## Section 1.5: Projectile Motion

### Mini Investigation: Analyzing the Range of a Projectile, page 38

Answers may vary. Sample answers:

**A.** There appears to be no relationship between the horizontal component of velocity and the maximum height of a projectile.

**B.** The maximum height of a projectile is greatest when the launch angle is largest.

**C.** A projectile has maximum range when launched at an intermediate angle, around 45°.

**D.** The range of a projectile is the same whether launched at an angle or at the complement of that angle. (Two angles are complementary if they add up to 90°; for example, 25° and 65° are complementary angles, as are 11° and 79°.)

### Tutorial 1 Practice, page 40

**1. (a) Given:** $v_{iy} = 0$ m/s; $\Delta d_y = 76.5$ cm $= 0.765$ m; $g = 9.8$ m/s$^2$

**Required:** $\Delta t$

**Analysis:** Set the table top as $d_i = 0$. Therefore $\Delta d_y = -0.765$ m. In the vertical direction, I know the displacement, initial velocity, and acceleration. Use down as positive, so the displacement will be negative. Use $\Delta d_y = v_{1y}\Delta t - \frac{1}{2}g\Delta t^2$ to determine $\Delta t$ using the quadratic formula.

$$\frac{1}{2}g\Delta t^2 - v_{1y}\Delta t + \Delta d_y = 0$$

$$\Delta t = \frac{v_{1y} \pm \sqrt{v_{1y}^2 - 4\left(\frac{1}{2}g\right)(\Delta d_y)}}{g}$$

**Solution:** $$\Delta t = \frac{v_{1y} \pm \sqrt{v_{1y}^2 - 4\left(\frac{1}{2}g\right)(\Delta d_y)}}{g}$$

$$\Delta t = \frac{0 \pm \sqrt{0 - 4\left(\frac{1}{2}(9.8\text{ m/s}^2)\right)(-0.765\text{ m})}}{9.8\text{ m/s}^2}$$

$$\Delta t = \pm 0.3951\text{ s (two extra digits carried)}$$

$$\Delta t = 0.40\text{ s}$$

**Statement:** The marble hits the floor after 0.40 s.

**(b) Given:** $v_x = 1.93$ m/s; $\Delta t = 0.3951$ s

**Required:** $\Delta d_x$

**Analysis:** Since I know the time of flight of the marble and its horizontal velocity. I can determine its horizontal range using $\Delta d_x = v_x\Delta t$.

**Solution:** $$\begin{aligned}\Delta d_x &= v_x\Delta t \\ &= (1.93\text{ m/}\cancel{\text{s}})(0.3951\text{ }\cancel{\text{s}}) \\ \Delta d_x &= 0.76\text{ m}\end{aligned}$$

**Statement:** The range of the marble is 0.76 m, or 76 cm.

**(c) Given:** $v_x = 1.93$ m/s; $v_{iy} = 0$ m/s; $\Delta t = 0.3951$ s; $g = 9.8$ m/s$^2$

**Required:** $\vec{v}_f$

**Analysis:** The horizontal component of the velocity is constant throughout. The vertical component changes with constant acceleration: $\Delta v_y = -g\Delta t$. Determine the vertical component of the final velocity and then construct the final velocity vector.

**Solution:** Determine the vertical component of the final velocity.

$$\begin{aligned}
\Delta v_y &= -g\Delta t \\
v_{fy} &= v_{iy} - g\Delta t \\
&= 0\text{ m/s} - (9.8\text{ m/s}^2)(0.3951\text{ s}) \\
v_{fy} &= -3.872\text{ m/s (two extra digits carried)}
\end{aligned}$$

Since $v_{fx} = v_x = 1.93$ m/s, I can combine the components to determine the magnitude of $\vec{v}_f$.

$$\begin{aligned}
\left|\vec{v}_f\right| &= \sqrt{(v_{fx})^2 + (v_{fy})^2} \\
&= \sqrt{(1.93\text{ m/s})^2 + (-3.872\text{ m/s})^2} \\
\left|\vec{v}_f\right| &= 4.3\text{ m/s}
\end{aligned}$$

The angle below the horizontal axis of $\vec{v}_f$ is

$$\begin{aligned}
\theta &= \tan^{-1}\left(\frac{\left|v_{fy}\right|}{\left|v_{fx}\right|}\right) \\
&= \tan^{-1}\left(\frac{3.872\ \cancel{\text{m/s}}}{1.93\ \cancel{\text{m/s}}}\right) \\
\theta &= 64^\circ
\end{aligned}$$

**Statement:** The final velocity of the marble is 4.3 m/s [64° below the horizontal].

**2. Given:** $v_{iy} = 0$ m/s; $\Delta d_y = -0.83$ m; $\Delta d_x = 18.4$ m; $g = 9.8$ m/s$^2$

**Required:** $v_x$

**Analysis:** I know the horizontal displacement and want to determine the constant horizontal speed. The appropriate formula is $\Delta d_x = v_x \Delta t$, but I do not know the time taken. Looking at the vertical motion, use $\Delta d_y = v_{1y}\Delta t - \frac{1}{2}g\Delta t^2$ to determine $\Delta t$ using the quadratic formula.

$$\frac{1}{2}g\Delta t^2 - v_{1y}\Delta t + \Delta d_y = 0$$

$$\Delta t = \frac{v_{1y} \pm \sqrt{v_{1y}^2 - 4\left(\frac{1}{2}g\right)(\Delta d_y)}}{g}$$

**Solution:** Using the vertical motion,

$$\Delta t = \frac{v_{1y} \pm \sqrt{v_{1y}^2 - 4\left(\frac{1}{2}g\right)(\Delta d_y)}}{g}$$

$$\Delta t = \frac{0 \pm \sqrt{0 - 4\left(\frac{1}{2}(9.8\ \text{m/s}^2)\right)(-0.83\ \text{m})}}{9.8\ \text{m/s}^2}$$

$\Delta t = \pm 0.4116\ \text{s}$ (two extra digits carried)

Using the horizontal motion,

$$\Delta d_x = v_x \Delta t$$

$$\begin{aligned} v_x &= \frac{\Delta d_x}{\Delta t} \\ &= \frac{18.4\ \text{m}}{0.4116\ \text{s}} \\ v_x &= 45\ \text{m/s} \end{aligned}$$

