SPH4U 5.1 Momentum and Impulse

1. Momentum

Linear momentum:	a measure of motion that is conserved in collisions. $\vec{p} = m\vec{v}$. Units: kg·m/s.

Calculate the momentum of a 2.5 kg rabbit travelling with a velocity of 2.0 m/s [E].

$\vec{p} = m\vec{v} = 2.5(2.0) = 5.0$ kg·m/s [E].

Calculate the momentum of a 5.0 kg groundhog travelling with a velocity of 1.0 m/s [S].

$\vec{p} = m\vec{v} = 5.0(1.0) = 5.0$ kg·m/s [S].

Compare the momentum and kinetic energies of the rabbit and the groundhog.

Rabbit: $\vec{p} = 5.0$ kg·m/s [E].

$E_k = \frac{1}{2}mv^2$
$= \frac{1}{2}(2.5)(2.0)^2$
$= 5.0$ J.

Groundhog: $\vec{p} = 5.0$ kg·m/s [S]

$E_k = \frac{1}{2}mv^2$
$= \frac{1}{2}(5.0)(1)^2$
$= 2.5$ J.

2. Impulse

Impulse:	change in momentum. $\Delta\vec{p} = m(\vec{v}_f - \vec{v}_i) = \vec{F}\Delta t$ Units: N·s

A 0.160 kg puck is travelling at 5.0 m/s [N]. A slapshot produces a collision that lasts for 0.0020 s and gives the puck a velocity of 40.0 m/s [S].

a. Calculate the impulse imparted by the hockey stick.

$\Delta\vec{p} = m(\vec{v}_f - \vec{v}_i) = 0.160(40\,[S] + 5\,[S])$
$= 0.160(45)$
$= 7.2$ N·s [S].

b. Determine the average ~~force applied by the stick~~ to the puck.

$\Delta\vec{p} = \vec{F}\Delta t = 7.2$ N·s [S].

$\vec{F} = \frac{\Delta\vec{p}}{\Delta t} = \frac{7.2}{0.002} = 3600$ N [S].

A volleyball player starts a serve by throwing the ball vertically upward. The 260 g volleyball comes to rest at its maximum height. The server then hits it and exerts an average horizontal force of magnitude 6.5 N on the ball.

a. Determine the speed of the ball after the player hits it if the average force is exerted on the ball for 615 ms.

$$\Delta\vec{p} = m(\vec{v}_f - \vec{v}_i) = \vec{F}\Delta t$$

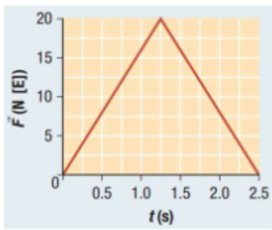
$$\vec{v}_f - \vec{v}_i = \frac{\vec{F}\Delta t}{m}$$

$$\vec{v}_f = \frac{\vec{F}\Delta t}{m} + \vec{v}_i = \frac{6.5(0.615)}{0.260} + 0 = \underline{15\ \text{m/s}}.$$

b. On the next serve, the volleyball player hits the ball with the same amount of horizontal force, but the time interval is 875 ms. Determine the speed of the ball.

$$\vec{v}_f = \frac{\vec{F}\Delta t}{m} + \vec{v}_i = \frac{6.5(0.875)}{0.260} + 0 = \underline{22\ \text{m/s}}.$$

Two figure skaters approach each other in a straight line. They meet hand to hand and then push off in opposite directions. The increase and decrease of force are both linear, which produces a force-time curve that is in the shape of a triangle. Determine the impulse for this interaction.



$$\Delta\vec{p} = \vec{F}\Delta t$$

→ area under the graph.

$$\Delta\vec{p} = \frac{bh}{2} = \frac{(2.5)(20)}{2} = \underline{25\ \text{N}\cdot\text{s}}.$$

Homework: page 227: #1, 4, 6, 10-11

SPH4U 5.2 Conservation of Momentum in One Direction

1. The law of conservation of momentum

Law of conservation of momentum:	When two objects collide in an isolated system, the total momentum is conserved.
equation	$\vec{p}_f = \vec{p}_i$ $\quad m_1\vec{v}_{i1} + m_2\vec{v}_{i2} = m_1\vec{v}_{f1} + m_2\vec{v}_{f2}$
interactions	collisions and explosions.
real life	momentum is lost to outside influences.

