Chapter 2: Motion in Two Dimensions

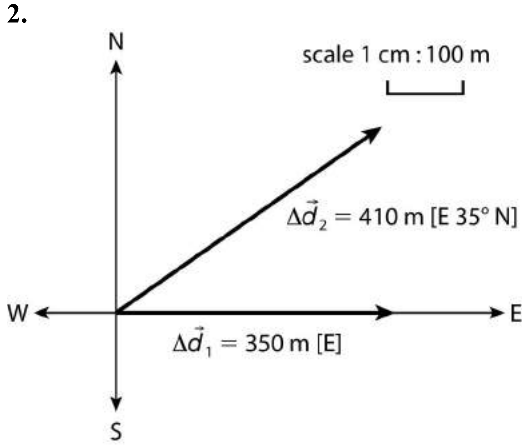
Mini Investigation: Garbage Can Basketball, page 59

A. Answers may vary. Sample answer:
When launched from knee height, an underhand upward thrust is used such that the ball of paper travels in a parabolic arc. When launched from waist height, a downward arm motion is used to direct the ball of paper to move in a curved trajectory downward into the trash can. When launched from shoulder height, an overhand upward thrust is used to launch the paper ball in a parabolic arc that allows it to land in the trash can.

Section 2.1: Motion in Two Dimensions—A Scale Diagram Approach

Tutorial 1 Practice, page 61

Note: after the first printing, the given value of 410 cm in Question 1 was changed to 410 m. The answer below reflects this change.
1. Answers may vary. Sample answer:
I think a suitable scale would have the vectors be about 5 cm long. Looking at the smaller displacement, if I divide 350 by 100, I get 3.5. Then if I divide 410 by 100, I get 4.1. Since 3.5 cm and 4.1 cm are both suitable lengths for my vectors I choose a scale of 1 cm : 100 m.
2.



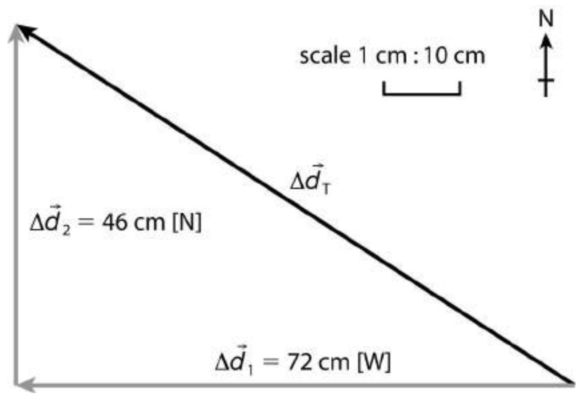
Tutorial 2 Practice, page 64

1. (a) Given: $\Delta\vec{d}_1 = 72$ cm [W]; $\Delta\vec{d}_2 = 46$ cm [N]

Required: $\Delta\vec{d}_T$

Analysis: $\Delta\vec{d}_T = \Delta\vec{d}_1 + \Delta\vec{d}_2$

Solution:



This figure shows the given vectors, with the tip of $\Delta\vec{d}_1$ joined to the tail of $\Delta\vec{d}_2$. The resultant vector $\Delta\vec{d}_T$ is drawn in black from the tail of $\Delta\vec{d}_1$ to the tip of $\Delta\vec{d}_2$. Using a compass, the direction of $\Delta\vec{d}_T$ is [W 33° N]. $\Delta\vec{d}_T$ measures 8.5 cm in length, so using the scale of 1 cm : 10 cm, the actual magnitude of $\Delta\vec{d}_T$ is 85 cm.

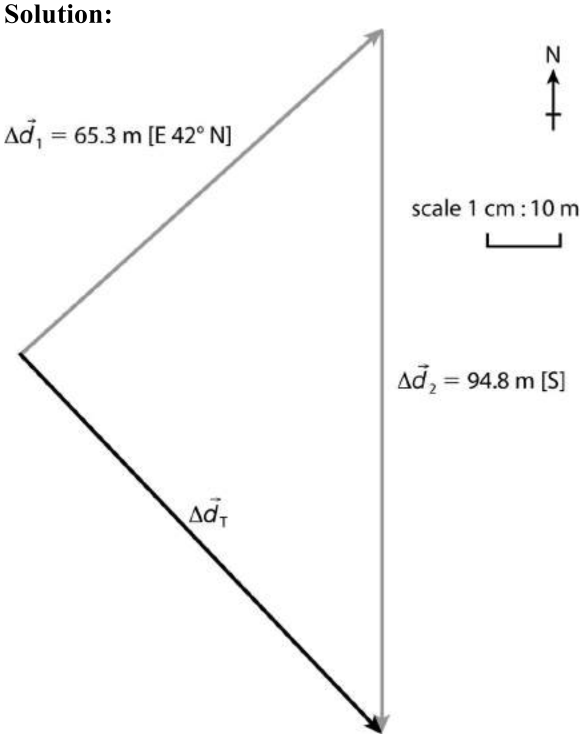
Statement: The sum of the two vectors is 85 cm [W 33° N].

(b) Given: $\Delta\vec{d}_1 = 65.3$ m [E 42° N];
$\Delta\vec{d}_2 = 94.8$ m [S]

Required: $\Delta\vec{d}_T$

Analysis: $\Delta\vec{d}_T = \Delta\vec{d}_1 + \Delta\vec{d}_2$

Solution:



This figure shows the given vectors, with the tip of $\Delta\vec{d}_1$ joined to the tail of $\Delta\vec{d}_2$. The resultant vector $\Delta\vec{d}_T$ is drawn in black from the tail of $\Delta\vec{d}_1$ to the tip of $\Delta\vec{d}_2$. Using a compass, the direction of $\Delta\vec{d}_T$ is [W 46° S]. $\Delta\vec{d}_T$ measures 7.05 cm in length, so using the scale of 1 cm : 10 m, the actual magnitude of $\Delta\vec{d}_T$ is 70.5 m.

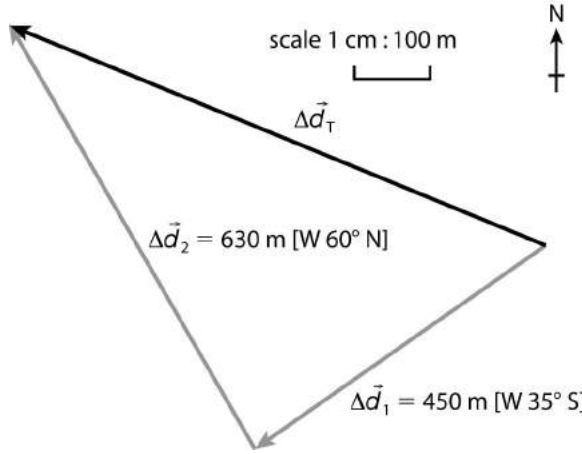
Statement: The sum of the two vectors is 70.5 m [W 46° S].

2. (a) Given: $\Delta\vec{d}_1$ = 450 m [W 35° S]; $\Delta\vec{d}_2$ = 630 m [W 60° N]

Required: $\Delta\vec{d}_T$

Analysis: $\Delta\vec{d}_T = \Delta\vec{d}_1 + \Delta\vec{d}_2$

Solution:



This figure shows the given vectors, with the tip of $\Delta\vec{d}_1$ joined to the tail of $\Delta\vec{d}_2$. The resultant vector $\Delta\vec{d}_T$ is drawn in black from the tail of $\Delta\vec{d}_1$ to the tip of $\Delta\vec{d}_2$. Using a compass, the direction of $\Delta\vec{d}_T$ is [W 23° N]. $\Delta\vec{d}_T$ measures 7.4 cm in length, so using the scale of 1 cm : 100 m, the actual magnitude of $\Delta\vec{d}_T$ is 740 m.

Statement: The sum of the two vectors is 740 m [W 23° N].

(b) Given: $\Delta\vec{d}$ = 740 m [W 23° N]; Δt = 77 s

Required: $\vec{v}_{av}$

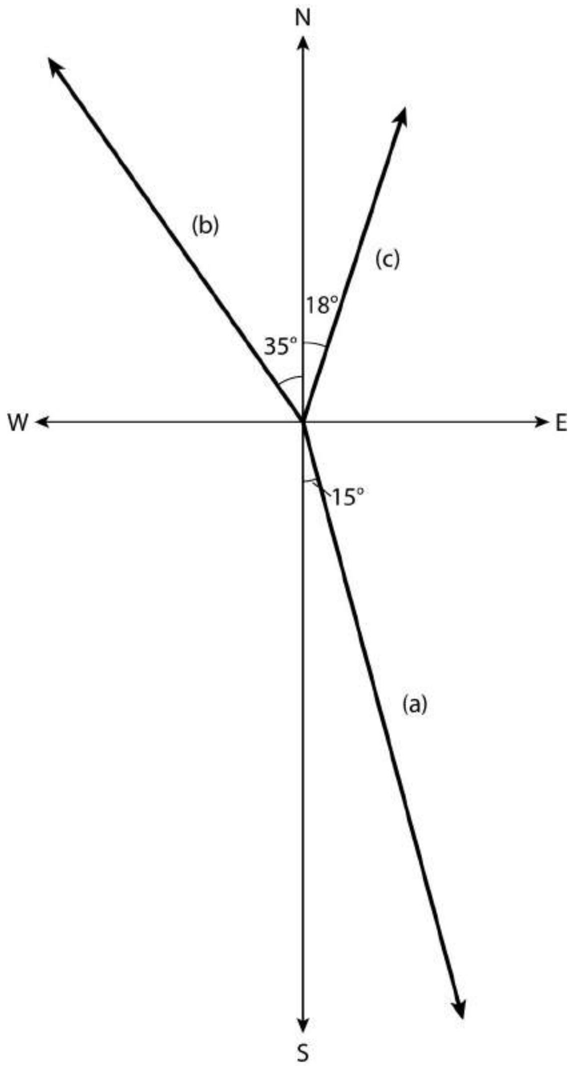
Analysis: $\vec{v}_{av} = \dfrac{\Delta\vec{d}}{\Delta t}$

Solution:
$$\begin{aligned}\vec{v}_{av} &= \frac{\Delta\vec{d}}{\Delta t}\\ &= \frac{740\text{ m [W 23° N]}}{77\text{ s}}\\ \vec{v}_{av} &= 9.6\text{ m/s [W 23° N]}\end{aligned}$$

Statement: The cyclist's average velocity is 9.6 m/s [W 23° N].

Section 2.1 Questions, page 65

1.



2. Each vector can be written with the second direction first. The angle will then change to the complementary angle, so subtract the angle from 90°. For example, [S 15° E] becomes [E 75° S].

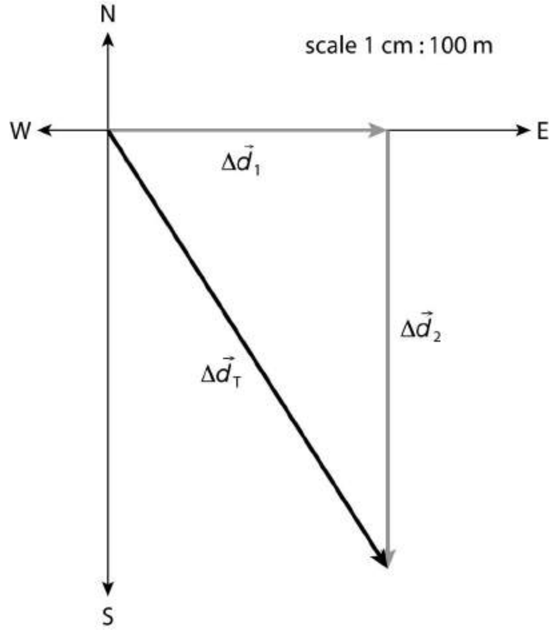
3. (a) The length of $\Delta\vec{d}_1$ is 3.6 cm and the length of $\Delta\vec{d}_2$ is 5.7 cm. Using the scale of 1 cm : 100 m, the actual vector of $\Delta\vec{d}_1$ is 360 m [E] and the actual vector of $\Delta\vec{d}_2$ is 570 m [S].

(b) Given: Figure 11

Required: $\Delta\vec{d}_T$

Analysis: $\Delta\vec{d}_T = \Delta\vec{d}_1 + \Delta\vec{d}_2$

Solution:



This figure shows the given vectors, with the tip of $\Delta\vec{d}_1$ joined to the tail of $\Delta\vec{d}_2$. The resultant vector $\Delta\vec{d}_T$ is drawn in black from the tail of $\Delta\vec{d}_1$ to the tip of $\Delta\vec{d}_2$. Using a compass, the direction of $\Delta\vec{d}_T$ is [E 58° S]. $\Delta\vec{d}_T$ measures 6.7 cm in length, so using the scale of 1 cm : 100 m, the actual magnitude of $\Delta\vec{d}_T$ is 670 m.

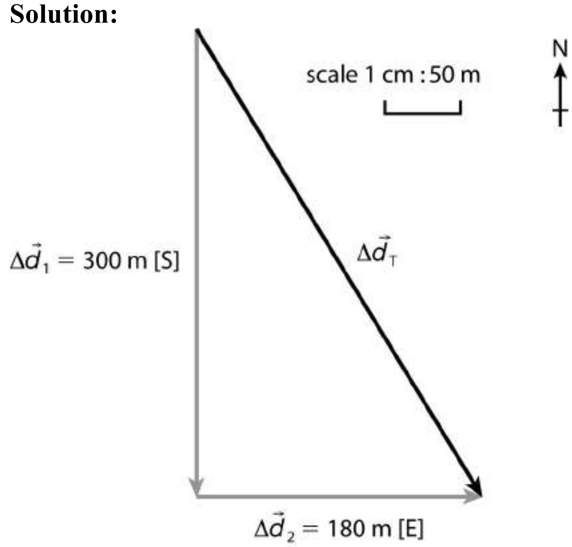
Statement: The sum of the two vectors is 670 m [E 58° S].

4. Given: $\Delta\vec{d}_1$ = 300.0 m [S]; $\Delta\vec{d}_2$ = 180.0 m [E]

Required: $\Delta\vec{d}_T$

Analysis: $\Delta\vec{d}_T = \Delta\vec{d}_1 + \Delta\vec{d}_2$

Solution:



This figure shows the given vectors, with the tip of $\Delta\vec{d}_1$ joined to the tail of $\Delta\vec{d}_2$. The resultant vector $\Delta\vec{d}_T$ is drawn in black from the tail of $\Delta\vec{d}_1$ to the tip of $\Delta\vec{d}_2$. Using a compass, the direction of $\Delta\vec{d}_T$ is [S 31° E]. $\Delta\vec{d}_T$ measures 7.0 cm in length, so using the scale of 1 cm : 50 m, the actual magnitude of $\Delta\vec{d}_T$ is 350 m.

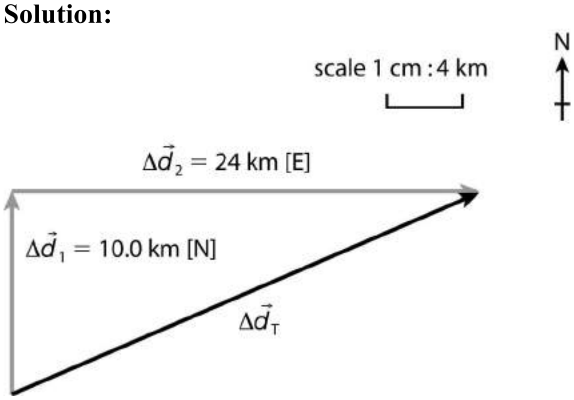
Statement: The taxi's total displacement is 350 m [S 31° E].

5. Given: $\Delta\vec{d}_1$ = 10.0 km [N]; $\Delta\vec{d}_2$ = 24 km [E]

Required: $\Delta\vec{d}_T$

Analysis: $\Delta\vec{d}_T = \Delta\vec{d}_1 + \Delta\vec{d}_2$

Solution:



This figure shows the given vectors, with the tip of $\Delta\vec{d}_1$ joined to the tail of $\Delta\vec{d}_2$. The resultant vector $\Delta\vec{d}_T$ is drawn in black from the tail of $\Delta\vec{d}_1$ to the tip of $\Delta\vec{d}_2$. Using a compass, the direction of $\Delta\vec{d}_T$ is [N 67° E]. $\Delta\vec{d}_T$ measures 6.5 cm in length, so using the scale of 1 cm : 4 km, the actual magnitude of $\Delta\vec{d}_T$ is 26 km.

Statement: The total displacement of the two trips is 26 km [N 67° E].

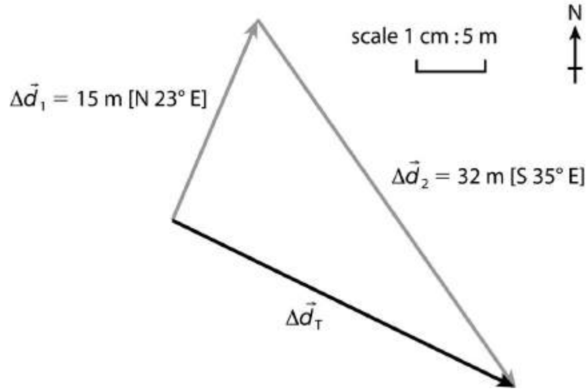
6. Yes, the answer would be the same. Whichever order the vectors are placed, the final position, which is what determines the sum of the vectors, stays the same.

7. Given: $\Delta\vec{d}_1$ = 15 m [N 23° E]; $\Delta\vec{d}_2$ = 32 m [S 35° E]

Required: $\Delta\vec{d}_T$

Analysis: $\Delta\vec{d}_T = \Delta\vec{d}_1 + \Delta\vec{d}_2$

Solution:



This figure shows the given vectors, with the tip of $\Delta\vec{d}_1$ joined to the tail of $\Delta\vec{d}_2$. The resultant vector $\Delta\vec{d}_T$ is drawn in black from the tail of $\Delta\vec{d}_1$ to the tip of $\Delta\vec{d}_2$. Using a compass, the direction of $\Delta\vec{d}_T$ is [E 27° S]. $\Delta\vec{d}_T$ measures 5.4 cm in length, so using the scale of 1 cm : 5 m, the actual magnitude of $\Delta\vec{d}_T$ is 27 m.

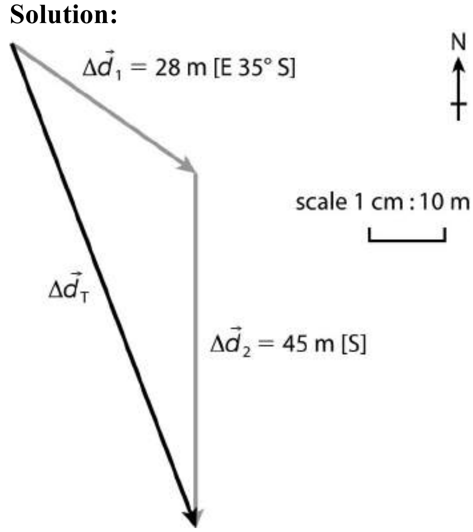
Statement: The horse's total displacement is 27 m [E 27° S].

8. (a) Given: $\Delta\vec{d}_1$ = 28 m [E 35° S]; $\Delta\vec{d}_2$ = 45 m [S]; Δt = 6.9 s

Required: $\vec{v}_{av}$

Analysis: $\vec{v}_{av} = \dfrac{\Delta\vec{d}_T}{\Delta t}$

Solution:



This figure shows the given vectors, with the tip of $\Delta\vec{d}_1$ joined to the tail of $\Delta\vec{d}_2$. The resultant vector $\Delta\vec{d}_T$ is drawn in black from the tail of $\Delta\vec{d}_1$ to the tip of $\Delta\vec{d}_2$. Using a compass, the direction of $\Delta\vec{d}_T$

is [E 69° S]. $\Delta \vec{d}_T$ measures 6.55 cm in length, so using the scale of 1 cm : 10 m, the actual magnitude of $\Delta \vec{d}_T$ is 65.5 m.

$$\begin{aligned}\vec{v}_{av} &= \frac{\Delta \vec{d}_T}{\Delta t} \\ &= \frac{65.5 \text{ m [E 69° S]}}{6.9 \text{ s}} \\ \vec{v}_{av} &= 9.5 \text{ m/s [E 69° S]}\end{aligned}$$

Statement: The car's average velocity is 9.5 m/s [E 69° S].

(b) Given: d_1 = 28 m; d_2 = 45 m; Δt = 6.9 s

Required: v_{av}

Analysis: $v_{av} = \frac{\Delta d_T}{\Delta t}$

$$v_{av} = \frac{d_1 + d_2}{\Delta t}$$

Solution: $v_{av} = \frac{d_1 + d_2}{\Delta t}$

$$\begin{aligned} &= \frac{28 \text{ m} + 45 \text{ m}}{6.9 \text{ s}} \\ v_{av} &= 11 \text{ m/s}\end{aligned}$$

Statement: The car's average speed is 11 m/s.

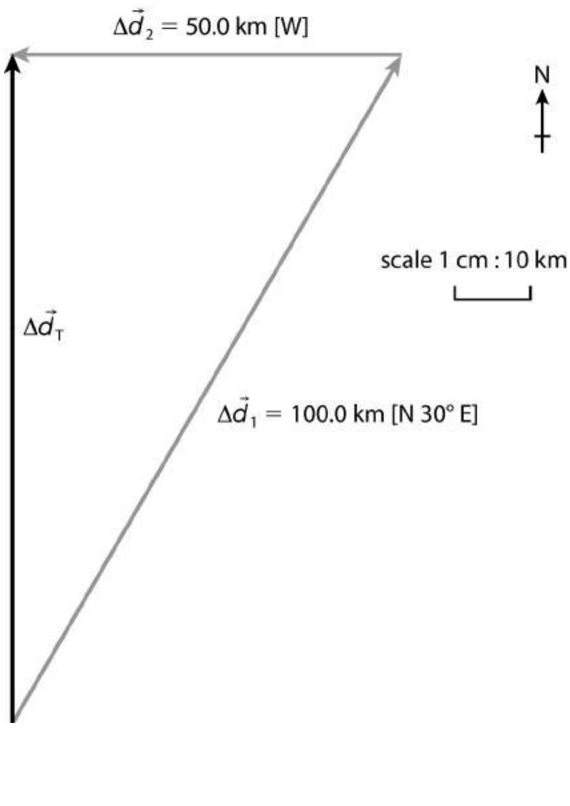
9. (a) Given: $\Delta \vec{d}_1$ = 100.0 km [N 30° E];

$\Delta \vec{d}_2$ = 50.0 km [W]

Required: $\Delta \vec{d}_T$

Analysis: $\Delta \vec{d}_T = \Delta \vec{d}_1 + \Delta \vec{d}_2$

Solution:



This figure shows the given vectors, with the tip of $\Delta \vec{d}_1$ joined to the tail of $\Delta \vec{d}_2$. The resultant vector $\Delta \vec{d}_T$ is drawn in black from the tail of $\Delta \vec{d}_1$ to the tip of $\Delta \vec{d}_2$. Using a compass, the direction of $\Delta \vec{d}_T$ is [N]. $\Delta \vec{d}_T$ measures 8.7 cm in length, so using the scale of 1 cm : 10 km, the actual magnitude of $\Delta \vec{d}_T$ is 87 km.

Statement: The aircraft's total displacement is 87 km [N].

(b) Given: $\Delta \vec{d}_T$ = 87 km [N]; Δt = 10.0 min

Required: $\vec{v}_{av}$

Analysis: $\vec{v}_{av} = \frac{\Delta \vec{d}_T}{\Delta t}$

Solution: $\vec{v}_{av} = \frac{\Delta \vec{d}_T}{\Delta t}$

$$\begin{aligned} &= \frac{87 \text{ km [N]}}{10.0 \ \cancel{\text{min}}}\left(\frac{60 \ \cancel{\text{min}}}{1 \text{ h}}\right) \\ \vec{v}_{av} &= 5.2 \times 10^2 \text{ km/h [N]}\end{aligned}$$

Statement: The aircraft's average velocity is 5.2×10^2 km/h [N] or 520 km/h [N].

Section 2.2: Motion in Two Dimensions—An Algebraic Approach

Tutorial 1 Practice, page 67

1. Given: $\Delta\vec{d}_1 = 27$ m [W]; $\Delta\vec{d}_2 = 35$ m [S]

Required: $\Delta\vec{d}_T$

Analysis: $\Delta\vec{d}_T = \Delta\vec{d}_1 + \Delta\vec{d}_2$

Solution: Let ϕ represent the angle $\Delta\vec{d}_T$ makes with the x-axis.

$$\begin{aligned}\Delta\vec{d}_T &= \Delta\vec{d}_1 + \Delta\vec{d}_2\\ \Delta d_T^{\,2} &= \Delta d_1^{\,2} + \Delta d_2^{\,2}\\ \Delta d_T &= \sqrt{\Delta d_1^{\,2} + \Delta d_2^{\,2}}\\ &= \sqrt{(27\text{ m})^2 + (35\text{ m})^2}\\ \Delta d_T &= 44\text{ m}\end{aligned}$$

$$\begin{aligned}\tan\phi &= \frac{\Delta d_2}{\Delta d_1}\\ \tan\phi &= \frac{35\not{\text{m}}}{27\not{\text{m}}}\\ \tan\phi &= 1.296\\ \phi &= \tan^{-1}(1.296)\\ \phi &= 52°\end{aligned}$$

Statement: The sum of the two vectors is 44 m [W 52° S].

2. Given: $\Delta\vec{d}_1 = 13.2$ m [S]; $\Delta\vec{d}_2 = 17.8$ m [E]

Required: $\Delta\vec{d}_T$

Analysis: $\Delta\vec{d}_T = \Delta\vec{d}_1 + \Delta\vec{d}_2$

Solution: Let ϕ represent the angle $\Delta\vec{d}_T$ makes with the y-axis.

$$\begin{aligned}\Delta\vec{d}_T &= \Delta\vec{d}_1 + \Delta\vec{d}_2\\ \Delta d_T^{\,2} &= \Delta d_1^{\,2} + \Delta d_2^{\,2}\\ \Delta d_T &= \sqrt{\Delta d_1^{\,2} + \Delta d_2^{\,2}}\\ &= \sqrt{(13.2\text{ m})^2 + (17.8\text{ m})^2}\\ \Delta d_T &= 22.2\text{ m}\end{aligned}$$

$$\begin{aligned}\tan\phi &= \frac{\Delta d_2}{\Delta d_1}\\ \tan\phi &= \frac{17.8\not{\text{m}}}{13.2\not{\text{m}}}\\ \tan\phi &= 1.348\\ \phi &= \tan^{-1}(1.348)\\ \phi &= 53°\end{aligned}$$

The sum of the two vectors is 22.2 m [S 53° E] or [E 37° S].