**Statement:** The initial horizontal speed of the ball is 45 m/s.

**3. (a) Given:** $\vec{v}_\text{i} = 12$ m/s [42° above the horizontal]; $\Delta d_y = -9.5$ m; $g = 9.8\ \text{m/s}^2$

**Required:** $\Delta t$

**Analysis:** First, determine the components of the initial velocity. Then, use the vertical motion and $\Delta d_y = v_{1y}\Delta t - \frac{1}{2}g\Delta t^2$ to solve for the time taken: $\Delta t = \dfrac{v_{1y} \pm \sqrt{v_{1y}^2 - 4\left(\frac{1}{2}g\right)(\Delta d_y)}}{g}$

**Solution:** The components of the initial velocity are

$v_{ix} = (12\ \text{m/s})(\cos 42°)$

$v_{ix} = 8.918\ \text{m/s}$ (two extra digits carried)

$v_{iy} = (12\ \text{m/s})(\sin 42°)$

$v_{iy} = 8.030\ \text{m/s}$ (two extra digits carried)

Using the vertical motion,

$$\begin{aligned} \Delta t &= \frac{v_{1y} \pm \sqrt{v_{1y}^2 - 4\left(\frac{1}{2}g\right)(\Delta d_y)}}{g} \\ &= \frac{8.030\ \text{m/s} \pm \sqrt{(-8.030\ \text{m/s})^2 - 4(4.9\ \text{m/s}^2)(-9.5\ \text{m})}}{2(4.9\ \text{m/s}^2)} \end{aligned}$$

$\Delta t = 2.435\ \text{s}$ or $-0.796\ \text{s}$ (two extra digits carried)

The time of flight cannot be a negative value.

$\Delta t = 2.4$ s

**Statement:** The rock's time of flight is 2.4 s.

**(b) Given:** $v_{ix} = 8.918$ m/s; $\Delta t = 2.435$ s; $g = 9.8$ m/s$^2$

**Required:** width of the moat, $\Delta d_x$

**Analysis:** Since I know the time of flight of the rock and its horizontal velocity, I can determine its horizontal range (the width of the moat) using $\Delta d_x = v_x \Delta t$.

**Solution:**
$$\begin{aligned}\Delta d_x &= v_x \Delta t \\ &= (8.918 \text{ m/s})(2.435 \text{ s}) \\ \Delta d_x &= 22 \text{ m}\end{aligned}$$

**Statement:** The width of the moat is 22 m.

**(c) Given:** $v_{ix} = 8.918$ m/s; $v_{iy} = 8.030$ m/s; $\Delta t = 2.435$ s; $g = 9.8$ m/s$^2$

**Required:** $\vec{v}_{\text{f}}$

**Analysis:** The horizontal component of the velocity is constant throughout. The vertical component changes with constant acceleration: $\Delta v_y = -g\Delta t$. Determine the vertical component of the final velocity and then construct the final velocity vector.

**Solution:** Determine the vertical component of the final velocity.

$$\begin{aligned}\Delta v_y &= -g\Delta t \\ v_{\text{f}y} &= v_{\text{i}y} - g\Delta t \\ &= 8.030 \text{ m/s} - (9.8 \text{ m/s}^{\cancel{2}})(2.435 \cancel{\text{s}}) \\ v_{\text{f}y} &= -15.83 \text{ m/s} \text{ (two extra digits carried)}\end{aligned}$$

Since $v_{\text{f}x} = v_x = 8.918$ m/s, I can combine the components to determine the magnitude of $\vec{v}_{\text{f}}$.

$$\begin{aligned}\left|\vec{v}_{\text{f}}\right| &= \sqrt{(v_{\text{f}x})^2 + (v_{\text{f}y})^2} \\ &= \sqrt{(8.918 \text{ m/s})^2 + (-15.83 \text{ m/s})^2} \\ &= 18.17 \text{ m/s} \\ \left|\vec{v}_{\text{f}}\right| &= 18 \text{ m/s}\end{aligned}$$

The angle below the horizontal axis of $\vec{v}_{\text{f}}$ is

$$\begin{aligned}\theta &= \tan^{-1}\left(\frac{\left|v_{\text{f}y}\right|}{\left|v_{\text{f}x}\right|}\right) \\ &= \tan^{-1}\left(\frac{15.83 \cancel{\text{m/s}}}{8.918 \cancel{\text{m/s}}}\right) \\ \theta &= 61^\circ\end{aligned}$$

**Statement:** The final velocity of the rock is 18 m/s [61° below the horizontal].

**4. (a) Given:** $\vec{v}_{\text{i}} = 4.3\text{ m/s [42° below the horizontal]}$; $\Delta d_y = -3.9\text{ m} + 1.4\text{ m} = -2.5\text{ m}$; $g = 9.8\text{ m/s}^2$

**Required:** $\Delta t$

**Analysis:** First determine the components of the initial velocity. Then use the vertical motion and $\Delta d_y = v_{1y}\Delta t - \frac{1}{2}g\Delta t^2$ to solve for the time taken: $\Delta t = \dfrac{v_{1y} \pm \sqrt{v_{1y}^2 - 4\left(\frac{1}{2}g\right)(\Delta d_y)}}{g}$

**Solution:** The components of the initial velocity are

$v_{\text{ix}} = (4.3\text{ m/s})(\cos 42°)$ $\quad$ $v_{\text{iy}} = -(4.3\text{ m/s})(\sin 42°)$

$v_{\text{ix}} = 3.196\text{ m/s}$ $\quad$ $v_{\text{iy}} = -2.877\text{ m/s}$

Using the vertical motion,

$$\Delta t = \frac{-2.877\text{ m/s} \pm \sqrt{(2.877\text{ m/s})^2 - 4(4.9\text{ m/s}^2)(-2.5\text{ m})}}{9.8\text{ m/s}^2}$$

$\Delta t = 0.4787\text{ s}$ or $-1.066\text{ s}$ (two extra digits carried)

The time the ball is in the air cannot be a negative value.

$\Delta t = 0.48\text{ s}$

**Statement:** The baseball's time of flight is 0.48 s.

**(b) Given:** $v_{\text{ix}} = 3.196\text{ m/s}$; $\Delta t = 0.4787\text{ s}$

**Required:** $\Delta d_x$

**Analysis:** Since I know the time of flight of the baseball and its horizontal velocity, I can determine its horizontal range using $\Delta d_x = v_x\Delta t$.