A hockey player of mass 97 kg skating with a velocity of 9.2 m/s [S] collides head-on with a defence player of mass 105 kg travelling with a velocity 6.5 m/s [N]. An instant after impact, the two skate together in the same direction. Calculate the final velocity of the two hockey players.

$\vec{p}_f = \vec{p}_i$

$\vec{p}_i = m_1\vec{v}_{i1} + m_2\vec{v}_{i2}$

$= 97(9.2) + 105(-6.5)$

$= 210$ kg·m/s [S].

$\vec{p}_f = 210$ kg·m/s [S].

$= (m_1 + m_2)\vec{v}_f$

$\vec{v}_f = \dfrac{\vec{p}_f}{m_1 + m_2}$

$= \dfrac{210}{97+105}$

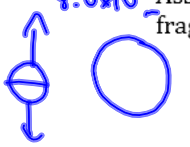
$= 1.0$ m/s [S]

In a science fiction novel, a large asteroid is approaching Earth. Scientists decide to explode it into two halves before it hits the Earth. The asteroid has a mass of 2.4 x 10^9 kg. For each half to safely miss Earth, they must travel a minimum of 8.0 x 10^6 m at a right angle away from Earth within 24 h.

24h.

8.0×10^6

Assume this is a one-dimensional problem. The magnitude of the impulse applied to each fragment is the same.



a. Calculate the momentum of each part of the asteroid after the explosion.

$$\vec{v} = \frac{8.0\times10^6\ m}{24\ h} \times \frac{1\ h}{60\ min} \times \frac{1\ min}{60\ s} = 92.6\ m/s.$$

$\vec{p} = m\vec{v}$ $\quad m = 1.2\times10^9$ kg.

$= (1.2\times10^9)(92.6)$

$= 1.1\times10^{11}$ kg·m/s.

b. Determine the impulse delivered to each part by the explosion.

$\Delta\vec{p} = \vec{p}_f - \vec{p}_i$

$= 1.1\times10^{11} - 0$

$= 1.1\times10^{11}$ kg·m/s.

$= 1.1\times10^{11}$ N·s

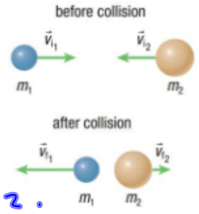
Rocket propulsion:	relies on conservation of momentum. mass decreases as you go up, which requires calculus.

Homework: page 232: #1, 3-4, 6-7

SPH4U 5.3 Collisions

1. Perfectly elastic collisions in one dimension

Elastic collision:	a collision where momentum and kinetic energy are conserved.
perfectly elastic collision	ideal collision with no external forces.
equations	$\vec{p}_{i1}+\vec{p}_{i2}=\vec{p}_{f1}+\vec{p}_{f2}$. $E_{ki1}+E_{ki2}=E_{kf1}+E_{kf2}$.



Suppose you have two balls with different masses involved in a perfectly elastic collision. Ball 1, with mass m_1 = 0.26 kg travelling at a velocity v_1 = 1.3 m/s [right], collides head-on with stationary ball 2, which has a mass of m_2 = 0.15 kg. Determine the final velocities of both balls after the collision.

This problem is way too much work. And 5.4 gives us a much easier way to solve these.
You don't need to do this.

Momentum: $m_1v_{i1}+\cancel{m_2v_{i2}}^{\,0}=m_1v_{f1}+m_2v_{f2}$

$$v_{f1}=\frac{m_1v_{i1}-m_2v_{f2}}{m_1}=v_{i1}-\frac{m_2}{m_1}v_{f2}$$

$$v_{f1}=1.3\text{ m/s}-0.58v_{f2}$$

E_k: $\frac{1}{2}mv_{i1}^2+\cancel{\frac{1}{2}mv_{i2}}^{\,0}=\frac{1}{2}mv_{f1}^2+\frac{1}{2}mv_{f2}^2$

→ substitute in our v_{f1} from above.

. . .

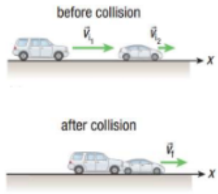
$$0=-0.39v_{f2}+0.24v_{f2}^2$$

$$0=v_{f2}(-0.39+0.24v_{f2})$$

$v_{f2}=0$, $v_{f2}=\frac{0.39}{0.24}=1.63\text{ m/s}.$

$$v_{f1}=1.3\text{ m/s}-0.58v_{f2}$$
$$=1.3-0.58(1.63)$$
$$=0.35\text{ m/s}.$$

2. Perfectly inelastic collisions in one dimension



Inelastic collision:	a collision where momentum is conserved, but kinetic energy is <u>not</u>.
perfectly inelastic collision	ideal collision where two objects stick together, have the same v_f.
equation	$\vec{v}_f = \dfrac{m_1\vec{v}_{i1} + m_2\vec{v}_{i2}}{m_1+m_2}$.

