Solving Equations/inequalities:

17) Determine the solution(s) of:

a) $349 = 7(1.49)^x$ b) $\log_2 8 = 3\log_2 x - \log_2 3$

a) $\frac{349}{7} = \frac{7(1.49)^x}{7}$

$49.8571 \doteq 1.49^x$

$\Leftrightarrow \log_{1.49} 49.8571 \doteq x$

$\frac{\log 49.8571}{\log 1.49} \doteq x$

$9.80 \doteq x$

b) $\log_2 8 = 3\log_2 x - \log_2 3$

$\log_2 8 = \log_2 x^3 - \log_2 3$

$\log_2 8 = \log_2 \left(\frac{x^3}{3}\right)$

$\therefore \quad 8 = \frac{x^3}{3}$

$24 = x^3$

$\sqrt[3]{24} = x$

$2.88 \doteq x, \quad x > 0$

c) $7^x = 3^{x^2-1}$

$\Leftrightarrow \log_3 7^x = x^2 - 1$

$x(\log_3 7) = x^2 - 1$

$1.7712x \doteq x^2 - 1$

$0 \doteq x^2 - 1.7712x - 1$

$x \doteq \frac{1.7712 \pm \sqrt{(-1.7712)^2 - 4(-1)}}{2}$

$x \doteq 2.22$ or $x \doteq -0.45$

$\rightarrow \underline{\underline{\text{or}}}$

$\therefore \log 7^x = \log 3^{x^2-1}$

$x \log 7 = (x^2 - 1)\log 3$

$0.8451x \doteq 0.4771x^2 - 0.4771$

$0 \doteq 0.4771x^2 - 0.8451x - 0.4771$

$x \doteq \frac{0.8451 \pm \sqrt{(-0.8451)^2 - 4(-0.4771)^2}}{2(0.4771)}$

$x \doteq 2.22$ or $x \doteq -0.45$

d) $x^3 - 6x^2 + 5x + 12 > 0$

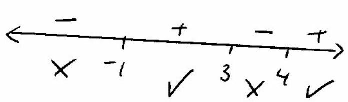
$x^3 - 6x^2 + 5x + 12 > 0$

let $P(x) = x^3 - 6x^2 + 5x + 12$

$P(-1) = 0 \therefore x + 1$ is a factor

$$\begin{array}{r|rrrr} -1 & 1 & -6 & 5 & 12 \\ & \downarrow & -1 & 7 & -12 \\ \hline & 1 & -7 & 12 & 0 \end{array}$$

$\therefore (x+1)(x^2 - 7x + 12) > 0$



when $-1 < x < 3$ and

when $4 < x$

or use factor chart

factors	$x < -1$	$-1 < x < 3$	$3 < x < 4$	$4 < x$
$x+1$	−	+	+	+
$x-4$	−	−	−	+
$x-3$	−	−	+	+
	−	+	−	+

or sketch!

e) $\dfrac{4x+9}{4x-1} \le \dfrac{x+5}{x}$

$$\frac{4x+9}{4x-1} - \frac{x+5}{x} \le 0$$

$$\frac{x(4x+9)-(4x-1)(x+5)}{x(4x-1)} \le 0$$

$$\frac{4x^2+9x-4x^2-19x+5}{x(4x-1)} \le 0$$

$$\frac{-10x+5}{x(4x-1)} \le 0$$

$$\frac{-5(2x-1)}{x(4x-1)} \le 0$$

zeros: $\frac{1}{2}$

VA: $x=0$ $\quad x=\frac{1}{4}$

	+		−		+		−
×		0 ✓		$\frac{1}{4}$ ×		$\frac{1}{2}$ ✓	

when $0 < x < \frac{1}{4}$

or $\frac{1}{2} \le x$

or

factors	$x<0$	$0<x<\frac{1}{4}$	$\frac{1}{4}<x<\frac{1}{2}$	$\frac{1}{2}<x$
-5	−	−	−	−
$2x-1$	−	−	−	+
x	−	+	+	+
$4x-1$	−	−	+	+
sign of fnct	+	−	+	−

f) $\cos 2\theta = -0.9541$

$\cos 2\theta = -0.9541$

Let $x = 2\theta$

$\cos x = -0.9541$

$\cos x_r = 0.9541$

$x_r \doteq 0.3042$

Period $= \frac{2\pi}{2}$

$= \pi$

$x_1 = \pi - x_r$

$x_1 \doteq 2.8374 \rightarrow \theta_1 = x_1 \div 2$ $\qquad \theta_3 = \theta_1 + \text{period}$

