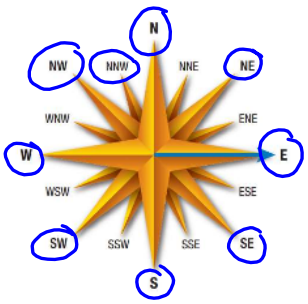
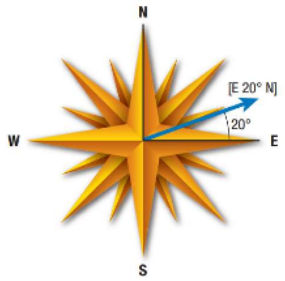
SPH3U: 2.1 Motion in Two Dimensions – Scale Diagrams

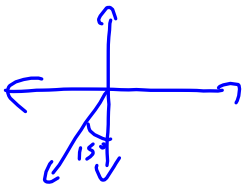
1. Compass directions

Compass:	North, South, East, West. (NESW).
compass rose	cross that shows directions (below).
Cartesian grid	N: +y, E: +x, S: -y, W: -x.
angles between NEWS	[E20°N], [N70°E], etc.

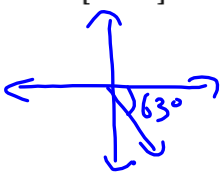




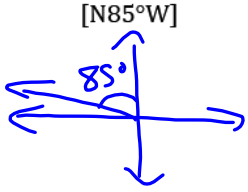
[S15°W]



[E63°S]

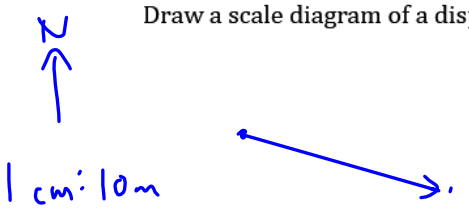


[N85°W]



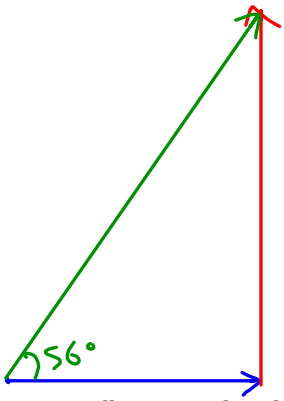
Scale diagram:	accurate drawing of a situation.
scale	a ratio between the size of your drawing and the size in real life. ex: 1cm : 10km.
resultant vector	the result of adding vectors.

Draw a scale diagram of a displacement vector of 41 m [E15°S].



Tip and tail:	tip: front of arrow, tail: back of arrow.
Adding vectors:	tip-to-tail: draw a scale diagram with vectors connected tip-to-tail.

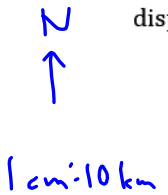
A cyclist rides her bicycle 50 m due east, and then turns a corner and rides 75 m due north. What is her total displacement?

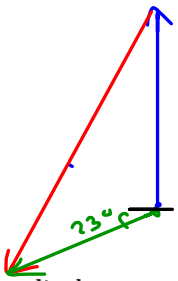


8.9cm → 89m.

$\therefore \vec{\Delta d} = 89\text{m}\ [E\,56°N]$.

While in a race, a sailboat travels a displacement of 40 m [N]. The boat then changes direction and travels a displacement of 60 m [S30°W]. What is the boat's total displacement?





3.2cm → 32m.

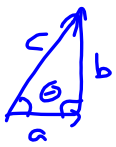
$\therefore \vec{\Delta d} = 32\text{m}\ [W23°S]$

A squash ball undergoes a displacement of 6.2 m [W25°S] as it approaches a wall. It bounces off the wall and experiences a displacement of 4.8 m [W25°N]. The whole motion takes 3.7 s. Determine the squash ball's total displacement and average velocity.

Homework: page 65: #1-4, 7-8

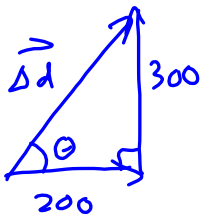
SPH3U: 2.2 Motion in Two Dimensions – Algebraic Approach

1. Adding displacements in two dimensions



Adding perpendicular vectors:	use basic trigonometry (Pythagorean Theorem and tan ratio).
magnitude	$a^2 + b^2 = c^2$
angle	$\tan\theta = \frac{b}{a} \rightarrow \theta = \tan^{-1}\left(\frac{b}{a}\right)$

A jogger runs 200.0 m [E], turns at an intersection, and continues for an additional displacement of 300.0 m [N]. What is the jogger's total displacement?