**Solution:**
$$\begin{aligned}\Delta d_x &= v_x\Delta t\\ &= (3.196\text{ m/s})(0.4787\text{ s})\\ \Delta d_x &= 1.5\text{ m}\end{aligned}$$

**Statement:** The horizontal distance from the window is 1.5 m.

**(c) Given:** $v_{\text{ix}} = 3.196\text{ m/s}$; $v_{\text{iy}} = -2.877\text{ m/s}$; $\Delta t = 0.4787\text{ s}$; $g = 9.8\text{ m/s}^2$

**Required:** $\vec{v}_{\text{f}}$

**Analysis:** The horizontal component of the velocity is constant throughout. The vertical component changes with constant acceleration: $\Delta v_y = -g\Delta t$. Determine the vertical component of the final velocity and then construct the final velocity vector.

**Solution:** Determine the vertical component of the final velocity.

$$\begin{aligned}\Delta v_y &= -g\Delta t\\ v_{\text{f}y} &= v_{\text{iy}} - g\Delta t\\ &= -2.877\text{ m/s} - (9.8\text{ m/s}^{\cancel{2}})(0.4787\text{ }\cancel{\text{s}})\\ v_{\text{f}y} &= -7.568\text{ m/s}\end{aligned}$$

Since $v_{fx} = v_x = 3.196\text{ m/s}$, I can combine the components to determine the magnitude of $\vec{v}_f$.

$$\left|\vec{v}_f\right| = \sqrt{(v_{fx})^2 + (v_{fy})^2}$$
$$= \sqrt{(3.196\text{ m/s})^2 + (-7.568\text{ m/s})^2}$$
$$\left|\vec{v}_f\right| = 8.2\text{ m/s}$$

**Statement:** The speed of the ball as you catch it is 8.2 m/s.

### Tutorial 2 Practice, page 42

**1. (a) Given:** $\vec{v}_i = 2.2\times10^2$ m/s [45° above the horizontal]; $d_{iy} = d_{fy}$; $g = 9.8\text{ m/s}^2$

**Required:** $\Delta t$

**Analysis:** Since the projectile (the marble) lands at the same height from which it was launched, the time taken is given by $\Delta t = \dfrac{2v_i \sin\theta}{g}$.

**Solution:** $\Delta t = \dfrac{2v_i \sin\theta}{g}$

$$= \frac{2(220\text{ m/s})(\sin 45°)}{9.8\text{ m/s}^2}$$
$$\Delta t = 32\text{ s}$$

**Statement:** The time of flight is 32 s.

**(b) Given:** $\vec{v}_i = 2.2\times10^2$ m/s [45° above the horizontal]; $g = 9.8\text{ m/s}^2$

**Required:** $\Delta d_x$

**Analysis:** Use the range formula, $\Delta d_x = \dfrac{v_i^2 \sin 2\theta}{g}$

**Solution:** $\Delta d_x = \dfrac{v_i^2 \sin 2\theta}{g}$

$$= \frac{(220\text{ m/s})^2(\sin 90°)}{9.8\text{ m/s}^2}$$
$$\Delta d_x = 4.9\times10^3\text{ m}$$

**Statement:** The horizontal range of the projectile is $4.9\times10^3$ m, or 4.9 km.

**(c) Given:** $\vec{v}_i = 2.2\times10^2$ m/s [45° above the horizontal]; $g = 9.8\text{ m/s}^2$

**Required:** maximum height, $d_y$

**Analysis:** The maximum height occurs when $v_y = 0$ m/s. Use the acceleration formula $v_{fy}^2 = v_{iy}^2 - 2g\Delta d_y$ to determine $d_y$.

**Solution:** Since $v_{iy} = v_i \sin\theta$ and $v_{fy} = 0$ m/s, $d_y$ can be calculated.

$$v_{fy}^2 = v_{iy}^2 - 2g\Delta d_y$$

$$\Delta d_y = \frac{v_{fy}^2}{2g}$$

$$\begin{aligned}\Delta d_y &= \frac{(v_i \sin\theta)^2}{2g}\\ &= \frac{((220\text{ m/s})\sin 45°)^2}{2(9.8\text{ m/s}^2)}\\ &= 1235\text{ m}\end{aligned}$$

$$\Delta d_y = 1.2 \times 10^3\text{ m}$$

**Statement:** The projectile's maximum height is $1.2 \times 10^3$ m above the ground.

**(d) Given:** $\vec{v}_i = 2.2 \times 10^2$ m/s [45° above the horizontal]; $d_{iy} = d_{fy}$

**Required:** velocity on impact, $\vec{v}_f$

**Analysis:** The projectile's flight is symmetric. Its final velocity is the same as the initial velocity except that the direction of its vertical component is reversed.

**Statement:** The velocity of the marble when it hits the floor is $2.2 \times 10^2$ m/s [45º below the horizontal].

**2. (a) Given:** $\vec{v}_i = 14.5$ m/s [35.0° above horizontal]; $d_{iy} = d_{fy}$; $g = 9.8$ m/s$^2$

**Required:** maximum height, $d_y$

**Analysis:** The maximum height occurs when $v_y = 0$ m/s. Use the acceleration formula $v_{fy}^2 = v_{iy}^2 - 2g\Delta d_y$ to determine $d_y$.

$$v_{fy}^2 = v_{iy}^2 - 2g\Delta d_y$$

$$\Delta d_y = \frac{v_{fy}^2}{2g}$$

**Solution:** Since $v_{iy} = v_i \sin\theta$ and $v_{fy} = 0$ m/s, $d_y$ can be calculated.

$$\begin{aligned}\Delta d_y &= \frac{(v_i \sin\theta)^2}{2g}\\ &= \frac{((14.5\text{ m/s})\sin 35°)^2}{2(9.8\text{ m/s}^2)}\end{aligned}$$

$$\Delta d_y = 3.5\text{ m}$$

**Statement:** The projectile's maximum height is 3.5 m above the ground.

**(b) Given:** $\vec{v}_\text{i} = 14.5$ m/s [35.0° above horizontal]; $g$ = 9.8 m/s$^2$

**Required:** horizontal range, $\Delta d_x$

**Analysis:** Use the range formula $\Delta d_x = \dfrac{v_\text{i}^2 \sin 2\theta}{g}$.

**Solution:**
$$\begin{aligned}\Delta d_x &= \frac{v_\text{i}^2 \sin 2\theta}{g}\\ &= \frac{(14.5\text{ m/s})^2(\sin 70.0°)}{9.8\text{ m/s}^2}\\ \Delta d_x &= 2.0\times 10^1\text{ m}\end{aligned}$$

**Statement:** The horizontal range of the projectile is $2.0 \times 10^1$ m.

**(c) Given:** $\vec{v}_\text{i} = 14.5$ m/s [35.0° above horizontal]**;** $g$ = 9.8 m/$^2$

**Required:** time to maximum height, $\Delta t$

**Analysis:** By symmetry, the time to maximum height is half of the total time $\Delta t = \dfrac{2v_\text{i}\sin\theta}{g}$.

