## Section 1.4: Velocity and Acceleration in Two Dimensions

### Tutorial 1 Practice, page 32

**1. (a) Given:** $\Delta\vec{d}_1 = 72.0$ km [W 30.0° S]; $\Delta\vec{d}_2 = 48.0$ km [S]; $\Delta\vec{d}_3 = 150.0$ km [W]; $\Delta t = 2.5$ h

**Required:** $\Delta\vec{d}$

**Analysis:** $\Delta\vec{d} = \Delta\vec{d}_1 + \Delta\vec{d}_2 + \Delta\vec{d}_3$. Determine the *x*- and *y*-components of the displacement vectors. Then determine the *x*- and *y*-components of the total displacement using $\Delta d_x = \Delta d_{1x} + \Delta d_{2x} + \Delta d_{3x}$ and $\Delta d_y = \Delta d_{1y} + \Delta d_{2y} + \Delta d_{3y}$. Finally, combine these components to determine the total displacement vector using the Pythagorean theorem and the tangent ratio. Use east and north as positive.

**Solution:** *x*-component of $\Delta\vec{d}$:

$$\begin{aligned}
\Delta d_x &= \Delta d_{1x} + \Delta d_{2x} + \Delta d_{3x} \\
&= -(72.0\text{ km})(\cos 30.0°) + 0\text{ km} + (-150.0\text{ km}) \\
&= -62.35\text{ km} + 0\text{ km} + (-150.0\text{ km})
\end{aligned}$$

$\Delta d_x = -212.4$ km (two extra digits carried)

*y*-component of $\Delta\vec{d}$:

$$\begin{aligned}
\Delta d_y &= \Delta d_{1y} + \Delta d_{2y} + \Delta d_{3y} \\
&= -(72.0\text{ km})(\sin 30.0°) + \left(-48.0\text{ km}\right) + 0\text{ km} \\
&= -36.0\text{ km} + (-48.0\text{ km}) + 0\text{ km}
\end{aligned}$$

$\Delta d_y = -84.0$ km (one extra digit carried)

Combine the total displacement components to determine the total displacement.

$$\begin{aligned}
\left|\Delta\vec{d}\right| &= \sqrt{\Delta d_x^2 + \Delta d_y^2} \\
&= \sqrt{(-212.4\text{ km})^2 + (-84.0\text{ km})^2} \\
&= 228.4\text{ km (two extra digits carried)}
\end{aligned}$$

$\left|\Delta\vec{d}\right| = 230$ km

$$\begin{aligned}
\theta &= \tan^{-1}\left(\frac{\left|\Delta d_y\right|}{\left|\Delta d_x\right|}\right) \\
&= \tan^{-1}\left(\frac{84.0\ \cancel{\text{km}}}{212.4\ \cancel{\text{km}}}\right)
\end{aligned}$$

$\theta = 22°$

**Statement:** The total displacement is 230 km [W 22° S].

**(b) Given:** $\Delta\vec{d} = 230$ km [W 22º S]; $\Delta t = 2.5$ h

**Required:** $\vec{v}_{\text{av}}$

**Analysis:** Use $\vec{v}_{\text{av}} = \dfrac{\Delta\vec{d}}{\Delta t}$ to determine the average velocity.

**Solution:** $\vec{v}_{av} = \dfrac{\Delta\vec{d}}{\Delta t}$

$= \dfrac{228.4 \text{ km [W } 22° \text{ S]}}{2.5 \text{ h}}$

$\vec{v}_{av} = 91 \text{ km/h [W } 22° \text{ S]}$

**Statement:** The average velocity of the plane is 91 km/h [W 22° S].

**(c) Given:** $\Delta\vec{d}_1 = 72.0 \text{ km [W } 30.0° \text{ S]}$; $\Delta\vec{d}_2 = 48.0 \text{ km [S]}$; $\Delta\vec{d}_3 = 150.0 \text{ km [W]}$; $\Delta t = 2.5 \text{ h}$

**Required:** $v_{av}$

**Analysis:** Calculate the total distance travelled and then use $v_{av} = \dfrac{\Delta d}{\Delta t}$ to determine the average speed.

**Solution:** $\Delta d = \Delta d_1 + \Delta d_2 + \Delta d_3$

$= 72.0 \text{ km} + 48.0 \text{ km} + 150.0 \text{ km}$

$\Delta d = 270.0 \text{ km}$ (two extra digits carried)

$v_{av} = \dfrac{\Delta d}{\Delta t}$

$= \dfrac{270.0 \text{ km}}{2.5 \text{ h}}$

$= 108 \text{ km/h}$

$v_{av} = 110 \text{ km/h}$

**Statement:** The average speed of the plane is 110 km/h.

**2. (a) Given:** $\Delta\vec{d}_1 = 25.0 \text{ km [E } 53.13° \text{ N]}$; $\Delta\vec{d}_2 = 20.0 \text{ km [S]}$; $\Delta\vec{d}_3 = 15.0 \text{ km [W]}$; $\Delta t = 12 \text{ h}$

**Required:** $\vec{v}_{av}$

**Analysis:** Calculate the total displacement using components and $\Delta\vec{d} = \Delta\vec{d}_1 + \Delta\vec{d}_2 + \Delta\vec{d}_3$. Then, calculate the average velocity using $\vec{v}_{av} = \dfrac{\Delta\vec{d}}{\Delta t}$. Use east and north as positive.

**Solution:** $x$-component of $\Delta\vec{d}$:

$\Delta d_x = \Delta d_{1x} + \Delta d_{2x} + \Delta d_{3x}$

$= (25.0 \text{ km})(\cos 53.13°) + 0 \text{ km} + (-15.0 \text{ km})$

$= 15.0 \text{ km} + 0 \text{ km} + (-15.0 \text{ km})$

$\Delta d_x = 0 \text{ km}$

$y$-component of $\Delta\vec{d}$:

$\Delta d_y = \Delta d_{1y} + \Delta d_{2y} + \Delta d_{3y}$

$= (25.0 \text{ km})(\sin 53.13°) + (-20.0 \text{ km}) + 0 \text{ km}$

$= 20.0 \text{ km} + (-20.0 \text{ km}) + 0 \text{ km}$

$\Delta d_y = 0 \text{ km}$

Both components of the total displacement are 0 km. Hence the total displacement is $\Delta\vec{d} = 0 \text{ km}$. As a result, the average velocity is 0 km/h.