$\theta_1 \doteq 1.4187$ $\qquad \theta_3 \doteq 4.5603$

$x_2 = \pi + x_r$

$x_2 \doteq 3.4458 \rightarrow \theta_2 = x_2 \div 2$ $\qquad \theta_4 = \theta_2 + \text{period}$

$\theta_2 \doteq 1.7229$ $\qquad \theta_4 \doteq 4.8645$

for more answers add/subtract $n\pi$

$n \in \mathbb{Z}$.

g) $\sin\theta - \sin\theta\tan\theta = 0$

$\sin\theta - \sin\theta\tan\theta = 0$

$\sin\theta(1-\tan\theta) = 0$

$\sin\theta = 0$ or $1-\tan\theta = 0$

$\theta = 0, \pi, 2\pi$

$1 = \tan\theta$

$\frac{\pi}{4} = \theta_1$

$\theta_3 = \frac{\pi}{4} + \pi$

$\theta_3 = \frac{5\pi}{4}$

$\theta = 0, \frac{\pi}{4}, \pi, \frac{5\pi}{4}, 2\pi$

h) $6\sin^2\theta - 5\cos\theta - 2 = 0$

) $6\sin^2\theta - 5\cos\theta - 2 = 0$

$6(1-\cos^2\theta) - 5\cos\theta - 2 = 0$

$6 - 6\cos^2\theta - 5\cos\theta - 2 = 0$

$-6\cos^2\theta - 5\cos\theta + 4 = 0$

$6\cos^2\theta + 5\cos\theta - 4 = 0$

$(2\cos\theta - 1)(3\cos\theta + 4) = 0$

$P = -24$
$S = 5$
$\frac{8}{6}, \frac{-3}{6}$
$\downarrow$
$\frac{4}{3}, \frac{-1}{2}$

$\cos\theta = \frac{1}{2}$

$\theta_1 = \frac{\pi}{3}$

$\theta_2 = 2\pi - \theta_1$

$\theta_2 = \frac{5\pi}{3}$

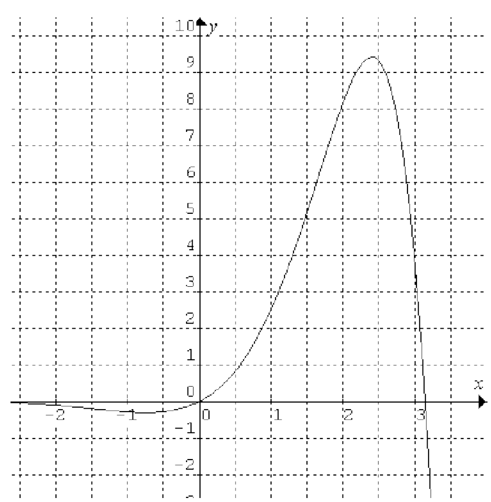
$\cos\theta = \frac{-4}{3}$

$\cos\theta_R = \frac{+4}{3}$

no sol^n $\frac{4}{3} > 1$

$\theta = \frac{\pi}{3}, \frac{5\pi}{3}$

18) The graph of the function $p(x) = 3^x \sin x$ is shown on the right. Use the graph to estimate the answer to the following questions then verify your answer(s) using the equation.



a) evaluate $p(1)$

a) $p(1)$ find y value when $x = 1$

$p(1) \doteq 2.6$

verify: $p(1) = 3^1 \sin 1$

$p(1) \doteq 3(0.8414\ldots)$

$p(1) \doteq 2.52$

b) solve for a if $p(a) = 8$

$p(a) = 8$ find x when $y = 8$

$a \doteq 1.99$ $a \doteq 2.7$

c) the interval for which $y \le 2$

$y \le 2$

find when $y = 2$

$x \doteq 0.85$ $x \doteq 3.07$

verify $p(0.85) \doteq 1.91$ $p(3.07) \doteq 2.09$

ok ok

$y \le 2$ when $x < 0.85$

or $x > 3.07$

19) For the function defined by $f(x) = k(x+1)^2(x-2)(x-4)$

a) Determine the value of k, if (1, -24) is a point on the graph of the function

a) $-24 = K(2)^2(-1)(-3)$

$-24 = 12K$

$-2 = K$

$f(x) = -2(x+1)^2(x-2)(x-4)$

b) solve for p if (3, p) is a point on the graph of the function

$p = f(3)$

$p = -2(4)^2(1)(-1)$

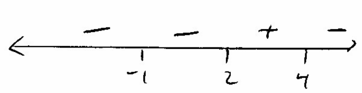
$p = 32$

c) considering the end behaviours and the zeros, state where $f(x) > 0$

c) zeros: -1 DR ∴ sign does not change

$2, 4$ SR ∴ sign changes

lead coeff. $-ve$ ∴ ends $-ve$



$f(x) > 0$ when $2 < x < 4$

Combination of functions:

