Section 5.1: Momentum and Impulse

Tutorial 1 Practice, page 223

1. Given: m = 160 g = 0.16 kg; $\vec{v}$ = 140 m/s [E]

Required: $\vec{p}$; E_k

Analysis: $\vec{p} = m\vec{v}$; $E_k = \frac{1}{2}mv^2$

Solution:
$$\begin{aligned}\vec{p} &= m\vec{v}\\ &= (0.16\text{ kg})(40.0\text{ m/s [E]})\\ \vec{p} &= 6.4\text{ kg}\cdot\text{m/s [E]}\end{aligned}$$

$$\begin{aligned}E_k &= \frac{1}{2}mv^2\\ &= \frac{1}{2}(0.16\text{ kg})(40.0\text{ m/s})^2\\ E_k &= 130\text{ J}\end{aligned}$$

Statement: The momentum of the puck is 6.4 kg·m/s [E], and the kinetic energy is 130 J.

2. Given: m_1 = 6.2 kg; v_1 = 1.6 m/s [E]; m_2 = 160 g = 0.16 kg; v_2 = 40.0 m/s [E]

Required: $\vec{p}_2 - \vec{p}_1$

Analysis: $\vec{p} = m\vec{v}$

Solution:
$$\begin{aligned}\vec{p}_1 - \vec{p}_2 &= m_1\vec{v}_1 - m_2\vec{v}_2\\ &= (6.2\text{ kg})(1.6\text{ m/s [E]}) - (0.16\text{ kg})(40.0\text{ m/s [E]})\\ \vec{p}_1 - \vec{p}_2 &= 3.5\text{ kg}\cdot\text{m/s [E]}\end{aligned}$$

Statement: The difference in the momenta is 3.5 kg·m/s [E].

Tutorial 2 Practice, page 226

1. (a) Given: $\vec{F}$ = 250 N [forward]; t = 0.0030 s

Required: $\Delta\vec{p}$

Analysis: $\Delta\vec{p} = \vec{F}\Delta t$

Solution:
$$\begin{aligned}\Delta\vec{p} &= \vec{F}\Delta t\\ &= (250\text{ N [forward]})(0.0030\text{ s})\\ \Delta\vec{p} &= 0.75\text{ N}\cdot\text{s [forward]}\end{aligned}$$

Statement: The impulse imparted by the hockey stick is 0.75 N·s [forward].

(b) Given: $\vec{F}\Delta t = 0.75\text{ N}\cdot\text{s}$; m = 180 g = 0.18 kg; $\vec{v}_i = 0$

Required: $\vec{v}_f$

Analysis: $\vec{F}\Delta t = \Delta\vec{p}$

$$\Delta\vec{p} = m(\vec{v}_{\text{f}} - \vec{v}_{\text{i}})$$

$$\vec{F}\Delta t = m(\vec{v}_{\text{f}} - \vec{v}_{\text{i}})$$

$$\vec{F}\Delta t = m\vec{v}_{\text{f}}$$

$$\vec{v}_{\text{f}} = \frac{\vec{F}\Delta t}{m} + \vec{v}_{\text{i}}$$

Solution: $\vec{v}_{\text{f}} = \dfrac{\vec{F}\Delta t}{m} + \vec{v}_{\text{i}}$

$$= \frac{0.75\ \cancel{\text{kg}} \cdot \text{m/s [forward]}}{0.18\ \cancel{\text{kg}}} + 0\ \text{m/s}$$

$$\vec{v}_{\text{f}} = 4.2\ \text{m/s [forward]}$$

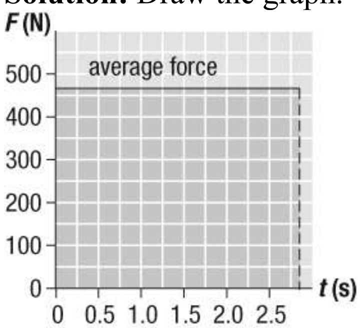
Statement: The final velocity is 4.2 m/s [forward].

2. Given: average force = 468 N; Δt = 2.9 s

Required: force–time graph; $\vec{F}\Delta t$

Analysis: The average force is 468 N, so the graph is a straight line at F = 468. Calculate the area under the curve.

Solution: Draw the graph.



$$\vec{F}\Delta t = bh$$

$$= (2.9\ \text{s})(468\ \text{N})$$

$$\vec{F}\Delta t = 1400\ \text{N} \cdot \text{s}$$

Statement: The impulse of the collision is 1400 N·s [away from the wall].

Section 5.1 Questions, page 227

1. (a) Given: $m = 4.25 \times 10^2$ kg; $\vec{v}$ = 6.9 m/s [N]

Required: $\vec{p}$

Analysis: $\vec{p} = m\vec{v}$

Solution: $\vec{p} = m\vec{v}$

$$= (4.25 \times 10^2\ \text{kg})(6.9\ \text{m/s [N]})$$

$$\vec{p} = 2900\ \text{kg} \cdot \text{m/s [N]}$$

Statement: The momentum of the moose is 2.9×10^3 kg·m/s [N].

(b) Given: $m = 9.97 \times 10^3$ kg; $\vec{v}$ = 5 km/h [forward]

Required: $\vec{p}$
Analysis: $\vec{p} = m\vec{v}$
Convert the velocity to metres per second.

$$\vec{v} = 5\ \frac{\cancel{\text{km}}}{\cancel{\text{h}}}\ [\text{forward}] \times \frac{1\ \cancel{\text{h}}}{3600\ \text{s}} \times \frac{1000\ \text{m}}{1\ \cancel{\text{km}}}$$

$\vec{v} = 1.4$ m/s [forward]

Solution: $\vec{p} = m\vec{v}$

$$\begin{aligned} &= (9.97 \times 10^3\ \text{kg})(1.4\ \text{m/s [forward]}) \\ \vec{p} &= 1.4 \times 10^4\ \text{kg} \cdot \text{m/s [forward]} \end{aligned}$$

Statement: The momentum of the bus is 1.4×10^4 kg·m/s [forward].
(c) Given: m = 995 g = 0.995 kg; $\vec{v} = 16$ m/s [S]
Required: $\vec{p}$
Analysis: $\vec{p} = m\vec{v}$
Solution: $\vec{p} = m\vec{v}$

$$\begin{aligned} &= (0.995\ \text{kg})(16\ \text{m/s [S]}) \\ \vec{p} &= 16\ \text{N} \cdot \text{s [S]} \end{aligned}$$

Statement: The momentum of the squirrel is 16 N·s [S].
2. Answers may vary. Sample answer: Impulse is the change in momentum of an object during a certain time. Its units are newton seconds (N·s).
3. Given: m = 79.3 kg; $\vec{p} = 2.16 \times 10^3$ kg · m/s [W]
Required: $\vec{v}$
Analysis: $\vec{p} = m\vec{v}$

$$\vec{v} = \frac{\vec{p}}{m}$$

Solution: $\vec{v} = \dfrac{\vec{p}}{m}$

$$\begin{aligned} &= \frac{2.16 \times 10^3\ \cancel{\text{kg}} \cdot \text{m/s [W]}}{79.3\ \cancel{\text{kg}}} \\ \vec{v} &= 27.2\ \text{m/s [W]} \end{aligned}$$

Statement: The velocity of the bicycle is 27.2 m/s [W].
4. Given: $\vec{v} = 9.0 \times 10^2$ m/s [W]; $\vec{p} = 4.5$ kg · m/s [W]
Required: m
Analysis: $\vec{p} = m\vec{v}$

$$m = \frac{\vec{p}}{\vec{v}}$$

Solution: $m = \dfrac{\vec{p}}{\vec{v}}$

$$= \frac{4.5\ \text{kg}\cdot\cancel{\text{m}}/\text{s [W]}}{9.0\times10^{2}\ \cancel{\text{m}}/\text{s [W]}}$$

$$m = 5.0\times10^{-3}\ \text{kg}$$

Statement: The mass of the projectile is 5.0×10^{-3} kg or 5.0 g.

5. Given: $\vec{v}$ = 29.5 m/s [forward]; $\vec{p} = 2.31\times10^{3}$ kg · m/s [forward]

Required: m

Analysis: $\vec{p} = m\vec{v}$

$$m = \frac{\vec{p}}{\vec{v}}$$

Solution: $m = \dfrac{\vec{p}}{\vec{v}}$

$$= \frac{2.31\times10^{3}\ \text{kg}\cdot\cancel{\text{m}}/\text{s [forward]}}{29.5\ \cancel{\text{m}}/\text{s [forward]}}$$

$$m = 78.3\ \text{kg}$$

Statement: The mass of the skier is 78.3 kg.

6. Answers may vary. Sample answer: In lacrosse, when a player follows through when striking a ball, this increases the time interval over which the collision between the ball and lacrosse stick occurs and thus contributes to an increase in the velocity change of the ball. Following through, a hitter can hit the ball in such a way that it is moving faster when it leaves the stick. This can improve performance in lacrosse by making the ball travel farther in a given amount of time and by making it harder for an opposing player to catch up with or react to the ball in time.

7. Both balls will hit the floor with the same speed, and both will rebound with this same speed. Since the basketball has more mass, the same speed means it has more momentum. To reverse its larger momentum requires a larger impulse on the basketball.

8. (a) Given: $\vec{F}$ = 1100.0 N [forward]; Δt = 5.0 ms = 5.0×10^{-3} s

Required: $\Delta\vec{p}$

Analysis: $\Delta\vec{p} = \vec{F}\Delta t$

Solution: $\Delta\vec{p} = \vec{F}\Delta t$

$$= (1100.0\ \text{N [forward]})(5.0\times10^{-3}\ \text{s})$$

$$\Delta\vec{p} = 5.5\ \text{N}\cdot\text{s [forward]}$$

Statement: The impulse is 5.5 N·s [forward].

(b) Given: $m = 0.12$ kg; $v_\text{i} = 0$ m/s; $\vec{F}\Delta t = 5.5\ \text{N}\cdot\text{s}$ [forward]

Required: v_f

Analysis: $\vec{F}\Delta t = \Delta\vec{p}$

$$\Delta\vec{p} = m(\vec{v}_\text{f} - \vec{v}_\text{i})$$
$$\vec{F}\Delta t = m(\vec{v}_\text{f} - \vec{v}_\text{i})$$
$$\vec{v}_\text{f} = \frac{\vec{F}\Delta t}{m} + \vec{v}_\text{i}$$

Solution: $v_\text{f} = \dfrac{F\Delta t}{m} + v_\text{i}$

$$= \frac{5.5\ \text{N}\cdot\text{s}}{0.12\ \text{kg}} + 0\ \text{m/s}$$
$$\vec{v}_\text{f} = 46\ \text{m/s}$$

Statement: The speed of the puck just after it leaves the stick is 46 m/s.

9. (a) Given: $m = 225$ g $= 0.225$ kg; $\Delta y = 74$ cm $= 0.74$ m

Required: $\vec{p}$

Analysis: The potential energy as you drop the phone is equal to the kinetic energy when it hits the ground. Use $E_\text{k} = E_\text{g}$, $E_\text{k} = \frac{1}{2}mv^2$ and $E_\text{g} = mg\Delta y$ to obtain $\frac{1}{2}mv^2 = mg\Delta y$. Isolate v in this equation: $v = \sqrt{2g\Delta y}$. Calculate the speed, and then use $\vec{p} = m\vec{v}$ to calculate the momentum.

Solution: Calculate the speed when the phone hits the ground.

$$v = \sqrt{2g\Delta y}$$
$$= \sqrt{2(9.8\ \text{m/s}^2)(0.74\ \text{m})}$$
$$v = 3.81\ \text{m/s (one extra digit carried)}$$

Calculate the momentum.

$$\vec{p} = m\vec{v}$$
$$= (0.225\ \text{kg})(3.81\ \text{m/s [down]})$$
$$\vec{p} = 0.86\ \text{kg}\cdot\text{m/s [down]}$$

Statement: The cellphone's momentum at the moment of impact is 0.86 kg·m/s [down].

(b) The surface of the impact makes no difference to the momentum. The speed and mass do not change, so the momentum does not change.

10. (a) Given: $m = 0.25$ kg; $\Delta y = 1.5$ m; $\vec{v}_\text{f} = 4.0$ m/s [up]

Required: impulse of the ball, $\Delta\vec{p}$

Analysis: The potential energy as you drop the ball is equal to the kinetic energy when it hits the ground. Use $E_\text{k} = E_\text{g}$, $E_\text{k} = \frac{1}{2}mv^2$ and $E_\text{g} = mg\Delta y$ to obtain $\frac{1}{2}mv^2 = mg\Delta y$. Isolate v in this equation: $v = \sqrt{2g\Delta y}$. Calculate the velocity, and then use $\Delta\vec{p} = m(\vec{v}_\text{f} - \vec{v}_\text{i})$ to calculate the impulse imparted by the floor to the ball.

Solution: Calculate the velocity of the ball just before it hits the ground.

$$
\begin{aligned}
v &= \sqrt{2g\Delta y} \\
&= \sqrt{2(9.8\ \text{m/s}^2)(1.5\ \text{m})}
\end{aligned}
$$

$v = 5.42$ m/s (one extra digit carried)

The velocity is downward, so $\vec{v}_\text{i} = 5.42\ \text{m/s [down]}$.

$$
\begin{aligned}
\Delta\vec{p} &= m(\vec{v}_\text{f} - \vec{v}_\text{i}) \\
&= (0.25\ \text{kg})(4.0\ \text{m/s [up]} - 5.42\ \text{m/s [down]}) \\
&= (0.25\ \text{kg})(4.0\ \text{m/s [up]} + 5.42\ \text{m/s [up]}) \\
&= 2.36\ \text{N}\cdot\text{s [up] (one extra digit carried)}
\end{aligned}
$$

$\Delta\vec{p} = 2.4\ \text{N}\cdot\text{s [up]}$

Statement: The impulse imparted by the floor to the ball is 2.4 N·s [up].

(b) Given: $\Delta\vec{p} = 2.36\ \text{N}\cdot\text{s [up]}$; $\vec{F} = 18\ \text{N [up]}$

Required: Δt

Analysis: $\vec{F}\Delta t = \Delta\vec{p}$, so $\Delta t = \dfrac{\Delta\vec{p}}{\vec{F}}$.

Solution:
$$
\begin{aligned}
\Delta t &= \frac{\Delta\vec{p}}{\vec{F}} \\
&= \frac{2.36\ \cancel{\text{N}}\cdot\text{s}}{18\ \cancel{\text{N}}} \\
\Delta t &= 0.13\ \text{s}
\end{aligned}
$$

Statement: The ball is in contact with the floor for 0.13 s.

11. (a) Given: $m = 0.030$ kg; $\vec{v} = 88\ \text{m/s [forward]}$

Required: $\Delta\vec{p}$

Analysis: The initial velocity of the arrow is 0 m/s. Use $\Delta\vec{p} = m\Delta\vec{v}$.

Solution:
$$
\begin{aligned}
\Delta\vec{p} &= m\Delta\vec{v} \\
&= (0.030\ \text{kg})(88\ \text{m/s [forward]}) \\
&= 2.64\ \text{N}\cdot\text{s [forward] (one extra digit carried)} \\
\Delta\vec{p} &= 2.6\ \text{N}\cdot\text{s [forward]}
\end{aligned}
$$

Statement: The impulse is 2.6 N·s [forward].

(b) Given: $\Delta t = 0.015$ s; $\Delta\vec{p} = 2.64\ \text{N}\cdot\text{s [forward]}$

Required: $\vec{F}$

Analysis: $\vec{F}\Delta t = \Delta\vec{p}$, so $\vec{F} = \dfrac{\Delta\vec{p}}{\Delta t}$.

Solution: $\vec{F} = \dfrac{\Delta\vec{p}}{\Delta t}$

$$= \frac{2.64 \text{ kg}\cdot\text{m/s [forward]}}{0.015 \text{ s}}$$

$$\vec{F} = 1.8\times10^2 \text{ N [forward]}$$

Statement: The average force of the bowstring on the arrow is 1.8×10^2 N [forward].

12. (a) Given: $\vec{v}_\text{i} = 63$ m/s [W]; $m = 0.057$ kg; $\vec{v}_\text{f} = 41$ m/s [E]

Required: $\Delta\vec{p}$

Analysis: $\Delta\vec{p} = m(\vec{v}_\text{f} - \vec{v}_\text{i})$

Solution: $\Delta\vec{p} = m(\vec{v}_\text{f} - \vec{v}_\text{i})$

$$= (0.057 \text{ kg})(41 \text{ m/s [E]} - 63 \text{ m/s [W]})$$

$$= (0.057 \text{ kg})(41 \text{ m/s [E]} + 63 \text{ m/s [E]})$$

$$\Delta\vec{p} = 5.93 \text{ N}\cdot\text{s [E] (one extra digit carried)}$$

Statement: The magnitude of the impulse is 5.9 N·s.

(b) Given: $\Delta\vec{p} = 5.93$ N · s [E]; $\Delta t = 0.023$ s

Required: $\vec{F}$

Analysis: $\vec{F}\Delta t = \Delta\vec{p}$, so $\vec{F} = \dfrac{\Delta\vec{p}}{\Delta t}$.

Solution: $\vec{F} = \dfrac{\Delta\vec{p}}{\Delta t}$

$$= \frac{5.93 \text{ N}\cdot\cancel{\text{s}}\text{ [E]}}{0.023\ \cancel{\text{s}}}$$

$$\vec{F} = 2.6\times10^2 \text{ N [E]}$$

Statement: The average force is 2.6×10^2 N [E].

Section 5.2: Conservation of Momentum in One Dimension

Tutorial 1 Practice, page 231

1. Given: m_1 = 1350 kg; $\vec{v}_1 = 72$ km/h [S]; m_2 = 1650 kg; $\vec{v}_\text{f} = 24$ km/h [S]

Required: $\vec{v}_2$

Analysis: The cars stick together after the collision, so they can be treated as a single object with mass $m_1 + m_2$, velocity $\vec{v}_\text{f}$, and momentum $\vec{p}_\text{f}$.

$p_\text{f} = (m_1 + m_2)v_\text{f}$

By conservation of momentum,

$\vec{p}_\text{f} = \vec{p}_1 + \vec{p}_2$

and $\vec{p} = m\vec{v}$; therefore,

$$\begin{aligned} \vec{p}_\text{f} &= \vec{p}_1 + \vec{p}_2 \\ (m_1 + m_2)\vec{v}_\text{f} &= m_1\vec{v}_1 + m_2\vec{v}_2 \\ \vec{v}_2 &= \frac{(m_1 + m_2)\vec{v}_\text{f} - m_1\vec{v}_1}{m_2} \end{aligned}$$

Solution: $\vec{v}_2 = \dfrac{(m_1 + m_2)\vec{v}_\text{f} - m_1\vec{v}_1}{m_2}$

$$\begin{aligned} &= \frac{(1350\text{ kg} + 1650\ \cancel{\text{kg}})(24\text{ km/h [S]}) - (1350\ \cancel{\text{kg}})(72\text{ km/h [S]})}{1650\ \cancel{\text{kg}}} \\ \vec{v}_2 &= -15\text{ km/h [S]} \\ \vec{v}_2 &= 15\text{ km/h [N]} \end{aligned}$$

Statement: The initial velocity of the second car is 15 km/h [N].

2. Given: m_1 = 28 g = 0.028 kg; $\vec{v}_1 = 92$ m/s [forward]; $\vec{v}_2 = 0.039$ m/s [backward]

Required: m_2

Analysis: The momentum of the arrow is the opposite of the momentum of the archer.

$m_1\vec{v}_1 = -m_2\vec{v}_2$

$m_2 = -\dfrac{m_1\vec{v}_1}{\vec{v}_2}$

Solution: $m_2 = -\dfrac{m_1\vec{v}_1}{\vec{v}_2}$

$$\begin{aligned} &= -\frac{(0.028\text{ kg})(92\text{ m/s [forward]})}{(0.039\text{ m/s [backward]})} \\ &= -\frac{(0.028\text{ kg})(92\ \cancel{\text{m/s}}\ \cancel{\text{[forward]}})}{(-0.039\ \cancel{\text{m/s}}\ \cancel{\text{[forward]}})} \\ m_2 &= 66\text{ kg} \end{aligned}$$

Statement: The mass of the archer and the bow is 6.6×10^1 kg.

Section 5.2 Questions, page 232

1. The total momentum of a system is conserved if there is no net force applied on the system.

2. Given: mass of student and surfboard, m_1 = 59.6 kg; mass of student, m_2 = 55 kg; velocity of surfboard relative to water, $\vec{v}_1$ = 2.0 m/s [E]; velocity of student relative to surfboard, $\vec{v}_2$ = 1.9 m/s [E]

Required: resultant velocity, $\vec{v}$

Analysis: The momentum of the student and surfboard before the student starts walking is $m_1\vec{v}_1$. The momentum of the system after the student starts walking is the momentum of the student, $m_2\vec{v}_2$, plus the momentum of the student and surfboard, $m_1\vec{v}$.

$$m_1\vec{v}_1 = m_2\vec{v}_2 + m_1\vec{v}$$

$$\vec{v} = \frac{m_1\vec{v}_1 - m_2\vec{v}_2}{m_1}$$

Solution: $\vec{v} = \dfrac{m_1\vec{v}_1 - m_2\vec{v}_2}{m_1}$

$$= \frac{(59.6\ \cancel{\text{kg}})(2.0\ \text{m/s [E]}) - (55\ \cancel{\text{kg}})(1.9\ \text{m/s [E]})}{59.6\ \cancel{\text{kg}}}$$

$$\vec{v} = 0.25\ \text{m/s [E]}$$

Statement: The final velocity of the surfboard is 0.25 m/s [E].

3. Given: m_1 = 35.6 kg; v_1 = 2.42 m/s; v_2 = 3.25 m/s

Required: m_2

Analysis: Assume the first hockey player is moving left after the collision and the second player is moving right. The momentum before the push is zero, so the total momentum after the push is also zero.

$$m_1\vec{v}_1 + m_2\vec{v}_2 = 0$$

$$m_2 = -\frac{m_1\vec{v}_1}{\vec{v}_2}$$

Solution: $m_2 = -\dfrac{m_1\vec{v}_1}{\vec{v}_2}$

$$= -\frac{(35.6\ \text{kg})(2.42\ \text{m/s [left]})}{3.25\ \text{m/s [right]}}$$

$$= -\frac{(35.6\ \text{kg})(-2.42\ \cancel{\text{m/s}}\ \cancel{[\text{right}]})}{3.25\ \cancel{\text{m/s}}\ \cancel{[\text{right}]}}$$

$$m_2 = 26.5\ \text{kg}$$

Statement: The mass of the other player is 26.5 kg.

4. Given: m_1 = 0.14 kg; $\vec{v}_1$ = 50 m/s [forward]; m_2 = 80 kg

Required: $\vec{v}_2$

Analysis: The momentum before the throw is zero, so the total momentum after the throw is also zero.

$$m_1\vec{v}_1 + m_2\vec{v}_2 = 0$$

$$\vec{v}_2 = -\frac{m_1\vec{v}_1}{m_2}$$

Solution: $\vec{v}_2 = -\frac{m_1\vec{v}_1}{m_2}$

$$= -\frac{(0.14\ \cancel{\text{kg}})(50\ \text{m/s [forward]})}{80\ \cancel{\text{kg}}}$$

$$= -9\times 10^{-2}\ \text{m/s [forward]}$$

$$\vec{v}_2 = 9\times 10^{-2}\ \text{m/s [backward]}$$

Statement: The recoil velocity of the pitcher is 9×10^{-2} m/s [backward].

5. Given: m_1 = 4.5 kg; m_2 = 6.2 kg; $\vec{v}_{i_1} = 16$ m/s [E]; $\vec{v}_{i_2} = 0$ m/s; $\vec{v}_{f_2} = 10$ m/s [E]

Required: $\vec{v}_{f_1}$

Analysis: Momentum is conserved.

$$m_1\vec{v}_{i_1} + m_2\vec{v}_{i_2} = m_1\vec{v}_{f_1} + m_2\vec{v}_{f_2}$$

$$\vec{v}_{f_1} = \frac{m_1\vec{v}_{i_1} + m_2\vec{v}_{i_2} - m_2\vec{v}_{f_2}}{m_1}$$

Solution: $\vec{v}_{f_1} = \frac{m_1\vec{v}_{i_1} + m_2\vec{v}_{i_2} - m_2\vec{v}_{f_2}}{m_1}$

$$= \frac{(4.5\ \cancel{\text{kg}})(16\ \text{m/s [E]}) + (6.2\ \cancel{\text{kg}})(0) - (6.2\ \cancel{\text{kg}})(10\ \text{m/s [E]})}{4.5\ \cancel{\text{kg}}}$$

$$\vec{v}_{f_1} = 2.2\ \text{m/s [E]}$$

Statement: The final velocity of the smaller object is 2.2 m/s [E].

6. Given: $m_1 = m$; $m_2 = 3m$; $\vec{v}_{i_1} = 3v$; $\vec{v}_{i_2} = -v$

Required: $\vec{v}_{f_1}$; $\vec{v}_{f_2}$

Analysis: Use conservation of momentum.

Solution: Assume the lighter mass is initially moving to the right. Consider the conservation of momentum.

$$m_1\vec{v}_{i_1} + m_2\vec{v}_{i_2} = m_1\vec{v}_{f_1} + m_2\vec{v}_{f_2}$$

$$\cancel{m}(3v) + 3\cancel{m}(-v) = \cancel{m}\vec{v}_{f_1} + 3\cancel{m}\vec{v}_{f_2}$$

$$0 = \vec{v}_{f_1} + 3\vec{v}_{f_2}$$

$$\vec{v}_{f_1} = -3\vec{v}_{f_2}$$

Statement: The final speed of the larger mass will be three times the final speed of the smaller mass, and in the opposite direction.

7. Given: m_1 = 2.5 kg; m_2 = 7.5 kg; $v_{1\,\mathrm{i}} = +6.0\ \text{m/s}$; $v_{2\,\mathrm{i}} = -15\ \text{m/s}$

Required: $\vec{v}_\mathrm{f}$

Analysis: The two objects stick together after the collision, with a mass of $m_1 + m_2$. Momentum is conserved.

$$m_1\vec{v}_{\mathrm{i}_1} + m_2\vec{v}_{\mathrm{i}_2} = (m_1 + m_2)\vec{v}_\mathrm{f}$$

$$\vec{v}_\mathrm{f} = \frac{m_1\vec{v}_{\mathrm{i}_1} + m_2\vec{v}_{\mathrm{i}_2}}{m_1 + m_2}$$

Solution: $\vec{v}_\mathrm{f} = \dfrac{m_1\vec{v}_{\mathrm{i}_1} + m_2\vec{v}_{\mathrm{i}_2}}{m_1 + m_2}$

$$= \frac{(2.5\ \cancel{\text{kg}})(6.0\ \text{m/s}) + (7.5\ \cancel{\text{kg}})(-15\ \text{m/s})}{2.5\ \cancel{\text{kg}} + 7.5\ \cancel{\text{kg}}}$$

$$\vec{v}_\mathrm{f} = -9.8\ \text{m/s}$$

Statement: The final velocity of the two objects is 9.8 m/s [left].