**Statement:** The average velocity of the elk is 0 km/h.

**(b) Given:** $\Delta d_1 = 25.0\text{ km}$; $\Delta d_2 = 20.0\text{ km}$; $\Delta d_3 = 15.0\text{ km}$; $\Delta t = 12\text{ h}$

**Required:** $v_{\text{av}}$

**Analysis:** Calculate the total distance travelled and then use $v_{\text{av}} = \dfrac{\Delta d}{\Delta t}$ to determine the average speed.

**Solution:**

$$\begin{aligned}\Delta d &= \Delta d_1 + \Delta d_2 + \Delta d_3\\ &= 25.0\text{ km} + 20.0\text{ km} + 15.0\text{ km}\\ \Delta d &= 60.0\text{ km (one extra digit carried)}\end{aligned}$$

$$\begin{aligned}v_{\text{av}} &= \frac{\Delta d}{\Delta t}\\ &= \frac{60.0\text{ km}}{18\text{ h}}\\ v_{\text{av}} &= 5.0\text{ km/h}\end{aligned}$$

**Statement:** The average speed of the elk is 5.0 km/h.

**(c)** Answers may vary. Sample answer: While the elk travelled 60.0 km, it returned to its starting place. Average velocity depends on displacement, a vector. From a vector point of view, the elk did not move, resulting in an average velocity of 0 km/h.

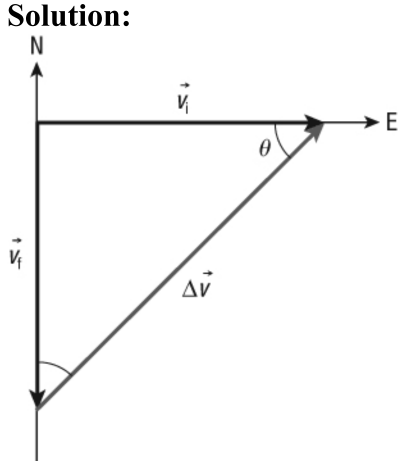
## Tutorial 2 Practice, page 34

**1. Given:** $\vec{v}_{\text{i}} = 20.0\text{ m/s [E]}$; $\vec{v}_{\text{f}} = 20.0\text{ m/s [S]}$; $\Delta t = 12\text{ s}$

**Required:** $\vec{a}_{\text{av}}$

**Analysis:** Draw a vector diagram of the situation. Calculate the change in velocity using components and $\Delta\vec{v} = \vec{v}_{\text{f}} - \vec{v}_{\text{i}}$. Then, determine the average acceleration using $\vec{a}_{\text{av}} = \dfrac{\Delta\vec{v}}{\Delta t}$. Use east and north as positive.

**Solution:**




*x*-component of $\Delta\vec{v}$:

$$\begin{aligned}\Delta v_x &= v_{\text{f}x} - v_{\text{i}x}\\ &= 0\text{ m/s} - 20.0\text{ m/s}\\ \Delta v_x &= -20.0\text{ m/s}\end{aligned}$$

$y$-component of $\Delta\vec{v}$ :

$$\Delta v_y = v_{fy} - v_{iy}$$
$$= (-20 \text{ m/s}) - 0 \text{ m/s}$$
$$\Delta v_y = -20.0 \text{ m/s}$$

Determine the change in velocity from its components.

$$\left|\Delta\vec{v}\right| = \sqrt{\Delta v_x^2 + \Delta v_y^2}$$
$$= \sqrt{(-20.0 \text{ m/s})^2 + (-20.0 \text{ m/s})^2}$$
$$\left|\Delta\vec{v}\right| = 28.28 \text{ m/s} \text{ (two extra digits carried)}$$

$$\theta = \tan^{-1}\left(\frac{\left|\Delta v_y\right|}{\left|\Delta v_x\right|}\right)$$
$$= \tan^{-1}\left(\frac{20.0 \cancel{\text{m/s}}}{20.0 \cancel{\text{m/s}}}\right)$$
$$\theta = 45°$$

Calculate the average acceleration.

$$\vec{a}_{av} = \frac{\Delta\vec{v}}{\Delta t}$$
$$= \frac{28.28 \text{ m/s [W 45° S]}}{12 \text{ s}}$$
$$\vec{a}_{av} = 2.4 \text{ m/s}^2 \text{ [W 45° S]}$$

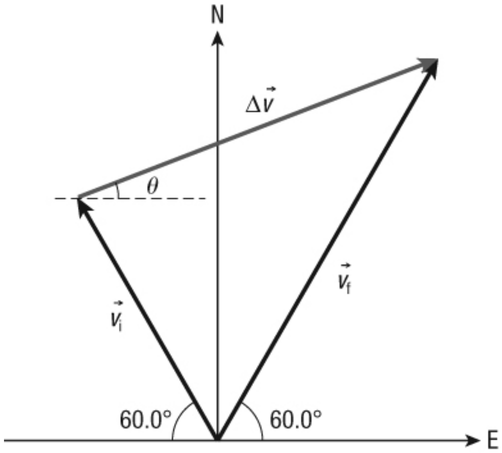
**Statement:** The average acceleration of the car is $2.4 \text{ m/s}^2$ [W 45° S].

