Section 1.2: Equations of Motion
Tutorial 1 Practice, page 19

1. (a) Given: $\vec{v}_i = 15.0$ m/s [forward]; $\vec{v}_f = 0$ m/s; $\vec{a} = 5.0$ m/s^2 [backward]

Required: $\Delta\vec{d}$

Analysis: Use $\vec{a} = \dfrac{\vec{v}_f - \vec{v}_i}{\Delta t}$ to determine the braking time, Δt. Then use $\Delta\vec{d} = \vec{v}_i\Delta t + \dfrac{1}{2}\vec{a}\Delta t^2$ to determine the braking distance. Choose forward to be the positive direction.

Solution:

$$a = \frac{v_f - v_i}{\Delta t}$$
$$\Delta t = \frac{v_f - v_i}{a}$$
$$= \frac{0\ \cancel{\text{m/s}} - 15.0\ \cancel{\text{m/s}}}{-5.0\ \cancel{\text{m}}/\text{s}^{\cancel{2}}}$$
$$\Delta t = 3.0\ \text{s}$$

$$\Delta d = v_i\Delta t + \frac{1}{2}a\Delta t^2$$
$$= (15.0\ \text{m}/\cancel{\text{s}})(3.0\ \cancel{\text{s}}) + \frac{1}{2}(-5.0\ \text{m}/\cancel{\text{s}^2})(3.0\ \cancel{\text{s}})^2$$
$$= 45.0\ \text{m} - 22.5\ \text{m}$$
$$\Delta d = 22\ \text{m}$$

Statement: The motorcycle's braking distance is 22 m [forward].

(b) Answers may vary. Sample answer: In Sample Problem 2 and part (a), I calculated the braking distance for a motorcyclist slowing down at 5.0 m/s^2. When the initial speed is high, it takes much longer and a much greater distance to stop, so a speeding vehicle has more difficulty stopping safely.

2. Given: $\vec{v}_i = 0$ m/s; $\Delta\vec{d} = 120$ m [N]; $\Delta t = 15$ s

Required: $\vec{a}$

Analysis: Use $\Delta\vec{d} = \vec{v}_i\Delta t + \dfrac{1}{2}\vec{a}\Delta t^2$ to determine the acceleration.

$$\Delta d = v_i\Delta t + \frac{1}{2}a\Delta t^2$$
$$\Delta d = (0\ \text{m/s})\Delta t + \frac{1}{2}a\Delta t^2$$
$$\Delta d = \frac{1}{2}a\Delta t^2$$
$$a = \frac{2\Delta d}{\Delta t^2}$$

Choose north to be the positive direction.

Solution: $a = \dfrac{2\Delta d}{\Delta t^2}$

$$= \frac{2(120\ \text{m})}{(15\ \text{s})^2}$$
$$a = 1.1\ \text{m/s}^2$$

Statement: The runner accelerates at 1.1 m/s^2 [N].

3. Given: $\vec{v}_1 = 22\text{ m/s [E]}$; $\Delta t_1 = 12\text{ s}$; $\vec{a} = 1.2\text{ m/s}^2\text{ [W]}$; $\vec{v}_\text{f} = 0\text{ m/s}$

Required: total displacement, $\Delta\vec{d}$

Analysis: Determine the displacement for the first part of the trip using

$$\vec{v}_1 = \frac{\Delta\vec{d}_1}{\Delta t_1}$$

$$\Delta d_1 = v_1 \Delta t_1$$

Determine the braking time using

$$\vec{a} = \frac{\vec{v}_\text{f} - \vec{v}_\text{i}}{\Delta t}$$

$$\Delta t = \frac{v_\text{f} - v_\text{i}}{a}$$

Then, use $\Delta\vec{d}_2 = \vec{v}_\text{i}\Delta t + \frac{1}{2}\vec{a}\Delta t^2$ to determine the displacement for the second part of the trip.

Finally, the total displacement is $\Delta\vec{d} = \Delta\vec{d}_1 + \Delta\vec{d}_2$. Use east as the positive direction.

Solution:

$$\begin{aligned}\Delta d_1 &= v_1 \Delta t_1 \\ &= (22\text{ m/}\cancel{\text{s}})(12\text{ }\cancel{\text{s}}) \\ \Delta d_1 &= 264\text{ m (one extra digit carried)}\end{aligned}$$

$$\begin{aligned}\Delta t &= \frac{v_\text{f} - v_\text{i}}{a} \\ &= \frac{0\text{ }\cancel{\text{m/s}} - 22\text{ }\cancel{\text{m/s}}}{-1.2\text{ }\cancel{\text{m}}/\text{s}^{\cancel{2}}} \\ \Delta t &= 18.33\text{ s (two extra digits carried)}\end{aligned}$$

$$\begin{aligned}\Delta d_2 &= v_1 \Delta t + \frac{1}{2}a\Delta t^2 \\ &= (22\text{ m/}\cancel{\text{s}})(18.33\text{ }\cancel{\text{s}}) + \frac{1}{2}(-1.2\text{ m/}\cancel{\text{s}^2})(18.33\text{ }\cancel{\text{s}})^{\cancel{2}} \\ \Delta d_2 &= 202\text{ m (one extra digit carried)}\end{aligned}$$

$$\begin{aligned}\Delta d &= \Delta d_1 + \Delta d_2 \\ &= 264\text{ m} + 202\text{ m} \\ \Delta d &= 470\text{ m}\end{aligned}$$

Statement: The total displacement of the bus is $4.7 \times 10^2\text{ m}$.

4. (a) Given: $\vec{v}_\text{i} = 0\text{ m/s}$; $\vec{v}_\text{f} = 9.6\text{ m/s [W]}$; $\Delta t = 4.2\text{ s}$

Required: $\vec{a}$

Analysis: Calculate the acceleration using $\vec{a} = \frac{\vec{v}_\text{f} - \vec{v}_\text{i}}{\Delta t}$. Use west as positive.

Solution: $a = \dfrac{v_f - v_i}{\Delta t}$

$$= \frac{9.6 \text{ m/s} - 0 \text{ m/s}}{4.2 \text{ s}}$$

$$= 2.286 \text{ m/s}^2 \text{ (two extra digits carried)}$$

$$a = 2.3 \text{ m/s}^2$$

Statement: The runner's acceleration is 2.3 m/s^2 [W].

(b) Given: $\vec{v}_i = 0$ m/s; $\Delta t = 4.2$ s; $\vec{a} = 2.286 \text{ m/s}^2$ [W]

Required: $\Delta\vec{d}$

Analysis: Calculate the displacement using $\Delta\vec{d} = \vec{v}_i\Delta t + \dfrac{1}{2}\vec{a}\Delta t^2$.

Solution: $\Delta d = v_i\Delta t + \dfrac{1}{2}a\Delta t^2$

$$= (0 \text{ m/}\cancel{\text{s}})(4.2 \cancel{\text{s}}) + \frac{1}{2}(2.286 \text{ m/}\cancel{\text{s}^2})(4.2 \cancel{\text{s}})^{\cancel{2}}$$

$$= 20.16 \text{ m (two extra digits carried)}$$

$$\Delta d = 20 \text{ m}$$

Statement: The displacement of the runner is 2.0×10^1 m [W] while accelerating.

