
COURSE REVIEW

Transformations:

1) State the transformation(s) that each function has undergone. Then state the transformations in function and in mapping notation.

a) $y = -\frac{1}{4}\sqrt{3(x-7)}$, base function: $f(x) = \sqrt{x}$

V. compression by a factor of 4
 reflection along the x-axis

$$y = -\frac{1}{2}f(3(x-7))$$

H. compression by a factor of 3

$$(x, y) \rightarrow \left(\frac{1}{3}x+7, -\frac{1}{2}y\right)$$

H. translation to the right 7 units

b) $y = 3(4-x)^3 - 6$, base function: $f(x) = x^3$

$$y = 3(-x+4)^3 - 6$$

$$y = 3[-1(x-4)]^3 - 6$$

V. stretch by a factor of 3
 reflection along the y-axis.

$$y = 3f(-(x-4)) - 6$$

H. shift right 4 units

$$(x, y) \rightarrow (-x+4, 3y-6)$$

V. translation 6 units down

c) $y = -3(4)^{x-1}$, compare to $f(x) = (4)^x$

reflection along the x-axis

V. stretch by a factor of 3

H. translation right 1 unit

fnct. notation: $y = -3f(x-1)$

mapping notation: $(x, y) \rightarrow (x+1, -3y)$

d) $y = \log(-x)$, compare to $f(x) = \log(x)$

reflection along the y-axis

fnct. notation: $y = f(-x)$

mapping notation: $(x, y) \rightarrow (-x, y)$

e) $y = -2\cos\left(\frac{1}{3}\left(\theta - \frac{\pi}{2}\right)\right) + 1$, compare to $f(\theta) = \cos(\theta)$

reflection along the x-axis

v. stretch by a factor of 2

H. stretch by a factor of 3

v. shift up 1 unit

fnct. notation: $y = -2 f\left(\frac{1}{3}\left(\theta - \frac{\pi}{2}\right)\right) + 1$

mapping notation: $(x, y) \rightarrow \left(3x + \frac{\pi}{2}, -2y + 1\right)$

2) The graph of $f(x) = x^4$ is horizontally stretched by a factor of 2, reflected in the y-axis, and shifted up 5 units. Find the equation of the transformed function.

H. stretch by a factor of 2

reflected in the y-axis

shifted up 5 units

$$y = \left(-\frac{1}{2}(x)\right)^4 + 5$$

$$y = \left(-\frac{1}{2}x\right)^4 + 5$$

note this would be the same graph as

$$y = \frac{1}{16}x^4 + 5$$

Inverses:

3) Determine the inverse of each of the following. State if the inverse is not a function and state any restrictions.

a) $y = x^2 - 10$

inverse: $x = y^2 - 10$

$$x + 10 = y^2$$

$$\pm \sqrt{x + 10} = y$$

not a fnct.

$D_{\text{inverse}}: \{x \mid x \geq -10, x \in \mathbb{R}\}$

extra: for the inverse to be a fnct $D_{\text{original}}: \{x \mid x \geq 0, x \in \mathbb{R}\}$
or $x \leq 0$

b) $f(x) = 2x + 1$

f: $y = 2x + 1$

inverse: $x = 2y + 1$

$$x - 1 = 2y$$

$$\frac{1}{2}x - \frac{1}{2} = y$$

$$\frac{1}{2}x - \frac{1}{2} = f^{-1}(x)$$

$D_{f^{-1}}: \{x \mid x \in \mathbb{R}\}$ \therefore inverse is a fnct.

$$c) y = \frac{1}{x+3} - 1$$

$$\text{inverse: } x = \frac{1}{y+3} - 1$$

$$x + 1 = \frac{1}{y+3}$$

$$y + 3 = \frac{1}{x+1}$$

$$y = \frac{1}{x+1} - 3$$

$$f^{-1}(x) = \frac{1}{x+1} - 3$$

$$D_{f^{-1}(x)}: \{x \mid x \neq -1, x \in \mathbb{R}\}$$

$$e) y = 10^{x-4} + 7$$

$$\text{inverse: } x = 10^{y-4} + 7$$

$$x - 7 = 10^{y-4}$$

$$\Leftrightarrow y - 4 = \log_{10}(x - 7)$$

$$y = \log_{10}(x - 7) + 4$$

$$f^{-1}(x) = \log_{10}(x - 7) + 4$$

The inverse is a fract!

