straight angle is.

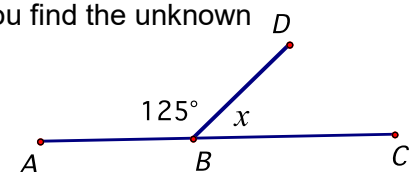
### Discovery

Examples of Supplementary Angles	Non Examples of Supplementary Angles

Describe what **supplementary angles** are:

Draw your own set of supplementary angles:

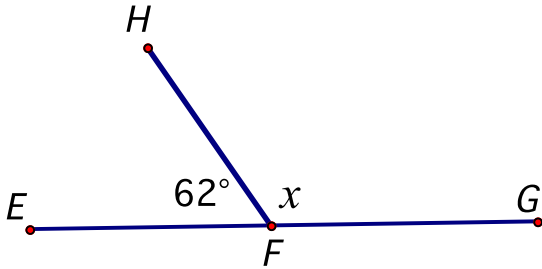
If two angles were supplementary, but only one was know, how could you find the unknown angle?



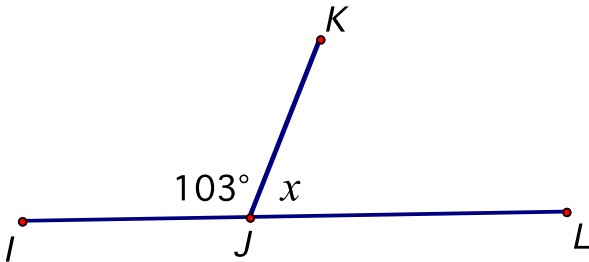
## Practice

For each of the following, determine the value of  $x$ ,  $y$  or  $z$ . Show your work or explain how you got your answer.

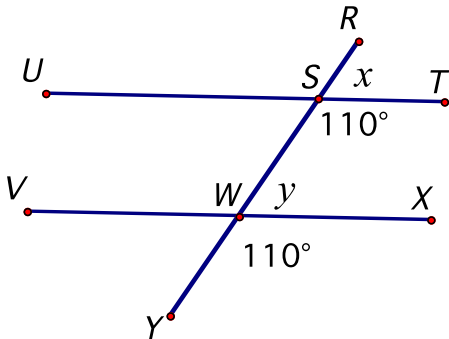
a)



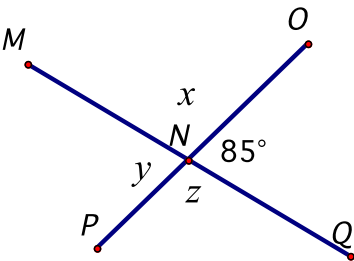
b)



c)



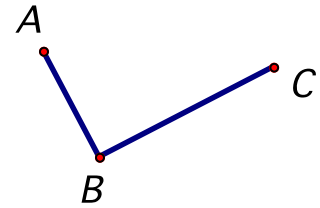
d)



## Complimentary Angles (Review)

### Recall

These two line segments are perpendicular. What does that mean?  
How do you indicate two lines are perpendicular on a diagram?



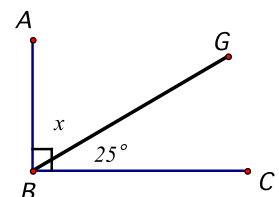
### Discovery

Examples of Complimentary Angles	Non Examples of Complimentary Angles

Describe what **Complimentary angles** are:

Draw your own set of complimentary angles:

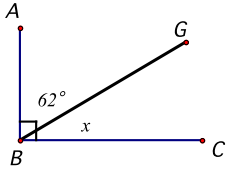
If two angles were complimentary, but only one was know, how could you find the unknown angle?



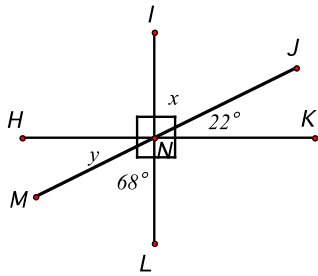
## Practice

For each of the following, determine the value of  $x$ ,  $y$  or  $z$ . Show your work or explain how you got your answer.

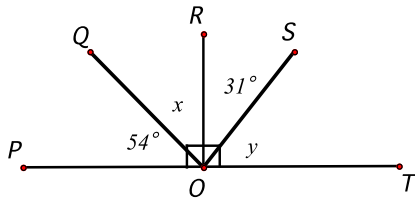
a)



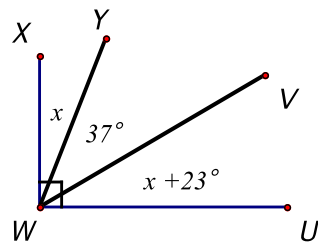
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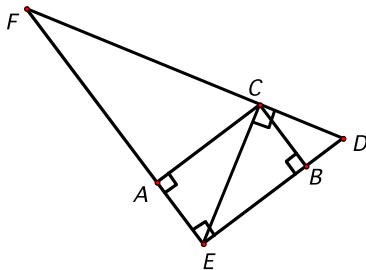
c)



d)



In the diagram below, which angles could be considered complimentary?

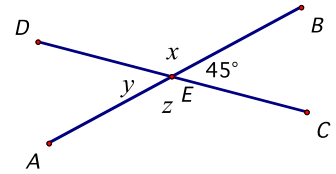


You may also use the GSP sketch **Opposite.gsp**

## Opposite Angles (Review)

### Discovery

Use supplementary angles to determine  $x$ , then  $y$  then  $z$ .



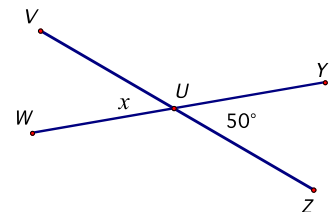
### Discovery

Examples of Opposite Angles	Non Examples of Opposite Angles

Describe what **opposite angles** are:

Draw your own set of opposite angles:

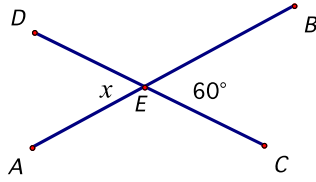
If two angles were opposite, but only one was know, how could you find the unknown angle?



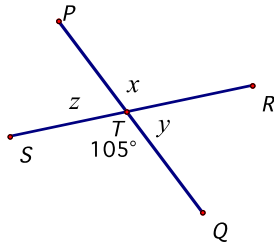
## Practice

For each of the following, determine the value of  $x$ ,  $y$  or  $z$ . Show your work or explain how you got your answer.