(c) Given: $\vec{v} = 9.6$ m/s [W]; $\Delta\vec{d} = 100.0$ m [W]; $\Delta\vec{d}_1 = 20.16$ m [W]; $\Delta t_1 = 4.2$ s

Required: total time, Δt

Analysis: Use $\Delta\vec{d} = \Delta\vec{d}_1 + \Delta\vec{d}_2$ to calculate the displacement, $\Delta\vec{d}_2$, for the second part of the race.

$$\Delta d = \Delta d_1 + \Delta d_2$$

$$\Delta d_2 = \Delta d - \Delta d_1$$

Then use the constant velocity formula, $\vec{v}_{av} = \dfrac{\Delta\vec{d}}{\Delta t}$, to determine the time, Δt_2, for the second part of the race.

$$v_2 = \frac{\Delta d_2}{\Delta t_2}$$

$$\Delta t_2 = \frac{\Delta d_2}{v_2}$$

Finally, the total time taken is the sum of the two times, $\Delta t = \Delta t_1 + \Delta t_2$.

Solution: $\Delta d_2 = \Delta d - \Delta d_1$

$$= 100.0 \text{ m} - 20.16 \text{ m}$$

$$\Delta d_2 = 79.84 \text{ m (two extra digits carried)}$$

$$\Delta t_2 = \frac{\Delta d_2}{v_2}$$

$$= \frac{79.84 \cancel{\text{m}}}{9.6 \cancel{\text{m}}\text{/s}}$$

$$\Delta t_2 = 8.317 \text{ s (two extra digits carried)}$$

$$\Delta t = \Delta t_1 + \Delta t_2$$
$$= 4.2 \text{ s} + 8.317 \text{ s}$$
$$\Delta t = 13 \text{ s}$$

Statement: The total time taken is 13 s.

5. (a) Given: player 1: $\vec{d}_{1i} = 0 \text{ m [right]}$; $\vec{v}_{1i} = 0 \text{ m/s}$; $a = 2.4 \text{ m/s}^2 \text{ [right]}$;

player 2: $\vec{d}_{2i} = 42 \text{ m [right]}$; $\vec{v}_2 = 5.4 \text{ m/s [left]}$

Required: time until the players meet, Δt

Analysis: Set up an equation for each player that relates position and time. The first player moves at constant acceleration, $\Delta \vec{d} = \vec{v}_i \Delta t + \frac{1}{2}\vec{a}\Delta t^2$.

The second player moves at a constant velocity:

$$\vec{v} = \frac{\Delta \vec{d}}{\Delta t}$$
$$v_2 = \frac{\Delta d_2}{\Delta t}$$
$$\Delta d_2 = v_2 \Delta t$$

Compare these equations to solve for either position or time. Use right as positive.

Solve the quadratic equation using the quadratic formula: $\Delta t = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Solution: player 1:

$$\Delta d = v_{1i}\Delta t + \frac{1}{2}a\Delta t^2$$
$$\Delta d_1 = (0 \text{ m/s})\Delta t + \frac{1}{2}(2.4 \text{ m/s}^2)\Delta t^2$$
$$d_{1f} - d_{1i} = (1.2 \text{ m/s}^2)\Delta t^2$$
$$d_{1f} = (1.2 \text{ m/s}^2)\Delta t^2$$

player 2:

$$\Delta d_2 = v_2 \Delta t$$
$$d_{2f} - d_{2i} = (-5.4 \text{ m/s})\Delta t$$
$$d_{2f} - 42 \text{ m} = (-5.4 \text{ m/s})\Delta t$$
$$d_{2f} = 42 \text{ m} - (5.4 \text{ m/s})\Delta t$$

The final positions of the two players are equal.

$$d_{1f} = d_{2f}$$
$$(1.2 \text{ m/s}^2)\Delta t^2 = 42 \text{ m} - (5.4 \text{ m/s})\Delta t$$
$$(1.2 \text{ m/s}^2)\Delta t^2 + (5.4 \text{ m/s})\Delta t - 42 \text{ m} = 0$$

This is a quadratic equation in Δt.

$$\Delta t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-5.4\text{ m/s} \pm \sqrt{(-5.4\text{ m/s})^2 - 4(1.2\text{ m/s}^2)(-42\text{ m})}}{2(1.2\text{ m/s}^2)}$$

$\Delta t = 4.1$ s or -8.8 s

The second solution is not possible, because the time taken to run is positive.

Statement: The players collide after 4.1 s.

(b) Given: equation for player 1: $d_1 = (1.2\text{ m/s}^2)\Delta t^2$; equation for player 2: $\Delta d_2 = v_2 \Delta t$;

$\Delta t = 4.1$ s

Required: $\Delta \vec{d}_1$; $\Delta \vec{d}_2$

Analysis: Use the time taken by the players and the player's equations of motion to determine their displacements.

Solution: player 1:

$$\Delta d_1 = (1.2\text{ m/s}^2)\Delta t^2$$
$$= (1.2\text{ m/s}^2)(4.1\text{ s})^2$$
$$\Delta d_1 = 20\text{ m}$$

player 2:

$$\Delta d_2 = v_2 \Delta t$$
$$= (-5.4\text{ m/s})(4.1\text{ s})$$
$$\Delta d_2 = -22\text{ m}$$

Statement: Player 1 ran 20 m [right] and player 2 ran 22 m [left].

(c) Given: $\vec{v}_{1i} = 0$ m/s; $\vec{a} = 2.4\text{ m/s}^2$; $\Delta t = 4.1$ s

Required: v_{1f}

Analysis: Use $\vec{a} = \dfrac{\vec{v}_f - \vec{v}_i}{\Delta t}$ to solve for the final speed of player 1.

$$v_{1f} - v_{1i} = a\Delta t$$
$$v_{1f} = v_{1i} + a\Delta t$$

Solution:

$$v_{1f} = v_{1i} + a\Delta t$$
$$= 0\text{ m/s} + (2.4\text{ m/s}^{\cancel{2}})(4.1\text{ }\cancel{\text{s}})$$
$$v_{1f} = 9.8\text{ m/s}$$

Statement: Player 1 is moving at 9.8 m/s when he collides with player 2.

6. (a) Given: $\vec{v}_1 = 110$ m/s [forward]; $\vec{v}_2 = 0$ m/s; $\vec{a} = 6.2\text{ m/s}^2$ [backward]

Required: minimum stopping time, Δt

Analysis: $\vec{a} = \dfrac{\vec{v}_f - \vec{v}_i}{\Delta t}$; Use forward as positive.

$$\Delta t = \frac{v_f - v_i}{a}$$

Solution: $\Delta t = \dfrac{v_f - v_i}{a}$

$$= \frac{0\ \cancel{m}/\cancel{s} - 110\ \cancel{m}/\cancel{s}}{-6.2\ \cancel{m}/s^{\cancel{2}}}$$

$$= 17.74 \text{ s (two extra digits carried)}$$

$$\Delta t = 18 \text{ s}$$

Statement: The airplane needs a minimum of 18 s to stop.