20) Determine $h(x) = (f \circ g)(x)$ when $f(x) = 2x^4 - 3x^2$ and $g(x) = \sqrt{x-3}$ and state the domain and range of *h(x)*.

a) $D_h \in D_g$ when $g(x) \in D_f$

$D_g: x \geq 3$ and all $g(x) \in D_f$

$D_h: \{x \mid x \geq 3,\ x \in \mathbb{R}\}$

b) $R_h \in R_f$ when $g(x) \in D_f$

$R_f: y \geq -1.125$ and all $g(x) \in D_f$

$R_h: \{y \mid y \geq -1.125,\ y \in \mathbb{R}\}$

↑ min. value of f(x)

21) Given $D_f = \left\{x \mid -5 \leq x \leq 8,\ x \in \Re\right\}$ and $D_g = \left\{x \mid -12 \leq x \leq 3,\ x \in \Re\right\}$ determine

a) D_{f+g} b) $D_{g \cdot f}$

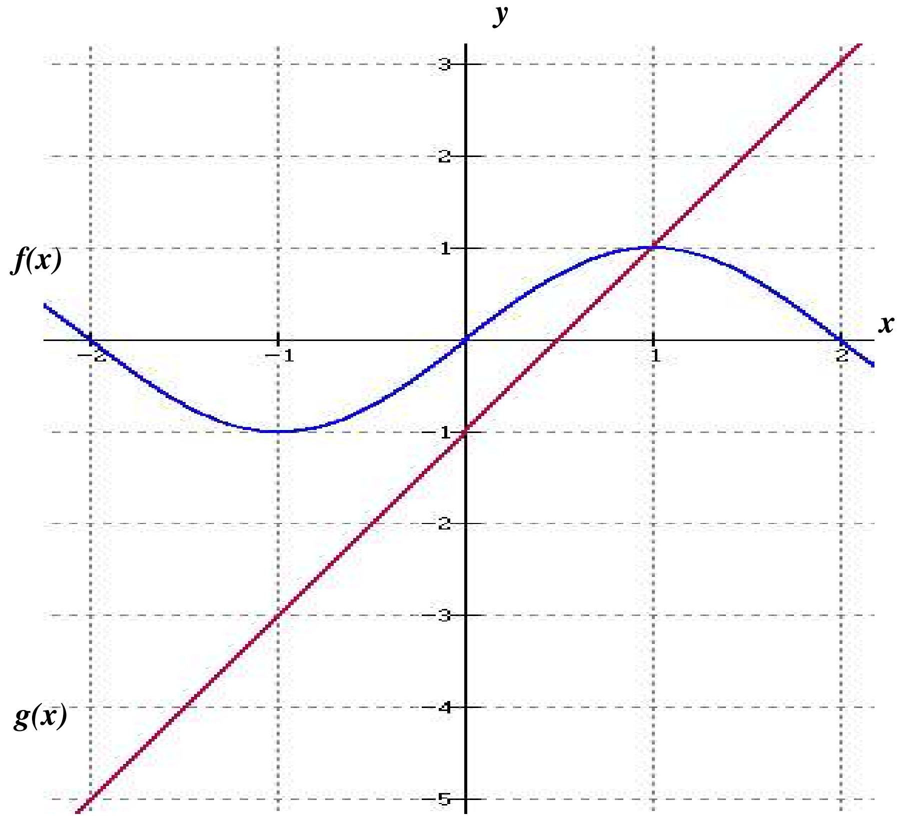
a) D_{f+g} : "overlap"

$D_{f+g} = \{x \mid -5 \leq x \leq 3,\ x \in \mathbb{R}\}$

b) $D_{g \cdot f}$: "overlap"

$D_{g \cdot f} = \{x \mid -5 \leq x \leq 3,\ x \in \mathbb{R}\}$

22) The Graphs of $y = f(x)$ and $y = g(x)$ are given below.