8. Answers may vary. Sample answer: The astronaut can throw the tool bag in a direction away from the International Space Station. Conservation of momentum means she will move in the opposite direction, toward the ISS.

Section 5.3: Collisions

Mini Investigation: Newton's Cradle, page 234

Answers may vary. Sample answers:

A. In Step 2, releasing one end ball caused the far ball on the other end to swing out at the same speed as the original ball, while the middle balls appeared to remain still. Changing the setup did not change the outcome as long as all balls were touching. When the middle ball was removed, momentum was not transferred all the way to the end of the line.

B. Yes, the collisions appear to conserve momentum, although the balls slow down after a while.

C. Yes, the collisions appear to conserve kinetic energy. The end ball moves at the same speed as the beginning ball, so kinetic energy is conserved (ignoring external forces).

D. The device as a whole does not appear to conserve mechanical energy. Energy is lost in the sound of the collisions, and friction between the balls and between the string and the supports.

Tutorial 1 Practice, page 236

1. Given: $m_1 = 3.5$ kg; $\vec{v}_{i_1} = 5.4 \text{ m/s [right]}$; $m_2 = 4.8$ kg; $\vec{v}_{i_2} = 0 \text{ m/s}$

Required: $\vec{v}_{f_2}$

Analysis: The collision is perfectly elastic, which means that kinetic energy is conserved. Apply conservation of momentum and conservation of kinetic energy to construct and solve a linear-quadratic system of two equations in two unknowns. First, use conservation of momentum to solve for v_{f_1} in terms of $\vec{v}_{f_2}$, remembering that $\vec{v}_{i_2} = 0 \text{ m/s}$. This is a one-dimensional problem, so omit the vector notation for velocities, recognizing that positive values indicate motion to the right and negative values indicate motion to the left.

$$m_1 v_{i_1} + m_2 v_{i_2} = m_1 v_{f_1} + m_2 v_{f_2}$$

$$m_1 v_{i_1} = m_1 v_{f_1} + m_2 v_{f_2}$$

$$v_{f_1} = \frac{m_1 v_{i_1} - m_2 v_{f_2}}{m_1}$$

Then, substitute the resulting equation into the conservation of kinetic energy equation to solve for the final velocity of ball 2.

Solution:

$$\begin{aligned} v_{f_1} &= \frac{m_1 v_{i_1} - m_2 v_{f_2}}{m_1} \\ &= \frac{(3.5\ \cancel{\text{kg}})(5.4 \text{ m/s}) - (4.8\ \cancel{\text{kg}})v_{f_2}}{3.5\ \cancel{\text{kg}}} \\ v_{f_1} &= \frac{18.9 \text{ m/s} - 4.8 v_{f_2}}{3.5} \end{aligned}$$

The conservation of kinetic energy equation can be simplified by multiplying both sides of the equation by 2 and noting that $\vec{v}_{i_2} = 0 \text{ m/s}$.

$$\cancel{\frac{1}{2}}m_1v_{i_1}^2 + \cancel{\frac{1}{2}}m_2v_{i_2}^2 = \cancel{\frac{1}{2}}m_1v_{f_1}^2 + \cancel{\frac{1}{2}}m_2v_{f_2}^2$$

$$m_1v_{i_2}^2 = m_1v_{f_1}^2 + m_2v_{f_2}^2$$

$$(3.5\ \cancel{\text{kg}})(5.4\ \text{m/s})^2 = (\cancel{3.5}\ \cancel{\text{kg}})\frac{(18.9\ \text{m/s} - 4.8v_{f_2})^2}{(3.5)^{\cancel{2}}} + (4.8\ \cancel{\text{kg}})v_{f_2}^2$$

$$(3.5)(3.5)(5.4\ \text{m/s})^2 = (18.9\ \text{m/s} - 4.8v_{f_2})^2 + (3.5)(4.8)v_{f_2}^2$$

$$357.21\ \text{m}^2/\text{s}^2 = 357.21\ \text{m}^2/\text{s}^2 - (181.44\ \text{m/s})v_{f_2} + 23.04v_{f_2}^2 + 16.8v_{f_2}^2$$

$$0 = 39.84v_{f_2}^2 - (181.44\ \text{m/s})v_{f_2}$$

$$0 = v_{f_2}(39.84v_{f_2} - 181.44\ \text{m/s})$$

The factor of v_{f_2} means the equation has a solution v_{f_2} = 0 m/s. This solution describes the system before the collision. The equation has a second solution describing the system after the collision.

$$0 = 39.84v_{f_2} - 181.44\ \text{m/s}$$

$$v_{f_2} = \frac{181.44\ \text{m/s}}{39.84}$$

$$v_{f_2} = 4.6\ \text{m/s}$$

Statement: The final velocity of ball 2 is 4.6 m/s [right].

2. Given: $\vec{v}_{i_1} = \vec{v}_1$; $\vec{v}_{i_2} = 0\ \text{m/s}$; $m_1 = m$; $m_2 = m$

Required: v_{f_1}; v_{f_2}

Analysis: $m_1v_{i_1} + m_2v_{i_2} = m_1v_{f_1} + m_2v_{f_2}$; $\frac{1}{2}m_1v_{i_1}^2 + \frac{1}{2}m_2v_{i_2}^2 = \frac{1}{2}m_1v_{f_1}^2 + \frac{1}{2}m_2v_{f_2}^2$

Solution: $m_1v_{i_1} + m_2v_{i_2} = m_1v_{f_1} + m_2v_{f_2}$

$$\cancel{m}v_1 + \cancel{m}(0\ \text{m/s}) = \cancel{m}v_{f_1} + \cancel{m}v_{f_2}$$

$$v_1 = v_{f_1} + v_{f_2}$$

$$v_{f_2} = v_1 - v_{f_1}$$

$$\cancel{\frac{1}{2}}m_1v_{i_1}^2 + \cancel{\frac{1}{2}}m_2v_{i_2}^2 = \cancel{\frac{1}{2}}m_1v_{f_1}^2 + \cancel{\frac{1}{2}}m_2v_{f_2}^2$$

$$\cancel{m}v_1^2 + \cancel{m}(0\ \text{m/s})^2 = \cancel{m}v_{f_1}^2 + \cancel{m}(v_1 - v_{f_1})^2$$

$$v_1^2 = v_{f_1}^2 + (v_1 - v_{f_1})^2$$

$$v_1^2 = v_{f_1}^2 + v_1^2 - 2v_1v_{f_1} + v_{f_1}^2$$

$$0 = 2v_{f_1}^2 - 2v_1v_{f_1}$$

$$0 = 2v_{f_1}(v_{f_1} - v_1)$$

$v_{f_1} = 0$ or $v_{f_1} = v_1$

The final speed of the first stone cannot be the same as its initial speed, so v_{f_1} = 0. Substitute v_{f_1} = 0 in the equation for v_{f_2}.

$$\begin{aligned} v_{f_2} &= v_1 - v_{f_1} \\ &= v_1 - 0 \\ v_{f_2} &= v_1 \end{aligned}$$

Statement: The final speed of the first stone is 0 m/s. The final speed of the second stone is v_1.

Tutorial 2 Practice, page 238

1. Given: m_1 = 4.0 kg; m_2 = 2.0 kg; $\vec{v}_{i_1} = 6.0 \text{ m/s [forward]}$; $\vec{v}_{i_2} = 0 \text{ m/s}$

Required: $\vec{v}_f$

Analysis: Use $\vec{v}_f = \dfrac{m_1\vec{v}_{i_1} + m_2\vec{v}_{i_2}}{m_1 + m_2}$.

Solution:
$$\begin{aligned} \vec{v}_f &= \frac{m_1\vec{v}_{i_1} + m_2\vec{v}_{i_2}}{m_1 + m_2} \\ &= \frac{(4.0 \cancel{\text{ kg}})(6.0 \text{ m/s [forward]}) + (2.0 \cancel{\text{ kg}})(0 \text{ m/s})}{4.0 \text{ kg} + 2.0 \cancel{\text{ kg}}} \\ \vec{v}_f &= 4.0 \text{ m/s [forward]} \end{aligned}$$

Statement: The velocity of the balls is 4.0 m/s [forward] after the collision.

2. (a) Given: m_1 = 2200 kg; $\vec{v}_{i_1} = 60.0 \text{ km/h [E]}$; m_2 = 1300 kg; $\vec{v}_{i_2} = 30.0 \text{ km/h [E]}$

Required: $\vec{v}_f$

Analysis: Convert the velocities to metres per second and use $\vec{v}_f = \dfrac{m_1\vec{v}_{i_1} + m_2\vec{v}_{i_2}}{m_1 + m_2}$.

$$\vec{v}_{i_1} = 60.0 \frac{\cancel{\text{km}}}{\cancel{\text{h}}} \times \frac{1000 \text{ m}}{1 \cancel{\text{ km}}} \times \frac{1 \cancel{\text{ h}}}{3600 \text{ s}} \text{ [E]}$$

$\vec{v}_{i_1} = 16.7 \text{ m/s [E]}$ (one extra digit carried)

$$\vec{v}_{i_2} = 30.0 \frac{\cancel{\text{km}}}{\cancel{\text{h}}} \times \frac{1000 \text{ m}}{1 \cancel{\text{ km}}} \times \frac{1 \cancel{\text{ h}}}{3600 \text{ s}} \text{ [E]}$$

$\vec{v}_{i_2} = 8.33 \text{ m/s [E]}$ (one extra digit carried)

Solution: $\vec{v}_{\text{f}} = \dfrac{m_1\vec{v}_{\text{i}_1} + m_2\vec{v}_{\text{i}_2}}{m_1 + m_2}$

$$= \frac{(2200\ \cancel{\text{kg}})(16.7\ \text{m/s [E]}) + (1300\ \cancel{\text{kg}})(8.33\ \text{m/s [E]})}{2200\ \cancel{\text{kg}} + 1300\ \cancel{\text{kg}}}$$

$\vec{v}_{\text{f}} = 13.6\ \text{m/s [E]}$ (one extra digit carried)

Statement: The final velocity of the vehicles is 14 m/s [E].

(b) Given: m_1 = 2200 kg; m_2 = 1300 kg; $\vec{v}_{\text{f}}$ = 13.6 m/s [E]

Required: p

Analysis: The momentum before the collision is equal to the momentum after the collision. Use the answer from (a) to determine the momentum after the collision.

$\vec{p} = (m_1 + m_2)\vec{v}_{\text{f}}$

Solution: $\vec{p} = (m_1 + m_2)\vec{v}_{\text{f}}$

$= (2200\ \text{kg} + 1300\ \text{kg})(13.6\ \text{m/s [E]})$

$\vec{p} = 4.8 \times 10^4\ \text{kg} \cdot \text{m/s}$

Statement: The momentum before and after the collision is 4.8×10^4 kg·m/s.

(c) Given: m_1 = 2200 kg; $\vec{v}_{\text{i}_1}$ = 60.0 km/h [E] = 16.7 m/s [E]; m_2 = 1300 kg;

$\vec{v}_{\text{i}_2} = 30.0\ \text{km/h [E]} = 8.33\ \text{m/s [E]}$; $\vec{v}_{\text{f}} = 13.6\ \text{m/s [E]}$

Required: ΔE_{k}

Analysis: $\Delta E_{\text{k}} = E_{\text{k}_\text{f}} - E_{\text{k}_\text{i}}$; $E_{\text{k}} = \dfrac{1}{2}mv^2$

Solution:

$$\begin{aligned}
\Delta E_{\text{k}} &= E_{\text{k}_\text{f}} - E_{\text{k}_\text{i}} \\
&= \frac{1}{2}(m_1 + m_2)v_{\text{f}}^2 - \left(\frac{1}{2}m_1 v_{\text{i}_1}^2 + \frac{1}{2}m_2 v_{\text{i}_2}^2\right) \\
&= \frac{1}{2}(2200\ \text{kg} + 1300\ \text{kg})(13.6\ \text{m/s})^2 - \frac{1}{2}[(2200\ \text{kg})(16.7\ \text{m/s})^2 + (1300\ \text{kg})(8.33\ \text{m/s})^2] \\
\Delta E_{\text{k}} &= -2.8 \times 10^4\ \text{J}
\end{aligned}$$

Statement: The decrease in kinetic energy is 2.8×10^4 J.

3. Given: m_1 = 66 kg; Δy = 25 m; m_2 = 72 kg; v_{i_2} = 0 m/s

Required: v_{f}

Analysis: Use conservation of energy to determine the speed of the snowboarder at the bottom of the hill.

$m_1 g\Delta y = \dfrac{1}{2}m_1 v_{\text{i}_1}^2$

Then, use $\vec{v}_{\text{f}} = \dfrac{m_1\vec{v}_{\text{i}_1} + m_2\vec{v}_{\text{i}_2}}{m_1 + m_2}$ to calculate the final velocity of both people after the collision.

Solution: $\cancel{m_1} g\Delta y = \frac{1}{2}\cancel{m_1} v_{i_1}^2$

$$v_{i_1} = \sqrt{2g\Delta y}$$

$$= \sqrt{2(9.8\ \text{m/s}^2)(25\ \text{m})}$$

$$v_{i_1} = 22.1\ \text{m/s (one extra digit carried)}$$

$$\vec{v}_f = \frac{m_1\vec{v}_{i_1} + m_2\vec{v}_{i_2}}{m_1 + m_2}$$

$$= \frac{(66\ \cancel{\text{kg}})(22.1\ \text{m/s}) + (72\ \cancel{\text{kg}})(0\ \text{m/s})}{66\ \cancel{\text{kg}} + 72\ \cancel{\text{kg}}}$$

$$v_f = 11\ \text{m/s}$$

Statement: The final speed of each person after the collision is 11 m/s.

Section 5.3 Questions, page 239

1. Answers may vary. Sample answer:

(a) Since the boxes stick together after the collision, we know this is an inelastic collision. Momentum is conserved in an inelastic collision. Momentum is always conserved if there are no external forces acting on the system.

(b) Kinetic energy is not conserved in an inelastic collision. In an inelastic collision, some kinetic energy is absorbed by one or both objects, causing the kinetic energy after the collision to be less than the kinetic energy before the collision.

2. Given: m_1 = 85 kg; m_2 = 8.0 kg; $\vec{v}_{i_2} = 0$ m/s; $\vec{v}_f = 3.0$ m/s [forward]

Required: v_{i_1}

Analysis: Use $v_f = \frac{m_1v_{i_1} + m_2v_{i_2}}{m_1 + m_2}$, rearranged to isolate v_{i_1}.

$$v_f = \frac{m_1v_{i_1} + m_2v_{i_2}}{m_1 + m_2}$$

$$(m_1 + m_2)v_f = m_1v_{i_1} + m_2v_{i_2}$$

$$m_1v_{i_1} = (m_1 + m_2)v_f - m_2v_{i_2}$$

$$v_{i_1} = \frac{(m_1 + m_2)v_f - m_2v_{i_2}}{m_1}$$

Solution: $v_{i_1} = \frac{(m_1 + m_2)v_f - m_2v_{i_2}}{m_1}$

$$= \frac{(85\ \cancel{\text{kg}} + 8.0\ \cancel{\text{kg}})(3.0\ \text{m/s}) - (8.0\ \cancel{\text{kg}})(0\ \text{m/s})}{85\ \cancel{\text{kg}}}$$

$$v_{i_1} = 3.3\ \text{m/s}$$

Statement: The speed of the skateboarder just before he landed on the skateboard was 3.3 m/s.

3. Given: $m_T = 3.0$ kg; $m_1 = 2.0$ kg; $\vec{v}_i = 0$ m/s; $\vec{v}_{f_1} = 2.5$ m/s [S]

Required: $\vec{v}_{f_2}$

Analysis: Since the total mass is 3.0 kg, and the mass of the first cart is 2.0 kg, the mass of the second cart, m_2, is 1.0 kg. The carts are at rest before they are released, so the initial momentum of the system is zero. The final momentum must equal the initial momentum.

$\vec{p} = m\vec{v}$

Solution: $\vec{p}_f = m_1\vec{v}_{f_1} + m_2\vec{v}_{f_2}$

$$0 = (2.0\ \cancel{\text{kg}})(2.5\text{ m/s [S]}) + (1.0\ \cancel{\text{kg}})\vec{v}_{f_2}$$

$$\vec{v}_{f_2} = -5.0\text{ m/s [S]}$$

$$\vec{v}_{f_2} = 5.0\text{ m/s [N]}$$

Statement: The final velocity of the other cart is 5.0 m/s [N].

4. (a) Yes. Both momentum and kinetic energy are conserved in a perfectly elastic collision.

(b) Given: $m_1 = 85$ kg; $\vec{v}_{i_1} = 6.5$ m/s; $m_2 = 120$ kg; $\vec{v}_{i_2} = 0$ m/s; $\vec{v}_{f_1} = -1.1$ m/s

Required: $\vec{v}_{f_2}$

Analysis: Use the conservation of momentum equation to solve for $\vec{v}_{f_2}$.

$m_1\vec{v}_{i_1} + m_2\vec{v}_{i_2} = m_1\vec{v}_{f_1} + m_2\vec{v}_{f_2}$

Solution:

$$m_1\vec{v}_{i_1} + m_2\vec{v}_{i_2} = m_1\vec{v}_{f_1} + m_2\vec{v}_{f_2}$$

$$(85\ \cancel{\text{kg}})(6.5\text{ m/s}) + (120\ \cancel{\text{kg}})(0\text{ m/s}) = (85\ \cancel{\text{kg}})(-1.1\text{ m/s}) + (120\ \cancel{\text{kg}})\vec{v}_{f_2}$$

$$552.5\text{ m/s} = -93.5\text{ m/s} + 120\vec{v}_{f_2}$$

$$\vec{v}_{f_2} = 5.4\text{ m/s}$$

Statement: The final velocity of the second person is 5.4 m/s in the direction that the first person was originally travelling.

5. (a) It is an inelastic collision because the two skaters stick together after the collision.

(b) Given: $m_1 = 95$ kg; $v_{i_1} = 5.0$ m/s; $m_2 = 130$ kg; $v_{i_2} = 0$ m/s

Required: v_f

Analysis: Use $v_f = \dfrac{m_1 v_{i_1} + m_2 v_{i_2}}{m_1 + m_2}$.

Solution:

$$v_f = \frac{m_1 v_{i_1} + m_2 v_{i_2}}{m_1 + m_2}$$

$$= \frac{(95\ \cancel{\text{kg}})(5.0\text{ m/s}) + (130\ \cancel{\text{kg}})(0\text{ m/s})}{95\ \cancel{\text{kg}} + 130\ \cancel{\text{kg}}}$$

$$v_f = 2.1\text{ m/s}$$

Statement: The final speed of the skaters is 2.1 m/s [initial direction of the first skater].

6. Given: $m_1 = m_2 = 1250$ kg; $\vec{v}_{i_1} = 12$ m/s [E]; $\vec{v}_{i_2} = 12$ m/s [W]

Required: $\vec{v}_f$

Analysis: Use $\vec{v}_f = \dfrac{m_1\vec{v}_{i_1} + m_2\vec{v}_{i_2}}{m_1 + m_2}$.

Solution:
$$\begin{aligned}
\vec{v}_f &= \frac{m_1\vec{v}_{i_1} + m_2\vec{v}_{i_2}}{m_1 + m_2} \\
&= \frac{(1250\text{ kg})(12\text{ m/s [E]}) + (1250\text{ kg})(12\text{ m/s [W]})}{1250\text{ kg} + 1250\text{ kg}} \\
&= \frac{(1250\ \cancel{\text{kg}})(12\text{ m/s [E]}) - (1250\ \cancel{\text{kg}})(12\text{ m/s [E]})}{1250\ \cancel{\text{kg}} + 1250\ \cancel{\text{kg}}} \\
\vec{v}_f &= 0\text{ m/s}
\end{aligned}$$

Statement: The velocity of both cars is 0 m/s after the collision. They come to a complete stop.

7. Answers may vary. Sample answer: No, if a moving object collides with a stationary object in a perfectly elastic collision, is it not possible for both objects to be at rest after the collision. Because kinetic energy is conserved in a perfectly elastic collision, and there was kinetic energy before the collision, there must also be kinetic energy after the collision. Therefore, one of the objects must be moving after the collision.

8. (a) Given: $m_1 = 1.3 \times 10^4$ kg; $\vec{v}_{i_1} = 9.0 \times 10^1$ km/h [N]; $m_2 = 1.1 \times 10^3$ kg; $\vec{v}_{i_2} = 3.0 \times 10^1$ km/h [N]

Required: $\vec{v}_f$

Analysis: Convert the velocities to metres per second and then use $\vec{v}_f = \dfrac{m_1\vec{v}_{i_1} + m_2\vec{v}_{i_2}}{m_1 + m_2}$.

$$\vec{v}_{i_1} = 9.0 \times 10^1\ \frac{\cancel{\text{km}}}{\cancel{\text{h}}} \times \frac{1000\text{ m}}{1\ \cancel{\text{km}}} \times \frac{1\ \cancel{\text{h}}}{3600\text{ s}}\ \text{[N]}$$

$$\vec{v}_{i_1} = 25\text{ m/s [N]}$$

$$\vec{v}_{i_2} = 3.0 \times 10^1\ \frac{\cancel{\text{km}}}{\cancel{\text{h}}} \times \frac{1000\text{ m}}{1\ \cancel{\text{km}}} \times \frac{1\ \cancel{\text{h}}}{3600\text{ s}}\ \text{[N]}$$

$$\vec{v}_{i_2} = 8.33\text{ m/s [N]}$$

Solution:
$$\begin{aligned}
\vec{v}_f &= \frac{m_1\vec{v}_{i_1} + m_2\vec{v}_{i_2}}{m_1 + m_2} \\
&= \frac{(1.3 \times 10^4\ \cancel{\text{kg}})(2.5 \times 10^1\text{ m/s [N]}) + (1.1 \times 10^3\ \cancel{\text{kg}})(8.33\text{ m/s [N]})}{1.3 \times 10^4\ \cancel{\text{kg}} + 1.1 \times 10^3\ \cancel{\text{kg}}} \\
\vec{v}_f &= 23.7\text{ m/s [N]}
\end{aligned}$$

Convert back to kilometres per hour.

$$\vec{v}_\mathrm{f} = 23.7\ \frac{\cancel{\mathrm{m}}}{\cancel{\mathrm{s}}} \times \frac{1\ \mathrm{km}}{1000\ \cancel{\mathrm{m}}} \times \frac{3600\ \cancel{\mathrm{s}}}{1\ \mathrm{h}}$$

$$\vec{v}_\mathrm{f} = 85\ \mathrm{km/h}$$

Statement: The velocity of the vehicles after the collision is 85 km/h [N].

(b) Given: $m_1 = 1.3 \times 10^4$ kg; $\vec{v}_{\mathrm{i}_1} = 25$ m/s [N]; $m_2 = 1.1 \times 10^3$ kg; $\vec{v}_{\mathrm{i}_2} = 8.33$ m/s [N];

$\vec{v}_\mathrm{f} = 23.7$ m/s [N]

Required: $E_{\mathrm{k}_\mathrm{i}}$; $E_{\mathrm{k}_\mathrm{f}}$

Analysis: $E_\mathrm{k} = \frac{1}{2}mv^2$

Solution:

$$\begin{aligned} E_{\mathrm{k}_\mathrm{i}} &= \frac{1}{2}(m_1 v_{\mathrm{i}_1}^2 + m_2 v_{\mathrm{i}_2}^2) \\ &= \frac{1}{2}[(1.3\times10^4\ \mathrm{kg})(25\ \mathrm{m/s})^2 + (1.1\times10^3\ \mathrm{kg})(8.33\ \mathrm{m/s})^2] \\ E_{\mathrm{k}_\mathrm{i}} &= 4.1\times10^6\ \mathrm{J} \end{aligned}$$

$$\begin{aligned} E_{\mathrm{k}_\mathrm{f}} &= \frac{1}{2}(m_1 + m_2)v_\mathrm{f}^2 \\ &= \frac{1}{2}(1.3\times10^4\ \mathrm{kg} + 1.1\times10^3\ \mathrm{kg})(23.7\ \mathrm{m/s})^2 \\ E_{\mathrm{k}_\mathrm{f}} &= 3.96\times10^6\ \mathrm{J}\ \text{(one extra digit carried)} \end{aligned}$$

Statement: The total kinetic energy before the collision was 4.1×10^6 J, and the total kinetic energy after the collision was 4.0×10^6 J.

(c) Given: $E_{\mathrm{k}_\mathrm{i}} = 4.1 \times 10^6$ J; $E_{\mathrm{k}_\mathrm{f}} = 3.96 \times 10^6$ J

Required: ΔE_k

Analysis: $\Delta E_\mathrm{k} = E_{\mathrm{k}_\mathrm{f}} - E_{\mathrm{k}_\mathrm{i}}$

Solution:

$$\begin{aligned} \Delta E_\mathrm{k} &= E_{\mathrm{k}_\mathrm{f}} - E_{\mathrm{k}_\mathrm{i}} \\ &= (3.96\times10^6\ \mathrm{J}) - (4.1\times10^6\ \mathrm{J}) \\ \Delta E_\mathrm{k} &= -1.4\times10^6\ \mathrm{J} \end{aligned}$$

Statement: The decrease in kinetic energy during the collision is 1.4×10^5 J.

Section 5.4: Collisions

Tutorial 1 Practice, page 243

1. Given: m_1 = 80.0 g = 0.0800 kg; $\vec{v}_{i_1} = 7.0 \text{ m/s [W]}$; m_2 = 60.0 g = 0.0600 kg; $\vec{v}_{i_2} = 0$ m/s

Required: $\vec{v}_{f_1}$; $\vec{v}_{f_2}$

Analysis: Since the second ball is initially at rest in the head-on elastic collision, use the simplified equations, $\vec{v}_{f_1} = \left(\dfrac{m_1 - m_2}{m_1 + m_2}\right)\vec{v}_{i_1}$ and $\vec{v}_{f_2} = \left(\dfrac{2m_1}{m_1 + m_2}\right)\vec{v}_{i_1}$.