(b) Given: $\vec{v}_i = 110$ m/s; $\Delta t = 17.74$ s; $\vec{a} = 6.2$ m/s^2 [backward]

Required: minimum stopping distance, Δd

Analysis: Use $\Delta d = v_i \Delta t + \frac{1}{2} a \Delta t^2$ to determine the distance travelled while stopping.

Solution: $\Delta d = v_i \Delta t + \dfrac{1}{2} a \Delta t^2$

$$= (110\ \text{m}/\cancel{s})(17.74\ \cancel{s}) + \frac{1}{2}(-6.2\ \text{m}/\cancel{s^2})(17.74\ \cancel{s})^{\cancel{2}}$$

$$\Delta d = 9.8 \times 10^2 \text{ m}$$

Statement: The minimum safe length of the runway is 9.8×10^2 m.

(c) Answers may vary. Sample answer: The runway should be much longer than the minimum length because the airplane may land partway along the runway. If the weather is bad, the usual braking force needed to slow the plane to a stop may be insufficient and more time and distance may be needed to stop the plane. Also, it is uncomfortable for the passengers to slow a plane down using the maximum allowed braking force.

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1. Given: $\vec{v}_i = 22$ m/s [up]; $\vec{v}_f = 0$ m/s; $\vec{a} = 9.8$ m/s^2 [down]

Required: maximum height, Δd

Analysis: Use $v_f^2 = v_i^2 + 2a\Delta d$ to calculate Δd.

$$v_f^2 = v_i^2 + 2a\Delta d$$

$$\Delta d = \frac{v_f^2 - v_i^2}{2a}$$

Use up as positive and down as negative.

Solution: $\Delta d = \dfrac{v_f^2 - v_i^2}{2a}$

$$= \frac{(0\ \text{m}/\cancel{s})^{\cancel{2}} - (22\ \text{m}/\cancel{s})^{\cancel{2}}}{2(-9.8\ \text{m}/\cancel{s^2})}$$

$$\Delta d = 25 \text{ m}$$

Statement: The maximum height of the apple is 25 m.

2. Given: $\vec{v}_{\mathrm{i}} = 0$ m/s; $\Delta\vec{d} = 10.0$ m [down]; $\vec{a} = \dfrac{1}{6}(9.8\text{ m/s}^2\text{ [down]})$

Required: time for an object to fall, Δt

Analysis: Calculate the acceleration and then use $\Delta d = v_{\mathrm{i}}\Delta t + \dfrac{1}{2}a\Delta t^2$ to determine Δt.

Solve the quadratic equation using the quadratic formula: $\Delta t = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Use up as positive.

Solution: $a = \dfrac{-9.8\text{ m/s}^2}{6}$

$a = -1.633\text{ m/s}^2$ (two extra digits carried)

$$\Delta d = v_{\mathrm{i}}\Delta t + \frac{1}{2}a\Delta t^2$$

$$-10.0\text{ m} = (0\text{ m/s})\Delta t + \frac{1}{2}(-1.633\text{ m/s}^2)\Delta t^2$$

$$(-0.8165\text{ m/s}^2)\Delta t^2 + (0\text{ m/s})\Delta t + 10.0\text{ m} = 0$$

$$\begin{aligned}\Delta t &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{0\text{ m/s} \pm \sqrt{(0\text{ m/s})^2 - 4\left(-0.8165\text{ m/s}^2\right)(10.0\text{ m})}}{-1.633\text{ m/s}^2}\end{aligned}$$

$\Delta t = 3.5$ s

Statement: It would take 3.5 s for an object to fall 10.0 m if *g* were one-sixth the value of Earth's *g*.

3. (a) Given: $\vec{v}_{\mathrm{i}} = 12$ m/s [down]; $\Delta\vec{d} = 45$ m [down]; $\vec{a} = 9.8\text{ m/s}^2$ [down]

Required: time to fall, Δt

Analysis: Use up as positive. Use $\Delta d = v_{\mathrm{i}}\Delta t + \dfrac{1}{2}a\Delta t^2$ to determine Δt.

$$\frac{1}{2}a\Delta t^2 + v_{\mathrm{i}}\Delta t - \Delta d = 0$$

Use the quadratic formula to determine Δt.

$$\Delta t = \frac{-v_{\mathrm{i}} \pm \sqrt{v_{\mathrm{i}}^2 - 4\left(\frac{1}{2}a\right)(-\Delta d)}}{a}$$

Solution: $\Delta t = \dfrac{-v_i \pm \sqrt{v_i^2 - 4\left(\dfrac{1}{2}a\right)(-\Delta d)}}{a}$

$$= \frac{-(-12\text{ m/s}) \pm \sqrt{(-12\text{ m/s})^2 - 4(-4.9\text{ m/s}^2)(45\text{ m})}}{-9.8\text{ m/s}^2}$$

$$= 2.044\text{ s or } -4.493\text{ s (two extra digits carried)}$$

$$\Delta t = 2.0\text{ s}$$

Statement: The ball takes 2.0 s to land.

(b) Given: $\vec{v}_i = 12$ m/s [down]; $\Delta\vec{d} = 45$ m [down]; $\vec{a} = 9.8\text{ m/s}^2$ [down]; $\Delta t = 2.044$ s

Required: v_f

Analysis: Use $a = \dfrac{v_f - v_i}{\Delta t}$ or $\Delta d = \left(\dfrac{v_i + v_f}{2}\right)\Delta t$ to determine v_f. Both solutions are provided.

$$a = \frac{v_f - v_i}{\Delta t}$$
$$v_f - v_i = a\Delta t$$
$$v_f = v_i + a\Delta t$$

$$\Delta d = \left(\frac{v_i + v_f}{2}\right)\Delta t$$
$$v_i + v_f = \frac{2\Delta d}{\Delta t}$$
$$v_f = \frac{2\Delta d}{\Delta t} - v_i$$

Solution: $v_f = v_i + a\Delta t$

$$= (-12\text{ m/s}) + (-9.8\text{ m/s}^{\cancel{2}})(2.044\ \cancel{\text{s}})$$
$$= -12\text{ m/s} - 20.03\text{ m/s}$$
$$v_f = -32\text{ m/s}$$

$$v_f = \frac{2\Delta d}{\Delta t} - v_i$$
$$= \frac{2(-45\text{ m})}{2.044\text{ s}} - (-12\text{ m/s})$$
$$= -44.03\text{ m/s} + 12\text{ m/s}$$
$$v_f = -32\text{ m/s}$$

Statement: The ball is moving at 32 m/s when it lands.

(c) Answers may vary. Sample answer: If the ball were thrown up at 12 m/s, it would rise to a maximum height and then fall. It would be moving at 12 m/s down as it passed its initial position. It would then continue down as in the question. The ball would be moving at the same speed as above, 32 m/s.