**2. Given:** $\vec{v}_i = 50.0$ km/h [W 60.0° N]; $\vec{v}_f = 80.0$ km/h [E 60.0° N]; $\Delta t = 15.0 \text{ min} = 0.250 \text{ h}$

**Required:** $\vec{a}_{av}$

**Analysis:** Draw a vector diagram of the situation. Calculate the change in velocity using components and $\Delta\vec{v} = \vec{v}_f - \vec{v}_i$. Then, determine the average acceleration using $\vec{a}_{av} = \frac{\Delta\vec{v}}{\Delta t}$. Use east and north as positive.

**Solution:**




The $x$-component of $\Delta\vec{v}$ is

$$\begin{aligned}\Delta v_x &= v_{fx} - v_{ix}\\ &= (80.0\text{ km/h})(\cos 60.0°) - (-(50.0\text{ km/h})\cos 60.0°)\\ &= 40.0\text{ km/h} - (-25.0\text{ km/h})\\ \Delta v_x &= 65.0\text{ km/h}\end{aligned}$$

The $y$-component of $\Delta\vec{v}$ is

$$\begin{aligned}\Delta v_y &= v_{fy} - v_{iy}\\ &= (80.0\text{ km/h})(\sin 60.0°) - (50.0\text{ km/h})(\sin 60.0°)\\ &= 69.28\text{ km/h} - 43.30\text{ km/h}\\ \Delta v_y &= 25.98\text{ km/h (one extra digit carried)}\end{aligned}$$

Determine the change in velocity from its components.

$$\begin{aligned}\left|\Delta\vec{v}\right| &= \sqrt{\Delta v_x^2 + \Delta v_y^2}\\ &= \sqrt{(65.0\text{ km/h})^2 + (25.98\text{ km/h})^2}\\ \left|\Delta\vec{v}\right| &= 70.00\text{ km/h (one extra digit carried)}\end{aligned}$$

$$\begin{aligned}\theta &= \tan^{-1}\left(\frac{\left|\Delta v_y\right|}{\left|\Delta v_x\right|}\right)\\ &= \tan^{-1}\left(\frac{25.98\ \cancel{\text{km/h}}}{65.0\ \cancel{\text{km/h}}}\right)\\ \theta &= 21.8°\end{aligned}$$

Calculate the average acceleration.

$$\vec{a}_{\text{av}} = \frac{\Delta\vec{v}}{\Delta t}$$
$$= \frac{70.00 \text{ km/h [E 21.8° N]}}{0.250 \text{ h}}$$
$$\vec{a}_{\text{av}} = 2.80 \times 10^2 \text{ km/h}^2 \text{ [E 21.8° N]}$$

**Statement:** The average acceleration of the truck is $2.80 \times 10^2$ km/h$^2$ [E 21.8° N].

**3. (a) Given:** $\Delta\vec{d}_1 = 800.0$ km [E 7.5° S]; $\Delta\vec{d}_2 = 400.0$ km [E 51° S]; $\Delta t = 18.0$ h

**Required:** $\Delta d$

**Analysis:** Calculate the total distance travelled by adding the individual distances travelled, $\Delta d = \Delta d_1 + \Delta d_2$.

**Solution:** $\Delta d = \Delta d_1 + \Delta d_2$

$$= 800.0 \text{ km} + 400.0 \text{ km}$$
$$\Delta d = 1.2 \times 10^3 \text{ km}$$

**Statement:** The total distance travelled by the bird is $1.2 \times 10^3$ km.

**(b) Given:** $\Delta\vec{d}_1 = 800.0$ km [E 7.5° S]; $\Delta\vec{d}_2 = 400.0$ km [E 51° S]

**Required:** $\Delta\vec{d}$

**Analysis:** $\Delta\vec{d} = \Delta\vec{d}_1 + \Delta\vec{d}_2$. Determine the *x*- and *y*-components of the displacement vectors. Then determine the *x*- and *y*-components of the total displacement using $\Delta d_x = \Delta d_{1x} + \Delta d_{2x}$ and $\Delta d_y = \Delta d_{1y} + \Delta d_{2y}$. Finally, combine these components to determine the total displacement vector using the Pythagorean theorem and the tangent ratio. Use east and north as positive.

**Solution:** The *x*-component of $\Delta\vec{d}$ is

$$\Delta d_x = \Delta d_{1x} + \Delta d_{2x}$$
$$= (800.0 \text{ km})(\cos 7.5°) + (400.0 \text{ km})(\cos 51°)$$
$$= 793.1 \text{ km} + 251.7 \text{ km}$$
$$\Delta d_x = 1045 \text{ km (two extra digits carried)}$$

The *y*-component of $\Delta\vec{d}$ is:

$$\Delta d_y = \Delta d_{1y} + \Delta d_{2y}$$
$$= -(400.0 \text{ km})(\sin 7.5°) + (-(800.0 \text{ km})\sin 51°)$$
$$= -104.4 \text{ km} + (-310.9 \text{ km})$$
$$\Delta d_y = -415.3 \text{ km (two extra digits carried)}$$

Combine the total displacement components to determine the total displacement.

$$
\begin{aligned}
\left|\Delta\vec{d}\right| &= \sqrt{\Delta d_x^2 + \Delta d_y^2} \\
&= \sqrt{(-1045\text{ km})^2 + (-415.3\text{ km})^2} \\
&= 1124\text{ km}
\end{aligned}
$$

$$\left|\Delta\vec{d}\right| = 1.1\times10^3\text{ km}$$

$$
\begin{aligned}
\theta &= \tan^{-1}\left(\frac{\left|\Delta d_y\right|}{\left|\Delta d_x\right|}\right) \\
&= \tan^{-1}\left(\frac{415.3\ \cancel{\text{km}}}{1045\ \cancel{\text{km}}}\right)
\end{aligned}
$$

$$\theta = 22°$$

**Statement:** The total displacement is $1.1\times10^3$ km [E 22° S].

**(c) Given:** $\Delta t = 18.0$ h; $\Delta d = 1200$ km

**Required:** $v_{\text{av}}$

**Analysis:** Use $v_{\text{av}} = \dfrac{\Delta d}{\Delta t}$ to determine the average speed.

**Solution:**
$$
\begin{aligned}
v_{\text{av}} &= \frac{\Delta d}{\Delta t} \\
&= \frac{1200\text{ km}}{18.0\text{ h}} \\
v_{\text{av}} &= 67\text{ km/h}
\end{aligned}
$$