**Solution:**
$$\begin{aligned}\Delta t &= \frac{v_\text{i}\sin\theta}{g}\\ &= \frac{(14.5\text{ m/s})(\sin 35.0°)}{9.8\text{ m/s}^2}\\ \Delta t &= 0.85\text{ s}\end{aligned}$$

**Statement:** The time for the projectile to reach its maximum height is 0.85 s.

**3.** Solutions may vary. Sample answer:

**(a)** The time of flight is given by $\Delta t = \dfrac{2v_\text{i}\sin\theta}{g}$. So $\Delta t$ is proportional to $v_\text{i}$. When $v_\text{i}$ doubles, $\Delta t$ doubles.

**(b)** The range is given by $\Delta d_x = \dfrac{v_\text{i}^2\sin 2\theta}{g}$. So $\Delta d_x$ is proportional to $v_\text{i}^2$. When $v_\text{i}$ doubles, $\Delta d_x$ increases by a factor of four.

**(c)** The maximum height is given by $\Delta d_y = \dfrac{(v_\text{i}\sin\theta)^2}{2g}$. So $\Delta d_y$ is proportional to $v_\text{i}^2$. When $v_\text{i}$ doubles, $\Delta d_y$ increases by a factor of four.

## Section 1.5 Questions, page 43

**1. Given:** $v_{\text{i}y} = 0$ m/s; $\Delta d_y = -1.5$ m; $\Delta d_x = 8.3$ m; $g = 9.8$ m/s$^2$

**Required:** rock's initial speed, $v_x$

**Analysis:** I know the horizontal displacement and want to determine the constant horizontal speed. The appropriate formula is $\Delta d_x = v_x\Delta t$, but I do not know the time taken.

$\Delta d_x = v_x \Delta t$

$$v_x = \frac{\Delta d_x}{\Delta t}$$

Looking at the vertical motion, determine $\Delta t$ from

$$\Delta d_y = v_{1y}\Delta t - \frac{1}{2}g\Delta t^2 \quad .$$

$$\Delta t = \frac{v_{1y} \pm \sqrt{v_{1y}^2 - 4\left(\frac{1}{2}g\right)(\Delta d_y)}}{g}$$

**Solution:** Using the vertical motion,

$$\Delta t = \frac{v_{1y} \pm \sqrt{v_{1y}^2 - 4\left(\frac{1}{2}g\right)(\Delta d_y)}}{g}$$

$$= \frac{0 \text{ m/s } \pm \sqrt{(0 \text{ m/s})^2 - 4(4.9 \text{ m/s}^2)(-1.5 \text{ m})}}{9.8 \text{ m/s}^2}$$

$\Delta t = \pm 0.5533$ s (two extra digits carried)

Using the horizontal motion,

$$v_x = \frac{\Delta d_x}{\Delta t}$$

$$= \frac{8.3 \text{ m}}{0.5533 \text{ s}}$$

$v_x = 15$ m/s

**Statement:** The initial speed of the rock is 15 m/s.

**2. (a) Given:** $\vec{v}_i = 1.1 \times 10^3$ m/s [45° above the horizontal]; $d_{iy} = d_{fy}$; $g = 9.8$ m/s$^2$

**Required:** $\Delta t$

**Analysis:** Since the projectile lands at the same height from which it was launched, the time taken is given by $\Delta t = \dfrac{2v_i \sin\theta}{g}$.

**Solution:** $\Delta t = \dfrac{2v_i \sin\theta}{g}$

$$= \frac{2(1.1\times 10^3 \ \cancel{\text{m}}/\cancel{\text{s}})(\sin 45°)}{9.8 \ \cancel{\text{m}}/\text{s}^{\cancel{2}}}$$

$$= 158.7 \text{ s}$$

$$\Delta t = 1.6 \times 10^2 \text{ s}$$

**Statement:** The object is in the air for $1.6 \times 10^2$ s.

**(b) Given:** $\vec{v}_{\mathrm{i}} = 1.1\times10^3$ m/s [45° above the horizontal]; $g$ = 9.8 m/s$^2$

**Required:** $\Delta d_x$

**Analysis:** Use the range formula $\Delta d_x = \dfrac{v_{\mathrm{i}}^2 \sin 2\theta}{g}$.

**Solution:**
$$\begin{aligned}
\Delta d_x &= \frac{v_{\mathrm{i}}^2 \sin 2\theta}{g} \\
&= \frac{(1.1\times10^3\ \text{m/s})^2(\sin 90°)}{9.8\ \text{m/s}^2} \\
&= \frac{(1.1\times10^3\ \text{m}/\cancel{\text{s}})(1.1\times10^3\ \cancel{\text{m}}/\cancel{\text{s}})(\sin 90°)}{9.8\ \cancel{\text{m}}/\cancel{\text{s}^2}} \\
&= 1.2\times10^5\ \text{m} \\
\Delta d_x &= 1.2\times10^2\ \text{km}
\end{aligned}$$

**Statement:** The horizontal range of the projectile is $1.2 \times 10^2$ km.

**(c) Given:** $\vec{v}_{\mathrm{i}} = 1.1\times10^3$ m/s [45° above the horizontal]; $g = 9.8$ m/s$^2$

**Required:** maximum height, $d_y$

**Analysis:** Use the formula $\Delta d_y = \dfrac{(v_{\mathrm{i}} \sin\theta)^2}{2g}$

**Solution:**
$$\begin{aligned}
\Delta d_y &= \frac{(v_{\mathrm{i}} \sin\theta)^2}{2g} \\
&= \frac{((1100\ \text{m/s})\sin 45°)^2}{2(9.8\ \text{m/s}^2)} \\
&= 3.1\times10^4\ \text{m} \\
\Delta d_y &= 31\ \text{km}
\end{aligned}$$

**Statement:** The projectile's maximum height is 31 km above the ground.

**3. (a) Given:** $\vec{v}_{\mathrm{i}} = 6.0$ m/s [32° below the horizontal]; $\Delta t = 3.4$ s; $g = 9.8$ m/s$^2$

**Required:** $\Delta d_y$

**Analysis:** First determine the components of the initial velocity. Then use the vertical motion and $\Delta d_y = v_{1y}\Delta t - \dfrac{1}{2} g\Delta t^2$ to solve for the vertical displacement.