4. (a) Given: $\Delta\vec{d} = 32$ m [down]; $\Delta t = 1.5$ s; $\vec{a} = 9.8\text{ m/s}^2$ [down]

Required: velocity at 32 m, $\vec{v}_i$

Analysis: The displacement and time taken are known. Calculate average velocity using $\Delta\vec{d} = \left(\dfrac{\vec{v}_i + \vec{v}_f}{2}\right)\Delta t$. This gives the sum of the velocities.

$$\Delta d = \left(\frac{v_\text{i} + v_\text{f}}{2}\right)\Delta t$$

$$v_\text{i} + v_\text{f} = \frac{2\Delta d}{\Delta t}$$

The acceleration is known, and the difference in the (squares of the) velocities can be determined using $v_\text{f}^2 = v_\text{i}^2 + 2a\Delta d$. The initial velocity of the ball can then be determined.

$$v_\text{f}^2 = v_\text{i}^2 + 2a\Delta d$$

$$v_\text{f}^2 - v_\text{i}^2 = 2a\Delta d$$

Although it is not usual, for convenience, use down as the positive direction.

Solution: $v_\text{i} + v_\text{f} = \dfrac{2\Delta d}{\Delta t}$

$$= \frac{2(32\text{ m})}{1.5\text{ s}}$$

$v_\text{i} + v_\text{f} = 42.67\text{ m/s}$ (two extra digits carried) (Equation 1)

$$v_\text{f}^2 - v_\text{i}^2 = 2a\Delta d$$

$$= 2\left(9.8\text{ m/s}^2\right)(32\text{ m})$$

$v_\text{f}^2 - v_\text{i}^2 = 627.2\text{ (m/s)}^2$ (two extra digits carried) (Equation 2)

The left side of Equation 2 can be used to determine the difference of the speeds.

$$v_\text{f}^2 - v_\text{i}^2 = 627.2\text{ (m/s)}^2$$

$$(v_\text{f} - v_\text{i})(v_\text{f} + v_\text{i}) = 627.2\text{ (m/s)}^2$$

$$(v_\text{f} - v_\text{i})(42.67\text{ m/s}) = 627.2\text{ (m/s)}^2$$

$v_\text{f} - v_\text{i} = 14.70\text{ m/s}$ (two extra digits carried) (Equation 3)

Finally, use equations 1 and 3 to determine v_i.

$v_\text{f} - v_\text{i} = 14.70\text{ m/s}$ (Equation 3)

$$v_\text{i} - v_\text{f} = -14.70\text{ m/s}$$

$v_\text{i} + v_\text{f} = 42.67\text{ m/s}$ (Equation 1)

$$2v_\text{i} = -14.70\text{ m/s} + 42.67\text{ m/s}$$

$$v_\text{i} = 14\text{ m/s}$$

Statement: The ball is moving 14 m/s [downward] when the timing starts at 32 m.

(b) Solutions may vary. Sample answer:

Given: $\vec{v}_\text{i} = 0\text{ m/s}$; $\vec{v}_\text{f} = 14\text{ m/s}$ [downward]; $\Delta d_2 = 32\text{ m}$; $\vec{a} = 9.8\text{ m/s}^2$ [downward]

Required: total displacement, $\Delta\vec{d}$

Analysis: Calculate the displacement for the first part of the trip using:

$$v_f^2 = v_i^2 + 2a\Delta d$$

$$\Delta d_1 = \frac{v_f^2 - v_i^2}{2a}$$

Then calculate the total displacement using $\Delta d = \Delta d_1 + \Delta d_2$. Continue to use down as the positive direction.

Solution:

$$\Delta d_1 = \frac{v_f^2 - v_i^2}{2a} = \frac{(14\ \text{m}/\cancel{\text{s}})^2 - (0\ \text{m}/\cancel{\text{s}})^2}{2(9.8\ \text{m}/\cancel{\text{s}^2})}$$

$$\Delta d_1 = 10\ \text{m}$$

$$\Delta d = \Delta d_1 + \Delta d_2 = 10\ \text{m} + 32\ \text{m}$$

$$\Delta d = 42\ \text{m}$$

Statement: The ball's displacement is 42 m [downward].

5. (a) Given: $\Delta\vec{d} = 14$ m [up]; $\Delta t = 1.1$ s; $\vec{a} = 9.8\ \text{m/s}^2$ [down]

Required: $\vec{v}_i$

Analysis: Use up as positive. Calculate the initial velocity using

$$\Delta d = v_i\Delta t + \frac{1}{2}a\Delta t^2$$

$$v_i = \frac{\Delta d - \frac{1}{2}a\Delta t^2}{\Delta t}$$

Solution:

$$\begin{aligned} v_i &= \frac{\Delta d - \frac{1}{2}a\Delta t^2}{\Delta t} \\ &= \frac{14\ \text{m} - \frac{1}{2}(-9.8\ \text{m/s}^2)(1.1\ \text{s})^2}{1.1\ \text{s}} \\ &= \frac{14\ \text{m} + 5.93\ \text{m}}{1.1\ \text{s}} \\ &= 18.12\ \text{m/s (two extra digits carried)} \\ v_i &= 18\ \text{m/s} \end{aligned}$$

Statement: The initial velocity of the keys was 18 m/s [up].

(b) Solutions may vary. Sample answer:

Given: $\Delta\vec{d} = 14$ m [up]; $\vec{a} = 9.8\ \text{m/s}^2$ [downward]; $\vec{v}_i = 18.12$ m/s

Required: $\vec{v}_f$

Analysis: Use $v_f^2 = v_i^2 + 2a\Delta d$ to determine v_f. (Any of the equations containing v_f could be used.)

Solution: $v_f^2 = v_i^2 + 2a\Delta d$

$$= (18.12 \text{ m/s})^2 + 2(-9.8 \text{ m/s}^2)(14 \text{ m})$$
$$= 328.33 \text{ (m/s)}^2 - 274.4 \text{ (m/s)}^2$$
$$= 53.93 \text{ (m/s)}^2$$
$$v_f = 7.3 \text{ m/s}$$

Statement: The velocity of the keys when they were caught was 7.3 m/s [up].

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1. (a) Given: $\vec{v}_i = 0$ m/s; $\Delta t = 5.2$ s; $\vec{a} = 4.1 \text{ m/s}^2$ [forward]

Required: $\Delta\vec{d}$

Analysis: Calculate the displacement using $\Delta\vec{d} = \vec{v}_i\Delta t + \frac{1}{2}\vec{a}\Delta t^2$. Use forward as the positive direction.

Solution: $\Delta d = v_i\Delta t + \frac{1}{2}a\Delta t^2$

$$= (0 \text{ m/}\cancel{\text{s}})(5.2 \cancel{\text{s}}) + \frac{1}{2}(4.1 \text{ m/}\cancel{\text{s}^2})(5.2 \cancel{\text{s}})^{\cancel{2}}$$
$$\Delta d = 55 \text{ m}$$

Statement: The racehorse's displacement is 55 m [forward].

(b) Given: $\vec{v}_i = 0$ m/s; $\Delta t = 5.2$ s; $\vec{a} = 4.1 \text{ m/s}^2$ [forward]

Required: $\vec{v}_f$

Analysis: Use $a = \frac{v_f - v_i}{\Delta t}$ to determine v_f:

$$a = \frac{v_f - v_i}{\Delta t}$$
$$v_f = v_i + a\Delta t$$

(Any of the equations containing v_f could be used.)

Solution: $v_f = v_i + a\Delta t$

$$= 0 \text{ m/s} + (4.1 \text{ m/s}^{\cancel{2}})(5.2 \cancel{\text{s}})$$
$$v_f = 21 \text{ m/s}$$

Statement: The horse's final velocity is 21 m/s [forward].