**Statement:** The average speed of the bird is 67 km/h.

**(d) Given:** $\Delta t = 18.0$ h; $\Delta\vec{d} = 1124$ km [E 22° S]

**Required:** $\vec{v}_{\text{av}}$

**Analysis:** Use $\vec{v}_{\text{av}} = \dfrac{\Delta\vec{d}}{\Delta t}$ to determine the average velocity.

**Solution:**
$$
\begin{aligned}
\vec{v}_{\text{av}} &= \frac{\Delta\vec{d}}{\Delta t} \\
&= \frac{1124\text{ km [E 22° S]}}{18.0\text{ h}} \\
\vec{v}_{\text{av}} &= 62\text{ km/h [E 22° S]}
\end{aligned}
$$

**Statement:** The average velocity of the bird is 62 km/h [E 22° S].

## Section 1.4 Questions, page 35

**1.** Answers may vary. Sample answer: The average speed is based on the distance travelled. Changes in direction do not affect the average speed. The average velocity is based on the displacement. The displacement is affected by changes in direction: it acts like the short-cut distance from start to finish. For example, a dog walks 40 m [E] and then 20 m [W] in 60 s. It travels a total of 60 m, giving an average speed of 1 m/s. The dog's displacement is 20 m [E] because it backtracked. You do not see the individual displacements of its walk. The dog's average velocity is 0.33 m/s [E]. Based just on the start and finish, the dog appears to move quite slowly.

**2. (a) Given:** $\Delta\vec{d}_1 = 25.0$ m [E 30.0° N]; $\Delta\vec{d}_2 = 75.0$ km [E 45.0° S]; $\Delta t = 4.0 \text{ min} = 240 \text{ s}$

**Required:** $\Delta d$

**Analysis:** Calculate the total distance travelled by adding the individual distances travelled, $\Delta d = \Delta d_1 + \Delta d_2$.

**Solution:**
$$\begin{aligned}\Delta d &= \Delta d_1 + \Delta d_2\\ &= 25.0 \text{ m} + 75.0 \text{ m}\\ \Delta d &= 1.0 \times 10^2 \text{ m}\end{aligned}$$

**Statement:** The total distance travelled by the loon is $1.0 \times 10^2$ m.

**(b) Given:** $\Delta\vec{d}_1 = 25.0$ m [E 30.0° N]; $\Delta\vec{d}_2 = 75.0$ km [E 45.0° S]

**Required:** $\Delta\vec{d}$

**Analysis:** $\Delta\vec{d} = \Delta\vec{d}_1 + \Delta\vec{d}_2$; Determine the *x*- and *y*-components of the displacement vectors. Then determine the *x*- and *y*-components of the total displacement using $\Delta d_x = \Delta d_{1x} + \Delta d_{2x}$ and $\Delta d_y = \Delta d_{1y} + \Delta d_{2y}$. Finally, combine these components to determine the total displacement vector using the Pythagorean theorem and the tangent ratio. Use east and north as positive.

**Solution:** The *x*-component of $\Delta\vec{d}$ is
$$\begin{aligned}\Delta d_x &= \Delta d_{1x} + \Delta d_{2x}\\ &= (25.0 \text{ m})(\cos 30.0°) + (75.0 \text{ m})(\cos 45.0°)\\ &= 21.6506 \text{ m} + 53.0330 \text{ m}\\ \Delta d_x &= 74.684 \text{ m (two extra digits carried)}\end{aligned}$$

The *y*-component of $\Delta\vec{d}$ is
$$\begin{aligned}\Delta d_y &= \Delta d_{1y} + \Delta d_{2y}\\ &= (25.0 \text{ m})(\sin 30.0°) + (-(75.0 \text{ m})\sin 45.0°)\\ &= 12.5 \text{ m} + \left(-53.0330 \text{ m}\right)\\ \Delta d_y &= -40.533 \text{ m (two extra digits carried)}\end{aligned}$$

Combine the total displacement components to determine the total displacement.

$$\left|\Delta\vec{d}\right| = \sqrt{\Delta d_x^2 + \Delta d_y^2}$$
$$= \sqrt{(74.684 \text{ m})^2 + (-40.533 \text{ m})^2}$$
$$= 84.974 \text{ m (two extra digits carried)}$$
$$\left|\Delta\vec{d}\right| = 85.0 \text{ m}$$
$$\theta = \tan^{-1}\left(\frac{\left|\Delta d_y\right|}{\left|\Delta d_x\right|}\right)$$
$$= \tan^{-1}\left(\frac{40.533 \ \cancel{\text{km}}}{74.684 \ \cancel{\text{km}}}\right)$$
$$\theta = 28.5°$$

**Statement:** The loon's total displacement is 85.0 m [E 28.5° S].

**(c) Given:** $\Delta d = 1.0 \times 10^2$ m; $\Delta t = 4.0$ min $= 240$ s

**Required:** $v_{\text{av}}$

**Analysis:** Use $v_{\text{av}} = \frac{\Delta d}{\Delta t}$ to determine the average speed.

**Solution:** $v_{\text{av}} = \frac{\Delta d}{\Delta t}$
$$= \frac{1.0 \times 10^2 \text{ m}}{240 \text{ s}}$$
$$v_{\text{av}} = 0.42 \text{ m/s}$$

