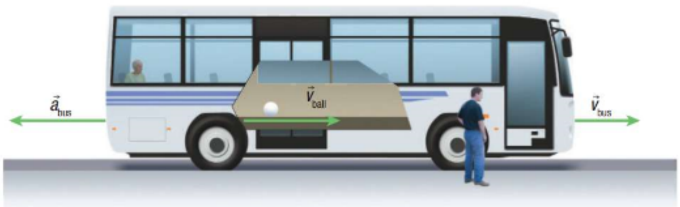
SPH4U 3.1 Inertial and Non-Inertial Frames of Reference

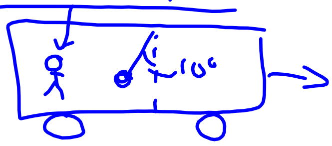
1. Frames of reference

Law of inertia:	Newton's 1st Law. No net force = no accel = no change in $\vec{v}$.
inertial frame of reference	Frame of reference that is not accelerating. ($\vec{v}$ is 0 or constant). Law of inertia holds.
non-inertial frame of reference	Frame that is accelerating. Law of Inertia does not hold.
fictitious forces	non-existant forces that explain the motion of objects in non-inertial frames.



A teacher suspends a small cork ball from the ceiling of a bus. When the bus accelerates at a constant rate forward, the string suspending the ball makes an angle of 10.0° with the vertical. Calculate the magnitude of the acceleration of the bus.

Non-Inertial



Inertial:

$$mg = F_T \cos 10° \rightarrow F_T = \frac{mg}{\cos 10°}$$

$$\Sigma F = F_T \sin 10° = ma.$$

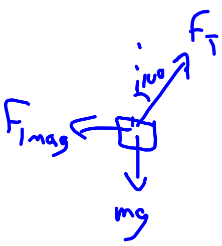
$$\cancel{m}a = \frac{\cancel{m}g}{\cos 10°} \cdot \sin 10°$$

$$a = g \cdot \frac{\sin 10°}{\cos 10°} = g \tan 10°$$

$$= 9.8 \tan 10°$$

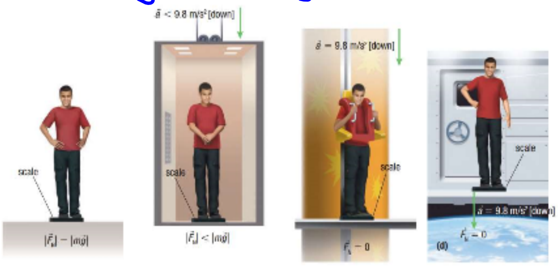
$$a = 1.7 \text{ m/s}^2$$

Non-Inertial:



2. Apparent weight

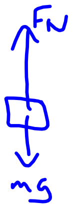
Apparent weight:	the magnitude of the normal force acting on an object in a non-inertial frame.



An elevator accelerates upward with an acceleration of magnitude 1.5 m/s^2, after which it moves with a constant velocity. As the elevator approaches its stopping point, it undergoes a downward acceleration of magnitude 0.9 m/s^2. Calculate the apparent weight of a passenger with a mass of 75 kg when:

$1.5\ m/s^2$

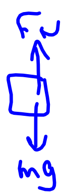
a. the elevator undergoes positive acceleration



$\Sigma F = ma$
$= F_N - mg$
$ma = F_N - mg$

$F_N = ma + mg = m(a+g)$
$= 75(1.5+9.8)$
$= 850\ N.$

b. the elevator moves at constant velocity



$\Sigma F = ma$
$= F_N - mg$
$ma = F_N - mg$

$F_N = ma + mg = m(a+g)$
$= 75(0+9)$
$= 740\ N.$

c. the elevator undergoes negative acceleration

$0.9\ m/s^2$

$\Sigma F = ma$
$= F_N - mg$
$ma = F_N - mg$

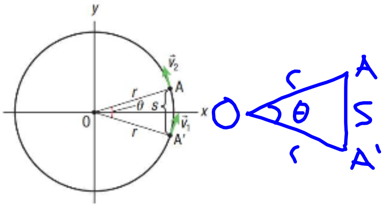
$F_N = ma + mg = m(a+g)$
$= 75(-0.9+9.8)$
$= 670\ N.$