$$D_{f^{-1}}: \{x \mid x > 7, x \in \mathbb{R}\}$$

$$d) y = \log_2 x$$

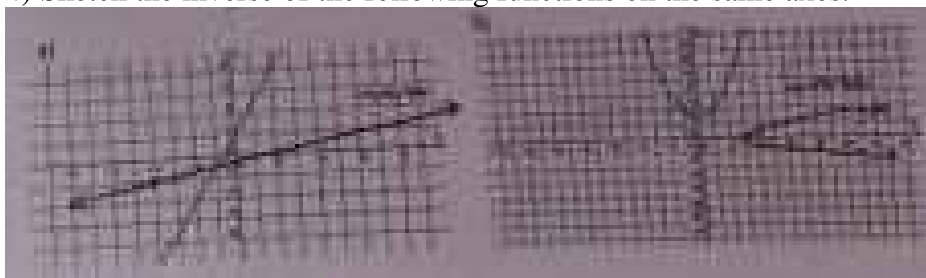
$$\text{inverse: } x = \log_2 y$$

$$\Leftrightarrow 2^x = y$$

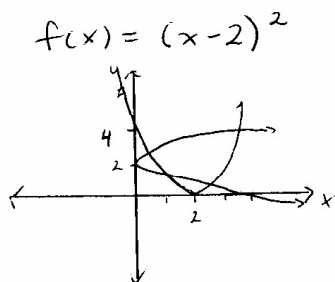
$$2^x = f^{-1}(x)$$

$$D_{f^{-1}}: \{x \mid x \in \mathbb{R}\}$$

4) Sketch the inverse of the following functions on the same axes.



5) Sketch $f(x) = (x-2)^2$ and its inverse. What would the domain have to be so that the inverse is a function?



$D_f: \{x \mid x \geq 0, x \in \mathbb{R}\}$
so that inverse is a funct.

$D_f: \{x \mid x \leq 0, x \in \mathbb{R}\}$
so that inverse is a funct.

Functions, properties and their graphs:

6) Given the graph of $y = f(x)$ shown on the right,

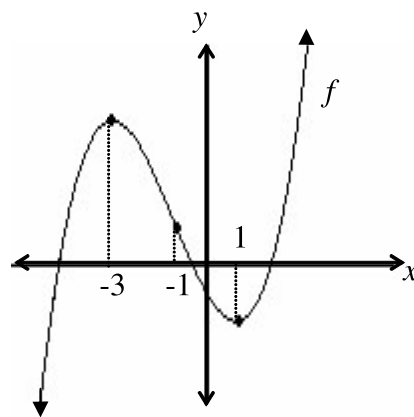
state the intervals of x for which

(a) the function is decreasing

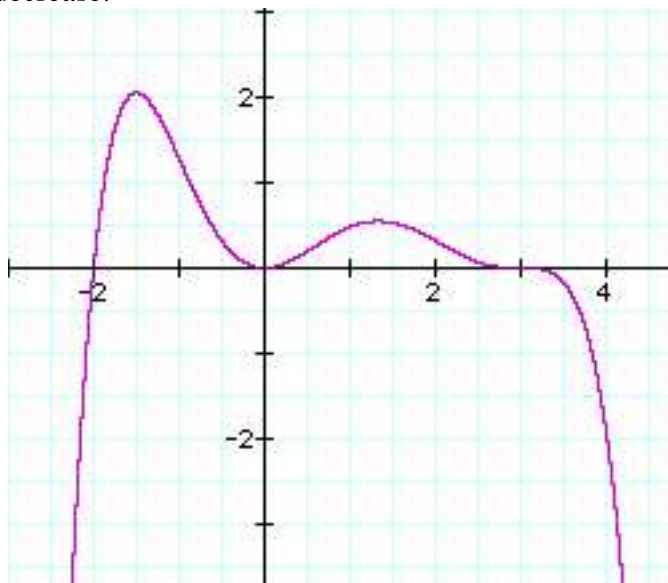
(b) the function is concave up

a) funct is decr.: $-3 < x < 1$

b) funct is conc. down: $x < -1$



7) For the following graph, state the number of turning points, the number of inflection points, and the intervals of increase and decrease:



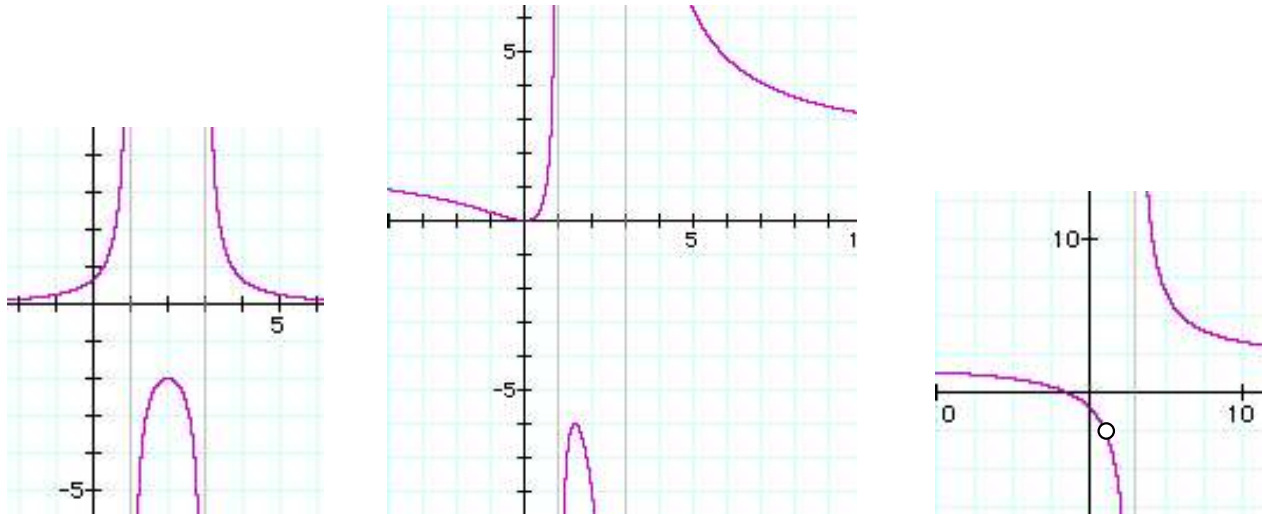
3 turning pts (2 max and one min.)