Statement: The sum of the two vectors is 22.2 m [E 37° S].

Tutorial 2 Practice, page 69

1. Given: $\Delta\vec{d}_T = 15$ m [W 35° N]

Required: $\Delta\vec{d}_x$; $\Delta\vec{d}_y$

Analysis: $\Delta\vec{d}_T = \Delta\vec{d}_x + \Delta\vec{d}_y$

Solution: Since the direction of $\Delta\vec{d}_T$ is between west and north, the direction of $\Delta\vec{d}_x$ is [W] and the direction of $\Delta\vec{d}_y$ is [N].

$$\begin{aligned}\sin\theta &= \frac{\Delta d_y}{\Delta d_T}\\ \Delta d_y &= \Delta d_T \sin\theta\\ &= (15\text{ m})(\sin 35°)\\ &= 8.604\text{ m}\\ \Delta d_y &= 8.6\text{ m}\end{aligned}$$

$$\begin{aligned}\cos\theta &= \frac{\Delta d_x}{\Delta d_T}\\ \Delta d_x &= \Delta d_T \cos\theta\\ &= (15\text{ m})(\cos 35°)\\ &= 12.29\text{ m}\\ \Delta d_x &= 12\text{ m}\end{aligned}$$

Statement: The vector has a horizontal or x-component of 12 m [W] and a vertical or y-component of 8.6 m [N].

2. Given: $\Delta\vec{d}_x = 27.2$ m [E]; $\Delta\vec{d}_y = 12.7$ m [N]

Required: $\Delta\vec{d}_T$

Analysis: $\Delta\vec{d}_T = \Delta\vec{d}_1 + \Delta\vec{d}_2$

Solution: Let ϕ represent the angle $\Delta\vec{d}_T$ makes with the x-axis.

$$\begin{aligned}\Delta\vec{d}_T &= \Delta\vec{d}_x + \Delta\vec{d}_y\\ \Delta d_T^{\,2} &= \Delta d_x^{\,2} + \Delta d_y^{\,2}\\ \Delta d_T &= \sqrt{\Delta d_x^{\,2} + \Delta d_y^{\,2}}\\ &= \sqrt{(27.2\text{ m})^2 + (12.7\text{ m})^2}\\ \Delta d_T &= 30.0\text{ m}\end{aligned}$$

$$\begin{aligned}\tan\phi &= \frac{\Delta d_y}{\Delta d_x}\\ \tan\phi &= \frac{12.7\not{\text{m}}}{27.2\not{\text{m}}}\\ \tan\phi &= 0.4669\\ \phi &= \tan^{-1}(0.4669)\\ \phi &= 25°\end{aligned}$$

Statement: The sum of the two vectors is 30.0 m [E 25° N], which is the original vector from Sample Problem 1.

Tutorial 3 Practice, page 71

1. Given: $\Delta\vec{d}_1 = 2.78$ cm [W];

$\Delta\vec{d}_2 = 6.25$ cm [S 40° E]

Required: $\Delta\vec{d}_T$

Analysis: $\Delta\vec{d}_T = \Delta\vec{d}_1 + \Delta\vec{d}_2$

Solution: Determine the total x-component and y-component of $\Delta\vec{d}_T$:

$$\begin{aligned}
\vec{d}_{Tx} &= \Delta\vec{d}_{1x} + \Delta\vec{d}_{2x} \\
&= 2.78\text{ cm [W]} + (6.25\text{ cm})(\sin 40°)\text{ [E]} \\
&= 2.78\text{ cm [W]} + 4.0174\text{ cm [E]} \\
&= -2.78\text{ cm [E]} + 4.0174\text{ cm [E]} \\
&= 1.2374\text{ cm [E]} \\
\vec{d}_{Tx} &= 1.24\text{ cm [E]}
\end{aligned}$$

$$\begin{aligned}
\vec{d}_{Ty} &= \Delta\vec{d}_{1y} + \Delta\vec{d}_{2y} \\
&= 0\text{ cm} + (6.25\text{ cm})(\cos 40°)\text{ [S]} \\
&= 4.7878\text{ cm [S]} \\
\vec{d}_{Ty} &= 4.79\text{ cm [S]}
\end{aligned}$$

Determine the magnitude of $\Delta\vec{d}_T$:

$$\begin{aligned}
\Delta d_T^{\,2} &= d_{Tx}^{\,2} + d_{Ty}^{\,2} \\
\Delta d_T &= \sqrt{d_{Tx}^{\,2} + d_{Ty}^{\,2}} \\
&= \sqrt{(1.2374\text{ cm})^2 + (4.7878\text{ cm})^2} \\
&\quad\text{(two extra digits carried)} \\
\Delta d_T &= 4.95\text{ cm}
\end{aligned}$$

Let ϕ represent the angle $\Delta\vec{d}_T$ makes with the x-axis.

$$\begin{aligned}
\tan\phi &= \frac{\Delta d_{Ty}}{\Delta d_{Tx}} \\
\tan\phi &= \frac{4.7878\ \cancel{\text{cm}}}{1.2374\ \cancel{\text{cm}}}\ \text{(two extra digits carried)} \\
\tan\phi &= 3.869 \\
\phi &= \tan^{-1}(3.869) \\
\phi &= 76°
\end{aligned}$$

Statement: The ant's total displacement is 4.95 cm [E 76° S].

2. Given: $\Delta\vec{d}_1 = 2.64$ m [W 26° N];

$\Delta\vec{d}_2 = 3.21$ m [S 12° E]

Required: $\Delta\vec{d}_T$

Analysis: $\Delta\vec{d}_T = \Delta\vec{d}_1 + \Delta\vec{d}_2$

Solution: Determine the total x-component and y-component of $\Delta\vec{d}_T$:

$$\begin{aligned}
\vec{d}_{Tx} &= \Delta\vec{d}_{1x} + \Delta\vec{d}_{2x} \\
&= (2.64\text{ m})(\cos 26°)\text{ [W]} + (3.21\text{ m})(\sin 12°)\text{ [E]} \\
&= 2.3728\text{ m [W]} + 0.6674\text{ m [E]} \\
&= 2.3728\text{ m [W]} - 0.6674\text{ m [W]} \\
&= 1.7054\text{ m [W]} \\
\vec{d}_{Tx} &= 1.71\text{ m [W]}
\end{aligned}$$

$$\begin{aligned}
\vec{d}_{Ty} &= \Delta\vec{d}_{1y} + \Delta\vec{d}_{2y} \\
&= (2.64\text{ m})(\sin 26°)\text{ [N]} + (3.21\text{ m})(\cos 12°)\text{ [S]} \\
&= 1.1573\text{ m [N]} + 3.1399\text{ m [S]} \\
&= -1.1573\text{ m [S]} + 3.1399\text{ m [S]} \\
&= 1.9826\text{ m [S]} \\
\vec{d}_{Ty} &= 1.98\text{ m [S]}
\end{aligned}$$

Determine the magnitude of $\Delta\vec{d}_T$:

$$\begin{aligned}
\Delta d_T^{\,2} &= d_{Tx}^{\,2} + d_{Ty}^{\,2} \\
\Delta d_T &= \sqrt{d_{Tx}^{\,2} + d_{Ty}^{\,2}} \\
&= \sqrt{(1.7054\text{ m})^2 + (1.9826\text{ m})^2}\ \text{(two extra digits carried)} \\
\Delta d_T &= 2.62\text{ m}
\end{aligned}$$

Let ϕ represent the angle $\Delta\vec{d}_T$ makes with the x-axis.

$$\begin{aligned}
\tan\phi &= \frac{\Delta d_{Ty}}{\Delta d_{Tx}} \\
\tan\phi &= \frac{1.9826\ \cancel{\text{m}}}{1.7054\ \cancel{\text{m}}}\ \text{(two extra digits carried)} \\
\tan\phi &= 1.163 \\
\phi &= \tan^{-1}(1.163) \\
\phi &= 49°
\end{aligned}$$

Statement: The total displacement of the paper airplane is 2.62 m [W 49° S].

Tutorial 4 Practice, page 74

1. Answers may vary. Sample answer:
Imagine the river current is flowing south and the canoe is pointed east. As long as the boat is pointed perpendicular to the current, the current has no effect on the time it takes to cross the river. Think about the component vectors. Even though the canoe will be travelling in a direction between south and east, all its eastbound velocity is the same, no matter how fast the current is, or if there's any current at all.

2. (a) Given: Δd = 20.0 m; v = 1.3 m/s
Required: Δt

Analysis: $v = \frac{\Delta d}{\Delta t}$

$$\Delta t = \frac{\Delta d}{v}$$

Solution:
$$\begin{aligned}\Delta t &= \frac{\Delta d}{v} \\ &= \frac{20.0\ \not{m}}{1.3\ \frac{\not{m}}{s}} \\ &= 15.38\ \text{s} \\ \Delta t &= 15\ \text{s}\end{aligned}$$

Statement: It will take the swimmer 15 s to cross the river.

(b) Given: $\vec{v}_x$ = 2.7 m/s [W]

Required: $\Delta\vec{d}_x$

Analysis:
$$\vec{v}_x = \frac{\Delta\vec{d}_x}{\Delta t}$$
$$\Delta\vec{d}_x = \vec{v}_x \Delta t$$

Solution:
$$\begin{aligned}\Delta\vec{d}_x &= \vec{v}_x \Delta t \\ &= \left(2.7\ \frac{\text{m}}{\not{s}}\ [\text{W}]\right)(15.38\ \not{s})\ \text{(two extra digits carried)} \\ \Delta\vec{d}_x &= 42\ \text{m [W]}\end{aligned}$$

Statement: The swimmer lands 42 m downstream from his intended location.

Section 2.2 Questions, page 75

1. (a) Given: $\Delta\vec{d}_T$ = 20 km [W 50° N]

Required: $\Delta\vec{d}_x$; $\Delta\vec{d}_y$

Analysis: $\Delta\vec{d}_T = \Delta\vec{d}_x + \Delta\vec{d}_y$

Solution: Since the direction of $\Delta\vec{d}_T$ is between west and north, the direction of $\Delta\vec{d}_x$ is [W] and the direction of $\Delta\vec{d}_y$ is [N].

$$\begin{aligned}\sin\theta &= \frac{\Delta d_y}{\Delta d_T} \\ \Delta d_y &= \Delta d_T \sin\theta \\ &= (20\ \text{km})(\sin 50°) \\ \Delta d_y &= 15\ \text{km}\end{aligned}$$

$$\begin{aligned}\cos\theta &= \frac{\Delta d_x}{\Delta d_T} \\ \Delta d_x &= \Delta d_T \cos\theta \\ &= (20\ \text{km})(\cos 50°) \\ \Delta d_x &= 13\ \text{km}\end{aligned}$$

Statement: The vector has a vertical or y-component of 15 km [N] and a horizontal or x-component of 13 km [W].

(b) Given: $\Delta\vec{d}_T$ = 15 km [W 80° S]

Required: $\Delta\vec{d}_x$; $\Delta\vec{d}_y$

Analysis: $\Delta\vec{d}_T = \Delta\vec{d}_x + \Delta\vec{d}_y$

Solution: Since the direction of $\Delta\vec{d}_T$ is between west and south, the direction of $\Delta\vec{d}_x$ is [W] and the direction of $\Delta\vec{d}_y$ is [S].

$$\begin{aligned}\sin\theta &= \frac{\Delta d_y}{\Delta d_T} \\ \Delta d_y &= \Delta d_T \sin\theta \\ &= (15\ \text{km})(\sin 80°) \\ &= 14.77\ \text{km} \\ \Delta d_y &= 15\ \text{km}\end{aligned}$$

$$\begin{aligned}\cos\theta &= \frac{\Delta d_x}{\Delta d_T} \\ \Delta d_x &= \Delta d_T \cos\theta \\ &= (15\ \text{km})(\cos 80°) \\ \Delta d_x &= 2.6\ \text{km}\end{aligned}$$

Statement: The vector has a vertical or y-component of 15 km [S] and a horizontal or x-component of 2.6 km [W].

(c) Given: $\Delta\vec{d}_T$ = 40 km [N 65° E]

Required: $\Delta\vec{d}_x$; $\Delta\vec{d}_y$

Analysis: $\Delta\vec{d}_T = \Delta\vec{d}_x + \Delta\vec{d}_y$

Solution: Since the direction of $\Delta\vec{d}_T$ is between north and east, the direction of $\Delta\vec{d}_x$ is [E] and the direction of $\Delta\vec{d}_y$ is [N].

$$\begin{aligned}\cos\theta &= \frac{\Delta d_y}{\Delta d_T} \\ \Delta d_y &= \Delta d_T \cos\theta \\ &= (40\ \text{km})(\cos 65°) \\ \Delta d_y &= 17\ \text{km}\end{aligned}$$

$$\begin{aligned}\sin\theta &= \frac{\Delta d_x}{\Delta d_T} \\ \Delta d_x &= \Delta d_T \sin\theta \\ &= (40\ \text{km})(\sin 65°) \\ \Delta d_x &= 36\ \text{km}\end{aligned}$$

Statement: The vector has a vertical or y-component of 17 km [N] and a horizontal or x-component of 36 km [E].

2. Given: $\Delta\vec{d}_1 = 5.1$ km [E]; $\Delta\vec{d}_2 = 14$ km [N]

Required: $\Delta\vec{d}_T$

Analysis: $\Delta\vec{d}_T = \Delta\vec{d}_1 + \Delta\vec{d}_2$

Solution: Let ϕ represent the angle $\Delta\vec{d}_T$ makes with the x-axis.

$$\begin{aligned}\Delta\vec{d}_T &= \Delta\vec{d}_1 + \Delta\vec{d}_2\\ \Delta d_T^{\,2} &= \Delta d_1^{\,2} + \Delta d_2^{\,2}\\ \Delta d_T &= \sqrt{\Delta d_1^{\,2} + \Delta d_2^{\,2}}\\ &= \sqrt{(5.1\text{ km})^2 + (14\text{ km})^2}\\ \Delta d_T &= 15\text{ km}\end{aligned}$$

$$\begin{aligned}\tan\phi &= \frac{\Delta d_2}{\Delta d_1}\\ \tan\phi &= \frac{14\ \cancel{\text{km}}}{5.1\ \cancel{\text{km}}}\\ \tan\phi &= 2.745\\ \phi &= \tan^{-1}(2.745)\\ \phi &= 70°\end{aligned}$$

Statement: The sum of the two vectors is 15 km [E 70° N].

3. Given: $\Delta\vec{d}_1 = 11$ m [N 20° E]; $\Delta\vec{d}_2 = 9.0$ m [E]

Required: $\Delta\vec{d}_T$

Analysis: $\Delta\vec{d}_T = \Delta\vec{d}_1 + \Delta\vec{d}_2$

Solution: Determine the total x-component and y-component of $\Delta\vec{d}_T$:

$$\begin{aligned}\vec{d}_{Tx} &= \Delta\vec{d}_{1x} + \Delta\vec{d}_{2x}\\ &= (11\text{ m})(\sin 20°)\ [\text{E}] + 9.0\text{ m [E]}\\ &= 3.762\text{ m [E]} + 9.0\text{ m [E]}\\ &= 12.76\text{ m [E]}\\ \vec{d}_{Tx} &= 13\text{ m [E]}\end{aligned}$$

$$\begin{aligned}\vec{d}_{Ty} &= \Delta\vec{d}_{1y} + \Delta\vec{d}_{2y}\\ &= (11\text{ m})(\cos 20°)\ [\text{N}] + 0\text{ m}\\ &= 10.34\text{ m [N]}\\ \vec{d}_{Ty} &= 10\text{ m [N]}\end{aligned}$$

Determine the magnitude of $\Delta\vec{d}_T$:

$$\begin{aligned}\Delta d_T^{\,2} &= d_{Tx}^{\,2} + d_{Ty}^{\,2}\\ \Delta d_T &= \sqrt{d_{Tx}^{\,2} + d_{Ty}^{\,2}}\\ &= \sqrt{(12.76\text{ m})^2 + (10.34\text{ m})^2}\quad\text{(two extra digits carried)}\\ \Delta d_T &= 16\text{ m}\end{aligned}$$

Let ϕ represent the angle $\Delta\vec{d}_T$ makes with the y-axis.

$$\begin{aligned}\tan\phi &= \frac{\Delta d_{Tx}}{\Delta d_{Ty}}\\ \tan\phi &= \frac{12.76\ \cancel{\text{m}}}{10.34\ \cancel{\text{m}}}\quad\text{(two extra digits carried)}\\ \tan\phi &= 1.234\\ \phi &= \tan^{-1}(1.234)\\ \phi &= 51°\end{aligned}$$

Statement: The total displacement of the football player is 16 m [N 51° E].

4. Given: $\Delta\vec{d}_1 = 200.0$ m [S 25° W]; $\Delta\vec{d}_2 = 150.0$ m [N 30° E]

Required: $\Delta\vec{d}_T$

Analysis: $\Delta\vec{d}_T = \Delta\vec{d}_1 + \Delta\vec{d}_2$

Solution: Determine the total x-component and y-component of $\Delta\vec{d}_T$:

$$\begin{aligned}\vec{d}_{Tx} &= \Delta\vec{d}_{1x} + \Delta\vec{d}_{2x}\\ &= (200.0\text{ m})(\sin 25°)\ [\text{W}] + (150.0\text{ m})(\sin 30°)\ [\text{E}]\\ &= 84.5236\text{ m [W]} + 75\text{ m [E]}\\ &= 84.5236\text{ m [W]} - 75\text{ m [W]}\\ &= 9.5236\text{ m [W]}\\ \vec{d}_{Tx} &= 9.524\text{ m [W]}\end{aligned}$$

$$\begin{aligned}\vec{d}_{Ty} &= \Delta\vec{d}_{1y} + \Delta\vec{d}_{2y}\\ &= (200.0\text{ m})(\cos 25°)\ [\text{S}] + (150.0\text{ m})(\cos 30°)\ [\text{N}]\\ &= 181.262\text{ m [S]} + 129.904\text{ m [N]}\\ &= 181.262\text{ m [S]} - 129.904\text{ m [S]}\\ &= 51.358\text{ m [S]}\\ \vec{d}_{Ty} &= 51.36\text{ m [S]}\end{aligned}$$

Determine the magnitude of $\Delta\vec{d}_T$:

$$\begin{aligned}\Delta d_T^{\,2} &= d_{Tx}^{\,2} + d_{Ty}^{\,2}\\ \Delta d_T &= \sqrt{d_{Tx}^{\,2} + d_{Ty}^{\,2}}\\ &= \sqrt{(9.5236\text{ m})^2 + (51.358\text{ m})^2}\quad\text{(one extra digit carried)}\\ \Delta d_T &= 52.23\text{ m}\end{aligned}$$

Let ϕ represent the angle $\Delta\vec{d}_T$ makes with the y-axis.

$$\begin{aligned}\tan\phi &= \frac{\Delta d_{Tx}}{\Delta d_{Ty}}\\ \tan\phi &= \frac{9.5236\ \cancel{\text{m}}}{51.358\ \cancel{\text{m}}}\quad\text{(one extra digit carried)}\end{aligned}$$

$\tan\phi = 0.1854$
$\phi = \tan^{-1}(0.1854)$
$\phi = 11°$

Statement: The total displacement of the boat is 52.23 m [S 11° W].

5. Given: $\Delta\vec{d}_1$ = 25 m [N 20° W];

$\Delta\vec{d}_2$ = 35 m [S 15° E]

Required: $\Delta\vec{d}_T$

Analysis: $\Delta\vec{d}_T = \Delta\vec{d}_1 + \Delta\vec{d}_2$

Solution: Determine the total x-component and y-component of $\Delta\vec{d}_T$:

$$\begin{aligned}\vec{d}_{Tx} &= \Delta\vec{d}_{1x} + \Delta\vec{d}_{2x}\\ &= (25\text{ m})(\sin 20°)\text{ [W]} + (35\text{ m})(\sin 15°)\text{ [E]}\\ &= 8.551\text{ m [W]} + 9.059\text{ m [E]}\\ &= -8.551\text{ m [E]} + 9.059\text{ m [E]}\\ &= 0.508\text{ m [E]}\\ \vec{d}_{Tx} &= 0.51\text{ m [E]}\end{aligned}$$

$$\begin{aligned}\vec{d}_{Ty} &= \Delta\vec{d}_{1y} + \Delta\vec{d}_{2y}\\ &= (25\text{ m})(\cos 20°)\text{ [N]} + (35\text{ m})(\cos 15°)\text{ [S]}\\ &= 23.49\text{ m [N]} + 33.81\text{ m [S]}\\ &= -23.49\text{ m [S]} + 33.81\text{ m [S]}\\ &= 10.32\text{ m [S]}\\ \vec{d}_{Ty} &= 1.0\times 10\text{ m [S]}\end{aligned}$$

Determine the magnitude of $\Delta\vec{d}_T$:

$$\begin{aligned}\Delta d_T^{\,2} &= d_{Tx}^{\,2} + d_{Ty}^{\,2}\\ \Delta d_T &= \sqrt{d_{Tx}^{\,2} + d_{Ty}^{\,2}}\\ &= \sqrt{(0.508\text{ m})^2 + (10.32\text{ m})^2}\ \text{(two extra digits carried)}\\ \Delta d_T &= 1.0\times 10\text{ m}\end{aligned}$$

Let ϕ represent the angle $\Delta\vec{d}_T$ makes with the y-axis.

$$\begin{aligned}\tan\phi &= \frac{\Delta d_{Tx}}{\Delta d_{Ty}}\\ \tan\phi &= \frac{0.508\ \cancel{\text{m}}}{10.32\ \cancel{\text{m}}}\ \text{(one extra digit carried)}\\ \tan\phi &= 0.0492\\ \phi &= \tan^{-1}(0.0492)\\ \phi &= 3°\end{aligned}$$

Statement: The total displacement of the object is 1.0 × 10 m [S 3° E].

6. (a) Given: $\Delta\vec{d}_1$ = 4.3 km [W];

$\Delta\vec{d}_2$ = 8.0 km [W 54° N]

Required: $\Delta\vec{d}_T$

Analysis: $\Delta\vec{d}_T = \Delta\vec{d}_1 + \Delta\vec{d}_2$

Solution: Determine the total x-component and y-component of $\Delta\vec{d}_T$:

$$\begin{aligned}\vec{d}_{Tx} &= \Delta\vec{d}_{1x} + \Delta\vec{d}_{2x}\\ &= 4.3\text{ km [W]} + (8.0\text{ km})(\cos 54°)\text{ [W]}\\ &= 4.3\text{ km [W]} + 4.702\text{ km [W]}\\ &= 9.002\text{ km [W]}\\ \vec{d}_{Tx} &= 9.0\text{ km [W]}\end{aligned}$$

$$\begin{aligned}\vec{d}_{Ty} &= \Delta\vec{d}_{1y} + \Delta\vec{d}_{2y}\\ &= 0\text{ km} + (8.0\text{ km})(\sin 54°)\text{ [N]}\\ &= 6.472\text{ km [N]}\\ \vec{d}_{Ty} &= 6.5\text{ km [N]}\end{aligned}$$

Determine the magnitude of $\Delta\vec{d}_T$:

$$\begin{aligned}\Delta d_T^{\,2} &= d_{Tx}^{\,2} + d_{Ty}^{\,2}\\ \Delta d_T &= \sqrt{d_{Tx}^{\,2} + d_{Ty}^{\,2}}\\ &= \sqrt{(9.002\text{ km})^2 + (6.472\text{ km})^2}\ \text{(two extra digits carried)}\\ \Delta d_T &= 11\text{ km}\end{aligned}$$

Let ϕ represent the angle $\Delta\vec{d}_T$ makes with the x-axis.

$$\begin{aligned}\tan\phi &= \frac{\Delta d_{Ty}}{\Delta d_{Tx}}\\ \tan\phi &= \frac{6.472\ \cancel{\text{km}}}{9.002\ \cancel{\text{km}}}\ \text{(two extra digits carried)}\\ \tan\phi &= 0.7190\\ \phi &= \tan^{-1}(0.7190)\\ \phi &= 36°\end{aligned}$$

Statement: The total displacement given by the two vectors is 11 km [W 36° N].

(b) Given: $\Delta\vec{d}_1$ = 35 m [E 65° N];

$\Delta\vec{d}_2$ = 22 m [E 37° S]

Required: $\Delta\vec{d}_T$

Analysis: $\Delta\vec{d}_T = \Delta\vec{d}_1 + \Delta\vec{d}_2$

Solution: Determine the total x-component and y-component of $\Delta\vec{d}_T$:

$$\begin{aligned}\vec{d}_{Tx} &= \Delta\vec{d}_{1x} + \Delta\vec{d}_{2x}\\ &= (35\text{ m})(\cos 65°)\text{ [E]} + (22\text{ m})(\cos 37°)\text{ [E]}\\ &= 14.79\text{ m [E]} + 17.57\text{ m [E]}\\ &= 32.36\text{ m [E]}\\ \vec{d}_{Tx} &= 32\text{ m [E]}\end{aligned}$$

$$\vec{d}_{Ty} = \Delta\vec{d}_{1y} + \Delta\vec{d}_{2y}$$
$$= (35 \text{ m})(\sin 65°) \text{ [N]} + (22 \text{ m})(\sin 37°) \text{ [S]}$$
$$= 31.72 \text{ m [N]} + 13.24 \text{ m [S]}$$
$$= 31.72 \text{ m [N]} - 13.24 \text{ m [N]}$$
$$= 18.48 \text{ m [N]}$$
$$\vec{d}_{Ty} = 18 \text{ m [N]}$$

Determine the magnitude of $\Delta\vec{d}_T$:

$$\Delta d_T^{\,2} = d_{Tx}^{\,2} + d_{Ty}^{\,2}$$
$$\Delta d_T = \sqrt{d_{Tx}^{\,2} + d_{Ty}^{\,2}}$$
$$= \sqrt{(32.36 \text{ m})^2 + (18.48 \text{ m})^2} \text{ (two extra digits carried)}$$
$$\Delta d_T = 37 \text{ m}$$

Let ϕ represent the angle $\Delta\vec{d}_T$ makes with the x-axis.

$$\tan\phi = \frac{\Delta d_{Ty}}{\Delta d_{Tx}}$$
$$\tan\phi = \frac{18.48 \cancel{\text{m}}}{32.36 \cancel{\text{m}}} \text{ (one extra digit carried)}$$
$$\tan\phi = 0.5711$$
$$\phi = \tan^{-1}(0.5711)$$
$$\phi = 30°$$

Statement: The total displacement of the boat is 37 m [E 30° N].