**Statement:** The average speed of the loon is 0.42 m/s.

**(d) Required:** $\vec{v}_{\text{av}}$

**Analysis:** Use $\vec{v}_{\text{av}} = \frac{\Delta\vec{d}}{\Delta t}$ to determine the average velocity.

**Solution:** $\vec{v}_{\text{av}} = \frac{\Delta\vec{d}}{\Delta t}$
$$= \frac{84.9739 \text{ m [E 28.5° S]}}{240 \text{ s}}$$
$$\vec{v}_{\text{av}} = 0.35 \text{ m/s [E 28.5° S]}$$

**Statement:** The average velocity of the loon is 0.35 m/s [E 28.5° S].

**3. (a) Given:** $\Delta\vec{d}_1 = 15.0$ km [W 30.0° N]; $\Delta\vec{d}_2 = 10.0$ km [W 75.0° N]; $\Delta\vec{d}_3 = 10.0$ km [E 70.0° N]; $\Delta t = 0.50$ h

**(b) Required:** $\vec{v}_{\text{av}}$

**Analysis:** Calculate the total displacement using components and $\Delta\vec{d} = \Delta\vec{d}_1 + \Delta\vec{d}_2 + \Delta\vec{d}_3$. Then, calculate the average velocity using $\vec{v}_{\text{av}} = \dfrac{\Delta\vec{d}}{\Delta t}$. Use east and north as positive.

**Solution:** The $x$-component of $\Delta\vec{d}$ is

$$\begin{aligned}\Delta d_x &= \Delta d_{1x} + \Delta d_{2x} + \Delta d_{3x}\\ &= -(15.0\text{ km})(\cos 30.0°) + (-(10.0\text{ km})\cos 75.0°) + (10.0\text{ km})(\cos 70.0°)\\ &= -12.990\text{ km} + \left(-2.588\text{ km}\right) + 3.420\text{ km}\\ \Delta d_x &= -12.158\text{ km (two extra digits carried)}\end{aligned}$$

The $y$-component of $\Delta\vec{d}$ is

$$\begin{aligned}\Delta d_y &= \Delta d_{1y} + \Delta d_{2y} + \Delta d_{3y}\\ &= +(15.0\text{ km})(\sin 30.0°) + (10.0\text{ km})(\sin 75.0°) + (10.0\text{ km})(\sin 70.0°)\\ &= +7.5\text{ km} + 9.659\text{ km} + 9.397\text{ km}\\ \Delta d_y &= +26.556\text{ km (two extra digits carried)}\end{aligned}$$

Combine the total displacement components to determine the total displacement.

$$\begin{aligned}\left|\Delta\vec{d}\right| &= \sqrt{\Delta d_x^2 + \Delta d_y^2}\\ &= \sqrt{(-12.158\text{ km})^2 + (26.556\text{ km})^2}\\ \left|\Delta\vec{d}\right| &= 29.207\text{ km}\end{aligned}$$

$$\begin{aligned}\theta &= \tan^{-1}\left(\frac{\left|\Delta d_y\right|}{\left|\Delta d_x\right|}\right)\\ &= \tan^{-1}\left(\frac{26.556\ \cancel{\text{km}}}{12.158\ \cancel{\text{km}}}\right)\\ \theta &= 65.4°\end{aligned}$$

The average velocity is

$$\begin{aligned}\vec{v}_{\text{av}} &= \frac{\Delta\vec{d}}{\Delta t}\\ &= \frac{29.207\text{ km [W 65.4° N]}}{0.50\text{ h}}\\ \vec{v}_{\text{av}} &= 58\text{ km/h [W 65.4° N]}\end{aligned}$$

**Statement:** The driver's average velocity is 58 km/h [W 65.4° N].

**4.** Answers may vary. Sample answer: Acceleration results from a change in velocity. The change may involve a change in speed or a change in direction. Going around a curve at constant speed is a situation where there is average acceleration and no change in speed.

**5.** Answers may vary. Sample answers: One way to subtract vectors is to use components. Determine the components of each vector. Subtract the *x*-components and then the *y*-components. Build the resulting vector out of its components. A second way is similar to adding vectors using the cosine and sine laws. Draw the first vector. Add to it the negative of the second vector. Draw the resulting vector to complete the triangle. Then solve the triangle.

**6. Given:** $\Delta\vec{d}_1 = 150$ km [E 12° N]; $\Delta\vec{v}_{\text{av}} = 130$ km/h [N 32° E]; $\Delta t_1 = 1.0$ h; $\Delta t = 3.0$ h

**Required:** $\Delta\vec{d}_2$

**Analysis:** Use $\vec{v}_{\text{av}} = \dfrac{\Delta\vec{d}}{\Delta t}$ to calculate the total displacement. Then determine the *x*- and *y*-components of the first and total displacement vectors. Subtract these *x*- and *y*-components to determine the *x*- and *y*-components of the second displacement, $\Delta\vec{d} = \Delta\vec{d}_1 + \Delta\vec{d}_2$. Finally, use the Pythagorean theorem and tangent ratio to determine the second displacement vector. Use east and north as positive.

**Solution:** Determine the total displacement $\Delta\vec{d}$.

$$\vec{v}_{\text{av}} = \frac{\Delta\vec{d}}{\Delta t}$$

$$\begin{aligned}\Delta\vec{d} &= \vec{v}_{\text{av}}\Delta t \\ &= (130\ \text{km}/\cancel{\text{h}}\ [\text{N } 32^\circ\ \text{E}])(3.0\ \cancel{\text{h}}) \\ \Delta\vec{d} &= 390\ \text{km [N } 32^\circ \text{ E]}\end{aligned}$$

The *x*-component of $\Delta\vec{d}_2$ is

$$\begin{aligned}\Delta d_{2x} &= \Delta d_x - \Delta d_{1x} \\ &= (390\ \text{km})(\sin 32^\circ) - (150\ \text{km})(\cos 12^\circ) \\ &= 206.67\ \text{km} - 146.72\ \text{km} \\ \Delta d_{2x} &= 59.95\ \text{km (two extra digits carried)}\end{aligned}$$