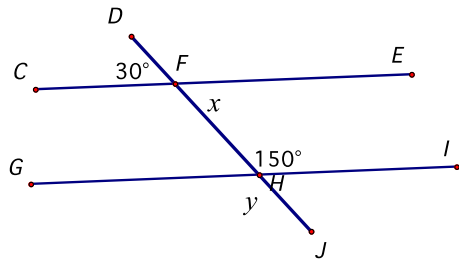
a)



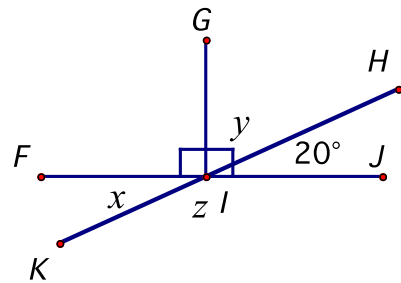
b)



c)



d)





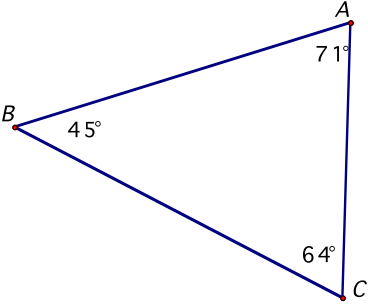
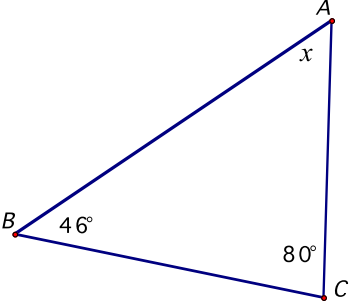
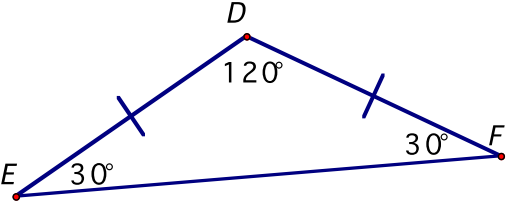
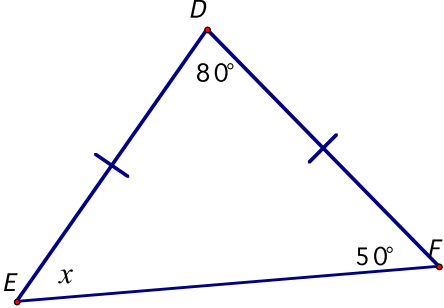
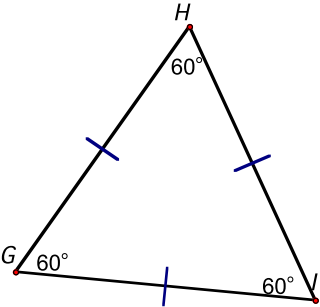
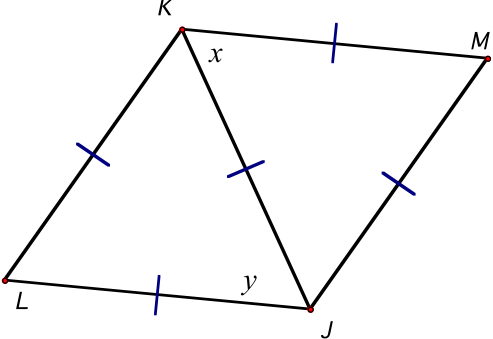
You may also use the GSP sketch **triangle.gsp**

## Triangle Interiors (Review)

### Define

Draw a picture of each of the following types of triangles: Scalene, Isosceles, and Equilateral

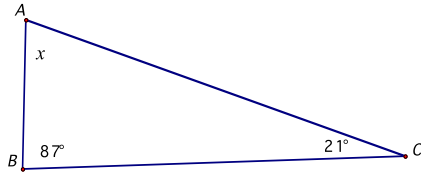
### Reminder

Angle relationships	Example
<p data-bbox="365 642 570 674">Scalene Triangle</p> 	
<p data-bbox="365 1041 570 1073">Isosceles Triangle</p> 	
<p data-bbox="365 1346 570 1377">Equilateral Triangle</p> 	

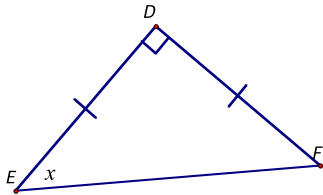
## Practice

For each of the following, determine the value of  $x$ ,  $y$  or  $z$ . Show your work or explain how you got your answer.

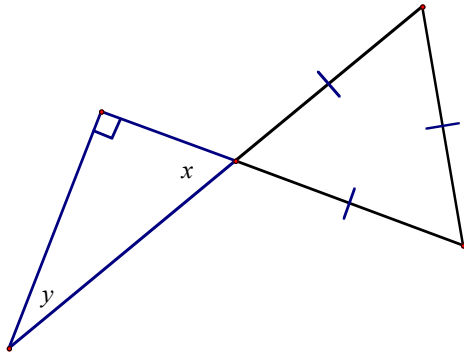
a)



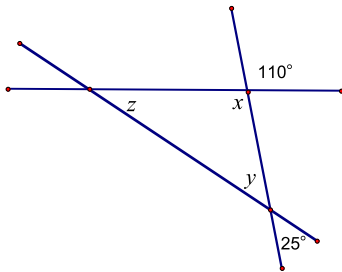
b)



c)



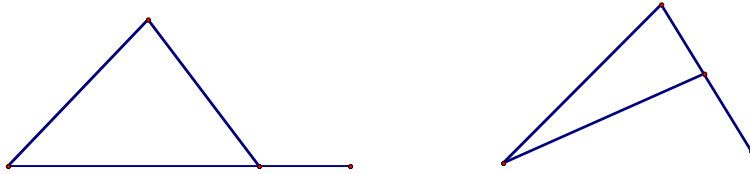
d)



## External Angle Theorem (EAT)

**Notation** (Note: in previous activities, vertices were labeled. Here, they have been left unlabeled for you to label on your own)

Exterior angles are angles that occur outside of a figure. Which are the exterior angles of the following triangles?



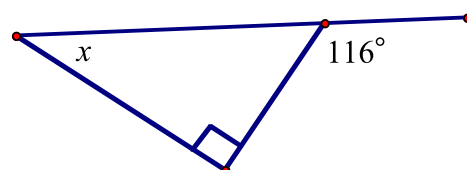
### Discovery

Examples of External Angle Theorem	Non Examples of External Angle Theorem

Describe what **Exterior Angle Theorem** is:

Draw your own set of exterior angles:

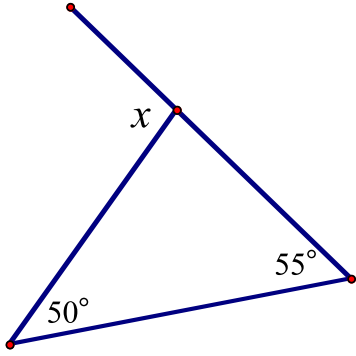
If these angles followed EAT, but only two were known, how could you find the unknown angle?



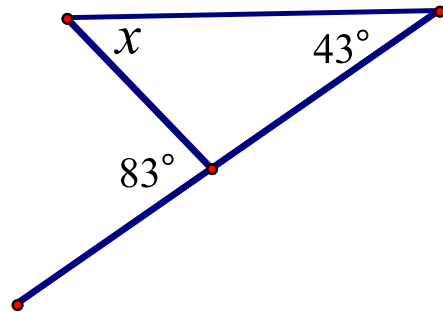
## Practice

For each of the following, determine the value of  $x$ ,  $y$  or  $z$ . Show your work or explain how you got your answer.