4 points of inflections

incr. $x < -1.5$ or $0 < x < 1.3$

decr. $-1.5 < x < 0$ or $1.3 < x < 3$ or $x > 3$

8) Which graph best matches the equation: $y = \frac{2x^2 + x - 3}{x^2 - 4x + 3}$?



8) $y = \frac{2x^2 + x - 3}{x^2 - 4x + 3}$

$P = -6 \quad 3, -2$
 $S = 1$

$P = 3 \quad -3, -1$
 $S = -4$

factor to determine zeros, VA, holes, HA/OA

$$y = \frac{(2x+3)(x-1)}{(x-3)(x-1)}$$

$$y = \frac{2x+3}{x-3}, \quad x \neq 1$$

\therefore hole at $x=1$

let $x=1$

$$y = \frac{2(1)+3}{1-3}$$

$$y = -\frac{5}{2}$$

\therefore hole $(1, -\frac{5}{2})$

VA: let $x-3=0$
 $x=3$

zero: let $2x+3=0$
 $2x=-3$
 $x = -\frac{3}{2}$

HA: equal degrees
 \therefore divide coeff.

$$y = 2$$

These properties match the 3rd graph.

9) Sketch each of the following functions labelling: intercepts and asymptotes and stating the domain.

a) $y = -(x+2)^3$

→ special cubic, no asymptotes.

→ -ve lead coeff

∴ as $x \rightarrow \infty, y \rightarrow -\infty$

→ shifted left 2 units

→ zero -2, pt. of inflection

x-int (let $y=0$) y-int. (let $x=0$)

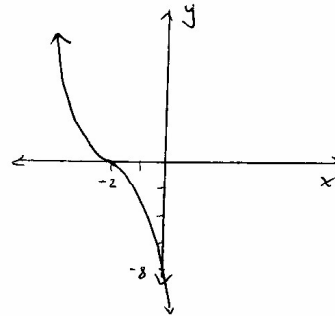
$0 = -(x+2)^3$ $y = -(2)^3$

$0 = (x+2)^3$ $y = -8$

$\sqrt[3]{0} = x+2$

$0 = x+2$

$-2 = x$



$D: \{x \mid x \in \mathbb{R}\}$

b) $y = (x-3)^2(x^2 - 4x - 7)$

→ quartic, no asymptotes

→ +ve lead coeff

∴ as $x \rightarrow \infty, y \rightarrow \infty$

→ DR at $x=3$, turning pt.

x-int (let $y=0$)

$(x-3)^2 = 0$ $x^2 - 4x - 7 = 0$

$x = 3$ $x = \frac{4 \pm \sqrt{16+28}}{2}$

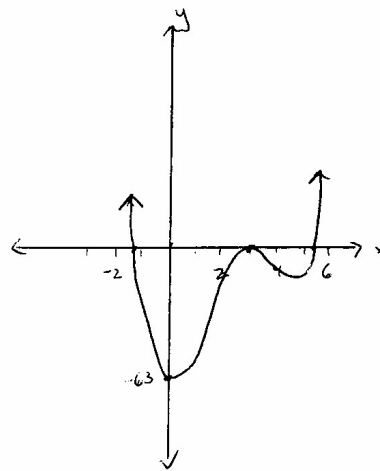
$x = 5.32$

$x = -1.32$

y-int (let $x=0$)

$y = (-3)^2(-7)$

$y = -63$



$D: \{x \mid x \in \mathbb{R}\}$

c) $y = x^4 + 2x^3 + x^2 + 2x$

→ quartic, no asymptotes

→ +ve lead coeff.

