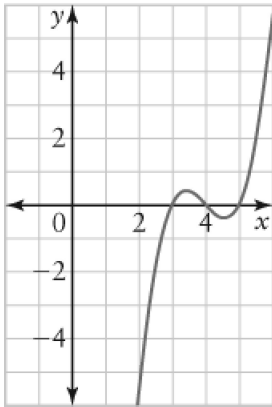




9. Based on the graph of  $f(x) = x^3 - 12x^2 + 47x - 60$  shown, what are the real roots of  $x^3 - 12x^2 + 47x - 60 = 0$ ?



- a. 3, 4, 5
- b. -3, -4, -5
- c. impossible to determine
- d. no real roots

10. Which of the following is a quartic polynomial function with zeros 1 (order 2), 2, and -5?

- a.  $y = (x - 1)^2(x - 2)(x + 5)$
- b.  $y = -\frac{5}{8}(x - 1)^2(x - 2)(x + 5)$
- c.  $y = -0.0034(x - 1)^2(x - 2)(x + 5)$
- d. all of the above

11. Examine the following functions. Which function does not belong to the same family?

- a.  $y = \frac{1}{3}(x - 2)^3(x + 7)$
- b.  $y = -0.00001(x + 7)(x - 2)^3$
- c.  $y = \frac{1}{3}(3x - 6)^3(x + 7)$
- d.  $y = 104935(x - 2)^3(x + 7)$

12. A family of polynomials has equation  $y = k(x - 4)(x + 1)^2$ . What is an equation for the family member whose graph passes through the point  $(2, -54)$ ?

- a.  $y = 3(x - 4)(x + 1)^2$
- b.  $y = -3(x - 4)(x + 1)^2$
- c.  $y = \frac{1}{70225}(x - 4)(x + 1)^2$
- d. none of the above

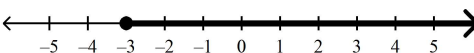
13. The solution to  $x^4 - 5x^2 + 4 \geq 0$  is

- a.  $x = -2, x = -1, x = 1, \text{ or } x = 2$
- b.  $-2 \leq x \leq -1 \text{ or } 1 \leq x \leq 2$
- c.  $x \leq -2, -1 \leq x \leq 1, \text{ or } x \geq 2$
- d.  $x \leq -2, -1 \leq x \leq 1, \text{ or } x \geq 2$

14. Which of the following number lines depicts the solution to  $3x(x^2 - 4) + 2x > 3x^3 + 5(x + 1)$ ?

- a.
- b.
- c.
- d.



24. a) Explain why 0, 1, 2, 3 is a bad list of possible values of  $x$  that make the function  $f(x) = x^3 - 4x^2 + x + 6$  equal to zero.  
 b) Use the factor theorem and synthetic division to determine the factors of  $f(x) = x^3 - 4x^2 + x + 6$ .
25. a) Explain how you would check if  is the solution to the inequality  $24 \leq -14x - 18$ .  
 b) Use your method to test the solution given above. If the solution does not satisfy the inequality, provide the correct solution and an explanation of what mistakes were made.
26. a) Determine and graph the solution to the inequality  $3x + 4 > 5$ .  
 b) The graph of this solution only has one axis. Comment on and try to explain the differences between the graph of an inequality and the graph of an equation.  
 c) Use your answer in part b) to make a conjecture about the graph of the inequality  $y < x + 4$ .
27. a) The polynomial function  $f(x) = 4x^2 - 6x - 33.75$  is cannot be factored easily. Describe two different methods of solving the inequality  $4x^2 - 6x - 33.75 < 0$ .  
 b) If you were asked to 'provide the exact solution' to the inequality from part a), which method would you use and why? What about if you were asked to 'provide an approximate solution'?  
 c) Choose a method and provide a solution to the inequality from part a).
28. Solve  $2x^4 - x^3 = 7x^2 - 9x + 3$ .
29. The height of a square-based box is 4 cm more than the side length of its square base. If the volume of the box is  $225 \text{ cm}^3$ , what are its dimensions?
30. The height,  $h$ , in metres, of a weather balloon above the ground after  $t$  seconds can be modelled by the function  $h(t) = -2t^3 + 3t^2 + 149t + 410$ , for  $0 \leq t \leq 10$ . When is the balloon exactly 980 m above the ground?
31. Amit has designed a rectangular storage unit to hold large factory equipment. His scale model has dimensions 1 m by 2 m by 4 m. By what amount should he increase each dimension to produce an actual storage unit that is 9 times the volume of his scale model?
32. Solve  $-x^3 + 5x^2 - 8x + 4 \geq 0$  algebraically and graphically.