2. (a) Given: $\vec{v}_i = 7.72 \times 10^6$ m/s [E]; $\vec{v}_f = 2.46 \times 10^6$ m/s [E]; $\Delta\vec{d} = 0.478$ m [E]

Required: $\vec{a}$

Analysis: Use east as the positive direction.

Use $v_f^2 = v_i^2 + 2a\Delta d$ to calculate the acceleration:

$$v_f^2 = v_i^2 + 2a\Delta d$$
$$a = \frac{v_f^2 - v_i^2}{2\Delta d}$$

Solution: $a = \dfrac{v_f^2 - v_i^2}{2\Delta d}$

$= \dfrac{(2.46 \times 10^6 \text{ m/s})^2 - (7.72 \times 10^6 \text{ m/s})^2}{2(0.478 \text{ m})}$

$= \dfrac{-5.355 \times 10^{13} \text{ m}^{\cancel{2}}/\text{s}^2}{0.956 \cancel{\text{m}}}$

$a = -5.60 \times 10^{13} \text{ m/s}^2$

Statement: The electron's acceleration is $5.60 \times 10^{13} \text{ m/s}^2$ [W].

(b) Given: $\vec{v}_i = 7.72 \times 10^6$ m/s [E]; $\vec{v}_f = 2.46 \times 10^6$ m/s [E]; $\vec{a} = 5.60 \times 10^{13} \text{ m/s}^2$ [W]

Required: Δt

Analysis: Use $a = \dfrac{v_f - v_i}{\Delta t}$ to determine Δt: $\Delta t = \dfrac{v_f - v_i}{a}$

(Any of the equations containing Δt could be used.)

Solution: $\Delta t = \dfrac{v_f - v_i}{a}$

$= \dfrac{(2.46 \times 10^6 \text{ m/s}) - (7.72 \times 10^6 \text{ m/s})}{-5.60 \times 10^{13} \text{ m/s}^2}$

$= \dfrac{-5.26 \times 10^6 \cancel{\text{m}}/\cancel{\text{s}}}{-5.60 \times 10^{13} \cancel{\text{m}}/\text{s}^{\cancel{2}}}$

$\Delta t = 9.39 \times 10^{-8}$ s

Statement: The acceleration occurs over a time interval of 9.39×10^{-8} s.

3. Solutions may vary. Sample answer:

(a) Given: cruiser: $\vec{d}_{1i} = 0$ m [forward]; $\vec{v}_{1i} = 0$ m/s; $a = 3.0 \text{ m/s}^2$ [forward];

car: $\vec{d}_{2i} = 0$ m [forward]; $\vec{v}_2 = 62$ km/h [forward]

Required: time until the cruiser catches up with the car, Δt

Analysis: The vehicles start from the same position. The displacement and time when the two vehicles meet are not known. Set up an equation for each vehicle that relates position and time. The cruiser moves at constant acceleration, $\Delta\vec{d} = \vec{v}_i\Delta t + \frac{1}{2}\vec{a}\Delta t^2$, and the car at constant velocity, $\vec{v} = \dfrac{\Delta\vec{d}}{\Delta t}$. Compare these equations to solve for time and, later, displacement. Use forward as positive.

$\vec{v}_2 = \dfrac{62 \cancel{\text{ km}}}{1 \cancel{\text{h}}} \times \dfrac{1000 \cancel{\text{ m}}}{1 \cancel{\text{ km}}} \times \dfrac{1 \cancel{\text{h}}}{60 \cancel{\text{ min}}} \times \dfrac{1 \cancel{\text{ min}}}{60 \text{ s}}$

$\vec{v}_2 = 17.22$ m/s (two extra digits carried)

Solution: cruiser:

$$\Delta d_1 = v_i\Delta t + \frac{1}{2}a\Delta t^2$$
$$= (0\text{ m/s})\Delta t + \frac{1}{2}(3.0\text{ m/s}^2)\Delta t^2$$
$$\Delta d_1 = (1.5\text{ m/s}^2)\Delta t^2$$

car:

$$v_2 = \frac{\Delta d_2}{\Delta t}$$
$$\Delta d_2 = v_2\Delta t$$
$$\Delta d_2 = (17.22\text{ m/s})\Delta t$$

The displacements of the two vehicles are equal.

$$\Delta d_1 = \Delta d_2$$
$$(1.5\text{ m/s}^2)\Delta t^2 = (17.22\text{ m/s})\Delta t$$
$$(1.5\text{ m/s}^2)\Delta t^{\cancel{2}} = (17.22\text{ m/s})\cancel{\Delta t}$$
$$\Delta t = \frac{17.22\ \cancel{\text{m}}/\cancel{\text{s}}}{1.5\ \cancel{\text{m}}/\text{s}^{\cancel{2}}}$$
$$= 11.48\text{ s (two extra digits carried)}$$
$$\Delta t = 11\text{ s}$$

Statement: The cruiser catches up 11 s after starting.

(b) Given: car: $\vec{v}_2 = 17.22$ m/s [forward]; $\Delta t = 11.48$ s

Required: common displacement of the vehicles, Δd

Analysis: Substitute the value of Δt in the displacement equation of either the cruiser or the car. Use the equation of the car.

Solution:
$$\Delta d_2 = (17.22\text{ m/s})\Delta t$$
$$= (17.22\text{ m/}\cancel{\text{s}})(11.48\ \cancel{\text{s}})$$
$$\Delta d_2 = 2.0\times 10^2\text{ m}$$

Statement: The cruiser catches up to the car 2.0×10^2 m [forward] from the cruiser's initial position.

(c) Given: cruiser: $\vec{v}_{1i} = 0$ m/s; $a = 3.0$ m/s^2 [forward]; $\Delta t = 11.48$ s

Required: v_{1f}

Analysis: Use $\vec{a} = \dfrac{\vec{v}_f - \vec{v}_i}{\Delta t}$ to solve for the final speed of the cruiser.

$$a = \frac{v_{1f} - v_{1i}}{\Delta t}$$
$$v_{1f} - v_{1i} = a\Delta t$$
$$v_{1f} = v_{1i} + a\Delta t$$

Solution: $v_{1f} = v_{1i} + a\Delta t$

$$= (0\ \text{m/s}) + (3.0\ \text{m/s}^{\cancel{2}})(11.48\ \cancel{\text{s}})$$
$$= 34\ \text{m/s}$$
$$= \frac{34\ \cancel{\text{m}}}{\cancel{\text{s}}} \times \frac{1\ \text{km}}{1000\ \cancel{\text{m}}} \times \frac{60\ \cancel{\text{s}}}{1\ \cancel{\text{min}}} \times \frac{60\ \cancel{\text{min}}}{1\ \text{h}}$$
$$v_{1f} = 120\ \text{km/h}$$

Statement: The cruiser is moving at 120 km/h when it catches up with the car. This is an unsafe speed in a school zone.