∴ as $x \rightarrow \infty, y \rightarrow \infty$

x-int. (let $y=0$)

$0 = x(x^3 + 2x^2 + x + 2)$

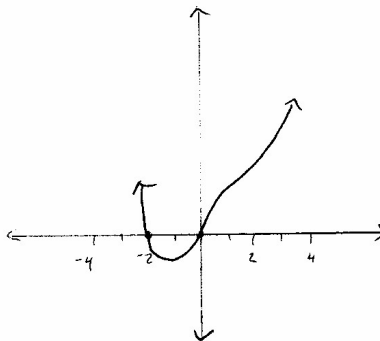
$0 = x[x^2(x+2) + 1(x+2)]$

$0 = x(x+2)(x^2+1)$

$x = 0, x = -2$ no real solⁿ

y-int (let $x=0$)

$y = 0$



$D: \{x \mid x \in \mathbb{R}\}$

d) $f(x) = \frac{x^2+3}{x+4}$

d) $f(x) = \frac{x^2+3}{x+4}$

→ rational frct
 → deg numerator > deg denom.
 by \div \therefore OA
 → +ve lead. coeff.
 \therefore ends in QI

x-int (let $y=0$)

$$0 = \frac{x^2+3}{x+4}$$

\Rightarrow numerator = 0
 $x^2+3=0$
 no real solⁿ
 \therefore no x-int

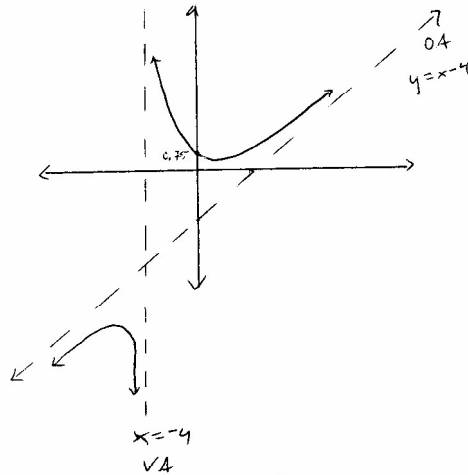
y-int (let $x=0$)

$$y = \frac{3}{4}$$

VA: $x = -4$

OA:
$$\begin{array}{r|rrr} -4 & 1 & 0 & 3 \\ & & -4 & 16 \\ \hline & 1 & -4 & 19 \neq 0 \end{array}$$

OA: $y = x - 4$



D: $\{x \mid x \neq -4, x \in \mathbb{R}\}$

e) $f(x) = \log(x+2)$

9e) $f(x) = \log(x+2)$

→ logarithmic frct, VA
 → shifted left 2 units

x-int (let $y=0$)

$$0 = \log(x+2)$$

$$\Leftrightarrow 10^0 = x+2$$

$$1 = x+2$$

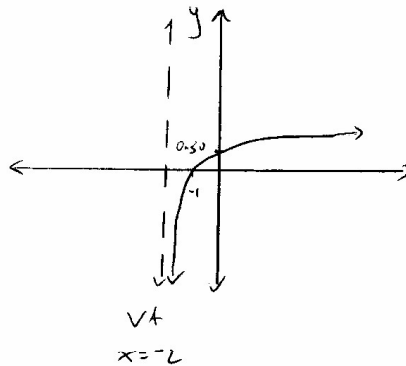
$$-1 = x$$

y-int (let $x=0$)

$$y = \log 2$$

$$y \approx 0.3010$$

VA let $x+2=0$
 $x = -2$



D: $\{x \mid x > -2, x \in \mathbb{R}\}$

f) $f(x) = 3 \cdot 5^{-x} + 4$

- exponential fct, HA
- v. stretch by a factor of 3
- reflection along y-axis
- v. shift up 4 units

x-int (let $y=0$)

$$0 = 3 \cdot 5^{-x} + 4$$

$$-4 = 3 \cdot 5^{-x}$$

$$-\frac{4}{3} = 5^{-x} \quad (\text{no sol}^n)$$

$$\Leftrightarrow \log_5 \frac{-4}{3} = -x \quad \leftarrow \text{check}$$

$$\frac{\log \frac{-4}{3}}{\log 5} = -x$$

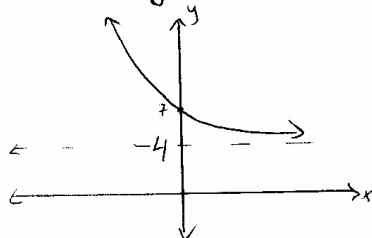
no solⁿ

y-int (let $x=0$)

$$f(0) = 3 \cdot 5^0 + 4$$

$$f(0) = 7$$

HA: $y = 4$



$$D: \{x \mid x \in \mathbb{R}\}$$

g) $f(x) = \frac{10 - 10x}{(x - 4)^2}$

- rational fct
- deg numerator < deg of denom
∴ HA
- -ve lead coeff
∴ ends in QIV