On the same grid sketch a) $y = f(x) + g(x)$ b) $y = f(x) \cdot g(x)$

23) Given $s(x) = \sin x + 2\cos x$,

a) determine D_s b) at most, what is the range of the function?

a) $D_s = \{x \mid x \in \mathbb{R}\}$ (both fnct have the same domain, $x \in \mathbb{R}$)

b) at most $R_s = R_{\sin x} + R_{2\cos x}$
but since phase shift, never equal to the actual sum!
$R_s : \{y \mid -3 < y < 3, y \in \mathbb{R}\}$

24) Given $d(x) = \tan x + \log x$, determine D_d

$D_d : \{x \mid x > 0, \ x \neq \frac{\pi}{2} + n\pi, \ n \in N, \ x \in \mathbb{R}\}$

↑ for $\log x$ ↑ restrictions on $\tan x$

25) What is the maximum number of zeros possible for $p(x)=(x+2)(x+3)(x-4)\log x$? Do you think there will actually be that many zeros? Justify your answers.

$p(x) = (x+2)(x+3)(x-4)\log x$

$p(x) =$ cubic · logarithm

The fnct could have 4 zeros (3 from cubic an 1 from log)

This will not be the actual # of zeros since two of the zeros of the cubic are negative and the domain – because of $\log x$ – is restricted to $x > 0$.

actual zeros = 1, 4

26) Given $f(x)=\dfrac{x}{x+1}$ and $g(x)=\cos x$, determine (Keep your answers within $[0, 2\pi]$.)

a) $D_{f \cdot g}$ b) $D_{f \div g}$ c) zeros, holes and vertical asymptotes of $g \div f$

1a) $D_{f \cdot g}$: overlap of D_f and D_g

$D_{f \cdot g} = \{x \mid 0 \le x \le 2\pi, x \in \mathbb{R}\}$

$x \neq -1$ not needed ∵ not in domain

b) $D_{f \div g}$: overlap of D_f and D_g additional restrictions $g(x) \neq 0$

$D_{f \div g} = \{x \mid x \neq \frac{\pi}{2}, \frac{3\pi}{2}, 0 \le x \le 2\pi, x \in \mathbb{R}\}$

restriction $x \neq -1$ not needed ∵ not in D_g

c) zeros: $\frac{\pi}{2}, \frac{3\pi}{2}$; hole: none since x = -1 is not in the domain; VA x = 0

Word Problems:

27) Distance in kilometres above sea level is given by the formula $d=\dfrac{500(\log P-2)}{27}$, where P is the atmospheric pressure measured in kiloPascals, kPa.

a) At the top of the highest mountain in Shelbyville, the atmospheric pressure was recorded as being 220 kPa. Calculate the height of the mountain above sea level.

$P = 220$

$$d = \frac{500(\log 220 - 2)}{27}$$

$d \doteq 6.34$

The highest mountain in Shelbyville is approx. 6.34 km above sea level.

b) The town of Springfield has a mountain with a peak 4.5 km above sea level. Calculate the atmospheric pressure at the top of the mountain.

$$4.5 = \frac{500(\log P - 2)}{27}$$

$121.5 = 500(\log P - 2)$

$0.243 = \log P - 2$

$2.243 = \log P$

$\Leftrightarrow \quad P = 10^{2.243}$

$P \doteq 174.98$

The atmospheric pressure at the top of the mountain is approx. 175 kPa.

c) In the year 1980, both towns had an earthquake. Springfield's earthquake measured 7.5 on the Richter Scale. The magnitude of the earthquake was $10^{7.5}$. The earthquake in Shelbyville measured 6.4. How many times more intense was Springfield's earthquake when compared to Shelbyville's earthquake. Recall: $M = \log\left(\frac{I}{I_o}\right)$

c) $M = \log \frac{I}{I_o}$

$\Delta M = \log \frac{I_{springfield}}{I_{shelbyville}}$

$7.5 - 6.4 = \log \frac{I_{spring}}{I_{shelby}}$

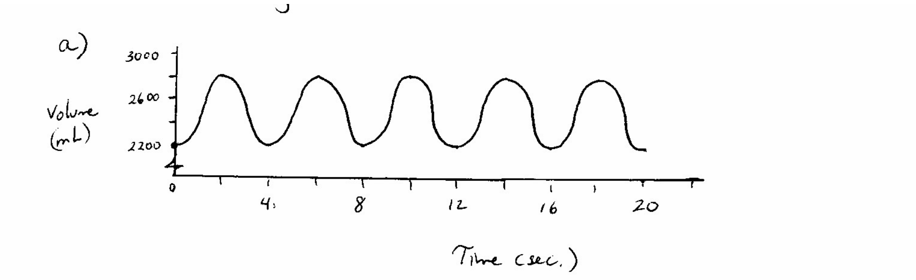
$\Leftrightarrow \quad \frac{I_{spring}}{I_{shelby}} = 10^{1.1}$