7. Given: $\Delta\vec{d}_1 = 25$ m [N 30° W]; $\Delta\vec{d}_2 = 30.0$ m [N 40° E]; $\Delta\vec{d}_3 = 35$ m [S 25° W]

Required: $\Delta\vec{d}_T$

Analysis: $\Delta\vec{d}_T = \Delta\vec{d}_1 + \Delta\vec{d}_2 + \Delta\vec{d}_3$

Solution: Determine the total x-component and y-component of $\Delta\vec{d}_T$:

$$\vec{d}_{Tx} = \Delta\vec{d}_{1x} + \Delta\vec{d}_{2x} + \Delta\vec{d}_{3x}$$
$$= (25 \text{ m})(\sin 30°) \text{ [W]} + (30.0 \text{ m})(\sin 40°) \text{ [E]}$$
$$+ (35 \text{ m})(\sin 25°) \text{ [W]}$$
$$= 12.5 \text{ m [W]} + 19.28 \text{ m [E]} + 14.79 \text{ m [W]}$$
$$= 12.5 \text{ m [W]} - 19.28 \text{ m [W]} + 14.79 \text{ m [W]}$$
$$= 8.01 \text{ m [W]}$$
$$\vec{d}_{Tx} = 8.0 \text{ m [W]}$$

$$\vec{d}_{Ty} = \Delta\vec{d}_{1y} + \Delta\vec{d}_{2y} + \Delta\vec{d}_{3y}$$
$$= (25 \text{ m})(\cos 30°) \text{ [N]} + (30.0 \text{ m})(\cos 40°) \text{ [N]}$$
$$+ (35 \text{ m})(\cos 25°) \text{ [S]}$$
$$= 21.65 \text{ m [N]} + 22.98 \text{ m [N]} + 31.72 \text{ m [S]}$$
$$= 21.65 \text{ m [N]} + 22.98 \text{ m [N]} - 31.72 \text{ m [N]}$$
$$= 12.91 \text{ m [N]}$$
$$\vec{d}_{Ty} = 13 \text{ m [N]}$$

Determine the magnitude of $\Delta\vec{d}_T$:

$$\Delta d_T^{\,2} = d_{Tx}^{\,2} + d_{Ty}^{\,2}$$
$$\Delta d_T = \sqrt{d_{Tx}^{\,2} + d_{Ty}^{\,2}}$$
$$= \sqrt{(8.01 \text{ m})^2 + (12.91 \text{ m})^2} \text{ (two extra digits carried)}$$
$$\Delta d_T = 15 \text{ m}$$

Let ϕ represent the angle $\Delta\vec{d}_T$ makes with the y-axis.

$$\tan\phi = \frac{\Delta d_{Tx}}{\Delta d_{Ty}}$$
$$\tan\phi = \frac{8.01 \cancel{\text{m}}}{12.91 \cancel{\text{m}}} \text{ (two extra digits carried)}$$
$$\tan\phi = 0.6204$$
$$\phi = \tan^{-1}(0.6204)$$
$$\phi = 32°$$

Statement: The total displacement of the vectors is 15 m [N 32° W].

8. (a) Given: $\Delta d = 5.1$ km; $v = 0.87$ km/h

Required: Δt

Analysis: $v = \dfrac{\Delta d}{\Delta t}$

$$\Delta t = \frac{\Delta d}{v}$$

Solution: $\Delta t = \dfrac{\Delta d}{v}$

$$= \frac{5.1 \cancel{\text{km}}}{0.87 \dfrac{\cancel{\text{km}}}{\text{h}}}$$
$$= 5.862 \text{ h}$$
$$\Delta t = 5.9 \text{ h}$$

Statement: It will take the swimmer 5.9 h to cross the river.

(b) Given: $\vec{v}_x = 2.0$ km/h [W]

Required: $\Delta\vec{d}_x$

Analysis: $\vec{v}_x = \dfrac{\Delta\vec{d}_x}{\Delta t}$

$$\Delta\vec{d}_x = \vec{v}_x \Delta t$$

Solution:

$$\Delta\vec{d}_x = \vec{v}_x \Delta t$$
$$= \left(2.0 \frac{\text{km}}{\cancel{\text{h}}} \text{ [W]}\right)(5.862 \cancel{\text{h}}) \text{ (two extra digits carried)}$$
$$\Delta\vec{d}_x = 12 \text{ km [W]}$$

Statement: The current has moved the swimmer 12 km downstream by the time she reaches the other side.

9. Time for the conductor to reach the other side:
Given: Δd = 4.0 m; v = 1.2 m/s
Required: Δt

Analysis: $v = \dfrac{\Delta d}{\Delta t}$

$$\Delta t = \frac{\Delta d}{v}$$

Solution: $\Delta t = \dfrac{\Delta d}{v}$

$$= \frac{4.0 \ \cancel{\text{m}}}{1.2 \ \frac{\cancel{\text{m}}}{\text{s}}}$$

$$\Delta t = 3.3 \text{ s}$$

Statement: It will take the conductor 3.3 s to cross the railcar.

The conductor's velocity relative to the ground:
Given: $\vec{v}_1$ = 4.0 m/s [N]; $\vec{v}_2$ = 1.2 m/s [E]
Required: $\vec{v}_T$
Analysis: $\vec{v}_T = \vec{v}_1 + \vec{v}_2$
Solution: Let ϕ represent the angle $\Delta\vec{v}_T$ makes with the y-axis.

$$\vec{v}_T = \vec{v}_1 + \vec{v}_2$$
$$v_T^2 = v_1^2 + v_2^2$$
$$v_T = \sqrt{v_1^2 + v_2^2}$$
$$= \sqrt{(4.0 \text{ m/s})^2 + (1.2 \text{ m/s})^2}$$
$$v_T = 4.2 \text{ m/s}$$

$$\tan\phi = \frac{v_2}{v_1}$$
$$\tan\phi = \frac{1.2 \text{ m/s}}{4.0 \text{ m/s}}$$
$$\phi = 17°$$

Statement: The velocity of the conductor relative to the ground is 4.2 m/s [N 17° E].

10. Answers may vary. Sample answer:
(a) I prefer the algebraic method of adding vectors. It takes more time, but I think the answers are more accurate because there is no chance of making errors measuring. I prefer to use a diagram only to double check my answer by sketching the vectors.
(b) If the navigator on a boat were working on a large map, it would probably be more useful to plot the vectors directly on the map. There would be no need for calculations and the navigator could use other map features to double check the resultant (like comparing distances and directions to landmarks on the map).

Section 2.3: Projectile Motion

Tutorial 1 Practice, page 78

1. (a) Given: $\Delta d_y = -32$ m; $a_y = -9.8$ m/s^2; $v_y = 0$ m/s

Required: Δt

Analysis:
$$\begin{aligned}\Delta d_y &= v_y \Delta t + \frac{1}{2}a_y \Delta t^2\\ \Delta d_y &= 0 + \frac{1}{2}a_y \Delta t^2\\ \Delta t^2 &= \frac{2\Delta d_y}{a_y}\\ \Delta t &= \sqrt{\frac{2\Delta d_y}{a_y}}\end{aligned}$$

Solution:
$$\begin{aligned}\Delta t &= \sqrt{\frac{2\Delta d_y}{a_y}}\\ &= \sqrt{\frac{2(-32\ \cancel{\text{m}})}{\left(-9.8\ \frac{\cancel{\text{m}}}{\text{s}^2}\right)}}\\ &= 2.556\text{ s}\\ \Delta t &= 2.6\text{ s}\end{aligned}$$

Statement: The hockey puck is in flight for 2.6 s.

(b) Given: $\Delta t = 2.6$ s; $a_x = 0$ m/s^2; $v_x = 8.6$ m/s

Required: Δd_x

Analysis: $\Delta d_x = v_x \Delta t$

Solution:
$$\begin{aligned}\Delta d_x &= v_x \Delta t\\ &= \left(8.6\ \frac{\text{m}}{\cancel{\text{s}}}\right)(2.556\ \cancel{\text{s}})\ \text{(two extra digits carried)}\\ \Delta d_x &= 22\text{ m}\end{aligned}$$

Statement: The range of the hockey puck is 22 m.

2. Since the velocity of the puck does not affect its vertical motion, it would still take 2.6 s to hit the ground. The range of the puck would be half because it is travelling at half the velocity (horizontally). That means the range would be 11 m.

Tutorial 2 Practice, page 81

1. Given: $\Delta d_y = -17$ m; $a_y = -9.8$ m/s^2; $v_i = 7.3$ m/s; $\theta = 25°$

First determine the time of flight:

Required: Δt

Analysis: $\Delta d_y = v_y \Delta t + \frac{1}{2}a_y \Delta t^2$

Solution:
$$\begin{aligned}\Delta d_y &= v_y \Delta t + \frac{1}{2}a_y \Delta t^2\\ &= v_i(\sin\theta)\Delta t + \frac{1}{2}a_y \Delta t^2\\ -17 &= (7.3)(\sin 25°)\Delta t + \frac{1}{2}(-9.8)\Delta t^2\\ 0 &= 3.085\Delta t - 4.9\Delta t^2 + 17\\ \Delta t &= \frac{-3.085 \pm \sqrt{3.085^2 - 4(-4.9)(17)}}{2(-4.9)}\\ &= \frac{-3.085 \pm 18.51}{-9.8}\\ \Delta t &= -1.574\text{ s or } \Delta t = 2.204\text{ s}\end{aligned}$$

The answer must be the positive value.

Statement: The superhero is in flight for 2.2 s.

Determine the range:

Required: Δd_x

Analysis: $\Delta d_x = v_x \Delta t$

Solution:
$$\begin{aligned}\Delta d_x &= v_x \Delta t\\ &= v_i \cos\theta \Delta t\\ &= \left(7.3\ \frac{\text{m}}{\cancel{\text{s}}}\right)(\cos 25°)(2.204\ \cancel{\text{s}})\ \text{(two extra digits carried)}\\ \Delta d_x &= 15\text{ m}\end{aligned}$$

Statement: The superhero travels 15 m horizontally before landing.

Determine the final velocity:

Required: $\vec{v}_f$

Analysis: $\vec{v}_f = \vec{v}_{fx} + \vec{v}_{fy}$

Solution:
$$\begin{aligned}\vec{v}_f &= \vec{v}_{fx} + \vec{v}_{fy}\\ &= (v_i \cos\theta) + (\vec{v}_{iy} + \vec{a}_y \Delta t)\\ &= v_i\cos\theta + v_i\sin\theta + \vec{a}_y \Delta t\\ &= \left(7.3\ \frac{\text{m}}{\text{s}}\right)(\cos 25°)\ [\text{right}] + \left(7.3\ \frac{\text{m}}{\text{s}}\right)(\sin 25°)\ [\text{up}]\\ &\quad + \left(9.8\ \frac{\text{m}}{\text{s}^{\cancel{2}}}\ [\text{down}]\right)(2.204\ \cancel{\text{s}})\ \text{(two extra digits carried)}\\ &= 6.616\text{ m/s [right]} + 3.085\text{ [up]} + 21.60\text{ m/s [down]}\\ &= 6.616\text{ m/s [right]} - 3.085\text{ [down]} + 21.60\text{ m/s [down]}\\ \vec{v}_f &= 6.616\text{ m/s [right]} + 18.52\text{ m/s [down]}\end{aligned}$$

Use the Pythagorean theorem:
$$\begin{aligned}{v_f}^2 &= {v_{fx}}^2 + {v_{fy}}^2\\ v_f &= \sqrt{{v_{fx}}^2 + {v_{fy}}^2}\\ &= \sqrt{(6.616\text{ m/s})^2 + (18.52\text{ m/s})^2}\ \text{(two extra digits carried)}\\ v_f &= 2.0 \times 10\text{ m/s}\end{aligned}$$

Let ϕ represent the angle $\vec{v}_f$ makes with the x-axis.

$$\tan\phi = \frac{v_{fy}}{v_{fx}}$$
$$= \frac{18.52\ \frac{\text{m}}{\text{s}}}{6.616\ \frac{\text{m}}{\text{s}}} \text{ (two extra digits carried)}$$
$$= 2.799$$
$$\phi = \tan^{-1}(2.799)$$
$$\phi = 70°$$

Statement: The superhero's final velocity is 2.0×10 m/s [right 70° down].

2. The ball thrown at an angle above the horizontal will take longer to reach the ground. It has an initial velocity with a vertical component, so it will take more time for it to reach the ground due to gravity. The difference in the times to reach the ground depends on the initial height and the initial velocity of the first ball.

Section 2.3 Questions, page 81

1. The horizontal and vertical motions of a projectile take the same amount of time.

2. Given: $\Delta d_x = 20.0$ m; $a_y = -9.8$ m/s^2; $v_x = 10.0$ m/s; $v_y = 0$ m/s

Determine the time of flight first:

Required: Δt

Analysis: $v_x = \frac{\Delta d_x}{\Delta t}$

$$\Delta t = \frac{\Delta d_x}{v_x}$$

Solution: $\Delta t = \frac{\Delta d_x}{v_x}$

$$= \frac{20.0\ \text{m}}{10.0\ \frac{\text{m}}{\text{s}}}$$
$$\Delta t = 2.00 \text{ s}$$

Statement: It takes the tennis ball 2.00 s to reach the ground.

Determine the vertical displacement:

Required: Δd_y

Analysis: $\Delta d_y = v_y \Delta t + \frac{1}{2} a_y \Delta t^2$

Solution: $\Delta d_y = v_y \Delta t + \frac{1}{2} a_y \Delta t^2$

$$= 0 + \frac{1}{2}\left(9.8\ \frac{\text{m}}{\text{s}^2}\right)(2.00\ \text{s})^2$$
$$\Delta d_y = 2.0 \times 10 \text{ m}$$

Statement: The water tower is 2.0×10 m high.

3. (a) To give a projectile the greatest time of flight, launch it at 90° from the ground because this angle maximizes the vertical component of the velocity. At 90° from the ground, all the velocity is straight up instead of some of the velocity going into the horizontal component.

(b) To give a projectile the greatest time of flight, launch it at 45°. If the angle is less than 45°, the flight will be too short to travel any farther. If the angle is greater than 45°, the horizontal component of the velocity is too short to travel any farther.

4. (a) The ball experiences no horizontal acceleration. The ball accelerates 9.8 m/s^2 down due to gravity.

(b) The ball experiences no horizontal acceleration. The ball accelerates 9.8 m/s^2 down due to gravity.

(c) The ball experiences no horizontal acceleration. The ball accelerates 9.8 m/s^2 down due to gravity.

5. The arrow strikes the ground before reaching the target. It would take the arrow more than 1 s to travel to 60 m to the target when travelling at 55 m/s. But in 1 s, the arrow would fall more than 1.5 m due to gravity. For her next shot, the archer should increase the initial velocity, aim higher, or a combination of the two.

6. (a) Given: $v_i = 26$ m/s; $\theta = 60°$; $a_y = -9.8$ m/s^2; $v_{fy} = 0$ m/s

Required: Δt

Analysis: $v_{fy} = v_{iy} + a_y \Delta t$

$$\Delta t = \frac{v_{fy} - v_{iy}}{a_y}$$

Solution: $\Delta t = \frac{v_{fy} - v_{iy}}{a_y}$

$$= \frac{0 - (26 \text{ m/s})(\sin 60°)}{-9.8 \text{ m/s}^2}$$
$$= \frac{22.52\ \frac{\text{m}}{\text{s}}}{9.8\ \frac{\text{m}}{\text{s}^2}}$$
$$= 2.298 \text{ s}$$
$$\Delta t = 2.3 \text{ s}$$

Statement: It takes the acrobat 2.3 s to reach his maximum height.

(b) Given: $v_i = 26$ m/s; $\theta = 60°$; $a_y = -9.8$ m/s^2; $v_{fy} = 0$ m/s

Determine the maximum height:

Required: Δd_y

Analysis: $\Delta d_y = \frac{v_{fy} + v_{iy}}{2} \Delta t$

Solution:

$$\Delta d_y = \frac{v_{fy} + v_{iy}}{2}\Delta t$$
$$= \frac{0 - 22.52\ \frac{\text{m}}{\cancel{\text{s}}}}{2}(2.298\ \cancel{\text{s}})\ \text{(two extra digits carried)}$$
$$= 25.88\ \text{m}$$
$$\Delta d_y = 26\ \text{m}$$

Statement: The maximum height of the acrobat is 26 m.

Determine the time to reach half the maximum height (13 m) the second time:

Required: Δt

Analysis: $\Delta d_y = v_{iy}\,\Delta t + \frac{1}{2}a_y\,\Delta t^2$

Solution: $\Delta d_y = v_{iy}\,\Delta t + \frac{1}{2}a_y\,\Delta t^2$

$$13 = (22.52)\Delta t + \frac{1}{2}(-9.8)\Delta t^2$$
$$0 = 22.52\Delta t - 4.9\Delta t^2 - 13$$
$$\Delta t = \frac{-22.52 \pm \sqrt{22.52^2 - 4(-4.9)(-13)}}{2(-4.9)}$$
$$= \frac{-22.52 \pm 15.89}{-9.8}$$
$$\Delta t = 0.68\ \text{s or } \Delta t = 3.9\ \text{s}$$

The first time is on his way up, so the correct time is 3.9 s.

Statement: It takes the acrobat 3.9 s to reach a point halfway back down to the ground.

7. Given: $v_i = 20$ m/s; $\theta = 45°$; $a_y = -9.8$ m/s^2; $v_{fy} = 0$ m/s

Determine the time of flight:

Required: Δt

Analysis: $\Delta d_y = v_{iy}\,\Delta t + \frac{1}{2}a_y\,\Delta t^2$

Solution:

$$\Delta d_y = v_{iy}\,\Delta t + \frac{1}{2}a_y\,\Delta t^2$$
$$0 = v_i(\sin 45°)\Delta t + \frac{1}{2}a_y\,\Delta t^2$$
$$0 = v_i(\sin 45°) + \frac{1}{2}a_y\,\Delta t,\ \ \Delta t \neq 0$$
$$\frac{1}{2}a_y\,\Delta t = -v_i(\sin 45°)$$
$$\Delta t = \frac{-2v_i(\sin 45°)}{a_y}$$
$$= \frac{-2\left(20\ \frac{\cancel{\text{m}}}{\cancel{\text{s}}}\right)(\sin 45°)}{\left(-9.8\ \frac{\cancel{\text{m}}}{\text{s}^{\cancel{2}}}\right)}$$
$$= 2.886\ \text{s}$$
$$\Delta t = 2.9\ \text{s}$$

Statement: The time of flight of the golf ball is 2.9 s.

Determine the horizontal distance:

Required: Δd_x

Analysis: $\Delta d_x = v_{ix}\,\Delta t$

Solution: $\Delta d_x = v_{ix}\,\Delta t$

$$= v_i(\cos 45°)\Delta t$$
$$= \left(20\ \frac{\text{m}}{\cancel{\text{s}}}\right)(\sin 45°)(2.9\ \cancel{\text{s}})$$
$$\Delta d_x = 41\ \text{m}$$

Statement: The golfer was 41 m from the hole when he hit the ball.

Determine the maximum height:

Required: Δd_y

Analysis: $v_{fy}{}^2 = v_{iy}{}^2 + 2a_y\,\Delta d_y$

$$\Delta d_y = \frac{v_{fy}{}^2 - v_{iy}{}^2}{2a_y}$$

Solution: $\Delta d_y = \frac{v_{fy}{}^2 - v_{iy}{}^2}{2a_y}$

$$= \frac{0 - (v_i \sin 45°)^2}{2(-9.8\ \text{m/s}^2)}$$
$$= \frac{0 - [(20\ \text{m/s})(\sin 45°)]^2}{-19.6\ \text{m/s}^2}$$
$$= \frac{200\ \frac{\text{m}^{\cancel{2}}}{\cancel{\text{s}^2}}}{19.6\ \frac{\cancel{\text{m}}}{\cancel{\text{s}^2}}}$$
$$\Delta d_y = 1.0 \times 10\ \text{m}$$

Statement: The golf ball reached a maximum height of 1.0 × 10 m.

8. Given: $\Delta d_y = -12$ m; $a_y = -9.8$ m/s^2; $v_i = 4.5$ m/s; $\theta = 25°$

First determine the time of flight:

Required: Δt

Analysis: $\Delta d_y = v_y\,\Delta t + \frac{1}{2}a_y\,\Delta t^2$

Solution: $\Delta d_y = v_y \Delta t + \frac{1}{2}a_y \Delta t^2$

$$= v_i(\sin\theta)\Delta t + \frac{1}{2}a_y \Delta t^2$$

$$-12 = (4.5)(\sin 25°)\Delta t + \frac{1}{2}(-9.8)\Delta t^2$$

$$0 = 1.902\Delta t - 4.9\Delta t^2 + 12$$

$$\Delta t = \frac{-1.902 \pm \sqrt{1.902^2 - 4(-4.9)(12)}}{2(-4.9)}$$

$$= \frac{-1.902 \pm 15.45}{-9.8}$$

$$\Delta t = -1.382 \text{ s or } \Delta t = 1.771 \text{ s}$$

The answer must be the positive value.

Statement: The beanbag is in flight for 1.8 s.

Determine the range:

Required: Δd_x

Analysis: $\Delta d_x = v_x \Delta t$

Solution:

$$\Delta d_x = v_x \Delta t$$

$$= v_i \cos\theta \Delta t$$

$$= \left(4.5 \frac{\text{m}}{\cancel{\text{s}}}\right)(\cos 25°)(1.771 \cancel{\text{s}}) \text{ (two extra digits carried)}$$

$$\Delta d_x = 7.2 \text{ m}$$

Statement: The student's friend must stand 7.2 m from the building to catch the beanbag.

Section 2.4: Physics Journal

Section 2.4 Questions, page 83

1. Answers may vary. Sample answer:
Albert Einstein considered Galileo to be the "father of modern science" because Galileo was the first to explore science through experiments instead of "purely logical means."

2. Galileo chose to use a ramp to perform his acceleration experiment because objects in free-fall increase their speed far too quickly to measure accurately.

3. Answers may vary. Sample answer could include one of the following:
Galileo discovered four of Jupiter's satellites, using a telescope that he devised. He was also the first to detect sunspots. He refuted Aristotle's theory that the moon was a smooth sphere. With the help of his telescope, Galileo showed that the Moon's surface had valleys and pot marks.

4. Answers may vary. Sample answers could include three of any of the following:
Until the mid nineteenth century, it was widely believed that there were only nine planets in our solar system. But due to discoveries of other bodies in our solar system that have the traits of a planet but were larger than Pluto. Pluto was reclassified as a dwarf planet in 2006. Other newly discovered bodies were also classified as dwarf planets. So officially today, there are only eight planets in our solar system.
Before the Human Genome Project started in 1984, it was widely believed that humans shared 98.5 % of their genes with monkeys, that percentage is now 96. However, it was discovered, that humans share a great percentage of the genes with other species that are very unlike humans. In fact, 61 % of human genes are a match to genes of a household fly.
Prior to the twenty-first century, it was believed that the ice caps of the polar region did not change much in size or mass over time. However, recent studies have shown that these masses are actually decreasing at a dramatic rate much faster than in previous years. With global warming, new phenomena are emerging such as the shrinking polar ice cap, more violent weather patterns, and odd weather patterns for various regions around the globe.

5. Answers may vary. Sample answer:
In the sixteenth and seventeenth centuries, Galileo's experiments and published works were often criticized by fellow scholars who followed Aristotle or by the Church.
In 1612, he published *Discourse on Floating Objects*, which refuted Aristotle's views on velocity of objects and their mass, through experiments. Galileo's work was challenged in four print articles.
In 1613, he published *Letters on Sunspots*. With the aid of his telescope, he showed that the Moon's surface was not smooth and hypothesized that the Earth was not stationary. This was in opposition to widely held views by the Church and he was called a heretic.
In 1614, he was denounced by some members of the Catholic church when he suggested that the Church's teachings and science be separate subjects.
In 1624, he wrote *Dialogue on the Tides*, which discussed Ptolemy's and Copernicus's theories on the physics of tides. The Church censors changed the title to *Dialogue on the Two Chief World Systems* and allowed it to be printed in 1632.

Chapter 2 Investigations

Investigation 2.3.1: Modelling Projectile Motion, page 87

Analyze and Evaluate

(a) Increasing (or decreasing) the vertical displacement of a projectile increases (or decreases) the range and time of flight.
(b) In theory, the square root of the vertical displacement is proportional to the time of flight.
(c) In theory, the square root of the vertical displacement is proportional to the range.
(d) Answers may vary. Sample answer:
My experimental range data was higher than my experimental range data.
(e) Answers may vary. Sample answer:
Some sources of uncertainty could be due to flaws in the uniformity of the ball, the surface of the ramp, and the surface the ball rolled onto.
(f) Answers may vary. Sample answer:
I would make sure the ball was smooth all around, and that the ramp and surface were smooth and clear of dirt, each time I carried out a trial.
(g) Answers may vary. Sample answer:
The data supported my prediction that increasing the vertical displacement of a projectile increases the range and time of flight.

Apply and Extend

(h) Answers may vary. Sample answer:
This equipment could be used to study projectiles with some initial vertical velocity, either down, by pointing the ramp over the edge of the table, or up, by adding a second ramp for the ball to roll up. The equipment could also be used to copy Galileo's study of two balls with different masses.

Chapter 2 Review, pages 90–95

Note: After the first printing, directions were added to each answer option for Question 6. The correct answer is still (c).

Knowledge

1. (b)
2. (d)
3. (b)
4. (a)
5. (b)
6. (c)
7. (c)
8. (a)
9. (a)
10. False. A diagram with a scale of 1 cm : 10 cm means that *1 cm* on the diagram represents *10 cm* in real life.
11. True
12. False. To add two vectors on a diagram, join them tip to *tail*.
13. False. The resultant vector is the vector that results from *adding* the given vectors.
14. False. When given the *x*- and *y*-component vectors, the Pythagorean theorem should be used to determine the *magnitude* of the displacement vector.
15. True
16. False. The *x*-component of the vector 8.0 m [S 45° W] is *5.7 m* [W].
17. True
18. True
19. False. When two objects are dropped from the same height at the same time, *both objects will land at the same time* when there is no air resistance.
20. (a)(ii)
(b)(i)
(c)(iv)
(d)(v)
(e)(iii)

Understanding

21. Answers may vary. Sample answers:
(a) A map or a building blueprints would have a scale that is smaller than the real-world measurement because you would want to see the whole area at a reasonable size.
(a) A diagram of a cell, atoms, or a microchip would have a scale that is larger than the real-world measurement because the original objects are too small to see in detail.