x-int

$$\text{let } -10x + 10 = 0$$

$$10 = 10x$$

$$1 = x$$

y-int, let $x=0$

$$f(0) = \frac{10}{(-4)^2}$$

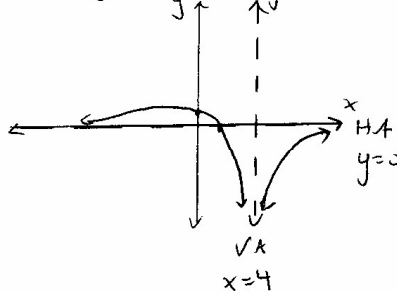
$$f(0) = \frac{5}{8}$$

VA let $(x - 4)^2 = 0$

$$x = 4 \quad \text{DR}$$

∴ arrows in same direction

HA $y = 0$ ∴ degrees



$$D: \{x \mid x \neq 4, x \in \mathbb{R}\}$$

$$h) f(x) = \frac{x^2+4}{x^2-4}$$

$$h) f(x) = \frac{x^2+4}{x^2-4}$$

→ rational fract

→ deg numerator = deg denominator
∴ HA

→ we lead coeff.
∴ ends Q I

x-int let $x^2+4=0$
no solⁿ

y-int let $x=0$

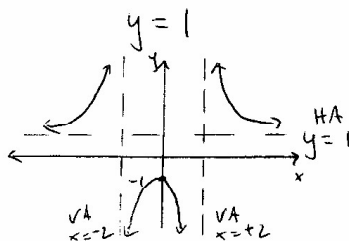
$$f(0) = \frac{4}{-4}$$

$$f(0) = -1$$

$$VA \text{ let } x^2-4=0$$

$$x = \pm 2$$

HA ∴ coeff ∴ deg equal



$$i) y = 2\sin\left(\theta - \frac{\pi}{2}\right) + 1$$

→ sine fract

→ Amp: 2

$$P.S: \frac{\pi}{2}$$

V. disp = 1

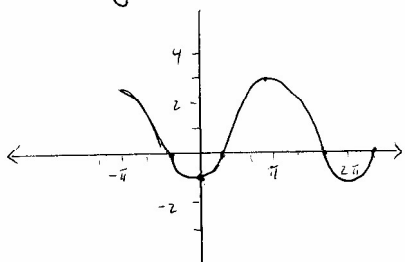
Period = 2π

y-int (let $\theta=0$)

$$y = 2\sin\left(-\frac{\pi}{2}\right) + 1$$

$$y = -2 + 1$$

$$y = -1$$



$$D: \{x \mid x \in \mathbb{R}\}$$

x-int (let $y=0$)

$$0 = 2\sin\left(\theta - \frac{\pi}{2}\right) + 1$$

$$-1 = 2\sin\left(\theta - \frac{\pi}{2}\right)$$

$$-\frac{1}{2} = \sin\left(\theta - \frac{\pi}{2}\right)$$

$$\text{let } B = \theta - \frac{\pi}{2}$$

$$\frac{1}{2} = \sin B_r$$

$$\frac{\pi}{6} = B_r$$

$$B_1 = \pi + \frac{\pi}{6}$$

$$B_2 = 2\pi - \frac{\pi}{6}$$

$$\theta_1 - \frac{\pi}{2} = \frac{7\pi}{6}$$

$$\theta_2 - \frac{\pi}{2} = \frac{11\pi}{6}$$

$$\theta_1 = \frac{7\pi}{6} + \frac{\pi}{2}$$

$$\theta_2 = \frac{11\pi}{6} + \frac{\pi}{2}$$

$$\theta_1 = \frac{10\pi}{6}$$

$$\theta_2 = \frac{14\pi}{6}$$

$$\theta_1 = \frac{5\pi}{3}$$

$$\theta_2 = \frac{7\pi}{3}$$

$$\theta_3 = \frac{5\pi}{3} \pm 2n\pi$$

$$\theta_2 = \frac{7\pi}{3} \pm 2n\pi$$

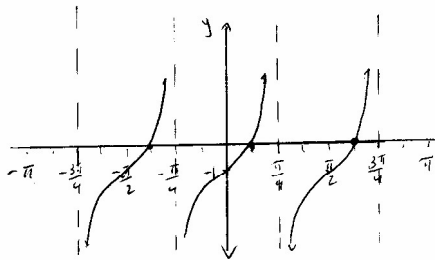
j) $y = \tan 2\theta - 1$

j) $y = \tan 2\theta - 1$
 → tangent funct, VA
 → Period = $\frac{\pi}{2}$
 v. shift down 1

y-int (let $\theta=0$)
 $y = \tan 0 - 1$
 $y = -1$

x-int (let $y=0$)
 $0 = \tan 2\theta - 1$
 $1 = \tan 2\theta$
 let $\beta = 2\theta$

$1 = \tan \beta$
 $\frac{\pi}{4} = \beta_1, \quad \beta_2 = \frac{5\pi}{4}$
 $\frac{\pi}{8} = \theta_1, \quad \theta_2 = \frac{5\pi}{8}$
 $\frac{\pi}{8} \pm \frac{\pi}{2} = \theta_3, \quad \theta_4 = \frac{5\pi}{8} \pm \frac{\pi}{2}$



VA for $y = \tan \theta$
 VA $x = \frac{\pi}{2}$
 # compression
 $x = \frac{\pi}{4}$