Solution:

$$\begin{aligned}\vec{v}_{f_1} &= \left(\frac{m_1 - m_2}{m_1 + m_2}\right)\vec{v}_{i_1} \\ &= \left(\frac{0.0800\ \cancel{\text{kg}} - 0.0600\ \cancel{\text{kg}}}{0.0800\ \cancel{\text{kg}} + 0.0600\ \cancel{\text{kg}}}\right)(7.0 \text{ m/s [W]}) \\ \vec{v}_{f_1} &= 1.0 \text{ m/s [W]}\end{aligned}$$

$$\begin{aligned}\vec{v}_{f_2} &= \left(\frac{2m_1}{m_1 + m_2}\right)\vec{v}_{i_1} \\ &= \left(\frac{2(0.0800\ \cancel{\text{kg}})}{0.0800\ \cancel{\text{kg}} + 0.0600\ \cancel{\text{kg}}}\right)(7.0 \text{ m/s [W]}) \\ \vec{v}_{f_1} &= 8.0 \text{ m/s [W]}\end{aligned}$$

Statement: The final velocity of ball 1 is 1.0 m/s [W], and the final velocity of ball 2 is 8.0 m/s [W].

2. Given: m_1 = 1.5 kg; $\vec{v}_{i_1} = 36.5 \text{ cm/s [E]} = 0.365 \text{ m/s [E]}$; m_2 = 5 kg;
$\vec{v}_{i_2} = 42.8 \text{ cm/s [W]} = -0.428 \text{ m/s [E]}$

Required: $\vec{v}_{f_1}$; $\vec{v}_{f_2}$

Analysis: $\vec{v}_{f_1} = \left(\dfrac{m_1 - m_2}{m_1 + m_2}\right)\vec{v}_{i_1} + \left(\dfrac{2m_2}{m_1 + m_2}\right)\vec{v}_{i_2}$; $\vec{v}_{f_2} = \left(\dfrac{m_2 - m_1}{m_1 + m_2}\right)\vec{v}_{i_2} + \left(\dfrac{2m_1}{m_1 + m_2}\right)\vec{v}_{i_1}$

Solution:

$$\begin{aligned}\vec{v}_{f_1} &= \left(\frac{m_1 - m_2}{m_1 + m_2}\right)\vec{v}_{i_1} + \left(\frac{2m_2}{m_1 + m_2}\right)\vec{v}_{i_2} \\ &= \left(\frac{1.5\ \cancel{\text{kg}} - 5\ \cancel{\text{kg}}}{1.5\ \cancel{\text{kg}} + 5\ \cancel{\text{kg}}}\right)(0.365 \text{ m/s [E]}) + \left(\frac{2(5\ \cancel{\text{kg}})}{1.5\ \cancel{\text{kg}} + 5\ \cancel{\text{kg}}}\right)(-0.428 \text{ m/s [E]}) \\ \vec{v}_{f_1} &= -0.9 \text{ m/s}\end{aligned}$$

$$\vec{v}_{f_2} = \left(\frac{m_2 - m_1}{m_1 + m_2}\right)\vec{v}_{i_2} + \left(\frac{2m_1}{m_1 + m_2}\right)\vec{v}_{i_1}$$

$$= \left(\frac{5\ \cancel{kg} - 1.5\ \cancel{kg}}{1.5\ \cancel{kg} + 5\ \cancel{kg}}\right)(-0.428 \text{ m/s [E]}) + \left(\frac{2(1.5\ \cancel{kg})}{1.5\ \cancel{kg} + 5\ \cancel{kg}}\right)(0.365 \text{ m/s [E]})$$

$$\vec{v}_{f\,2} = -0.06 \text{ m/s [E]}$$

Statement: The final velocity of cart 1 is 90 cm/s [W], and the final velocity of cart 2 is 6 cm/s [W].

Tutorial 2 Practice, page 247

1. (a) Given: m_1 = 1.2 kg; $\vec{v}_{i_1} = 3.0$ m/s [right]; m_2 = 1.2 kg; $\vec{v}_{i_2} = 3.0$ m/s [left]; $\vec{v}_{f_2} = 1.5$ m/s [right]

Required: x

Analysis: Use the conservation of momentum equation to determine the velocity of glider 1 during the collision, when glider 2 is moving at 1.5 m/s [right]. Rearrange this equation to express the final velocity of glider 1 in terms of the other given values.

$$m_1 v_{i_1} + m_2 v_{i_2} = m_1 v_{f_1} + m_2 v_{f_2}$$

$$v_{f_1} = \frac{m_1 v_{i_1} + m_2 v_{i_2} - m_2 v_{f_2}}{m_1}$$

Then, apply conservation of mechanical energy to determine the compression of the spring at this particular moment during the collision. Consider right to be positive and left to be negative, and omit the vector notation. Clear fractions first, and then isolate x.

Solution: $v_{f_1} = \dfrac{m_1 v_{i_1} + m_2 v_{i_2} - m_2 v_{f_2}}{m_1}$

$$= \frac{(\cancel{1.2\ kg})(3.0 \text{ m/s}) + (\cancel{1.2\ kg})(-3.0 \text{ m/s}) - (\cancel{1.2\ kg})(1.5 \text{ m/s})}{\cancel{1.2\ kg}}$$

$$v_{f_1} = -1.5 \text{ m/s}$$

$$\cancel{\frac{1}{2}} m_1 v_{i_1}^2 + \cancel{\frac{1}{2}} m_2 v_{i_2}^2 = \cancel{\frac{1}{2}}(m_1 v_{f_1}^2 + m_2 v_{f_2}^2) + \cancel{\frac{1}{2}} kx^2$$

$$m_1 v_{i_1}^2 + m_2 v_{i_2}^2 - (m_1 v_{f_1}^2 + m_2 v_{f_2}^2) = kx^2$$

$$x = \sqrt{\frac{m_1 v_{i_1}^2 + m_2 v_{i_2}^2 - (m_1 v_{f_1}^2 + m_2 v_{f_2}^2)}{k}}$$

$$= \sqrt{\frac{(1.2 \text{ kg})(3.0 \text{ m/s})^2 + (1.2 \text{ kg})(-3.0 \text{ m/s})^2 - (1.2 \text{ kg})(-1.5 \text{ m/s})^2 + (1.2 \text{ kg})(1.5 \text{ m/s})^2}{6.0 \times 10^4 \text{ N/m}}}$$

$$x = 0.016 \text{ m}$$

Statement: The compression of the spring when the second glider is moving at 1.5 m/s [right] is 1.6 cm.

(b) Given: m_1 = 1.2 kg; $\vec{v}_{\mathrm{i}_1} = 3.0$ m/s [right]; m_2 = 1.2 kg; $\vec{v}_{\mathrm{i}_2} = 3.0$ m/s [left]; $\vec{v}_{\mathrm{f}_2} = 1.5$ m/s [right]

Required: x

Analysis: At the beginning of the collision, as the gliders come together and the spring is being compressed, glider 1 and glider 2 are moving at the same speed, in opposite directions. Immediately after the collision, the gliders will reverse direction, but still have the same speed. At the point of maximum compression of the spring, the two gliders will have the same velocity, $\vec{v}_{\mathrm{f}}$. Use the conservation of momentum equation to determine this velocity.

$$m_1 v_{\mathrm{i}_1} + m_2 v_{\mathrm{i}_2} = (m_1 + m_2) v_{\mathrm{f}}$$

$$v_{\mathrm{f}_1} = \frac{m_1 v_{\mathrm{i}_1} + m_2 v_{\mathrm{i}_2}}{m_1 + m_2}$$

Then, apply the conservation of mechanical energy to calculate the maximum compression of the spring.

Solution: $$v_{\mathrm{f}} = \frac{m_1 v_{\mathrm{i}_1} + m_2 v_{\mathrm{i}_2}}{m_1 + m_2}$$

$$= \frac{(1.2\ \cancel{\text{kg}})(3.0\ \text{m/s}) + (1.2\ \cancel{\text{kg}})(-3.0\ \text{m/s})}{1.2\ \cancel{\text{kg}} + 1.2\ \cancel{\text{kg}}}$$

$$v_{\mathrm{f}} = 0\ \text{m/s}$$

Now use the law of conservation of mechanical energy to determine the maximum compression of the spring, using the fact that v_{f} = 0 m/s.

$$\frac{\cancel{1}}{\cancel{2}} m_1 v_{\mathrm{i}_1}^2 + \frac{\cancel{1}}{\cancel{2}} m_2 v_{\mathrm{i}_2}^2 = \frac{\cancel{1}}{\cancel{2}} (m_1 + m_2) v_{\mathrm{f}}^2 + \frac{\cancel{1}}{\cancel{2}} k x^2$$

$$m_1 v_{\mathrm{i}_1}^2 + m_2 v_{\mathrm{i}_2}^2 = k x^2$$

$$x = \sqrt{\frac{m_1 v_{\mathrm{i}_1}^2 + m_2 v_{\mathrm{i}_2}^2}{k}}$$

$$= \sqrt{\frac{(1.2\ \text{kg})(3.0\ \text{m/s})^2 + (1.2\ \text{kg})(-3.0\ \text{m/s})^2}{6.0 \times 10^4\ \text{N/m}}}$$

$$x = 0.019\ \text{m}$$

Statement: The maximum compression of the spring is 1.9 cm.

2. Given: $m_1 = 4.4 \times 10^2$ kg; $\vec{v}_{i_1} = 3.0 \text{ m/s [E]}$; $m_2 = 4.0 \times 10^2$ kg; $\vec{v}_{i_2} = 3.3 \text{ m/s [W]}$;
$\Delta x = 44$ cm = 0.44 m
Required: k
Analysis: At the point of maximum compression of the spring, the two carts will have the same velocity, $\vec{v}_f$. Use the conservation of momentum equation to determine this velocity.

$$m_1 v_{i_1} + m_2 v_{i_2} = (m_1 + m_2) v_f$$

$$v_{f_1} = \frac{m_1 v_{i_1} + m_2 v_{i_2}}{m_1 + m_2}$$

Then apply the conservation of mechanical energy to calculate the spring constant.

Solution:
$$\begin{aligned} v_f &= \frac{m_1 v_{i_1} + m_2 v_{i_2}}{m_1 + m_2} \\ &= \frac{(4.4 \times 10^2 \ \cancel{\text{kg}})(3.0 \text{ m/s}) + (4.0 \times 10^2 \ \cancel{\text{kg}})(-3.3 \text{ m/s})}{4.4 \times 10^2 \ \cancel{\text{kg}} + 4.0 \times 10^2 \ \cancel{\text{kg}}} \\ v_f &= 0 \text{ m/s} \end{aligned}$$

Now use the law of conservation of mechanical energy to determine the spring constant, using the fact that $v_f = 0$.

$$\begin{aligned} \cancel{\frac{1}{2}} m_1 v_{i_1}^2 + \cancel{\frac{1}{2}} m_2 v_{i_2}^2 &= \cancel{\frac{1}{2}} (m_1 + m_2) v_f^2 + \cancel{\frac{1}{2}} k (\Delta x)^2 \\ m_1 v_{i_1}^2 + m_2 v_{i_2}^2 &= k(\Delta x)^2 \\ k &= \frac{m_1 v_{i_1}^2 + m_2 v_{i_2}^2}{(\Delta x)^2} \\ &= \frac{(4.4 \times 10^2 \text{ kg})(3.0 \text{ m/s})^2 + (4.0 \times 10^2 \text{ kg})(-3.3 \text{ m/s})^2}{(0.44 \text{ m})^2} \\ k &= 4.3 \times 10^4 \text{ N/m} \end{aligned}$$

Statement: The spring constant is 4.3×10^4 N/m.

Section 5.4 Questions, page 248

1. Answers may vary. Sample answer: Elastic collision: No, it is not possible for two moving masses to undergo an elastic head-on collision and both be at rest immediately after the collision. In an elastic collision, kinetic energy is conserved. Therefore, if the objects were moving before the collision, at least one of the objects has to be moving after the collision.
Inelastic collision: Yes, it is possible for two moving masses to undergo an elastic head-on collision and both be at rest immediately after the collision. Kinetic energy is not conserved in an inelastic collision. If the masses have equal but opposite momentum before the collision, then the total momentum is zero. After the collision, they could both be at rest and still conserve momentum.

2. Answers may vary. Sample answer: The two curling stones have the same mass. In an elastic collision, the momentum of the first object can transfer completely to the other object if the objects have the same mass. Thus, the speed of the first object is zero, and the speed of the second object is equal to the initial speed of the first object.

3. Given: m_1 = 1.5 g = 0.0015 kg; m_2 = 3.5 g = 0.0035 kg; $\vec{v}_{i_1}$ = 12 m/s [right];

$\vec{v}_{i_2}$ = 7.5 m/s [left]

Required: $\vec{v}_{f_1}$; $\vec{v}_{f_2}$

Analysis: $\vec{v}_{f_1} = \left(\dfrac{m_1 - m_2}{m_1 + m_2}\right)\vec{v}_{i_1} + \left(\dfrac{2m_2}{m_1 + m_2}\right)\vec{v}_{i_2}$; $\vec{v}_{f_2} = \left(\dfrac{m_2 - m_1}{m_1 + m_2}\right)\vec{v}_{i_2} + \left(\dfrac{2m_1}{m_1 + m_2}\right)\vec{v}_{i_1}$

Solution:
$$\begin{aligned}\vec{v}_{f_1} &= \left(\frac{m_1 - m_2}{m_1 + m_2}\right)\vec{v}_{i_1} + \left(\frac{2m_2}{m_1 + m_2}\right)\vec{v}_{i_2}\\ &= \left(\frac{0.0015\ \cancel{\text{kg}} - 0.0035\ \cancel{\text{kg}}}{0.0015\ \cancel{\text{kg}} + 0.0035\ \cancel{\text{kg}}}\right)(12\text{ m/s}) + \left(\frac{2(0.0035\ \cancel{\text{kg}})}{0.0015\ \cancel{\text{kg}} + 0.0035\ \cancel{\text{kg}}}\right)(-7.5\text{ m/s})\\ \vec{v}_{f_1} &= -15\text{ m/s}\end{aligned}$$

$$\begin{aligned}\vec{v}_{f_2} &= \left(\frac{m_2 - m_1}{m_1 + m_2}\right)\vec{v}_{i_2} + \left(\frac{2m_1}{m_1 + m_2}\right)\vec{v}_{i_1}\\ &= \left(\frac{0.0035\ \cancel{\text{kg}} - 0.0015\ \cancel{\text{kg}}}{0.0015\ \cancel{\text{kg}} + 0.0035\ \cancel{\text{kg}}}\right)(-7.5\text{ m/s}) + \left(\frac{2(0.0015\ \cancel{\text{kg}})}{0.0015\ \cancel{\text{kg}} + 0.0035\ \cancel{\text{kg}}}\right)(12\text{ m/s})\\ \vec{v}_{f_2} &= 4.2\text{ m/s}\end{aligned}$$

Statement: The velocity of particle 1 after the collision is 15 m/s [left]. The velocity of particle 2 after the collision is 4.2 m/s [right].

4. Given: m_1 = 2.67 kg; m_2 = 5.83 kg; $\vec{v}_{f_1} = 185$ m/s [right]; $\vec{v}_{f_2} = 172$ m/s [right]

Required: $\vec{v}_{i_1}$; $\vec{v}_{i_2}$

Analysis: Consider right to be positive, and let the direction of the chunks' final motion be positive. Use the final velocity equations, $\vec{v}_{f_1} = \left(\dfrac{m_1 - m_2}{m_1 + m_2}\right)\vec{v}_{i_1} + \left(\dfrac{2m_2}{m_1 + m_2}\right)\vec{v}_{i_2}$ and

$\vec{v}_{f_2} = \left(\dfrac{m_2 - m_1}{m_1 + m_2}\right)\vec{v}_{i_2} + \left(\dfrac{2m_1}{m_1 + m_2}\right)\vec{v}_{i_1}$. Then, solve the resulting linear system.

Solution:

$$\vec{v}_{f_1} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)\vec{v}_{i_1} + \left(\frac{2m_2}{m_1 + m_2}\right)\vec{v}_{i_2}$$

$$185 \text{ m/s} = \left(\frac{2.67 \cancel{\text{kg}} - 5.83 \cancel{\text{kg}}}{2.67 \cancel{\text{kg}} + 5.83 \cancel{\text{kg}}}\right)\vec{v}_{i_1} + \left(\frac{2(5.83 \cancel{\text{kg}})}{2.67 \cancel{\text{kg}} + 5.83 \cancel{\text{kg}}}\right)\vec{v}_{i_2}$$

$$185 \text{ m/s} = \left(\frac{-3.16}{8.50}\right)\vec{v}_{i_1} + \left(\frac{11.66}{8.50}\right)\vec{v}_{i_2}$$

$$1572.5 \text{ m/s} = -3.16\vec{v}_{i_1} + 11.66\vec{v}_{i_2}$$

$$\vec{v}_{f_2} = \left(\frac{m_2 - m_1}{m_1 + m_2}\right)\vec{v}_{i_2} + \left(\frac{2m_1}{m_1 + m_2}\right)\vec{v}_{i_1}$$

$$172 \text{ m/s} = \left(\frac{5.83 \cancel{\text{kg}} - 2.67 \cancel{\text{kg}}}{2.67 \cancel{\text{kg}} + 5.83 \cancel{\text{kg}}}\right)\vec{v}_{i_2} + \left(\frac{2(2.67 \cancel{\text{kg}})}{2.67 \cancel{\text{kg}} + 5.83 \cancel{\text{kg}}}\right)\vec{v}_{i_1}$$

$$172 \text{ m/s} = \left(\frac{5.83 \cancel{\text{kg}} - 2.67 \cancel{\text{kg}}}{8.50}\right)\vec{v}_{i_2} + \left(\frac{2(2.67 \cancel{\text{kg}})}{8.50}\right)\vec{v}_{i_1}$$

$$1462 \text{ m/s} = 3.16\vec{v}_{i_2} + 5.34\vec{v}_{i_1}$$

Solve the linear system.

$$-3.16\vec{v}_{i_1} + 11.66\vec{v}_{i_2} = 1572.5 \text{ m/s} \quad \text{Equation 1}$$

$$5.34\vec{v}_{i_1} + 3.16\vec{v}_{i_2} = 1462 \text{ m/s} \quad \text{Equation 2}$$

Multiply Equation 1 by $\frac{534}{316}$ and add.

$$-5.34\vec{v}_{i_1} + 19.704\vec{v}_{i_2} = 2657.3 \text{ m/s} \quad \text{Equation 1}$$

$$5.34\vec{v}_{i_1} + 3.16\vec{v}_{i_2} = 1462 \text{ m/s} \quad \text{Equation 2}$$

$$22.864\vec{v}_{i_2} = 4119.3$$

$$= 180.2 \text{ m/s [right] (one extra digit carried)}$$

$$\vec{v}_{i_2} = 1.80 \times 10^2 \text{ m/s [right]}$$

Substitute $\vec{v}_{i_2} = 180.2$ into Equation 2.

$$5.34\vec{v}_{i_1} + 3.16\vec{v}_{i_2} = 1462 \text{ m/s}$$

$$5.34\vec{v}_{i_1} + (3.16)(180.2 \text{ m/s}) = 1462 \text{ m/s}$$

$$5.34\vec{v}_{i_1} = 892.568 \text{ m/s}$$

$$\vec{v}_{i_1} = 167 \text{ m/s [right]}$$

Statement: The initial velocity of the more massive chunk is 1.80×10^2 m/s [right], and the initial velocity of the less massive chunk is 167 m/s [right]. (Since all initial and final velocities are in the same direction, the more massive chunk overtakes the less massive one and imparts some of its momentum to it.)

5. (a) Given: $m_1 = 0.84$ kg; $\vec{v}_{i_1} = 4.2$ m/s [right]; $m_2 = 0.48$ kg; $\vec{v}_{i_2} = 2.4$ m/s [left];

$k = 8.0 \times 10^3$ N/m

Required: $\vec{v}_{f_1}$; $\vec{v}_{f_2}$

Analysis: $\vec{v}_{f_1} = \left(\dfrac{m_1 - m_2}{m_1 + m_2}\right)\vec{v}_{i_1} + \left(\dfrac{2m_2}{m_1 + m_2}\right)\vec{v}_{i_2}$; $\vec{v}_{f_2} = \left(\dfrac{m_2 - m_1}{m_1 + m_2}\right)\vec{v}_{i_2} + \left(\dfrac{2m_1}{m_1 + m_2}\right)\vec{v}_{i_1}$

Solution: $\vec{v}_{f_1} = \left(\dfrac{m_1 - m_2}{m_1 + m_2}\right)\vec{v}_{i_1} + \left(\dfrac{2m_2}{m_1 + m_2}\right)\vec{v}_{i_2}$

$$= \left(\frac{0.84\ \cancel{\text{kg}} - 0.48\ \cancel{\text{kg}}}{0.84\ \cancel{\text{kg}} + 0.48\ \cancel{\text{kg}}}\right)(4.2\text{ m/s}) + \left(\frac{2(0.48\ \cancel{\text{kg}})}{0.84\ \cancel{\text{kg}} + 0.48\ \cancel{\text{kg}}}\right)(-2.4\text{ m/s})$$

$$\vec{v}_{f_1} = -0.60\text{ m/s}$$

$$\vec{v}_{f_2} = \left(\frac{m_2 - m_1}{m_1 + m_2}\right)\vec{v}_{i_2} + \left(\frac{2m_1}{m_1 + m_2}\right)\vec{v}_{i_1}$$

$$= \left(\frac{0.48\ \cancel{\text{kg}} - 0.84\ \cancel{\text{kg}}}{0.84\ \cancel{\text{kg}} + 0.48\ \cancel{\text{kg}}}\right)(-2.4\text{ m/s}) + \left(\frac{2(0.84\ \cancel{\text{kg}})}{0.84\ \cancel{\text{kg}} + 0.48\ \cancel{\text{kg}}}\right)(4.2\text{ m/s})$$

$$\vec{v}_{f_2} = 6.0\text{ m/s}$$

Statement: The velocity of cart 1 after the collision is 0.60 m/s [left]. The velocity of cart 2 after the collision is 6.0 m/s [right].

(b) Given: $m_1 = 0.84$ kg; $\vec{v}_{i_1} = 4.2$ m/s [right]; $m_2 = 0.48$ kg; $\vec{v}_{i_2} = 2.4$ m/s [left];

$\vec{v}_{f_1} = 3.0$ m/s [right]; $k = 8.0 \times 10^3$ N/m

Analysis: Use the conservation of momentum to determine the velocity of cart 2 during the collision, when cart 2 is moving 3.0 m/s [right]. Rearrange the conservation of momentum equation to express the final velocity of cart 2 in terms of the other given values.

$$m_1 v_{i_1} + m_2 v_{i_2} = m_1 v_{f_1} + m_2 v_{f_2}$$

$$v_{f_2} = \frac{m_1 v_{i_1} + m_2 v_{i_2} - m_1 v_{f_1}}{m_2}$$

Then, use conservation of mechanical energy to determine the compression of the spring at this particular moment during the collision. Consider right to be positive, and omit the vector notation.

Solution: $v_{f_2} = \dfrac{m_1 v_{i_1} + m_2 v_{i_2} - m_1 v_{f_1}}{m_2}$

$$= \frac{(0.84\text{ kg})(4.2\text{ m/s}) + (0.48\text{ kg})(-2.4\text{ m/s}) - (0.84\text{ kg})(3.0\text{ m/s})}{0.48\text{ kg}}$$

$$v_{f_2} = -0.30\text{ m/s}$$

Now use the law of conservation of mechanical energy to determine the compression of the spring. Clear fractions first, and then isolate *x*.

$$\cancel{\frac{1}{2}} m_1 v_{i_1}^2 + \cancel{\frac{1}{2}} m_2 v_{i_2}^2 = \cancel{\frac{1}{2}} (m_1 v_{f_1}^2 + m_2 v_{f_2}^2) + \cancel{\frac{1}{2}} kx^2$$

$$m_1 v_{i_1}^2 + m_2 v_{i_2}^2 - (m_1 v_{f_1}^2 + m_2 v_{f_2}^2) = kx^2$$

$$x = \sqrt{\frac{m_1 v_{i_1}^2 + m_2 v_{i_2}^2 - (m_1 v_{f_1}^2 + m_2 v_{f_2}^2)}{k}}$$

$$= \sqrt{\frac{(0.84\text{ kg})(4.2\text{ m/s})^2 + (0.48\text{ kg})(-2.4\text{ m/s})^2 - (0.84\text{ kg})(3.0\text{ m/s})^2 + (0.48\text{ kg})(-0.30\text{ m/s})^2}{8.0 \times 10^3\text{ N/m}}}$$

$$x = 0.035\text{ m}$$

Statement: The compression of the spring when cart 1 is moving at 3.0 m/s [right] is 3.5×10^{-2} m.

(c) Given: $m_1 = 0.84$ kg; $\vec{v}_{i_1} = 4.2$ m/s [right]; $m_2 = 0.48$ kg; $\vec{v}_{i_2} = 2.4$ m/s [left]; $k = 8.0 \times 10^3$ N/m

Required: x

Analysis: At the point of maximum compression of the spring, the two carts will have the same velocity, $\vec{v}_f$. Use the conservation of momentum equation to determine this velocity.

$$m_1 v_{i_1} + m_2 v_{i_2} = (m_1 + m_2) v_f$$

$$v_{f_1} = \frac{m_1 v_{i_1} + m_2 v_{i_2}}{m_1 + m_2}$$

Then use the law of conservation of mechanical energy to determine the maximum compression of the spring.

Solution: $v_f = \dfrac{m_1 v_{i_1} + m_2 v_{i_2}}{m_1 + m_2}$

$$= \frac{(0.84\text{ kg})(4.2\text{ m/s}) + (0.48\text{ kg})(-2.4\text{ m/s})}{0.84\text{ kg} + 0.48\text{ kg}}$$

$$v_f = 1.8\text{ m/s}$$

$$\cancel{\frac{1}{2}}m_1v_{i_1}^2 + \cancel{\frac{1}{2}}m_2v_{i_2}^2 = \cancel{\frac{1}{2}}(m_1 + m_2)v_f^2 + \cancel{\frac{1}{2}}kx^2$$

$$m_1v_{i_1}^2 + m_2v_{i_2}^2 = (m_1 + m_2)v_f^2 + kx^2$$

$$kx^2 = m_1v_{i_1}^2 + m_2v_{i_2}^2 - (m_1 + m_2)v_f^2$$

$$x^2 = \frac{m_1v_{i_1}^2 + m_2v_{i_2}^2 - (m_1 + m_2)v_f^2}{k}$$

$$x = \sqrt{\frac{m_1v_{i_1}^2 + m_2v_{i_2}^2 - (m_1 + m_2)v_f^2}{k}}$$

$$= \sqrt{\frac{(0.84\text{ kg})(4.2\text{ m/s})^2 + (0.48\text{ kg})(-2.4\text{ m/s})^2 - (0.84\text{ kg} + 0.48\text{ kg})(1.8\text{ m/s})^2}{8.0 \times 10^3\text{ N/m}}}$$

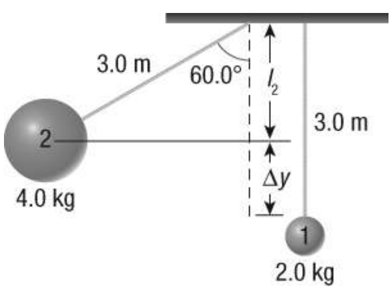
$$x = 0.041\text{ m}$$

Statement: The maximum compression of the spring is 4.1×10^{-2} m.