## Practice Test Answer Section

### MULTIPLE CHOICE

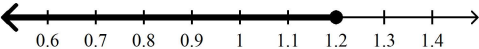
1. ANS: A                   PTS: 1                   DIF: 1                   REF: Knowledge and Understanding  
OBJ: Section 2.1   LOC: C3.1           TOP: Polynomial and Rational Functions  
KEY: restriction
2. ANS: D                   PTS: 1                   DIF: 1                   REF: Knowledge and Understanding  
OBJ: Section 2.1   LOC: C3.1           TOP: Polynomial and Rational Functions  
KEY: remainder theorem
3. ANS: A                   PTS: 1                   DIF: 2  
REF: Knowledge and Understanding; Application                   OBJ: Section 2.1  
LOC: C3.1           TOP: Polynomial and Rational Functions  
KEY: remainder theorem
4. ANS: C                   PTS: 1                   DIF: 1                   REF: Knowledge and Understanding  
OBJ: Section 2.2   LOC: C3.2           TOP: Polynomial and Rational Functions  
KEY: factor theorem, integral zero theorem
5. ANS: A                   PTS: 1                   DIF: 1  
REF: Knowledge and Understanding; Application                   OBJ: Section 2.2  
LOC: C3.2           TOP: Polynomial and Rational Functions  
KEY: factor theorem, integral zero theorem
6. ANS: B                   PTS: 1                   DIF: 1                   REF: Knowledge and Understanding  
OBJ: Section 2.2   LOC: C3.2           TOP: Polynomial and Rational Functions  
KEY: integral zero theorem
7. ANS: A                   PTS: 1                   DIF: 3  
REF: Knowledge and Understanding; Application                   OBJ: Section 2.2  
LOC: C3.2           TOP: Polynomial and Rational Functions  
KEY: factor theorem
8. ANS: C                   PTS: 1                   DIF: 2                   REF: Knowledge and Understanding  
OBJ: Section 2.2   LOC: C3.2           TOP: Polynomial and Rational Functions  
KEY: factored form
9. ANS: A                   PTS: 1                   DIF: 1                   REF: Knowledge and Understanding  
OBJ: Section 2.3   LOC: C3.3           TOP: Polynomial and Rational Functions  
KEY: polynomial equation, real roots
10. ANS: D                   PTS: 1                   DIF: 2                   REF: Knowledge and Understanding  
OBJ: Section 2.4   LOC: C1.7           TOP: Polynomial and Rational Functions  
KEY: family of polynomials
11. ANS: C                   PTS: 1                   DIF: 2                   REF: Knowledge and Understanding  
OBJ: Section 2.4   LOC: C1.8           TOP: Polynomial and Rational Functions  
KEY: family of polynomials
12. ANS: A                   PTS: 1                   DIF: 2                   REF: Knowledge and Understanding  
OBJ: Section 2.4   LOC: C1.8           TOP: Polynomial and Rational Functions  
KEY: family of polynomials
13. ANS: D                   PTS: 1                   DIF: 3                   REF: Knowledge and Understanding  
OBJ: Sections 2.5, 2.6           LOC: C4.2, C4.3   TOP: Polynomial and Rational Functions  
KEY: polynomial inequality

14. ANS: A                   PTS: 1                   DIF: 2                   REF: Knowledge and Understanding  
 OBJ: Sections 2.5, 2.6                   LOC: C4.3                   TOP: Polynomial and Rational Functions  
 KEY: number line, polynomial inequality
15. ANS: B                   PTS: 1                   DIF: 2                   REF: Knowledge and Understanding  
 OBJ: Sections 2.5, 2.6                   LOC: C4.2, C4.3                   TOP: Polynomial and Rational Functions  
 KEY: polynomial inequality