(d) Given: cruiser: $\vec{v}_{1i} = 0$ m/s; $\vec{v}_{1f} = 72$ km/h [forward]; $a = 3.0\ \text{m/s}^2$ [forward]; then at constant speed; car: $\vec{v}_2$ = 17.22 m/s [forward]

Required: time until the cruiser catches up with the car, Δt

Analysis: Build a new equation for the displacement of the cruiser and the car.

$$v_{1f} = \frac{72\ \cancel{\text{km}}}{\cancel{\text{h}}} \times \frac{1000\ \text{m}}{1\ \cancel{\text{km}}} \times \frac{1\ \cancel{\text{h}}}{60\ \cancel{\text{min}}} \times \frac{1\ \cancel{\text{min}}}{60\ \text{s}}$$
$$v_{1f} = 20\ \text{m/s}$$

Determine how long and how far the cruiser accelerates to reach the speed of 20 m/s. Use $v_f = v_i + a\Delta t$ and $v_f^2 = v_i^2 + 2a\Delta d$. Then write an equation for the cruiser's displacement while continuing at constant speed. Determine where the car is when the cruiser's motion changes, and then write a new equation for the car's displacement. The rest of the solution will be similar to part (a).

Solution: The cruiser's final speed is $\vec{v}_f = 20$ m/s [forward]. It reaches 20 m/s when

$$v_f = v_i + a\Delta t$$
$$\Delta t = \frac{v_f - v_i}{a}$$
$$= \frac{20\ \cancel{\text{m/s}} - 0\ \cancel{\text{m/s}}}{3.0\ \cancel{\text{m}}/\text{s}^{\cancel{2}}}$$
$$\Delta t = 6.67\ \text{s}$$

The cruiser's displacement is now

$$v_f^2 = v_i^2 + 2a\Delta d$$
$$\Delta d_1 = \frac{v_f^2 - v_i^2}{2a}$$
$$= \frac{(20\ \text{m/s})^2 - (0\ \text{m/s})^2}{2(3.0\ \text{m/s}^2)}$$
$$= \frac{400\ \text{m}^{\cancel{2}}/\cancel{\text{s}^2}}{6.0\ \cancel{\text{m}}/\cancel{\text{s}^2}}$$
$$d_1 = 66.7\ \text{m (one extra digit carried)}$$

At 6.67 s, the car's displacement is

$$\Delta d_2 = v_2 \Delta t$$
$$\Delta d_2 = (17.22 \text{ m/}\cancel{\text{s}})(6.67 \ \cancel{\text{s}})$$
$$d_2 = 114.9 \text{ m (two extra digits carried)}$$

Treat the motion as if it started at $t_i = 6.67$ s, after which both vehicles move at constant speed. The later positions of the vehicles are

cruiser:

$$\Delta d_1 = v_1 \Delta t$$
$$d_1 - 66.7 \text{ m} = (20 \text{ m/s})\Delta t$$
$$d_1 = 66.7 \text{ m} + (20 \text{ m/s})\Delta t$$

car:

$$\Delta d_2 = v_2 \Delta t$$
$$d_2 - 114.9 \text{ m} = (17.22 \text{ m/s})\Delta t$$
$$d_2 = 114.9 \text{ m} + (17.22 \text{ m/s})\Delta t$$

The cruiser catches up to the car when the positions are equal.

$$d_1 = d_2$$
$$66.7 \text{ m} + (20 \text{ m/s})\Delta t = 114.9 \text{ m} + (17.22 \text{ m/s})\Delta t$$
$$(20 \text{ m/s})\Delta t - (17.22 \text{ m/s})\Delta t = 114.9 \text{ m} - 66.7 \text{ m}$$
$$\Delta t = \frac{48.2 \ \cancel{\text{m}}}{2.78 \ \cancel{\text{m}}\text{/s}}$$
$$\Delta t = 17.34 \text{ s (two extra digits carried)}$$

$$d_1 = 66.7 \text{ m} + (20 \text{ m/s})\Delta t$$
$$= 66.7 \text{ m} + (20 \text{ m/}\cancel{\text{s}})(17.34 \ \cancel{\text{s}})$$
$$d_1 = 413.5 \text{ m (two extra digits carried)}$$

The total time elapsed from the start is

$$\Delta t = 6.67 \text{ s} + 17.34 \text{ s}$$
$$\Delta t = 24 \text{ s}$$

Statement: Under the second scenario, it takes 24 s for the cruiser to catch the car, by which time the cruiser and the car have travelled more than 400 m. They might be on the other side of the school zone and it would be too late to protect the children. Neither of these scenarios is very reasonable.

4. Answers may vary. Sample answer: I would stand at the top of the cliff with a small stone. My partner would be at the bottom of the cliff with a stopwatch. I would signal with my hand the instant I dropped the stone. My partner would time the stone's fall. Since I would now know the time interval taken, the acceleration due to gravity, and the initial velocity (0 m/s), I would use $\Delta \vec{d} = \vec{v}_i \Delta t + \frac{1}{2}\vec{a}\Delta t^2$ to determine the displacement. The magnitude of the displacement would be our estimate of the height of the cliff. The assumption I would have to make is that the height of the cliff is being measured from sea level.

5. Solutions may vary. Sample answer:
(a) Given: $\Delta t = 2.4\text{ s}$; $\vec{a} = 9.8\text{ m/s}^2\text{ [down]}$
Required: $\vec{v}_\text{i}$
Analysis: Since the ball is caught at the same height from which it is thrown, its total displacement is 0 m. From $\Delta\vec{d} = \left(\dfrac{\vec{v}_\text{i} + \vec{v}_\text{f}}{2}\right)\Delta t$, the final velocity is the opposite of the initial velocity. Use this fact and $\vec{a} = \dfrac{\vec{v}_\text{f} - \vec{v}_\text{i}}{\Delta t}$ to calculate $\vec{v}_\text{i}$.

$$\vec{a} = \frac{\vec{v}_\text{f} - \vec{v}_\text{i}}{\Delta t}$$
$$\vec{a} = \frac{-2\vec{v}_\text{i}}{\Delta t}$$
$$\vec{v}_\text{i} = -\frac{\vec{a}\Delta t}{2}$$

Use up as the positive direction.

Solution:
$$\begin{aligned}\vec{v}_\text{i} &= -\frac{\vec{a}\Delta t}{2}\\ &= -\frac{(-9.8\text{ m/s}^{\cancel{2}})(2.4\ \cancel{\text{s}})}{2}\\ &= 11.76\text{ m/s (two extra digits carried)}\\ \vec{v}_\text{i} &= 12\text{ m/s [up]}\end{aligned}$$

Statement: The ball's initial velocity is 12 m/s [up].
(b) Given: $\Delta t = 2.4\text{ s}$; $\vec{a} = 9.8\text{ m/s}^2\text{ [down]}$; $\vec{v}_i = 11.76\text{ m/s [up]}$
Required: maximum height, Δd
Analysis: The flight of the ball is symmetric going up and coming down, so the time to reach maximum height is one-half of the total time interval, 2.4 s: $\Delta t = 1.2\text{ s}$. The speed of the ball at maximum height is 0 m/s. Use any of the formulas that contain Δd to calculate the height. This solution uses $\Delta d = \left(\dfrac{v_\text{i} + v_\text{f}}{2}\right)\Delta t$.