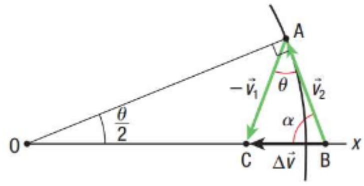
Homework: page 113: #1-3, 5-6

SPH4U 3.2 Centripetal Acceleration

1. Deriving the equations

Uniform circular motion:	motion of an object with constant speed along a circular path with constant radius.
centripetal acceleration	$\vec{a}_c$, instantaneous acceleration toward the centre of the circle.





v_1 and v_2 are related: $v_1 = v_2 = v$

s is the distance traveled in Δt: $s = v\Delta t$

BAC and AOA' are similar triangles. We can use this to find Δv:

$$\frac{\Delta v}{v} = \frac{s}{r} \qquad \frac{\Delta v}{v} = \frac{v\Delta t}{r} \rightarrow \underline{\Delta v = \frac{v^2 \Delta t}{r}}$$

We can use Δv to find a_{av}, which is equal to a_c when Δt is very small:

$$a_{av} = \frac{\Delta v}{\Delta t} = \frac{v^2 \Delta t / r}{\Delta t} = \frac{v^2}{r} \qquad \therefore a_c = \frac{v^2}{r} \quad (1)$$

Sometimes we don't know an object's speed. If we can measure the period T for one cycle, and the radius, we can determine the speed:

$$v = \frac{2\pi r}{T}$$

... and use this to find a_c:

$$a_c = \frac{v^2}{r} = \frac{\left(\frac{2\pi r}{T}\right)^2}{r} = \frac{4\pi^2 r^2}{T^2 r} = \frac{4\pi^2 r}{T^2} \quad (2)$$

Finally, sometimes we prefer to talk about an object's frequency, f, instead of its period T:

$$f = \frac{1}{T} \qquad a_c = \frac{4\pi^2 r}{T^2} = \frac{4\pi^2 r}{\left(\frac{1}{f}\right)^2} = 4\pi^2 r f^2 \quad (3)$$

$$T = \frac{1}{f}$$

So, there are three equations for centripetal acceleration:

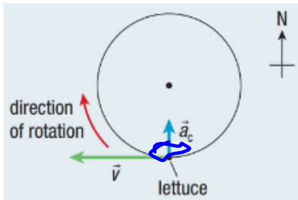
① $a_c = \frac{v^2}{r}$ ② $a_c = \frac{4\pi^2 r}{T^2}$ ③ $a_c = 4\pi^2 r f^2$.

2. Using the equations

A child rides a carousel with a radius of 5.1 m that rotates with a constant speed of 2.2 m/s. Calculate the magnitude of the centripetal acceleration of the child:

$$a_c = \frac{v^2}{r} = \frac{2.2^2}{5.1} = 0.95\ m/s^2.$$

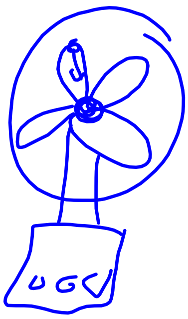
A salad spinner with a radius of 9.7 cm rotates clockwise with a frequency of 12 Hz. At a given instant, the lettuce in the spinner moves in the westward direction. Determine the magnitude and direction of the centripetal acceleration of the piece of lettuce in the salad spinner at the moment shown in the figure.



$$a_c = 4\pi^2 r f^2 = 4\pi^2(0.097\ m)(12)^2$$

$$\vec{a}_c = 550\ m/s^2\ [N].$$

The centripetal acceleration at the end of an electric fan blade has a magnitude of 1.75 x 10³ m/s². The distance between the tip of the fan blade and the centre is 12 cm. Calculate the frequency and the period of rotation of the fan.