22. For each vector, the magnitude stays the same, but the cardinal directions are replaced by their opposites.

(a) $\Delta\vec{d} = 17\text{ m [W 63° S]}$
$\Delta\vec{d}_{\text{opposite}} = 17\text{ m [E 63° N]}$

(b) $\Delta\vec{d} = 79\text{ cm [E 56° N]}$
$\Delta\vec{d}_{\text{opposite}} = 79\text{ cm [W 56° S]}$

(c) $\Delta\vec{d} = 44\text{ km [S 27° E]}$
$\Delta\vec{d}_{\text{opposite}} = 44\text{ km [N 27° W]}$

23.

Diagram size	Real-world size
3.4 cm	**170 m**
0.75 cm	37.5 m
85.0 mm	**425 m**
25.0 cm	1250 m

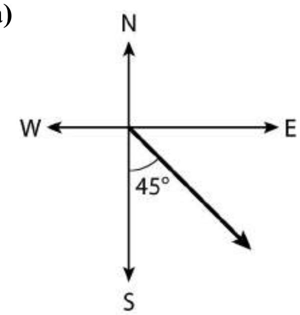
Row 1: Multiply by 50 and change the units to metres.
3.4 cm × 50 = 170 m

Row 2: Divide by 50 and change the units to cenitmetres.
37.5 m ÷ 50 = 0.75 cm

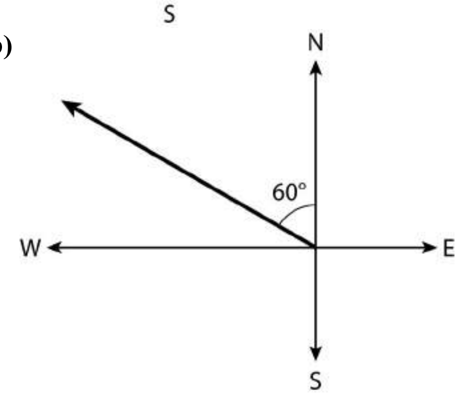
Row 3: Multiply by 50 and change the units to metres.
85.0 cm × 50 = 425 m

Row 4: Divide by 50 and change the units to cenitmetres.
1250 m ÷ 50 =25.0 cm

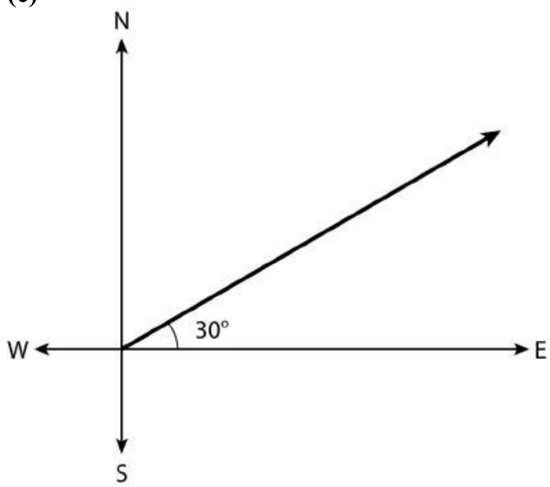
24. (a)



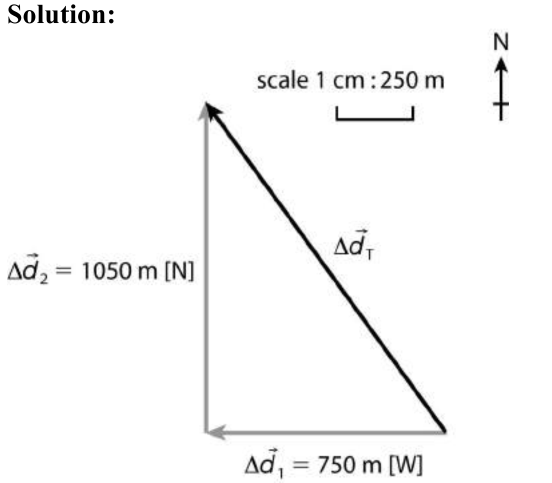
(b)



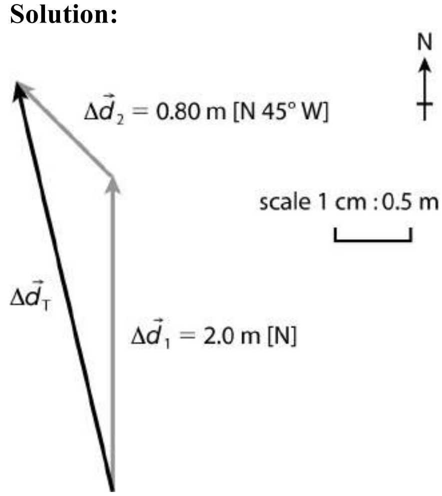
(c)



25. (a) Since 120 is 50 times 2.4, an appropriate scale is 1 cm : 50 m.
(b) Since 360 is 150 times 2.4, an appropriate scale is 1 cm : 150 km.
(c) Since 1200 is 500 times 2.4, an appropriate scale is 1 cm : 500 m.
26. For each vector, determine the complementary angle, then reverse the order of the directions.
(a) 90° – 18° = 72°
$\Delta\vec{d}$ = 566 m [W 18° N]
$\Delta\vec{d}$ = 566 m [N 72° W]
(b) 90° – 68° = 22°
$\Delta\vec{d}$ = 37 cm [E 68° S]
$\Delta\vec{d}$ = 37 cm [S 22° E]
(c) 90° – 38° = 52°
$\Delta\vec{d}$ = 7150 km [S 38° W]
$\Delta\vec{d}$ = 7150 km [W 52° S]
27. Given: $\Delta\vec{d}_1$ = 750 m [W]; $\Delta\vec{d}_2$ = 1050 m [N]
Required: $\Delta\vec{d}_T$
Analysis: $\Delta\vec{d}_T = \Delta\vec{d}_1 + \Delta\vec{d}_2$
Solution:



This figure shows the given vectors, with the tip of $\Delta\vec{d}_1$ joined to the tail of $\Delta\vec{d}_2$. The resultant vector $\Delta\vec{d}_T$ is drawn in black from the tail of $\Delta\vec{d}_1$ to the tip of $\Delta\vec{d}_2$. Using a compass, the direction of $\Delta\vec{d}_T$ is [W 54° N]. $\Delta\vec{d}_T$ measures 5.2 cm in length, so using the scale of 1 cm : 250 m, the actual magnitude of $\Delta\vec{d}_T$ is 1300 m.
Statement: The net displacement of the driver is 1300 m [W 54° N].
28. Given: $\Delta\vec{d}_1$ = 2.0 m [N];
$\Delta\vec{d}_2$ = 0.80 m [N 45° W]
Required: $\Delta\vec{d}_T$
Analysis: $\Delta\vec{d}_T = \Delta\vec{d}_1 + \Delta\vec{d}_2$
Solution:



This figure shows the given vectors, with the tip of $\Delta\vec{d}_1$ joined to the tail of $\Delta\vec{d}_2$. The resultant vector $\Delta\vec{d}_T$ is drawn in black from the tail of $\Delta\vec{d}_1$ to the tip of $\Delta\vec{d}_2$. Using a compass, the direction of $\Delta\vec{d}_T$ is [N 12° W]. $\Delta\vec{d}_T$ measures 5.2 cm in length, so using the scale of 1 cm : 0.5 m, the actual magnitude of $\Delta\vec{d}_T$ is 2.6 m.
Statement: The net displacement of the cue ball is 2.6 m [N 12° W].
29.

d_x	d_y	d_T
3.0	4.0	**5.0**
8.0	**6.0**	10.0
6.00	5.00	7.81
4.00	**7.00**	8.06

Use the Pythagorean theorem to determine each missing magnitude: $d_x^2 + d_y^2 = d_T^2$.

Row 1:

$$d_T^{\ 2} = d_x^{\ 2} + d_y^{\ 2}$$
$$d_T = \sqrt{d_x^{\ 2} + d_y^{\ 2}}$$
$$= \sqrt{(3.0)^2 + (4.0)^2}$$
$$d_T = 5.0$$

Row 2:

$$d_T^{\ 2} = d_x^{\ 2} + d_y^{\ 2}$$
$$d_y = \sqrt{d_T^{\ 2} - d_x^{\ 2}}$$
$$= \sqrt{(10.0)^2 - (8.0)^2}$$
$$d_y = 6.0$$

Row 3:

$$d_T^{\ 2} = d_x^{\ 2} + d_y^{\ 2}$$
$$d_x = \sqrt{d_T^{\ 2} - d_y^{\ 2}}$$
$$= \sqrt{(7.81)^2 - (5.00)^2}$$
$$d_x = 6.00$$

Row 4:

$$d_T^{\ 2} = d_x^{\ 2} + d_y^{\ 2}$$
$$d_y = \sqrt{d_T^{\ 2} - d_x^{\ 2}}$$
$$= \sqrt{(8.06)^2 - (4.00)^2}$$
$$d_y = 7.00$$

30.

$\vec{d}_x$	$\vec{d}_y$	ϕ
3.0 [E]	4.0 [N]	**E 53° N**
5.00 [W]	7.00 [N]	**W 54.5° N**
82.0 [E]	**21.1 [S]**	E 14.4° S
351 [W]	456 [N]	W 52.4° N

Use the tangent function: $\tan\phi = \dfrac{d_y}{d_x}$.

Row 1: Find the missing angle.

$$\tan\phi = \frac{d_y}{d_x}$$
$$\tan\phi = \frac{4.0}{3.0}$$
$$\tan\phi = 1.333 \text{ (two extra digits carried)}$$
$$\phi = \tan^{-1}(1.333)$$
$$\phi = 53°$$

Row 2: Find the missing angle.

$$\tan\phi = \frac{d_y}{d_x}$$
$$\tan\phi = \frac{7.00}{5.00}$$
$$\tan\phi = 1.40$$
$$\phi = \tan^{-1}(1.40)$$
$$\phi = 54.5°$$

Row 3: Find the missing component vector.

$$\tan\phi = \frac{d_y}{d_x}$$
$$\tan\ 14.4° = \frac{d_y}{82.0}$$
$$(0.257)(82.0) = d_y$$
$$21.1 = d_y$$

Row 4: Find the missing component vector.

$$\tan\phi = \frac{d_y}{d_x}$$
$$\tan\ 52.4° = \frac{456}{d_x}$$
$$d_x = \frac{256}{1.30}$$
$$d_x = 351$$

31. (a) Given: $\Delta\vec{d}_T$ = 52 m [W 72° S]

Required: $\Delta\vec{d}_x$; $\Delta\vec{d}_y$

Analysis: $\Delta\vec{d}_T = \Delta\vec{d}_x + \Delta\vec{d}_y$

Solution: Since the direction of $\Delta\vec{d}_T$ is between west and south, the direction of $\Delta\vec{d}_x$ is [W] and the direction of $\Delta\vec{d}_y$ is [S].

$$\sin\theta = \frac{\Delta d_y}{\Delta d_T}$$
$$\Delta d_y = \Delta d_T \sin\theta$$
$$= (52 \text{ m})(\sin 72°)$$
$$\Delta d_y = 49 \text{ m}$$

$$\cos\theta = \frac{\Delta d_x}{\Delta d_T}$$
$$\Delta d_x = \Delta d_T \cos\theta$$
$$= (52 \text{ m})(\cos 72°)$$
$$\Delta d_x = 16 \text{ m}$$

Statement: The vector has a horizontal or x-component of 16 m [W] and a vertical or y-component of 49 m [S].

(b) Given: $\Delta\vec{d}_T$ = 38 km [E 14° N]

Required: $\Delta\vec{d}_x$; $\Delta\vec{d}_y$

Analysis: $\Delta\vec{d}_T = \Delta\vec{d}_x + \Delta\vec{d}_y$

Solution: Since the direction of $\Delta\vec{d}_T$ is between east and north, the direction of $\Delta\vec{d}_x$ is [E] and the direction of $\Delta\vec{d}_y$ is [N].

$$\sin\theta = \frac{\Delta d_y}{\Delta d_T}$$
$$\begin{aligned}\Delta d_y &= \Delta d_T \sin\theta \\ &= (38\text{ km})(\sin 14°) \\ \Delta d_y &= 9.2\text{ km}\end{aligned}$$
$$\cos\theta = \frac{\Delta d_x}{\Delta d_T}$$
$$\begin{aligned}\Delta d_x &= \Delta d_T \cos\theta \\ &= (38\text{ km})(\cos 14°) \\ \Delta d_x &= 37\text{ km}\end{aligned}$$

Statement: The vector has a horizontal or *x*-component of 37 km [E] and a vertical or *y*-component of 9.2 km [N].

(c) Given: $\Delta\vec{d}_T$ = 92 m [S 82° W]

Required: $\Delta\vec{d}_x$; $\Delta\vec{d}_y$

Analysis: $\Delta\vec{d}_T = \Delta\vec{d}_x + \Delta\vec{d}_y$

Solution: Since the direction of $\Delta\vec{d}_T$ is between south and west, the direction of $\Delta\vec{d}_x$ is [W] and the direction of $\Delta\vec{d}_y$ is [S].

$$\cos\theta = \frac{\Delta d_y}{\Delta d_T}$$
$$\begin{aligned}\Delta d_y &= \Delta d_T \cos\theta \\ &= (92\text{ m})(\cos 82°) \\ \Delta d_y &= 13\text{ m}\end{aligned}$$

$$\sin\theta = \frac{\Delta d_x}{\Delta d_T}$$
$$\begin{aligned}\Delta d_x &= \Delta d_T \sin\theta \\ &= (92\text{ m})(\sin 82°) \\ \Delta d_x &= 91\text{ m}\end{aligned}$$

Statement: The vector has a horizontal or *x*-component of 91 m [W] and a vertical or *y*-component of 13 m [S].

32. (a) Given: $\Delta\vec{d}_x$ = 5.0 m [W]; $\Delta\vec{d}_y$ = 2.9 m [S]

Required: $\Delta\vec{d}_T$

Analysis: $\Delta\vec{d}_T = \Delta\vec{d}_x + \Delta\vec{d}_y$

Solution: Let ϕ represent the angle $\Delta\vec{d}_T$ makes with the *x*-axis.

$$\begin{aligned}\Delta\vec{d}_T &= \Delta\vec{d}_x + \Delta\vec{d}_y \\ \Delta d_T^{\,2} &= \Delta d_x^{\,2} + \Delta d_y^{\,2} \\ \Delta d_T &= \sqrt{\Delta d_x^{\,2} + \Delta d_y^{\,2}} \\ &= \sqrt{(5.0\text{ m})^2 + (2.9\text{ m})^2} \\ \Delta d_T &= 5.8\text{ m}\end{aligned}$$
$$\tan\phi = \frac{\Delta d_y}{\Delta d_x}$$
$$\tan\phi = \frac{2.9\ \cancel{\text{m}}}{5.0\ \cancel{\text{m}}}$$
$$\phi = 30°$$

Statement: The sum of the two vectors is 5.8 m [W 30° S].

(b) Given: $\Delta\vec{d}_x$ = 18 m [E]; $\Delta\vec{d}_y$ = 5.2 m [N]

Required: $\Delta\vec{d}_T$

Analysis: $\Delta\vec{d}_T = \Delta\vec{d}_x + \Delta\vec{d}_y$

Solution: Let ϕ represent the angle $\Delta\vec{d}_T$ makes with the *x*-axis.

$$\begin{aligned}\Delta\vec{d}_T &= \Delta\vec{d}_x + \Delta\vec{d}_y \\ \Delta d_T^{\,2} &= \Delta d_x^{\,2} + \Delta d_y^{\,2} \\ \Delta d_T &= \sqrt{\Delta d_x^{\,2} + \Delta d_y^{\,2}} \\ &= \sqrt{(18\text{ m})^2 + (5.2\text{ m})^2} \\ \Delta d_T &= 19\text{ m}\end{aligned}$$

$$\tan\phi = \frac{\Delta d_y}{\Delta d_x}$$
$$\tan\phi = \frac{5.2\ \cancel{\text{m}}}{18\ \cancel{\text{m}}}$$
$$\phi = 16°$$

Statement: The sum of the two vectors is 19 m [E 16° N].

(c) Given: $\Delta\vec{d}_x$ = 64 km [W]; $\Delta\vec{d}_y$ = 31 km [N]

Required: $\Delta\vec{d}_T$

Analysis: $\Delta\vec{d}_T = \Delta\vec{d}_x + \Delta\vec{d}_y$

Solution: Let ϕ represent the angle $\Delta\vec{d}_T$ makes with the *x*-axis.

$$\begin{aligned}\Delta\vec{d}_T &= \Delta\vec{d}_x + \Delta\vec{d}_y \\ \Delta d_T^{\,2} &= \Delta d_x^{\,2} + \Delta d_y^{\,2} \\ \Delta d_T &= \sqrt{\Delta d_x^{\,2} + \Delta d_y^{\,2}} \\ &= \sqrt{(64\text{ km})^2 + (31\text{ km})^2} \\ \Delta d_T &= 71\text{ km}\end{aligned}$$

$$\tan\phi = \frac{\Delta d_y}{\Delta d_x}$$

$$\tan\phi = \frac{31\ \cancel{\text{km}}}{64\ \cancel{\text{km}}}$$

$$\phi = 26°$$

Statement: The sum of the two vectors is 71 km [W 26° N].

Note: After the first printing, the unit "m" was added to the displacement labels on Figures 2, 3, and 4 in the Student Book. The solutions to Questions 33, 34, and 35 reflect these changes.

33. Given: $\Delta\vec{d}_1 = 2.0$ m [left]; $\Delta\vec{d}_2 = 5.0$ m [up]

Required: Δd_T

Analysis: $\Delta\vec{d}_T = \Delta\vec{d}_1 + \Delta\vec{d}_2$

Solution:

$$\begin{aligned}\Delta\vec{d}_T &= \Delta\vec{d}_1 + \Delta\vec{d}_2\\ \Delta d_T^{\,2} &= \Delta d_1^{\,2} + \Delta d_2^{\,2}\\ \Delta d_T &= \sqrt{\Delta d_1^{\,2} + \Delta d_2^{\,2}}\\ &= \sqrt{(2.0\text{ m})^2 + (5.0\text{ m})^2}\\ \Delta d_T &= 5.4\text{ m}\end{aligned}$$

Statement: The magnitude of the resultant vector is 5.4 m.

34. Given: $\Delta\vec{d}_1 = 2.0$ m [left]; $\Delta\vec{d}_2 = 6.0$ m [up]

Required: Δd_T

Analysis: $\Delta\vec{d}_T = \Delta\vec{d}_1 + \Delta\vec{d}_2$

Solution:

$$\begin{aligned}\Delta\vec{d}_T &= \Delta\vec{d}_1 + \Delta\vec{d}_2\\ \Delta d_T^{\,2} &= \Delta d_1^{\,2} + \Delta d_2^{\,2}\\ \Delta d_T &= \sqrt{\Delta d_1^{\,2} + \Delta d_2^{\,2}}\\ &= \sqrt{(2.0\text{ m})^2 + (6.0\text{ m})^2}\\ \Delta d_T &= 6.3\text{ m}\end{aligned}$$

Statement: The magnitude of the resultant vector is 6.3 m.

35. Given: $\Delta\vec{d}_1 = 7.0$ m [down];
$\Delta\vec{d}_2 = 4.0$ m [right]

Required: Δd_T

Analysis: $\Delta\vec{d}_T = \Delta\vec{d}_1 + \Delta\vec{d}_2$

Solution:

$$\begin{aligned}\Delta\vec{d}_T &= \Delta\vec{d}_1 + \Delta\vec{d}_2\\ \Delta d_T^{\,2} &= \Delta d_1^{\,2} + \Delta d_2^{\,2}\\ \Delta d_T &= \sqrt{\Delta d_1^{\,2} + \Delta d_2^{\,2}}\\ &= \sqrt{(7.0\text{ m})^2 + (4.0\text{ m})^2}\\ \Delta d_T &= 8.1\text{ m}\end{aligned}$$

Statement: The magnitude of the resultant vector is 8.1 m.

36. Given: $\Delta\vec{d}_1 = 24$ m [W 12° S];
$\Delta\vec{d}_2 = 33$ m [E 52° S]

Required: $\Delta\vec{d}_T$

Analysis: $\Delta\vec{d}_T = \Delta\vec{d}_1 + \Delta\vec{d}_2$

Solution: Determine the total *x*-component and *y*-component of $\Delta\vec{d}_T$:

$$\begin{aligned}\vec{d}_{Tx} &= \Delta\vec{d}_{1x} + \Delta\vec{d}_{2x}\\ &= (24\text{ m})(\cos 12°)\text{ [W]} + (33\text{ m})(\cos 52°)\text{ [E]}\\ &= 23.48\text{ m [W]} + 20.32\text{ m [E]}\\ &= 23.48\text{ m [W]} - 20.32\text{ m [W]}\\ &= 3.16\text{ m [W]}\\ \vec{d}_{Tx} &= 3.2\text{ m [W]}\end{aligned}$$

$$\begin{aligned}\vec{d}_{Ty} &= \Delta\vec{d}_{1y} + \Delta\vec{d}_{2y}\\ &= (24\text{ m})(\sin 12°)\text{ [S]} + (33\text{ m})(\sin 52°)\text{ [S]}\\ &= 4.99\text{ m [S]} + 26.00\text{ m [S]}\\ &= 30.99\text{ m [S]}\\ \vec{d}_{Ty} &= 31\text{ m [S]}\end{aligned}$$

Determine the magnitude of $\Delta\vec{d}_T$:

$$\begin{aligned}\Delta d_T^2 &= d_{Tx}^2 + d_{Ty}^2\\ \Delta d_T &= \sqrt{d_{Tx}^2 + d_{Ty}^2}\\ &= \sqrt{(3.16\text{ m})^2 + (30.99\text{ m})^2}\ \text{(two extra digits carried)}\\ \Delta d_T &= 31\text{ m}\end{aligned}$$

Let ϕ represent the angle $\Delta\vec{d}_T$ makes with the *x*-axis.

$$\tan\phi = \frac{\Delta d_{Ty}}{\Delta d_{Tx}}$$

$$\tan\phi = \frac{30.99\ \cancel{\text{m}}}{3.16\ \cancel{\text{m}}}\ \text{(two extra digits carried)}$$

$$\phi = 84°$$

Statement: The dog's displacement is 31 m [W 84° S].

37. (a) Given: $\Delta d = 36$ m; $v = 2.0$ m/s

Required: Δt

Analysis: $v = \dfrac{\Delta d}{\Delta t}$

$$\Delta t = \frac{\Delta d}{v}$$

Solution:

$$\begin{aligned}\Delta t &= \frac{\Delta d}{v}\\ &= \frac{36\ \cancel{\text{m}}}{2.0\ \dfrac{\cancel{\text{m}}}{\text{s}}}\\ \Delta t &= 18\text{ s}\end{aligned}$$

Statement: It will take the student 18 s to cross the river.

(b) Given: $\vec{v}_1$ = 6.2 m/s [W]; $\vec{v}_2$ = 2.0 m/s [N]

Required: $\vec{v}_T$

Analysis: $\vec{v}_T = \vec{v}_1 + \vec{v}_2$

Solution: Let ϕ represent the angle $\Delta\vec{v}_T$ makes with the *x*-axis.

$$\begin{aligned}\vec{v}_T &= \vec{v}_1 + \vec{v}_2\\ {v_T}^2 &= {v_1}^2 + {v_2}^2\\ v_T &= \sqrt{{v_1}^2 + {v_2}^2}\\ &= \sqrt{(6.2\text{ m/s})^2 + (2.0\text{ m/s})^2}\\ v_T &= 6.5\text{ m/s}\end{aligned}$$

$$\begin{aligned}\tan\phi &= \frac{v_2}{v_1}\\ \tan\phi &= \frac{2.0\ \cancel{\text{m/s}}}{6.2\ \cancel{\text{m/s}}}\\ \phi &= 18°\end{aligned}$$

Statement: The resulting velocity of the boat is 6.5 m/s [W 18° N].

(c) Given: $\vec{v}_x$ = 6.2 m/s [W]

Required: $\Delta\vec{d}_x$

Analysis:
$$\begin{aligned}\vec{v}_x &= \frac{\Delta\vec{d}_x}{\Delta t}\\ \Delta\vec{d}_x &= \vec{v}_x\,\Delta t\end{aligned}$$

Solution:
$$\begin{aligned}\Delta\vec{d}_x &= \vec{v}_x\,\Delta t\\ &= \left(6.2\ \frac{\text{m}}{\cancel{\text{s}}}\ [\text{W}]\right)(18\ \cancel{\text{s}})\\ &= 111.6\text{ m [W]}\\ \Delta\vec{d}_x &= 1.1\times10^2\text{ m [W]}\end{aligned}$$

Statement: The student lands 1.1×10^2 m or 110 m downstream from her destination.

38. The beanbag thrown at an angle will hit the ground first. Both beanbags have the same initial velocity but different vertical components of the velocity. Since both accelerate down at the same rate (gravity), the beanbag thrown at angle will have a shorter time of flight because its initial vertical velocity was less.

39. Given: v_i = 15 m/s; θ = 50°

Required: $\vec{v}_{ix}$; $\vec{v}_{iy}$

Analysis: $\vec{v}_i = \vec{v}_{ix} + \vec{v}_{iy}$

Solution:
$$\begin{aligned}\sin\theta &= \frac{\vec{v}_{iy}}{\vec{v}_i}\\ \vec{v}_{iy} &= \vec{v}_i\sin\theta\\ &= (15\text{ m/s})(\sin 50°)\\ \vec{v}_{iy} &= 11\text{ m/s}\end{aligned}$$

$$\begin{aligned}\cos\theta &= \frac{\vec{v}_{ix}}{\vec{v}_i}\\ \vec{v}_{ix} &= \vec{v}_i\cos\theta\\ &= (15\text{ m/s})(\cos 50°)\\ \vec{v}_{ix} &= 9.6\text{ m/s}\end{aligned}$$

Statement: The initial velocity has a horizontal or *x*-component of 9.6 m/s and a vertical or *y*-component of 11 m/s.

40. (a) Given: Δd_y = –1.3 m; a_y = –9.8 m/s^2; v_y = 0 m/s

Required: Δt

Analysis:
$$\begin{aligned}\Delta d_y &= v_y\,\Delta t + \frac{1}{2}a_y\,\Delta t^2\\ \Delta d_y &= 0 + \frac{1}{2}a_y\,\Delta t^2\\ \Delta t^2 &= \frac{2\,\Delta d_y}{a_y}\\ \Delta t &= \sqrt{\frac{2\,\Delta d_y}{a_y}}\end{aligned}$$

Solution:
$$\begin{aligned}\Delta t &= \sqrt{\frac{2\,\Delta d_y}{a_y}}\\ &= \sqrt{\frac{2(-1.3\ \cancel{\text{m}})}{\left(-9.8\ \frac{\cancel{\text{m}}}{\text{s}^2}\right)}}\\ &= 0.5151\text{ s}\\ \Delta t &= 0.52\text{ s}\end{aligned}$$

Statement: The time of flight of the beanbag should be 0.52 s.