**COMPLETION**

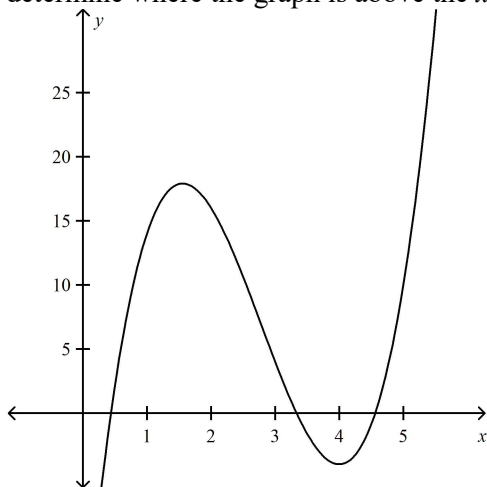
16. ANS:  
 $y = k(x + 4)(x - 2)(x - 3)$   
 $f(x) = k(x + 4)(x - 2)(x - 3)$
- PTS: 1                   DIF: 2                   REF: Knowledge and Understanding  
 OBJ: Section 2.4   LOC: C1.8                   TOP: Polynomial and Rational Functions  
 KEY: family of polynomials

**SHORT ANSWER**

17. ANS:  
 $f(x) = (x + 3)(2x + 10)(4x + 5)$
- PTS: 1                   REF: Knowledge and Understanding                   OBJ: 4.1 - Solving Polynomial Equations
18. ANS:
- 
- PTS: 1                   REF: Knowledge and Understanding                   OBJ: 4.2 - Solving Linear Inequalities
19. ANS:  
 $x < -2, -2 < x < -\frac{3}{7}, -\frac{3}{7} < x < 1, x < 1$
- PTS: 1                   REF: Knowledge and Understanding  
 OBJ: 4.3 - Solving Polynomial Inequalities
20. ANS:  
 The function  $f(x) = 3x^3 + 4x^2 - 59x - 13$  is less than 7 for  $x < -5$  and  $-\frac{1}{3} < x < 4$ .
- PTS: 1                   REF: Knowledge and Understanding  
 OBJ: 4.3 - Solving Polynomial Inequalities

21. ANS:

You can either graph both  $y = 3x^3 - 25x^2 + 56x + 7$  and  $y = 27$ , or just graph  $y = 3x^3 - 25x^2 + 56x - 20$  and determine where the graph is above the  $x$ -axis.

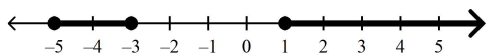


The graph is positive on  $\frac{1}{2} < x < 3\frac{1}{3}$  and  $x > 4\frac{1}{2}$ .

PTS: 1 REF: Knowledge and Understanding

OBJ: 4.3 - Solving Polynomial Inequalities

22. ANS:



PTS: 1 REF: Knowledge and Understanding

OBJ: 4.3 - Solving Polynomial Inequalities

## PROBLEM

23. ANS:

a) The list of possible values of  $x$  that could make  $f(x) = 0$  can be generated by using  $\pm \frac{p}{q}$ , where  $p$  is a factor of the constant and  $q$  is a factor of the leading coefficient.

For  $f(x) = 10x^3 + 41x^2 + x - 12$ , the list of possible values is

$$\pm 1, \pm \frac{1}{2}, \pm \frac{1}{5}, \pm \frac{1}{10}, \pm 2, \pm \frac{2}{5}, \pm 3, \pm \frac{3}{2}, \pm \frac{3}{5}, \pm \frac{3}{10}, \pm 4, \pm \frac{4}{5}, \pm 6, \pm \frac{6}{5}, \pm 12, \pm \frac{12}{5}.$$

To make things easier, begin with the whole numbers.

$$f(2) = 234$$

$$f(-2) = 70$$

$$f(3) = 630$$

$$f(-3) = 84$$

$$f(4) = 1288$$

$$f(-4) = 0$$

By the factor theorem, we know that  $(x + 4)$  is a factor of  $f(x) = 10x^3 + 41x^2 + x - 12$ .