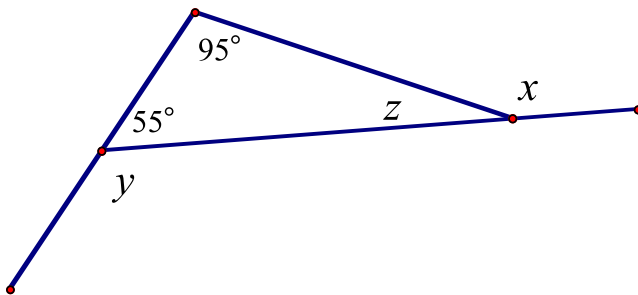
a)



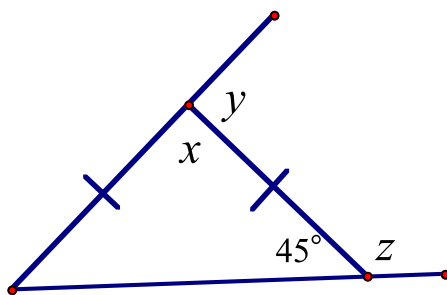
b)



c)



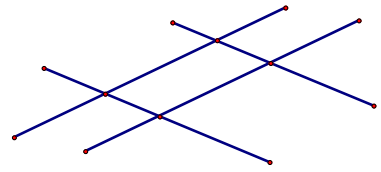
d)



You may also use the GSP sketch **Corresponding.gsp**

## Corresponding Angle Theorem (TPT-F)

**Notation** (Note: in previous activities, vertices were labeled. Here, they have been left unlabeled for you to label on your own)  
How are parallel lines indicated on the following diagram?



**Discovery** (Note: Each of the following 6 diagrams has two parallel lines, indicate which they are)

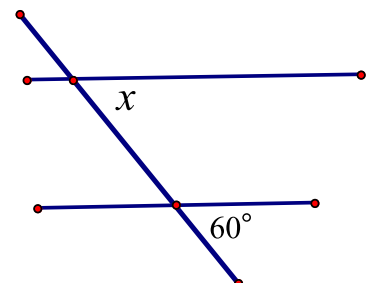
Examples of External Angle Theorem	Non Examples of External Angle Theorem

Describe what the **Corresponding Angle Theorem** is:

Draw your own set of corresponding angles:

This theorem is sometimes referred to as TPT-F. Explain why.

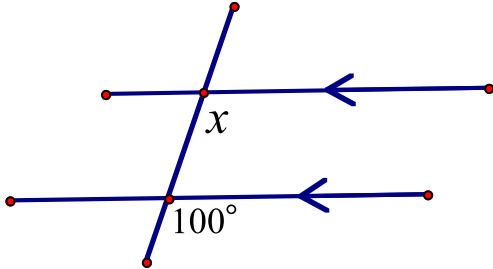
Explain how you can determine the value of  $x$  if you knew two lines were parallel.



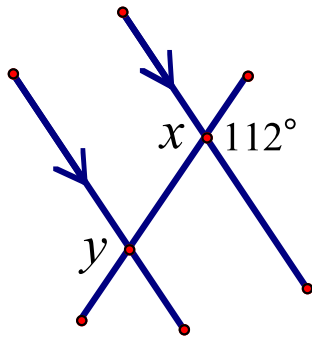
## Practice

For each of the following, determine the value of  $x$ ,  $y$  or  $z$ . Show your work or explain how you got your answer.

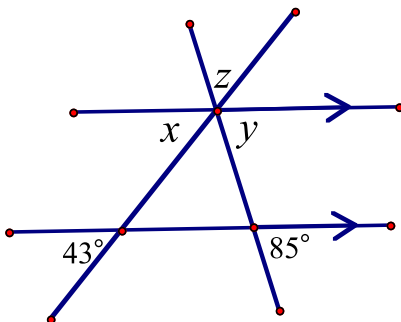
a)



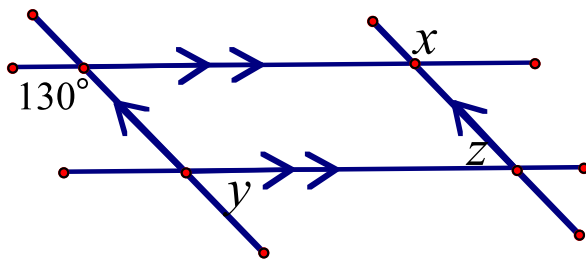
b)



c)

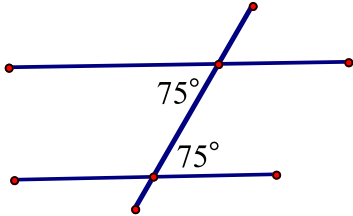
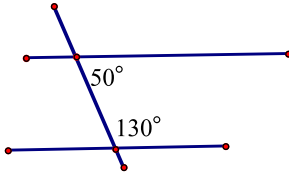
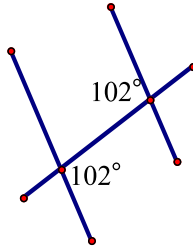
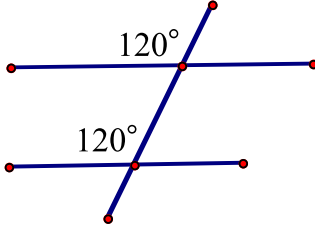
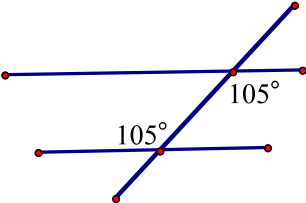
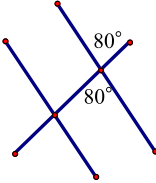


d)



## Alternate Angle Theorem (TPT-Z)

**Discovery** (Note: Each of the following 6 diagrams has two parallel lines, indicate which they are)

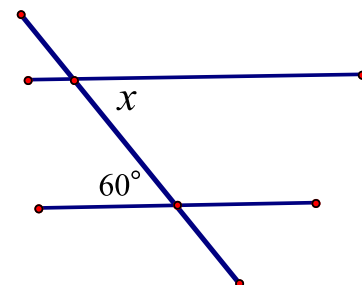
Examples of External Angle Theorem	Non Examples of External Angle Theorem
	
	
	

Describe what the **Alternate Angle Theorem** is:

Draw your own set of alternate angles:

This theorem is sometimes referred to as TPT-Z. Explain why.

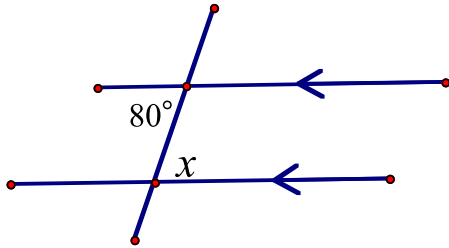
Explain how you can determine the value of  $x$  if you knew two of the lines were parallel.