$$a_c = \frac{4\pi^2 r}{T^2} \rightarrow T = \sqrt{\frac{4\pi^2 r}{a_c}}$$

$$= \sqrt{\frac{4\pi^2(0.12)}{1.75 \times 10^3}}$$

$$= 0.052\ s.$$

$$f = \frac{1}{T} = \frac{1}{0.052} = 19\ Hz.$$

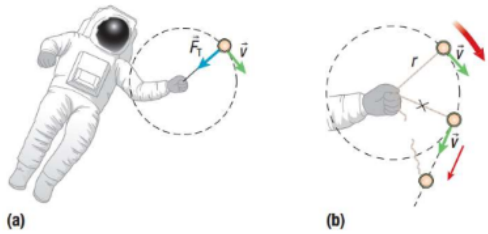
Homework: page 119: #1, 3-5, 7, 9

SPH4U 3.3 Centripetal Force

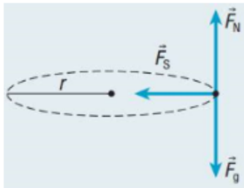
1. Centripetal force

Centripetal force:	$\vec{F}_c$, net force that causes $\vec{a}_c$ could be tension, normal force, gravity, ...
banked curves	a surface is on an angle to help something go around a curve.

$$\vec{F}_c = m\vec{a}_c = \frac{mv^2}{r}$$



Suppose a bug is sitting on the edge of a horizontal DVD. The bug has a mass of 5.0 g, and the DVD has a radius of 6.0 cm. The DVD is spinning such that the bug travels around its circular path three times per second. Calculate the centripetal acceleration of the bug, the net force on the bug, and the force responsible for the centripetal force.



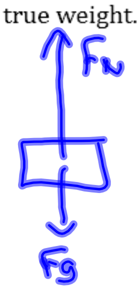
$$a_c = 4\pi^2 r f^2 = 4\pi^2(0.06)(3)^2$$
$$= \underline{21.3\,m/s^2} = \underline{21\,m/s^2}$$

$$F_c = ma_c = 0.005(21.3) = 0.1066\,N = \underline{0.11\,N}$$

F_c is caused by static friction.

2. Calculating speed

A roller coaster car is at the lowest point on its circular track. The radius of curvature is 22 m. The apparent weight of one of the passengers in the roller coaster car is 3.0 times her true weight. Determine the speed of the roller coaster.

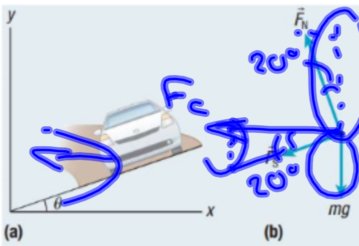


$F_c = F_N - F_g = 3F_g - F_g = 2F_g = \underline{2mg}$.

$F_c = \frac{mv^2}{r}$ $\quad \frac{\cancel{m}v^2}{r} = 2\cancel{m}g \quad \frac{v^2}{r} = 2g$

$v = \sqrt{2gr} = \sqrt{2(9.8)(22)} = \underline{21\ m/s}$.

A car making a turn on a dry, banked highway ramp is experiencing friction. The coefficient of static friction between the tires and the road is 0.60. Determine the maximum speed at which the car can safely negotiate a turn of radius 2.0 x 10² m with a banking angle of 20.0°.



$F_c = F_N \sin 20° + F_s \cos 20°$

$F_{s\,max} = \mu_s F_N$. $\quad F_c = F_N(\sin 20° + \mu \cos 20°)$

$F_{Ny} = mg + F_s \sin 20°$

$F_N \cos 20° = mg + \mu F_N \sin 20° \quad F_N(\mu \sin 20° - \cos 20°) = -mg$

$F_N = \frac{mg}{\cos 20° - \mu \sin 20°} \quad F_c = \left(\frac{mg}{\cos 20 - \mu \sin 20}\right)(\sin 20 + \mu \cos 20)$.

$F_c = \frac{mv^2}{r} \quad v = \sqrt{\frac{F_c r}{m}} = \sqrt{\frac{\cancel{m}gr(\sin 20 + \mu\cos 20)}{\cancel{m}(\cos 20 - \mu \sin 20)}} = 49.17\ m/s$

$= \underline{49\ m/s}$.