Solution:
$$\begin{aligned}\Delta d &= \left(\frac{v_\text{i} + v_\text{f}}{2}\right)\Delta t\\ &= \frac{(11.76\text{ m/}\cancel{\text{s}} + 0\text{ m/}\cancel{\text{s}})(1.2\ \cancel{\text{s}})}{2}\\ \Delta d &= 7.1\text{ m}\end{aligned}$$

Statement: The maximum height of the ball is 7.1 m.

6. Solutions may vary. Sample answer:

(a) Given: $\vec{v}_{\mathrm{i}} = 18$ m/s [up]; $\Delta\vec{d} = 32$ m [down]; $\vec{a} = 9.8$ m/s^2 [down]

Required: time to reach the ground, Δt

Analysis: Use up as positive. Use $\Delta d = v_{\mathrm{i}}\Delta t + \frac{1}{2}a\Delta t^2$ to calculate Δt:

$$\frac{1}{2}a\Delta t^2 + v_{\mathrm{i}}\Delta t - \Delta d = 0$$

Use the quadratic formula to determine Δt.

$$\Delta t = \frac{-v_{\mathrm{i}} \pm \sqrt{v_{\mathrm{i}}^2 - 4\left(\frac{1}{2}a\right)(-\Delta d)}}{a}$$

Solution:
$$\begin{aligned}\Delta t &= \frac{-v_{\mathrm{i}} \pm \sqrt{v_{\mathrm{i}}^2 - 4(\frac{1}{2}a)(-\Delta d)}}{a} \\ &= \frac{-18\text{ m/s} \pm \sqrt{(18\text{ m/s})^2 - 4(-4.9\text{ m/s}^2)(32\text{ m})}}{-9.8\text{ m/s}^2} \\ &= 4.984\text{ s (or } -1.310\text{ s) (two extra digits carried)} \\ \Delta t &= 5.0\text{ s}\end{aligned}$$

Statement: The ball takes 5.0 s to hit the ground.

(b) Given: $\vec{v}_{\mathrm{i}} = 18$ m/s [up]; $\Delta\vec{d} = 32$ m [down]; $\vec{a} = 9.8$ m/s^2 [down]; $\Delta t = 4.984$ s

Required: v_{f}

Analysis: Use $a = \frac{v_{\mathrm{f}} - v_{\mathrm{i}}}{\Delta t}$ to calculate $v_{\mathrm{f}} = v_{\mathrm{i}} + a\Delta t$.

Solution:
$$\begin{aligned}v_{\mathrm{f}} &= v_{\mathrm{i}} + a\Delta t \\ &= (18\text{ m/s}) + (-9.8\text{ m/s}^2)(4.984\text{ s}) \\ &= 18\text{ m/s} - 48.80\text{ m/s} \\ v_{\mathrm{f}} &= -31\text{ m/s}\end{aligned}$$

Statement: The velocity of the ball when it hits the ground is 3.1×10^1 m/s [down].

(c) Given: $\vec{v}_{\mathrm{i}} = 18$ m/s [up]; $\vec{v}_{\mathrm{f}} = 0$ m/s ; $\vec{a} = 9.8$ m/s^2 [down]

Required: maximum height, $d_{\max}$

Analysis: Use $v_{\mathrm{f}}^2 = v_{\mathrm{i}}^2 + 2a\Delta d$ to solve for $\Delta d = \frac{v_{\mathrm{f}}^2 - v_{\mathrm{i}}^2}{2a}$.

Then $\Delta d = d_{\max} - d_i$ to solve for $d_{\max}$.

Solution: $\Delta d = \dfrac{v_f^2 - v_i^2}{2a}$

$$= \frac{(0 \text{ m/s})^2 - (18 \text{ m/s})^2}{2(-9.8 \text{ m/s}^2)}$$

$$= \frac{324 \text{ m}^{\cancel{2}}/\cancel{s^2}}{19.6 \cancel{\text{m}}/\cancel{s^2}}$$

$\Delta d = 16.53$ m (two extra digits carried)

$d_{max} = d_i + \Delta d$

$= 32 \text{ m} + 16.53 \text{ m}$

$d_{max} = 49$ m

Statement: The ball reaches a maximum height of 49 m.

(d) The flight of the ball is not symmetric. The time and distance travelled from my hand to the maximum are smaller than the time and distance travelled from the maximum to the ground. So, half the total time does not correspond to the time for either part of the whole flight.

7. Solutions may vary. Sample answer:

(a) Given: $\vec{v}_i = 0$ m/s; $\Delta t = 5.0$ s; $\vec{a} = 39.2 \text{ m/s}^2$ [up]

Required: $\vec{v}_f$

Analysis: Use $a = \dfrac{v_f - v_i}{\Delta t}$ to determine $v_f = v_i + a\Delta t$.

Use up as the positive direction.

Solution: $v_f = v_i + a\Delta t$

$= 0 \text{ m/s} + (39.2 \text{ m/s}^{\cancel{2}})(5.0 \cancel{s})$

$= 196$ m/s (one extra digit carried)

$v_f = 2.0 \times 10^2$ m/s

Statement: The rocket is moving at 2.0×10^2 m/s [up] when the engines stop.

(b) Given: $\vec{v}_i = 196$ m/s [up]; $\vec{v}_f = 0$ m/s; $\vec{a} = 9.8 \text{ m/s}^2$ [down]

Required: maximum height of the rocket, d_f

Analysis: Determine the rocket's height when the engine shuts off using $\Delta d = v_i\Delta t + \dfrac{1}{2}a\Delta t^2$.

Then determine the displacement of the rocket while it slows down using $v_f^2 = v_i^2 + 2a\Delta d$.

$v_f^2 = v_i^2 + 2a\Delta d$

$\Delta d = \dfrac{v_f^2 - v_i^2}{2a}$

Together, these will give us the maximum height. Continue to use up as positive.

Solution: When the engine has finished firing, the rocket has risen $\Delta d = v_{\text{i}}\Delta t + \frac{1}{2}a\Delta t^2$.

$$\begin{aligned}\Delta d &= v_{\text{i}}\Delta t + \frac{1}{2}a\Delta t^2\\ &= (0\ \text{m}/\cancel{\text{s}})(5.0\ \cancel{\text{s}}) + \frac{1}{2}(39.2\ \text{m}/\cancel{\text{s}^2})(5.0\ \cancel{\text{s}})^{\cancel{2}}\\ \Delta d &= 490\ \text{m}\end{aligned}$$

From when the engines stops to maximum height, the rocket goes up.

$$\begin{aligned}\Delta d &= \frac{v_{\text{f}}^2 - v_{\text{i}}^2}{2a}\\ &= \frac{(0\ \text{m}/\cancel{\text{s}})^{\cancel{2}} - (196\ \text{m}/\cancel{\text{s}})^{\cancel{2}}}{2(-9.8\ \text{m}/\cancel{\text{s}^2})}\\ \Delta d &= 1960\ \text{m}\end{aligned}$$

The maximum height is

$$\begin{aligned}d_{\text{f}} &= d_{\text{i}} + \Delta d\\ &= 490\ \text{m} + 1960\ \text{m}\\ &= 2450\ \text{m}\\ d_{\text{f}} &= 2.4\times10^3\ \text{m}\end{aligned}$$

Statement: The maximum height of the rocket is 2.4×10^3 m.