$\frac{I_{spring}}{I_{shelby}} \doteq 12.59$

Springfield's earthquake is approx. 12.59 times more intense than Shelbyville's earthquake.

28) The volume of air in the lungs during normal breathing can be modeled by a sinusoidal function of time. Suppose a person's lungs contain from 2200 mL to 2800 mL of air during normal breathing. Suppose a normal breath takes 4 seconds, and that $t = 0\ s$ corresponds to a minimum volume.

a) Let V represent the volume of air in a person's lungs. Draw a graph of Volume versus time for 20 seconds.



b) State the period, amplitude, phase shift and vertical translation for the function.

Period $= 4$ Amplitude $= \frac{|Max - Min|}{2}$ P.S: 0 for $-$cosine

$K = \frac{2\pi}{4}$ $= 300$ V. translation $= \frac{Max + Min}{2}$

$K = \frac{\pi}{2}$ $= 2500$

c) Write a possible equation for the volume of air as a function of time.

$V(t) = -300 \cos \frac{\pi}{2} t + 2500$

where t rep. the time in seconds and $V(t)$ is the volume in mL.

d) Describe how the graph would change if the person breaths more rapidly.

If a person breaths more rapidly the period will be shorter ∴ K would be larger.

e) Describe how the graph would change if the person took bigger breaths.

If a person took deeper breaths the amplitude would be greater since they would breath in more air.
(The minimum amount would probaby stay the same ∴ graph would also shift up).

f) Determine the amount of air in the lungs after 8 seconds.

$V(8) = -300 \cos \frac{\pi}{2} \cdot 8 + 2500$

$V(8) = -300 \cos 4\pi + 2500$

$V(8) = 2200$ (matches graph)

After 8 seconds the is 2200mL of air in the lungs.

g) Determine when, within the first 8 seconds, the volume is 2400 mL.

$V(t) = 2400$

$2400 = -300 \cos \frac{\pi}{2}t + 2500$

$2400 - 2500 = -300 \cos \frac{\pi}{2}t$

$-100 = -300 \cos \frac{\pi}{2}t$

$\frac{1}{3} = \cos \frac{\pi}{2}t$

Let $\theta = \frac{\pi}{2}t$

$\frac{1}{3} = \cos\theta$

$1.2310 \doteq \theta_1 \rightarrow t_1 = \theta_1 \div \frac{\pi}{2}$

$t_1 = 0.7837$

$\theta_2 = 2\pi - \theta_1$

$\theta_2 \doteq 5.0522 \rightarrow t_2 \doteq 3.2163$

$t_3 = t_1 + \text{period}$

$t_3 = 4.7837$

$t_4 = t_2 + \text{period}$

$t_4 = 7.2163$

$t_5 = t_3 + \text{period}$

> 8 second $\therefore$ inadmissible

The volume is 2400 mL when $t \doteq 0.78$ sec, 3.22 sec, 4.78 sec, and 7.22 sec.

29) You have just walked out the front door of your home. You notice that it closes quickly at first and then closes more slowly. In fact, a model of the movement of a closing door is given by $d(t) = 200t(2)^{-t}$, where d represents the width of the opening in cm t seconds after opening the door.

a) determine the width of the opening after 2 sec., 4 sec., 6 sec, 10 sec.

a) $d(2) = 200(2)(2)^{-2}$

$d(2) = 100$

$d(4) = 200(4)(2)^{-4}$

$d(4) = 50$

$d(6) = 200(6)(2)^{-6}$

$d(6) = 18.75$

$d(10) = 200(10)(2)^{-10}$

$d(10) \doteq 1.95$

The door opening is 100 cm after 2 sec, 50 cm after 4 sec, 18.75 cm after 6 sec and 1.95 cm after 10 sec.

b) Determine the average rate of change from t = 0 sec. to t = 1.5 sec.. What does this tell you about the movement of the door.