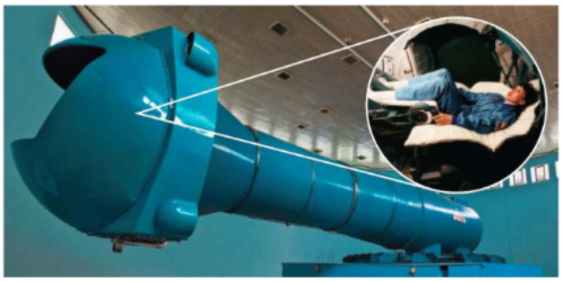
3. Summary

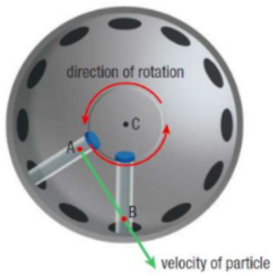
Homework: page 124: #1-4, 6

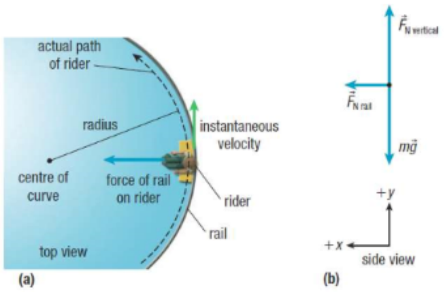
SPH4U 3.4 Rotating Frames of Reference

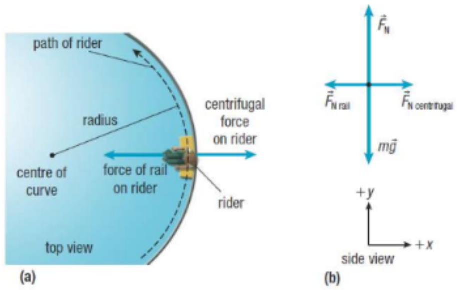
1. Rotating frames of reference

Centrifuge:	a rapidly rotating device used to separate substances and simulate gravity.
centripetal force	always acts inwards to cause centripetal acceleration. it is real.
centrifugal force	Fictitious force. acts opposite the centripetal Force (outwards). It's what we feel.

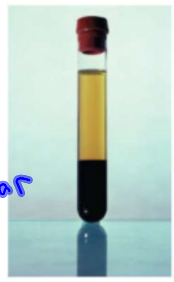








Centrifuges and test tubes:	dense particles settle to the bottom because of centrifugal force (or inertia).
Centrifugal force and Earth's surface:	① lighter at the equator (0.34%) because of centrifugal force ② also squishes Earth.
Coriolis force:	Fictitious force that acts perpendicular to the velocity of an object and the axis of rotation in a rotating frame.



2. Artificial gravity

Spacecraft and extended freefall:	simulate weightlessness by freefalling it is unhealthy - bodies aren't used to weightlessness.
Artificial gravity:	simulate a gravitational force, i.e. by using a rotating reference frame.

Consider a rotating space station similar to the one to the right. The radius of the station is 40.0 m. How many times per minute must the space station rotate to produce a force due to artificial gravity equal to 30.0% of Earth's gravity?

$F_N = 0.3F_g = 0.3mg$

$F_N = F_c = m \cdot 4\pi^2 r f^2$

$0.3\cancel{m}g = \cancel{m} \cdot 4\pi^2 r f^2$

$f^2 = \dfrac{0.3g}{4\pi^2 r}$

$f = \sqrt{\dfrac{0.3(9.8)}{4\pi^2(40)}} = 0.043\ Hz$

$0.043\ Hz \times \dfrac{60s}{min} = \underline{2.59\ rpm}$

3. Summary

Homework: page 130: #1-4, 6

SPH4U 6.1 Newtonian Gravitation

1. Universal gravitation

Universal law of gravitation:	any two objects experience a gravitational attraction related to their mass and distance.
equation	$F_g = \frac{Gm_1m_2}{r^2}$, $G = 6.67\times10^{-11}\ N\cdot m^2/kg^2$.
inverse-square law	when one value decreases with the square of another. $F_g \propto \frac{1}{r^2}$.

The centres of two uniformly dense spheres are separated by 50.0 cm. Each sphere has a mass of 2.00 kg.

a. Calculate the magnitude of the gravitational force between the two spheres.

$$F_g = \frac{Gm_1m_2}{r^2} = \frac{(6.67\times10^{-11})(2)(2)}{(0.5)^2} = \underline{1.07\times10^{-9}\ N}.$$

b. How much of an effect will this force have on the spheres?

Not much.

Eris, a dwarf planet, is the ninth most massive body orbiting the Sun. It is more massive than Pluto and three times farther away from the Sun. Eris has a radius of approximately 1200 km.

a. An astronaut stands on Eris and drops a rock from a height of 0.30 m. The rock takes 0.87 s to reach the surface. Calculate the value of *g* on Eris.