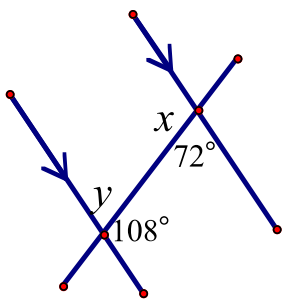
## Practice

For each of the following, determine the value of  $x$ ,  $y$  or  $z$ . Show your work or explain how you got your answer.

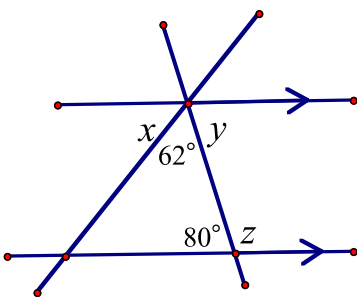
a)



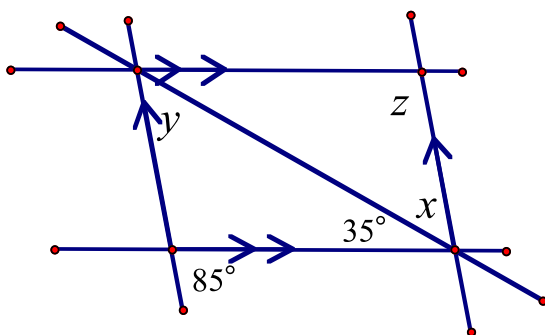
b)



c)



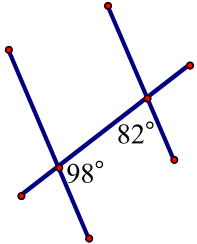
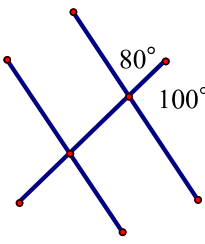
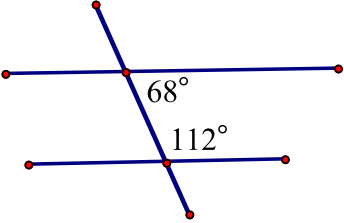
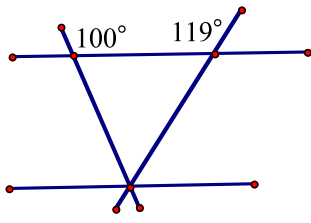
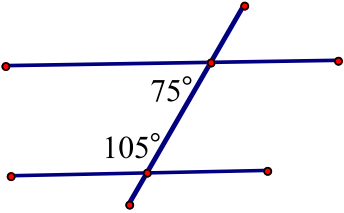
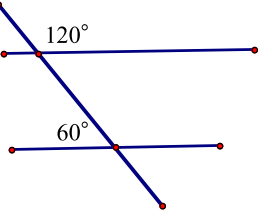
d)





## Co Interior Angle Theorem (TPT-C)

**Discovery** (Note: Each of the following 6 diagrams has two parallel lines, indicate which they are)

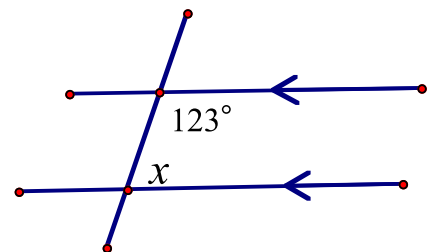
Examples of External Angle Theorem	Non Examples of External Angle Theorem
	
	
	

Describe what the **ColInterior Angle Theorem** is:

Draw your own set of ColInterior angles:

This theorem is sometimes referred to as TPT-C. Explain why.

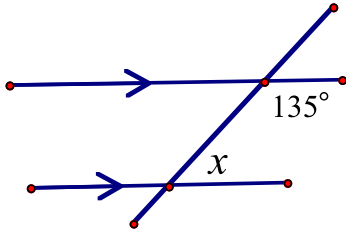
Explain how you can determine the value of  $x$ .



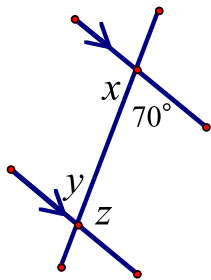
## Practice

For each of the following, determine the value of  $x$ ,  $y$  or  $z$ . Show your work or explain how you got your answer.

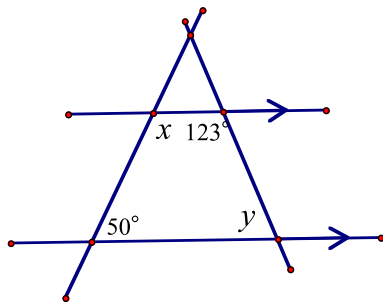
a)



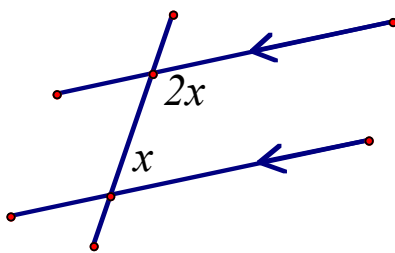
b)



c)


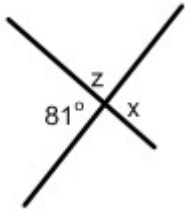
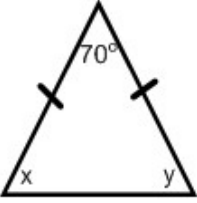
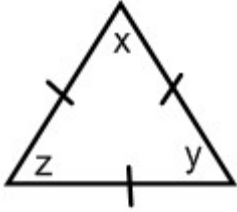
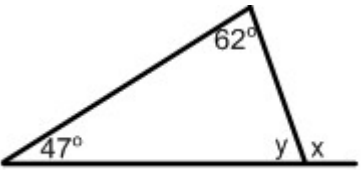
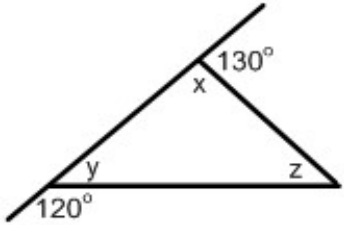
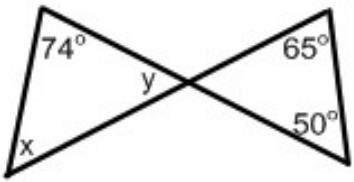
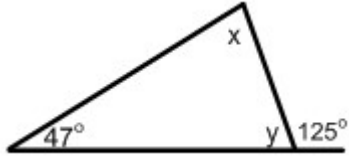
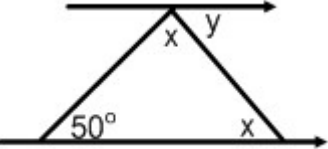
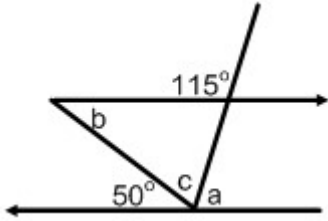
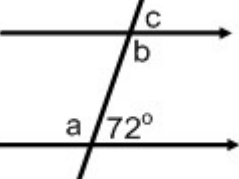
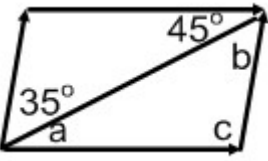


d)