(c) Given: $\vec{v}_{\text{i}} = 0$ m/s ; $\vec{d}_{\text{i}} = 2450$ m [up] from part (b); $\vec{a} = 9.8\ \text{m/s}^2$ [down]

Required: time to reach the ground, Δt, and total time of the flight, t_{tot}

Analysis: Use $v_{\text{f}} = v_{\text{i}} + a\Delta t$ to calculate the time taken from the engine stopping to maximum: $\Delta t = \frac{v_{\text{f}} - v_{\text{i}}}{a}$. Then, use $\Delta d = v_{\text{i}}\Delta t + \frac{1}{2}a\Delta t^2$ to determine the time to fall to the ground:

$$\Delta t = \frac{-v_{\text{i}} \pm \sqrt{v_{\text{i}}^2 - 4\left(\frac{1}{2}a\right)(-\Delta d)}}{a}$$

Lastly, add the times needed for the three parts of the flight to determine the total time.

Solution:
$$\begin{aligned}\Delta t &= \frac{v_{\text{f}} - v_{\text{i}}}{a}\\ &= \frac{0\ \cancel{\text{m}}/\cancel{\text{s}} - 196\ \cancel{\text{m}}/\cancel{\text{s}}}{-9.8\ \cancel{\text{m}}/\text{s}^{\cancel{2}}}\\ \Delta t &= 20\ \text{s}\end{aligned}$$

For the rocket to fall to the ground from its maximum height:

$$\begin{aligned}\Delta t &= \frac{-v_i \pm \sqrt{v_i^2 - 4\left(\frac{1}{2}a\right)(-\Delta d)}}{a}\\ &= \frac{0 \pm \sqrt{0^2 - 4(-4.9)(2450)}}{-9.8}\\ &= \pm 22.36 \text{ s (two extra digits carried)}\end{aligned}$$

$t_{tot} = 5.0 \text{ s} + 20 \text{ s} + 22.36 \text{ s}$

$t_{tot} = 47 \text{ s}$

Statement: The time for the rocket to fall from rest at the maximum is 22 s. The total time from being at rest initially is 47 s.

8. Solutions may vary. Sample answer:

Table 2

	Acceleration (m/s^2)	Reaction time (s)	Speed (km/h)	Braking distance (m)
(i)	9.5	0.80	60.0	28
(ii)	9.5	0.80	120.0	85
(iii)	9.5	2.0	60.0	48

(a) (i) Given: $\vec{v}_1 = 60.0$ km/h [forward]; $\Delta t_1 = 0.80$ s; $\vec{a} = 9.5 \text{ m/s}^2$ [backward]

Required: braking distance, d

Analysis: The car moves at constant speed during the reaction time. Use $v_{av} = \frac{\Delta d}{\Delta t}$ to determine the distance covered during this time: $\Delta d = v\Delta t$.

The car then slows to a stop. Determine the distance covered during this time using:

$$v_f^2 = v_i^2 + 2a\Delta d$$

$$\Delta d = \frac{v_f^2 - v_i^2}{2a}$$

Then, determine the total of these distances travelled. Use forward as the positive direction.

$$\vec{v}_1 = \frac{60.0 \cancel{\text{ km}}}{1 \text{ h}} \times \frac{1000 \text{ m}}{1 \cancel{\text{ km}}} \times \frac{1 \cancel{\text{h}}}{60 \cancel{\text{ min}}} \times \frac{1 \cancel{\text{ min}}}{60 \cancel{\text{s}}}$$

$\vec{v}_1 = 16.67$ m/s [forward] (two extra digits carried)

Solution:

$$\begin{aligned}\Delta d_1 &= v_1 \Delta t\\ &= (16.67 \text{ m}/\cancel{\text{s}})(0.80 \cancel{\text{s}})\\ \Delta d_1 &= 13.34 \text{ m (two extra digits carried)}\end{aligned}$$

$$\Delta d_2 = \frac{v_f^2 - v_i^2}{2a}$$
$$= \frac{(0\ \text{m/}\cancel{\text{s}})^2 - (16.67\ \text{m/}\cancel{\text{s}})^2}{2(-9.5\ \cancel{\text{m}}/\cancel{\text{s}^2})}$$
$\Delta d_2 = 14.63$ m (two extra digits carried)

$$\Delta d = \Delta d_1 + \Delta d_2$$
$$= 13.34\ \text{m} + 14.63\ \text{m}$$
$$\Delta d = 28\ \text{m}$$

Statement: The braking distance is 28 m.

(ii) Given: $\vec{v}_2 = 120.0$ km/h [forward] $= 33.33$ m/s [forward]; $\Delta t_2 = 0.80$ s;
$\vec{a} = 9.5\ \text{m/s}^2$ [backward]

Required: braking distance, d

Analysis: Repeat the steps of part (i).

Solution: $\Delta d_3 = v_2 \Delta t_2$
$$= (33.33\ \text{m/}\cancel{\text{s}})(0.80\ \cancel{\text{s}})$$
$\Delta d_3 = 26.66$ m (two extra digits carried)

$$\Delta d_4 = \frac{v_f^2 - v_i^2}{2a}$$
$$= \frac{(0\ )^2 - (33.33\ \text{m/}\cancel{\text{s}})^2}{2(-9.5\ \cancel{\text{m}}/\cancel{\text{s}^2})}$$
$\Delta d_4 = 58.47$ m (two extra digits carried)

$$d = \Delta d_3 + \Delta d_4$$
$$= 26.66\ \text{m} + 58.47\ \text{m}$$
$$d = 85\ \text{m}$$

Statement: The braking distance is 85 m.

(iii) Given: $\vec{v}_3 = 60.0$ km/h [forward] $= 16.67$ m/s [forward]; $\Delta t_3 = 2.0$ s; $\vec{a} = 9.5\ \text{m/s}^2$ [backward]

Required: braking distance, d

Analysis: Repeat the steps of part (i).

Solution: $\Delta d_5 = v_3 \Delta t_3$
$$= (16.67\ \text{m/}\cancel{\text{s}})(2.0\ \cancel{\text{s}})$$
$\Delta d_5 = 33.34$ m (two extra digits carried)

$$\Delta d_6 = \frac{v_f^2 - v_i^2}{2a}$$
$$= \frac{(0\ \text{m/}\cancel{\text{s}})^2 - (16.67\ \text{m/}\cancel{\text{s}})^2}{2(-9.5\ \cancel{\text{m}}/\cancel{\text{s}^2})}$$
$\Delta d_6 = 14.63$ m (two extra digits carried)

$$d = \Delta d_5 + \Delta d_6$$
$$= 33.34\ \text{m} + 14.63\ \text{m}$$
$$d = 48\ \text{m}$$

Statement: The braking distance is 48 m.

(b) If a driver uses a cellphone while driving, it is likely that the driver's reaction time and braking distance will increase. The table shows that at 60 km/h, increasing the reaction time from 0.80 s to 2.0 s causes the braking distance to increase from 28 m to 48 m (almost double). Similarly, the driving speed affects the braking distance. For a reaction time of 0.80 s, increasing the speed from 60 km/h to 120 km/h results in the braking distance increasing from 28 m to 85 m (about triple).