$D: \{x \mid x \neq \frac{\pi}{4} + n\pi, n \in \mathbb{Z}, x \in \mathbb{R}\}$

k) $y = 0.5 \sec\left(\theta + \frac{\pi}{4}\right)$

→ secant funct.
 → reciprocal of cosine
 → Amp not applicable
 H. shift left $\frac{\pi}{4}$

y-int (let $\theta=0$)
 $y = 0.5 \sec\left(\frac{\pi}{4}\right)$

$y = 0.5 \cdot \frac{1}{\cos \frac{\pi}{4}}$

$y = 0.5 \cdot \frac{\sqrt{2}}{1}$

$y = \frac{\sqrt{2}}{2}$

$y \approx 0.7071$

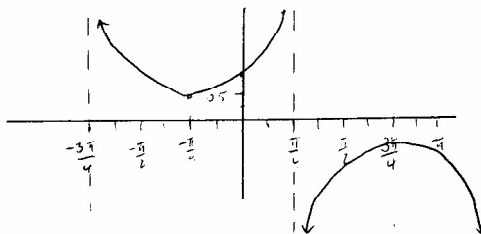
x-int (let $y=0$)
 $0 = 0.5 \sec\left(\theta + \frac{\pi}{4}\right)$
 $0 = \sec\left(\theta + \frac{\pi}{4}\right)$
 $0 = \frac{1}{\cos\left(\theta + \frac{\pi}{4}\right)}$

$\cos\left(\theta + \frac{\pi}{4}\right) = \text{undef.}$
 not possible.

VA for $y = \sec \theta$
 when $\theta = \frac{\pi}{2} \pm n\pi$
 (zeros of $y = \cos \theta$)
 H. shift left $\frac{\pi}{4}$

∴ VA $\theta = \frac{\pi}{2} - \frac{\pi}{4} \pm n\pi$

$\theta = \frac{\pi}{4} \pm n\pi$



$D: \{x \mid x \neq \frac{\pi}{4} \pm n\pi, n \in \mathbb{Z}, x \in \mathbb{R}\}$

10) Analyze and sketch the following, using intercepts, asymptotes, and end behaviours:

$$y = \frac{3x^3 + 10x^2 + 3x}{x^2 + 5x + 6}$$

zeros: let $3x^3 + 10x^2 + 3x = 0$
 $x(3x^2 + 10x + 3) = 0$
 $x(3x+1)(x+3) = 0$
 $x=0, x=-\frac{1}{3}, x=-3$

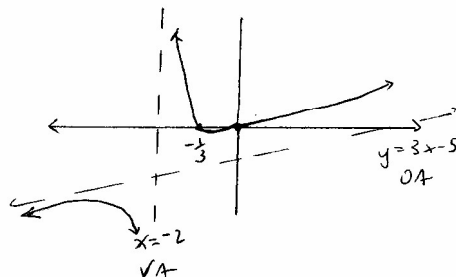
y-int let $x=0$
 $y = \frac{0}{6}$
 $y=0$

O.A.:
$$\begin{array}{r} 3x - 5 \\ x^2 + 5x + 6 \overline{) 3x^3 + 10x^2 + 3x + 0} \\ \underline{-(3x^3 + 15x^2)} \\ -5x^2 + 3x + 0 \\ \underline{-(-5x^2 - 25x - 30)} \\ 28x + 30 \end{array}$$

O.A. $y = 3x - 5$

VA let $x^2 + 5x + 6 = 0$
 $(x+3)(x+2) = 0$
 $x = -3, x = -2$
 \uparrow VA
 same as zero
 \therefore hole

hole: $y = \frac{x(3x+1)}{x+2}$
 $y = \frac{-3(-8)}{-1}$
 $y = -24$
 $(-3, -24)$



11) A polynomial of degree 5 has a negative leading coefficient.

- How many turning points could the polynomial have?
- How many zeros could the function have?
- Describe the end behaviour.
- Sketch two possible graphs, each passing through the point $(1, -2)$.

poly of deg 5

$\Rightarrow n=5$

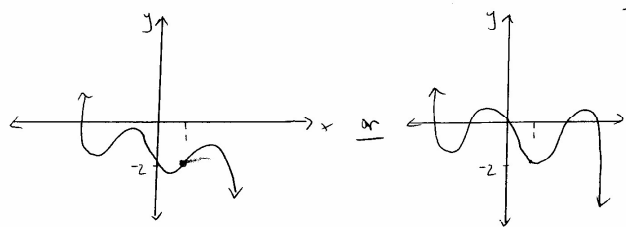
a) max # of turning pts = $n-1$
 $= 4$

\therefore 0, 2 or 4 turning pts

b) max # of zeros = n odd deg.
 $= 5$

zeros 1, 2, 3, 4 or 5

c) -ve lead. coeff \therefore as $x \rightarrow \infty, y \rightarrow -\infty$
 odd deg as $x \rightarrow -\infty, y \rightarrow \infty$

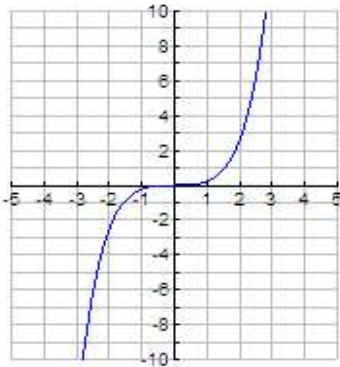


many answers.