Use synthetic division to factor out  $(x + 4)$ .

$$\begin{array}{r} 10x^2 + x - 3 \\ x + 4 \overline{) 10x^3 + 41x^2 + x - 12} \\ \underline{10x^3 + 40x^2} \phantom{+ x - 12} \\ x^2 + x \phantom{- 12} \\ \underline{x^2 + 4x} \phantom{- 12} \\ -3x - 12 \\ \underline{-3x - 12} \\ 0 \end{array}$$

$$f(x) = (10x^2 + x - 3)(x + 4)$$

Now factor  $(10x^2 + x - 3)$ .

$$(10x^2 + x - 3) = (2x - 1)(5x + 3)$$

The completely factored form of  $f(x) = 10x^3 + 41x^2 + x - 12$  is  $f(x) = (x + 4)(2x - 1)(5x + 3)$ .

b) To solve the equation  $0 = 10x^3 + 41x^2 + x - 12$  set each of the factors equal to zero and solve.

$$0 = x + 4, x = -4$$

$$0 = 2x - 1, x = \frac{1}{2}$$

$$0 = 5x + 3, x = -\frac{3}{5}$$

PTS: 1

REF: Knowledge and Understanding

OBJ: 4.1 - Solving Polynomial Equations



24. ANS:

a) Numbers that could make the function  $f(x) = x^3 - 4x^2 + x + 6$  equal to 0 are of the form  $\frac{p}{q}$ , where  $p$  is a factor of the constant term and  $q$  is a factor of the leading coefficient. So,  $p = \pm 1, \pm 2, \pm 3, \pm 6$  and  $q = \pm 1$ .

Therefore, the list of possible values that could make  $f(x) = 0$  is  $\frac{\pm 6}{\pm 1}, \frac{\pm 3}{\pm 1}, \frac{\pm 2}{\pm 1}, \frac{\pm 1}{\pm 1}$  which is equal to  $\pm 6, \pm 3, \pm 2, \pm 1$ . The list given is a bad list of possible values because it contains only positive numbers and because it contains zero. Zero is not a factor of either 6 or 1.

b)  $f(1) = 4$

$$f(-1) = 0$$

According to the factor theorem,  $(x + 1)$  is a factor of  $f(x) = x^3 - 4x^2 + x + 6$ . Use synthetic division to determine partially factored form of the function.

$$\begin{array}{r|rrrr} -1 & 1 & -4 & 1 & 6 \\ & \downarrow & -1 & 5 & -6 \\ \hline & 1 & -5 & 6 & 0 \end{array}$$

The partially factored form of  $f(x) = x^3 - 4x^2 + x + 6$  is  $f(x) = (x + 1)(x^2 - 5x + 6)$ . Now factor  $(x^2 - 5x + 6)$ .  
 $(x^2 - 5x + 6) = (x - 2)(x - 3)$

The fully factored form of  $f(x) = x^3 - 4x^2 + x + 6$  is  $f(x) = (x + 1)(x - 2)(x - 3)$ .

PTS: 1

REF: Knowledge and Understanding

OBJ: 4.1 - Solving Polynomial Equations

25. ANS:

a) Begin by checking the inequality sign. Because the number line includes a closed circle, the inequality sign should be an 'or equal to' symbol. Next, choose a number on the side of the circle indicated by the arrow. For the specific inequality given you could choose 0. Substitute this number for  $x$ . If the resulting inequality is correct, then the solution given is correct.

b) The inequality sign in the inequality is 'less than or equal to.' This matches the filled in circle in the solution. Next, substitute 0 for  $x$  in the inequality  $24 \leq -14x - 18$ .

$$24 \leq -14x - 18$$

$$24 \leq -14(0) - 18$$

$$24 \leq -18$$

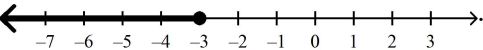
24 is NOT less than or equal to  $-18$ , and so this must not be a correct solution for the inequality.

$$24 \leq -14x - 18$$

$$42 \leq -14x$$

$$\frac{42}{-14} \geq x$$

$$-3 \geq x$$

This inequality, expressed as a number line, is 

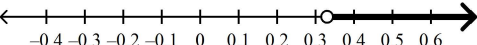
The incorrect solution given previously was most likely caused by forgetting to reverse the inequality sign when dividing by  $-14$ .