**Solution:** The components of the initial velocity are

$v_{\mathrm{i}x} = (6.0\ \text{m/s})(\cos 32°)$

$v_{\mathrm{i}x} = 5.088$ m/s (two extra digits carried)

$v_{\mathrm{i}y} = -(6.0\ \text{m/s})(\sin 32°)$

$v_{\mathrm{i}y} = -3.180$ m/s (two extra digits carried)

Using the vertical motion,

$$\Delta d_y = v_{1y}\Delta t - \frac{1}{2}g\Delta t^2$$

$$= (-3.180\ \text{m}/\cancel{\text{s}})(3.4\ \cancel{\text{s}}) - \frac{1}{2}(9.8\ \text{m}/\cancel{\text{s}^2})(3.4\ \cancel{\text{s}})^2$$

$$\Delta d_y = -67\ \text{m}$$

**Statement:** The ball fell 67 m, so the window was 67 m above the ground.

**(b) Given:** $v_{ix} = 5.088$ m/s; $v_{iy} = -3.180$ m/s; $\Delta t = 3.4$ s; $g = 9.8$ m/s$^2$

**Required:** $\vec{v}_f$

**Analysis:** The horizontal component of the velocity is constant throughout. The vertical component changes with constant acceleration:

$$\Delta v_y = -g\Delta t$$

$$v_{fy} = v_{iy} - g\Delta t$$

Determine the vertical component of the final velocity and then construct the final velocity vector.

**Solution:** Determine the vertical component of the final velocity.

$$v_{fy} = v_{iy} - g\Delta t$$

$$= -3.180\ \text{m/s} - (9.8\ \text{m/s}^{\cancel{2}})(3.4\ \cancel{\text{s}})$$

$$v_{fy} = -36.50\ \text{m/s}\ \text{(two extra digits carried)}$$

Since $v_{fx} = v_x = 5.088$ m/s, I can combine the components to determine the magnitude of $\vec{v}_f$.

$$\left|\vec{v}_f\right| = \sqrt{(v_{fx})^2 + (v_{fy})^2}$$

$$= \sqrt{(5.088\ \text{m/s})^2 + (-36.50\ \text{m/s})^2}$$

$$\left|\vec{v}_f\right| = 37\ \text{m/s}$$

The angle below the horizontal axis of $\vec{v}_f$ is

$$\theta = \tan^{-1}\left(\frac{\left|v_{fy}\right|}{\left|v_{fx}\right|}\right)$$

$$= \tan^{-1}\left(\frac{36.50\ \cancel{\text{m/s}}}{5.088\ \cancel{\text{m/s}}}\right)$$

$$\theta = 82°$$

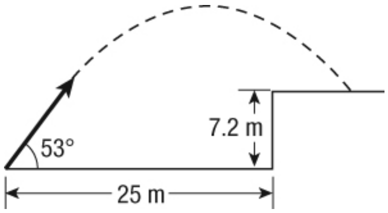
**Statement:** The final velocity of the ball is 37 m/s [82° below the horizontal].

**4. (a) Given:** ball's initial direction, $\theta = 53°$ above the horizontal; $\Delta d_x = 25$ m; $\Delta t = 2.1$ s

**Required:** initial velocity, $\vec{v}_i$

**Analysis:** Draw a diagram of the situation. The horizontal component of velocity is constant. Determine it from the time interval and the horizontal distance using $\Delta d_x = v_x \Delta t$. Then use the cosine of the initial angle to determine the initial speed.

**Solution:**




The $x$-component of $\vec{v}_i$ is

$$\Delta d_x = v_x \Delta t$$

$$v_x = \frac{\Delta d_x}{\Delta t}$$

$$= \frac{25 \text{ m}}{2.1 \text{ s}}$$

$$v_x = 11.91 \text{ m/s} \text{ (two extra digits carried)}$$

Determine the initial speed.

$$\cos\theta = \frac{|v_{ix}|}{v_i}$$

$$v_i = \frac{|v_{ix}|}{\cos\theta}$$

$$= \frac{11.91 \text{ m/s}}{\cos 53°}$$

$$= 19.79 \text{ m/s (two extra digits carried)}$$

$$v_i = 2.0 \times 10^1 \text{ m/s}$$

**Statement:** The initial velocity of the soccer ball is $2.0 \times 10^1$ m/s [53° above the horizontal].

**(b) Given:** $\vec{v}_i = 19.79$ m/s [53° above the horizontal]; $\Delta d_y = 7.2$ m; $g = 9.8$ m/s$^2$

**Required:** horizontal range, $\Delta d_x$

**Analysis:** I know the horizontal component of the initial velocity and want the horizontal distance. I can use $\Delta d_x = v_x \Delta t$ once I know the time of flight. For this I need to look at the vertical motion, $v_{iy} = v_i \sin\theta$ .

Use $\Delta d_y = v_{1y}\Delta t - \frac{1}{2} g \Delta t^2$ to determine $\Delta t = \dfrac{v_{1y} \pm \sqrt{v_{1y}^2 - 4\left(\frac{1}{2}g\right)(\Delta d_y)}}{g}$.

Then solve the horizontal motion.

**Solution:** The vertical component of the initial velocity is

$$v_{iy} = v_i \sin\theta$$

$$= (19.79 \text{ m/s})(\sin 53°)$$

$$v_{iy} = 15.80 \text{ m/s} \text{ (two extra digits carried)}$$

$$\Delta t = \frac{v_{iy} \pm \sqrt{v_{iy}^2 - 4\left(\frac{1}{2}g\right)(\Delta d_y)}}{g}$$

$$= \frac{15.80 \text{ m/s} \pm \sqrt{(-15.80 \text{ m/s})^2 - 4(4.9 \text{ m/s}^2)(7.2 \text{ m})}}{9.8 \text{ m/s}^2}$$

$\Delta t = 2.675$ s or 0.5495 s (two extra digits carried)

The ball lands on the building at about 2.7 s. The other time corresponds to the ball moving through a height of 7.2 m on the way up.

The horizontal displacement of the ball is

$$\Delta d_x = v_x \Delta t$$

$$= (11.91 \text{ m/}\cancel{\text{s}})(2.675 \cancel{\text{s}})$$

$$\Delta d_x = 32 \text{ m}$$

**Statement:** The ball's horizontal range is 32 m.

**(c) Given:** $\Delta d_x = 25$ m; $v_x = 11.91$ m/s; $v_{iy} = 15.80$ m/s ; $g = 9.8$ m/s$^2$

**Required:** ball's clearance above wall based on $\Delta d_y$

**Analysis:** I have calculated the components of the initial velocity above. Use the *x*-component and $\Delta d_x = v_x \Delta t$ to determine the time for the ball to reach the building. Use the *y*-component and $\Delta d_y = v_{1y}\Delta t - \frac{1}{2}g\Delta t^2$ to determine the ball's height at that time. Then calculate how much above 7.2 m the ball is.