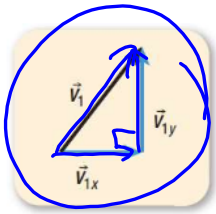


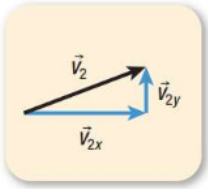
$\Delta d = \sqrt{200^2 + 300^2} = 360\ m$

$\theta = \tan^{-1}\left(\frac{300}{200}\right) = 56°$

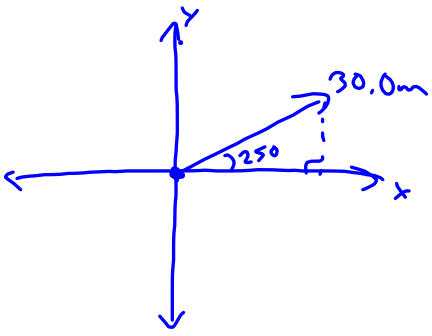
$\therefore \vec{\Delta d} = 360\ m\ [E56°N]$.

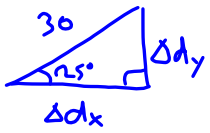
Component vectors:	perpendicular (x and y) vectors that add up to the original vector.





Break the displacement vector 30.0 m [E25°N] down into perpendicular component vectors.





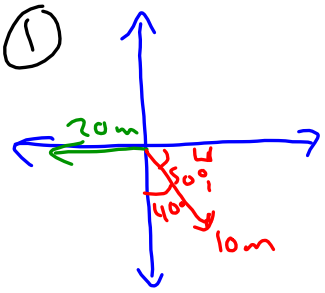
$\cos 25° = \frac{\Delta d_x}{30}$

$\rightarrow \vec{\Delta d_x} = 30\cos 25° = 27.19\ m\ [E]$

$\sin 25° = \frac{\Delta d_y}{30}$

$\rightarrow \vec{\Delta d_y} = 30\sin 25° = 12.68\ m\ [N]$

A cat walks 20.0 m [W] and then turns and walks a further 10.0 m [S40°E]. What is the cat's total displacement?

① 

② $\vec{\Delta d_x} = \vec{\Delta d_{1x}} + \vec{\Delta d_{2x}}$

$= 20\text{m [W]} + 10\cos 50^\circ \text{ [E]}.$

$= -20\text{m} + 10\cos 50^\circ$

$= -13.57\text{ m}$

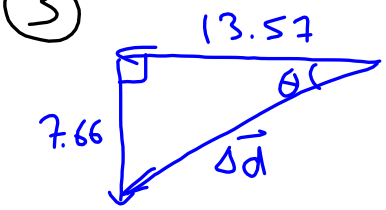
$= 13.57\text{ m [W]}.$

$\vec{\Delta d_y} = \vec{\Delta d_{1y}} + \vec{\Delta d_{2y}}$

$= 0\text{ m} - 10\sin 50^\circ$

$= -7.66\text{ m}$

$= 7.66\text{ m [S]}.$

③ 

④ $\Delta d = \sqrt{13.57^2 + 7.66^2}$

$= 15.58\text{m}$

$\theta = \tan^{-1}\left(\frac{7.66}{13.57}\right)$

$= 29^\circ$

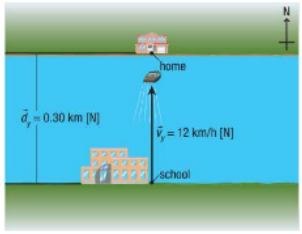
⑤ ∴ the total displacement is $\vec{\Delta d} = 15.6\text{ m [W29°S]}.$

A hockey puck travels a displacement of 4.2 m [S38°W]. It is then struck by a hockey player's stick and undergoes a displacement of 2.7 m [E25°N]. What is the puck's total displacement?

2. Adding velocities in two dimensions

River crossing problems:	2-dimensional problems with perpendicular vectors. the 2 dimensions are independent.