PTS: 1

REF: Communication

OBJ: 4.2 - Solving Linear Inequalities

26. ANS:

a)  $x > \frac{1}{3}$  

b) The graph of an equation will have two axes (or more, depending upon the number of variables involved in the equation), but number lines only involve one axis. This makes sense as the inequalities in this lesson only have an  $x$ -variable. You could graph the inequality  $x > \frac{1}{3}$  on a graph with two axes: it would just be a straight line on the  $x$ -axis, as no matter what the value of  $x$ -there will be no associated  $y$  value. In other words,  $3x + 4 > 5$  is the same thing as  $0y + 3x + 4 > 5$ .

c) Like an equation, an inequality with two variables would have a variable on either side of the inequality symbol. An example would be  $y < x + 4$ . The solution would be the ordered pairs that satisfy the inequality, such as  $(1, 4)$ . In this case, the inequality is true so long as  $y$  isn't 4 or more greater than the value of  $x$ . Some other values that would make this inequality true would be  $(0, 4)$ ,  $(-10, -25)$ , and  $(3, 4)$ . Values that would not work are  $(1, 7)$ ,  $(1, 5)$ , and  $(25, 35)$ . It would appear that the solutions to the inequality would be all of the values that fall below, but do not include the line  $y = x + 4$ .

PTS: 1

REF: Thinking

OBJ: 4.2 - Solving Linear Inequalities

27. ANS:

a) One method would involve the quadratic formula. and the other would involve graphing the function  $f(x)$ . You could use the quadratic formula to find the zeros of the function and break it up into intervals. Then you would use a table to help you organize your data and determine where the function is positive and where it is negative. Graphing the function would allow you to approximately see where the function was positive and where it was negative.

b) If you were asked to 'provide the exact solution' to an inequality you would use the quadratic formula. If you were asked to 'provide an approximate solution' you could use the graph.

c) Use the quadratic formula to help solve the inequality.

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\frac{6 \pm \sqrt{(-6)^2 - 4(4)(-33.75)}}{2(4)}$$

$$x = 3.71, x = -2.25$$

	$x < -2.25$	$-2.25 < x < 3.71$	$x > 3.71$
$(x + 2.25)$	-	+	+
$(x - 3.71)$	-	-	+
$(x - 3.71)(x + 2.25)$	+	-	+

The inequality is true on  $-2.25 < x < 3.71$ .

PTS: 1

REF: Thinking

OBJ: 4.3 - Solving Polynomial Inequalities

28. ANS:

Rewrite in the form  $P(x) = 0$ .

$$2x^4 - x^3 - 7x^2 + 9x - 3 = 0$$

By the rational zero theorem, possible values of  $\frac{b}{a}$  are  $\pm 1, \pm 3, \pm \frac{1}{2},$  and  $\pm \frac{3}{2}$ . Test the values to find a zero.

Since  $x = 1$  is a zero of  $P(x)$ ,  $x - 1$  is a factor. Divide to determine the other factor.

$$2x^4 - x^3 - 7x^2 + 9x - 3 = (x - 1)(2x^3 + x^2 - 6x + 3)$$

Factor  $2x^3 + x^2 - 6x + 3$  using a similar method.

$$2x^3 + x^2 - 6x + 3 = (x - 1)(2x^2 + 3x - 3)$$

So,  $2x^4 - x^3 - 7x^2 + 9x - 3 = (x - 1)^2(2x^2 + 3x - 3)$ . Solve  $(x - 1)^2(2x^2 + 3x - 3) = 0$ .

$$x - 1 = 0 \text{ or } 2x^2 + 3x - 3 = 0$$

$$x = 1 \text{ or } x = \frac{-3 \pm \sqrt{3^2 - 4(2)(-3)}}{2(2)}$$

$$= \frac{-3 \pm \sqrt{33}}{4}$$

PTS: 1

DIF: 3

REF: Knowledge and Understanding

OBJ: Section 2.3

LOC: C3.4

TOP: Polynomial and Rational Functions

KEY: real roots, polynomial equation

29. ANS:

Let  $x$  represent the side length of the base. Then,  $V(x) = x^2(x + 4)$ .

Solve  $x^2(x + 4) = 225$ .

$$x^2(x + 4) - 225 = 0$$

$$x^3 + 4x^2 - 225 = 0$$

Factor the corresponding polynomial function. Use the integral zero theorem to determine the possible values of  $b$  are  $\pm 1, \pm 3, \pm 5, \pm 9, \pm 15, \pm 25, \pm 45, \pm 75$ , and  $\pm 225$ . Test only positive values since  $x$  represents a side length.