6. (a) Given: $m_1 = 2.0$ kg; $m_2 = 4.0$ kg; $\theta = 60.0°$; length of two strings = 3.0 m; $\vec{v}_{i_1} = 0$ m/s

Required: $\vec{v}_{f_1}$; $\vec{v}_{f_2}$

Analysis: Draw a diagram of the situation. Let the $y = 0$ reference point be the vertical position of ball 1 before the collision. Let Δy be the vertical height of ball 2 above ball 1. Let l_2 be the vertical distance of ball 2 from the post.



Use conservation of energy to find the velocity of ball 2 just before the collision, at $y = 0$. Then, use the equations for perfectly elastic collisions to find the velocities of the balls just after the collision.

$E_k = E_g$; $\vec{v}_{f_1} = \left(\dfrac{m_1 - m_2}{m_1 + m_2}\right)\vec{v}_{i_1} + \left(\dfrac{2m_2}{m_1 + m_2}\right)\vec{v}_{i_2}$; $\vec{v}_{f_2} = \left(\dfrac{m_2 - m_1}{m_1 + m_2}\right)\vec{v}_{i_2} + \left(\dfrac{2m_1}{m_1 + m_2}\right)\vec{v}_{i_1}$

Solution: From the diagram, l_2 = (3.0 m)cos 60° = 1.5 m. Thus, Δy = 1.5 m.
Use conservation of energy.

$$E_k = E_g$$

$$\frac{1}{2}\cancel{m_2}v_{i_2}^2 = \cancel{m_2}g\Delta y$$

$$v_{i_2} = \sqrt{2g\Delta y}$$

$$= \sqrt{2(9.8\text{ m/s}^2)(1.5\text{ m})}$$

$$v_{i_2} = 5.42\text{ m/s (one extra digit carried)}$$

Now, use the equations for perfectly elastic collisions, using the fact that $\vec{v}_{i\,1} = 0\text{ m/s}$.

$$\vec{v}_{f_1} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)\vec{v}_{i_1} + \left(\frac{2m_2}{m_1 + m_2}\right)\vec{v}_{i_2}$$

$$= \left(\frac{2m_2}{m_1 + m_2}\right)\vec{v}_{i_2}$$

$$= \left(\frac{2(4.0\ \cancel{\text{kg}})}{2.0\ \cancel{\text{kg}} + 4.0\ \cancel{\text{kg}}}\right)(5.42\text{ m/s})$$

$$\vec{v}_{f_1} = 7.23\text{ m/s (one extra digit carried)}$$

$$\vec{v}_{f_2} = \left(\frac{m_2 - m_1}{m_1 + m_2}\right)\vec{v}_{i_2}$$

$$= \left(\frac{m_2 - m_1}{m_1 + m_2}\right)\vec{v}_{i_2}$$

$$= \left(\frac{4.0\ \cancel{\text{kg}} - 2.0\ \cancel{\text{kg}}}{2.0\ \cancel{\text{kg}} + 4.0\ \cancel{\text{kg}}}\right)(5.42\text{ m/s})$$

$$\vec{v}_{f_2} = 1.81\text{ m/s (one extra digit carried)}$$

Statement: The speed of ball 1 is 7.2 m/s, and the speed of ball 2 is 1.8 m/s.
(b) Given: m_1 = 2.0 kg; m_2 = 4.0 kg; $\vec{v}_{f_1} = 7.23\text{ m/s}$; $\vec{v}_{f_2} = 1.81\text{ m/s}$

Required: $h_{max\,1}$; $h_{max\,2}$
Analysis: Use conservation of energy, $E_g = E_k$.

Solution: $E_{g_1} = E_{k_1}$

$$\cancel{m_1} g h_{max\ 1} = \frac{1}{2} \cancel{m_1} v_{f_1}^2$$

$$h_{max\ 1} = \frac{v_{f_1}^2}{2g}$$

$$= \frac{(7.23\ \text{m}/\cancel{\text{s}})^{\cancel{2}}}{2(9.8\ \cancel{\text{m}}/\cancel{\text{s}^2})}$$

$$h_{max\ 1} = 2.7\ \text{m}$$

$$E_{g_2} = E_{k_2}$$

$$\cancel{m_2} g h_{max\ 2} = \frac{1}{2} \cancel{m_2} v_{f_2}^2$$

$$h_{max\ 2} = \frac{v_{f_2}^2}{2g}$$

$$= \frac{(1.81\ \text{m}/\cancel{\text{s}})^{\cancel{2}}}{2(9.8\ \cancel{\text{m}}/\cancel{\text{s}^2})}$$

$$h_{max\ 2} = 0.17\ \text{m}$$

Statement: After the first collision, the maximum height of ball 1 is 2.7 m, and the maximum height of ball 2 is 0.17 m.

Section 5.5: Collisions in Two Dimensions: Glancing Collisions

Mini Investigation: Glancing Collisions, page 249

Answers may vary. Sample answers:

A. When one puck collides with a second puck at an angle, the speed of the second puck will be less than the initial speed of the first puck, and as the angle of the collision increases, the speed of the second puck will decrease.

B. There is less friction with pucks on an air table than with billiard balls or marbles, which will affect the results. The results for pucks on an air table may be closer to those for an ideal, frictionless system.

Tutorial 1 Practice, page 252

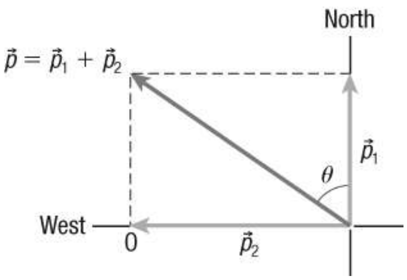
1. Given: Inelastic collision; $m_1 = 1.4 \times 10^4$ kg; $m_2 = 1.5 \times 10^4$ kg; $\vec{v}_{i_1} = 45$ km/h [N];

$\vec{v}_{i_2} = 53$ km/h [W]

Required: $\vec{v}_f$

Analysis: According to the law of conservation of momentum, $\vec{p}_{T_i} = \vec{p}_{T_f}$. Since the initial velocities are at right angles to each other, as shown in the figure below, you can calculate the total velocity and momentum using the Pythagorean theorem and trigonometry:

$p^2 = p_1^2 + p_2^2$, and $\tan\theta = \dfrac{p_2}{p_1}$



First, convert the velocities to metres per second.

$$\vec{v}_1 = 45\ \frac{\cancel{\text{km}}}{\cancel{\text{h}}} \times \frac{1000\ \text{m}}{1\ \cancel{\text{km}}} \times \frac{1\ \cancel{\text{h}}}{3600\ \text{s}}$$

$\vec{v}_1 = 12.5$ m/s (one extra digit carried)

$$\vec{v}_2 = 53\ \frac{\cancel{\text{km}}}{\cancel{\text{h}}} \times \frac{1000\ \text{m}}{1\ \cancel{\text{km}}} \times \frac{1\ \cancel{\text{h}}}{3600\ \text{s}}$$

$\vec{v}_2 = 14.72$ m/s (two extra digits carried)

Solution: Engine 1's momentum is

$$\begin{aligned}\vec{p}_1 &= m_1\vec{v}_1 \\ &= (1.4 \times 10^4\ \text{kg})(12.5\ \text{m/s})\ [\text{N}]\end{aligned}$$

$\vec{p}_1 = 1.75 \times 10^5\ \text{kg}\cdot\text{m/s}$ [N] (one extra digit carried)

Engine 2's momentum is

$$\vec{p}_2 = m_2\vec{v}_2$$
$$= (1.5\times10^4\ \text{kg})(14.7\ \text{m/s})\ [\text{W}]$$
$$\vec{p}_2 = 2.208\times10^5\ \text{kg}\cdot\text{m/s [W] (two extra digits carried)}$$

Calculate the magnitude of the total momentum by applying the Pythagorean theorem:

$$p^2 = p_1^2 + p_2^2$$
$$p = \sqrt{p_1^2 + p_2^2}$$
$$= \sqrt{(1.75\times10^5\ \text{kg}\cdot\text{m/s})^2 + (2.208\times10^5\ \text{kg}\cdot\text{m/s})^2}$$
$$p = 2.817\times10^5\ \text{kg}\cdot\text{m/s (two extra digits carried)}$$

Determine the direction by applying the tangent ratio:

$$\tan\theta = \frac{p_2}{p_1}$$
$$\theta = \tan^{-1}\left(\frac{p_2}{p_1}\right)$$
$$= \tan^{-1}\left(\frac{2.208\times10^5\ \cancel{\text{kg}\cdot\text{m/s}}}{1.75\times10^5\ \cancel{\text{kg}\cdot\text{m/s}}}\right)$$
$$\theta = 52^\circ$$

The direction of the two engines is [N 52° W].

By conservation of momentum, the final total momentum of the engines must equal the initial momentum. Since the collision is perfectly inelastic, both engines have the same final velocity:

$$\vec{p}_\text{f} = m_1\vec{v}_\text{f} + m_2\vec{v}_\text{f}$$
$$\vec{p}_\text{f} = (m_1 + m_2)\vec{v}_\text{f}$$
$$\vec{v}_\text{f} = \frac{\vec{p}_\text{f}}{m_1 + m_2}$$
$$= \frac{2.817\times10^5\ \cancel{\text{kg}}\cdot\text{m/s [N 52° W]}}{(1.4\times10^4\ \cancel{\text{kg}} + 1.5\times10^4\ \cancel{\text{kg}})}$$
$$\vec{v}_\text{f} = 9.7\ \text{m/s [N 52° W]}$$

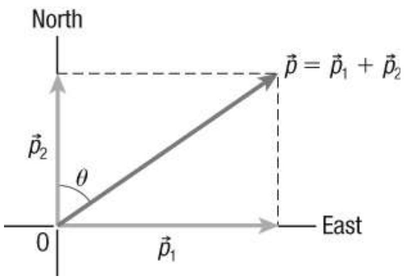
Statement: After the collision, the two engines are travelling together at a velocity of 9.7 m/s [N 52° W].

2. Given: $m_1 = 2\times10^{30}$ kg; $\vec{v}_1 = 2\times10^4$ m/s [E]; $m_2 = 5\times10^{30}$ kg; $\vec{v}_2 = 3\times10^4$ m/s [at right angle to star 1]

Required: $\vec{v}_\text{f}$

Analysis: Assume that the direction of star 2 is north. According to the law of conservation of momentum, $\vec{p}_{T_i} = \vec{p}_{T_f}$. Since the initial velocities are at right angles to each other, as shown in the figure below, you can calculate the total velocity and momentum using the Pythagorean theorem and trigonometry:

$p^2 = p_1^2 + p_2^2$, and $\tan\theta = \dfrac{p_2}{p_1}$



Solution: Star 1's momentum is

$$\begin{aligned}
\vec{p}_1 &= m_1\vec{v}_1 \\
&= (2\times10^{30}\ \text{kg})(2\times10^4\ \text{m/s})\ [\text{E}] \\
\vec{p}_1 &= 4\times10^{34}\ \text{kg}\cdot\text{m/s}\ [\text{E}]
\end{aligned}$$

Star 2's momentum is

$$\begin{aligned}
\vec{p}_2 &= m_2\vec{v}_2 \\
&= (5\times10^{30}\ \text{kg})(3\times10^4\ \text{m/s})\ [\text{N}] \\
\vec{p}_2 &= 1.5\times10^{35}\ \text{kg}\cdot\text{m/s}\ [\text{N}]\ \text{(one extra digit carried)}
\end{aligned}$$

Calculate the magnitude of the total momentum by applying the Pythagorean theorem:

$$\begin{aligned}
p^2 &= p_1^2 + p_2^2 \\
p &= \sqrt{p_1^2 + p_2^2} \\
&= \sqrt{(4\times10^{34}\ \text{kg}\cdot\text{m/s})^2 + (1.5\times10^{35}\ \text{kg}\cdot\text{m/s})^2} \\
p &= 1.6\times10^{35}\ \text{kg}\cdot\text{m/s}\ \text{(one extra digit carried)}
\end{aligned}$$

Determine the direction by applying the tangent ratio:

$$\begin{aligned}
\tan\theta &= \frac{p_1}{p_2} \\
\theta &= \tan^{-1}\left(\frac{p_1}{p_2}\right) \\
&= \tan^{-1}\left(\frac{4\times10^{34}\ \cancel{\text{kg}\cdot\text{m/s}}}{1.5\times10^{35}\ \cancel{\text{kg}\cdot\text{m/s}}}\right) \\
\theta &= 14.9^\circ\ \text{(two extra digits carried)}
\end{aligned}$$

The two stars' direction is [N 10° E].

By conservation of momentum, the final total momentum of the stars must equal the initial momentum. Since the collision is perfectly inelastic, both stars have the same final velocity:

$$\vec{p}_{\text{f}} = m_1\vec{v}_{\text{f}} + m_2\vec{v}_{\text{f}}$$

$$\vec{p}_{\text{f}} = (m_1 + m_2)\vec{v}_{\text{f}}$$

$$\begin{aligned}\vec{v}_{\text{f}} &= \frac{\vec{p}_{\text{f}}}{m_1 + m_2} \\ &= \frac{1.6\times 10^{35}\ \text{kg}\cdot\text{m/s [N 10° E]}}{(2\times 10^{30}\ \text{kg} + 5\times 10^{30}\ \text{kg})}\end{aligned}$$

$$\vec{v}_{\text{f}} = 2\times 10^{4}\ \text{m/s [N 10° E]}$$

Statement: After the collision, the two stars are travelling together at a velocity of 2×10^4 m/s, at 10º to the initial path of the second star.

Section 5.5 Questions, page 253

1. Given: $m_1 = m_2 = m$; $\vec{v}_{\text{i}_1} = 10.0$ m/s [right]; $\vec{v}_{\text{i}_2} = 0$ m/s; $\vec{v}_{\text{f}_1} = 4.7$ m/s; $\theta = 60.0°$

Required: $\vec{v}_{\text{f}_2}$

Analysis: Choose a coordinate system to identify directions: let positive x be to the right and negative x be to the left. Let positive y be up and negative y be down.
Apply conservation of momentum independently in the x-direction and the y-direction to determine the magnitude and direction of the final velocity of ball 2.
Solution: In the y-direction, the total momentum before and after the collision is zero:

$$p_{\text{T}_{\text{i}y}} = p_{\text{T}_{\text{f}y}} = 0$$

Therefore, after the collision:

$$mv_{\text{f}_{1y}} + mv_{\text{f}_{2y}} = 0$$

Divide both sides by m and substitute the vertical component of each velocity vector. The vertical component of the velocity vector for ball 1 is $v_{\text{f}_1}\sin\theta$. The vertical component of the velocity vector for ball 2 is $v_{\text{f}_2}\sin\phi$

$$\begin{aligned} v_{\text{f}_1}\sin\theta + v_{\text{f}_2}\sin\phi &= 0 \\ (-4.7\ \text{m/s})(\sin 60.0°) + v_{\text{f}_2}\sin\phi &= 0 \\ v_{\text{f}_2}\sin\phi &= (4.7\ \text{m/s})(\sin 60.0°)\end{aligned}$$

In the x-direction, the total momentum before the collision is equal to the total momentum after the collision. Only ball 1 has momentum in the x-direction before the collision, but both balls have momentum in the x-direction after the collision.

$$\cancel{m}v_{\text{i}_{1x}} = \cancel{m}v_{\text{f}_{1x}} + \cancel{m}v_{\text{f}_{2x}}$$

$$v_{\text{i}_{1x}} = v_{\text{f}_{1x}} + v_{\text{f}_{2x}}$$

The horizontal component of the velocity vector of ball 1 after the collision is $v_{f_1} \cos 60.0°$. The horizontal component of the velocity vector of ball 2 after the collision is $v_{f_2} \cos\phi$. The horizontal component of the velocity vector of ball 1 before the collision is 10.0 m/s.

$$10.0 \text{ m/s} = v_{f_1} \cos 60.0° + v_{f_2} \cos\phi$$

$$10.0 \text{ m/s} = (4.7 \text{ m/s})\left(\frac{1}{2}\right) + v_{f_2} \cos\phi$$

$$v_{f_2} \cos\phi = 7.65 \text{ m/s}$$

$$v_{f_2} = \frac{7.65 \text{ m/s}}{\cos\phi}$$

Substitute this result into the previous result for v_{f_2}:

$$v_{f_2} \sin\phi = (4.7 \text{ m/s})(\sin 60.0°)$$

$$\frac{(7.65 \text{ m/s})\sin\phi}{\cos\phi} = (4.7 \text{ m/s})(\sin 60.0°)$$

$$\frac{\sin\phi}{\cos\phi} = \frac{(4.7\ \cancel{\text{m/s}})(\sin 60.0°)}{(7.65\ \cancel{\text{m/s}})}$$

$$\tan\phi = 0.532$$

$$\phi = 28°$$

Now, substitute to determine v_{f_2}:

$$v_{f_2} = \frac{7.65 \text{ m/s}}{\cos\phi}$$

$$= \frac{7.65 \text{ m/s}}{\cos 28°}$$

$$v_{f_2} = 8.7 \text{ m/s}$$

Statement: The velocity of ball 2 after the collision is 8.7 m/s, 28° above the horizontal (above the initial path of ball 1).

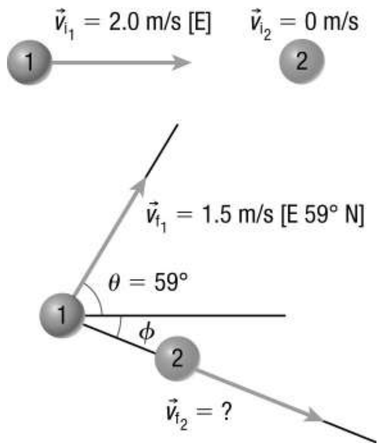
2. Given: m_1 = 0.16 kg; m_2 = 0.17 kg; $\vec{v}_{i_1} = 2.0$ m/s [E]; $\vec{v}_{i_2} = 0$ m/s;

$\vec{v}_{f_1} = 1.5$ m/s [N 31° E] = 1.5 m/s [E 59° N]

Required: $\vec{v}_{f_2}$

Analysis: Choose a coordinate system to identify directions: let positive *x* be to the right and negative *x* be to the left. Let positive *y* be up and negative *y* be down.

Apply conservation of momentum independently in the *x*-direction and the *y*-direction to determine the magnitude and direction of the final velocity of puck 2.



Solution: In the *y*-direction, the total momentum before and after the collision is zero:

$$p_{T_{iy}} = p_{T_{fy}} = 0$$

Therefore, after the collision:

$$mv_{f_{1y}} + mv_{f_{2y}} = 0$$

Divide both sides by *m* and substitute the vertical component of each velocity vector. The vertical component of the velocity vector for puck 1 is $v_{f_1} \sin\theta$. The vertical component of the velocity vector for puck 2 is $v_{f_2} \sin\phi$.

$$v_{f_1} \sin\theta + v_{f_2} \sin\phi = 0$$

$$(1.5 \text{ m/s})(\sin 59°) + v_{f_2} \sin\phi = 0$$

$$v_{f_2} \sin\phi = (-1.5 \text{ m/s})(\sin 59°)$$

In the *x*-direction, the total momentum before the collision is equal to the total momentum after the collision. Only puck 1 has momentum in the *x*-direction before the collision, but both pucks have momentum in the *x*-direction after the collision.

$$\cancel{m}v_{i_{1x}} = \cancel{m}v_{f_{1x}} + \cancel{m}v_{f_{2x}}$$

$$v_{i_{1x}} = v_{f_{1x}} + v_{f_{2x}}$$

The horizontal component of the velocity vector of puck 1 after the collision is $v_{f_1} \cos 59°$. The horizontal component of the velocity vector of puck 2 after the collision is $v_{f_2} \cos\phi$. The horizontal component of the velocity vector of puck 1 before the collision is 2.0 m/s.

$$2.0 \text{ m/s} = v_{f_1} \cos 59° + v_{f_2} \cos\phi$$

$$2.0 \text{ m/s} = (1.5 \text{ m/s})(\cos 59°) + v_{f_2} \cos\phi$$

$$v_{f_2} \cos\phi = 1.23 \text{ m/s}$$

$$v_{f_2} = \frac{1.23 \text{ m/s}}{\cos\phi} \text{ (one extra digit carried)}$$

Substitute this result into the previous result for v_{f_2}:

$$v_{f2}\sin\phi = (-1.5 \text{ m/s})(\sin 59°)$$

$$\frac{(1.23 \text{ m/s})\sin\phi}{\cos\phi} = (-1.5 \text{ m/s})(\sin 59°)$$

$$\frac{\sin\phi}{\cos\phi} = \frac{(-1.5 \cancel{\text{m/s}})(\sin 59°)}{(1.23 \cancel{\text{m/s}})}$$

$$\tan\phi = -1.05$$

$$\phi = -46.4° \text{ (one extra digit carried)}$$

The direction of the final velocity of puck 2 is 46° south of east, or [S 44° E].

Now, substitute to determine v_{f_2}:

$$v_{f_2} = \frac{1.23 \text{ m/s}}{\cos\phi}$$

$$= \frac{1.23 \text{ m/s}}{\cos 46.4°}$$

$$v_{f_2} = 1.8 \text{ m/s}$$

Statement: The velocity of puck 2 after the collision is 1.8 m/s [S 44° E].

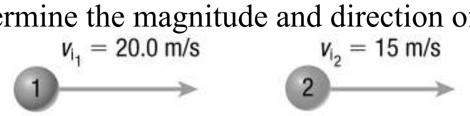
3. (a) Given: $m_1 = m_2 = m$; $\vec{v}_{i_1} = 20.0 \text{ m/s [S 45° E]}$; $\vec{v}_{i_2} = 15 \text{ m/s [S 45° W]}$;

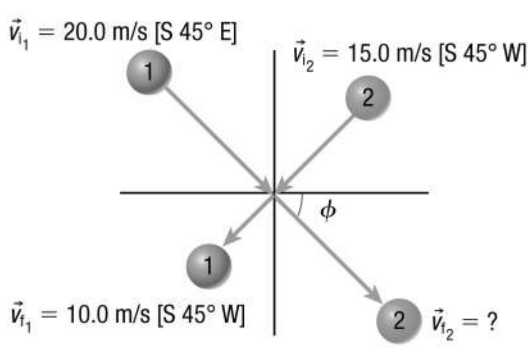
$\vec{v}_{f_1} = 10.0 \text{ m/s [S 45° W]}$

Required: $\vec{v}_{f_2}$

Analysis: Choose a coordinate system to identify directions: let positive *x* be to the right and negative *x* be to the left. Let positive *y* be up and negative *y* be down.

Apply conservation of momentum independently in the *x*-direction and the *y*-direction to determine the magnitude and direction of the final velocity of puck 2.





Solution: In the *y*-direction, the total momentum before and after the collision is equal:

$$p_{T_{iy}} = p_{T_{fy}}$$

Therefore, after the collision:

$$\cancel{m}v_{i_{1y}} + \cancel{m}v_{i_{2y}} = \cancel{m}v_{f_{1y}} + \cancel{m}v_{f_{2y}}$$

$$v_{i_{1y}} + v_{i_{2y}} = v_{f_{1y}} + v_{f_{2y}}$$

Substitute the vertical component of each velocity vector. The vertical component of the initial velocity vector for puck 1 is $-v_{i_1}$ sin 45°. The vertical component of the initial velocity vector for puck 2 is $-v_{i_2}$ sin 45°. The vertical component of the final velocity vector for puck 1 is $-v_{f1}\sin 45^{\circ}$. The vertical component of the final velocity vector for puck 2 is $-v_{f_2}\sin\phi$.

$$-v_{i_1}\sin 45^\circ + (-v_{i_2}\sin 45^\circ) = -v_{f_1}\sin 45^\circ - v_{f_2}\sin\phi$$

$$v_{f_2}\sin\phi = \sin 45^\circ(v_{i_1} + v_{i_2} - v_{f_1})$$

$$v_{f_2} = \frac{\sin 45^\circ(v_{i_1} + v_{i_2} - v_{f_1})}{\sin\phi}$$

$$= \frac{\sin 45^\circ(20.0\text{ m/s} + 15\text{ m/s} - 10.0\text{ m/s})}{\sin\phi}$$

$$v_{f_2} = \frac{(25\text{ m/s})\sin 45^\circ}{\sin\phi}$$

In the *x*-direction, the total momentum before the collision is equal to the total momentum after the collision.

$$\cancel{m}v_{i_{1x}} + \cancel{m}v_{i_{2x}} = \cancel{m}v_{f_{1x}} + \cancel{m}v_{f_{2x}}$$

$$v_{i_{1x}} + v_{i_{2x}} = v_{f_{1x}} + v_{f_{2x}}$$

The horizontal component of the initial velocity vector of puck 1 is v_{i_1} cos 45°. The horizontal component of the initial velocity vector of puck 2 is $-v_{i_2}$ cos 45°. The horizontal component of the final velocity vector of puck 1 is $-v_{f_1}$ cos 45°. The horizontal component of the final velocity vector of puck 2 is v_{f_2} cos ϕ.

$$v_{i_1}\cos 45^\circ + (-v_{i_2}\cos 45^\circ) = -v_{f_1}\cos 45^\circ + v_{f_2}\cos\phi$$

$$v_{f_2} = \frac{(\cos 45^\circ)(v_{i_1} - v_{i_2} + v_{f_1})}{\cos\phi}$$

$$= \frac{(\cos 45^\circ)(20.0\text{ m/s} - 15\text{ m/s} + 10.0\text{ m/s})}{\cos\phi}$$

$$v_{f_2} = \frac{(15\text{ m/s})(\cos 45^\circ)}{\cos\phi}$$

Substitute this result into the previous result for v_{f_2}:

$$\frac{(\cos 45°)(15\text{ m/s})}{\cos\phi} = \frac{(25\text{ m/s})(\sin 45°)}{\sin\phi}$$

$$\frac{\frac{\cancel{\sqrt{2}}}{\cancel{2}}(15\text{ m/s})}{\cos\phi} = \frac{(25\text{ m/s})\frac{\cancel{\sqrt{2}}}{\cancel{2}}}{\sin\phi}$$

$$\frac{\sin\phi}{\cos\phi} = \frac{25\ \cancel{\text{m/s}}}{15\ \cancel{\text{m/s}}}$$

$$\tan\phi = \frac{5}{3}$$

$$\phi = 59°$$

The direction of the final velocity of puck 2 is 59° south of east, or [S 31° E].