**Solution:** Calculate the time for the ball to reach the wall.

$$\Delta d_x = v_x \Delta t$$

$$\Delta t = \frac{\Delta d_x}{v_x}$$

$$= \frac{25 \cancel{\text{m}}}{11.91 \cancel{\text{m}}\text{/s}}$$

$$\Delta t = 2.101 \text{ s (two extra digits carried)}$$

The vertical displacement of the ball at this time is

$$\Delta d_y = v_{1y}\Delta t - \frac{1}{2}g\Delta t^2$$

$$= (15.80 \text{ m/s})(2.101 \text{ s}) - (4.9 \text{ m/s}^2)(2.101 \text{ s})^2$$

$$= 33.19 \text{ m} - 21.63 \text{ m}$$

$$\Delta d_y = 11.56 \text{ m}$$

Determine the distance by which the ball clears the wall.

$$\Delta h = 11.56 \text{ m} - 7.2 \text{ m}$$

$$\Delta h = 4.4 \text{ m}$$

**Statement:** The ball clears the wall by 4.4 m.

**5. (a) Given:** $\vec{v}_{\text{i}} = 26$ m/s [52° above horizontal]; $v_{\text{f}y} = 0$ m/s; $g = 3.7\ \text{m/s}^2$; $g = 9.8\ \text{m/s}^2$

**Required:** maximum height, $d_{\text{f}y}$, based on $\Delta d_y$

**Analysis:** Determine the vertical component of the initial velocity using $v_{\text{i}y} = v_{\text{i}} \sin\theta$.

Then use $v_{\text{f}y}^2 = v_{\text{i}y}^2 - 2g\Delta d_y$.

**Solution:** The vertical component of the initial velocity is

$$\begin{aligned} v_{\text{i}y} &= v_{\text{i}} \sin\theta \\ &= (26\ \text{m/s})(\sin 52°) \\ v_{\text{i}y} &= 20.49\ \text{m/s} \ \ \text{(two extra digits carried)} \end{aligned}$$

The vertical displacement is

$$\begin{aligned} v_{\text{f}y}^2 &= v_{\text{i}y}^2 - 2g\Delta d_y \\ (0\ \text{m/s})^2 &= (20.49\ \text{m/s})^2 - 2(3.7\ \text{m/s}^2)\Delta d_y \\ \Delta d_y &= \frac{(20.49\ \text{m/}\cancel{\text{s}})^{\cancel{2}}}{2(3.7\ \cancel{\text{m}}/\cancel{\text{s}^2})} \\ \Delta d_y &= 57\ \text{m} \end{aligned}$$

**Statement:** The rock rises to a maximum height of 57 m.

**(b) Given:** $\vec{v}_{\text{i}} = 26$ m/s [52° above horizontal]; $\Delta d_y = 12$ m; $g = 3.7\ \text{m/s}^2$

**Required:** time of flight, $\Delta t$

**Analysis:** Use $\Delta d_y = v_{1y}\Delta t - \frac{1}{2}g\Delta t^2$ to determine $\Delta t = \dfrac{v_{1y} \pm \sqrt{v_{1y}^2 - 4\left(\frac{1}{2}g\right)(\Delta d_y)}}{g}$.

**Solution:** $\Delta t = \dfrac{20.488\ \text{m/s} \pm \sqrt{(-20.488\ \text{m/s})^2 - 4(1.85\ \text{m/s}^2)(12\ \text{m})}}{2(1.85\ \text{m/s}^2)}$

$$\begin{aligned} &= 10.45\ \text{s or } 0.621\ \text{s} \ \ \text{(two extra digits carried)} \\ \Delta t &= 1.0 \times 10^1\ \text{s} \end{aligned}$$

**Statement:** The rock strikes the hill after $1.0 \times 10^1$ s.

**(c) Given:** $\vec{v}_{\text{i}} = 26$ m/s [52° above horizontal]; $\Delta = 10.45$ s

**Required:** horizontal range, $\Delta d_x$

**Analysis:** Calculate the horizontal component of the initial velocity using $v_{\text{i}x} = v_{\text{i}} \cos\theta$.

Using $\Delta d_x = v_x \Delta t$, determine the horizontal displacement.

**Solution:** The horizontal component of $\vec{v}_{\text{i}}$ is

$$\begin{aligned} v_{\text{i}x} &= v_{\text{i}} \cos\theta \\ &= (26\ \text{m/s})(\cos 52°) \\ v_{\text{i}x} &= 16.01\ \text{m/s} \ \ \text{(two extra digits carried)} \end{aligned}$$

The horizontal distance travelled is

$$\Delta d_x = v_x \Delta t$$
$$= \left(16.01\ \text{m}/\cancel{\text{s}}\right)(10.45\ \cancel{\text{s}})$$
$$\Delta d_x = 1.7 \times 10^2\,\text{m}$$

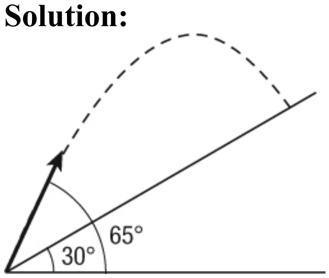
**Statement:** The rock's range is $1.7 \times 10^2\,\text{m}$.

**6. Given:** $\vec{v}_\text{i} = 16$ m/s [65° above horizontal]; angle of inclination of hill $\phi = 30°$; $g = 9.8$ m/s$^2$

**Required:** distance up the hill where rock lands, $l$

**Analysis:** Draw a diagram of the situation. I know the initial velocity of the rock. From $\Delta d_x = v_x \Delta t$ and $\Delta d_y = v_{1y}\Delta t - \frac{1}{2}g\Delta t^2$, I can determine where the rock is at any later time. The rock will hit the hill when its displacement components are in the correct ratio: $\tan\phi = \dfrac{\Delta d_y}{\Delta d_x}$.