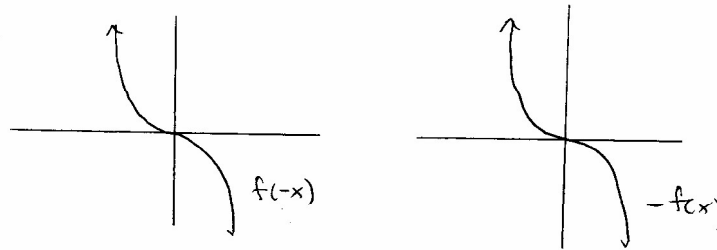
Symmetry:

12) Determine whether each of the following functions is even, odd, or neither. Justify your answer.

a)

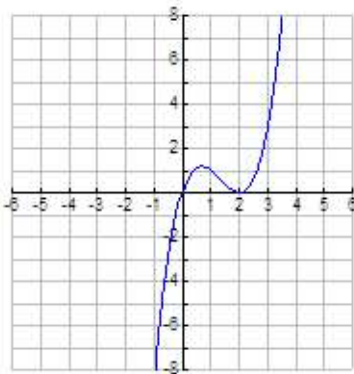


a)

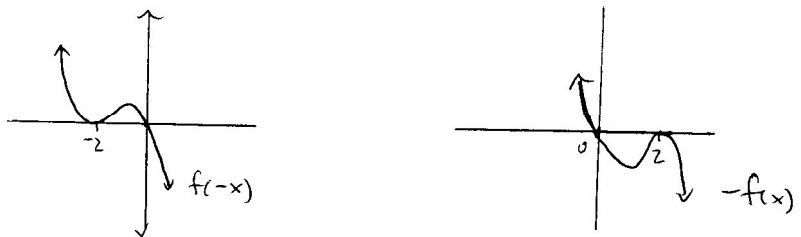


$$f(-x) = -f(x) \therefore \text{odd funct.}$$

b)

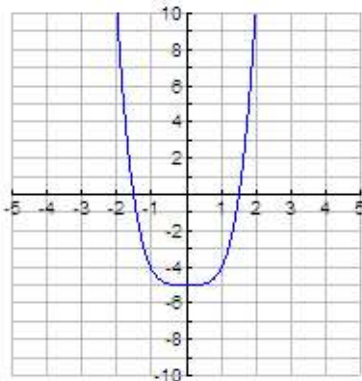


b)

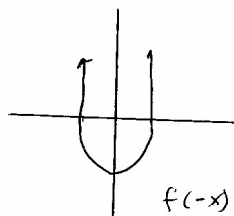


$$f(x) \neq f(-x), \quad f(-x) \neq -f(x) \therefore \text{neither odd nor even}$$

c)



c)



$$f(-x) = f(x) \therefore \text{funct is even}$$

d) $f(x) = 3x^2 + 4$

$$\begin{aligned} d) \quad f(x) &= 3x^2 + 4 \\ f(-x) &= 3(-x)^2 + 4 \\ f(-x) &= 3x^2 + 4 \\ f(-x) &= f(x) \\ \therefore \text{funct is even} \end{aligned}$$

e) $f(x) = -3x^3 + x$

$$\begin{aligned} e) \quad f(x) &= -3x^3 + x \\ f(-x) &= -3(-x)^3 + (-x) \\ f(-x) &= 3x^3 - x \\ -f(x) &= -(-3x^3 + x) \\ -f(x) &= 3x^3 - x \\ -f(x) &= f(-x) \\ \therefore \text{odd funct.} \end{aligned}$$

$$f) f(x) = \tan x$$

$$f) f(x) = \tan x, \text{ assume } x \in \text{QI}$$

$$f(-x) = \tan(-x) \quad -x \in \text{QIV}$$

$$f(-x) = -\tan x \quad \begin{array}{l} \text{tan ratio is} \\ \text{-ve in QIV} \end{array}$$

$$-f(x) = -\tan x$$

$$-f(x) = f(-x) \therefore \text{odd fnct}$$

$$g) y = 3^x + 1$$

$$g) y = 3^x + 1$$

$$\text{let } y = f(x)$$

$$f(x) = 3^x + 1$$

$$f(-x) = 3^{-x} + 1$$

$$-f(x) = -(3^x + 1)$$

$$-f(x) = -3^x - 1$$

$$f(-x) \neq f(x)$$

$$f(-x) \neq -f(x)$$

\therefore fnct is neither odd nor even.