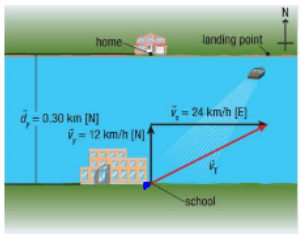
A physics student hops into her motorboat and steers straight across a river at a constant velocity of 12 km/h [N]. If the river is 0.30 km across and has no current, how long will it take her to cross the river?



$\vec{v} = \frac{\vec{\Delta d}}{\Delta t} \rightarrow \Delta t = \frac{\vec{\Delta d}}{\vec{v}} = \frac{0.30\,km}{12\,km/h}$

$= 0.025\,h \times \frac{3600\,s}{h} = 90\,s.$

Most rivers have a current moving in the direction of the river. The river now has a current of 24 km/h [E], as shown to the right. How long does it now take the boat to cross the river?



Still 90s (x and y are independent).

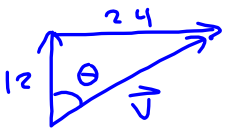
How far downstream does the boat land?

$\vec{v} = \frac{\vec{\Delta d}}{\Delta t} \quad \vec{\Delta d}_x = \vec{v}_x \Delta t$

$= (24\,km/h\,[E])(0.025\,h)$

$= 0.6\,km\,[E].$

What is the boat's resultant velocity?



$v = \sqrt{12^2 + 24^2} = 27\,km/h.$

$\theta = \tan^{-1}\left(\frac{24}{12}\right) = 63°$

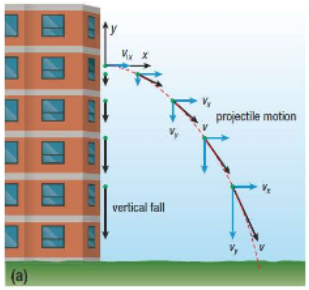
$\therefore \vec{v} = 27\,km/h\,[N63°E].$

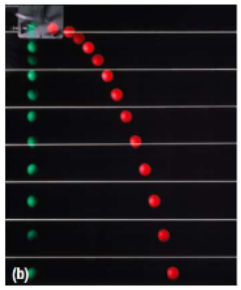
Homework: page 75: #1-3, 6b, 8-9

SPH3U: 2.3 Projectile Motion

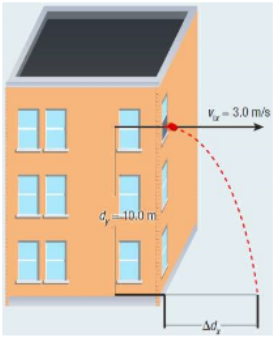
1. Projectile motion

Projectile:	
projectile motion	
projectile motion vs. river crossing	
range	
convention	





A beanbag is thrown from a window 10.0 m above the ground with an initial horizontal velocity of 3.0 m/s.

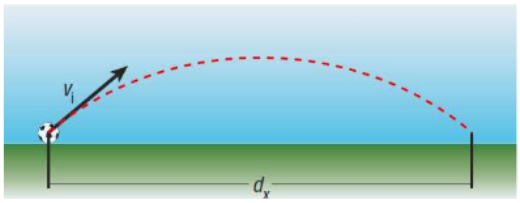


a. How long will it take the beanbag to reach the ground (what is its time of flight)?

b. How far will the beanbag travel horizontally (what is its range)?

2. Launching a projectile at an angle

A soccer player running on a level playing field kicks a soccer ball with a velocity of 9.4 m/s at an angle of 40° above the horizontal. Determine the soccer ball's:

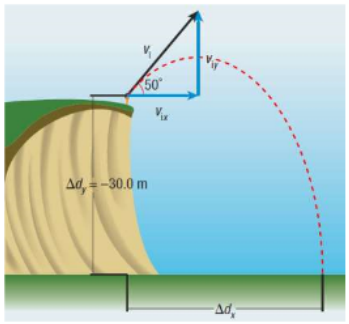


a. time of flight

b. range

c. maximum height

A golfer is trying to improve the range of her shot. To do so she drives a golf ball from the top of a steep cliff, 30.0 m above the ground where the ball will land. If the ball has an initial velocity of 25 m/s and is launched at an angle of 50° above the horizontal, determine the ball's:



a. time of flight

b. range

c. final velocity (just before it hits the ground)

Homework: page 81: #1-2, 4, 6