# Plane Geometry Practice

Determine the missing angles

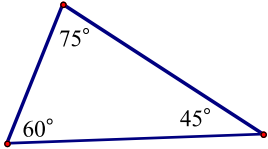
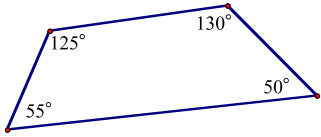
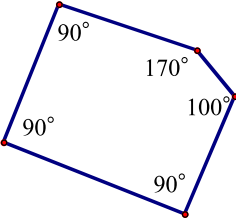
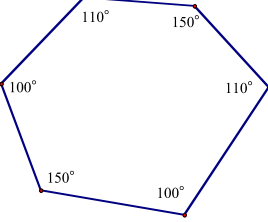
<p>1.</p> 	<p>2.</p> 
<p>3.</p> 	<p>4.</p> 
<p>5.</p> 	<p>6.</p> 
<p>7.</p> 	<p>8.</p> 
<p>9.</p> 	<p>10.</p> 
<p>11.</p> 	<p>12.</p> 

You may also use the GSP sketch [InteriorPolygon.gsp](#)

## Sum of Interior Angles in a Polygon Theorem

### Discovery

Complete the following table:

Diagram	# of sides	Sum of Interior Angles	No of Unique Triangles
			
			
			
			

How does the sum of the interior angles change as the number of sides increase?

How does the number of unique interior triangles change as the number of sides increases?

Explain how you could determine the sum of the interior angles of a polygon if all you knew was the number of sides?

## Practice

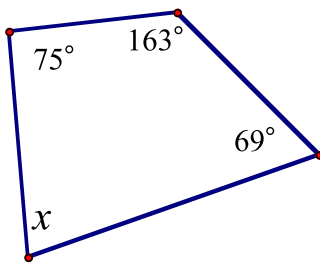
1. What will the sum of the angles be in a polygon with? Show your work

a) 8 sides

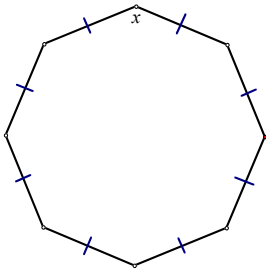
b) 15 sides

2. For each of the following, determine the value of  $x$ ,  $y$  or  $z$ . Show your work or explain how you got your answer.

a)



b)



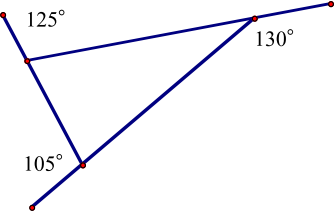
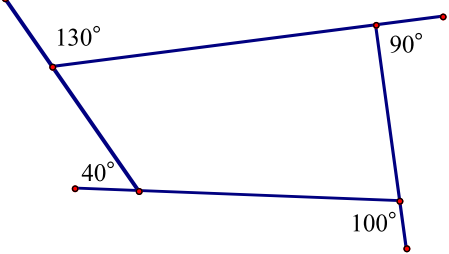
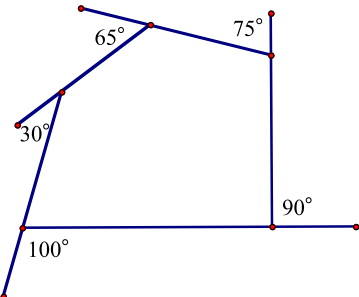
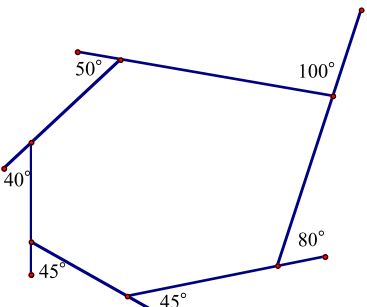
3. The world's largest nickel is found in Sudbury, Ontario. What is the measure of the interior angle in this polygon?



## Sum of Exterior Angles in a Polygon Theorem

### Discovery

Complete the following table:

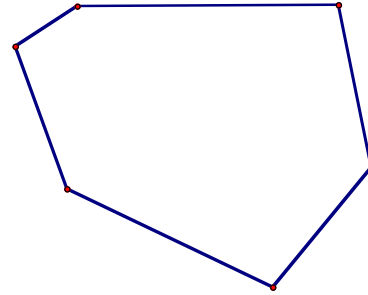
Diagram	Number of sides	Sum of Exterior Angles
		
		
		
		

How does the sum of the exterior angles change as the number of sides increase?

Explain how you could determine the sum of the interior angles of a polygon if all you knew was the number of sides?

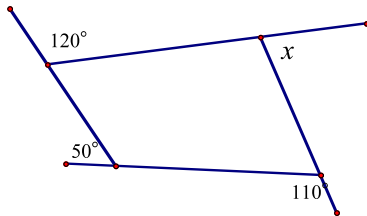
## Practice

1. Draw all of the external angles on this polygon

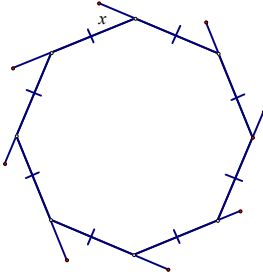


2. For each of the following, determine the value of  $x$ . Show your work or explain how you got your answer.

a)



b)

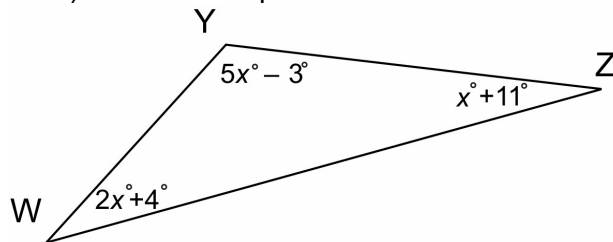


3. Determine the external angle size for a regular polygon with 10 sides. Show your work.

4. A regular polygon has an external angle of  $20^\circ$ , how many sides does it have? Show your work.

## Connecting Algebra to Geometry

- The sum of the interior angles in a triangle is:
  - An equation that models the sum of the interior angles in this triangle is:
  - Solve the equation to determine the value of  $x$ .