$$h) f(x) = 3 \log x - 1$$

$$h) f(x) = 3 \log x - 1$$

$$f(-x) = 3 \log(-x) - 1$$

$$-f(x) = -(3 \log x - 1)$$

$$-f(x) = -3 \log x + 1$$

$$f(-x) \neq f(x)$$

$$f(-x) \neq -f(x)$$

\therefore fnct neither odd nor even

$$i) y = \frac{1}{x^2 - 4}$$

$$i) y = \frac{1}{x^2 - 4}$$

$$\text{let } y = f(x)$$

$$f(x) = \frac{1}{(x)^2 - 4}$$

$$f(-x) = \frac{1}{(-x)^2 - 4}$$

$$f(-x) = f(x)$$

\therefore fnct is even

$$j) f(x) = 2^x + 2^{-x}$$

$$j) f(x) = 2^x + 2^{-x}$$

$$f(-x) = 2^{-x} + 2^{-(-x)}$$

$$f(-x) = 2^{-x} + 2^x$$

\therefore addition is commutative

$$f(-x) = 2^x + 2^{-x}$$

$$f(-x) = f(x)$$

\therefore fnct is even

$$k) f(x) = \frac{\sin x}{x^2 - 4}$$

(use combinations of functions to justify)

$$k) f(x) = \frac{\sin x}{x^2 - 4}$$

$y = \sin x$ is an odd fnct

$y = x^2 - 4$ is an even fnct.

The quotient of an odd and an even fnct is odd

$$l) f(x) = x \cdot \log x$$

(use combinations of functions to justify)

$$l) f(x) = x \cdot \log x$$

$y = x$ is an odd fnct

$y = \log x$ is neither odd nor even

The product of an odd and a neither is a neither.

Rates of Change:

13) The position in kilometres of a particle at t hours is given by $d(t) = t^3 - 12t^2 + 34t + 75$, where $t \geq 0$.

- What is the initial position of the particle?
- What is the particle's average velocity from 3 hours to 5 hours?
- What is the particle's instantaneous velocity at 7 hours?

a) initial position $\Rightarrow t=0$

$$d(0) = 75$$

The initial position is 75 km.

b) avg. vel'y = avg. RoC of $d(t)$

$$\text{avg. vel'y} = \frac{d(5) - d(3)}{5 - 3}$$

$$\text{avg. vel'y} = \frac{70 - 96}{2}$$

$$\text{avg. vel'y} = -13$$

The ptcl's avg. vel'y from 3 hrs. to 5 hrs. is -13 km/hr.

or The position is decr. at an avg. RoC of 13 km/hr.

c)

nbhd Pt	Pt.	Δt	$\Delta d(t)$	$\frac{\Delta d(t)}{\Delta t}$
(6.9, 66.789)	(7, 68)	0.1	1.211	12.11
(6.99, 67.8709)	(7, 68)	0.01	0.1291	12.91
(7.01, 68.1309)	(7, 68)	-0.01	-0.1309	13.09
(7.1, 69.391)	(7, 68)	-0.1	-1.391	13.91

The inst. rate of change, at $t=7$ hrs, is 13 km/hr.

14) Find the slope of the secant of $y = 2^x - 3$ that passes through the points where $x = -3$ and $x = 1$.

$$y = 2^x - 3$$

$$m_{\text{sec}} = \frac{\Delta y}{\Delta x}$$

$$= \frac{y|_{x=1} - y|_{x=-3}}{1 - (-3)}$$

$$= \frac{-1 + 2.875}{4}$$

$$\hat{=} 0.47$$

15) The concentration of medicine in a patient's bloodstream is given by $C(t) = \frac{0.4t}{(0.3t + 2)^3}$, $t \geq 0$, where

C is measured in milligrams per cubic centimetre and t is the time in hours after the medicine was taken. Determine:

- the concentration in the bloodstream 3 hours after the medicine was taken.
- the average rate at which the concentration is decreasing from 4 hours after taking the medicine to 7 hours after taking the medicine.
- the instantaneous rate of change for the concentration 2 hours after the medicine was taken. Interpret the meaning of your answer.

$$a) \quad C(3) = \frac{0.4(3)}{[(0.3)(3) + 2]^3}$$

$$C(3) \doteq 0.0492$$

After 3 hrs. The conc. is 0.0492 mg/cm³

$$b) \quad \text{avg. RoC} = \frac{\Delta C(t)}{\Delta t}$$

$$\text{avg. RoC} = \frac{C(7) - C(4)}{7 - 4}$$

$$\text{avg. RoC} \doteq \frac{0.0406 - 0.0488}{3}$$

$$\text{avg. RoC} \doteq -0.0027$$

The conc. is decr. at an avg. RoC of 0.0027 mg/cm³/hr.

c)

nbhd pt	pt.	Δt	$\Delta C(t)$	$\frac{\Delta C(t)}{\Delta t}$
(1.9, 0.0448)	(2, 0.0455)	0.1	0.0007	0.007
(1.99, 0.04545)	(2, 0.04552)	0.01	0.00007	0.007
(2.01, 0.04559)	(2, 0.04552)	-0.01	-0.00007	0.007
(2.1, 0.0462)	(2, 0.0455)	-0.1	-0.0007	0.007

The inst. RoC is approx. 0.007 mg/cm³/hr. The conc. is increasing slightly.

16) Complete the table