**Solution:**




Determine the $x$- and $y$-components of the initial velocity.

$$v_{\text{i}x} = v_\text{i}\cos\theta$$
$$= (16\ \text{m/s})(\cos 65°)$$
$$v_{\text{i}x} = 6.762\ \text{m/s}\ \ \text{(two extra digits carried)}$$
$$v_{\text{i}y} = v_\text{i}\cos\theta$$
$$= (16\ \text{m/s})(\sin 65°)$$
$$v_{\text{i}y} = 14.50\ \text{m/s}\ \ \text{(two extra digits carried)}$$

Write equations for the $x$- and $y$-components of the displacement after time $\Delta t$.

$$\Delta d_x = v_x \Delta t$$
$$\Delta d_x = (6.762\ \text{m/s})\Delta t$$

$$\Delta d_y = v_{1y}\Delta t - \frac{1}{2}g\Delta t^2$$
$$\Delta d_y = (14.50\ \text{m/s})\Delta t - (4.9\ \text{m/s}^2)\Delta t^2$$

Use $\tan\phi = \dfrac{\Delta d_y}{\Delta d_x}$ to build an equation for the time taken to reach the hill, $\Delta t$.

$$\tan\phi = \frac{\Delta d_y}{\Delta d_x}$$

$$\tan 30° = \frac{(14.50\text{ m/s})\Delta t - (4.9\text{ m/s}^2)\Delta t^2}{(6.762\text{ m/s})\Delta t}$$

$$(0.5774)(6.762\text{ m/s})\Delta t = (14.50\text{ m/s})\Delta t - (4.9\text{ m/s}^2)\Delta t^2$$

$$(4.9\text{ m/s}^2)\Delta t^2 + (3.904\text{ m/s})\Delta t - (14.50\text{ m/s})\Delta t = 0$$

$$(4.9\text{ m/s}^2)\Delta t^2 - (10.60\text{ m/s})\Delta t = 0$$

$$[(4.9\text{ m/s}^2)\Delta t - (10.60\text{ m/s})]\Delta t = 0$$

One solution is $\Delta t = 0$ s, when the rock is thrown from the bottom of the hill. The other solution is as follows:

$$(4.9\text{ m/s}^2)\Delta t - (10.60\text{ m/s}) = 0$$

$$\Delta t = \frac{10.60\ \cancel{\text{m}}/\cancel{\text{s}}}{4.9\ \cancel{\text{m}}/\text{s}^{\cancel{2}}}$$

$$\Delta t = 2.163\text{ s (two extra digits carried)}$$

The $x$- and $y$-components of the displacement at this time are

$$\Delta d_x = (6.762\text{ m/s})\Delta t$$
$$= (6.762\text{ m/}\cancel{\text{s}})(2.163\ \cancel{\text{s}})$$
$$\Delta d_x = 14.63\text{ m (two extra digits carried)}$$

$$\Delta d_y = (14.50\text{ m/s})\Delta t - (4.9\text{ m/s}^2)\Delta t^2$$
$$= (14.50\text{ m/}\cancel{\text{s}})(2.163\ \cancel{\text{s}}) - (4.9\text{ m/}\cancel{\text{s}^2})(2.163\ \cancel{\text{s}})^2$$
$$= 31.37\text{ m} - 22.92\text{ m}$$
$$\Delta d_y = 8.45\text{ m}$$

The distance up the hill is

$$l = \sqrt{(14.63\text{ m})^2 + (8.45\text{ m})^2}$$
$$l = 17\text{ m}$$

**Statement:** The rock lands 17 m up the hill.

**7.** Solutions may vary. Sample answer:

**Given:** $\vec{v}_i = 45$ m/s [35° above horizontal]; $d_{ix} = 0$ m; $d_{iy} = 12$ m; $g = 9.8\text{ m/s}^2$

**Required:** whether the snowball lands on a 25 m high, 35 m wide building, 150 m away

**Analysis:** One approach is to determine the horizontal displacement when the snowball is 25 m up. If the snowball is between 150 m and 185 m horizontally from its launch position, then it is over the building—and lands on the building.

Determine the components of the initial velocity, calculate the time to reach $d_y = 25$ m, and then determine $d_x$ at that time.

**Solution:** The *x*- and *y*-components of the initial velocity are

$$v_{ix} = v_i \cos\theta$$
$$= (45 \text{ m/s})(\cos 35°)$$
$$v_{ix} = 36.86 \text{ m/s} \text{ (two extra digits carried)}$$
$$v_{iy} = v_i \cos\theta$$
$$= (45 \text{ m/s})(\sin 35°)$$
$$v_{iy} = 25.81 \text{ m/s} \text{ (two extra digits carried)}$$

The time to reach a height of 35 m starting at 12 m is

$$\Delta d_y = v_{1y}\Delta t - \frac{1}{2}g\Delta t^2$$
$$13 \text{ m} = (25.81 \text{ m/s})\Delta t - \frac{1}{2}(9.8 \text{ m/s}^2)\Delta t^2$$
$$(4.9 \text{ m/s}^2)\Delta t^2 - (25.81 \text{ m/s})\Delta t + 13 \text{ m} = 0$$

Solve the quadratic equation for $\Delta t$.

$$\Delta t = \frac{(25.81 \text{ m/s}) \pm \sqrt{(25.81 \text{ m/s})^2 - 4(4.9 \text{ m/s}^2)(13 \text{ m})}}{9.8 \text{ m/s}^2}$$
$$\Delta t = 0.564 \text{ s or } 4.703 \text{ s} \text{ (two extra digits carried)}$$

The first solution represents the snowball passing through the height of 25 m on its way up. The second solution is the one where the snowball may be over the building. Determine the horizontal displacement when $\Delta t = 4.703$ s.

$$\Delta d_x = (36.86 \text{ m/s})\Delta t$$
$$= (36.86 \text{ m/s})(4.703 \text{ s})$$
$$\Delta d_x = 170 \text{ m}$$

The horizontal displacement is indeed between 150 m and 185 m.
**Statement:** Yes, the snowball lands on top of building 2.
**8.** Solutions may vary. A written explanation and an algebraic solution are presented.
**Written Explanation:** All objects fall with the same acceleration regardless of mass. When a projectile is fired at a target, its path is formed by the action of gravity together with its given velocity. If gravity were to stop acting, the projectile would follow a straight line directly from the launcher to the target. When gravity is acting, both the projectile and falling target fall at the same speed, even though the projectile is travelling faster horizontally than the target. So at some point along the path of the dropped target, the projectile will hit it because both the projectile and the target are falling at the same speed. Given enough distance, they will both hit the floor at the same time. If the projectile is fired with a greater velocity, the target will not fall as far before it is hit by the projectile. In this case the projectile will follow a straighter path to the target. If the projectile is fired more slowly, it will follow a more curved path and hit the target farther down toward the ground. As long as the projectile launcher is aimed directly at the target and the projectile has enough velocity to reach the target before it hits the ground, the projectile will hit the target as it falls.

