SPH4U 6.1 Newtonian Gravitation

1. Universal gravitation

Universal law of gravitation:	any two objects experience a gravitational attraction related to their mass and distance.
equation	$F_g = \frac{Gm_1m_2}{r^2}$, $G = 6.67\times10^{-11}\ N\cdot m^2/kg^2$.
inverse-square law	when one value decreases with the square of another. $F_g \propto \frac{1}{r^2}$.

The centres of two uniformly dense spheres are separated by 50.0 cm. Each sphere has a mass of 2.00 kg.

a. Calculate the magnitude of the gravitational force between the two spheres.

$$F_g = \frac{Gm_1m_2}{r^2} = \frac{(6.67\times10^{-11})(2)(2)}{(0.5)^2} = \underline{1.07\times10^{-9}\ N}.$$

b. How much of an effect will this force have on the spheres?

Not much.

Eris, a dwarf planet, is the ninth most massive body orbiting the Sun. It is more massive than Pluto and three times farther away from the Sun. Eris has a radius of approximately 1200 km.

a. An astronaut stands on Eris and drops a rock from a height of 0.30 m. The rock takes 0.87 s to reach the surface. Calculate the value of *g* on Eris.

$\Delta d = 0.3m,\ \Delta t = 0.87s,\ v_i = 0,\ g = ?$

$\Delta d = \cancel{v_i \Delta t}^{0} + \frac{1}{2}g\Delta t^2$ $\qquad g = \frac{2\Delta d}{\Delta t^2} = \frac{2(0.3)}{0.87^2} = 0.7927\ m/s^2$.

$= \underline{0.79\ m/s^2}$.

b. Calculate the mass of Eris.

$F_g = \cancel{m_1}g = \frac{G\cancel{m_1}m_2}{r^2}$

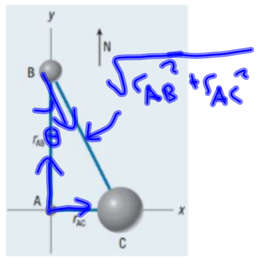
$g = \frac{Gm_2}{r^2} \rightarrow m_2 = \frac{gr^2}{G} = \frac{(0.7927)(1200000)^2}{6.67\times10^{-11}} = \underline{1.7\times10^{22}\ kg}.$

c. An astronaut stands on Eris and drops a rock from a height of 2.50 m. Calculate how long it would take the rock to reach the surface.

$\Delta d = 2.5m,\ v_i = 0,\ g = 0.7927,\ \Delta t = ?$

$\Delta d = \cancel{v_i \Delta t}^{0} + \frac{1}{2}g\Delta t^2$

$\Delta t^2 = \frac{2\Delta d}{g}$ $\qquad \Delta t = \sqrt{\frac{2\Delta d}{g}} = \sqrt{\frac{2(2.5)}{0.7927}} = \underline{2.5\ s}.$



Three large, spherical asteroids are arranged in space at the corners of a right triangle ABC. Asteroid A has a mass of 1.0 x 10^{20} kg. Asteroid B has a mass of 2.0 x 10^{20} kg and is 5.0 x 10^{10} m from asteroid A. Asteroid C has a mass of 4.0 x 10^{20} kg and is 2.5 x 10^{10} m away from asteroid A along the other side of the triangle.

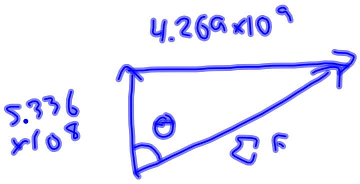
a. Determine the net force on asteroid A from asteroids B and C.

$$\vec{F}_{BA} = \frac{G m_B m_A}{r_{BA}^2} = \frac{(6.67\times10^{-11})(2\times10^{20})(1\times10^{20})}{(5\times10^{10})^2}$$

$$= 5.336\times10^{8}\ \text{N [N]}$$

$$\vec{F}_{CA} = \frac{G m_C m_A}{r_{CA}^2} = \frac{(6.67\times10^{-11})(4\times10^{20})(1\times10^{20})}{(2.5\times10^{10})^2}$$

$$= 4.269\times10^{9}\ \text{N [E]}.$$



$$\Sigma\vec{F} = \sqrt{(5.336\times10^{8})^2 + (4.269\times10^{9})^2}$$

$$= 4.3\times10^{9}\ \text{N}$$

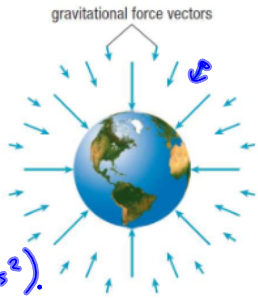
$$\theta = \tan^{-1}\left(\frac{4.269\times10^{9}}{5.336\times10^{8}}\right) = 83^\circ$$

$$\boxed{\therefore\ \Sigma\vec{F} = 4.3\times10^{9}\ \text{N [N83E]}}$$

b. Determine the net force on asteroid B from asteroid C.