(b) Given: Δt = 0.5151 s; v_x = 4.2 m/s

Required: Δd_x

Analysis: $\Delta d_x = v_x\,\Delta t$

Solution:
$$\begin{aligned}\Delta d_x &= v_x\,\Delta t\\ &= \left(4.2\ \frac{\text{m}}{\cancel{\text{s}}}\right)(0.5151\ \cancel{\text{s}})\ \text{(two extra digits carried)}\\ \Delta d_x &= 2.2\text{ m}\end{aligned}$$

Statement: The range of the beanbag should be 2.2 m.

41. (a) Given: a_y = –9.8 m/s^2; Δt = 3.78 s; θ = 45.0°

Required: v_i

Analysis: $v_i = v_{iy}\sin\theta$

Solution: Determine the y-component:

$$v_{fy} = v_{iy} + a_y \Delta t$$
$$v_{iy} = v_{fy} - a_y \Delta t$$
$$= 0 - (-9.8 \text{ m/s}^2)\left(\frac{3.78 \text{ s}}{2}\right)$$
$$= 18.52 \text{ m/s}$$
$$v_{iy} = 19 \text{ m/s}$$

Use the sine function:

$$\sin\theta = \frac{v_{iy}}{v_i}$$
$$v_i = \frac{v_{iy}}{\sin\theta}$$
$$= \frac{18.52 \text{ m/s}}{\sin 45.0°} \text{ (two extra digits carried)}$$
$$v_i = 26.2 \text{ m/s}$$

Statement: The cannon is fired with an initial velocity of 26.2 m/s.

(b) Given: v_{iy} = 18.52 m/s; Δt = 3.78 s

Required: Δd_y

Analysis: $v_{ix} = \frac{\Delta d_x}{\Delta t}$

$$\Delta d_x = v_{ix} \Delta t$$

Solution: Since the initial angle is 45.0°, the horizontal and vertical components of the velocity have the same magnitude.

$$\Delta d_x = v_{ix} \Delta t$$
$$\Delta d_x = v_{iy} \Delta t$$
$$= \left(18.52 \frac{\text{m}}{\text{s}}\right)(3.78 \text{ s}) \text{ (two extra digits carried)}$$
$$\Delta d_x = 70.0 \text{ m}$$

Statement: The range of the cannon is 70.0 m.

Analysis and Application

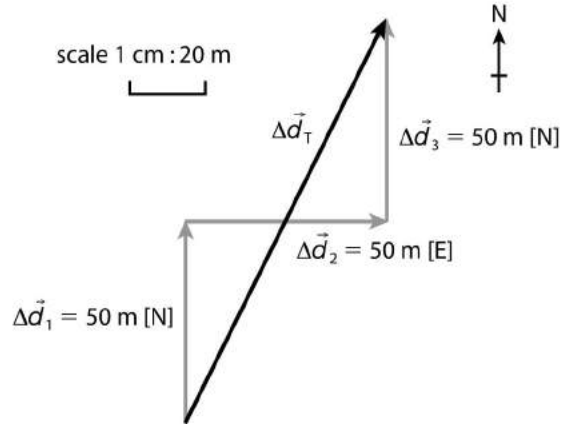
42. (a) The trip would be represented by three vectors: a 50 m [N] vector, a 50 m [E] vector, and another 50 m [N] vector. A possible scale would be 1 cm : 20 m, so each vector is 2.5 cm long.

(b) Given: $\Delta\vec{d}_1$ = 50 m [N]; $\Delta\vec{d}_2$ = 50 m [E]; $\Delta\vec{d}_3$ = 50 m [N]

Required: $\Delta\vec{d}_T$

Analysis: $\Delta\vec{d}_T = \Delta\vec{d}_1 + \Delta\vec{d}_2 + \Delta\vec{d}_3$

Solution:



This figure shows the given vectors, with the tip of $\Delta\vec{d}_1$ joined to the tail of $\Delta\vec{d}_2$, then the tip of $\Delta\vec{d}_2$ joined to the tail of $\Delta\vec{d}_3$. The resultant vector $\Delta\vec{d}_T$ is drawn in black from the tail of $\Delta\vec{d}_1$ to the tip of $\Delta\vec{d}_3$. Using a compass, the direction of $\Delta\vec{d}_T$ is [N 30° E]. $\Delta\vec{d}_T$ measures 5.6 cm in length, so using the scale of 1 cm : 20 m, the actual magnitude of $\Delta\vec{d}_T$ is 112 m.

Statement: The total displacement for his trip is 112 m [N 30° E].

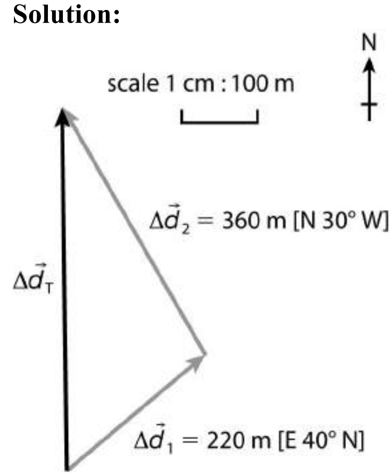
43. Given: $\Delta\vec{d}_1$ = 220 m [E 40° N]; $\Delta\vec{d}_2$ = 360 m [N 30° W]; Δt = 22 s

Determine the displacement of the boat:

Required: $\Delta\vec{d}_T$

Analysis: $\Delta\vec{d}_T = \Delta\vec{d}_1 + \Delta\vec{d}_2$

Solution:



This figure shows the given vectors, with the tip of $\Delta\vec{d}_1$ joined to the tail of $\Delta\vec{d}_2$. The resultant vector $\Delta\vec{d}_T$ is drawn in black from the tail of $\Delta\vec{d}_1$ to the tip of $\Delta\vec{d}_2$. Using a compass, the direction of $\Delta\vec{d}_T$

is [N 1.4° W]. $\Delta \vec{d}_T$ measures 4.5 cm in length, so using the scale of 1 cm : 100 m, the actual magnitude of $\Delta \vec{d}_T$ is 450 m.

Statement: The boat's displacement is 450 m [N 1.4° W].

Determine the average velocity of the boat:

Required: $\vec{v}_{av}$

Analysis: $\vec{v}_{av} = \dfrac{\Delta \vec{d}}{\Delta t}$

Solution:
$$\begin{aligned} \vec{v}_{av} &= \frac{\Delta \vec{d}}{\Delta t} \\ &= \frac{450 \text{ m [N 1.4° W]}}{22 \text{ s}} \\ &= 20.45 \text{ m/s [N 1.4° W]} \\ \vec{v}_{av} &= 2.0 \times 10 \text{ m/s [N 1.4° W]} \end{aligned}$$

Statement: The boat's average velocity is 2.0×10 m/s [N 1.4° W].

44. Answers may vary. Sample answer:

(a) Since I would draw $\vec{c}$ from the tail of $\vec{a}$ to the tip of $\vec{b}$ (when they were tip to tail), I would draw $\vec{a}$ and $\vec{c}$ touching tails, and the displacement from the tip of $\vec{a}$ to the tip of $\vec{c}$ would represent $\vec{b}$.

(b) As in part **(a)**, I would draw what I knew of the vector addition to determine the magnitude and direction of the missing vector. This time, I would put the tips of $\vec{b}$ and $\vec{c}$ together, and $\vec{a}$ would be the vector from the tail of $\vec{c}$ to the tail of $\vec{b}$.

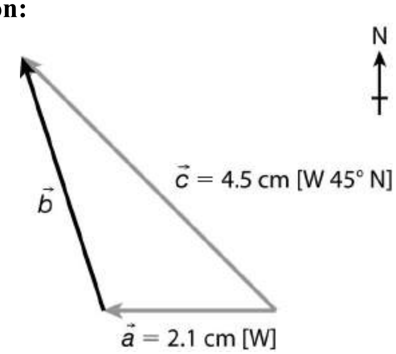
(c) It does not matter which component vector because vectors can be added in either order and you would always get the same resultant vector.

(d) Given: $\vec{a}$ = 2.1 cm [W]; $\vec{c}$ = 4.3 cm [W 45° N]

Required: $\Delta \vec{d}_T$

Analysis: $\Delta \vec{d}_T = \Delta \vec{d}_1 + \Delta \vec{d}_2$

Solution:



This figure shows the given vectors, with the tail of $\vec{a}$ joined to the tail of $\vec{c}$. The missing vector $\vec{b}$ is drawn in black from the tip of $\vec{a}$ to the tip of $\vec{c}$. Using a compass, the direction of $\vec{b}$ is [W 74° N]. $\vec{b}$ measures 3.4 cm in length.

Statement: Vector $\vec{b}$ is 3.4 m [W 74° N].

Note: Figure 5 in the Student Book was modified after the first printing. The solutions below reflect this change.

45. Given: $\Delta \vec{d}_1$ = 5 blocks [N]; $\Delta \vec{d}_2$ = 5 blocks [E]

Required: Δd_T

Analysis: $\Delta d_T = \Delta d_1 + \Delta d_2$

Solution:
$$\begin{aligned} \Delta d_T &= \Delta d_1 + \Delta d_2 \\ &= 5 \text{ blocks} + 5 \text{ blocks} \\ &= (10 \cancel{\text{ blocks}})\left(\frac{250 \text{ m}}{1 \cancel{\text{ block}}}\right) \\ \Delta d_T &= 2500 \text{ m} \end{aligned}$$

Statement: The total distance she travels is 2500 m or 2.5 km.

46. (a) The most direct route is 1.5 blocks [E], 9 blocks [S], and another 1.5 blocks [E].

(b) Given: v = 40.0 km/h; $\Delta \vec{d}_1$ = 1.5 blocks [E]; $\Delta \vec{d}_2$ = 9 blocks [S]; $\Delta \vec{d}_3$ = 1.5 blocks [E]

Required: Δt

Analysis:
$$\begin{aligned} v &= \frac{\Delta d}{\Delta t} \\ \Delta t &= \frac{\Delta d}{v} \end{aligned}$$

Solution:
$$\begin{aligned} \Delta t &= \frac{\Delta d}{v} \\ &= \frac{1.5 \text{ blocks} + 9 \text{ blocks} + 1.5 \text{ blocks}}{40.0 \text{ km/h}} \\ &= \frac{12 \cancel{\text{ blocks}}}{40.0 \text{ km/h}}\left(\frac{250 \text{ m}}{1 \cancel{\text{ block}}}\right) \\ &= \frac{3000 \cancel{\text{ m}}}{40.0 \dfrac{\cancel{\text{km}}}{\text{h}}}\left(\frac{1 \cancel{\text{ km}}}{1000 \cancel{\text{ m}}}\right) \\ &= 0.075 \cancel{\text{ h}}\left(\frac{60 \text{ min}}{1 \cancel{\text{ h}}}\right) \\ \Delta t &= 4.5 \text{ min} \end{aligned}$$

Statement: It will take the student 4.5 min to get to the market.

47. Given: $\Delta \vec{d}_1$ = 3 blocks [N]; $\Delta \vec{d}_2$ = 5 blocks [W]

Required: $\Delta \vec{d}_T$

Analysis: $\Delta \vec{d}_T = \Delta \vec{d}_1 + \Delta \vec{d}_2$

Solution: Let ϕ represent the angle $\Delta \vec{d}_T$ makes with the x-axis.

3 blocks [N] = 750 m [N]

5 blocks [W] = 1250 m [W]

$$\Delta\vec{d}_T = \Delta\vec{d}_1 + \Delta\vec{d}_2$$
$$\Delta d_T^2 = \Delta d_1^2 + \Delta d_2^2$$
$$\Delta d_T = \sqrt{\Delta d_1^2 + \Delta d_2^2}$$
$$= \sqrt{(750\text{ m})^2 + (1250\text{ m})^2}$$
$$\Delta d_T = 1460\text{ m}$$

$$\tan\phi = \frac{\Delta d_2}{\Delta d_1}$$
$$\tan\phi = \frac{1250\not{\text{m}}}{750\not{\text{m}}}$$
$$\phi = 59°$$

Statement: The student's net displacement is 1460 m [N 59° W].

48. Given: $\Delta\vec{d}_1$ = 5 blocks [W]; $\Delta\vec{d}_2$ = 11 blocks [S]; $\Delta\vec{d}_3$ = 2 blocks [W]

Required: $\Delta\vec{d}_T$

Analysis: $\Delta\vec{d}_T = \Delta\vec{d}_x + \Delta\vec{d}_y$

Solution: Let ϕ represent the angle $\Delta\vec{d}_T$ makes with the *x*-axis.

5 blocks [W] = 1250 m [W]
11 blocks [S] = 2750 m [S]
2 blocks [W] = 500 m [W]

$$\Delta\vec{d}_x = 1250\text{ m [W]} + 500\text{ m [W]}$$
$$= 1750\text{ m [W]}$$
$$\Delta\vec{d}_y = 2750\text{ m [S]}$$
$$\Delta\vec{d}_T = \Delta\vec{d}_x + \Delta\vec{d}_y$$
$$\Delta d_T^2 = \Delta d_x^2 + \Delta d_y^2$$
$$\Delta d_T = \sqrt{\Delta d_x^2 + \Delta d_y^2}$$
$$= \sqrt{(1750\text{ m})^2 + (2750\text{ m})^2}$$
$$\Delta d_T = 3260\text{ m}$$

$$\tan\phi = \frac{\Delta d_y}{\Delta d_x}$$
$$\tan\phi = \frac{2750\not{\text{m}}}{1750\not{\text{m}}}$$
$$\phi = 57.5°$$

Statement: The student's net displacement is 3260 m [W 57.5° S].

49. (a) Given: v = 25 km/h; $\Delta\vec{d}_1$ = 5 blocks [W]; $\Delta\vec{d}_2$ = 5 blocks [S]

Required: Δt

Analysis: $v = \frac{\Delta d}{\Delta t}$

$$\Delta t = \frac{\Delta d}{v}$$

Solution: $\Delta t = \frac{\Delta d}{v}$

$$= \frac{5\text{ blocks} + 5\text{ blocks}}{25\text{ km/h}}$$
$$= \frac{10\not{\text{blocks}}}{25\text{ km/h}}\left(\frac{250\text{ m}}{1\not{\text{block}}}\right)$$
$$= \frac{2500\not{\text{m}}}{25\frac{\not{\text{km}}}{\text{h}}}\left(\frac{1\not{\text{km}}}{1000\not{\text{m}}}\right)$$
$$= 0.1\not{\text{h}}\left(\frac{60\text{ min}}{1\not{\text{h}}}\right)$$
$$\Delta t = 6.0\text{ min}$$

Statement: It takes her 6.0 min to drive home from the school.

(b) Given: $\Delta\vec{d}_1$ = 5 blocks [W]; $\Delta\vec{d}_2$ = 5 blocks [S]; Δt = 6.0 min

Determine the total displacement:

Required: $\Delta\vec{d}_T$

Analysis: $\Delta\vec{d}_T = \Delta\vec{d}_1 + \Delta\vec{d}_2$

Solution: Let ϕ represent the angle $\Delta\vec{d}_T$ makes with the *x*-axis.

5 blocks = 1250 m

$$\Delta\vec{d}_T = \Delta\vec{d}_1 + \Delta\vec{d}_2$$
$$\Delta d_T^2 = \Delta d_1^2 + \Delta d_2^2$$
$$\Delta d_T = \sqrt{\Delta d_1^2 + \Delta d_2^2}$$
$$= \sqrt{(1250\text{ m})^2 + (1250\text{ m})^2}$$
$$\Delta d_T = 1770\text{ m}$$

$$\tan\phi = \frac{\Delta d_2}{\Delta d_1}$$
$$\tan\phi = \frac{1250\not{\text{m}}}{1250\not{\text{m}}}$$
$$\phi = 45°$$

Statement: The net displacement from school to home is 1770 m [W 45° S].

Determine the average velocity:

Required: $\vec{v}_{av}$

Analysis: $\vec{v}_{av} = \frac{\Delta\vec{d}}{\Delta t}$

Solution: $\vec{v}_{av} = \frac{\Delta\vec{d}}{\Delta t}$

$$= \frac{1770\not{\text{m}}\text{ [W 45° S]}}{6.0\not{\text{min}}}\left(\frac{60\not{\text{min}}}{1\text{ h}}\right)\left(\frac{1\text{ km}}{1000\not{\text{m}}}\right)$$
$$\vec{v}_{av} = 18\text{ km/h}$$

Statement: Her average velocity from school to home is 18 km/h [W 45° S].

50. Given: v = 25.2 km/h; $\Delta\vec{d}_1$ = 5 blocks [W]; $\Delta\vec{d}_2$ = 2 blocks [S]; $\Delta\vec{d}_3$ = 5 blocks [E]

Required: Δt

Analysis:
$$v = \frac{\Delta d}{\Delta t}$$
$$\Delta t = \frac{\Delta d}{v}$$

Solution:
$$\begin{aligned}\Delta t &= \frac{\Delta d}{v}\\ &= \frac{5\text{ blocks} + 2\text{ blocks} + 5\text{ blocks}}{25.2\text{ km/h}}\\ &= \frac{12\ \cancel{\text{blocks}}}{25.2\text{ km/h}}\left(\frac{250\text{ m}}{1\ \cancel{\text{block}}}\right)\\ &= \frac{3000\ \cancel{\text{m}}}{25.2\ \frac{\cancel{\text{km}}}{\cancel{\text{h}}}}\left(\frac{1\ \cancel{\text{km}}}{1000\ \cancel{\text{m}}}\right)\left(\frac{60\text{ min}}{1\ \cancel{\text{h}}}\right)\\ \Delta t &= 7.14\text{ min}\end{aligned}$$

Statement: It took the student 7.14 min to get from school to the library.

51. Given: Δd_x = 750 m; Δd_T = 1100 m

Determine how far north the boat travelled:

Required: Δd_x

Analysis:
$$\begin{aligned}\Delta d_\text{T}{}^2 &= \Delta d_x{}^2 + \Delta d_y{}^2\\ \Delta d_y{}^2 &= \Delta d_\text{T}{}^2 - \Delta d_x{}^2\\ \Delta d_y &= \sqrt{\Delta d_\text{T}{}^2 - \Delta d_x{}^2}\end{aligned}$$

Solution:
$$\begin{aligned}\Delta d_y &= \sqrt{\Delta d_\text{T}{}^2 - \Delta d_x{}^2}\\ &= \sqrt{(1100\text{ m})^2 + (750\text{ m})^2}\\ \Delta d_y &= 800\text{ m}\end{aligned}$$

Statement: The boat travelled 800 m north.

Determine the direction the boat travelled:

Required: θ

Analysis: $\sin\theta = \dfrac{\Delta d_x}{\Delta d_\text{T}}$

Solution: Let θ represent the angle $\Delta\vec{d}_\text{T}$ makes with the y-axis.

$$\begin{aligned}\sin\theta &= \frac{\Delta d_x}{\Delta d_\text{T}}\\ &= \frac{750\ \cancel{\text{m}}}{1100\ \cancel{\text{m}}}\\ \theta &= 43°\end{aligned}$$

Statement: The boat travelled in the direction [N 43° E]

52. Given: Δd_x = 13 m; θ = [N 32° W]

Required: Δd_T

Analysis:
$$\sin\theta = \frac{\Delta d_x}{\Delta d_\text{T}}$$
$$\Delta d_\text{T} = \frac{\Delta d_x}{\sin\theta}$$

Solution:
$$\begin{aligned}\Delta d_\text{T} &= \frac{\Delta d_x}{\sin\theta}\\ &= \frac{13\text{ m}}{\sin 32°}\\ \Delta d_\text{T} &= 25\text{ m}\end{aligned}$$

Statement: He needs to kick the ball at least 25 m in order to make it through the centre of the posts.

53. Given: $\Delta\vec{d}_1$ = 11 m [N]; Δt_1 = 0.55 s; $\Delta\vec{d}_2$ = 26 m [W 42° N]; Δt_2 = 1.2 s

Required: $\vec{v}_\text{av}$

Analysis: $\vec{v}_\text{av} = \dfrac{\Delta\vec{d}}{\Delta t}$

Solution: Determine the total x-component and y-component of $\Delta\vec{d}_\text{T}$:

$$\begin{aligned}\vec{d}_{\text{T}x} &= \Delta\vec{d}_{1x} + \Delta\vec{d}_{2x}\\ &= 0 + (26\text{ m})(\cos 42°)\ [\text{W}]\\ &= 19.32\text{ m [W]}\\ \vec{d}_{\text{T}x} &= 19\text{ m [W]}\end{aligned}$$

$$\begin{aligned}\vec{d}_{\text{T}y} &= \Delta\vec{d}_{1y} + \Delta\vec{d}_{2y}\\ &= 11\text{ m [N]} + (26\text{ m})(\sin 42°)\ [\text{N}]\\ &= 11\text{ m [N]} + 17.40\text{ m [N]}\\ &= 28.40\text{ m [N]}\\ \vec{d}_{\text{T}y} &= 28\text{ m [N]}\end{aligned}$$

Determine the magnitude of $\Delta\vec{d}_\text{T}$:

$$\begin{aligned}\Delta d_\text{T}{}^2 &= d_{\text{T}x}{}^2 + d_{\text{T}y}{}^2\\ \Delta d_\text{T} &= \sqrt{d_{\text{T}x}{}^2 + d_{\text{T}y}{}^2}\\ &= \sqrt{(19.32\text{ m})^2 + (28.40\text{ m})^2}\ \text{(two extra digits carried)}\\ \Delta d_\text{T} &= 34\text{ m}\end{aligned}$$

Let ϕ represent the angle $\Delta\vec{d}_\text{T}$ makes with the y-axis.

$$\begin{aligned}\tan\phi &= \frac{\Delta d_{\text{T}x}}{\Delta d_{\text{T}y}}\\ \tan\phi &= \frac{19.32\ \cancel{\text{m}}}{28.40\ \cancel{\text{m}}}\ \text{(one extra digit carried)}\\ \phi &= 34°\end{aligned}$$

Determine the average velocity:

$$\vec{v}_{av} = \frac{\Delta\vec{d}}{\Delta t}$$
$$= \frac{\Delta\vec{d}_T}{\Delta t_1 + \Delta t_2}$$
$$= \frac{34.35 \text{ m [N 34° W]}}{0.55 \text{ s} + 1.2 \text{ s}}$$
$$= \frac{34.35 \text{ m [N 34° W]}}{1.75 \text{ s}}$$
$$\vec{v}_{av} = 20 \text{ m/s [N 34° W]}$$

Statement: The puck's average velocity is 20 m/s [N 34° W].

54. Given: v_x = 5.2 m/s; Δd_x = 35 m; Δd_y = 25 m

Required: v_y

Analysis: $v_y = \frac{\Delta d_y}{\Delta t}$

Solution: Determine the time to cross the river:

$$v_x = \frac{\Delta d_x}{\Delta t}$$
$$\Delta t = \frac{\Delta d_x}{v_x}$$
$$= \frac{35 \text{ m}}{5.2 \frac{\text{m}}{\text{s}}}$$
$$= 6.731 \text{ s}$$
$$\Delta t = 6.7 \text{ s}$$

Determine the speed of the river current:

$$v_y = \frac{\Delta d_y}{\Delta t}$$
$$= \frac{25 \text{ m}}{6.731 \text{ s}} \text{ (two extra digits carried)}$$
$$v_y = 3.7 \text{ m/s}$$

Statement: The speed of the current is 3.7 m/s.

55. (a) The student is on the south bank of an east-west river, so the resulting direction of the velocity of the boat must be north.

(b) The student should point the boat north, then turn it west into the current at an angle that will result in the boat travelling directly across the river.

(c) Given: Δd_y = 50 m; v_x = 1.1 m/s; v_T = 3.8 m/s

Required: θ

Analysis: $\sin\theta = \frac{v_y}{v_T}$

Solution: $\sin\theta = \frac{v_y}{v_T}$

$$= \frac{1.1 \frac{\text{m}}{\text{s}}}{3.8 \frac{\text{m}}{\text{s}}}$$
$$\theta = 17°$$

Statement: The student should point the boat [N 17° W].

(d) Given: Δd_y = 50 m; v_T = 3.8 m/s

Required: Δt

Analysis: $v = \frac{\Delta d}{\Delta t}$

$$\Delta t = \frac{\Delta d}{v}$$

Solution: $\Delta t = \frac{\Delta d}{v}$

$$= \frac{\left(\frac{50 \text{ m}}{\cos 17°}\right)}{3.8 \frac{\text{m}}{\text{s}}}$$
$$\Delta t = 14 \text{ s}$$

Statement: It takes the student 14 s to cross the river.

56. Given: Δd_y = –1.2 m; a_y = –9.8 m/s^2; v_x = 1.5 m/s; v_y = 0 m/s

Determine the time of flight:

Required: Δt

Analysis: $\Delta d_y = v_y \Delta t + \frac{1}{2} a_y \Delta t^2$

$$\Delta d_y = 0 + \frac{1}{2} a_y \Delta t^2$$
$$\Delta t^2 = \frac{2\Delta d_y}{a_y}$$
$$\Delta t = \sqrt{\frac{2\Delta d_y}{a_y}}$$

Solution: $\Delta t = \sqrt{\frac{2\Delta d_y}{a_y}}$

$$= \sqrt{\frac{2(-1.2 \text{ m})}{\left(-9.8 \frac{\text{m}}{\text{s}^2}\right)}}$$
$$= 0.4949 \text{ s}$$
$$\Delta t = 0.49 \text{ s}$$

Statement: The hockey puck is in flight for 0.49 s.