$\Delta d = 0.3m,\ \Delta t = 0.87s,\ v_i = 0,\ g = ?$

$$\Delta d = \cancel{v_i\Delta t}^{\,0} + \frac{1}{2}g\Delta t^2 \qquad g = \frac{2\Delta d}{\Delta t^2} = \frac{2(0.3)}{0.87^2} = \underline{0.7927\ m/s^2}. = \underline{0.79\ m/s^2}.$$

b. Calculate the mass of Eris.

$$F_g = \cancel{m_1}g = \frac{G\cancel{m_1}m_2}{r^2}$$

$$g = \frac{Gm_2}{r^2} \rightarrow m_2 = \frac{gr^2}{G} = \frac{(0.7927)(1200000)^2}{6.67\times10^{-11}} = \underline{1.7\times10^{22}\ kg}.$$

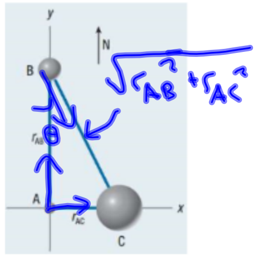
c. An astronaut stands on Eris and drops a rock from a height of 2.50 m. Calculate how long it would take the rock to reach the surface.

$\Delta d = 2.5m,\ v_i = 0,\ g = 0.7927,\ \Delta t = ?$

$$\Delta d = \cancel{v_i\Delta t}^{\,0} + \frac{1}{2}g\Delta t^2$$

$$\Delta t^2 = \frac{2\Delta d}{g} \qquad \Delta t = \sqrt{\frac{2\Delta d}{g}} = \sqrt{\frac{2(2.5)}{0.7927}} = \underline{2.5s}.$$

Three large, spherical asteroids are arranged in space at the corners of a right triangle ABC. Asteroid A has a mass of 1.0 x 10^{20} kg. Asteroid B has a mass of 2.0 x 10^{20} kg and is 5.0 x 10^{10} m from asteroid A. Asteroid C has a mass of 4.0 x 10^{20} kg and is 2.5 x 10^{10} m away from asteroid A along the other side of the triangle.



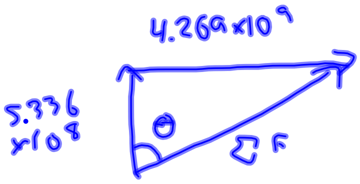
a. Determine the net force on asteroid A from asteroids B and C.

$$\vec{F}_{BA} = \frac{G m_B m_A}{r_{BA}^2} = \frac{(6.67\times10^{-11})(2\times10^{20})(1\times10^{20})}{(5\times10^{10})^2}$$

$$= 5.336\times10^{8}\ \text{N [N]}$$

$$\vec{F}_{CA} = \frac{G m_C m_A}{r_{CA}^2} = \frac{(6.67\times10^{-11})(4\times10^{20})(1\times10^{20})}{(2.5\times10^{10})^2}$$

$$= 4.269\times10^{9}\ \text{N [E]}.$$



$$\Sigma \vec{F} = \sqrt{(5.336\times10^{8})^2 + (4.269\times10^{9})^2}$$

$$= 4.3\times10^{9}\ \text{N}$$

$$\theta = \tan^{-1}\left(\frac{4.269\times10^{9}}{5.336\times10^{8}}\right)$$

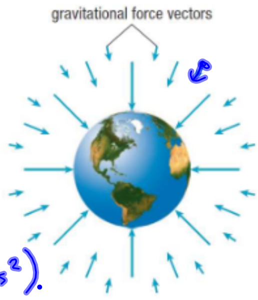
$$= 83^\circ$$

$$\boxed{\therefore \Sigma\vec{F} = 4.3\times10^{9}\ \text{N [N83E]}}$$

b. Determine the net force on asteroid B from asteroid C.

2. Gravitational fields

Gravitational field:	vectors at all points in space that indicate the magnitude and direction of F_g at that point.
gravitational field strength	the force of gravity per kg at a point in space. $g = \frac{F_g}{m}$ Units: N/kg (m/s²).