Now, substitute to determine v_{f_2}:

$$v_{f_2} = \frac{(15\text{ m/s})(\cos 45°)}{\cos 59°}$$

$$v_{f_2} = 21\text{ m/s}$$

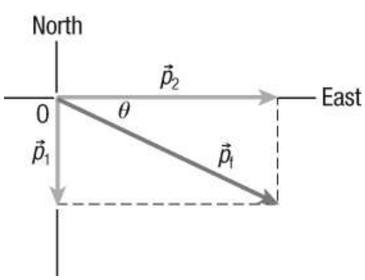
Statement: The velocity of puck 2 after the collision is 21 m/s [S 31° E].

(b) The collision is non-perfectly inelastic, because kinetic energy is not conserved, but the pucks do not move together after the collision.

4. Given: $m_1 = 1.4 \times 10^3$ kg; $m_2 = 2.6 \times 10^4$ kg; $\vec{v}_{i_1} = 32\text{ km/h [S]}$; $\vec{v}_{i_2} = 48\text{ km/h [E]}$

Required: $\vec{v}_f$

Analysis: The vehicles have the same velocity after the collision. Momentum is conserved in the inelastic collision. Use the Pythagorean theorem and trigonometry to determine $\vec{p}_f$. Then, use $\vec{p}_f = m\vec{v}_f$ to determine $\vec{v}_f$.



First, convert the speeds to metres per second.

$$v_1 = 32\ \frac{\cancel{\text{km}}}{\cancel{\text{h}}} \times \frac{1000\text{ m}}{1\ \cancel{\text{km}}} \times \frac{1\ \cancel{\text{h}}}{3600\text{ s}}$$

$v_1 = 8.889$ m/s (two extra digits carried)

$$v_2 = 48\ \frac{\cancel{\text{km}}}{\cancel{\text{h}}} \times \frac{1000\ \text{m}}{1\ \cancel{\text{km}}} \times \frac{1\ \cancel{\text{h}}}{3600\ \text{s}}$$

$v_2 = 13.3$ m/s (one extra digit carried)

Solution: $$\begin{aligned} p_\text{f} &= \sqrt{p_1^2 + p_2^2} \\ &= \sqrt{(m_1 v_1)^2 + (m_2 v_2)^2} \\ &= \sqrt{[(1.4\times10^3\ \text{kg})(8.89\ \text{m/s})]^2 + [(2.6\times10^4\ \text{kg})(13.33\ \text{m/s})]^2} \\ p_\text{f} &= 346\,000\ \text{kg}\cdot\text{m/s (two extra digits carried)} \end{aligned}$$

$$p_\text{f} = (m_1 + m_2)v_\text{f}$$

$$\begin{aligned} v_\text{f} &= \frac{p_\text{f}}{m_1 + m_2} \\ &= \frac{346\ 000\ \cancel{\text{kg}}\cdot\text{m/s}}{1.4\times10^3\ \cancel{\text{kg}} + 2.6\times10^4\ \cancel{\text{kg}}} \end{aligned}$$

$v_\text{f} = 13$ m/s

Convert the speed to kilometres per hour:

$$v_1 = 13\ \frac{\cancel{\text{m}}}{\cancel{\text{s}}} \times \frac{1\ \text{km}}{1000\ \cancel{\text{m}}} \times \frac{3600\ \cancel{\text{s}}}{1\ \text{h}}$$

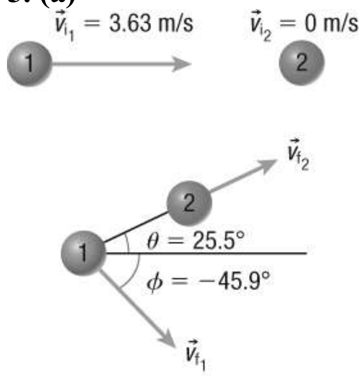
$v_1 = 47$ km/h

$$\begin{aligned} \tan\theta &= \frac{p_1}{p_2} \\ &= \frac{m_1 v_1}{m_2 v_2} \\ &= \frac{(1.4\times10^3\ \cancel{\text{kg}})(8.89\ \cancel{\text{m/s}})}{(2.6\times10^4\ \cancel{\text{kg}})(13.3\ \cancel{\text{m/s}})} \\ \tan\theta &= 0.0359 \\ \theta &= 2.1° \end{aligned}$$

The angle is 2.1° south of east, so it is 90° – 2.1° or 88° east of south.

Statement: The velocity of the vehicles after the collision is 47 km/h [S 88º E].

5. (a)



(b) Given: $\theta = 25.5°$; $\phi = -45.9°$; $\vec{v}_{i_1} = 3.63$ m/s; $\vec{v}_{i_2} = 0$ m/s

Required: v_{f_1}; v_{f_2}

Analysis: The momentum before and after the collision is equal. The momentum before the collision consists of the momentum of ball 1 only, which is in the *x*-direction. The momentum after the collision consists of the momentum of both balls, in both the *x*-direction and the *y*-direction. Consider the momentum in the *x*-direction and the *y*-direction separately. Use trigonometry to determine the components.

Solution: Consider momentum in the *x*-direction first.

$$p_{ix} = p_{fx}$$

$$\cancel{m}v_{i_{1x}} = \cancel{m}v_{f_{1x}} + \cancel{m}v_{f_{2x}}$$

$$v_{i_{1x}} = v_{f_{1x}} + v_{f_{2x}}$$

$$3.63 \text{ m/s} = v_{f_1}\cos\phi + v_{f_2}\cos\theta$$

$$3.63 \text{ m/s} = v_{f_1}\cos(-45.9°) + v_{f_2}\cos 25.5°$$

Now, consider momentum in the *y*-direction. There is no momentum in the *y*-direction before the collision.

$$p_{iy} = p_{fy}$$

$$0 = \cancel{m}v_{f_{1y}} + \cancel{m}v_{f_{2y}}$$

$$0 = v_{f_{1y}} + v_{f_{2y}}$$

$$0 = v_{f_1}\sin\phi + v_{f_2}\sin\theta$$

$$0 = v_{f_1}\sin(-45.9°) + v_{f_2}\sin 25.5°$$

$$v_{f_2} = \frac{-v_{f_1}\sin(-45.9°)}{\sin 25.5°}$$

$$v_{f_2} = \frac{v_{f_1}\sin 45.9°}{\sin 25.5°}$$

Substitute this expression into the other equation for v_{f_2} and v_{f_1}:

$$3.63 \text{ m/s} = v_{f_1} \cos(-45.9°) + v_{f_2} \cos 25.5°$$

$$3.63 \text{ m/s} = v_{f_1} \cos(-45.9°) + \left(\frac{v_{f_1} \sin 45.9°}{\sin 25.5°}\right)\cos 25.5°$$

$$v_{f_1} = 1.65 \text{ m/s}$$

Substitute again to determine v_{f_2}:

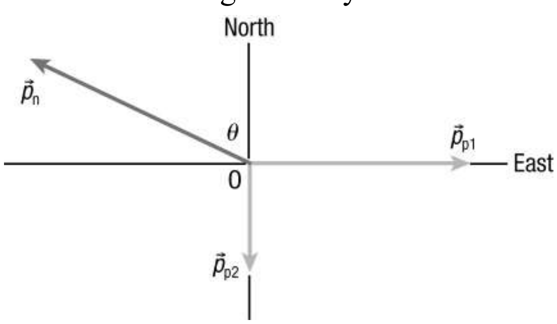
$$\begin{aligned} v_{f_2} &= \frac{v_{f_1} \sin 45.9°}{\sin 25.5°} \\ &= \frac{(1.65 \text{ m/s}) \sin 45.9°}{\sin 25.5°} \\ v_{f_2} &= 2.75 \text{ m/s} \end{aligned}$$

Statement: The final speed of ball 1 is 1.65 m/s, and the final speed of ball 2 is 2.75 m/s.

6. (a) Given: $\vec{p}_{p_1} = 7.8 \times 10^{-21}$ kg · m/s [E]; $\vec{p}_{p_2} = 3.5 \times 10^{-21}$ kg · m/s [S]

Required: θ

Analysis: This is like a perfectly inelastic collision, with the particles as the objects that collide, and the nucleus as the objects sticking together, except that the nucleus fires off in the opposite direction. Use trigonometry to determine θ.



Solution: $\tan\theta = \dfrac{p_{p_1}}{p_{p_2}}$

$$\begin{aligned} \theta &= \tan^{-1}\left(\frac{p_{p_1}}{p_{p_2}}\right) \\ &= \tan^{-1}\left(\frac{7.8 \times 10^{-21} \ \cancel{\text{kg} \cdot \text{m/s}}}{3.5 \times 10^{-21} \ \cancel{\text{kg} \cdot \text{m/s}}}\right) \\ \theta &= 66° \end{aligned}$$

Statement: The direction of the nucleus is [W 24° N].

(b) Given: $\vec{p}_{p_1} = 7.8 \times 10^{-21}$ kg·m/s [E]; $\vec{p}_{p_2} = 3.5 \times 10^{-21}$ kg·m/s [S]

Analysis: Use the Pythagorean theorem to determine the final momentum of the nucleus.

Solution: $p_n^2 = p_{p_1}^2 + p_{p_2}^2$

$$p_n = \sqrt{p_{p_1}^2 + p_{p_2}^2}$$

$$= \sqrt{(7.8 \times 10^{-21}\ \text{kg} \cdot \text{m/s})^2 + (3.5 \times 10^{-21}\ \text{kg} \cdot \text{m/s})^2}$$

$$p_n = 8.5 \times 10^{-21}\ \text{kg} \cdot \text{m/s}$$

Statement: The final momentum is 8.5×10^{-21} kg·m/s [W 24° N].

(c) Given: $p_n = 8.5 \times 10^{-21}$ kg·m/s; $m = 2.3 \times 10^{-26}$ kg

Analysis: $p = mv$

Solution: $p_n = mv_n$

$$v_n = \frac{p_n}{m}$$

$$= \frac{8.5 \times 10^{-21}\ \cancel{\text{kg}} \cdot \text{m/s}}{2.3 \times 10^{-26}\ \cancel{\text{kg}}}$$

$$v_n = 3.7 \times 10^5\ \text{m/s}$$

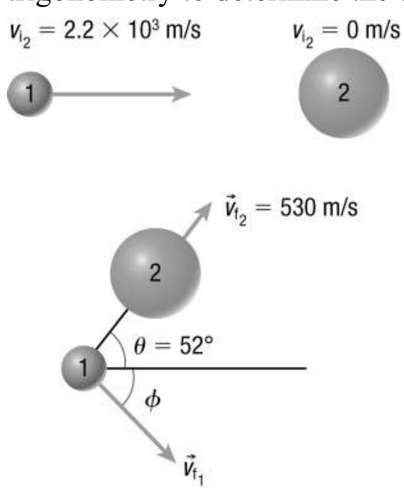
Statement: The final velocity of the nucleus is 3.7×10^5 m/s [W 24° N].

7. Given: $m_1 = 1.7 \times 10^{-27}$ kg; $v_{i_1} = 2.2$ km/s $= 2.2 \times 10^3$ m/s; $m_2 = 6.6 \times 10^{-27}$ kg; $v_{i_2} = 0$ m/s;

$v_{f_2} = 0.53$ km/s $= 530$ m/s; $\theta = 52°$

Required: $\vec{v}_{f_2}$

Analysis: The momentum before and after the collision is equal. The momentum before the collision consists of the momentum of ball 1 only, which is in the x-direction. The momentum after the collision consists of the momentum of both balls, in both the x-direction and the y-direction. Consider the momentum in the x-direction and the y-direction separately. Use trigonometry to determine the components.



Solution: Consider momentum in the x-direction first.

$$p_{\mathrm{i}_x} = p_{\mathrm{f}_x}$$

$$m_1 v_{\mathrm{i}_{1x}} = m_1 v_{\mathrm{f}_{1x}} + m_2 v_{\mathrm{f}_{2x}}$$

$$(1.7\times10^{-27}\ \text{kg})(2.2\times10^3\ \text{m/s}) = (1.7\times10^{-27}\ \text{kg})v_{\mathrm{f}_{1x}} + (6.6\times10^{-27}\ \text{kg})(530\ \text{m/s})(\cos 52°)$$

$$v_{\mathrm{f}_{1x}} = \frac{(1.7\times10^{-27}\ \cancel{\text{kg}})(2.2\times10^3\ \text{m/s}) - (6.6\times10^{-27}\ \cancel{\text{kg}})(530\ \text{m/s})(\cos 52°)}{(1.7\times10^{-27}\ \cancel{\text{kg}})}$$

$$v_{\mathrm{f}_{1x}} = 933.2\ \text{m/s (two extra digits carried)}$$

Now, consider momentum in the y-direction. There is no momentum in the y-direction before the collision.

$$p_{\mathrm{i}_y} = p_{\mathrm{f}_y}$$

$$0 = m_1 v_{\mathrm{f}_{1y}} + m_2 v_{\mathrm{f}_{2y}}$$

$$0 = m_1 v_{\mathrm{f}_{1y}} + m_2 v_{\mathrm{f}_2} \sin\theta$$

$$0 = (1.7\times10^{-27}\ \text{kg})v_{\mathrm{f}_{1y}} + (6.6\times10^{-27}\ \text{kg})(530\ \text{m/s})(\sin 52°)$$

$$v_{\mathrm{f}_{1y}} = \frac{-(6.6\times10^{-27}\ \cancel{\text{kg}})(530\ \text{m/s})(\sin 52°)}{1.7\times10^{-27}\ \cancel{\text{kg}}}$$

$$v_{\mathrm{f}_{1y}} = -1.621\times10^3\ \text{m/s (two extra digits carried)}$$

Use the Pythagorean theorem to determine the final speed of the neutron and trigonometry to determine its direction.

$$v_{\mathrm{f}_1}^2 = v_{\mathrm{f}_{1x}}^2 + v_{\mathrm{f}_{1y}}^2$$

$$\begin{aligned} v_{\mathrm{f}_1} &= \sqrt{v_{\mathrm{f}_{1x}}^2 + v_{\mathrm{f}_{1y}}^2} \\ &= \sqrt{(933.2\ \text{m/s})^2 + (1.621\times10^3\ \text{m/s})^2} \\ &= 1.9\times10^3\ \text{m/s} \end{aligned}$$

$$v_{\mathrm{f}_1} = 1.9\ \text{km/s}$$

$$\tan\theta = \frac{v_{\mathrm{f}_{1y}}}{v_{\mathrm{f}_{1x}}}$$

$$\begin{aligned} \theta &= \tan^{-1}\left(\frac{v_{\mathrm{f}_{1y}}}{v_{\mathrm{f}_{1x}}}\right) \\ &= \tan^{-1}\left(\frac{1.621\times10^3\ \cancel{\text{m/s}}}{933.2\ \cancel{\text{m/s}}}\right) \\ \theta &= 60° \end{aligned}$$

Statement: The final velocity of the neutron is 1.9 km/s, 60° below its original direction.

8. The statement should be rewritten as: "For a head-on elastic collision between two objects of equal mass, the after-collision velocities of the objects are at an 180° angle to each other."

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Knowledge

1. (d)
2. (d)
3. (b)
4. (c)
5. False. You can determine the average force exerted on an object during a collision if you know the object's momentum before and after the collision *and the time of the collision*.
6. True
7. True
8. False. When two objects undergo a perfectly elastic head-on collision, each object will always have a final velocity equal to the initial velocity of the other object *if the two objects have the same mass*.
9. True
10. False. In glancing collisions in two dimensions, momentum *is* conserved.
11. True

Understanding

12. The formula for momentum is $\vec{p} = m\vec{v}$. Thus, the units are $[\text{kg}]\cdot\left[\frac{\text{m}}{\text{s}}\right]$ or $\left[\text{kg}\cdot\text{m/s}\right]$.

The units for force multiplied by time are $\left[\text{kg}\cdot\text{m/s}^{\cancel{2}}\right]\cdot[\cancel{\text{s}}]$ or $\left[\text{kg}\cdot\text{m/s}\right]$. The units are equal.
13. Answers may vary. Sample answer: The stretchiness of the cloth lengthens the time interval of the collision, reducing the average force exerted on it by the watermelon.
14. Answers may vary. Sample answer: The more successful worker may be using a heavier hammer, which has more momentum and thus exerts more force on the nail during the collision.
15. Given: m = 57 g = 0.057 kg; v_1 = 6.0 m/s; v_2 = −7.0 m/s; Δt = 4.0 ms = 4.0×10^{-3} s
Required: F
Analysis:
$$\begin{aligned} F\Delta t &= \Delta p \\ &= m\Delta v \\ F\Delta t &= m(v_2 - v_1) \\ F &= \frac{m(v_2 - v_1)}{\Delta t} \end{aligned}$$
Solution:
$$\begin{aligned} F &= \frac{m(v_2 - v_1)}{\Delta t} \\ &= \frac{(0.057\text{ kg})(-7.0\text{ m/s} - 6.0\text{ m/s})}{4.0 \times 10^{-3}\text{ s}} \\ F &= -1.9 \times 10^2\text{ N} \end{aligned}$$
Statement: The player must apply an average force of 1.9×10^2 N.

16. Given: m = 1100 kg; v = 33 m/s
Required: p
Analysis: $p = mv$
Solution: $p = mv$

$$= (1100 \text{ kg})(33 \text{ m/s})$$

$$p = 3.6 \times 10^4 \text{ kg} \cdot \text{m/s}$$

Statement: The magnitude of the total momentum is 3.6×10^4 kg·m/s.
17. Answers may vary. Sample answer: That a meteor slows down when it enters Earth's atmosphere does not violate conservation of momentum because the meteor collides with air molecules, and some of the meteor's momentum is transferred to the air molecules.
18. Answers may vary. Sample answers:
(a) Conservation of momentum appears to fail when two race cars collide and come to a stop on a stretch of gravel because you would expect that either the cars would stick together and continue to travel together or would bounce apart, with any momentum lost by one car in the collision gained by the other. Because of the external force of friction acting on the cars, however, conservation of momentum does not apply.
(b) Conservation of momentum appears to fail when a stick repeatedly bumps the shore in a flowing river because you would expect the stick to continue to travel away from the shore after it bumps into the shore and bounces away. However, the river currents exert a force on the stick that keeps pushing it back.
(c) Conservation of momentum appears to fail when a bus slows down, picks up a passenger, and speeds back up again because the bus gains momentum after coming to a stop but nothing can be seen to have transferred the momentum to the bus. The bus speeds back up again because the bus's engine exerts a force on the bus.
19. Answers may vary. Sample answer: The cup will slow down as it gains mass. Momentum is conserved, so the speed decreases as the mass increases.
20. Answers may vary. Sample answer: The speed of the sled will increase because it loses mass. Momentum is conserved, so the speed increases as mass decreases.
21. Answers may vary. Sample answer: After the collision, the two balls will move with the same speeds, but their directions of motion will be reversed.
22. Answers may vary. Sample answer: This is impossible. If it were true, the total momentum after the collision would be double the total momentum before the collision, which violates the law of conservation of momentum.
23. Given: $m_1 = m$; $m_2 = 1.2m$; $v_{f_1} = v_{f_2} = 0$ m/s
Required: relationship of initial speeds, v_{i_1} and v_{i_2}
Analysis: The total momentum before the collision is equal to the total momentum after the collision.

$p_i = p_f$

Because both players are stopped, the total momentum after the collision is zero.

$$p_i = 0$$

$$m_1 v_{i_1} + m_2 v_{i_2} = 0$$

Solution: $m_1v_{i_1} + m_2v_{i_2} = 0$

$$mv_{i_1} + 1.2mv_{i_2} = 0$$

$$v_{i_1} + 1.2v_{i_2} = 0$$

$$v_{i_1} = -1.2v_{i_2}$$

Statement: The initial speed of the lighter player is 1.2 times the initial speed of the heavier player, and in the opposite direction.

24. (a) inelastic

(b) elastic

(c) perfectly inelastic

(d) elastic

25. (a) Given: m_1 = 25 kg; $\vec{v}_{i_1}$ = 6.0 m/s [E]; m_2 = 15 kg; $\vec{v}_{i_2}$ = 0 m/s

Required: $\vec{v}_{f_1}$; $\vec{v}_{f_2}$

Analysis: Use the equations for $\vec{v}_{f_1}$ and $\vec{v}_{f_2}$ for perfectly elastic collisions with $\vec{v}_{i_2}$ = 0 m/s.

$$\vec{v}_{f_1} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)\vec{v}_{i_1}; \ \vec{v}_{f_2} = \left(\frac{2m_1}{m_1 + m_2}\right)\vec{v}_{i_1}$$

Solution: $\vec{v}_{f_1} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)\vec{v}_{i_1}$

$$= \left(\frac{25\ \cancel{\text{kg}} - 15\ \cancel{\text{kg}}}{25\ \cancel{\text{kg}} + 15\ \cancel{\text{kg}}}\right)(6.0\text{ m/s})$$

$$\vec{v}_{f_1} = 1.5\text{ m/s}$$

$$\vec{v}_{f_2} = \left(\frac{2m_1}{m_1 + m_2}\right)\vec{v}_{i_1}$$

$$= \left(\frac{2(25\ \cancel{\text{kg}})}{25\ \cancel{\text{kg}} + 15\ \cancel{\text{kg}}}\right)(6.0\text{ m/s})$$

$$\vec{v}_{f_2} = 7.5\text{ m/s}$$

Statement: The final velocity for the first mass is 1.5 m/s [E], and the final velocity for the second mass is 7.5 m/s [E].

(b) Given: m_1 = 12 kg; $\vec{v}_{i_1}$ = 8.0 m/s [E]; m_2 = 22 kg; $\vec{v}_{i_2}$ = 2.0 m/s [E]

Required: $\vec{v}_{f_1}$; $\vec{v}_{f_2}$

Analysis: Use the equations for $\vec{v}_{f_1}$ and $\vec{v}_{f_2}$ for perfectly elastic collisions.

$$\vec{v}_{f_1} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)\vec{v}_{i_1} + \left(\frac{2m_2}{m_1 + m_2}\right)\vec{v}_{i_2}; \ \vec{v}_{f_2} = \left(\frac{m_2 - m_1}{m_1 + m_2}\right)\vec{v}_{i_2} + \left(\frac{2m_1}{m_1 + m_2}\right)\vec{v}_{i_1}$$

Solution: $\vec{v}_{f_1} = \left(\dfrac{m_1 - m_2}{m_1 + m_2}\right)\vec{v}_{i_1} + \left(\dfrac{2m_2}{m_1 + m_2}\right)\vec{v}_{i_2}$

$$= \left(\frac{12\ \cancel{kg} - 22\ \cancel{kg}}{12\ \cancel{kg} + 22\ \cancel{kg}}\right)(8.0\text{ m/s}) + \left(\frac{2(22\ \cancel{kg})}{12\ \cancel{kg} + 22\ \cancel{kg}}\right)(2.0\text{ m/s})$$

$$\vec{v}_{f_1} = 0.24\text{ m/s}$$

$$\vec{v}_{f_2} = \left(\frac{m_2 - m_1}{m_1 + m_2}\right)\vec{v}_{i_2} + \left(\frac{2m_1}{m_1 + m_2}\right)\vec{v}_{i_1}$$

$$= \left(\frac{22\ \cancel{kg} - 12\ \cancel{kg}}{12\ \cancel{kg} + 22\ \cancel{kg}}\right)(2.0\text{ m/s}) + \left(\frac{2(12\ \cancel{kg})}{12\ \cancel{kg} + 22\ \cancel{kg}}\right)(8.0\text{ m/s})$$

$$\vec{v}_{f_2} = 6.2\text{ m/s}$$

Statement: The final velocity for the first mass is 0.24 m/s [E], and the final velocity for the second mass is 6.2 m/s [E].

(c) Given: m_1 = 150 kg; $\vec{v}_{i_1}$ = 2.0 m/s [N]; m_2 = 240 kg; $\vec{v}_{i_2}$ = 3.0 m/s [S]

Required: $\vec{v}_{f_1}$; $\vec{v}_{f_2}$

Analysis: Use the equations for $\vec{v}_{f_1}$ and $\vec{v}_{f_2}$ for perfectly elastic collisions.

$$\vec{v}_{f_1} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)\vec{v}_{i_1} + \left(\frac{2m_2}{m_1 + m_2}\right)\vec{v}_{i_2};\ \vec{v}_{f_2} = \left(\frac{m_2 - m_1}{m_1 + m_2}\right)\vec{v}_{i_2} + \left(\frac{2m_1}{m_1 + m_2}\right)\vec{v}_{i_1}$$

Solution: $\vec{v}_{f_1} = \left(\dfrac{m_1 - m_2}{m_1 + m_2}\right)\vec{v}_{i_1} + \left(\dfrac{2m_2}{m_1 + m_2}\right)\vec{v}_{i_2}$

$$= \left(\frac{150\ \cancel{kg} - 240\ \cancel{kg}}{150\ \cancel{kg} + 240\ \cancel{kg}}\right)(2.0\text{ m/s}) + \left(\frac{2(240\ \cancel{kg})}{150\ \cancel{kg} + 240\ \cancel{kg}}\right)(-3.0\text{ m/s})$$

$$\vec{v}_{f_1} = -4.2\text{ m/s}$$

$$\vec{v}_{f_2} = \left(\frac{m_2 - m_1}{m_1 + m_2}\right)\vec{v}_{i_2} + \left(\frac{2m_1}{m_1 + m_2}\right)\vec{v}_{i_1}$$

$$= \left(\frac{240\ \cancel{kg} - 150\ \cancel{kg}}{240\ \cancel{kg} + 150\ \cancel{kg}}\right)(-3.0\text{ m/s}) + \left(\frac{2(150\ \cancel{kg})}{240\ \cancel{kg} + 150\ \cancel{kg}}\right)(2.0\text{ m/s})$$

$$\vec{v}_{f_2} = 0.85\text{ m/s}$$

Statement: The final velocity for the first mass is 4.2 m/s [S], and the final velocity for the second mass is 0.85 m/s [N].