- Use the value of  $x$  to calculate the size of:  
 $\angle W$ :                       $\angle Y$ :                       $\angle Z$ :
- The sum of the angles in a right angle is:
  - Write 2 equations to model the sums of the 2 sets of angles that add to  $90^\circ$ :
    - 
    -
  - Solve these equations to determine the values.
    - solve for  $x^\circ$
    - solve for  $y^\circ$
  - Use the values of  $x$  and  $y$  to calculate the size of:  
 $\angle CBP$ :                       $\angle ABQ$ :

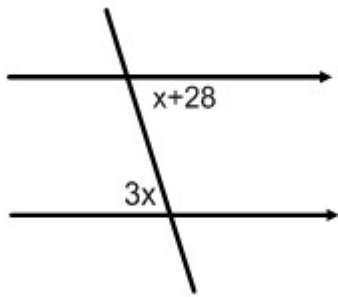


## Connecting Algebra to Geometry (continued)

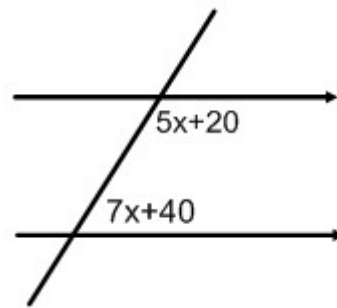
3. Write an equation and solve for the unknown. State the theorem used to make the equation.

a)	b)
c)	d)

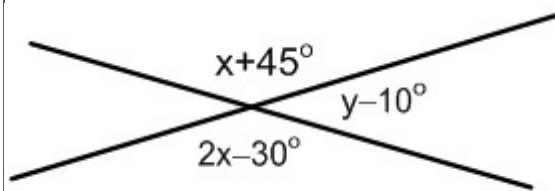
e)



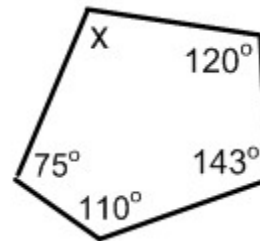
f)



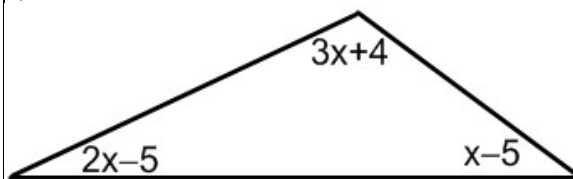
g)



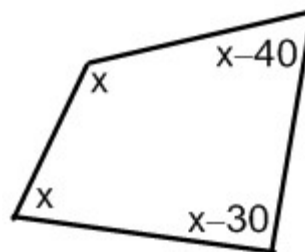
h)



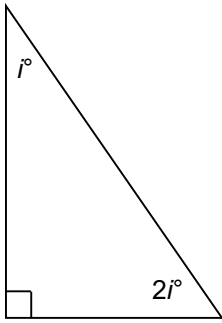
i)



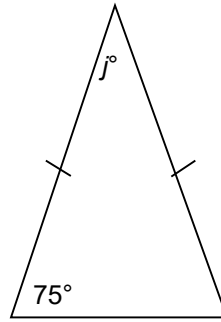
j)



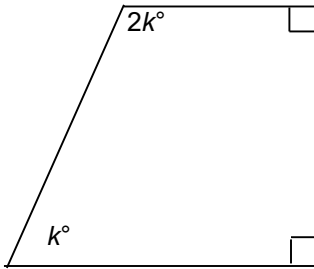
k)



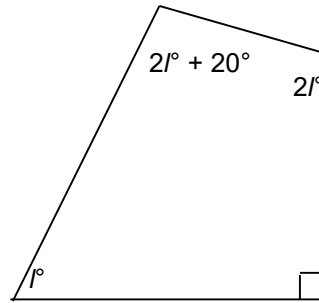
l)



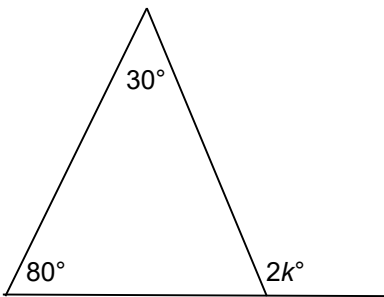
m)



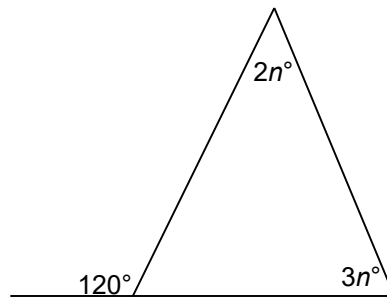
n)



o)

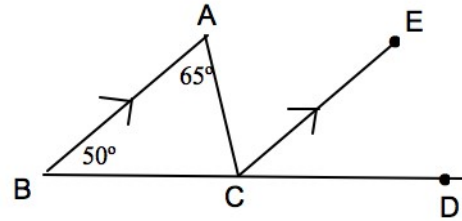


p)



## Review

1. Look at this diagram.  
(a) Name two parallel line segments.

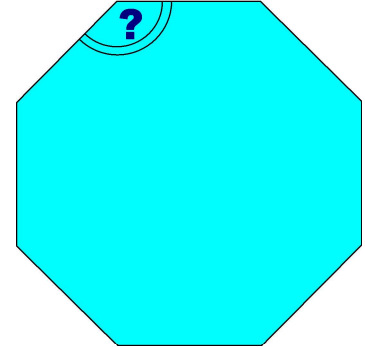


- (b) Name two transversals.
- © Name two corresponding angles.
- (d) Name two alternate angles.
- (e) Find the measures of  $\angle ECD$ ,  $\angle ACE$ , and  $\angle BCA$ .
2. (a) Draw parallelogram ABCD with  $\angle A = 51^\circ$ .

- (b) How can you use what you know about parallel line segments and a transversal to find the measures of the other 3 angles in the parallelogram? Explain your work.

- c) When is a quadrilateral a parallelogram? Explain.

3. Derek is building a deck for his summer job in the shape of a regular octagon.
- a) Define: regular octagon



- b) Determine the measure of the interior angles of the deck.  
Show your work.

4. A Canadian \$1 coin, known as a loonie, is a regular polygon with 11 sides, called an *undecagon*.

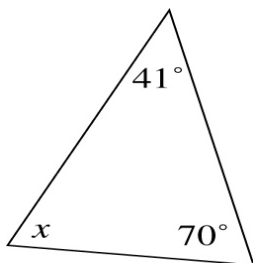
- a) Define a regular polygon with 11 sides.
- b) Determine the sum of the interior angles of the loonie.



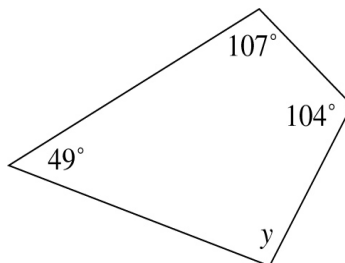
- c) What is the size of one of the interior angles?