$\text{avg. RoC} = \frac{d(1.5) - d(0)}{1.5 - 0}$

$\text{avg. RoC} \doteq \frac{106.07 - 0}{1.5}$

$\text{avg. RoC} \doteq 70.71$

The door is opening at a rate of 70.71 cm/sec.

c) Determine the average rate of change from t = 3 sec. to t = 6 sec.. What does this tell you about the movement of the door.

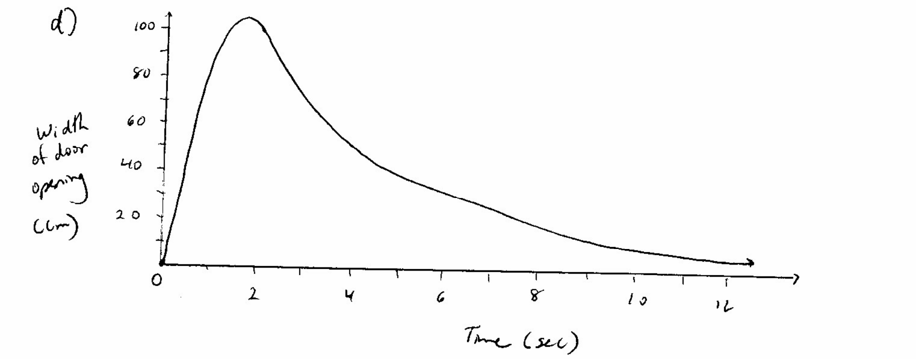
$\text{avg. RoC} = \frac{d(6) - d(3)}{6 - 3}$

$\text{avg. RoC} = \frac{18.75 - 75}{3}$

$\text{avg. RoC} = -18.75$

The door is closing at a rate of 18.75 cm/sec.

d) sketch a graph to model the movement of the door. How does your sketch support the conclusions you reached in b) and c)?



<u>Identities</u>:

30) Prove the following identities.

a) $\dfrac{\cos\theta}{1+\sin\theta}+\dfrac{\cos\theta}{1-\sin\theta}=\dfrac{2}{\cos\theta}$

a) $\dfrac{\cos\theta}{1+\sin\theta}+\dfrac{\cos\theta}{1-\sin\theta}=\dfrac{2}{\cos\theta}$

LS: $\dfrac{\cos\theta}{1+\sin\theta}+\dfrac{\cos\theta}{1-\sin\theta}$

$=\dfrac{\cos\theta(1-\sin\theta)+\cos\theta(1+\sin\theta)}{(1+\sin\theta)(1-\sin\theta)}$

$=\dfrac{\cos\theta-\sin\theta\cos\theta+\cos\theta+\sin\theta\cos\theta}{1-\sin^2\theta}$