26. Given: $\vec{v}_1 = 12$ m/s [N]; $\vec{v}_2 = 18$ m/s [S]; $\vec{v}_\text{f} = -4.0$ m/s; $m_1 = 120$ kg

Required: m_2

Analysis: This is a perfectly elastic collision, so momentum is conserved. The momentum before the collision is equal to the momentum after the collision. Let north be positive and south be negative.

$$m_1\vec{v}_1 + m_2\vec{v}_2 = (m_1 + m_2)\vec{v}_\text{f}$$

$$m_1\vec{v}_1 + m_2\vec{v}_2 = m_1\vec{v}_\text{f} + m_2\vec{v}_\text{f}$$

$$m_1(\vec{v}_1 - \vec{v}_\text{f}) = m_2(\vec{v}_\text{f} - \vec{v}_2)$$

$$m_2 = \frac{m_1(\vec{v}_1 - \vec{v}_\text{f})}{\vec{v}_\text{f} - \vec{v}_2}$$

Solution: $m_2 = \dfrac{m_1(\vec{v}_1 - \vec{v}_\text{f})}{\vec{v}_\text{f} - \vec{v}_2}$

$$= \frac{(120\text{ kg})[12\ \cancel{\text{m/s}} - (-4.0\ \cancel{\text{m/s}})]}{[-4.0\ \cancel{\text{m/s}} - (-18\ \cancel{\text{m/s}})]}$$

$$m_2 = 140\text{ kg}$$

Statement: Hockey player 2 has a mass of 140 kg.

27. Given: $m_1 = 1.2$ kg; $\Delta y = 2.4$ m; $m_2 = 1.4$ kg; $v_{\text{i}_2} = 0$ m/s

Required: v_{f_1}; v_{f_2}

Analysis: Determine the velocity of cart 1 just before the collision using conservation of energy.

$$\cancel{m}g\Delta y = \frac{1}{2}\cancel{m}v_{\text{i}_1}^2$$

$$v_{\text{i}_1} = \sqrt{2g\Delta y}$$

Then, use conservation of momentum and conservation of kinetic energy to determine the velocities of the carts just after the collision. Let right be positive.

Solution: $v_{\text{i}_1} = \sqrt{2g\Delta y}$

$$= \sqrt{2(9.8\text{ m/s}^2)(2.4\text{ m})}$$

$$v_{\text{i}_1} = 6.9\text{ m/s}$$

$$m_1v_{\text{i}_1} + m_2v_{\text{i}_2} = m_1v_{\text{f}_1} + m_2v_{\text{f}_2}$$

$$(1.2\ \cancel{\text{kg}})(6.9\text{ m/s}) = (1.2\ \cancel{\text{kg}})v_{\text{f}_1} + (1.4\ \cancel{\text{kg}})v_{\text{f}_2}$$

$$8.28\text{ m/s} = 1.2v_{\text{f}_1} + 1.4v_{\text{f}_2}$$

$$v_{\text{f}_2} = \frac{(8.28\text{ m/s}) - 1.2v_{\text{f}_1}}{1.4}$$

$$\cancel{\frac{1}{2}}m_1v_{i_1}^2 + \cancel{\frac{1}{2}}m_2v_{i_2}^2 = \cancel{\frac{1}{2}}m_1v_{f_1}^2 + \cancel{\frac{1}{2}}m_2v_{f_2}^2$$

$$m_1v_{i_1}^2 = m_1v_{f_1}^2 + m_2v_{f_2}^2$$

$$(1.2\ \cancel{\text{kg}})(6.9\text{ m/s})^2 = (1.2\ \cancel{\text{kg}})v_{f_1}^2 + (1.4\ \cancel{\text{kg}})v_{f_2}^2$$

$$v_{f_2}^2 = \frac{(1.2)(6.9\text{ m/s})^2 - 1.2v_{f_1}^2}{1.4}$$

Substitute the expression for v_{f_2} from above into this equation:

$$v_{f_2}^2 = \frac{(1.2)(6.9\text{ m/s})^2 - 1.2v_{f_1}^2}{1.4}$$

$$\left(\frac{8.28\text{ m/s} - 1.2v_{f_1}}{1.4}\right)^2 = \frac{(1.2)(6.9\text{ m/s})^2 - 1.2v_{f_1}^2}{1.4}$$

$$\frac{(68.56\text{ m}^2/\text{s}^2) - (19.87\text{ m/s})v_{f_1} + 1.44v_{f_1}^2}{1.4^{\cancel{2}}} = \frac{(57.132\text{ m}^2/\text{s}^2) - 1.2v_{f_1}^2}{\cancel{1.4}}$$

$$(68.56\text{ m}^2/\text{s}^2) - (19.87\text{ m/s})v_{f_1} + 1.44v_{f_1}^2 = 1.4(57.132\text{ m}^2/\text{s}^2 - 1.2v_{f_1}^2)$$

$$(68.56\text{ m}^2/\text{s}^2) - (19.87\text{ m/s})v_{f_1} + 1.44v_{f_1}^2 = 79.98\text{ m}^2/\text{s}^2 - 1.68v_{f_1}^2$$

$$3.12v_{f_1}^2 - (19.87\text{ m/s})v_{f_1} - (11.42\text{ m}^2/\text{s}^2) = 0$$

Use the quadratic formula to solve for v_{f1}:

$$\begin{aligned} v_{f_1} &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-(-19.87\text{ m/s}) \pm \sqrt{(-19.87\text{ m/s})^2 - 4(3.12)(-11.42\text{ m}^2/\text{s}^2)}}{2(3.12)} \end{aligned}$$

$$v_{f_1} = 6.9\text{ m/s or } -0.53\text{ m/s}$$

Cart 1 will bounce off cart 2, so its initial velocity must be negative, so $v_{f_1} = -0.53$ m/s.

Substitute this in the equation for v_{f_2}:

$$\begin{aligned} v_{f_2} &= \frac{(8.28\text{ m/s}) - 1.2v_{f_1}}{1.4} \\ &= \frac{(8.28\text{ m/s}) - 1.2(-0.53\text{ m/s})}{1.4} \end{aligned}$$

$$v_{f_2} = 6.4\text{ m/s}$$

Statement: Cart 1's final speed is 0.53 m/s, and cart 2's final speed is 6.4 m/s.

28. (a) Given: m_1 = 1.2 kg; Δy = 1.8 m; m_2 = 2.0 kg; $\vec{v}_{i_2} = 0$ m/s; Δx = 2.0 cm = 0.020 m

Required: k

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Analysis: Determine the velocity of cart 1 just before the collision using conservation of energy. The gravitational potential energy at the top of the ramp is equal to the kinetic energy at the bottom of the ramp.

$$\cancel{m}g\Delta y = \frac{1}{2}\cancel{m}v_{i_1}^2$$
$$v_{i_1} = \sqrt{2g\Delta y}$$

At the point of maximum compression of the spring, the two carts will have the same velocity, $\vec{v}_f$. Use the conservation of momentum equation to determine this velocity, rearranged to isolate v_f.

$$m_1 v_{i_1} + m_2 v_{i_2} = (m_1 + m_2)v_f$$
$$v_f = \frac{m_1 v_{i_1} + m_2 v_{i_2}}{m_1 + m_2}$$

Then, apply conservation of mechanical energy to calculate the spring constant.

$$\cancel{\frac{1}{2}}m_1 v_{i_1}^2 + \cancel{\frac{1}{2}}m_2 v_{i_2}^2 = \cancel{\frac{1}{2}}(m_1 + m_2)v_f^2 + \cancel{\frac{1}{2}}k(\Delta x)^2$$
$$m_1 v_{i_1}^2 + m_2 v_{i_2}^2 = (m_1 + m_2)v_f^2 + k(\Delta x)^2$$
$$k = \frac{m_1 v_{i_1}^2 + m_2 v_{i_2}^2 - (m_1 + m_2)v_f^2}{(\Delta x)^2}$$

Solution: $v_{i_1} = \sqrt{2g\Delta y}$

$$= \sqrt{2(9.8\text{ m/s}^2)(1.8\text{ m})}$$
$$v_{i_1} = 5.94\text{ m/s (one extra digit carried)}$$

$$v_f = \frac{m_1 v_{i_1} + m_2 v_{i_2}}{m_1 + m_2}$$
$$= \frac{(1.2\ \cancel{\text{kg}})(5.94\text{ m/s}) + (2.0\ \cancel{\text{kg}})(0\text{ m/s})}{1.2\ \cancel{\text{kg}} + 2.0\ \cancel{\text{kg}}}$$
$$v_f = 2.23\text{ m/s (one extra digit carried)}$$

$$k = \frac{m_1 v_{i_1}^2 + m_2 v_{i_2}^2 - (m_1 + m_2)v_f^2}{(\Delta x)^2}$$
$$= \frac{(1.2\text{ kg})(5.94\text{ m/s})^2 + (2.0\text{ kg})(0\text{ m/s})^2 - (1.2\text{ kg} + 2.0\text{ kg})(2.23\text{ m/s})^2}{(0.02\text{ m})^2}$$
$$k = 6.6 \times 10^4\text{ N/m}$$

Statement: The spring constant is 6.6×10^4 N/m.

(b) Given: m_1 = 1.2 kg; m_2 = 2.0 kg; v_{i_1} = 5.94 m/s; v_{i_2} = 0 m/s

Required: $\vec{v}_{f_1}$; $\vec{v}_{f_2}$

Analysis: Use the formulas for $\vec{v}_{f_1}$ and $\vec{v}_{f_2}$ when $\vec{v}_{i_2}$ = 0 m/s:

$$\vec{v}_{f_1} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)\vec{v}_{i_1};\ \vec{v}_{f_2} = \left(\frac{2m_1}{m_1 + m_2}\right)\vec{v}_{i_1}$$

Solution:
$$\begin{aligned}\vec{v}_{f_1} &= \left(\frac{m_1 - m_2}{m_1 + m_2}\right)\vec{v}_{i_1}\\ &= \left(\frac{1.2\ \cancel{\text{kg}} - 2.0\ \cancel{\text{kg}}}{1.2\ \cancel{\text{kg}} + 2.0\ \cancel{\text{kg}}}\right)(5.94\ \text{m/s})\\ \vec{v}_{f_1} &= -1.5\ \text{m/s}\end{aligned}$$

$$\begin{aligned}\vec{v}_{f_2} &= \left(\frac{2m_1}{m_1 + m_2}\right)\vec{v}_{i_1}\\ &= \left(\frac{2(1.2\ \cancel{\text{kg}})}{1.2\ \cancel{\text{kg}} + 2.0\ \cancel{\text{kg}}}\right)(5.94\ \text{m/s})\\ \vec{v}_{f_2} &= 4.5\ \text{m/s}\end{aligned}$$

Statement: The final velocity of cart 1 is 1.5 m/s [W], and the final velocity of cart 2 is 4.5 m/s [E].

(c) Given: m_1 = 1.2 kg; $\vec{v}_{f_1} = -1.5$ m/s

Required: Δy

Analysis: The kinetic energy at the bottom of the ramp transforms to gravitational potential energy at the top of the ramp.

$$\frac{1}{2}\cancel{m_1}v_{f_1}^2 = \cancel{m_1}g\Delta y$$

$$\Delta y = \frac{v_{f_1}^2}{2g}$$

Solution:
$$\begin{aligned}\Delta y &= \frac{v_{f_1}^2}{2g}\\ &= \frac{(-1.5\ \text{m/s})^2}{2(9.8\ \text{m/s}^2)}\\ &= \frac{(-1.5)^2\ \text{m}^{\cancel{2}}/\cancel{\text{s}^2}}{2(9.8\ \cancel{\text{m}}/\cancel{\text{s}^2})}\\ \Delta y &= 0.11\ \text{m}\end{aligned}$$

Statement: The maximum height reached by the cart is 0.11 m.

29. (a) Given: $m_1 = 81$ kg; $m_2 = 93$ kg; $\vec{v}_{f_1} = 1.7$ m/s [N]; $\vec{v}_{f_2} = 1.1$ m/s [S]

Required: $\vec{v}_{i_1}$; $\vec{v}_{i_2}$

Analysis: Use the formulas for $\vec{v}_{f_1}$ and $\vec{v}_{f_2}$.

$$\vec{v}_{f_1} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)\vec{v}_{i_1} + \left(\frac{2m_2}{m_1 + m_2}\right)\vec{v}_{i_2};\ \vec{v}_{f_2} = \left(\frac{m_2 - m_1}{m_1 + m_2}\right)\vec{v}_{i_2} + \left(\frac{2m_1}{m_1 + m_2}\right)\vec{v}_{i_1}$$

Solution:

$$\vec{v}_{f_1} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)\vec{v}_{i_1} + \left(\frac{2m_2}{m_1 + m_2}\right)\vec{v}_{i_2}$$

$$1.7\text{ m/s} = \left(\frac{81\ \cancel{\text{kg}} - 93\ \cancel{\text{kg}}}{81\ \cancel{\text{kg}} + 93\ \cancel{\text{kg}}}\right)\vec{v}_{i_1} + \left(\frac{2(93\ \cancel{\text{kg}})}{81\ \cancel{\text{kg}} + 93\ \cancel{\text{kg}}}\right)\vec{v}_{i_2}$$

$$1.7\text{ m/s} = \frac{-12}{174}\vec{v}_{i_1} + \frac{186}{174}\vec{v}_{i_2}$$

$$1.7\text{ m/s} = -\frac{2}{29}\vec{v}_{i_1} + \frac{31}{29}\vec{v}_{i_2}$$

$$49.3\text{ m/s} = -2\vec{v}_{i_1} + 31\vec{v}_{i_2}$$

$$\vec{v}_{f_2} = \left(\frac{m_2 - m_1}{m_1 + m_2}\right)\vec{v}_{i_2} + \left(\frac{2m_1}{m_1 + m_2}\right)\vec{v}_{i_1}$$

$$-1.1\text{ m/s} = \left(\frac{93\text{ kg} - 81\text{ kg}}{81\text{ kg} + 93\text{ kg}}\right)\vec{v}_{i_2} + \left(\frac{2(81\text{ kg})}{81\text{ kg} + 93\text{ kg}}\right)\vec{v}_{i_1}$$

$$-1.1\text{ m/s} = \frac{12\ \cancel{\text{kg}}}{174\ \cancel{\text{kg}}}\vec{v}_{i_2} + \frac{162\ \cancel{\text{kg}}}{174\ \cancel{\text{kg}}}\vec{v}_{i_1}$$

$$-1.1\text{ m/s} = \frac{2}{29}\vec{v}_{i_2} + \frac{27}{29}\vec{v}_{i_1}$$

$$-31.9\text{ m/s} = 2\vec{v}_{i_2} + 27\vec{v}_{i_1}$$

$$\vec{v}_{i_2} = \frac{-27\vec{v}_{i_1} - 31.9\text{ m/s}}{2}$$

Substitute this expression into the equation above:

$$49.3\text{ m/s} = -2\vec{v}_{i_1} + 31\vec{v}_{i_2}$$

$$49.3\text{ m/s} = -2\vec{v}_{i_1} + 31\left(\frac{-27\vec{v}_{i_1} - 31.9\text{ m/s}}{2}\right)$$

$$98.6\text{ m/s} = -4\vec{v}_{i_1} - 837\vec{v}_{i_1} - 988.9\text{ m/s}$$

$$841\vec{v}_{i_1} = -1087.5\text{ m/s}$$

$$\vec{v}_{i_1} = -1.29\text{ m/s (one extra digit carried)}$$

Substitute this in the expression for $\vec{v}_{i_2}$:

$$\vec{v}_{i_2} = \frac{-27\vec{v}_{i_1} - 31.9 \text{ m/s}}{2}$$
$$= \frac{-27(-1.29 \text{ m/s}) - 31.9 \text{ m/s}}{2}$$
$$\vec{v}_{i_2} = 1.5 \text{ m/s}$$

Statement: The initial velocity of cart 1 is 1.3 m/s [S], and the initial velocity of cart 2 is 1.5 m/s [N].

(b) Given: m_1 = 81 kg; m_2 = 93 kg; $\vec{v}_{f_1} = 1.7$ m/s [N]; $\vec{v}_{f_2} = 1.1$ m/s [S]

Required: E_{k_T}

Analysis: Since kinetic energy is conserved in an elastic collision, the total kinetic energy of the inner tubes and riders is equal to their total kinetic energy after the collision.

$$E_k = \frac{1}{2}mv^2$$

Solution: $E_{k_T} = \frac{1}{2}(m_1 v_{f_1}^2 + m_2 v_{f_2}^2)$

$$= \frac{1}{2}[(81 \text{ kg})(1.7 \text{ m/s})^2 + (9.3 \text{ kg})(-1.1 \text{ m/s})^2]$$
$$E_{k_T} = 1.7 \times 10^2 \text{ J}$$

Statement: The total kinetic energy of the inner tubes and riders is 1.7×10^2 J.

(c) Given: m_1 = 81 kg; m_2 = 93 kg; $\vec{v}_{f_1} = 1.7$ m/s [N]; $\vec{v}_{f_2} = 1.1$ m/s [S]

Required: $\vec{p}_T$

Analysis: Since momentum is conserved in an elastic collision, the total momentum of the inner tubes and riders is equal to their total momentum after the collision.

$\vec{p} = m\vec{v}$

Solution: $\vec{p}_T = m_1\vec{v}_{f_1} + m_2\vec{v}_{f_2}$

$$= (81 \text{ kg})(1.7 \text{ m/s}) + (93 \text{ kg})(-1.1 \text{ m/s})$$
$$\vec{p}_T = 35 \text{ kg} \cdot \text{m/s}$$

Statement: The total momentum of the inner tubes and riders is 35 kg·m/s [N].

30. Given: m = 240 kg; v = 4.3 m/s; Δt = 3 s

Required: $\vec{F}$

Analysis: $F\Delta t = p$ and $p = mv$, so

$F\Delta t = mv$

$$F = \frac{mv}{\Delta t}$$

Solution: $F = \dfrac{mv}{\Delta t}$

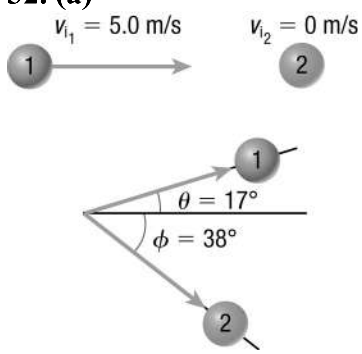
$$= \frac{(240\text{ kg})(4.3\text{ m/s})}{3\text{ s}}$$

$$F = 3\times 10^{2}\text{ N}$$

Statement: The average force of friction is 3×10^{2} N [backward].

31. Answers may vary. Sample answer: The more massive ball is moving more slowly after the collision.

32. (a)



(b) Given: $m_1 = m_2 = m$; $v_{\text{i}_1} = 5.0$ m/s; $v_{\text{i}_2} = 0$ m/s; $\theta = 17°$; $\phi = 38°$

Required: v_{f_1}; v_{f_2}

Analysis: The momentum in the x-direction and in the y-direction is conserved before and after the collision. Consider the two directions separately.

Solution: Start with the x-direction.

$$p_{\text{i}_x} = p_{\text{f}_x}$$

$$m_1 v_{\text{i}_{1x}} + m_2 v_{\text{i}_{2x}} = m_1 v_{\text{f}_{1x}} + m_2 v_{\text{f}_{2x}}$$

$$\cancel{m} v_{\text{i}_{1x}} = \cancel{m} v_{\text{f}_{1x}} + \cancel{m} v_{\text{f}_{2x}}$$

$$v_{\text{i}_{1x}} = v_{\text{f}_{1x}} + v_{\text{f}_{2x}}$$

$$5.0\text{ m/s} = v_{\text{f}_1}\cos 17° + v_{\text{f}_2}\cos 38°$$

Now, consider the y-direction. There is no initial momentum in the y-direction, since the only motion is in the x-direction.

$$0 = p_{\text{f}_y}$$

$$0 = m_1 v_{\text{f}_{1y}} + m_2 v_{\text{f}_{2y}}$$

$$0 = \cancel{m} v_{\text{f}_{1y}} + \cancel{m} v_{\text{f}_{2y}}$$

$$0 = v_{\text{f}_{1y}} + v_{\text{f}_{2y}}$$

$$0 = v_{\text{f}_1}\sin 17° - v_{\text{f}_2}\sin 38°$$

$$v_{\text{f}_1} = \frac{v_{\text{f}_2}\sin 38°}{\sin 17°}$$

Substitute this into the equation above.

$$5.0 \text{ m/s} = v_{f_1} \cos 17° + v_{f_2} \cos 38°$$

$$5.0 \text{ m/s} = \left(\frac{v_{f_2} \sin 38°}{\sin 17°}\right)(\cos 17°) + v_{f_2} \cos 38°$$

$$5.0 \text{ m/s} = v_{f_2}\left(\frac{\sin 38° \cos 17°}{\sin 17°} + \cos 38°\right)$$

$$v_{f_2} = 1.785 \text{ m/s (two extra digits carried)}$$

Substitute this into the equation for v_{f_1}:

$$v_{f_1} = \frac{v_{f_2} \sin 38°}{\sin 17°}$$

$$= \frac{(1.785 \text{ m/s})(\sin 38°)}{\sin 17°}$$

$$v_{f_1} = 3.8 \text{ m/s}$$

Statement: The speed of stone 1 is 3.8 m/s, and the speed of stone 2 is 1.8 m/s.

33. They all have the same mass, because of the conservation of momentum.

Analysis and Application

34. (a) Given: m_1; m_2; v_1; v_2; $E_{k_1} = E_{k_2}$; $p_1 = p_2$

Required: relationship between the velocities

Analysis: Set the momenta equal, and then solve for one of the masses. Substitute into the equation for the equal kinetic energies and simplify.

Solution:

$$m_1 v_1 = m_2 v_2$$

$$m_1 = \frac{m_2 v_2}{v_1}$$

$$\cancel{\frac{1}{2}} m_1 v_1^2 = \cancel{\frac{1}{2}} m_2 v_2^2$$

$$\left(\frac{\cancel{m_2}\,\cancel{v_2}}{\cancel{v_1}}\right) v_1^{\cancel{2}} = \cancel{m_2} v_2^{\cancel{2}}$$

$$v_1 = v_2$$

Statement: The two velocities are equal.

(b) Given: m_1; m_2; v_1; v_2; $E_{k_1} = E_{k_2}$; $p_1 = p_2$

Required: relationship between the masses

Analysis: Set the momenta equal, and then solve for one of the velocities. Substitute into the equation for the equal kinetic energies and simplify.

Solution: $m_1v_1 = m_2v_2$

$$v_1 = \frac{m_2v_2}{m_1}$$

$$\cancel{\frac{1}{2}}m_1v_1^2 = \cancel{\frac{1}{2}}m_2v_2^2$$

$$m_1\left(\frac{m_2v_2}{m_1}\right)^2 = m_2v_2^2$$

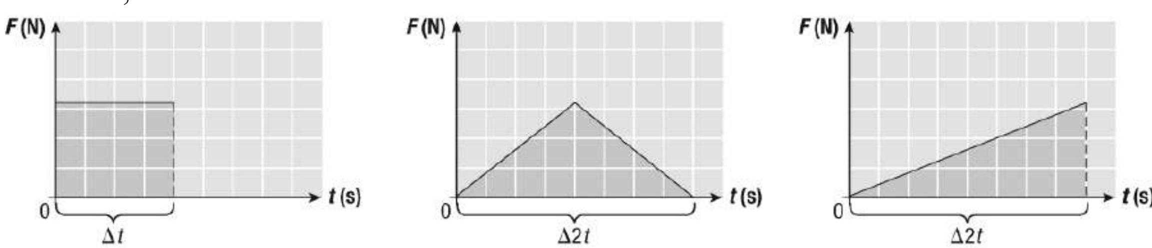
$$\frac{m_2^{\cancel{2}}\cancel{v_2^2}}{m_1} = \cancel{m_2}\cancel{v_2^2}$$

$$\frac{m_2}{m_1} = 1$$

$$m_2 = m_1$$

Statement: The masses are equal.

35. Answers may vary. Sample answer: In all three cases, the impulse is the area under the curve, or $F\Delta t$.



36. (a) Given: m = 0.152 kg; v_1 = 35 m/s; Δt = 1.6 ms = 1.6×10^{-3} s; v_2 = −29 m/s

Required: $\vec{F}$

Analysis: $F\Delta t = m\Delta v$

$$F = \frac{m(v_2 - v_1)}{\Delta t}$$

Solution: $F = \frac{m(v_2 - v_1)}{\Delta t}$

$$= \frac{(0.152\text{ kg})(-29\text{ m/s} - 35\text{ m/s})}{1.6 \times 10^{-3}\text{ s}}$$

$$F = -6.1 \times 10^3\text{ N}$$

Statement: The force is 6.1×10^3 N [away from the wall].

(b) Given: m = 0.125 kg; v_1 = 25 m/s; Δt = 0.25 s; v_2 = −23 m/s

Required: $\vec{F}$

Analysis: $F\Delta t = m\Delta v$

$$F = \frac{m(v_2 - v_1)}{\Delta t}$$

Solution: $F = \dfrac{m(v_2 - v_1)}{\Delta t}$

$$= \frac{(0.125\text{ kg})(-23\text{ m/s} - 25\text{ m/s})}{0.25\text{ s}}$$

$$F = -24\text{ N}$$

Statement: The force is 24 N [away from the wall].

(c) Given: m = 0.06 kg; v_1 = 340 m/s; Δt = 0.1 ms = 1×10^{-4} s; v_2 = −3 m/s

Required: $\vec{F}$

Analysis: $F\Delta t = m\Delta v$

$$F = \frac{m(v_2 - v_1)}{\Delta t}$$

Solution: $F = \dfrac{m(v_2 - v_1)}{\Delta t}$

$$= \frac{(0.06\text{ kg})(-3\text{ m/s} - 340\text{ m/s})}{1 \times 10^{-4}\text{ s}}$$

$$F = -2 \times 10^5\text{ N}$$

Statement: The force is 2×10^5 N [away from the wall].

37. Given: m_1 = 0.1 kg; $v_{i_1} = v$; $v_{f_1} = -\frac{1}{3}v$; v_{i_2} = 0 m/s

Required: m_2

Analysis: Use the equation for v_{f_1} where v_{i_2} = 0 m/s:

$$v_{f_1} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)v_{i_1}$$

Solution:

$$\begin{aligned} v_{f_1} &= \left(\frac{m_1 - m_2}{m_1 + m_2}\right)v_{i_1} \\ -\frac{1}{3}\cancel{v} &= \left(\frac{0.1\text{ kg} - m_2}{0.1\text{ kg} + m_2}\right)\cancel{v} \\ -0.1\text{ kg} - m_2 &= 0.3\text{ kg} - 3m_2 \\ 2m_2 &= 0.4\text{ kg} \\ m_2 &= 0.2\text{ kg} \end{aligned}$$

Statement: The mass of ball 2 is 0.2 kg.