5. Determine the measure of each indicated angle.

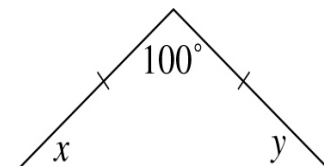
a)



b)

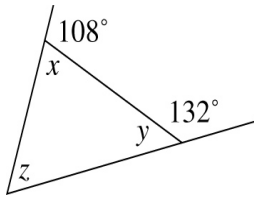


c)

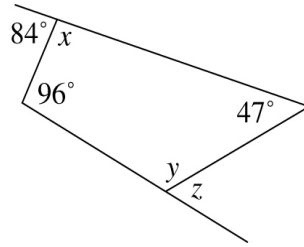


6. Determine the values of  $x$ ,  $y$ , and  $z$ .

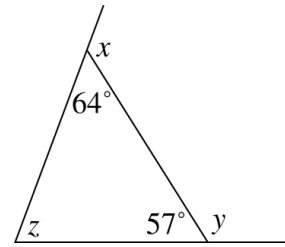
a)



b)

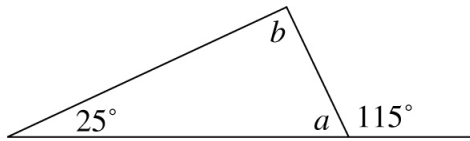


c)

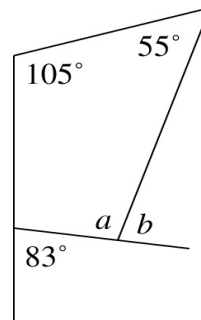


7. Determine the measures of  $a$  and  $b$ .

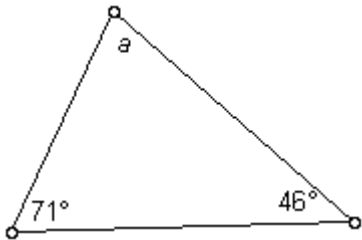
a)



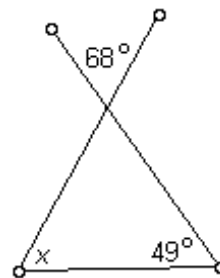
b)



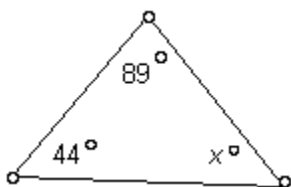
8. Find the measure of  $a$ .



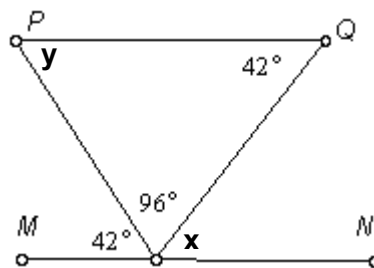
9. Find the value of  $x$ .



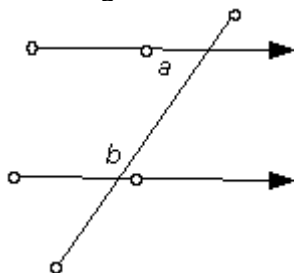
10. Find  $x$ .



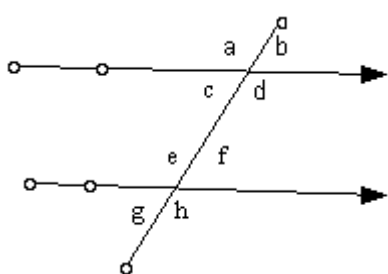
11. Find the values of the missing angles.



12. The diagram shows two parallel lines cut by a transversal. The measure of  $a + b$  is \_\_\_\_\_.



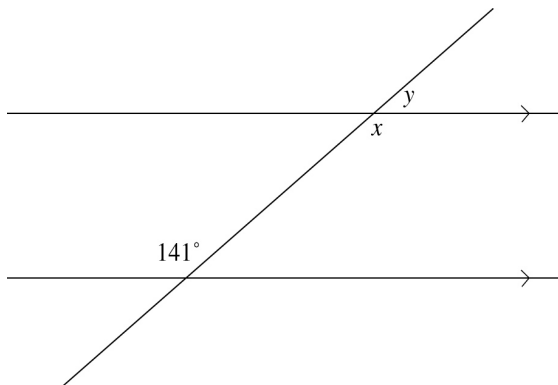
13. For the following diagram, list as many examples of each Angle Theorem as possible.



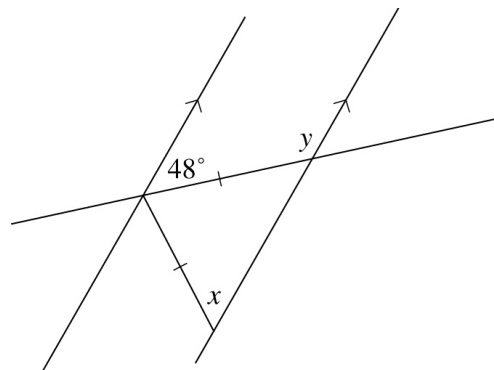
Z - pattern	C - pattern	F - pattern
		e.g. $\angle c = \angle g$

14. Solve for  $x$  and  $y$ .

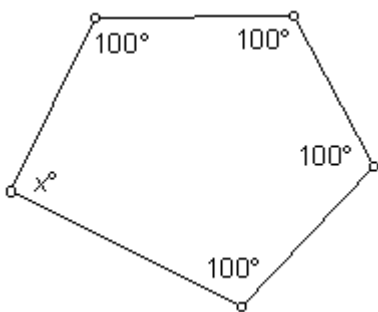
a)



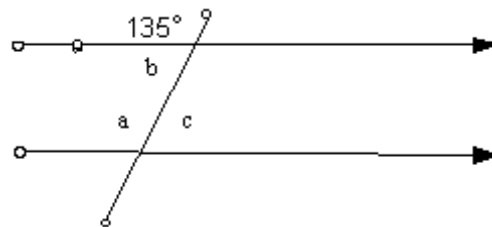
b)



15. Find the measure of  $x$  in the following pentagon.



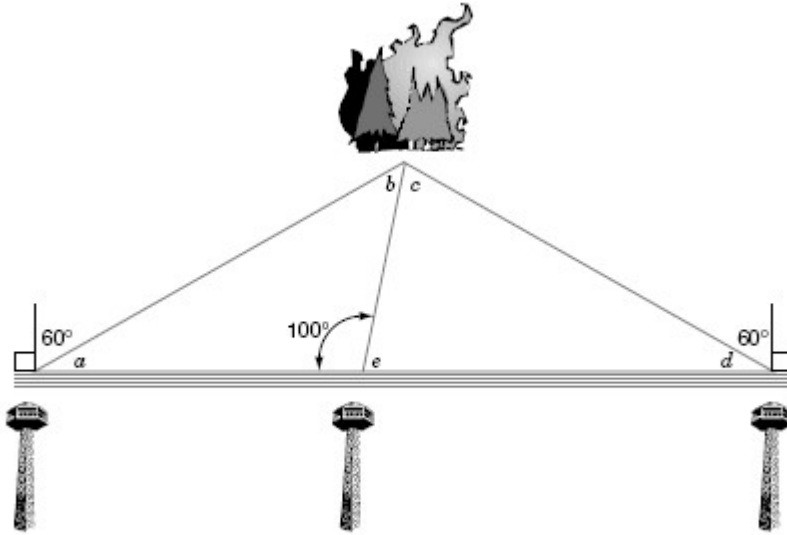
16. Find the measures of  $a$ ,  $b$ , and  $c$ .