The *y*-component of $\Delta\vec{d}_2$ is:

$$\begin{aligned}\Delta d_{2y} &= \Delta d_y - \Delta d_{1y} \\ &= (390\ \text{km})(\cos 32^\circ) - (150\ \text{km})(\sin 12^\circ) \\ &= 330.74\ \text{km} - 31.19\ \text{km} \\ \Delta d_{2y} &= 299.55\ \text{km (three extra digits carried)}\end{aligned}$$

Combine the displacement components of the second vector to determine the second displacement.

$$\begin{aligned}\left|\Delta\vec{d}_2\right| &= \sqrt{\Delta d_{2x}^2 + \Delta d_{2y}^2} \\ &= \sqrt{(59.95\ \text{km})^2 + (299.55\ \text{km})^2} \\ &= 305.49\ \text{km} \\ \left|\Delta\vec{d}_2\right| &= 3.1\times 10^2\ \text{km}\end{aligned}$$

$$\theta = \tan^{-1}\left(\frac{\left|\Delta d_{2y}\right|}{\left|\Delta d_{2x}\right|}\right)$$

$$= \tan^{-1}\left(\frac{299.55\ \cancel{\text{km}}}{59.95\ \cancel{\text{km}}}\right)$$

$$\theta = 79°$$

**Statement:** The second displacement is $3.1 \times 10^2$ km [N 11° E].

**7. Given:** $v_{\text{av}} = 3.5$ m/s; $\Delta\vec{d}_1 = 1.8$ km [E]; $\Delta\vec{d}_2 = 2.6$ km [N 35° E]

**Required:** $\Delta t$

**Analysis:** The jogger returns to his starting place, so the total displacement is 0 m, $\Delta\vec{d}_1 + \Delta\vec{d}_2 + \Delta\vec{d}_3 = 0$ m. Use vector subtraction by components to determine the magnitude of the third displacement vector. Use $\Delta d = \Delta d_1 + \Delta d_2 + \Delta d_3$ to calculate the total distance the jogger ran. Finally determine the time taken using $v_{\text{av}} = \dfrac{\Delta d}{\Delta t}$. Use east and north as positive.

**Solution:** The $x$-component of $\Delta\vec{d}_3$ is

$$\begin{aligned}
\Delta d_{1x} + \Delta d_{2x} + \Delta d_{3x} &= 0 \text{ m} \\
\Delta d_{3x} &= -\Delta d_{1x} - \Delta d_{2x} \\
&= -(1.8 \text{ km}) - (2.6 \text{ km})(\sin 35°) \\
&= -(1.8 \text{ km}) - 1.491 \text{ km} \\
\Delta d_{3x} &= -3.291 \text{ km (two extra digits carried)}
\end{aligned}$$

The $y$-component of $\Delta\vec{d}_3$ is

$$\begin{aligned}
\Delta d_{1y} + \Delta d_{2y} + \Delta d_{3y} &= 0 \text{ m} \\
\Delta d_{3y} &= -\Delta d_{1y} - \Delta d_{2y} \\
&= -(0 \text{ km}) - (2.6 \text{ km})(\cos 35°) \\
&= -(0 \text{ km}) - 2.130 \text{ km} \\
\Delta d_{3y} &= -2.130 \text{ km (two extra digits carried)}
\end{aligned}$$

Combine these displacement components to determine the magnitude of the third displacement.

$$\begin{aligned}
\left|\Delta\vec{d}\right| &= \sqrt{\Delta d_x^2 + \Delta d_y^2} \\
&= \sqrt{(-3.291 \text{ km})^2 + (2.130 \text{ km})^2} \\
\left|\Delta\vec{d}\right| &= 3.920 \text{ km}
\end{aligned}$$

The total distance run is

$$\begin{aligned}
\Delta d &= \Delta d_1 + \Delta d_2 + \Delta d_3 \\
&= (1.8 \text{ km}) + (2.6 \text{ km}) + (3.920 \text{ km}) \\
\Delta d &= 8.320 \text{ km}
\end{aligned}$$

The time taken is

$$\Delta t = \frac{\Delta d}{v_{\text{av}}}$$
$$= \frac{8.320\ \cancel{\text{km}}}{3.5\ \cancel{\text{m}}/\text{s}} \times \frac{1000\ \cancel{\text{m}}}{1\ \cancel{\text{km}}}$$
$$= 2377\ \text{s}$$
$$\Delta t = 2.4 \times 10^3\ \text{s}$$

**Statement:** The jogger's run takes $2.4 \times 10^3$ s.

**8. Given:** $\vec{v}_1 = 6.4$ m/s [E 30.0° S]; $\vec{v}_2 = 8.5$ m/s [E 30.0° N]; $\Delta t = 3.8$ s

**Required:** $\vec{a}_{\text{av}}$

**Analysis:** Calculate the change in velocity using components and $\Delta\vec{v} = \vec{v}_\text{f} - \vec{v}_\text{i}$. Then determine the average acceleration from $\vec{a}_{\text{av}} = \dfrac{\Delta\vec{v}}{\Delta t}$. Use east and north as positive.

**Solution:** The *x*-component of $\Delta\vec{v}$ is

$$\Delta v_x = v_{\text{f}x} - v_{\text{i}x}$$
$$= (8.5\ \text{m/s})(\cos 30.0°) - (6.4\ \text{m/s})(\cos 30.0°)$$
$$= 7.361\ \text{m/s} - 5.543\ \text{m/s}$$
$$\Delta v_x = 1.818\ \text{m/s (two extra digits carried)}$$

The *y*-component of $\Delta\vec{v}$ is

$$\Delta v_y = v_{\text{f}y} - v_{\text{i}y}$$
$$= (8.5\ \text{m/s})(\sin 30.0°) - (-(6.4\ \text{m/s})(\sin 30.0°))$$
$$= 4.25\ \text{m/s} - (-3.2\ \text{m/s})$$
$$\Delta v_y = 7.45\ \text{m/s (one extra digit carried)}$$