38. A lighter performer will have a greater speed on exiting the cannon, so they should use a heavier performer, since for constant kinetic energy E_k, speed is given in terms of mass by $v = \sqrt{\dfrac{2E_k}{m}}$, so that momentum is given by $mv = m\sqrt{\dfrac{2E_k}{m}} = \sqrt{2E_k m}$.

39. Answers may vary. Sample answer:
Given: $m_w = 2.0$ kg; $m_g = 0.8$ kg; $E_{k_T} = 10.5$ J; $p_T = 7.5$ kg·m/s; head-on elastic collision

Required: v_{i_w}; v_{i_g}; v_{f_w}; v_{f_g}

Analysis: Kinetic energy and momentum are conserved in a head-on elastic collision. Let right be positive.
Solution: Assume that both fruits are moving in the positive direction.
Consider the initial velocities. Start with momentum:

$$\begin{aligned}
m_1 v_{i_1} + m_2 v_{i_2} &= p_T \\
(2.0\ \cancel{\text{kg}})v_{i_w} + (0.8\ \cancel{\text{kg}})v_{i_g} &= 7.5\ \cancel{\text{kg}} \cdot \text{m/s} \\
2.0 v_{i_w} + 0.8 v_{i_g} &= 7.5\ \text{m/s} \\
2.0 v_{i_w} &= 7.5\ \text{m/s} - 0.8 v_{i\,g}
\end{aligned}$$

Now consider kinetic energy:

$$\begin{aligned}
\frac{1}{2}(m_w v_{i_1}^2 + m_2 v_{i_2}^2) &= E_{k_T} \\
m_w v_{i_w}^2 + m_g v_{i_g}^2 &= 2E_{k_T} \\
(2.0\ \text{kg})v_{i_w}^2 + (0.8\ \text{kg})v_{i_g}^2 &= 21\ \text{J} \\
v_{i_w}^2 + 0.4 v_{i_g}^2 &= 10.5\ \text{J/kg} \\
4 v_{i_w}^2 + 1.6 v_{i_g}^2 &= 42\ \text{J/kg} \\
(2 v_{i_w})^2 + 1.6 v_{i_g}^2 &= 42\ \text{J/kg}
\end{aligned}$$

In the second-last line above, the equation was multiplied by 4.
Substitute the expression above for $2.0\, v_{i_1}$:

$$\begin{aligned}
(2 v_{i_w})^2 + 1.6 v_{i_g}^2 &= 42\ \text{J/kg} \\
(7.5\ \text{m/s} - 0.8 v_{i_g})^2 + 1.6 v_{i_g}^2 &= 42\ \text{J/kg} \\
(56.25\ \text{m}^2/\text{s}^2) - (12\ \text{m/s})v_{i_g} + 0.64 v_{i_g}^2 + 1.6 v_{i_g}^2 &= 42\ \text{m}^2/\text{s}^2 \\
2.24 v_{i_g}^2 - (12\ \text{m/s})v_{i_g} + (14.25\ \text{m}^2/\text{s}^2) &= 0
\end{aligned}$$

Use the quadratic formula to solve for $v_{i\,g}$:

$$\begin{aligned}
v_{i_g} &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
&= \frac{-(-12\ \text{m/s}) \pm \sqrt{(-12\ \text{m/s})^2 - 4(2.24)(14.25\ \text{m}^2/\text{s}^2)}}{2(2.24)} \\
&= 1.78\ \text{m/s or } 3.58\ \text{m/s (one extra digit carried)} \\
v_{i_g} &= 1.8\ \text{m/s or } 3.6\ \text{m/s}
\end{aligned}$$

Suppose the initial velocity of the grapefruit is 1.78 m/s:
Substitute this value into the equation for the initial velocity of the watermelon:

$$v_{i_w} = \frac{7.5 \text{ m/s} - 0.8v_{i_g}}{2.0}$$

$$= \frac{7.5 \text{ m/s} - 0.8(1.78 \text{ m/s})}{2.0}$$

$$v_{i_w} = 3.0 \text{ m/s}$$

Suppose the initial velocity of the grapefruit is 3.58 m/s.
Substitute this value into the equation for the initial velocity of the watermelon:

$$2.0v_{i_w} = 7.5 \text{ m/s} - 0.8v_{i_g}$$

$$v_{i_w} = \frac{7.5 \text{ m/s} - 0.8v_{i_g}}{2.0}$$

$$= \frac{7.5 \text{ m/s} - 0.8(3.58 \text{ m/s})}{2.0}$$

$$v_{i_w} = 2.3 \text{ m/s}$$

The equations for final velocity will be exactly the same, with f subscripts instead of i subscripts, so the final velocities will be the same.
Statement: If the initial velocities of the grapefruit and watermelon are 1.8 m/s and 3.0 m/s, respectively, then the final velocities of the grapefruit and watermelon will be 3.6 m/s and 2.3 m/s, respectively. If the initial velocities of the grapefruit and watermelon are 3.6 m/s and 2.3 m/s, respectively, then the final velocities for the grapefruit and watermelon will be 1.8 m/s and 3.0 m/s, respectively.
40. (a) Given: $m_1 = m_2 = m_3 = m_4 = 63$ kg; $m_b = 210$ kg; $v_{i_b} = 3.0$ m/s; each bobsledder is moving at 2.0 m/s faster than the bobsled when he or she jumps in
Required: v_{f_b}
Analysis: Calculate the speed of the bobsled after each bobsledder jumps on, one by one, using conservation of momentum.
Solution: Start with the initial momentum of the sled.

$$m_b v_{i_b} = (210 \text{ kg})(3.0 \text{ m/s})$$

$$m_b v_{i_b} = 630 \text{ kg} \cdot \text{m/s}$$

Determine the total momentum after the first bobsledder jumps in:

$$\begin{aligned} p_{T_1} &= m_b v_{i_b} + m_1 v_1 \\ m_{T_1} v_{T_1} &= (630 \text{ kg} \cdot \text{m/s}) + (63 \text{ kg})(3.0 \text{ m/s} + 2.0 \text{ m/s}) \\ (210 \cancel{\text{kg}} + 63 \cancel{\text{kg}})v_{T_1} &= 945 \cancel{\text{kg}} \cdot \text{m/s} \\ 273v_{T_1} &= 945 \text{ m/s} \\ v_{T_1} &= 3.46 \text{ m/s (one extra digit carried)} \end{aligned}$$

Determine the total momentum after the second bobsledder jumps in:

$$p_{T_2} = m_{T_1}v_{T_1} + m_2v_2$$
$$m_{T_2}v_{T_2} = (945\ \text{kg}\cdot\text{m/s}) + (63\ \text{kg})(3.46\ \text{m/s} + 2.0\ \text{m/s})$$
$$(273\ \cancel{\text{kg}} + 63\ \cancel{\text{kg}})v_{T_2} = 1289\ \cancel{\text{kg}}\cdot\text{m/s}$$
$$336v_{T_2} = 1289\ \text{m/s}$$
$$v_{T_2} = 3.84\ \text{m/s}\ \ \text{(one extra digit carried)}$$

Determine the total momentum after the third bobsledder jumps in:

$$p_{T_3} = m_{T_2}v_{T_2} + m_3v_3$$
$$m_{T_3}v_{T_3} = (1289\ \text{kg}\cdot\text{m/s}) + (63\ \text{kg})(3.84\ \text{m/s} + 2.0\ \text{m/s})$$
$$(336\ \cancel{\text{kg}} + 63\ \cancel{\text{kg}})v_{T_3} = 1657\ \cancel{\text{kg}}\cdot\text{m/s}$$
$$399v_{T_3} = 1657\ \text{m/s}$$
$$v_{T_3} = 4.15\ \text{m/s (one extra digit carried)}$$

Determine the total momentum after the fourth bobsledder jumps in:

$$p_{T_4} = m_{T_3}v_{T_3} + m_4v_4$$
$$m_{T_4}v_{T_4} = (1657\ \text{kg}\cdot\text{m/s}) + (63\ \text{kg})(4.15\ \text{m/s} + 2.0\ \text{m/s})$$
$$(399\ \cancel{\text{kg}} + 63\ \cancel{\text{kg}})v_{T_4} = 2044\ \cancel{\text{kg}}\cdot\text{m/s}$$
$$462v_{T_4} = 2044\ \text{m/s}$$
$$v_{T_4} = 4.4\ \text{m/s}$$

Statement: The final speed of the sled is 4.4 m/s.
(b) From (a), the final momentum is 2.0×10^3 kg·m/s [forward].
41. (a) Given: graph of force versus time
Required: impulse, $F\Delta t$
Analysis: The impulse is the area under the curve in the graph, which is a triangle with base 3.0 s and height 2.0 N.

$$\text{impulse} = \frac{1}{2}bh$$

Solution:
$$\begin{aligned}\text{impulse} &= \frac{1}{2}bh \\ &= \frac{1}{2}(3.0\ \text{s})(2.0\ \text{N}) \\ \text{impulse} &= 3.0\ \text{N}\cdot\text{s}\end{aligned}$$

Statement: The impulse is 3.0 N·s.
(b) Given: impulse = 3.0 N·s; m = 2.4 kg; $v_i = -14$ m/s
Required: $\vec{v}_f$
Analysis: impulse = $m\Delta\vec{v}$

Solution: $\text{impulse} = m\Delta\vec{v}$

$$\text{impulse} = m(\vec{v}_{\text{f}} - \vec{v}_{\text{i}})$$

$$3.0\ \text{N}\cdot\text{s} = (2.4\ \text{kg})[v_{\text{f}} - (-14\ \text{m/s})]$$

$$1.25\ \text{m/s} = \vec{v}_{\text{f}} + 14\ \text{m/s}$$

$$\vec{v}_{\text{f}} = -13\ \text{m/s}$$

Statement: The final velocity of the object is −13 m/s.

42. (a) The first lily pad moves opposite to the frog's motion, and the second lily pad moves with the frog's motion.

(b) The first lily pad has a higher speed because of conservation of momentum. The momentum of the first lily pad must be equal and opposite to the momentum of the second lily pad plus the frog. To make up for the extra mass of the frog, the speed of the first lily pad must be faster.

43. Given: $m_1 = m$; $m_2 = \frac{2}{3}m_1 = \frac{2}{3}m$; $m_3 = \frac{1}{3}m_1 = \frac{1}{3}m$;

$\vec{v}_1$ = 12 km/s [forward] = 1.2×10^4 m/s [forward];

$\vec{v}_2 = 10.0$ km/s [backward] $= 1.00 \times 10^4$ m/s [backward] $= -1.00 \times 10^4$ m/s [forward]

Required: $\vec{v}_3$

Analysis: Momentum is conserved in this situation.

$$\vec{p}_{\text{i}} = \vec{p}_{\text{f}}$$

$$m_1\vec{v}_1 = m_2\vec{v}_2 + m_3\vec{v}_3$$

$$\vec{v}_3 = \frac{m_1\vec{v}_1 - m_2\vec{v}_2}{m_3}$$

Solution: $\vec{v}_3 = \dfrac{m_1\vec{v}_1 - m_2\vec{v}_2}{m_3}$

$$= \frac{\cancel{m}(1.2 \times 10^4\ \text{m/s}) - \frac{2}{3}\cancel{m}(-1.00 \times 10^4\ \text{m/s})}{\frac{1}{3}\cancel{m}}$$

$$\vec{v}_3 = 5.6 \times 10^4\ \text{m/s}$$

Statement: The rocket's velocity after shedding the rear stages is 56 km/s [forward].

44. Given: $m_1 = m_2 = 0.3$ kg; $E_{\text{k}_\text{T}} = 0.52$ J; $p_\text{T} = -0.12$ kg·m/s

Required: $\vec{v}_{\text{f}_1}$; $\vec{v}_{\text{f}_2}$

Analysis: Both momentum and kinetic energy are conserved in an elastic collision.

Solution: First consider momentum:

$$p_{\mathrm{T}} = -0.12 \text{ kg}\cdot\text{m/s}$$

$$m_1 v_{\mathrm{f}_1} + m_2 v_{\mathrm{f}_2} = -0.12 \text{ kg}\cdot\text{m/s}$$

$$(0.3 \text{ kg})v_{\mathrm{f}_1} + (0.3 \text{ kg})v_{\mathrm{f}_2} = -0.12 \text{ kg}\cdot\text{m/s}$$

$$v_{\mathrm{f}_1} + v_{\mathrm{f}_2} = -0.4 \text{ m/s}$$

$$v_{\mathrm{f}_1} = (-0.4 \text{ m/s}) - v_{\mathrm{f}_2}$$

Now consider kinetic energy:

$$E_{\mathrm{k}_{\mathrm{T}}} = 0.52 \text{ J}$$

$$\frac{1}{2}(m_1 v_{\mathrm{f}_1}^2 + m_2 v_{\mathrm{f}_2}^2) = 0.52 \text{ J}$$

$$(0.3 \text{ kg})v_{\mathrm{f}_1}^2 + (0.3 \text{ kg})v_{\mathrm{f}_2}^2 = 1.04 \text{ J}$$

$$v_{\mathrm{f}_1}^2 + v_{\mathrm{f}_2}^2 = \frac{1.04}{0.3} \text{ m}^2/\text{s}^2$$

Substitute the expression for v_{f_2} from above:

$$v_{\mathrm{f}_1}^2 + v_{\mathrm{f}_2}^2 = \frac{1.04}{0.3} \text{ m}^2/\text{s}^2$$

$$[(-0.4 \text{ m/s}) - v_{\mathrm{f}_2}]^2 + v_{\mathrm{f}_2}^2 = \frac{1.04}{0.3} \text{ m}^2/\text{s}^2$$

$$0.3\,[(0.16 \text{ m}^2/\text{s}^2) + (0.8 \text{ m/s})v_{\mathrm{f}_2} + v_{\mathrm{f}_2}^2] + 0.3v_{\mathrm{f}_2}^2 = 1.04 \text{ m}^2/\text{s}^2$$

$$(0.048 \text{ m}^2/\text{s}^2) + (0.24 \text{ m/s})v_{\mathrm{f}_2} + 0.3v_{\mathrm{f}_2}^2 + 0.3v_{\mathrm{f}_2}^2 = 1.04 \text{ m}^2/\text{s}^2$$

$$0.6v_{\mathrm{f}_2}^2 + (0.24 \text{ m/s})v_{\mathrm{f}_2} - (0.992 \text{ m}^2/\text{s}^2) = 0$$

Use the quadratic formula to solve for v_{f_2}:

$$\begin{aligned} v_{\mathrm{f}_2} &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-(0.24 \text{ m/s}) \pm \sqrt{(0.24 \text{ m/s})^2 - 4(0.6)(-0.992 \text{ m}^2/\text{s}^2)}}{2(0.6)} \end{aligned}$$

$$v_{\mathrm{f}_2} = 1.1 \text{ m/s or } -1.5 \text{ m/s}$$

Assume that v_{f_2} = 1.1 m/s. Substitute into the expression for $v_{\mathrm{f}1}$:

$$\begin{aligned} v_{\mathrm{f}_1} &= -0.4 \text{ m/s} - v_{\mathrm{f}_2} \\ &= -0.4 \text{ m/s} - 1.1 \text{ m/s} \\ v_{\mathrm{f}_1} &= -1.5 \text{ m/s} \end{aligned}$$

If you assume that v_{f_2} = −1.5 m/s, then v_{f_1} = 1.1 m/s. Either solution is correct, because they both give the same final momentum and final kinetic energy.

Statement: The final velocities of the carts must be 1.5 m/s [left] and 1.1 m/s [right].

45. Given: m_1 = 75 kg; m_2 = 0.02 kg; v_2 = 18 m/s
Required: v_1
Analysis: Momentum is conserved in this situation, so the momentum of the snowball equals the momentum of the boy.
Solution:

$$\begin{aligned} m_1 v_1 &= m_2 v_2 \\ (75\text{ kg})v_1 &= (0.02\text{ kg})(18\text{ m/s}) \\ v_1 &= \frac{(0.02\ \cancel{\text{kg}})(18\text{ m/s})}{75\ \cancel{\text{kg}}} \\ &= 4.8\times 10^{-3}\text{ m/s (one extra digit carried)} \\ v_1 &= 5\times 10^{-3}\text{ m/s} \end{aligned}$$

Statement: The boy's speed is 5×10^{-3} m/s.
46. (a) (i) There is no kinetic energy before the boy throws the snowball, because nothing is moving.
(ii) Given: m_1 = 75 kg; m_2 = 0.02 kg; $v_1 = 4.8 \times 10^{-3}$ m/s; v_2 = 18 m/s
Required: E_{k_T}
Analysis: The total kinetic energy of the system is the sum of the kinetic energy of the boy and the kinetic energy of the snowball.
Solution:

$$\begin{aligned} E_{k_T} &= \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 \\ &= \frac{1}{2}(75\text{ kg})(4.8\times 10^{-3}\text{ m/s})^2 + \frac{1}{2}(0.02\text{ kg})(18\text{ m/s})^2 \\ E_{k_T} &= 3\text{ J} \end{aligned}$$

Statement: The total kinetic energy after the boy throws the snowball is 3 J.
(b) Kinetic energy is not conserved in this case because the boy adds energy to the system by throwing the snowball.
(c) The energy in the boy's muscles is converted to kinetic energy.
47. (a) Given: m_1 = 78 kg; $E_{k_i} = 1.2 \times 10^5$ J; m_2 = 40.0 g = 0.0400 kg; $v_{f_1} = v_{i_1} - 0.1$ m/s; v_{f_2} = 0 m/s
Required: v_{f_2}
Analysis: Momentum is conserved in this situation. The total momentum before the performer punches the ball is the same as the total momentum after the performer punches the ball.

Solution:
$$p_{\text{T}_\text{i}} = p_{\text{T}_\text{f}}$$
$$m_1 v_{\text{i}_1} = m_1 v_{\text{f}_1} + m_2 v_{\text{f}_2}$$
$$m_1 v_{\text{i}_1} = m_1 (v_{\text{i}_1} - 0.1 \text{ m/s}) + m_2 v_{\text{f}_2}$$
$$m_1 v_{\text{i}_1} = m_1 v_{\text{i}_1} - (0.1 \text{ m/s}) m_1 + m_2 v_{\text{f}_2}$$
$$(0.1 \text{ m/s}) m_1 = m_2 v_{\text{f}_2}$$
$$v_{\text{f}_2} = \frac{(0.1 \text{ m/s}) m_1}{m_2}$$
$$= \frac{(0.1 \text{ m/s})(78 \cancel{\text{kg}})}{0.0400 \cancel{\text{kg}}}$$
$$= 195 \text{ m/s (two extra digits carried)}$$
$$v_{\text{f}_2} = 2 \times 10^2 \text{ m/s}$$

Statement: The speed of the beach ball is 2×10^2 m/s after the performer punches it.
(b) Given: v_i = 0 m/s; v_f = 195 m/s; Δt = 5 ms = 5×10^{-3} s; m = 40.0 g = 0.0400 kg
Required: F
Analysis: $F\Delta t = m\Delta v$
$$F = \frac{m\Delta v}{\Delta t}$$
Solution: $F = \dfrac{m\Delta v}{\Delta t}$
$$= \frac{(0.0400 \text{ kg})(195 \text{ m/s} - 0 \text{ m/s})}{5 \times 10^{-3} \text{ s}}$$
$$F = 2 \times 10^3 \text{ N}$$

Statement: The average force the performer exerted on the ball is 2×10^3 N .
(c) Given: m_1 = 78 kg; E_{k_i} = 1.2×10^5 J; $v_{\text{f}_1} = v_{\text{i}_1} - 0.1$ m/s

Required: v_{f_1}

Analysis: The performer's final speed is 0.1 m/s less than his initial speed. Determine his initial speed using his initial kinetic energy, and then subtract 0.1 m/s.

Solution:
$$E_{\text{k i}} = 1.2 \times 10^5 \text{ J}$$
$$\frac{1}{2}m_1 v_{\text{i}_1}^2 = 1.2 \times 10^5 \text{ J}$$
$$v_{\text{i}_1} = \sqrt{\frac{2(1.2 \times 10^5 \text{ J})}{m_1}}$$
$$= \sqrt{\frac{2(1.2 \times 10^5 \text{ J})}{78 \text{ kg}}}$$
$$v_{\text{i}_1} = 55.5 \text{ m/s}$$

Subtract 0.1 m/s:
$$v_{\text{f}_1} = v_{\text{i}_1} - 0.1 \text{ m/s}$$
$$= 55.5 \text{ m/s} - 0.1 \text{ m/s}$$
$$v_{\text{f}_1} = 55.4 \text{ m/s}$$

Statement: The final speed is 6×10^1 m/s.

(d) Yes, you can answer (a) and (b) without determining the performer's final speed, as shown in (a) and (b) above. In (a), expressing the performer's final speed in terms of his initial speed ($v_{\text{f}_1} = v_{\text{i}_1} - 0.1$ m/s) allows you to eliminate the performer's initial and final speeds as variables in the conservation of momentum equation, $m_1 v_{\text{i}_1} = m_1 v_{\text{f}_1} + m_2 v_{\text{f}_2}$, and then you can simply solve for the speed of the beach ball. In (b) you can use the impulse equation, $F\Delta t = m\Delta v$, to determine F, because you know that the beach ball was initially at rest, and you determined the final speed of the beach ball in (a), so the performer's final speed is not needed.

48. Given: m_1 = 810 kg; v_1 = 5.2 m/s; v_2 = 0 m/s; v_{f} = 4.85 m/s; perfectly inelastic collision

Required: m_2

Analysis: Only momentum is conserved in a perfectly inelastic collision.

$p_{\text{i}} = p_{\text{f}}$

Solution:
$$p_{\text{i}} = p_{\text{f}}$$
$$m_1 v_1 + m_2 v_2 = (m_1 + m_2)v_{\text{f}}$$
$$(810 \text{ kg})(5.2 \text{ m/s}) = (810 \text{ kg} + m_2)(4.85 \text{ m/s})$$
$$m_2 = \frac{(810 \text{ kg})(5.2 \text{ m/s})}{4.85 \text{ m/s}} - (810 \text{ kg})$$
$$m_2 = 58 \text{ kg}$$

Statement: The mass of the baby moose is 58 kg.

49. (a) Given: m_1 = 50.0 g = 0.0500 kg; v_1 = 0.30 m/s; m_2 = 100.0 g = 0.1000 kg; v_2 = 0.25 m/s

Required: v_{f}

Analysis: Momentum is conserved in this situation.

$p_{\text{i}} = p_{\text{f}}$

Solution:

$$p_\text{i} = p_\text{f}$$

$$m_1v_1 + m_2v_2 = (m_1 + m_2)v_\text{f}$$

$$(0.0500\ \cancel{\text{kg}})(0.30\text{ m/s}) + (0.1000\ \cancel{\text{kg}})(0.25\text{ m/s}) = (0.0500\ \cancel{\text{kg}} + 0.1000\ \cancel{\text{kg}})v_\text{f}$$

$$0.040\text{ m/s} = 0.1500v_\text{f}$$

$$v_\text{f} = 0.2667\text{ m/s (two extra digits carried)}$$

$$v_\text{f} = 0.27\text{ m/s}$$

Statement: The two blocks are moving at 0.27 m/s after the collision.
(b) Given: m_1 = 50.0 g = 0.0500 kg; v_1 = 0.30 m/s; m_2 = 100.0 g = 0.1000 kg; v_2 = 0.25 m/s; v_f = 0.2667 m/s
Required: ΔE_k
Analysis: Kinetic energy is not conserved in a perfectly inelastic collision. Calculate the change in energy from before the collision to after the collision.

$$E_\text{k} = \frac{1}{2}mv^2$$

$$\Delta E_\text{k} = E_\text{k f} - E_\text{k i}$$

$$\Delta E_\text{k} = \frac{1}{2}(m_1 + m_2)v_\text{f}^2 - \frac{1}{2}(m_1v_1^2 + m_2v_2^2)$$

Solution:

$$\begin{aligned}\Delta E_\text{k} &= \frac{1}{2}(m_1 + m_2)v_\text{f}^2 - \frac{1}{2}(m_1v_1^2 + m_2v_2^2)\\ &= \frac{1}{2}(0.0500\text{ kg} + 0.1000\text{ kg})(0.2667\text{ m/s})^2\\ &\qquad - \frac{1}{2}[(0.0500\text{ kg})(0.30\text{ m/s})^2 + (0.1000\text{ kg})(0.25\text{ m/s})^2]\\ \Delta E_\text{k} &= -4.0\times10^{-5}\text{ J}\end{aligned}$$

Statement: The kinetic energy lost is 4.0×10^{-5} J.
50. Given: m_1 = 810 kg; v_{i_1} = 5.2 m/s; m_2 = 58 kg; v_{i_2} = 0 m/s; v_{f_2} = 8.0 m/s
Required: $\vec{v}_{\text{f}_1}$
Analysis: Momentum is conserved in this situation.
$p_\text{i} = p_\text{f}$

Solution:

$$p_\text{i} = p_\text{f}$$

$$m_1v_{\text{i}_1} + m_2v_{\text{i}_2} = m_1v_{\text{f}_1} + m_2v_{\text{f}_2}$$

$$(810\text{ kg})(5.2\text{ m/s}) = (810\text{ kg})v_{\text{f}_1} + (58\text{ kg})(8.0\text{ m/s})$$

$$v_{\text{f}_1} = \frac{(810\text{ kg})(5.2\text{ m/s}) - (58\text{ kg})(8.0\text{ m/s})}{810\text{ kg}}$$

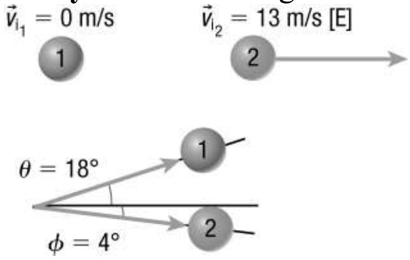
$$v_{\text{f}_1} = 4.6\text{ m/s}$$

Statement: The adult moose's final velocity is 4.6 m/s [in the original direction].

51. Given: $m_1 = m_2 = m$; $\vec{v}_{i_1} = 0$ m/s ; $\vec{v}_{i_2} = 13$ m/s [E] ; θ = [E 18° N]; ϕ = [E 4° S]

Required: $\vec{v}_{f_1}$; $\vec{v}_{f_2}$

Analysis: Draw a diagram.



Momentum is conserved in this situation. Consider the x-direction and the y-direction separately.