2. Gravitational fields

Gravitational field:	vectors at all points in space that indicate the magnitude and direction of F_g at that point.
gravitational field strength	the force of gravity per kg at a point in space. $g = \frac{F_g}{m}$ Units: N/kg (m/s^2).



Assume that Saturn is perfectly spherical with a radius of 6.03 x 10^7 m, and a mass of ~~.69 x~~ 5.69 x 10^{26} kg.

a. Calculate the magnitude of the gravitational field strength on the surface of Saturn.

$F_g = \frac{G \cancel{m_1} m_2}{r^2} = \cancel{m}g \rightarrow g = \frac{G m_2}{r^2}$

$= \frac{(6.67 \times 10^{-11})(5.69 \times 10^{26})}{(6.03 \times 10^7)^2}$

$= 10.438$ N/kg.

$\therefore$ $\underline{10.4 \text{ N/kg}}$.

b. Determine the ratio of Saturn's gravitational field strength to Earth's gravitational field strength (9.8 N/kg).

$\frac{g_{Saturn}}{g_{Earth}} = \frac{10.438}{9.8} = \underline{1.1}$

Ratio: 1.1 : 1 (Saturn : Earth).

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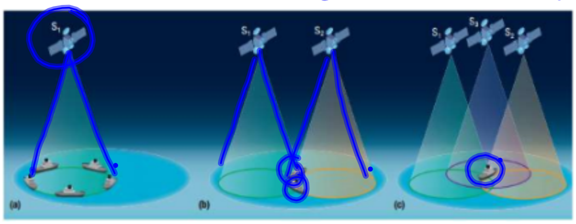
SPH4U 6.2 Orbits

1. Satellites and space stations

Satellite:	an object that revolves around another object because of gravity.
artificial satellite	an object intentionally put in orbit by humans.
GPS and triangulation	24 satellites (up to 30). 3 satellites are needed to "triangulate" an object's location.



Space station and microgravity:	microgravity = apparent weightlessness due to constant freefall (g on ISS = 8.7 N/kg).

2. Satellites in circular orbits

Orbital radius:	distance between a satellite and its parent body.
orbit shape	elliptical, but we assume they're circular.
geosynchronous orbit	orbit around Earth with a period of 1 day.
geostationary orbit	geosynchronous orbit over the equator, so that it always stays at the same point in the sky.

Gravitational field strength at a distance *r* above Earth: $g = \frac{Gm_E}{r^2}$

Centripetal acceleration: $a_c = \frac{v^2}{r}$.

$g = a_c$. $\frac{Gm_E}{r^2} = \frac{v^2}{r}$.

$v^2 = \frac{Gm_E}{r} \rightarrow \boxed{v = \sqrt{\frac{Gm_E}{r}}}$

Only variable.

The International Space Station (ISS) obits Earth at an altitude of about 350 km above Earth's surface.

a. Determine the speed needed by the ISS to maintain its orbit.

$$v = \sqrt{\frac{Gm}{r}} = \sqrt{\frac{(6.67\times10^{-11})(5.98\times10^{24})}{(350\times10^3 + 6.38\times10^6)}}$$

$$= 7.698\times10^3 \text{ m/s.}$$

$$\underline{= 7.7\times10^3 \text{ m/s}}$$

b. Determine the orbital period of the ISS in minutes.

$$T = \frac{2\pi r}{v} = \frac{2\pi(6.38\times10^6 + 350\times10^3)}{7.7\times10^3}$$

$$= 5491.7\text{ s} \times \frac{1 \text{ min}}{60 \text{ s}}$$

$$= 91.53 \text{ min}$$

$$\underline{= 92 \text{ min}}$$

Determine the speeds of Venus and Earth as they orbit the Sun. The Sun's mass is 1.99 x 10^{30} kg. Venus has an orbital radius of 1.08 x 10^{11} m, an dEarth has an orbital radius of 1.49 x 10^{11} m.

$$v = \sqrt{\frac{Gm}{r}}$$

<u>Venus:</u>

$$v = \sqrt{\frac{Gm}{r}}$$

$$= \sqrt{\frac{(6.67\times10^{-11})(1.99\times10^{30})}{1.08\times10^{11}}}$$

$$\underline{= 3.51\times10^4 \text{ m/s.}}$$

<u>Earth:</u>

$$v = \sqrt{\frac{Gm}{r}}$$

$$= \sqrt{\frac{(6.67\times10^{-11})(1.99\times10^{30})}{1.49\times10^{11}}}$$

$$\underline{= 2.98\times10^4 \text{ m/s.}}$$

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