**Algebraic Solution:**

**Given:** projectile's motion: $\vec{v}_{\text{i}}$ at $\theta$ above horizontal; $\vec{d}_{\text{i}} = 0$ m; target: $\vec{v}_{\text{i}} = 0$ m/s;

$d_{iy} = d_{ix} \tan\theta$; $g = 9.8 \text{ m/s}^2$

**Required:** Show that the projectile hits the target.

**Analysis:** Write the equations for the position of the projectile after time $\Delta t$. Also do this for the target. Then compare the equations to determine the time when the *x*-components of the two objects coincide. Check whether the *y*-components coincide at the same time.

**Solution:**

projectile:

$$\Delta d_x = v_{ix}\Delta t$$

$$\Delta d_x = v_{i}\cos\theta\Delta t$$

$$d_x = v_{i}\cos\theta\Delta t \quad \text{(Equation 1)}$$

$$\Delta d_y = v_{1y}\Delta t - \frac{1}{2}g\Delta t^2$$

$$d_y = v_1 \sin\theta\Delta t - \frac{1}{2}g\Delta t^2 \quad \text{(Equation 2)}$$

target:

$$\Delta d_x = v_{ix}\Delta t$$

$$\Delta d_x = 0 \text{ m}$$

$$d_x = d_{ix} \quad \text{(Equation 3)}$$

$$\Delta d_y = v_{1y}\Delta t - \frac{1}{2}g\Delta t^2$$

$$\Delta d_y = (0 \text{ m/s})\Delta t - \frac{1}{2}g\Delta t^2$$

$$d_y = d_{iy} - \frac{1}{2}g\Delta t^2 \quad \text{(Equation 4)}$$

Compare the horizontal positions of the projectile and the target using Equations 1 and 3.

$$v_{i}\cos\theta\Delta t = d_{ix}$$

$$\Delta t = \frac{d_{ix}}{v_{i}\cos}$$

Compare the vertical positions of the projectile and the target using Equations 2 and 3, and also the given fact $d_{iy} = d_{ix}\tan\theta$.

$$v_1 \sin\theta\,\Delta t - \frac{1}{2}g\Delta t^2 = d_{iy} - \frac{1}{2}g\Delta t^2$$

$$v_1 \sin\theta\,\Delta t = d_{iy}$$

$$\Delta t = \frac{d_{iy}}{v_1 \sin\theta}$$

$$= \frac{d_{ix}\tan\theta}{v_1 \sin\theta}$$

$$= \frac{d_{ix}\dfrac{\cancel{\sin\theta}}{\cos\theta}}{v_1 \cancel{\sin\theta}}$$

$$\Delta t = \frac{d_{ix}}{v_{i}\cos\theta}$$

$$\Delta t = \frac{d_{iy}}{v_1 \sin\theta}$$
$$= \frac{d_{ix} \tan\theta}{v_1 \sin\theta}$$
$$= \frac{d_{ix} \frac{\cancel{\sin\theta}}{\cos\theta}}{v_1 \cancel{\sin\theta}}$$
$$\Delta t = \frac{d_{ix}}{v_i \cos\theta}$$

The $x$-positions of the projectile and target are equal at the same time that the $y$-positions are equal. In other words, the projectile hits the target.

**Statement:** The equations of motion describe motion at constant velocity, except for the $-\frac{1}{2} g\Delta t^2$ term in the $y$-direction. Since this term is the same for both the projectile and target, they will be equally affected by gravity. The projectile was aimed directly at the target, so it will remain headed for the target as gravity pulls them both downward. So the projectile will hit the target.

**9.** Solutions may vary. Sample answer:

**Given:** football: $\vec{v}_i = 18$ m/s [39° above horizontal]; $d_{ix} = 0$ m; $d_{iy} = 22$ m;

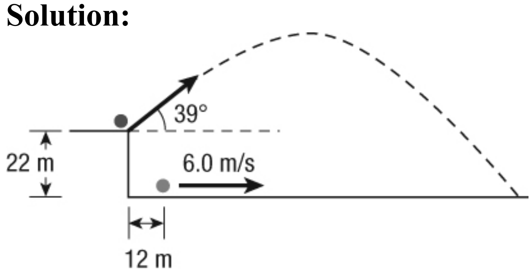
player: $\vec{v} = 6.0$ m/s [horizontally]; $d_{ix} = 12$ m; $d_{iy} = 0$ m; $g = 9.8$ m/s$^2$

**Required:** whether the player can catch the ball

**Analysis:** Draw a diagram for the situation. There is enough information to determine where and when the football would hit the ground. Use $\Delta d_y = v_{1y}\Delta t - \frac{1}{2} g\Delta t^2$ to determine $\Delta t = \dfrac{v_{1y} \pm \sqrt{v_{1y}^2 - 4\left(\frac{1}{2}g\right)(\Delta d_y)}}{g}$.

Check whether or not the player has enough time to get to the correct position while running at constant speed.

**Solution:**



Determine the vertical speed.

$$v_{1y} = v_1 \sin\theta$$
$$= (18 \text{ m/s})(\sin 39°)$$
$$v_{1y} = 11.33 \text{ m/s (two extra digits carried)}$$

Determine when the football hits the ground.

$$\Delta t = \frac{v_{1y} \pm \sqrt{v_{1y}^2 - 4\left(\frac{1}{2}g\right)(\Delta d_y)}}{g}$$
$$= \frac{(11.33 \text{ m/s}) \pm \sqrt{(11.33 \text{ m/s})^2 - 4(4.9 \text{ m/s}^2)(-22 \text{ m})}}{9.8 \text{ m/s}^2}$$
$$\Delta t = 3.570 \text{ s or } -1.258 \text{ s (two extra digits carried)}$$

Time must be a positive quantity, so the time when the football hits the ground is 3.570 s.

$$\Delta d_x = v_{ix}\Delta t$$
$$\Delta d_x = v_i \cos\theta \Delta t$$
$$= (18 \text{ m/s})(\cos 39°)(3.570 \text{ s})$$
$$\Delta d_x = 64 \text{ m}$$

The football will hit the ground about 64 m from the cliff, about 3.6 s after being thrown.
At this time the player can make it to the position

$$\Delta d_x = v_x \Delta t$$
$$= (6.0 \text{ m/s})(3.570 \text{ s})$$
$$\Delta d_x = 21.42 \text{ m}$$

$$d_x = 12 \text{ m} + 21.42 \text{ m}$$
$$d_x = 32 \text{ m}$$

The player is about 32 m short of catching the football.
**Statement:** No, the player cannot run far enough to catch the ball.