$p_i = p_f$

Solution: First consider the x-direction. Puck 2 has zero initial momentum in the x-direction.

$$p_{i_x} = p_{f_x}$$

$$m_1 v_{i_{1x}} + m_2 v_{i_{2x}} = m_1 v_{f_{1x}} + m_2 v_{f_{2x}}$$

$$\cancel{m}(13 \text{ m/s}) = \cancel{m} v_{f_1} \cos 18° + \cancel{m} v_{f_2} \cos 4°$$

$$13 \text{ m/s} = v_{f_1} \cos 18° + v_{f_2} \cos 4°$$

Now consider the y-direction. There is no initial momentum in the y-direction.

$$0 = p_{f_y}$$

$$0 = m_1 v_{f_{1y}} + m_2 v_{f_{2y}}$$

$$0 = \cancel{m} v_{f_1} \sin 18° + \cancel{m} v_{f_2} \sin(-4°)$$

$$0 = v_{f_1} \sin 18° - v_{f_2} \sin 4°$$

$$v_{f_1} = \frac{v_{f_2} \sin 4°}{\sin 18°}$$

Substitute this into the equation above:

$$13 \text{ m/s} = v_{f_1} \cos 18° + v_{f_2} \cos 4°$$

$$13 \text{ m/s} = \left(\frac{v_{f_2} \sin 4°}{\sin 18°} \right)(\cos 18°) + v_{f_2} \cos 4°$$

$$v_{f_2} = \frac{13 \text{ m/s}}{\frac{(\sin 4°)(\cos 18°)}{\sin 18°} + (\cos 4°)}$$

$$v_{f_2} = 10.7 \text{ m/s (one extra digit carried)}$$

Substitute this into the expression for v_{f_1}:

$$v_{f_1} = \frac{v_{f_2} \sin 4°}{\sin 18°}$$
$$= \frac{(10.7 \text{ m/s})(\sin 4°)}{\sin 18°}$$
$$v_{f_1} = 2.4 \text{ m/s}$$

Statement: After the collision, the velocity of puck 1 is 2.4 m/s [E 18° N], and the velocity of puck 2 is 11 m/s [E 4° S].

52. (a) Given: m = 1500 kg; Δd = 1.3 m; v_i = 32 m/s; v_f = 0 m/s

Required: Δt

Analysis: $\Delta d = \left(\frac{v_i + v_f}{2}\right)\Delta t$

Solution:
$$\Delta d = \left(\frac{v_i + v_f}{2}\right)\Delta t$$
$$1.3 \text{ m} = \left(\frac{32 \text{ m/s} + 0 \text{ m/s}}{2}\right)\Delta t$$
$$\Delta t = \frac{2(1.3 \cancel{\text{m}})}{32 \cancel{\text{m}}\text{/s}}$$
$$\Delta t = 8.1 \times 10^{-2} \text{ s}$$

Statement: The duration of the collision is 8.1×10^{-2} s.

(b) Given: m = 1500 kg; Δd = 1.3 m; v_i = 32 m/s; v_f = 0 m/s; Δt = 0.081 s

Required: $\vec{F}$

Analysis: Use the impulse equation:

$$m\Delta\vec{v} = \vec{F}\Delta t$$
$$\vec{F} = \frac{m\Delta\vec{v}}{\Delta t}$$
$$\vec{F} = \frac{m(\vec{v}_f - \vec{v}_i)}{\Delta t}$$

Solution:
$$\vec{F} = \frac{m(\vec{v}_f - \vec{v}_i)}{\Delta t}$$
$$= \frac{(1500 \text{ kg})(0 \text{ m/s} - 32 \text{ m/s})}{0.081 \text{ s}}$$
$$F = -5.9 \times 10^5 \text{ N}$$

Statement: The average force exerted on the car is 5.9×10^5 N [backward].

Evaluation

53. Answers may vary. Sample answers: A billiard ball drops onto a table and rebounds; the ball is an open system; the ball plus the table is a closed system.
Someone drops a plate on the floor of an airplane; the plate is an open system. Expand the system to interpret it as an inelastic collision between the airplane and the plate.

54. (a) Kinetic energy is conserved in a perfectly elastic head-on collision. Let the total kinetic energy at any time be a constant E_k. At any time other than the collision,

$$\frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2 = E_k$$

Multiply both sides of the equation by $\frac{2}{m}$:

$$\frac{\cancel{2}}{\cancel{m}}\left(\frac{\cancel{1}}{\cancel{2}}\cancel{m}v_1^2 + \frac{\cancel{1}}{\cancel{2}}\cancel{m}v_2^2\right) = \frac{2E_k}{m}$$

$$v_1^2 + v_2^2 = \frac{2E_k}{m}$$

Let C represent the constant $\frac{2E_k}{m}$. Therefore,

$$v_1^2 + v_2^2 = C$$

(b) Momentum is conserved in a perfectly elastic head-on collision. Let the total Momentum at any time be a constant p. At any time other than the collision,

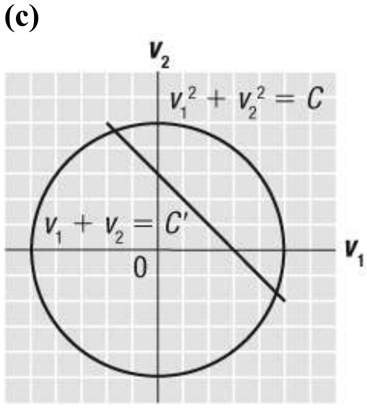
$$mv_1 + mv_2 = p$$

$$v_1 + v_2 = \frac{p}{m}$$

Let C' represent the constant $\frac{p}{m}$. Therefore,

$$v_1 + v_2 = C'$$

(c)



(d) The graphs intersect at two points. These points represent where the magnitudes of the total momentum and total kinetic energy are equal.

55. Consider the situation with two identical balls.

Given: $m_1 = m_2 = m$; $\vec{v}_{i_1} = v$; $\vec{v}_{i_2} = 0$ m/s

Required: $\vec{v}_{f_1}$; $\vec{v}_{f_2}$

Analysis: Use the equations for $\vec{v}_{i_1}$ and $\vec{v}_{f_2}$ when $\vec{v}_{i_2} = 0$ m/s:

$$\vec{v}_{f_1} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)\vec{v}_{i_1};\ \vec{v}_{f_2} = \left(\frac{2m_1}{m_1 + m_2}\right)\vec{v}_{i_1}$$

Solution:
$$\begin{aligned}\vec{v}_{f_1} &= \left(\frac{m_1 - m_2}{m_1 + m_2}\right)\vec{v}_{i_1}\\ &= \left(\frac{m - m}{m + m}\right)v\\ \vec{v}_{f_1} &= 0\end{aligned}$$

$$\begin{aligned}\vec{v}_{f_2} &= \left(\frac{2m_1}{m_1 + m_2}\right)\vec{v}_{i_1}\\ &= \left(\frac{2m}{m + m}\right)v\\ \vec{v}_{f_2} &= v\end{aligned}$$

Statement: The initial velocity of ball 1 transfers completely to ball 2, which means that the initial momentum and kinetic energy also transfer completely to ball 2. Ball 1 stops, and ball 2 moves forward with a velocity of v [forward].

Now consider the case of several balls in a row. Each ball after the first and before the last is blocked from moving by other balls and the sides of the gutter, so the momentum and kinetic energy are transferred ball to ball until the last ball, which then moves at velocity v [forward].

56. Predictions may vary. Sample answers:

(a) I predict that the final velocities of both objects will be forward, with the final velocity of the lighter object greater than the initial velocity of the heavier object, and the final velocity of the heavier object less than the initial velocity of the heavier object.

Given: $m_1 = 2m$; $m_2 = m$; $\vec{v}_{i_1} = 12$ m/s [E]; $\vec{v}_{i_2} = 0$ m/s

Required: $\vec{v}_{f_1}$; $\vec{v}_{f_2}$

Analysis: Use the equations for $\vec{v}_{f_1}$ and $\vec{v}_{f_2}$ when $\vec{v}_{i_2} = 0$ m/s:

$$\vec{v}_{f_1} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)\vec{v}_{i_1};\ \vec{v}_{f_2} = \left(\frac{2m_1}{m_1 + m_2}\right)\vec{v}_{i_1}$$

Solution: $\vec{v}_{f_1} = \left(\dfrac{m_1 - m_2}{m_1 + m_2}\right)\vec{v}_{i_1}$

$= \left(\dfrac{2m - m}{2m + m}\right)(12 \text{ m/s})$

$\vec{v}_{f_1} = 4.0 \text{ m/s}$

$$\vec{v}_{f_2} = \left(\frac{2m_1}{m_1 + m_2}\right)\vec{v}_{i_1}$$

$$= \left(\frac{2(2m)}{2m + m}\right)(12 \text{ m/s})$$

$$\vec{v}_{f_2} = 16 \text{ m/s}$$

Statement: The final velocity of the moving object is 4.0 m/s [E], and the final velocity of the stationary object is 16 m/s [E].

(b) I predict that the lighter, moving object will change directions and lose speed, while the heavier, stationary object will move in the original direction (forward) at a speed less than the initial speed of the moving object.

Given: $m_1 = m$; $m_2 = 2m$; $\vec{v}_{i_1} = 12$ m/s [forward]; $\vec{v}_{i_2} = 0$ m/s

Required: $\vec{v}_{f_1}$; $\vec{v}_{f_2}$

Analysis: Use the equations for $\vec{v}_{f_1}$ and $\vec{v}_{f_1}$ when $\vec{v}_{i_2} = 0$ m/s:

$$\vec{v}_{f_1} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)\vec{v}_{i_1}; \ \vec{v}_{f_2} = \left(\frac{2m_1}{m_1 + m_2}\right)\vec{v}_{i_1}$$

Solution: $\vec{v}_{f_1} = \left(\dfrac{m_1 - m_2}{m_1 + m_2}\right)\vec{v}_{i_1}$

$= \left(\dfrac{m - 2m}{m + 2m}\right)(12 \text{ m/s})$

$\vec{v}_{f_1} = -4.0 \text{ m/s}$

$$\vec{v}_{f_2} = \left(\frac{2m_1}{m_1 + m_2}\right)\vec{v}_{i_1}$$

$$= \left(\frac{2m}{2m + m}\right)(12 \text{ m/s})$$

$$\vec{v}_{f_2} = 8.0 \text{ m/s}$$

Statement: The final velocity of the moving object is 4.0 m/s [W], and the final velocity of the stationary object is 8.0 m/s [E].

(c) I predict that both objects will continue moving forward. The initially moving, much heavier object will lose a little bit of speed, and the much lighter object will gain a lot of speed.

Given: $m_1 = 106m$; $m_2 = m$; $\vec{v}_{i_1} = 12$ m/s [forward]; $\vec{v}_{i_2} = 0$ m/s

Required: $\vec{v}_{f_1}$; $\vec{v}_{f_2}$

Analysis: Use the equations for $\vec{v}_{f_1}$ and $\vec{v}_{f_2}$ when $\vec{v}_{i_2} = 0$ m/s:

$$\vec{v}_{f_1} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)\vec{v}_{i_1}; \ \vec{v}_{f_2} = \left(\frac{2m_1}{m_1 + m_2}\right)\vec{v}_{i_1}$$

Solution:
$$\begin{aligned}\vec{v}_{f_1} &= \left(\frac{m_1 - m_2}{m_1 + m_2}\right)\vec{v}_{i_1} \\ &= \left(\frac{106m - m}{106m + m}\right)(12\text{ m/s}) \\ \vec{v}_{f_1} &= 12\text{ m/s}\end{aligned}$$

$$\begin{aligned}\vec{v}_{f_2} &= \left(\frac{2m_1}{m_1 + m_2}\right)\vec{v}_{i_1} \\ &= \left(\frac{2(106m)}{106m + m}\right)(12\text{ m/s}) \\ \vec{v}_{f_2} &= 24\text{ m/s}\end{aligned}$$

Statement: The final velocity of the moving object is (slightly less than) 12 m/s [E], and the final velocity of the stationary object is 24 m/s [E].

(d) I predict that the lighter, moving object will reverse direction at a fairly high speed, and the heavier object will move forward at a slow speed.

Given: $m_1 = m$; $m_2 = 106m$; $\vec{v}_{i_1} = 12$ m/s [forward]; $\vec{v}_{i_2} = 0$ m/s

Required: $\vec{v}_{f_1}$; $\vec{v}_{f_2}$

Analysis: Use the equations for $\vec{v}_{f1}$ and $\vec{v}_{f2}$ when $\vec{v}_{i2} = 0$ m/s:

$$\vec{v}_{f_1} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)\vec{v}_{i_1}; \ \vec{v}_{f_2} = \left(\frac{2m_1}{m_1 + m_2}\right)\vec{v}_{i_1}$$

Solution:
$$\begin{aligned}\vec{v}_{f_1} &= \left(\frac{m_1 - m_2}{m_1 + m_2}\right)\vec{v}_{i_1} \\ &= \left(\frac{m - 106m}{m + 106m}\right)(12\text{ m/s}) \\ \vec{v}_{f_1} &= -12\text{ m/s}\end{aligned}$$

$$\vec{v}_{f_2} = \left(\frac{2m_1}{m_1 + m_2}\right)\vec{v}_{i_1}$$
$$= \left(\frac{2m}{m + 106m}\right)(12 \text{ m/s})$$
$$\vec{v}_{f_2} = 0.22 \text{ m/s}$$

Statement: The final velocity of the moving object is 12 m/s [W], and the final velocity of the stationary object is 0.22 m/s [E].

57. (a) Answers may vary. Sample answer: I predict that the final velocity of the first object will barely change, and the final velocity of the second object will be very fast—much faster than the initial velocity of the first object.

(b) Given: m_1 much greater than m_2; $\vec{v}_{i_1} = \vec{v}_i$

Required: $\vec{v}_{f_1}$; $\vec{v}_{f_2}$

Analysis: $\vec{v}_{f_1} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)\vec{v}_{i_1}$; $\vec{v}_{f_2} = \left(\frac{2m_1}{m_1 + m_2}\right)\vec{v}_{i_1}$

Solution: $\vec{v}_{f_1} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)\vec{v}_{i_1}$ $\quad$ $\vec{v}_{f_2} = \left(\frac{2m_1}{m_1 + m_2}\right)\vec{v}_{i_1}$

Since m_1 is much greater than m_2, $m_1 + m_2 \approx m_1$ and $m_1 - m_2 \approx m_1$. Therefore,

$$\vec{v}_{f_1} \approx \left(\frac{\cancel{m_1}}{\cancel{m_1}}\right)\vec{v}_{i_1} \qquad \vec{v}_{f_2} \approx \left(\frac{2\cancel{m_1}}{\cancel{m_1}}\right)\vec{v}_{i_1}$$

$$\vec{v}_{f_1} \approx \vec{v}_i \qquad \vec{v}_{f_2} \approx 2\vec{v}_{i_1}$$

Statement: The final velocity of the much more massive object is equal to its initial velocity. The final velocity of the very light object is twice the initial velocity of the much more massive object.

(c) Answers may vary. Sample answer: The results match my prediction.

Reflect on Your Learning

58. Students should describe what they found most surprising and most interesting about collisions.

59. (a) Answers may vary. Sample answer: Yes, I think I could explain momentum to a fellow student who has not taken physics. I would use real-life examples to communicate the concept. For example, I would ask them if they stood in front of a car that was rolling down a hill, would they be able to stop the car, and why not? (The car has a lot of momentum and a large force will be needed to bring it to a stop.) I would explain that both mass and speed affect momentum (I would probably avoid vectors in my explanation). A moving object with a lot of mass has more momentum, and the faster an object is moving, the more momentum it has, so momentum is determined by both mass and speed—in fact, it is the product of mass and speed. I would tell them that momentum describes not only how fast an object is moving, but also the effect the object has on other objects in its path of motion.

(b) Answers may vary. Sample answer: There are a couple of things about momentum and conservation of momentum that still confuse me. Sometimes I am not sure which direction to select as the positive reference direction when solving questions involving momentum. I know that in the end it does not matter which direction I chose to be positive, as long as I stick to that throughout the calculation, but I know that sometimes a different choice of direction might make calculations easier, and I am not always sure what that would be until after I have finished the calculations. Also, I like to use real-life situations to visualize physics concepts, but in a lot of real-world situations, even though momentum is conserved, you can't observe it because of external forces like friction, so sometimes when I try to use real-life situations I am familiar with to understand momentum problems it doesn't work that well.

Research

60. Answers may vary. Sample answer: Newton's cradle is a device that demonstrates the law of conservation of momentum and the law of conservation of energy. The device consists of a number of identical steel balls (usually an odd number) suspended from a sturdy frame. When the ball at one end is pulled away from the others and released, it will swing and hit the ball in front of it. The second ball will hit the third ball, and the third ball will hit the fourth ball, and so on until the last ball is hit. After the last ball is hit, it returns and hits the previous ball and the process is reversed back to the original ball. This process repeats until it finally stops due to losses from friction and the elasticity of the balls. Steel balls are used since they are highly elastic, so only a small amount of energy is lost in the collision.

When the end ball is pulled up and released, it swings and hits the next ball. The energy and momentum from that ball is transmitted through the balls at rest, all the way to the ball at the other end. The ball at the other end is sent forward with the same speed the first ball had because of the force of the first collision. The last ball reaches its peak and swings back down to hit the balls at rest and ultimately the first ball upward.

The last ball is knocked away from the others with the same speed the first ball had initially, while all of the other balls remain nearly at rest. Even if two balls are pulled back and allowed to hit the others, two balls are knocked from the other end and all of the other balls remain nearly at rest. This is the "standard" behaviour under ideal conditions (no mass differences between balls).

The law of conservation of energy applies where the kinetic energy of the moving ball(s) on impact equals the kinetic energy of the ball(s) moving toward the other end.

The law of conservation of momentum applies where the momentum of the balls on impact equals the momentum of the remaining balls after impact.

Because there is the conservation of energy and momentum in the system, the number of balls initially pulled up and allowed to hit the others will equal the number of balls knocked out at the other end.

61. Answers may vary. Sample answer: A Galilean cannon consists of a stack of balls, with the heaviest at the base and the lightest at the top. The stack of balls can be dropped to hit the ground so all of the kinetic energy in the lower balls will be transferred to the ball at the top. The ball at the top will rebound to a height that is many times the height it was dropped from.

A basketball and tennis ball can be used to demonstrate the Galilean cannon concept by stacking the tennis ball on top of the basketball. When the pair is dropped to the ground, the tennis ball will rebound to a height that is many times the height it was dropped from.

The law of conservation of momentum is observed in this demonstration. When the balls are dropped, they fall and build up speed. When the basketball hits the ground it rebounds and collides with the tennis ball. Both balls are moving at the same speed before the collision but the basketball has a larger mass, thus has more momentum than the lighter tennis ball. When the basketball transfers its momentum to the tennis ball during the collision, it will cause the tennis ball to rebound much higher than the height it was dropped from.

62. Answers may vary. Sample answer: Skydivers and paratroopers both use parachutes to land safely on the ground. Skydivers and paratroopers keep their knees bent and roll over to the side upon impact with the ground. Impulse is the product of force and time so extending the time of a collision will decrease the impact force. Landing with bent knees lengthens the amount of time force is experienced by the skydivers/paratroopers and reduces the force experienced. Landing with a straight knee will transmit impact to the skydivers'/paratroopers' spine, neck, ankles, hips, and knee joints. Landing with bent knees absorbs more impact through the muscles.

Conservative forces mean the work done by those forces depend on the initial and final conditions and is path independent. An example is gravity. Non-conservative forces mean the work done by those forces depends on the path taken between the initial and final positions of the mass. An example is air resistance. By ignoring any non-conservative forces, the law of conservation of energy states that the sum of the gravitation potential energy and kinetic energy of the sky divers/paratroopers is the same at the start and end of the jump.

The work–energy theorem states that the total work is equal to the change in kinetic energy. If air resistance is ignored, the work done by the parachute is equal to the changes in the kinetic energy of the skydivers/paratroopers during the jump.

63. Answers may vary. Sample answer: The height the bowling ball is dropped from determines the amount of potential energy in the ball. If the ball is bowled straight, then the potential energy will not affect the game and dropping the ball from a high height just makes a loud sound.

A straight-bowler should just release the ball as close to the lane as possible. Regardless of how long the ball is in the air, the ball will reach the pins in the same amount of time because the horizontal velocity is independent of the vertical velocity. Releasing the ball with a force that is perfectly parallel to the lane will result in the maximum impact.

The horizontal kinetic energy is the energy that the ball has to collide with the pins. Hooking the ball means the ball curves into the pins. Hooking is a good strategy because a ball bowled straight down the lane does not efficiently distribute its impact.

A ball that hits the head pin and a neighbouring pin at 30º will distribute the impact force evenly amongst the pins.

A strike requires the ball to hit one of the "pockets" halfway between the head pin and the pins on either side of it.

By hooking the ball, the ball impacts the head pin at a glancing collision and some of its momentum goes to the pin. A hook ball rebounds less than a straight ball and will continue moving forward after the collision, hitting other pins.

High-performance bowling balls have an asymmetrical core in the ball, causing it to naturally hook. Being able to hook the ball would make the game easier and allows for more strikes. Pins are spaced 12 inches apart in an equilateral-triangle formation. A regulation bowling ball has a radius of 4.25 inches.

64. Answers may vary. Sample answer: Crumple zones are designed to dissipate the force of a crash before it gets transferred onto the passengers. Cars have two crumple zones—one in the front and one in the back. Crumple zones use materials that compress during collisions to absorb the energy of the collision. A heavier car requires a longer crumple zone. The crumple zones lengthen the time it takes to change the car's momentum and will reduce the amount of force that is passed onto the passengers. The car is less likely to rebound on impact because it crumples and minimizes the momentum change and impulse. Crumple zones absorb some of the impact and transform some of the energy into sound and heat.

A frontal offset test places a dummy in the driver seat. A vehicle travels at 40 mph toward a barrier made of aluminum honeycomb. The test is equivalent to a frontal crash by two vehicles of the same weight, each going 40 mph. Safety devices such as crumple zones, airbags, and seat beats are common parameters varied in a crash test. The frontal ratings are determined by looking at the structural performance, injury measures, and dummy movement. A driver of a vehicle with a good rating means that he/she is 46 % less likely to die in a frontal crash compared to a driver of a vehicle with a poor rating. A driver of a vehicle with an acceptable or marginal rating is 33 % less likely to die.

65. Answers may vary. Sample answer: A five-story building can be levelled using excavators and wrecking balls. A 20-story building is levelled using explosive demolition. If a building is surrounded by other buildings, the demolition requires an implosion so that the building collapses straight down onto the base of the building. Explosives are loaded at different floors of the building and set off simultaneously, so the building falls down at multiple points. Sometimes 3-D computer models of the building may be used to test out how the blast will be conducted. The building can also be tipped over during demolition onto a parking lot or an open area. Explosives are detonated on the north side of the building to topple the building to the north.

Steel cables can be used to support columns in the building to ensure that the building falls in a controlled way. Concrete columns can be demolished using dynamite. Steel columns can be demolished using cyclotrimethylenetrinitramine (RDX). An example of improving demolition technology is preventing contaminants from being released during demolition by using real-time monitoring for demolishing contaminated structures.

66. Answers may vary. Sample answer: Both the car and child have the same momentum as the car travels. When the car collides, an unrestrained child will continue to travel until he/she hits something to stop that momentum.

In a collision, the change in momentum of the vehicle results in a large impulse. The child car seat restrains the child and increases the stopping distance and decreases the impact force.

A rear-facing car seat can protect the child's head, neck and spine in a frontal crash. The back of the rear-facing car seat absorbs the energy of the impact forces. The proper

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child car seat size is important, to ensure the child is fully restrained. The weight and height of the child must not exceed the child car seat.

The child car seat should be made of suitable padding material which allows some compression to absorbing the impact force of a collision. Design improvements include using restraints that can stretch a bit to lengthen the slowing-down period, and well as increasing the padding. Also, child car seats that are more rigidly attached to the vehicle using supporting rods or legs can reduce the amount the seat moves during a collision.

67. Answers may vary. Sample answer:

To test the protective systems and safety of cars, engineers and scientists developed test subjects (crash test dummies) that are similar to human drivers and passengers. The dummies are used to measure the human injury potential in vehicle collisions. They give vehicle designers a safe and repeatable test instrument.

Crash test dummies are modelled after male and female portions of the U.S. adult populations including: mid-size adult males, small adult females, and large adult males. Children dummies are modelled after the median heights and weights of children in specific age groups, but without regard to sex. Dummies are made to include heads, necks, joints, ribs, knees, thorax, spine, lumbar, pelvis, and legs. Early dummies only provided acceleration data which would correspond to a human response. Stresses experienced by humans that could result in severe or fatal injury were not measured.

Dummies can be classified as frontal impact, side impact, rear impact, children, and pedestrian, and each type of dummy is suited for different types of collision testing. The frontal impact dummies can be used to test restraints such as seat belts and airbags. Side impact dummies can be used to test the effectiveness of strengthened side doors and side curtain airbags to protect the head and upper body. Rear impact dummies can be used to test the effectiveness of headrests on the seats in preventing excess motion of the head and neck. Dummies modelled after children can be used to test the effectiveness of child restraints such as baby carriers, child seats, and booster seats. Pedestrian dummies can be used to determine the impact of injuries caused by rigid hood ornaments, bumper impact, high grilles, and hood/windshield impact.

Crash test dummies have played a major role in making cars safer. In the 1930s, the fatality rate was 15.6 per 100 million vehicle miles travelled. Presently the fatality rate is 1.8 per 100 million vehicle miles travelled, even though there are millions of more cars driven.

68. Answers may vary. Sample answers:

(a) Rube Goldberg was an engineering graduate from the University of California Berkley. He worked as an engineer for the City of San Francisco Water and Sewers Department. He later worked in the sports department of the *San Francisco Chronicle* where he submitted drawings and cartoons. He moved from San Francisco to New York to work for the *Evening Mail* drawing cartoons. His engineering background provided him with the knowledge and inspiration for his cartoons of "inventions."

(b) The core principle to Goldberg's designs is the law of conservation of energy. The design stores potential energy and then converts it into other forms of energy.

The "self-rolling rug" transforms the potential energy of the house maid as she falls back into her chair into kinetic energy through a series of steps performed by various animals until the elephant finally rolls up the rug.

A “simple fly swatter” transforms the elastic potential energy stored in the elastic bean shooter into other forms of energy until the fly is finally stomped on by a shoe.
(c) Although Rube Goldberg never built his machines, his cartoons inspire aspiring engineers and scientists worldwide. There are annual contests to build any of Rube Goldberg’s machines.