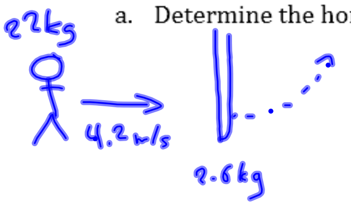
A large car of mass 2500 kg and a small car of mass 1200 kg are coasting at constant velocities along a straight road. The small car is moving at 10.0 m/s [W], and the large car is behind it moving at 40.0 m/s [W], catching up to it. When they collide, the cars lock bumpers. Determine the velocity of the cars just after the collision.

$$\vec{v}_f = \frac{m_1\vec{v}_{i1} + m_2\vec{v}_{i2}}{m_1+m_2} = \frac{2500(40)+1200(10)}{2500+1200}$$

$$= 30.3 \text{ m/s [W]}.$$

A child with a mass of 22 kg runs at a horizontal velocity of 4.2 m/s [forward] and jumps onto a stationary rope swing of mass 2.6 kg. The child "sticks" on the rope swing and swings forward.

a. Determine the horizontal velocity of the child plus the swing just after impact.



$$\vec{v}_f = \frac{m_1\vec{v}_{i1} + m_2\vec{v}_{i2}}{m_1+m_2}$$

$$= \frac{22(4.2)+0}{22+2.6} = 3.76 \text{ m/s [forward]}.$$

b. How high do the child and swing rise?

$E_k = \Delta E_g$

$$\tfrac{1}{2}\cancel{(m_1+m_2)}v_f^2 = \cancel{(m_1+m_2)}g\Delta y$$

$$\Delta y = \frac{v_f^2}{2g} = \frac{3.76^2}{2(9.8)} = 0.72\text{m}$$

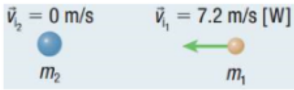
Homework: page 239: #1-2, 7-8

SPH4U 5.4 Head-on Elastic Collisions

1. Perfectly elastic head-on collisions in one dimension

Head-on elastic collision:	an impact where 2 objects approach from opposite directions, and $\vec{p}$ and E_k are conserved.
equation for $\vec{v}_{f1}$	$\vec{v}_{f1} = \left(\frac{m_1-m_2}{m_1+m_2}\right)\vec{v}_{i1} + \left(\frac{2m_2}{m_1+m_2}\right)\vec{v}_{i2}$
equation for $\vec{v}_{f2}$	$\vec{v}_{f2} = \left(\frac{m_2-m_1}{m_1+m_2}\right)\vec{v}_{i2} + \left(\frac{2m_1}{m_1+m_2}\right)\vec{v}_{i1}$
special case: $\vec{v}_{i2} = 0$	$\vec{v}_{f1} = \left(\frac{m_1-m_2}{m_1+m_2}\right)\vec{v}_{i1}$ $\quad \vec{v}_{f2} = \left(\frac{2m_1}{m_1+m_2}\right)\vec{v}_{i1}$

Consider an elastic head-on collision between two balls of different masses, as shown. The mass of ball 1 is 1.2 kg, and its velocity is 7.2 m/s [W]. The mass of ball 2 is 3.6 kg, and ball 2 is initially at rest. Determine the final velocity of each ball after the collision.



$$\vec{v}_{f1} = \left(\frac{m_1-m_2}{m_1+m_2}\right)\vec{v}_{i1} = \left(\frac{1.2-3.6}{1.2+3.6}\right)(-7.2) = \underline{3.6\text{ m/s [E]}}.$$

$$\vec{v}_{f2} = \left(\frac{2m_2}{m_1+m_2}\right)\vec{v}_{i1} = \left(\frac{2(1.2)}{1.2+3.6}\right)(-7.2) = -3.6 = \underline{3.6\text{ m/s [W]}}.$$

In a bumper car ride, bumper car 1 (m = 350 kg, $\vec{v}_i$ = 4.0 m/s [E]) hits bumper car 2 (m = 250 kg, $\vec{v}_i$ = 2.0 m/s [W]). Calculate the final velocity of each bumper car immediately after the collision.