$=\dfrac{2\cancel{\cos\theta}}{\cos^{\cancel{2}}\theta}$

$=\dfrac{2}{\cos\theta}$

$=RS$ QED

b) $\dfrac{1}{\sec\theta+\tan\theta}=\sec\theta-\tan\theta$

LS: $\dfrac{1}{\sec\theta+\tan\theta}$

$=1\div(\sec\theta+\tan\theta)$

$=1\div\left(\dfrac{1}{\cos\theta}+\dfrac{\sin\theta}{\cos\theta}\right)$

$=1\div\dfrac{1+\sin\theta}{\cos\theta}$

$=1\cdot\dfrac{\cos\theta}{1+\sin\theta}$

$=\dfrac{\cos\theta}{1+\sin\theta}\cdot\dfrac{1-\sin\theta}{1-\sin\theta}$

$=\dfrac{\cos\theta(1-\sin\theta)}{1-\sin^2\theta}$

$=\dfrac{\cancel{\cos\theta}(1-\sin\theta)}{\cos^{\cancel{2}}\theta}$

$=\dfrac{1-\sin\theta}{\cos\theta}$

$=\dfrac{1}{\cos\theta}-\dfrac{\sin\theta}{\cos\theta}$

$=\sec\theta-\tan\theta$

$=RS$ QED

c) $\dfrac{1}{1+\sin\theta} = \sec^2\theta - \dfrac{\tan\theta}{\cos\theta}$

RS: $\sec^2\theta - \dfrac{\tan\theta}{\cos\theta}$

$= \dfrac{1}{\cos^2\theta} - \tan\theta \cdot \dfrac{1}{\cos\theta}$

$= \dfrac{1}{\cos^2\theta} - \dfrac{\sin\theta}{\cos^2\theta}$

$= \dfrac{1-\sin\theta}{\cos^2\theta}$

$= \dfrac{1-\sin\theta}{1-\sin^2\theta}$

$= \dfrac{\cancel{1-\sin\theta}}{(1+\sin\theta)\cancel{(1-\sin\theta)}}$

$= \dfrac{1}{1+\sin\theta}$

$=$ LS QED

d) $(1-\cos\beta)^2 + \sin^2\beta = 2(1-\cos\beta)$

LS: $(1-\cos\beta)^2 + \sin^2\beta$

$= 1 - 2\cos\beta + \cos^2\beta + \sin^2\beta$

$= 1 - 2\cos\beta + 1$

$= 2 - 2\cos\beta$

$= 2(1-\cos\beta)$

$=$ RS QED

Miscellaneous:

31) $2x^3 + 3x^2 + kx - 5$ is divided by $x+2$ to give a remainder of 2. Determine k.

let $P(x) = 2x^3 + 3x^2 + Kx - 5$

remainder thm: $P(-2) = 2$

$2(-2)^3 + 3(-2)^2 + K(-2) - 5 = 2$

$-16 + 12 - 2K - 5 = 2$

$-2K = 11$

$K = \dfrac{-11}{2}$

32) State the quotient and remainder when $2x^3 + 5x^2 - x - 5$ is divided by $x + 2$.

$$\begin{array}{r l}
 & 2x^2 + x - 3 \\
x+2 \,\big)\!\! & 2x^3 + 5x^2 - x - 5 \\
 & -(2x^3 + 4x^2) \\
 & \quad x^2 - x \\
 & -(x^2 + 2x) \\
 & \quad -3x - 5 \\
 & -(-3x - 6) \\
 & \quad 1
\end{array}$$

quotient: $2x^2 + x - 3$

remainder: 1

33) Use the Factor Theorem to fully factor: $x^3 - 4x^2 - 11x + 30$

Let $P(x) = x^3 - 4x^2 - 11x + 30$

$P(2) = 8 - 16 - 22 + 30$

$P(2) = 0$ $\therefore$ $x - 2$ is a factor

$$\begin{array}{r|rrrr} 2 & 1 & -4 & -11 & 30 \\ & \downarrow & 2 & -4 & -30 \\ \hline & 1 & -2 & -15 & 0 \end{array}$$

$x^3 - 4x^2 - 11x + 30 = (x-2)(x^2 - 2x - 15)$

$x^3 - 4x^2 - 11x + 30 = (x-2)(x-3)(x+5)$

34) Convert the following radians to degrees. Round your answer to one decimal place, if necessary.

a) $\dfrac{5\pi}{6}$ b) $\dfrac{-3\pi}{8}$ c) 2.678

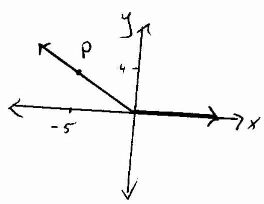
34) a) $\frac{5\pi}{6} = \frac{5}{6}\pi \cdot \frac{180}{\pi}$ b) $-\frac{3\pi}{8} = \frac{-3}{8}\pi \cdot \frac{180}{\pi}$ $2.678 = 2.678 \cdot \frac{180}{\pi}$

$= 150°$ $= -67.5°$ $\doteq 153.4°$

35) The point P(-5, 4) is on the terminal arm of an angle of measure θ in standard position.

a) Sketch the principal angle. b) Determine the exact value of $\sin\theta$.

a)



b) $x^2 + y^2 = r^2$

$25 + 16 = r^2$

$41 = r^2$

$\sin\theta = \frac{4}{\sqrt{41}}$

c) Determine the exact value of $\cos\left(\theta - \dfrac{\pi}{6}\right)$

c) $\cos\left(\theta - \frac{\pi}{6}\right)$

$= \cos\theta \cos\frac{\pi}{6} + \sin\theta \sin\frac{\pi}{6}$

$= \frac{-5}{\sqrt{41}} \cdot \frac{\sqrt{3}}{2} + \frac{4}{\sqrt{41}} \cdot \frac{1}{2}$

$= \frac{-5\sqrt{3} + 4}{2\sqrt{41}}$

d) Determine the value of θ, to the nearest degree, where $0 \le \theta \le 2\pi$.