Determine the range:
Required: Δd_x
Analysis: $\Delta d_x = v_x \Delta t$
Solution:

$$\Delta d_x = v_x \Delta t$$
$$= \left(1.5\ \frac{\text{m}}{\not{\text{s}}}\right)(0.4949\ \not{\text{s}})\ \text{(two extra digits carried)}$$
$$\Delta d_x = 0.74\ \text{m}$$

Statement: The range of the hockey puck is 0.74 m.
(b) Given: $\Delta d_y = -1.2$ m; $a_y = -9.8$ m/s^2; $v_x = 1.5$ m/s; $v_y = 0$ m/s
Required: $\vec{v}_\text{f}$
Analysis: $\vec{v}_\text{f} = \vec{v}_\text{fx} + \vec{v}_\text{fy}$

$$\vec{v}_\text{fy} = \vec{a}_y \Delta t$$
$$= (9.8\ \text{m/s}^2\ [\text{down}])(0.4949\ \text{s})\ \text{(two extra digits carried)}$$
$$\vec{v}_\text{fy} = 4.85\ \text{m/s}^2\ [\text{down}]$$

Solution: Use the Pythagorean theorem:

$$v_\text{f}^2 = v_\text{fx}^2 + v_\text{fy}^2$$
$$v_\text{f} = \sqrt{v_\text{fx}^2 + v_\text{fy}^2}$$
$$= \sqrt{(1.5\ \text{m/s})^2 + (4.85\ \text{m/s})^2}\ \text{(one extra digit carried)}$$
$$v_\text{f} = 5.1\ \text{m/s}$$

Let ϕ represent the angle $\vec{v}_\text{f}$ makes with the x-axis.

$$\tan\phi = \frac{v_\text{fy}}{v_\text{fx}}$$
$$= \frac{4.85\ \frac{\not{\text{m}}}{\not{\text{s}}}}{1.5\ \frac{\not{\text{m}}}{\not{\text{s}}}}\ \text{(one extra digit carried)}$$
$$\phi = 73°$$

Statement: The puck's final velocity is 5.1 m/s [73° above horizontal].
57. (a) Given: $a_y = -9.8$ m/s^2; $v_\text{i} = 16.5$ m/s; $\theta = 35°$
Required: Δt
Analysis: $\Delta d_y = v_y \Delta t + \frac{1}{2} a_y \Delta t^2$
Solution:

$$\Delta d_y = v_y \Delta t + \frac{1}{2} a_y \Delta t^2$$
$$= v_\text{i}(\sin\theta)\Delta t + \frac{1}{2} a_y \Delta t^2$$
$$0 = (16.5\ \text{m/s})(\sin 35°)\Delta t + \frac{1}{2}(-9.8\ \text{m/s}^2)\Delta t^2$$
$$0 = (9.464\ \text{m/s})\Delta t - (4.9\ \text{m/s}^2)\Delta t^2$$
$$0 = (9.464\ \text{m/s}) - (4.9\ \text{m/s}^2)\Delta t\ (\Delta t \neq 0)$$
$$\Delta t = \frac{9.464\ \frac{\not{\text{m}}}{\not{\text{s}}}}{4.9\ \frac{\not{\text{m}}}{\text{s}^{\not{2}}}}$$
$$= 1.931\ \text{s}$$
$$\Delta t = 1.9\ \text{s}$$

Statement: The soccer ball's time of flight is 1.9 s.
(b) Given: $v_\text{i} = 16.5$ m/s; $\theta = 35°$
Required: Δd_x
Analysis: $\Delta d_x = v_x \Delta t$
Solution:

$$\Delta d_x = v_x \Delta t$$
$$= v_\text{i} \cos\theta \Delta t$$
$$= \left(16.5\ \frac{\text{m}}{\not{\text{s}}}\right)(\cos 35°)(1.931\ \not{\text{s}})\ \text{(two extra digits carried)}$$
$$\Delta d_x = 26\ \text{m}$$

Statement: The soccer ball's range is 26 m.
(c) Given: $a_y = -9.8$ m/s^2; $v_\text{i} = 16.5$ m/s; $\theta = 35°$; $v_\text{fy} = 0$ m/s
Required: Δd_y
Analysis: $v_\text{fy}^2 = v_\text{iy}^2 + 2a_y \Delta d_y$

$$\Delta d_y = \frac{v_\text{fy}^2 - v_\text{iy}^2}{2a_y}$$

Solution: $\Delta d_y = \dfrac{v_\text{fy}^2 - v_\text{iy}^2}{2a_y}$

$$= \frac{0 - (v_\text{i} \sin 35°)^2}{2(-9.8\ \text{m/s}^2)}$$
$$= \frac{0 - [(16.5\ \text{m/s})(\sin 35°)]^2}{-19.6\ \text{m/s}^2}$$
$$= \frac{89.57\ \frac{\text{m}^{\not{2}}}{\not{\text{s}}^{\not{2}}}}{19.6\ \frac{\not{\text{m}}}{\not{\text{s}}^{\not{2}}}}$$
$$\Delta d_y = 4.6\ \text{m}$$

Statement: The soccer ball reached a maximum height of 4.6 m.
58. (a) Given: $a_y = -9.8$ m/s^2; $\Delta t = 2.2$ s; $\Delta d_x = 17$ m; $\Delta d_y = 5.2$ m
Required: v_i
Analysis: $\vec{v}_\text{i} = \vec{v}_\text{ix} + \vec{v}_\text{iy}$

Solution: Determine the *x*-component:

$$\begin{aligned} v_{ix} &= \frac{\Delta d_x}{\Delta t} \\ &= \frac{17\text{ m}}{2.2\text{ s}} \\ &= 7.727\text{ m/s} \\ v_{ix} &= 7.7\text{ m/s} \end{aligned}$$

Determine the *y*-component:

$$\begin{aligned} {v_{fy}}^2 &= {v_{iy}}^2 + 2a_y\,\Delta d_y \\ {v_{iy}}^2 &= {v_{fy}}^2 - 2a_y\,\Delta d_y \\ v_{iy} &= \sqrt{{v_{fy}}^2 - 2a_y\,\Delta d_y} \\ &= \sqrt{0 - 2\left(-9.8\text{ m/s}^2\right)\left(5.2\text{ m}\right)} \\ &= \sqrt{101.92\text{ m}^2/\text{s}^2} \\ &= 10.10\text{ m/s} \\ v_{iy} &= 10\text{ m/s} \end{aligned}$$

Use the Pythagorean theorem:

$$\begin{aligned} {v_f}^2 &= {v_{fx}}^2 + {v_{fy}}^2 \\ v_f &= \sqrt{{v_{fx}}^2 + {v_{fy}}^2} \\ &= \sqrt{\left(7.727\text{ m/s}\right)^2 + \left(10.10\text{ m/s}\right)^2} \text{ (two extra digits carried)} \\ v_f &= 13\text{ m/s} \end{aligned}$$

Statement: The soccer ball is kicked with an initial velocity of 13 m/s.

(b) Given: $a_y = -9.8\text{ m/s}^2$; $\Delta t = 2.2$ s; $\Delta d_x = 17$ m; $\Delta d_y = 5.2$ m

Required: θ

Analysis: $\tan\theta = \frac{v_{iy}}{v_{ix}}$

Solution: $\tan\theta = \frac{v_{iy}}{v_{ix}}$

$$= \frac{10.10\ \frac{\cancel{m}}{\cancel{s}}}{7.727\ \frac{\cancel{m}}{\cancel{s}}} \text{ (two extra digits carried)}$$

$$\theta = 53°$$

Statement: The soccer ball is kicked at an angle of 53°.

Evaluation

59. Answers may vary. Sample answer:
Scale drawings require accuracy in scale, drawing, and measurement in order to get the correct answer. It can also be time consuming. It is less effective than calculating a precise answer.

60. Answers may vary. Sample answer:
Time is a scalar because it always has the same direction: forward in time. The analogy in the question relates time to distance and ordering magnitudes only.

61. (a) The vertical distance is being manipulated to observe differences in the horizontal displacement (range) and time of flight. Initial velocity and angle of launch are being controlled, while acceleration due to gravity is constant.

(b) Student B's data will be the most valid because she is repeating her launches ten times to avoid any errors due to measurement or malfunction.

(c) Answers may vary. Sample answer:
Errors can happen due to bad measuring or problems keeping the initial velocity and angle controlled. Having more than one person measuring time and distance can prevent errors, as well as making sure the launch mechanism is working consistently.

Reflect on Your Learning

62. (a) When drawing two vectors that are added together, draw the first vector then draw the second vector, keeping its size and direction, starting at the tip of the first. The resultant vector starts at the tail of the first vector and ends at the tip of the second.

(b) Answers may vary. Sample answer:
To subtract two vectors, you could use the rules for vector addition in reverse. Rearrange your vector subtraction to look like an addition problem and apply the steps of addition to find the missing vector. First, draw the resultant vector. You know that this connects the tail of one vector to the tip of the other. Then draw the other known vector starting its tail at the same point as the tail of the resultant vector. The missing vector is the vector connecting the tip of the second vector to the tip of the resultant vector.

(c) Answers may vary. Sample answer:
I prefer the algebraic method of vector addition. Sometimes it seems like a lot more work since vectors must be first broken down into components, then added, then the resultant vector must be determined, but this method is more accurate. As long as you are careful with your algebra there are fewer errors and mistakes than using scale diagrams.

63. Students should discuss any gaps in their understanding of motion problems and how they could learn more about solving motion problems.

Research

64. Students' answers should be a few paragraphs that discuss the origin and development of the compass rose.

65. Students' papers should discuss the Cartesian coordinate system and its development. Descartes should be mentioned as well as a reference to at least one other type of coordinate system such as a three-dimensional rectilinear, polar, or spherical coordinate system.

66. Students' reports should discuss the uses of accelerometers in nature. The examples given include the study of shark mating behaviour, the study of gliding behaviour of lemurs, and turning laptops into earthquake sensors.

67. Students should describe both radar and laser devices. The physics behind each technology should be briefly outlined in one or two paragraphs.

Chapter 2 Self-Quiz, page 89

Note: After the first printing, Question 5 was modified to ask for the total **magnitude** of the displacement of the ocean liner. The correct answer is still (a).

1. (b)
2. (a)
3. (c)
4. (b)
5. (a)
6. (c)
7. (d)
8. (b)
9. (b)
10. True
11. True
12. False. To find the direction of the vector 30 m [N 22° W], point *north* and then turn 22° *west*.
13. False. The horizontal component of a vector using the cardinal directions is the component of that vector that points *east or west*.
14. False. The magnitude of a vector with components 4.0 m [W] and 7.0 m [S] is *8.1* m.
15. False. The direction of the resultant vector with components 5.2 m/s [S] and 8.5 m/s [E] is [*E* 31° *S*].
16. False. The vector 57.0 m [S 22° E] has an *x*-component of 21.4 m [*E*].

Unit 1 Review, pages 100–107

Knowledge

1. (c)
2. (c)
3. (b)
4. (d)
5. (b)
6. (c)
7. (d)
8. (b)
9. (d)
10. (b)
11. (b)
12. True
13. True
14. False. The average velocity of an object is the change in *displacement* divided by the change in *time*.
15. True
16. False. A runner that is veering left to pass another runner *is accelerating leftward*.
17. False. Since 1987, the annual number of automobile accident fatalities in Canada has *decreased* by 33%.
18. False. A diagram where 150 m in real life is represented as 1 cm on the diagram would have a scale of *1 cm : 150 m*.
19. False. When given only the *x*- component and *y*-component vectors, *the Pythagorean theorem* should be used to determine the magnitude of the displacement vector.
20. True
21. True
22. False. If a bowling ball and a feather are dropped from the same height in a vacuum at the same time, then *they will both hit the ground at the same time*.
23. True
24. (a) (vii)
(b) (iv)
(c) (v)
(d) (ii)
(e) (iii)
(f) (i)
(g) (vi)
25. Answers may vary. Sample answer:
Position is the location of an object relative to a given reference point. Displacement is the change in position of an object, while distance is the total length of the path travelled by an object. For example, an object that starts at 0 m, moves 20 m [E] and then 10 m [W] has moved a total distance of 30 m, but the displacement of the object from its starting point is only 10 m [E].
26. Answers may vary. Sample answer:
(a) A scalar is a quantity that only has magnitude, whereas a vector is a quantity that has magnitude and direction. For example 20 m/s is a scalar since it has no direction. 20 m/s [E] is a vector since it has magnitude (20 m/s) and direction (east).
(b) A vector is drawn as a directed line segment, which is a line segment between two points with an arrow at one end. The end with the arrow is called the tip and the end without the arrow is called the tail. When two vectors are added on a diagram, one vector is drawn from its starting point and the second vector is drawn, keeping its size and direction, but with its tail starting at the tip of the first vector. The tail of the first vector to the tip of the second vector is the sum of the two vectors.
27. Answers may vary. Sample answer:
One argument for using speed limiters for teenage drivers is that teenage drivers are more likely to speed and cause accidents. Using speed limiters would lower speeds and decrease the number of accidents. One of the arguments against using speed limiters is that by limiting the speed of some cars they could disrupt the flow of traffic and cause more problems.

Understanding

28. Given: $\vec{d}_{\text{inital}}$ = 1750 m [W]; $\vec{d}_{\text{final}}$ = 3250 m [W]

Required: $\Delta\vec{d}_{\text{T}}$

Analysis: $\Delta\vec{d}_{\text{T}} = \vec{d}_{\text{final}} - \vec{d}_{\text{inital}}$

Solution:
$$\begin{aligned}\Delta\vec{d}_{\text{T}} &= \vec{d}_{\text{final}} - \vec{d}_{\text{inital}}\\ &= 3250 \text{ m [W]} - 1750 \text{ m [W]}\\ \Delta\vec{d}_{\text{T}} &= 1500 \text{ m [W]}\end{aligned}$$

Statement: My displacement is 1500 m [W].

29. Given: $\vec{d}_{\text{inital}}$ = 2620 m [E]; $\vec{d}_{\text{final}}$ = 3250 m [W]

Required: $\Delta\vec{d}_{\text{T}}$

Analysis: $\Delta\vec{d}_{\text{T}} = \vec{d}_{\text{final}} - \vec{d}_{\text{inital}}$

Solution:
$$\begin{aligned}\Delta\vec{d}_{\text{T}} &= \vec{d}_{\text{final}} - \vec{d}_{\text{inital}}\\ &= 3250 \text{ m [W]} - 2620 \text{ m [E]}\\ &= 3250 \text{ m [W]} + 2620 \text{ m [W]}\\ \Delta\vec{d}_{\text{T}} &= 5870 \text{ m [W]}\end{aligned}$$

Statement: My displacement is 5870 m [W].

30. Given: $\vec{d}_{\text{inital}}$ = 1750 m [W]; $\vec{d}_{\text{final}}$ = 0 m (ignore the detail about the market because the girl's displacement only involves her initial and final positions)
Required: $\Delta\vec{d}_{\text{T}}$
Analysis: $\Delta\vec{d}_{\text{T}} = \vec{d}_{\text{final}} - \vec{d}_{\text{inital}}$
Solution: $\Delta\vec{d}_{\text{T}} = \vec{d}_{\text{final}} - \vec{d}_{\text{inital}}$

$$= 0\text{ m} - 1750\text{ m [W]}$$
$$= 0\text{ m} + 1750\text{ m [E]}$$
$$\Delta\vec{d}_{\text{T}} = 1750\text{ m [E]}$$

Statement: The girl's displacement is 1750 m [E].
31. Given: $\vec{d}_{\text{initial}}$ = 121 m [W]; $\vec{d}_{\text{final}}$ = 64 m [E]
Required: $\Delta\vec{d}_{\text{T}}$
Analysis: $\Delta\vec{d}_{\text{T}} = \vec{d}_{\text{final}} - \vec{d}_{\text{inital}}$
Solution: $\Delta\vec{d}_{\text{T}} = \vec{d}_{\text{final}} - \vec{d}_{\text{inital}}$

$$= 64\text{ m [E]} - 121\text{ m [W]}$$
$$= 64\text{ m [E]} + 121\text{ m [E]}$$
$$\Delta\vec{d}_{\text{T}} = 185\text{ m [E]}$$

Statement: The bird's displacement is 185 m [E].
32. Given: Δd = 280 m; Δt = 4.3 s
Required: v_{av}
Analysis: $v_{\text{av}} = \dfrac{\Delta d}{\Delta t}$
Solution: $v_{\text{av}} = \dfrac{\Delta d}{\Delta t}$

$$= \frac{280\text{ m}}{4.3\text{ s}}$$
$$v_{\text{av}} = 65\text{ m/s}$$

Statement: The race car's average speed is 65 m/s.
33. Given: $\Delta\vec{d}$ = 420 m [E]; Δt = 14.4 s
Required: $\vec{v}_{\text{av}}$
Analysis: $\vec{v}_{\text{av}} = \dfrac{\Delta\vec{d}}{\Delta t}$
Solution: $\vec{v}_{\text{av}} = \dfrac{\Delta\vec{d}}{\Delta t}$

$$= \frac{420\text{ m [E]}}{14.4\text{ s}}$$
$$\vec{v}_{\text{av}} = 29\text{ m/s [E]}$$

Statement: The average velocity of the bird is 29 m/s [E].
34. Given: $\vec{d}_{\text{initial}}$ = 32 km [W]; $\vec{d}_{\text{final}}$ = 27 km [E]; Δt = 1.8 h
Required: $\vec{v}_{\text{av}}$
Analysis: $\vec{v}_{\text{av}} = \dfrac{\Delta\vec{d}}{\Delta t}$

$$\vec{v}_{\text{av}} = \frac{\vec{d}_{\text{final}} - \vec{d}_{\text{initial}}}{\Delta t}$$

Solution: $\vec{v}_{\text{av}} = \dfrac{\vec{d}_{\text{final}} - \vec{d}_{\text{initial}}}{\Delta t}$

$$= \frac{27\text{ km [E]} - 32\text{ km [W]}}{1.8\text{ h}}$$
$$= \frac{27\text{ km [E]} + 32\text{ km [E]}}{1.8\text{ h}}$$
$$= \left(\frac{59\ \cancel{\text{km}}\text{ [E]}}{1.8\text{ h}}\right)\left(\frac{1000\text{ m}}{1\ \cancel{\text{km}}}\right)$$
$$= \left(\frac{59\ 000\text{ m [E]}}{1.8\ \cancel{\text{h}}}\right)\left(\frac{1\ \cancel{\text{h}}}{60\ \cancel{\text{min}}}\right)\left(\frac{1\ \cancel{\text{min}}}{60\text{ s}}\right)$$
$$\vec{v}_{\text{av}} = 9.1\text{ m/s [E]}$$

Statement: The velocity of the car is 9.1 m/s [E].
35. Given: v_{av} = 263 km/h; Δt = 13.7 s
Required: Δd
Analysis: $v_{\text{av}} = \dfrac{\Delta d}{\Delta t}$

$$\Delta d = v_{\text{av}}\,\Delta t$$

Solution:

$$\Delta d = v_{\text{av}}\,\Delta t$$
$$= \left(263\ \frac{\text{km}}{\cancel{\text{h}}}\right)(13.7\ \cancel{\text{s}})\left(\frac{1\ \cancel{\text{h}}}{60\ \cancel{\text{min}}}\right)\left(\frac{1\ \cancel{\text{min}}}{60\ \cancel{\text{s}}}\right)$$
$$\Delta d = 1.00\text{ km}$$

Statement: The length of the track is 1.00 km or 1000 m.
36. (a) The position–time graph is curved, so the object has non-uniform velocity. In the first half of the time, the displacement is more than twice the displacement in the second half of the time.
(b) The slope at all points of the position–time graph is negative, so the velocity is always negative. The slope is becoming less steep, so the magnitude of the velocity must also be decreasing. The object is slowing down as it heads west.
37. (a) Average velocity is the total distance divided by the total time. Instantaneous velocity is the velocity at a specific moment. It is possible for these two values to be different whenever an object has non-uniform velocity.
(b) Given a position–time graph, I would calculate the slope between two points to determine the average velocity between them. I would look at the slope of a tangent to the curve to determine the instantaneous velocity at that point.

38. Given: v_i = 0 m/s; Δt = 1.6 s; v_f = 2.8 m/s
Required: a_{av}

Analysis: $a_{av} = \frac{\Delta v}{\Delta t}$

$$a_{av} = \frac{v_f - v_i}{\Delta t}$$

Solution: $a_{av} = \frac{v_f - v_i}{\Delta t}$

$$= \frac{2.8\ \text{m/s} - 0\ \text{m/s}}{1.6\ \text{s}}$$

$$a_{av} = 1.8\ \text{m/s}^2$$

Statement: The average acceleration of the runner is 1.8 m/s^2.

39. Given: v_i = 0 m/s; a_{av} = 7.10 m/s^2; Δt = 2.20 s
Required: v_f

Analysis: $a_{av} = \frac{v_f - v_i}{\Delta t}$

$$a_{av}\,\Delta t = v_f - v_i$$

$$v_f = v_i + a_{av}\,\Delta t$$

Solution:

$$v_f = v_i + a_{av}\,\Delta t$$

$$= 0\ \text{m/s} + \left(7.10\ \frac{\text{m}}{\text{s}^{\not 2}}\right)\left(2.20\ \text{s}\right)$$

$$\vec{v}_f = 15.6\ \text{m/s}$$

Statement: The horse's final speed is 15.6 m/s.

40. Given: v_i = 0 m/s; v_f = 152 m/s; $a_{av} = 1.35 \times 10^4\ \text{m/s}^2$
Required: Δt

Analysis: $a_{av} = \frac{v_f - v_i}{\Delta t}$

$$\Delta t = \frac{v_f - v_i}{a_{av}}$$

Solution: $\Delta t = \frac{v_f - v_i}{a_{av}}$

$$= \frac{152\ \frac{\not\text{m}}{\not\text{s}} - 0\ \frac{\not\text{m}}{\not\text{s}}}{1.35 \times 10^4\ \frac{\not\text{m}}{\text{s}^{\not 2}}}$$

$$\Delta t = 1.13 \times 10^{-2}\ \text{s}$$

Statement: The arrow will take 1.13×10^{-2} s or 11.3 ms to accelerate from rest to a speed of 152 m/s.

41. Given: b = 4.0 s; h = 2.0 m/s [E]; l = 4.0 s; w = 2.0 m/s [E]
Required: $\Delta\vec{d}$

Analysis: Use the area under the graph to determine the position at t = 4.0 s:

$$\Delta\vec{d} = A_{\text{triangle}} + A_{\text{rectangle}}$$

Solution:

$$\Delta\vec{d} = A_{\text{triangle}} + A_{\text{rectangle}}$$

$$= \frac{1}{2}bh + lw$$

$$= \frac{1}{2}\left(4.0\ \not\text{s}\right)\left(2.0\ \frac{\text{m}}{\not\text{s}}\ [\text{E}]\right) + \left(4.0\ \not\text{s}\right)\left(2.0\ \frac{\text{m}}{\not\text{s}}\ [\text{E}]\right)$$

$$= 4.0\ \text{m}\ [\text{E}] + 8.0\ \text{m}\ [\text{E}]$$

$$\Delta\vec{d} = 12\ \text{m}\ [\text{E}]$$

Statement: The object has travelled 12 m [E] after 4.0 s.

42. (a) Given: $\vec{v}_i$ = 0 m/s; $\Delta\vec{d}$ = 15.0 m [down]; $\vec{a} = \vec{g}$ = 9.8 m/s^2 [down]
Required: Δt

Analysis: $\Delta\vec{d} = \vec{v}_i\,\Delta t + \frac{1}{2}\vec{a}\left(\Delta t\right)^2$

$$= \left(0\ \text{m/s}\right)\Delta t + \frac{1}{2}\vec{a}\left(\Delta t\right)^2$$

$$\left(\Delta t\right)^2 = \frac{2\Delta\vec{d}}{\vec{a}}$$

$$\Delta t = \sqrt{\frac{2\Delta\vec{d}}{\vec{a}}}$$

Solution: $\Delta t = \sqrt{\frac{2\Delta\vec{d}}{\vec{a}}}$

$$= \sqrt{\frac{2\left(15.0\ \not\text{m}\right)}{\left(9.8\ \frac{\not\text{m}}{\text{s}^2}\right)}}$$

$$\Delta t = 1.7\ \text{s}$$

Statement: The camera takes 1.7 s to hit the ground.

(b) Given: $\vec{v}_i$ = 0 m/s; $\Delta\vec{d}$ = 10.0 m [down]; $\vec{a} = \vec{g}$ = 9.8 m/s^2 [down]
Required: $\vec{v}_f$

Analysis: $v_f^{\,2} = v_i^{\,2} + 2a\,\Delta d$

$$v_f = \sqrt{v_i^{\,2} + 2a\,\Delta d}$$

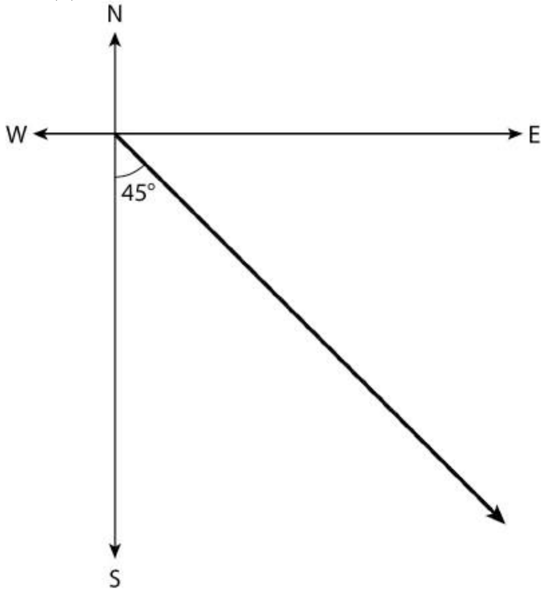
Solution: $v_f = \sqrt{v_i^{\,2} + 2a\,\Delta d}$

$$= \sqrt{\left(0\ \frac{\text{m}}{\text{s}}\right)^2 + 2\left(9.8\ \frac{\text{m}}{\text{s}^2}\right)\left(15.0\ \text{m}\right)}$$

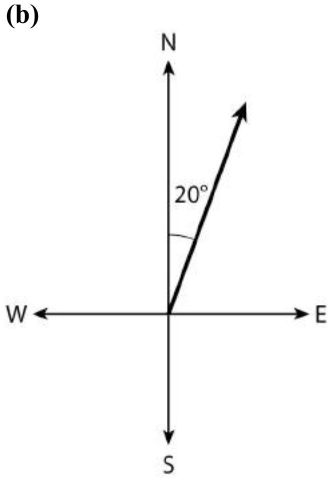
$$v_f = 17\ \text{m/s}$$

Statement: The final velocity of the camera is 17 m/s.