$$\vec{v}_{f1} = \left(\frac{m_1-m_2}{m_1+m_2}\right)\vec{v}_{i1} + \left(\frac{2m_2}{m_1+m_2}\right)\vec{v}_{i2} = \dots = -1.0\text{ m/s} = 1.0\text{ m/s [W]}.$$

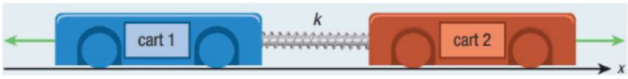
$$\vec{v}_{f2} = \left(\frac{m_2-m_1}{m_1+m_2}\right)\vec{v}_{i2} + \left(\frac{2m_1}{m_1+m_2}\right)\vec{v}_{i1} = \dots = \underline{5.0\text{ m/s [E]}}.$$

Special case: $m_1 = m_2$	$\vec{v}_{f1} = \left(\frac{m-m}{m+m}\right)\vec{v}_{i1} + \left(\frac{2m}{m+m}\right)\vec{v}_{i2}$ $= 0 + \left(\frac{2m}{2m}\right)\vec{v}_{i2}$ $\underline{\vec{v}_{f1} = \vec{v}_{i2}}$	$\vec{v}_{f2} = \left(\frac{m-m}{m+m}\right)\vec{v}_{i2} + \frac{2m}{2m}\vec{v}_{i1}$ $= 0 + \vec{v}_{i1}$ $\vec{v}_{f2} = \vec{v}_{i1}$
Special case: Light object + Stationary, very heavy object	$\vec{v}_{f1} = \left(\frac{0-m_2}{0+m_2}\right)\vec{v}_{i1}$ $= \frac{-m_2}{m_2}\vec{v}_{i1}$ $\underline{\vec{v}_{f1} = -\vec{v}_{i1}}$	$\vec{v}_{f2} = \left(\frac{2(0)}{0+m_2}\right)\vec{v}_{i1}$ $\underline{\vec{v}_{f2} = 0}$

2. Conservation of mechanical energy

Collision with a spring:	mechanical energy → E_e → mechanical energy. $\frac{1}{2}m_1v_{i1}^2 + \frac{1}{2}m_2v_{i2}^2 = \frac{1}{2}m_1v_{f1}^2 + \frac{1}{2}m_2v_{f2}^2 + \frac{1}{2}kx^2$
maximum compression	happens when $\vec{v}_{f1} = \vec{v}_{f2}$ (when spring isn't pushing apart). $\frac{1}{2}m_1v_{i1}^2 + \frac{1}{2}m_2v_{i2}^2 = \frac{1}{2}(m_1+m_2)v_f^2 + \frac{1}{2}kx^2$

Dynamics cart 1 has a mass of 1.8 kg and is moving with a velocity of 4.0 m/s [right] along a frictionless track. Dynamics cart 2 has a mass of 2.2 kg and is moving at 6.0 m/s [left]. The carts collide in a head-on elastic collision cushioned by a spring with spring constant k = 8.0 x 10^4 N/m.



a. Determine the compression of the spring, in centimetres, during the collision when cart 2 is moving at 4.0 m/s [left].

$m_1v_{i1} + m_2v_{i2} = m_1v_{f1} + m_2v_{f2}$

$v_{f1} = \dfrac{m_1v_{i1} + m_2v_{i2} - m_2v_{f2}}{m_1} = \dfrac{1.8(4.0) + 2.2(-6.0) - 2.2(-4.0)}{1.8} = 1.56\text{ m/s}.$

$\cancel{\frac{1}{2}}m_1v_{i1}^2 + \cancel{\frac{1}{2}}m_2v_{i2}^2 = \cancel{\frac{1}{2}}m_1v_{f1}^2 + \cancel{\frac{1}{2}}m_2v_{f2}^2 + \cancel{\frac{1}{2}}kx^2$

$x = \sqrt{\dfrac{m_1v_{i1}^2 + m_2v_{i2}^2 - m_1v_{f1}^2 - m_2v_{f2}^2}{k}} = 2.9\times10^{-2}\text{ m}$

$= 2.9\text{ cm}.$

b. Calculate the maximum compression of the spring, in centimetres.