Determine the change in velocity from its components.

$$\left|\Delta\vec{v}\right| = \sqrt{\Delta v_x^2 + \Delta v_y^2}$$
$$= \sqrt{(1.818\ \text{m/s})^2 + (7.45\ \text{m/s})^2}$$
$$\left|\Delta\vec{v}\right| = 7.669\ \text{m/s}$$
$$\theta = \tan^{-1}\left(\frac{\left|\Delta v_y\right|}{\left|\Delta v_x\right|}\right)$$
$$= \tan^{-1}\left(\frac{7.45\ \cancel{\text{m}}/\text{s}}{1.818\ \cancel{\text{m}}/\text{s}}\right)$$
$$\theta = 76°$$

Calculate the average acceleration.

$$\vec{a}_{av} = \frac{\Delta\vec{v}}{\Delta t}$$
$$= \frac{7.669 \text{ m/}\cancel{\text{s}} \text{ [E 76° N]}}{3.8 \text{ }\cancel{\text{s}}}$$
$$\vec{a}_{av} = 2.0 \text{ m/s}^2 \text{ [E 76° N]}$$

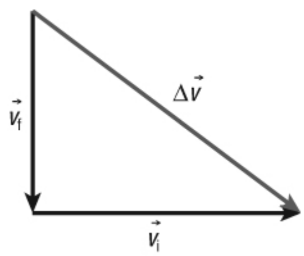
**Statement:** The average acceleration of the bird is $2.0 \text{ m/s}^2$ [E 76° N].

**9. Given:** $\vec{v}_i = 50.0$ m/s [W]; $\vec{v}_f = 35.0$ m/s [S]; $\Delta t = 45.0$ s

**Required:** $\vec{a}_{av}$

**Analysis:** Draw a vector diagram showing the change in velocity using $\Delta\vec{v} = \vec{v}_f - \vec{v}_i$. Solve the triangle using trigonometry. Then determine the average acceleration from $\vec{a}_{av} = \frac{\Delta\vec{v}}{\Delta t}$. Use east and north as positive.

**Solution:**




Use the Pythagorean theorem to determine $|\Delta\vec{v}|$.

$$|\Delta\vec{v}| = \sqrt{|\Delta\vec{v}_i|^2 + |\Delta\vec{v}_f|^2}$$
$$= \sqrt{(50.0 \text{ m/s})^2 + (35.0 \text{ m/s})^2}$$
$$= 61.033 \text{ m/s (two extra digits carried)}$$

$$\theta = \tan^{-1}\left(\frac{|\Delta\vec{v}_f|}{|\Delta\vec{v}_i|}\right)$$
$$= \tan^{-1}\left(\frac{35.0 \text{ }\cancel{\text{m/s}}}{50.0 \text{ }\cancel{\text{m/s}}}\right)$$
$$\theta = 35.0°$$

Calculate the average acceleration.

$$\vec{a}_{av} = \frac{\Delta\vec{v}}{\Delta t}$$
$$= \frac{61.033 \text{ m/}\cancel{\text{s}} \text{ [E 35.0° S]}}{45.0 \text{ }\cancel{\text{s}}}$$
$$\vec{a}_{av} = 1.36 \text{ m/s}^2 \text{ [E 35.0° S]}$$

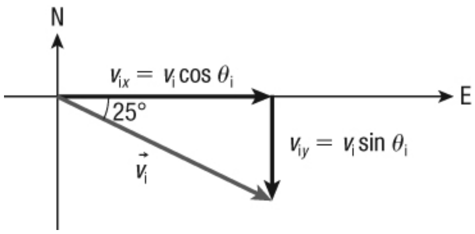
**Statement:** The average acceleration of the helicopter is $1.36 \text{ m/s}^2$ [E 35.0° S].

**10. Given:** $\vec{v}_i = 8.2$ m/s [E 25° S]; $\vec{v}_f = 8.2$ m/s [E 25° N]; $\Delta t = 3.2$ ms $= 3.2 \times 10^{-3}$ s

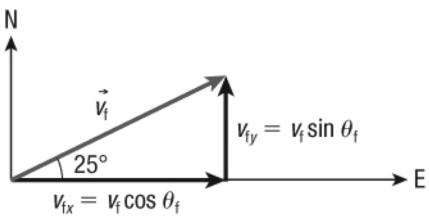
**Required:** $\vec{a}_{av}$

**Analysis:** Draw a vector diagram of the situation. Calculate the change in velocity using components and $\Delta\vec{v} = \vec{v}_f - \vec{v}_i$. Then, determine the average acceleration from $\vec{a}_{av} = \dfrac{\Delta\vec{v}}{\Delta t}$. Use east and north as positive.

**Solution:** Components for the initial velocity vector:




Components for the final velocity vector:




The $x$-component of $\Delta\vec{v}$ is

$$\begin{aligned}\Delta v_x &= v_{fx} - v_{ix} \\ &= (8.2 \text{ m/s})(\cos 25°) - (8.2 \text{ m/s})(\cos 25°) \\ &= 7.432 \text{ m/s} - 7.432 \text{ m/s} \\ \Delta v_x &= 0 \text{ m/s}\end{aligned}$$

The $y$-component of $\Delta\vec{v}$ is

$$\begin{aligned}\Delta v_y &= v_{fy} - v_{iy} \\ &= (8.2 \text{ m/s})(\sin 25°) - (-(8.2 \text{ m/s})(\sin 25°)) \\ &= 3.465 \text{ m/s} - (-3.465 \text{ m/s}) \\ \Delta v_y &= 6.930 \text{ m/s (two extra digits carried)}\end{aligned}$$

Determine the change in velocity from its components.

$$\begin{aligned}\left|\Delta\vec{v}\right| &= \sqrt{\Delta v_x^2 + \Delta v_y^2} \\ &= \sqrt{(0 \text{ m/s})^2 + (6.930 \text{ m/s})^2} \\ \left|\Delta\vec{v}\right| &= 6.930 \text{ m/s (two extra digits carried)}\end{aligned}$$

Since $\Delta\vec{v}_x = 0$ m/s, $\Delta\vec{v}$ points north.