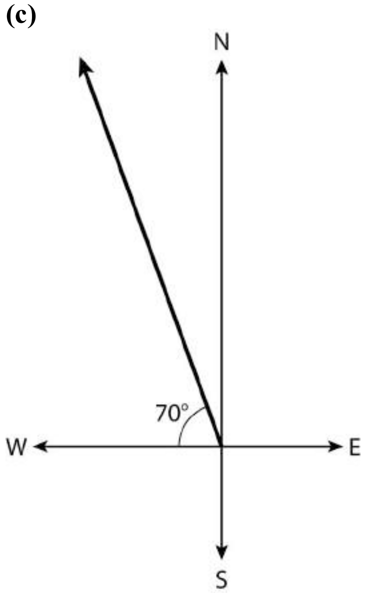
43. (a)



(b)



(c)



44. For each vector, determine the complementary angle, then reverse the order of the directions.

(a) 90° – 8° = 82°

$\Delta\vec{d}$ = 86 m [E 8° N]

$\Delta\vec{d}$ = 86 m [N 82° E]

(b) 90° – 23° = 67°

$\Delta\vec{d}$ = 97 cm [E 23° S]

$\Delta\vec{d}$ = 97 cm [S 67° E]

(c) 90° – 68° = 22°

$\Delta\vec{d}$ = 3190 km [S 68° W]

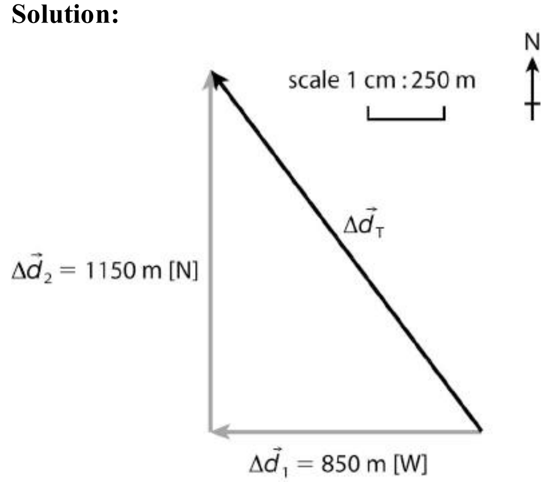
$\Delta\vec{d}$ = 3190 km [W 22° S]

45. Given: $\Delta\vec{d}_1$ = 850 m [W]; $\Delta\vec{d}_2$ = 1150 m [N]

Required: $\Delta\vec{d}_T$

Analysis: $\Delta\vec{d}_T = \Delta\vec{d}_1 + \Delta\vec{d}_2$

Solution:



This figure shows the given vectors, with the tip of $\Delta\vec{d}_1$ joined to the tail of $\Delta\vec{d}_2$. The resultant vector $\Delta\vec{d}_T$ is drawn in black from the tail of $\Delta\vec{d}_1$ to the tip of $\Delta\vec{d}_2$. Using a compass, the direction of $\Delta\vec{d}_T$ is [W 54° N]. $\Delta\vec{d}_T$ measures 5.7 cm in length, so using the scale of 1 cm : 250 m, the actual magnitude of $\Delta\vec{d}_T$ is 1400 m.

Statement: The student's net displacement is 1400 m [W 54° N].

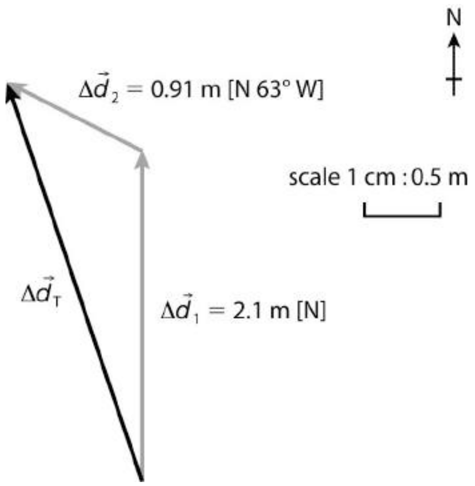
46. Given: $\Delta\vec{d}_1$ = 2.1 m [N];

$\Delta\vec{d}_2$ = 0.91 m [N 63° E]

Required: $\Delta\vec{d}_T$

Analysis: $\Delta\vec{d}_T = \Delta\vec{d}_1 + \Delta\vec{d}_2$

Solution:



This figure shows the given vectors, with the tip of $\Delta\vec{d}_1$ joined to the tail of $\Delta\vec{d}_2$. The resultant vector $\Delta\vec{d}_T$ is drawn in black from the tail of $\Delta\vec{d}_1$ to the tip of $\Delta\vec{d}_2$. Using a compass, the direction of $\Delta\vec{d}_T$ is [N 18° E]. $\Delta\vec{d}_T$ measures 5.2 cm in length, so using the scale of 1 cm : 0.5 m, the actual magnitude of $\Delta\vec{d}_T$ is 2.6 m.

Statement: The net displacement of the cue ball is 2.6 m [N 18° E].

47.

d_x	d_y	d_T
6	8	**10**
5.0	**12**	13
8.0	15	17
2.0	**7.0**	7.3
6.0	6.7	**9.0**

Use the Pythagorean theorem to determine teach missing magnitude: $d_x^2 + d_y^2 = d_T^2$.

Row 1:

$$d_T^2 = d_x^2 + d_y^2$$
$$d_T = \sqrt{d_x^2 + d_y^2}$$
$$= \sqrt{(6)^2 + (8)^2}$$
$$d_T = 10$$

Row 2:

$$d_T^2 = d_x^2 + d_y^2$$
$$d_y = \sqrt{d_T^2 - d_x^2}$$
$$= \sqrt{(13)^2 - (5.0)^2}$$
$$d_y = 12$$

Row 3:

$$d_T^2 = d_x^2 + d_y^2$$
$$d_x = \sqrt{d_T^2 - d_y^2}$$
$$= \sqrt{(17)^2 - (15)^2}$$
$$d_x = 8.0$$

Row 4:

$$d_T^2 = d_x^2 + d_y^2$$
$$d_y = \sqrt{d_T^2 - d_x^2}$$
$$= \sqrt{(7.3)^2 - (2.0)^2}$$
$$d_y = 7.0$$

Row 5:

$$d_T^2 = d_x^2 + d_y^2$$
$$d_T = \sqrt{d_x^2 + d_y^2}$$
$$= \sqrt{(6.0)^2 + (6.7)^2}$$
$$d_T = 9.0$$

48.

$\vec{d}_x$	$\vec{d}_y$	ϕ
5.0 [E]	12.0 [N]	**[E 67° N]**
15.00 [W]	8.00 [N]	**[W 28° N]**
91.0 [E]	**151 [S]**	[E 58.9° S]
640 [W]	213 [N]	[W 18.4° N]
0.051 [W]	**0.10 [S]**	[W 63° S]

Use the tangent function: $\tan\phi = \frac{d_y}{d_x}$.

Row 1: Find the missing angle.

$$\text{an}\,\phi = \frac{d_y}{d_x}$$
$$\tan\phi = \frac{12.0}{5.0}$$
$$\tan\phi = 2.4$$
$$\phi = \tan^{-1}(2.4)$$
$$\phi = 67°$$

Row 2: Find the missing angle.

$$\text{an}\,\phi = \frac{d_y}{d_x}$$
$$\tan\phi = \frac{8.00}{15.00}$$
$$\tan\phi = 5.33$$
$$\phi = \tan^{-1}(5.33)$$
$$\phi = 28°$$

Row 3: Find the missing component vector.

$$\tan\phi = \frac{d_y}{d_x}$$
$$\tan\ 58.9° = \frac{d_y}{91.0}$$
$$(1.659)(91.0) = d_y \quad \text{(two extra digits caried)}$$
$$151 = d_y$$

Row 4: Find the missing component vector.

$$\tan\phi = \frac{d_y}{d_x}$$
$$\tan\ 18.4° = \frac{213}{d_x}$$
$$d_x = \frac{213}{0.33} \quad \text{(one extra digit carried)}$$
$$d_x = 640$$

Row 5: Find the missing component vector.

$$\tan\phi = \frac{d_y}{d_x}$$
$$\tan\ 63° = \frac{d_y}{0.051}$$
$$(1.96078)(0.051) = d_y \quad \text{(two extra digits caried)}$$
$$0.10 = d_y$$

49. (a) Given: $\Delta\vec{d}_\text{T} = 82$ m [W 76° S]

Required: $\Delta\vec{d}_x$; $\Delta\vec{d}_y$

Analysis: $\Delta\vec{d}_\text{T} = \Delta\vec{d}_x + \Delta\vec{d}_y$

Solution: Since the direction of $\Delta\vec{d}_\text{T}$ is between west and south, the direction of $\Delta\vec{d}_x$ is [W] and the direction of $\Delta\vec{d}_y$ is [S].

$$\sin\theta = \frac{\Delta d_y}{\Delta d_\text{T}}$$
$$\Delta d_y = \Delta d_\text{T}\sin\theta$$
$$= (82\text{ m})(\sin 76°)$$
$$\Delta d_y = 80\text{ m}$$

$$\cos\theta = \frac{\Delta d_x}{\Delta d_\text{T}}$$
$$\Delta d_x = \Delta d_\text{T}\cos\theta$$
$$= (82\text{ m})(\cos 76°)$$
$$\Delta d_x = 20\text{ m}$$

Statement: The vector has a horizontal or x-component of 20 m [W] and a vertical or y-component of 80 m [S].

(b) Given: $\Delta\vec{d}_\text{T} = 34$ m [E 13° N]

Required: $\Delta\vec{d}_x$; $\Delta\vec{d}_y$

Analysis: $\Delta\vec{d}_\text{T} = \Delta\vec{d}_x + \Delta\vec{d}_y$

Solution: Since the direction of $\Delta\vec{d}_\text{T}$ is between east and north, the direction of $\Delta\vec{d}_x$ is [E] and the direction of $\Delta\vec{d}_y$ is [N].

$$\sin\theta = \frac{\Delta d_y}{\Delta d_\text{T}}$$
$$\Delta d_y = \Delta d_\text{T}\sin\theta$$
$$= (34\text{ m})(\sin 13°)$$
$$\Delta d_y = 7.6\text{ m}$$

$$\cos\theta = \frac{\Delta d_x}{\Delta d_\text{T}}$$
$$\Delta d_x = \Delta d_\text{T}\cos\theta$$
$$= (34\text{ m})(\cos 13°)$$
$$\Delta d_x = 33\text{ m}$$

Statement: The vector has a horizontal or x-component of 33 m [E] and a vertical or y-component of 7.6 m [N].

(c) Given: $\Delta\vec{d}_\text{T} = 97$ m [S 65° W]

Required: $\Delta\vec{d}_x$; $\Delta\vec{d}_y$

Analysis: $\Delta\vec{d}_\text{T} = \Delta\vec{d}_x + \Delta\vec{d}_y$

Solution: Since the direction of $\Delta\vec{d}_\text{T}$ is between south and west, the direction of $\Delta\vec{d}_x$ is [W] and the direction of $\Delta\vec{d}_y$ is [S].

$$\cos\theta = \frac{\Delta d_x}{\Delta d_\text{T}}$$
$$\Delta d_x = \Delta d_\text{T}\cos\theta$$
$$= (97\text{ m})(\cos 65°)$$
$$\Delta d_x = 41\text{ m}$$

$$\sin\theta = \frac{\Delta d_y}{\Delta d_\text{T}}$$
$$\Delta d_y = \Delta d_\text{T}\sin\theta$$
$$= (97\text{ m})(\sin 65°)$$
$$\Delta d_y = 88\text{ m}$$

Statement: The vector has a horizontal or x-component of 41 m [W] and a vertical or y-component of 88 m [S].

50. (a) Given: $\Delta\vec{d}_x = 4.0$ m [W]; $\Delta\vec{d}_y = 1.9$ m [S]

Required: $\Delta\vec{d}_{\mathrm{T}}$

Analysis: $\Delta\vec{d}_{\mathrm{T}} = \Delta\vec{d}_x + \Delta\vec{d}_y$

Solution: Let ϕ represent the angle $\Delta\vec{d}_{\mathrm{T}}$ makes with the x-axis.

$$\begin{aligned}\Delta\vec{d}_{\mathrm{T}} &= \Delta\vec{d}_x + \Delta\vec{d}_y\\ \Delta d_{\mathrm{T}}{}^2 &= \Delta d_x{}^2 + \Delta d_y{}^2\\ \Delta d_{\mathrm{T}} &= \sqrt{\Delta d_x{}^2 + \Delta d_y{}^2}\\ &= \sqrt{(4.0\text{ m})^2 + (1.9\text{ m})^2}\\ \Delta d_{\mathrm{T}} &= 4.4\text{ m}\end{aligned}$$

$$\begin{aligned}\tan\phi &= \frac{\Delta d_y}{\Delta d_x}\\ \tan\phi &= \frac{1.9\ \cancel{\text{m}}}{4.0\ \cancel{\text{m}}}\\ \phi &= 25^\circ\end{aligned}$$

Statement: The sum of the two vectors is 4.4 m [W 25° S].

(b) Given: $\Delta\vec{d}_x = 1.9$ m [E]; $\Delta\vec{d}_y = 7.6$ m [N]

Required: $\Delta\vec{d}_{\mathrm{T}}$

Analysis: $\Delta\vec{d}_{\mathrm{T}} = \Delta\vec{d}_x + \Delta\vec{d}_y$

Solution: Let ϕ represent the angle $\Delta\vec{d}_{\mathrm{T}}$ makes with the x-axis.

$$\begin{aligned}\Delta\vec{d}_{\mathrm{T}} &= \Delta\vec{d}_x + \Delta\vec{d}_y\\ \Delta d_{\mathrm{T}}{}^2 &= \Delta d_x{}^2 + \Delta d_y{}^2\\ \Delta d_{\mathrm{T}} &= \sqrt{\Delta d_x{}^2 + \Delta d_y{}^2}\\ &= \sqrt{(1.9\text{ m})^2 + (7.6\text{ m})^2}\\ \Delta d_{\mathrm{T}} &= 7.8\text{ m}\end{aligned}$$

$$\begin{aligned}\tan\phi &= \frac{\Delta d_y}{\Delta d_x}\\ \tan\phi &= \frac{7.6\ \cancel{\text{m}}}{1.9\ \cancel{\text{m}}}\\ \phi &= 76^\circ\end{aligned}$$

Statement: The sum of the two vectors is 7.8 m [E 76° N].

(c) Given: $\Delta\vec{d}_x = 72$ m [W]; $\Delta\vec{d}_y = 15$ m [N]

Required: $\Delta\vec{d}_{\mathrm{T}}$

Analysis: $\Delta\vec{d}_{\mathrm{T}} = \Delta\vec{d}_x + \Delta\vec{d}_y$

Solution: Let ϕ represent the angle $\Delta\vec{d}_{\mathrm{T}}$ makes with the x-axis.

$$\begin{aligned}\Delta\vec{d}_{\mathrm{T}} &= \Delta\vec{d}_x + \Delta\vec{d}_y\\ \Delta d_{\mathrm{T}}{}^2 &= \Delta d_x{}^2 + \Delta d_y{}^2\\ \Delta d_{\mathrm{T}} &= \sqrt{\Delta d_x{}^2 + \Delta d_y{}^2}\\ &= \sqrt{(72\text{ m})^2 + (15\text{ m})^2}\\ \Delta d_{\mathrm{T}} &= 74\text{ m}\end{aligned}$$

$$\begin{aligned}\tan\phi &= \frac{\Delta d_y}{\Delta d_x}\\ \tan\phi &= \frac{15\ \cancel{\text{m}}}{72\ \cancel{\text{m}}}\\ \phi &= 12^\circ\end{aligned}$$

Statement: The sum of the two vectors is 74 m [W 12° N].

51. Given: $\Delta\vec{d}_1 = 32$ m [W 14° S]; $\Delta\vec{d}_2 = 15$ m [E 62° S]

Required: $\Delta\vec{d}_{\mathrm{T}}$

Analysis: $\Delta\vec{d}_{\mathrm{T}} = \Delta\vec{d}_1 + \Delta\vec{d}_2$

Solution: Determine the total x-component and y-component of $\Delta\vec{d}_{\mathrm{T}}$:

$$\begin{aligned}\vec{d}_{\mathrm{T}x} &= \Delta\vec{d}_{1x} + \Delta\vec{d}_{2x}\\ &= (32\text{ m})(\cos 14^\circ)\text{ [W]} + (15\text{ m})(\cos 62^\circ)\text{ [E]}\\ &= 31.05\text{ m [W]} + 7.04\text{ m [E]}\\ &= 31.05\text{ m [W]} - 7.04\text{ m [W]}\\ &= 24.01\text{ m [W]}\\ \vec{d}_{\mathrm{T}x} &= 24\text{ m [W]}\end{aligned}$$

$$\begin{aligned}\vec{d}_{\mathrm{T}y} &= \Delta\vec{d}_{1y} + \Delta\vec{d}_{2y}\\ &= (32\text{ m})(\sin 14^\circ)\text{ [S]} + (15\text{ m})(\sin 62^\circ)\text{ [S]}\\ &= 7.74\text{ m [S]} + 13.24\text{ m [S]}\\ &= 20.98\text{ m [S]}\\ \vec{d}_{\mathrm{T}y} &= 21\text{ m [S]}\end{aligned}$$

Determine the magnitude of $\Delta\vec{d}_T$:

$$\Delta d_T{}^2 = d_{Tx}{}^2 + d_{Ty}{}^2$$
$$\Delta d_T = \sqrt{d_{Tx}{}^2 + d_{Ty}{}^2}$$
$$= \sqrt{(24.01\text{ m})^2 + (20.98\text{ m})^2} \text{ (two extra digits carried)}$$
$$\Delta d_T = 32\text{ m}$$

Let ϕ represent the angle $\Delta\vec{d}_T$ makes with the x-axis.

$$\tan\phi = \frac{\Delta d_{Ty}}{\Delta d_{Tx}}$$
$$\tan\phi = \frac{20.98\ \cancel{\text{m}}}{24.01\ \cancel{\text{m}}} \text{ (two extra digits carried)}$$
$$\phi = 41°$$

Statement: The net displacement of the disc is 32 m [W 41° S].

52. (a) Given: Δd = 64 m; v = 0.2 m/s

Required: Δt

Analysis: $v = \frac{\Delta d}{\Delta t}$

$$\Delta t = \frac{\Delta d}{v}$$

Solution: $\Delta t = \frac{\Delta d}{v}$

$$= \frac{64\ \cancel{\text{m}}}{0.2\ \frac{\cancel{\text{m}}}{\text{s}}}$$
$$\Delta t = 3.2 \times 10^2\text{ s}$$

Statement: It takes the fish 3.2×10^2 s to cross the river.

(b) Given: $\vec{v}_1$ = 0.90 m/s [S]; $\vec{v}_2$ = 0.2 m/s [E]

Required: $\vec{v}_T$

Analysis: $\vec{v}_T = \vec{v}_1 + \vec{v}_2$

Solution: Let ϕ represent the angle $\Delta\vec{v}_T$ makes with the y-axis.

$$\vec{v}_T = \vec{v}_1 + \vec{v}_2$$
$$v_T{}^2 = v_1{}^2 + v_2{}^2$$
$$v_T = \sqrt{v_1{}^2 + v_2{}^2}$$
$$= \sqrt{(0.90\text{ m/s})^2 + (0.2\text{ m/s})^2}$$
$$v_T = 0.9\text{ m/s}$$

$$\tan\phi = \frac{v_2}{v_1}$$
$$\tan\phi = \frac{0.2\text{ m/s}}{0.9\text{ m/s}}$$
$$\phi = 13°$$

Statement: The resulting velocity of the fish is 0.9 m/s [S 13° E].

(c) Given: $\vec{v}_x$ = 0.90 m/s [S], $\Delta t = 3.2 \times 10^2$ s

Required: $\Delta\vec{d}_x$

Analysis: $\vec{v}_x = \frac{\Delta\vec{d}_x}{\Delta t}$

$$\Delta\vec{d}_x = \vec{v}_x\,\Delta t$$

Solution: $\Delta\vec{d}_x = \vec{v}_x\,\Delta t$

$$= \left(0.90\ \frac{\text{m}}{\cancel{\text{s}}}\text{ [S]}\right)(320\ \cancel{\text{s}})$$
$$= 288\text{ m [S]}$$
$$\Delta\vec{d}_x = 2.9 \times 10^2\text{ m [S]}$$

Statement: The fish arrives 2.9×10^2 m downstream from being directly across from where it started.

53. The pens will hit the ground at the same time. Both pens have no vertical component to their initial velocities: the first starts with no velocity at all and the second starts with only horizontal velocity. Since both accelerate down at the same rate (gravity), both pens will land at the same time.

54. Given: v_i = 22 m/s; θ = 62°

Required: $\vec{v}_{ix}$; $\vec{v}_{iy}$

Analysis: $\vec{v}_i = \vec{v}_{ix} + \vec{v}_{iy}$

Solution: $\sin\theta = \frac{\vec{v}_{iy}}{\vec{v}_i}$

$$\vec{v}_{iy} = \vec{v}_i \sin\theta$$
$$= (22\text{ m/s})(\sin 62°)$$
$$\vec{v}_{iy} = 19\text{ m/s}$$

$$\cos\theta = \frac{\vec{v}_{ix}}{\vec{v}_i}$$
$$\vec{v}_{ix} = \vec{v}_i \cos\theta$$
$$= (22\text{ m/s})(\cos 62°)$$
$$\vec{v}_{ix} = 10\text{ m/s}$$

Statement: The initial velocity has a horizontal or x-component of 10 m/s and a vertical or y-component of 19 m/s.

55. (a) Given: $\Delta d_y = -1.2$ m; $a_y = -9.8$ m/s^2; $v_y = 0$ m/s
Required: Δt

Analysis:
$$\Delta d_y = v_y \Delta t + \frac{1}{2} a_y \Delta t^2$$
$$\Delta d_y = 0 + \frac{1}{2} a_y \Delta t^2$$
$$\Delta t^2 = \frac{2\Delta d_y}{a_y}$$
$$\Delta t = \sqrt{\frac{2\Delta d_y}{a_y}}$$

Solution:
$$\begin{aligned}\Delta t &= \sqrt{\frac{2\Delta d_y}{a_y}}\\ &= \sqrt{\frac{2(-1.2\ \cancel{\text{m}})}{\left(-9.8\ \frac{\cancel{\text{m}}}{\text{s}^2}\right)}}\\ &= 0.4949\ \text{s}\\ \Delta t &= 0.49\ \text{s}\end{aligned}$$

Statement: The time of flight of the tennis ball will be 0.49 s.
(b) Given: $a_x = 0$ m/s^2; $v_x = 5.3$ m/s
Required: Δd_x
Analysis: $\Delta d_x = v_x \Delta t$
Solution:
$$\begin{aligned}\Delta d_x &= v_x \Delta t\\ &= \left(5.3\ \frac{\text{m}}{\cancel{\text{s}}}\right)(0.4949\ \cancel{\text{s}})\ \text{(two extra digits carried)}\\ \Delta d_x &= 2.6\ \text{m}\end{aligned}$$

Statement: The range of the tennis ball will be 2.6 m.

Analysis and Application

56. Given: $\vec{v}_\text{i} = 0.60$ m/s [up]; $\vec{v}_\text{f} = 27.0$ m/s [down]; $\Delta t = 5.50$ s
Required: $\vec{a}_\text{av}$
Analysis: $\vec{a}_\text{av} = \dfrac{\vec{v}_\text{f} - \vec{v}_\text{i}}{\Delta t}$
Solution:
$$\begin{aligned}\vec{a}_\text{av} &= \frac{\vec{v}_\text{f} - \vec{v}_\text{i}}{\Delta t}\\ &= \frac{27.0\ \text{m/s [down]} - 0.60\ \text{m/s [up]}}{5.50\ \text{s}}\\ &= \frac{27.0\ \text{m/s [down]} + 0.60\ \text{m/s [down]}}{5.50\ \text{s}}\\ \vec{a}_\text{av} &= 5.0\ \text{m/s}^2\ \text{[down]}\end{aligned}$$

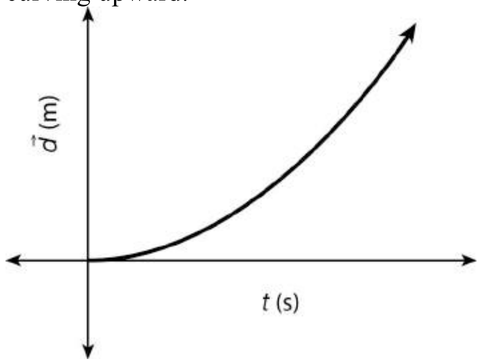
Statement: The average acceleration of the roller coaster is 5.0 m/s^2 [down].
57. (a) Given: $\Delta\vec{v} = 6.0$ m/s [S]; $\Delta t = 3.0$ s
Required: $\vec{a}_\text{av}$
Analysis: $\vec{a}_\text{av} = \dfrac{\Delta\vec{v}}{\Delta t}$
Solution:
$$\begin{aligned}\vec{a}_\text{av} &= \frac{\Delta\vec{v}}{\Delta t}\\ &= \frac{6.0\ \text{m/s [S]}}{3.0\ \text{s}}\\ \vec{a}_\text{av} &= 2.0\ \text{m/s}^2\ \text{[S]}\end{aligned}$$

Statement: The average acceleration from 0 s to 3.0 s is 2.0 m/s^2 [S].
(b) Given: $\Delta\vec{v} = 6.0$ m/s; $\Delta t = 4.0$ s
Required: $\vec{a}_\text{av}$
Analysis: $\vec{a}_\text{av} = \dfrac{\Delta\vec{v}}{\Delta t}$
Solution:
$$\begin{aligned}\vec{a}_\text{av} &= \frac{\Delta\vec{v}}{\Delta t}\\ &= \frac{6.0\ \text{m/s [S]}}{4.0\ \text{s}}\\ \vec{a}_\text{av} &= 1.5\ \text{m/s}^2\ \text{[S]}\end{aligned}$$

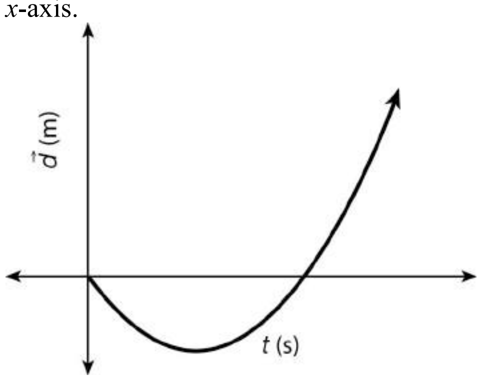
Statement: The average acceleration from 2.0 s to 6.0 s is 1.5 m/s^2 [S].
(c) Given: $b = 5.0$ s; $h = 10.0$ m/s [W]; $l = 1.0$ s
Required: $\Delta\vec{d}$
Analysis: $\Delta\vec{d} = A_\text{triangle} + A_\text{rectangle}$
Solution:
$$\begin{aligned}\Delta\vec{d} &= A_\text{triangle} + A_\text{rectangle}\\ &= \frac{1}{2}bh + lh\\ &= \frac{1}{2}(5.0\ \cancel{\text{s}})\left(10.0\ \frac{\text{m}}{\cancel{\text{s}}}\ \text{[S]}\right) + (1.0\ \cancel{\text{s}})\left(10.0\ \frac{\text{m}}{\cancel{\text{s}}}\ \text{[S]}\right)\\ &= 25\ \text{m [S]} + 10\ \text{m [S]}\\ \Delta\vec{d} &= 35\ \text{m [S]}\end{aligned}$$

Statement: The object has travelled 35 m [S] after 6.0 s.
58. Graph (a) shows uniformly increasing velocity. Graph (b) shows uniformly decreasing velocity. Every second, the velocity in graph (a) changes by 1 m/s while the velocity in graph (b) changes by 1.5 m/s^2. So, graph (b) must have the greater acceleration in magnitude since its velocity is changing faster.
59. (a) The object has constant positive acceleration, so the graph will be curved up. Since the object started from rest at the reference point,

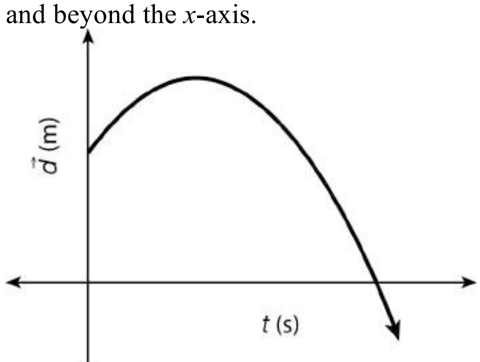
the graph will start at the origin and increase, curving upward.