Since  $x = 5$  is a zero of the function,  $x - 5$  is a factor. Divide to determine the other factor.

$$x^3 + 4x^2 - 225 = (x - 5)(x^2 + 9x + 45)$$

Solve  $(x - 5)(x^2 + 9x + 45) = 0$ .

$$x - 5 = 0 \text{ or } x^2 + 9x + 45 = 0$$

$x = 5$  or no solution

The dimensions of the box are 5 cm by 5 cm by 9 cm.

PTS: 1

DIF: 3

REF: Knowledge and Understanding; Thinking; Application

OBJ: Section 2.3

LOC: C3.7

TOP: Polynomial and Rational Functions

KEY: polynomial equation, real roots

30. ANS:

Solve  $-2t^3 + 3t^2 + 149t + 410 = 980$ .

$$-2t^3 + 3t^2 + 149t - 570 = 0$$

Factor the corresponding polynomial function using the factor theorem.

$$-2t^3 + 3t^2 + 149t - 570 = -(t - 6)(t - 5)(2t + 19)$$

Solve  $-(t - 6)(t - 5)(2t + 19) = 0$ .

$$t - 6 = 0 \text{ or } t - 5 = 0 \text{ or } 2t + 19 = 0$$

$$t = 6 \text{ or } t = 5 \text{ or } t = -\frac{19}{2}$$

Since  $0 \leq t \leq 10$ , the balloon is exactly 980 m above the ground at 5 s and 6 s.

PTS: 1

DIF: 3

REF: Knowledge and Understanding; Thinking; Application

OBJ: Section 2.3

LOC: C3.7

TOP: Polynomial and Rational Functions

KEY: real roots, polynomial equation

31. ANS:

Let  $x$  represent the increase in each dimension. Then  $V(x) = (1 + x)(2 + x)(4 + x)$ .

Solve  $(1 + x)(2 + x)(4 + x) = 9(1)(2)(4)$ .

$$x^3 + 7x^2 + 14x + 8 = 72$$

$$x^3 + 7x^2 + 14x - 64 = 0$$

Factor the corresponding polynomial function using the factor theorem.

$$x^3 + 7x^2 + 14x - 64 = (x - 2)(x^2 + 9x + 32)$$

Solve  $(x - 2)(x^2 + 9x + 32) = 0$ .

$$x - 2 = 0 \text{ or } x^2 + 9x + 32 = 0$$

$x = 2$  or no solution

Each dimension should be increased by 2 m.

PTS: 1

DIF: 3

REF: Knowledge and Understanding; Thinking; Application

OBJ: Section 2.3

LOC: C3.7

TOP: Polynomial and Rational Functions

KEY: real roots, polynomial equation

32. ANS:

Factor  $-x^3 + 5x^2 - 8x + 4$  using the factor theorem.

$$-x^3 + 5x^2 - 8x + 4 = -(x-2)(x-2)(x-1)$$

So, the inequality becomes  $-(x-2)(x-2)(x-1) \geq 0$ .

The three factors are  $-(x-2)$ ,  $x-2$ , and  $x-1$ . There are four cases to consider.

Case 1:  $-(x-2) \geq 0, x-2 \geq 0, x-1 \geq 0$

So,  $x = 2$  is a solution.

Case 2:  $-(x-2) \leq 0, x-2 \leq 0, x-1 \geq 0$

So,  $x = 2$  is a solution.

Case 3:  $-(x-2) \leq 0, x-2 \geq 0, x-1 \leq 0$

No solution.

Case 4:  $-(x-2) \geq 0, x-2 \leq 0, x-1 \leq 0$

So,  $x \leq 1$  is a solution.

Combining the results, the solution is  $x \leq 1$  or  $x = 2$ .

Use a graphing calculator to graph the corresponding polynomial function  $y = -x^3 + 5x^2 - 8x + 4$ . Then, use the **Zero** operation.

The zeros are 1 and 2. From the graph,  $-x^3 + 5x^2 - 8x + 4 \geq 0$  when  $x \leq 1$  or  $x = 2$ .

PTS: 1                      DIF: 3

REF: Knowledge and Understanding

OBJ: Sections 2.5, 2.6

LOC: C4.2, C4.3      TOP: Polynomial and Rational Functions

KEY: polynomial inequality