Calculate the average acceleration.

$$\vec{a}_{av} = \frac{\Delta\vec{v}}{\Delta t}$$
$$= \frac{6.930 \text{ m/}\cancel{\text{s}} \text{ [N]}}{3.2\times10^{-3} \ \cancel{\text{s}}}$$
$$\vec{a}_{av} = 2.2\times10^{3} \text{ m/s}^2 \text{ [N]}$$

**Statement:** The average acceleration of the pool ball is $2.2\times10^{3} \text{ m/s}^2 \text{ [N]}$.

**11. Given:** $\vec{v}_i = 6.4$ m/s [W 35° N]; $\vec{a}_{av} = 2.2 \text{ m/s}^2$ [S]; $\Delta t = 4.0$ s

**Required:** $\vec{v}_f$

**Analysis:** Calculate the change in velocity from the acceleration and time interval:

$$\vec{a}_{av} = \frac{\Delta\vec{v}}{\Delta t}$$
$$\Delta\vec{v} = \vec{a}_{av}\Delta t$$

Using components and $\Delta\vec{v} = \vec{v}_f - \vec{v}_i$, determine $\vec{v}_f$. Use east and north as positive.

**Solution:** The change in velocity is

$$\Delta\vec{v} = \vec{a}_{av}\Delta t$$
$$= \left(2.2 \text{ m/s}^{\cancel{2}} \text{ [S]}\right)(4.0 \ \cancel{\text{s}})$$
$$\Delta\vec{v} = 8.8 \text{ m/s [S]}$$

The $x$-component of $\vec{v}_f$ is

$$v_{fx} = v_{ix} + \Delta v_x$$
$$= -(6.4 \text{ m/s})(\cos 35°) + 0 \text{ m/s}$$
$$= -5.246 \text{ m/s} + 0 \text{ m/s}$$
$$v_{fx} = -5.246 \text{ m/s}$$

The $y$-component of $\vec{v}_f$ is

$$v_{fy} = v_{iy} + \Delta v_y$$
$$= (6.4 \text{ m/s})(\sin 35°) + \left(-8.8 \text{ m/s}\right)$$
$$= 3.671 \text{ m/s} + (-8.8 \text{ m/s})$$
$$v_{fy} = -5.129 \text{ m/s}$$

Determine the final velocity from its components.

$$\left|\vec{v}_f\right| = \sqrt{\left|v_{fx}\right|^2 + \left|v_{fy}\right|^2}$$
$$= \sqrt{(-5.246 \text{ m/s})^2 + (-5.129 \text{ m/s})^2}$$
$$\left|\vec{v}_f\right| = 7.3 \text{ m/s}$$

$$\theta = \tan^{-1}\left(\frac{|v_{fy}|}{|v_{fx}|}\right)$$
$$= \tan^{-1}\left(\frac{5.129\ \cancel{\text{m/s}}}{5.246\ \cancel{\text{m/s}}}\right)$$
$$\theta = 44°$$

**Statement:** The final velocity of the boat is 7.3 m/s [W 44° S].

**12. Given:** $\vec{v}_f = 3.6\times10^2$ km/h [N]; $\vec{a}_{av} = 5.0\ \text{m/s}^2$ [W]; $\Delta t = 9.2$ s

**Required:** $\vec{v}_f$

**Analysis:** Convert the final velocity to metres per second.

$$\vec{v}_f = 3.6\times10^2\ \cancel{\text{km}}/\cancel{\text{h}} \times \frac{1\cancel{\text{h}}}{3600\ \text{s}} \times \frac{1000\ \text{m}}{1\ \cancel{\text{km}}}$$
$$\vec{v}_f = 1.0\times10^2\ \text{m/s [N]}$$

Calculate the change in velocity from the acceleration and time interval:

$$\vec{a}_{av} = \frac{\Delta\vec{v}}{\Delta t}$$
$$\Delta\vec{v} = \vec{a}_{av}\Delta t$$

Using components and $\Delta\vec{v} = \vec{v}_f - \vec{v}_i$, determine $\vec{v}_i$. Use east and north as positive.

**Solution:** The change in velocity is

$$\Delta\vec{v} = \vec{a}_{av}\Delta t$$
$$= \left(5.0\ \text{m/s}^{\cancel{2}}\ [\text{W}]\right)(9.2\ \cancel{\text{s}})$$
$$\Delta\vec{v} = 46\ \text{m/s [W]}$$

The $x$-component of $\vec{v}_i$ is

$$v_{ix} = v_{fx} - \Delta v_x$$
$$= 0\ \text{m/s} - (-46\ \text{m/s})$$
$$v_{ix} = 46\ \text{m/s}$$

The $y$-component of $\vec{v}_i$ is

$$v_{iy} = v_{fy} - \Delta v_y$$
$$= 100\ \text{m/s} - 0\ \text{m/s}$$
$$v_{iy} = 100\ \text{m/s}$$

Determine the initial velocity from its components.

$$|\vec{v}_i| = \sqrt{|v_{ix}|^2 + |v_{iy}|^2}$$
$$= \sqrt{(46\ \text{m/s})^2 + (100\ \text{m/s})^2}$$
$$= 110.1\ \text{m/s}$$
$$|\vec{v}_i| = 1.1\times10^2\ \text{m/s}$$

$$\theta = \tan^{-1}\left(\frac{|v_{iy}|}{|v_{ix}|}\right)$$

$$= \tan^{-1}\left(\frac{100\ \cancel{\text{m/s}}}{46\ \cancel{\text{m/s}}}\right)$$

$$\theta = 65^\circ$$

**Statement:** The initial velocity of the boat is $1.1\times10^2$ m/s [E 65° N].