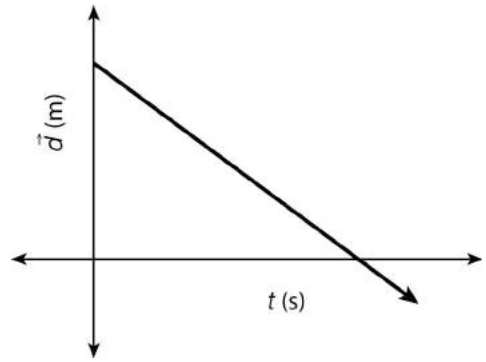
(b) The object has constant positive acceleration, so the graph will be curved up. Since the object started with negative velocity at the reference point, the graph will start at the origin, decrease, level out, then increase up to and beyond the *x*-axis.



(c) The object has constant negative acceleration, so the graph will be curved down. Since the object started with positive velocity away from the reference point, the graph will start high on the *y*-axis, increase, level out, then decrease down to and beyond the *x*-axis.



(d) The object has no acceleration, so the graph will be a straight line. Since the object started with negative velocity away from the reference point, the graph will start high on the *y*-axis, then decrease down to and beyond the *x*-axis.



60. (a) Given: v_i = 145 km/h; v_f = 0 m/s; a = 10.4 m/s^2

Required: Δt

Analysis: $v_f = v_i + a_{av}\,\Delta t$

$$\Delta t = \frac{v_f - v_i}{a_{av}}$$

Solution: Convert v_i to metres per second:

$$v_i = \left(145\ \frac{\cancel{\text{km}}}{\cancel{\text{h}}}\right)\left(\frac{1000\ \text{m}}{1\ \cancel{\text{km}}}\right)\left(\frac{1\ \cancel{\text{h}}}{60\ \cancel{\text{min}}}\right)\left(\frac{1\ \cancel{\text{min}}}{60\ \text{s}}\right)$$

$$v_i = 40.278\ \text{m/s}$$

$$\Delta t = \frac{v_f - v_i}{a_{av}}$$

$$= \frac{0\ \text{km/h} - 40.278\ \text{m/s}}{-10.4\ \text{m/s}^2}\ \text{(two extra digits carried)}$$

$$= \frac{-40.278\ \frac{\cancel{\text{m}}}{\cancel{\text{s}}}}{-10.4\ \frac{\cancel{\text{m}}}{\text{s}^{\cancel{2}}}}$$

$$\Delta t = 3.87\ \text{s}$$

Statement: The car takes 3.87 s to stop.

(b) Given: v_f = 0 m/s; a = 11.0 m/s^2

Required: Δd

Analysis: $v_f^{\,2} = v_i^{\,2} + 2a\Delta d$

$$\Delta d = \frac{v_f^{\,2} - v_i^{\,2}}{2a}$$

Solution:

$$\Delta d = \frac{v_f^{\,2} - v_i^{\,2}}{2a}$$

$$= \frac{(0\ \text{m/s})^2 - (40.278\ \text{m/s})^2}{2(-10.4\ \text{m/s}^2)}\ \text{(two extra digits carried)}$$

$$= \frac{-1622.3\ \frac{\text{m}^{\cancel{2}}}{\cancel{\text{s}^2}}}{-20.8\ \frac{\cancel{\text{m}}}{\cancel{\text{s}^2}}}$$

$$\Delta d = 78.0\ \text{m}$$

Statement: The car travels 78.0 m while slowing down.

61. (a) Given: v_i = 54.0 km/h; a_{av} = 1.90 m/s^2; Δt = 7.50 s

Required: $\Delta\vec{d}$

Analysis: $\Delta\vec{d} = \vec{v}_i\,\Delta t + \frac{1}{2}\vec{a}_{av}\,\Delta t^2$

Solution: Convert v_i to metres per second:

$$v_i = \left(54.0\ \frac{\text{km}}{\text{h}}\right)\left(\frac{1000\ \text{m}}{1\ \text{km}}\right)\left(\frac{1\ \text{h}}{60\ \text{min}}\right)\left(\frac{1\ \text{min}}{60\ \text{s}}\right)$$

$$v_i = 15\ \text{m/s}$$

$$\Delta d = v_i\,\Delta t + \frac{1}{2}a_{av}\,\Delta t^2$$

$$= \left(15\ \frac{\text{m}}{\text{s}}\right)(7.50\ \text{s}) + \frac{1}{2}\left(1.90\ \frac{\text{m}}{\text{s}^2}\right)(7.50\ \text{s})^2$$

$$\Delta d = 166\ \text{m}$$

Statement: The displacement of the vehicle is 166 m.

(b) Given: v_i = 15 m/s; a = 1.90 m/s^2; Δt = 7.50 s

Required: v_f

Analysis: $v_f = v_i + a\,\Delta t$

Solution:

$$v_f = v_i + a\,\Delta t$$

$$= 15\ \frac{\text{m}}{\text{s}} + \left(1.90\ \frac{\text{m}}{\text{s}^2}\right)(7.50\ \text{s})$$

$$= \left(29.25\ \frac{\text{m}}{\text{s}}\right)\left(\frac{1\ \text{km}}{1000\ \text{m}}\right)\left(\frac{60\ \text{s}}{1\ \text{min}}\right)\left(\frac{60\ \text{min}}{1\ \text{h}}\right)$$

$$\vec{v}_f = 105\ \text{km/h}$$

Statement: The van's final velocity is 105 km/h.

62.

	Equation	Uses
Equation 1	$\Delta\vec{d} = \left(\frac{\vec{v}_1 + \vec{v}_2}{2}\right)\Delta t$	Solving for $\Delta\vec{d}$, $\vec{v}_1$, $\vec{v}_2$, or Δt when the other three are known and acceleration is not known.
Equation 2	$\vec{v}_f = \vec{v}_i + \vec{a}\,\Delta t$	Solving for $\vec{v}_i$, $\vec{v}_f$, $\vec{a}$, or Δt when the other three are known and displacement is not known.
Equation 3	$\Delta\vec{d} = \vec{v}_i\,\Delta t + \frac{1}{2}\vec{a}_{av}\,\Delta t^2$	Solving for $\Delta\vec{d}$, $\vec{v}_i$, $\vec{a}$, or Δt when the other three are known and the final velocity is not known.
Equation 4	$\vec{v}_f^{\,2} = \vec{v}_i^{\,2} + 2\vec{a}_{av}\,\Delta d$	Solving for $\vec{v}_i$, $\vec{v}_f$, $\vec{a}$, or $\Delta\vec{d}$ when the other three are known and the time is not known.
Equation 5	$\Delta\vec{d} = \vec{v}_f\,\Delta t - \frac{1}{2}\vec{a}_{av}\,\Delta t^2$	Solving for $\Delta\vec{d}$, $\vec{v}_f$, $\vec{a}$, or Δt when the other three are known and the initial velocity is not known.

63. Given: $\Delta\vec{d}_1 = 240$ m [W 11° N];

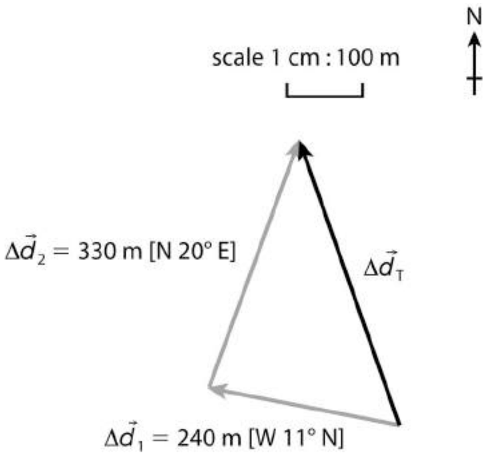
$\Delta\vec{d}_2 = 330$ m [N 20° E]; $\Delta t = 22$ s

Determine the displacement of the boat:

Required: $\Delta\vec{d}_T$

Analysis: $\Delta\vec{d}_T = \Delta\vec{d}_1 + \Delta\vec{d}_2$

Solution:



This figure shows the given vectors, with the tip of $\Delta\vec{d}_1$ joined to the tail of $\Delta\vec{d}_2$. The resultant vector $\Delta\vec{d}_T$ is drawn in black from the tail of $\Delta\vec{d}_1$ to the tip of $\Delta\vec{d}_2$. Using a compass, the direction of $\Delta\vec{d}_T$ is [N 1.4° W]. $\Delta\vec{d}_T$ measures 3.8 cm in length, so using the scale of 1 cm : 100 m, the actual magnitude of $\Delta\vec{d}_T$ is 380 m.

Statement: Her displacement is 380 m [W 71° N].

Determine the average velocity of the boat:

Required: $\vec{v}_{av}$

Analysis: $\vec{v}_{av} = \dfrac{\Delta\vec{d}}{\Delta t}$

Solution:
$$\begin{aligned}\vec{v}_{av} &= \frac{\Delta\vec{d}}{\Delta t}\\ &= \frac{380 \text{ m [W 71° N]}}{22 \text{ s}}\\ &= 17.27 \text{ m/s [W 71° N]}\\ \vec{v}_{av} &= 17 \text{ m/s [W 71° N]}\end{aligned}$$

Statement: Her average velocity is 17 m/s [W 71° N].

64. Answers may vary. Sample answer:
I would draw what I knew of the vector addition to determine the magnitude and direction of the missing vector. For example, I would put the tips of the two vectors together, and the missing component vector would be the vector from the tail of the other component vector to the tip of the resultant vector.

65. Answers may vary. Sample answer:

(a) Vectors can be added using a scale diagram or algebraically.

To add vectors using a scale diagram in one or two dimensions, draw the second vector starting at the tip of the first vector. The distance from the tail of the first to the tip of the second is the resultant vector.

Adding vectors algebraically is different depending on the number of dimensions. With one dimension, just add the values together once they have the same direction. In two dimensions, add the x-components and y-components separately, then use the tangent function to determine the direction of the resultant.

(b) Adding vectors using a scale diagram is the same in one and two dimensions. Using algebra, you must break the vectors into their components, then solve each dimension by adding as you would in one dimension.

66. Given: $\Delta d_x = 1250$ m; $\Delta d_T = 1550$ m

Determine how far north the ranger travelled:

Required: Δd_x

Analysis:
$$\begin{aligned}\Delta d_T^2 &= \Delta d_x^2 + \Delta d_y^2\\ \Delta d_y^2 &= \Delta d_T^2 - \Delta d_x^2\\ \Delta d_y &= \sqrt{\Delta d_T^2 - \Delta d_x^2}\end{aligned}$$

Solution:
$$\begin{aligned}\Delta d_y &= \sqrt{\Delta d_T^2 - \Delta d_x^2}\\ &= \sqrt{(1550 \text{ m})^2 - (1250 \text{ m})^2}\\ \Delta d_y &= 917 \text{ m}\end{aligned}$$

Statement: The ranger travelled 917 m north.

Determine the direction the ranger travelled:

Required: θ

Analysis: $\sin\theta = \dfrac{\Delta d_x}{\Delta d_T}$

Solution: Let θ represent the angle $\Delta\vec{d}_T$ makes with the y-axis.
$$\begin{aligned}\sin\theta &= \frac{\Delta d_x}{\Delta d_T}\\ &= \frac{1250 \cancel{\text{m}}}{1550 \cancel{\text{m}}}\\ \theta &= 53.8°\end{aligned}$$

Statement: The ranger travelled in the direction [N 53.8° E].

67. Given: $\Delta d_x = 11$ m; $\theta =$ [N 28° W]
Required: Δd_T
Analysis: $\sin\theta = \dfrac{\Delta d_x}{\Delta d_T}$

$$\Delta d_T = \frac{\Delta d_x}{\sin\theta}$$

Solution: $\Delta d_T = \dfrac{\Delta d_x}{\sin\theta}$

$$= \frac{11\text{ m}}{\sin 28°}$$
$$\Delta d_T = 23\text{ m}$$

Statement: The archer is 23 m from her target.

68. (a) Given: $\Delta t = 3.0$ s; $\Delta\vec{d}_x = 9.0$ m [E]; $\vec{v}_y = 4.0$ m/s [S]

Required: $\vec{v}_T$

Analysis: $\vec{v}_T = \vec{v}_x + \vec{v}_y$

Solution: Determine $\vec{v}_x$, which is constant:

$$\vec{v}_x = \frac{\Delta\vec{d}_x}{\Delta t} = \frac{9.0\text{ m [E]}}{3.0\text{ s}}$$
$$\vec{v}_x = 3.0\text{ m/s [E]}$$

Use the Pythagorean theorem:

$$v_T^2 = v_x^2 + v_y^2$$
$$v_T = \sqrt{v_x^2 + v_y^2} = \sqrt{(3.0\text{ m/s})^2 + (4.0\text{ m/s})^2}$$
$$v_T = 5.0\text{ m/s}$$

Let ϕ represent the angle $\vec{v}_T$ makes with the x-axis.

$$\tan\phi = \frac{v_y}{v_x} = \frac{4.0\ \cancel{\text{m/s}}}{3.0\ \cancel{\text{m/s}}}$$
$$\phi = 53°$$

Statement: The velocity of the object at $t = 3.0$ s is 5.0 m/s [E 53° S].

(b) Given: $\Delta t = 3.0$ s; $\Delta\vec{d}_x = 6.0$ m [E]; $\vec{a}_y = 2.0$ m/s^2 [S]

Required: $\vec{v}_T$

Analysis: $\vec{v}_T = \vec{v}_x + \vec{v}_y$

Solution: Determine $\vec{v}_x$, which is constant:

$$\vec{v}_x = \frac{\Delta\vec{d}_x}{\Delta t} = \frac{6.0\text{ m [E]}}{3.0\text{ s}}$$
$$\vec{v}_x = 2.0\text{ m/s [E]}$$

Determine $\vec{v}_y$:

$$\vec{v}_y = \vec{a}_y\,\Delta t = \left(2.0\ \frac{\text{m}}{\text{s}^{\cancel{2}}}\text{ [S]}\right)(3.0\ \cancel{\text{s}})$$
$$\vec{v}_y = 6.0\text{ m/s [S]}$$

Use the Pythagorean theorem:

$$v_T^2 = v_x^2 + v_y^2$$
$$v_T = \sqrt{v_x^2 + v_y^2} = \sqrt{(2.0\text{ m/s})^2 + (6.0\text{ m/s})^2}$$
$$v_T = 6.3\text{ m/s}$$

Let ϕ represent the angle $\vec{v}_T$ makes with the x-axis.

$$\tan\phi = \frac{v_y}{v_x} = \frac{6.0\ \cancel{\text{m/s}}}{2.0\ \cancel{\text{m/s}}}$$
$$\phi = 72°$$

Statement: The velocity of the object at $t = 3.0$ s is 6.3 m/s [E 72° S].

69. (a) Given: $a_y = -9.8$ m/s^2; $v_i = 27.5$ m/s; $\theta = 41°$
Determine the time of flight:
Required: Δt

Analysis: $\Delta d_y = v_y\,\Delta t + \frac{1}{2}a_y\,\Delta t^2$

Solution:

$$\Delta d_y = v_y\,\Delta t + \frac{1}{2}a_y\,\Delta t^2$$
$$= v_i(\sin\theta)\Delta t + \frac{1}{2}a_y\,\Delta t^2$$
$$0 = (27.5\text{ m/s})(\sin 41°)\Delta t + \frac{1}{2}(-9.8\text{ m/s}^2)\Delta t^2$$
$$0 = (18.04\text{ m/s})\Delta t - (4.9\text{ m/s}^2)\Delta t^2$$
$$0 = (18.04\text{ m/s}) - (4.9\text{ m/s}^2)\Delta t\ \ (\Delta t \neq 0)$$

$$\Delta t = \frac{18.04\ \frac{\cancel{m}}{\cancel{s}}}{4.9\ \frac{\cancel{m}}{s^{\cancel{2}}}}$$
$$= 3.682\ \text{s}$$
$$\Delta t = 3.7\ \text{s}$$

Statement: The football's time of flight is 3.7 s.

Determine the range:

Required: Δd_x

Analysis: $\Delta d_x = v_x \Delta t$

Solution:

$$\Delta d_x = v_x \Delta t$$
$$= v_i \cos\theta \Delta t$$
$$= \left(27.5\ \frac{\text{m}}{\cancel{s}}\right)(\cos 41°)(3.682\ \cancel{s})\ \text{(two extra digits carried)}$$
$$\Delta d_x = 76\ \text{m}$$

Statement: The football's range is 76 m.

Determine the maximum height:

Required: Δd_y

Analysis: $v_{fy}^{\ 2} = v_{iy}^{\ 2} + 2a_y \Delta d_y$

$$\Delta d_y = \frac{v_{fy}^{\ 2} - v_{iy}^{\ 2}}{2a_y}$$

Solution: $\Delta d_y = \frac{v_{fy}^{\ 2} - v_{iy}^{\ 2}}{2a_y}$

$$= \frac{0 - (v_i \sin 41°)^2}{2(-9.8\ \text{m/s}^2)}$$
$$= \frac{0 - \left[(27.5\ \text{m/s})(\sin 41°)\right]^2}{-19.6\ \text{m/s}^2}$$
$$= \frac{325.5\ \frac{\text{m}^{\cancel{2}}}{\cancel{s^2}}}{19.6\ \frac{\cancel{m}}{\cancel{s^2}}}$$
$$\Delta d_y = 17\ \text{m}$$

Statement: The football reached a maximum height of 17 m.

(b) Given: $a_y = -9.8\ \text{m/s}^2$; $\Delta t = 3.2\ \text{s}$; $\Delta d_x = 29\ \text{m}$

Determine the initial velocity:

Required: v_i

Analysis: $\vec{v}_i = \vec{v}_{ix} + \vec{v}_{iy}$

Solution: Determine the x-component:

$$v_{ix} = \frac{\Delta d_x}{\Delta t}$$
$$= \frac{29\ \text{m}}{3.2\ \text{s}}$$
$$= 9.0625\ \text{m/s}$$
$$v_{ix} = 9.1\ \text{m/s}$$

Determine the y-component:

$$\Delta d_y = v_{iy} \Delta t + \frac{1}{2} a_y \Delta t^2$$
$$v_{iy} = \frac{\Delta d_y - \frac{1}{2} a_y \Delta t^2}{\Delta t}$$
$$= \frac{0 - \frac{1}{2}(-9.8\ \text{m/s}^2)(3.2\ \text{s})^2}{3.2\ \text{s}}$$
$$= \frac{\left(4.9\ \frac{\text{m}}{\cancel{s^2}}\right)(10.24\ \cancel{s^2})}{3.2\ \text{s}}$$
$$= 15.68\ \text{m/s}$$
$$v_{iy} = 16\ \text{m/s}$$

Use the Pythagorean theorem:

$$v_i^2 = v_{ix}^{\ 2} + v_{iy}^{\ 2}$$
$$v_i = \sqrt{v_{ix}^{\ 2} + v_{iy}^{\ 2}}$$
$$= \sqrt{(9.062\ \text{m/s})^2 + (15.68\ \text{m/s})^2}\ \text{(two extra digits carried)}$$
$$v_f = 18\ \text{m/s}$$

Statement: The football is kicked with an initial velocity of 18 m/s.

Determine the initial angle:

Required: θ

Analysis: $\tan\theta = \frac{v_{iy}}{v_{ix}}$

$$\tan\theta = \frac{v_{iy}}{v_{ix}}$$
$$= \frac{15.68\ \frac{\cancel{m}}{\cancel{s}}}{9.062\ \frac{\cancel{m}}{\cancel{s}}}\ \text{(two extra digits carried)}$$
$$\theta = 60°$$

Statement: The football is kicked at an angle of 60°.

70. **(a)** If two objects were dropped at the same time from the same height, but one was on the Moon and the other here on Earth, then the one that is dropped on Earth will hit the ground first since the acceleration due to Earth's gravity is greater than the acceleration due to the Moon's gravity.
(b) If one beanbag was launched horizontally and another of equal mass was dropped from the same height at the same time, then they would both hit the ground at the same time. If this experiment were performed on the Moon the results would be the same since both beanbags would still experience the same vertical acceleration.
(c) For the beanbags launched horizontally in part (b), the beanbag on the Moon would have the larger range. Since both beanbags are launched with the same horizontal velocity, the one with the larger time of flight will have the larger range. Since the Moon has less gravity, the beanbag on the Moon will fall more slowly and have a longer time of flight than the one on Earth.
71. Suppose a projectile in a parabolic path was surrounded by a room. Within that room, the projectile would appear to be free-floating without gravity. So, by flying an airplane in the parabolic path of a projectile, the people in the airplane will experience what feels like a gravity-free environment inside the airplane.

Evaluation

72. Answers may vary. Sample answer:
(a) Drawing vectors in three dimensions would use the same directed line segments, except that instead of only pointing in directions on a surface, they would point in any direction in a space.
(b) Adding two vectors on a three dimensional diagram would apply the same methods used for one and two dimensions. The first vector would be drawn and the second added by joining its tail to the tip of the first vector. The resultant vector would then be the vector drawn from the tail of the first to the tip of the second.
(c) Three dimensional vectors would have three component vectors. These components would correspond to the normal x- and y-axes vectors: east and west for the x-directions, north and south for the y-axis vectors, and then there would be an additional direction on a vertical z-axis corresponding to up and down.
73. Answers may vary. Sample answer:
Yes, Galileo's experiments had a major impact and lead to the discovery of modern kinematics. Not only did the experiments lead to important scientific discoveries that enabled Newton to formulate his theories, but he was also one of the first to challenge the scientific notions of the day. Without his leadership, people may have been left with their false understandings for another century.
74. Answers may vary. Sample answer:
Accelerometers could be used in wireless mice to adjust the power needed for the signal. Wireless mice already use blue tooth technology to allow you to control your computer remotely, but this is usually limited to a short distance. Accelerometers could be used in the mouse to measure its position from the receiver. If the distance increases, the mouse could draw more power. If the distance decreases, this could also be used as a power saving technology because the mouse is close to the receiver and does not need as strong of a signal.

Reflect on Your Learning

75. Answers may vary.
(a) Students should discuss any material in the unit that they found illuminating or insightful.
(b) Students should discuss their understanding of trigonometry and its application in direction and projectiles.
76. Answers may vary. Students should discuss their understanding of gravity and projectiles.

Research

77. Answers may vary. Students' answers should discuss the use of high-speed rails around the world. They may wish to include which countries have the most high-speed trains, and which countries have the fastest trains. They will probably discuss Canada's proposed high-speed rail locations and the issues involved.
78. Answers may vary. Students' answers should discuss a sport and the world record speeds and distances involved. They should also include their group work and calculations performed to determine how their values compare with the record values.
79. Answers may vary. Students' answers should discuss the future of space travel and any new technologies that may make it more easily accessible. Topics include but are not limited to space elevators, warp drives, the future of private space flight, and designs for space shuttles that carry their own fuel.

Unit 1 Self-Quiz, pages 98–99

1. (c)
2. (b)
3. (c)
4. (b)
5. (b)
6. (d)
7. (c)
8. (b)
9. (b)
10. (a)
11. (b)
12. (a)
13. (c)
14. (d)
15. (c)
16. (b)
17. (c)
18. (c)
19. False. If your position is 20 m [W] and you change the reference point to a location that is 20 m [E] of the previous reference point, then your new position would be *40 m [W]*.
OR If your position is 20 m [W] and you change the reference point to a location that is 20 m *[W]* of the previous reference point, then your new position would be 0 m.
20. False. *Only the term vector refers* to quantities that have magnitude and direction.
21. True
22. False. If a squirrel runs 5.0 m [S] and then runs 7.0 m [N], then the displacement of the squirrel is 2.0 m *[N]*.
23. False. If you were to measure the *displacement* of an object and divide by the time it took, then you would get the average velocity.
24. False. An object in motion that is changes direction *cannot* have uniform velocity.
25. True
26. True
27. False. When comparing two velocity-time graphs, the one with the steeper slope will have a *larger* acceleration.
28. False. When an object reaches terminal velocity, it will fall at a constant *velocity*.
29. False. According to Transport Canada, fuel consumption could drop by as much as *20 %* if drivers reduce their speed on the highway from 120 km/h to 100 km/h.
30. False. A diagram depicting a human tissue cell would have a diagram scale on the order of *1 cm : 1 nm*.
31. True
32. False. The resultant vector is determined by taking the *sum of* two vectors.
33. False. When you are given two component vectors and you want to determine the angle of the resultant vector, you should use the inverse *tangent* function.
34. False. A vector with components of 11 m [E] and 20 m [S] has the direction of *E* 61° *S*.
35. False. A vector that has a magnitude of 8.9 m and an *x*-component of 5.2 m will have a *y*-component of *7.2 m*.
36. True
37. False. The time of flight for a projectile that is dropped from a height of 3.0 m is *0.78 s*.
38. True
39. True

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Analyze and Evaluate

(a) Answers may vary. Students should discuss methods of changing angles, initial height, or initial velocity to affect the range of the launcher.
(b) Answers may vary. Students should use their understanding of kinematics to explain how their changes achieved the different ranges for the launcher.
(c) Answers may vary. Sample answer:
For each launch in the final trial, we calibrated the launcher using the final settings from each previous trial. In those trials, we had adjusted the launcher to those settings to best hit the targets.

Apply and Extend

(d) Answers may vary. Students should discuss any aspects of waste-reduction or recycling that would improve their design's environmental impact.
(e) Answers may vary. Students should analyze their design and suggest improvements to help with the accuracy and precision of the launcher.