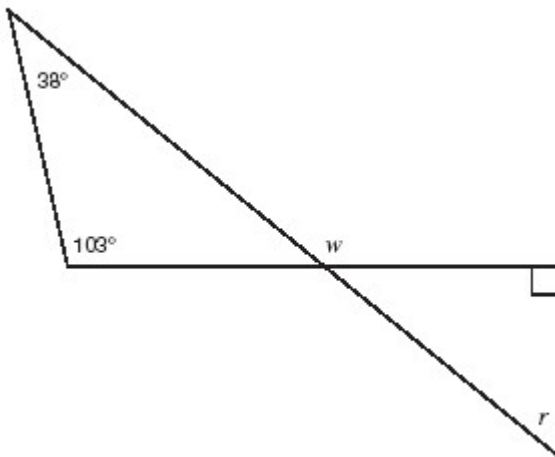


## EQAQ Questions

1. Three forest ranger stations are located along a road. The ranger in each of the stations spots a fire. The diagram below shows the positions of the stations as well as the location of the fire. Calculate the value of  $a$ ,  $b$  and  $c$ . Show your work.



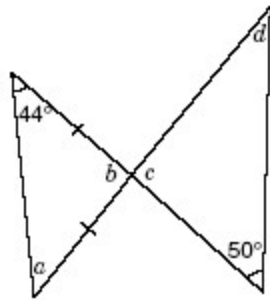
2. Consider the following diagram.



Determine the values of  $r$  and  $w$ .

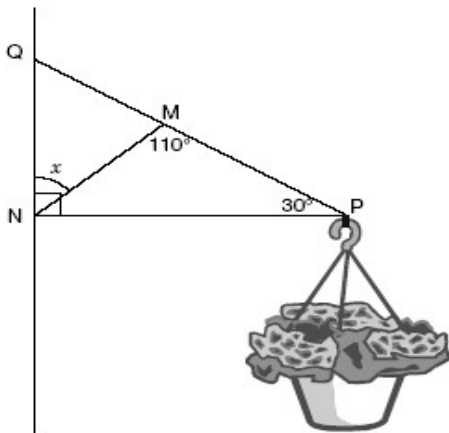
Justify your answer.

3. What is the **sum** of the measures of angles  $a$ ,  $b$ ,  $c$  and  $d$ ?



- a  $136^\circ$
- b  $166^\circ$
- c  $180^\circ$
- d  $266^\circ$

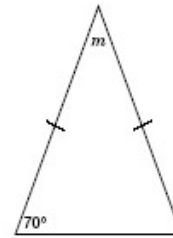
4. A flowerpot hangs from a brace.  $\triangle MNQ$  and  $\triangle MNP$  form the brace.



What is the value of  $x$ ?

- a  $22^\circ$
- b  $30^\circ$
- c  $40^\circ$
- d  $50^\circ$

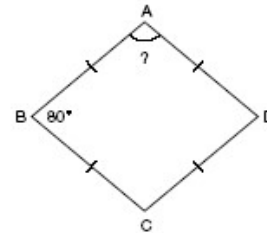
5. The figure below shows an isosceles triangle.



What is the value of  $m$ ?

- a  $40^\circ$
- b  $50^\circ$
- c  $60^\circ$
- d  $70^\circ$

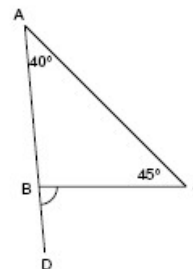
6. ABCD is a quadrilateral with all sides the same length.  $\angle B = 80^\circ$ .



What is the measure of  $\angle A$ ?

- a  $80^\circ$
- b  $90^\circ$
- c  $100^\circ$
- d  $110^\circ$

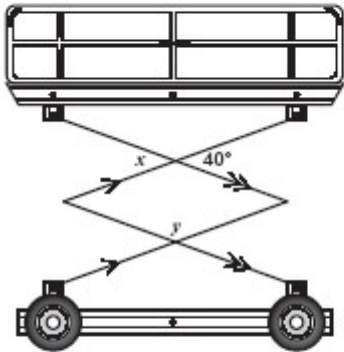
7. ABC is a triangle. AB is extended to D.



What type of angle is  $\angle CBD$ ?

- a straight angle
- b obtuse angle
- c acute angle
- d reflex angle

8. A custodian uses a lift to change light bulbs in the gym. A cross-section of the lift is shown below.

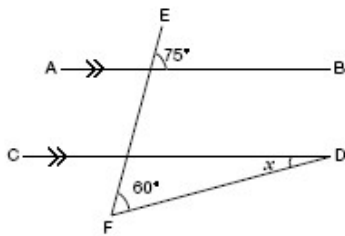


What are the values of  $x$  and  $y$ ?

- a  $x = 40^\circ, y = 100^\circ$
- b  $x = 40^\circ, y = 140^\circ$
- c  $x = 50^\circ, y = 130^\circ$
- d  $x = 50^\circ, y = 140^\circ$

9.

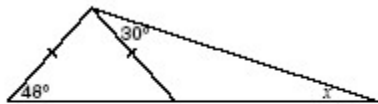
What is the value of  $x$ ?



- A  $15^\circ$
- B  $30^\circ$
- C  $45^\circ$
- D  $60^\circ$

10.

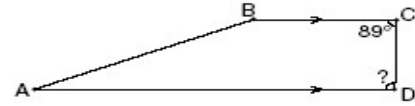
Determine the value of  $x$ .



- F  $x = 18^\circ$
- G  $x = 30^\circ$
- H  $x = 48^\circ$
- J  $x = 78^\circ$

11. The figure ABCD is a trapezoid. BC is parallel to AD.

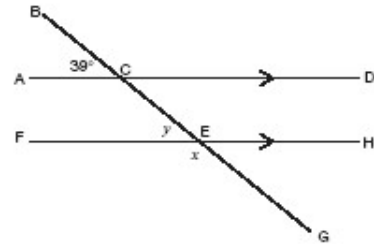
The measure of angle BCD is  $89^\circ$ .  
What is the measure of angle CDA?



**Hint:**  
The diagram is **not** drawn to scale.

- a  $21^\circ$
- b  $86^\circ$
- c  $90^\circ$
- d  $91^\circ$

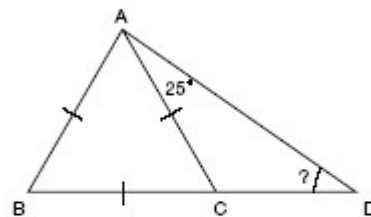
12. The measure of  $\angle ACB$  is  $39^\circ$ .



What are the values of  $x$  and  $y$ ?

- a  $x = 39^\circ$  and  $y = 141^\circ$
- b  $x = 39^\circ$  and  $y = 39^\circ$
- c  $x = 141^\circ$  and  $y = 141^\circ$
- d  $x = 141^\circ$  and  $y = 39^\circ$

13. ABC is an equilateral triangle. BC is extended to D so that  $\angle CAD = 25^\circ$ .



What is the measure of  $\angle ADC$ ?

- a  $25^\circ$
- b  $35^\circ$
- c  $45^\circ$
- d  $55^\circ$