Assume that Saturn is perfectly spherical with a radius of 6.03 x 10^7 m, and a mass of ~~.69 x~~ 5.69 x 10^{26} kg.

a. Calculate the magnitude of the gravitational field strength on the surface of Saturn.

$$F_g = \frac{G\cancel{m_1}m_2}{r^2} = \cancel{m}g \rightarrow g = \frac{Gm_2}{r^2}$$

$$= \frac{(6.67\times10^{-11})(5.69\times10^{26})}{(6.03\times10^{7})^2}$$

$$= 10.438 \text{ N/kg}.$$

$$\doteq \underline{10.4 \text{ N/kg}}.$$

b. Determine the ratio of Saturn's gravitational field strength to Earth's gravitational field strength (9.8 N/kg).

$$\frac{g_{Saturn}}{g_{Earth}} = \frac{10.438}{9.8} = \underline{1.1}$$

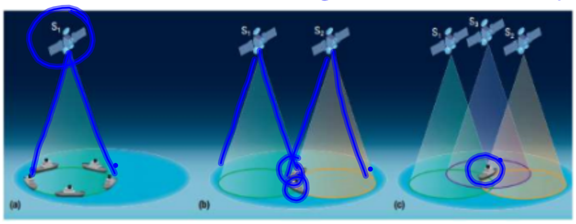
<u>Ratio:</u> 1.1 : 1 (Saturn : Earth).

Homework: page 296: #1-3, 5-6, 8, 11

SPH4U 6.2 Orbits

1. Satellites and space stations

Satellite:	an object that revolves around another object because of gravity.
artificial satellite	an object intentionally put in orbit by humans.
GPS and triangulation	24 satellites (up to 30). 3 satellites are needed to "triangulate" an object's location.



Space station and microgravity:	microgravity = apparent weightlessness due to constant freefall (g on ISS = 8.7 N/kg).

2. Satellites in circular orbits

Orbital radius:	distance between a satellite and its parent body.
orbit shape	elliptical, but we assume they're circular.
geosynchronous orbit	orbit around Earth with a period of 1 day.
geostationary orbit	geosynchronous orbit over the equator, so that it always stays at the same point in the sky.

Gravitational field strength at a distance *r* above Earth: $g = \frac{Gm_E}{r^2}$

Centripetal acceleration: $a_c = \frac{v^2}{r}$.

$g = a_c$. $\frac{Gm_E}{r^{\cancel{2}}} = \frac{v^2}{\cancel{r}}$.

$v^2 = \frac{Gm_E}{r} \rightarrow \boxed{v = \sqrt{\frac{Gm_E}{r}}}$

Only variable.

The International Space Station (ISS) obits Earth at an altitude of about 350 km above Earth's surface.

a. Determine the speed needed by the ISS to maintain its orbit.

$$v = \sqrt{\frac{Gm}{r}} = \sqrt{\frac{(6.67\times10^{-11})(5.98\times10^{24})}{(350\times10^{3}+6.38\times10^{6})}}$$

$$= 7.698\times10^{3}\ m/s.$$

$$= \underline{7.7\times10^{3}\ m/s}$$

b. Determine the orbital period of the ISS in minutes.

$$T = \frac{2\pi r}{v} = \frac{2\pi(6.38\times10^{6}+350\times10^{3})}{7.7\times10^{3}}$$

$$= 5491.7s \times \frac{1\ min}{60s}$$

$$= 91.53\ min$$

$$= \underline{92\ min}$$

Determine the speeds of Venus and Earth as they orbit the Sun. The Sun's mass is 1.99 x 10^{30} kg. Venus has an orbital radius of 1.08 x 10^{11} m, an dEarth has an orbital radius of 1.49 x 10^{11} m.

$$v = \sqrt{\frac{Gm}{r}}$$

<u>Venus:</u> $v = \sqrt{\frac{Gm}{r}}$

$$= \sqrt{\frac{(6.67\times10^{-11})(1.99\times10^{30})}{1.08\times10^{11}}}$$

$$= \underline{3.51\times10^{4}\ m/s.}$$

<u>Earth:</u> $v = \sqrt{\frac{Gm}{r}}$

$$= \sqrt{\frac{(6.67\times10^{-11})(1.99\times10^{30})}{1.49\times10^{11}}}$$

$$= \underline{2.98\times10^{4}\ m/s.}$$

Homework: page 303: #4-7, 9, 14