$\theta = \sin^{-1}\left(\frac{4}{\sqrt{41}}\right)$

$\theta \doteq 0.6747$

36) Determine the smallest positive co-terminal angle to $\frac{13\pi}{5}$. Determine a co-terminal angle that is larger than $\frac{13\pi}{5}$. Determine a negative co-terminal angle.

+ve co-terminal $= \frac{13\pi}{5} - 2\pi$

$= \frac{3\pi}{5}$

larger co-terminal $= \frac{13\pi}{5} + 2\pi$

$= \frac{23\pi}{5}$

-ve coterminal

$= \frac{3\pi}{5} - 2\pi$

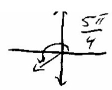
$= -\frac{7\pi}{5}$

37) Determine the exact value for each of the following:

a) $\sin\frac{5\pi}{4}$

b) $\cos\frac{11\pi}{6}$

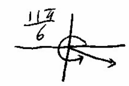
a) $\sin\frac{5\pi}{4}$



$= -\sin\frac{\pi}{4}$

$= -\frac{\sqrt{2}}{2}$

b) $\cos\frac{11\pi}{6}$



$= \cos\frac{\pi}{6}$

$= \frac{\sqrt{3}}{2}$

c) $\tan\frac{\pi}{8}$

$\tan\frac{\pi}{4} = \frac{2\tan\frac{\pi}{8}}{1-\tan^2\frac{\pi}{8}}$

$1 = \frac{2\tan\frac{\pi}{8}}{1-\tan^2\frac{\pi}{8}}$

$1-\tan^2\frac{\pi}{8} = 2\tan\frac{\pi}{8}$

$0 = \tan^2\frac{\pi}{8} + 2\tan\frac{\pi}{8} - 1$

$\tan\frac{\pi}{8} = \frac{-2 \pm \sqrt{4+4}}{2}$

$\tan\frac{\pi}{8} = \frac{-2 \pm \sqrt{8}}{2}$

$\tan\frac{\pi}{8} = \frac{-2 \pm 2\sqrt{2}}{2}$

$\tan\frac{\pi}{8} = \frac{-2 \pm \sqrt{2}}{2}$

d) $\csc\frac{7\pi}{12}$

d) $\csc \frac{7\pi}{12}$ = reciprocal of $\sin \frac{7\pi}{12}$

$$\sin \frac{7\pi}{12} = \sin\left(\frac{3\pi}{12} + \frac{4\pi}{12}\right)$$

$$\sin \frac{7\pi}{12} = \sin\left(\frac{\pi}{4} + \frac{\pi}{3}\right)$$

$$\sin \frac{7\pi}{12} = \sin\frac{\pi}{4} \cdot \cos\frac{\pi}{3} + \cos\frac{\pi}{4} \cdot \sin\frac{\pi}{3}$$

$$\sin \frac{7\pi}{12} = \frac{\sqrt{2}}{2} \cdot \frac{1}{2} + \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2}$$

$$\sin \frac{7\pi}{12} = \frac{\sqrt{2} + \sqrt{6}}{4}$$

$$\csc \frac{7\pi}{12} = \frac{4}{\sqrt{2} + \sqrt{6}}$$

38) Express each of the following as a co-function

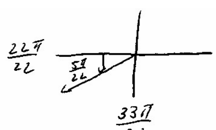
a) $\sin\frac{27\pi}{22}$

b) $\sec\frac{17\pi}{30}$

a) $\sin \frac{27\pi}{22}$

$= \cos \frac{6\pi}{22}$

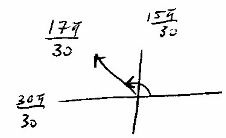
$= \cos \frac{3\pi}{11}$



b) $\sec \frac{17\pi}{30}$

$= -\csc \frac{2\pi}{30}$

$= -\csc \frac{\pi}{15}$



39) Express $\log_{11} 3 + 2\log_{11} 5 - \log_{11} 7$ as a single logarithm.

$$\log_{11} 3 + 2\log_{11} 5 - \log_{11} 7$$

$$= \log_{11} 3 + \log_{11} 25 - \log_{11} 7$$

$$= \log_{11} \frac{75}{7}$$

40) Express $\log_4 7$ as a single logarithm with base 2.

$$\log_4 7 = \frac{\log_2 7}{\log_2 4}$$

$$\log_4 7 = \frac{\log_2 7}{2}$$

$$\log_4 7 = \frac{1}{2} \log_2 7$$

$$\log_4 7 = \log_2 \sqrt{7}$$