$m_1v_{i1} + m_2v_{i2} = (m_1+m_2)v_f$

$v_f = \dfrac{m_1v_{i1} + m_2v_{i2}}{m_1+m_2} = \dfrac{1.8(4.0) + 2.2(-6.0)}{1.8+2.2} = -1.5\text{ m/s}.$

$x = \sqrt{\dfrac{m_1v_{i1}^2 + m_2v_{i2}^2 - (m_1+m_2)v_f^2}{k}} = 3.5\times10^{-2}\text{ m}.$

$= 3.5\text{ cm}$

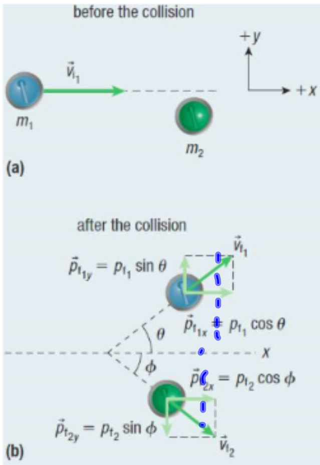
Homework: page 248: #1-3, 5

SPH4U 5.5 Collisions in Two Dimensions: Glancing Collisions

1. Components of momentum

Components of momentum:	can consider $\vec{\Delta p_x}$ $\vec{\Delta p_x}$ indendently. $\vec{\Delta p_x} = \sum \vec{F_x} \Delta t$ $\quad \vec{p_{1ix}} + \vec{p_{2ix}} = \vec{p_{1fx}} + \vec{p_{2fx}}$ $\vec{\Delta p_y} = \sum \vec{F_y} \Delta t$ $\quad \vec{p_{1iy}} + \vec{p_{2iy}} = \vec{p_{1fy}} + \vec{p_{2fy}}$
Glancing collision:	a collision where the first object, after impact, travels at an angle from its original direction.

In a game of curling, a collision occurs between two stones of equal mass. The object stone is initally at rest. After the collision, the stone that is thrown has a speed of 0.56 m/s at an angle of θ, and the object stone has a velocity of 0.42 m/s at an angle of 30.0° from the original direction of motion. Determine the initial velocity of the thrown stone.



$$p_{Tiy} = p_{Tfy} = 0$$

$$p_{Tfy} = m v_{f1} \sin\theta - m v_{f2} \sin\phi = 0.$$

$$\cancel{m} v_{f1} \sin\theta = \cancel{m} v_{f2} \sin\phi$$

$$\theta = \sin^{-1}\left(\frac{v_{f2} \sin\phi}{v_{f1}}\right) = \sin^{-1}\left(\frac{0.42 \sin 30°}{0.56}\right)$$

$$= 22.0°.$$

$$p_{Tix} = p_{Tfx}$$

$$\cancel{m_1} v_{1ix} + \cancel{m_2 v_{2ix}} = \cancel{m} v_{f1x} + \cancel{m} v_{f2x}.$$

$$v_{1ix} = v_{f1x} + v_{f2x}$$

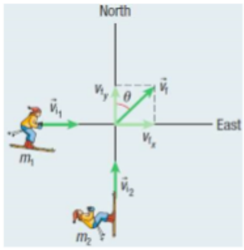
$$= v_{f1} \cos\theta + v_{f2} \cos\phi$$

$$= 0.56 \cos 22° + 0.42 \cos 30°$$

$$= 0.88 \text{ m/s [right]}.$$

2. Inelastic glancing collisions

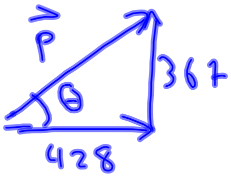
Two cross-country skiers collide at a right angle, locking their skis together. Skier 1 has a mass of 84 kg and was travelling east, and skier 2 has a mass of 72 kg and was travelling north. Both skiers were travelling with an initial speed of 5.1 m/s. Calculate the final velocity of the two skiers.



$$\vec{p}_{Tf} = \vec{p}_{Ti} = \vec{p}_1 + \vec{p}_2$$

$$\vec{p}_1 = m_1\vec{v}_{i1} = 84(5.1) = 428 \text{ kgm/s [E]}.$$

$$\vec{p}_2 = m_2\vec{v}_{i2} = 72(5.1) = 367 \text{ kgm/s. [N]}.$$



$$p = \sqrt{367^2 + 428^2}$$
$$= 564 \text{ kgm/s}$$
$$\theta = \tan^{-1}\left(\frac{367}{428}\right) = 41^\circ.$$

$$\vec{p}_f = (m_1 + m_2)\vec{v}_f$$

$$\vec{v}_f = \frac{\vec{p}_f}{m_1 + m_2} = \frac{564}{84+72} = 3.6 \text{ m/s [E41°N]}.$$

Homework: page 253: #1, 4, 6