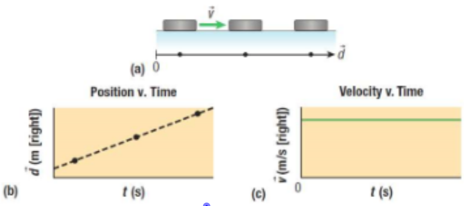
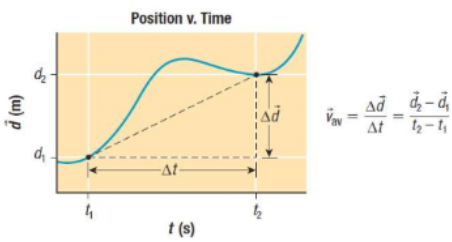
SPH4U 1.1 Motion and Motion Graphs

1. Kinematics

Kinematics:	study of motion. → ignore forces
Dynamics:	study of the causes of motion. → lots of forces!

	Scalar: has magnitude, (size) no direction just a number and units	Vector: has magnitude and direction variables have arrows on them
d	distance usually longer than displacement (because of indirect routes)	$\vec{d}$: position magnitude and direction straight line from reference point
Δd		$\Delta\vec{d}$: displacement change in position $\Delta\vec{d} = \vec{d}_f - \vec{d}_i$ or $\vec{d}_2 - \vec{d}_1$
v	v: speed (how fast) $v_{av} = \frac{\Delta d}{\Delta t}$	$\vec{v}$: velocity (instantaneous) $\vec{v}_{av} = \frac{\Delta\vec{d}}{\Delta t}$
a	a: acceleration. $a_{av} = \frac{\Delta v}{\Delta t}$	$\vec{a}$: acceleration. (vector) $\vec{a}_{av} = \frac{\Delta\vec{v}}{\Delta t}$





0 → a

A jogger takes 25.1 s to run a total distance of 165 m by running 140 m [E] and then 25 m [W]. The displacement is 115 m [E].

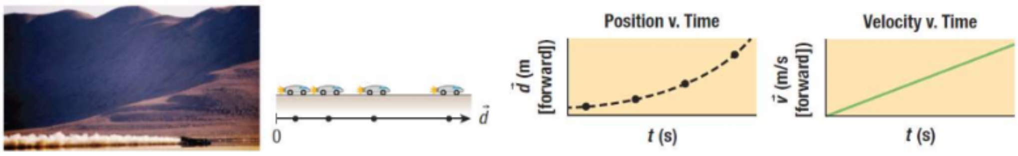
a. Calculate the jogger's average velocity.

$$\vec{v}_{av} = \frac{\Delta\vec{d}}{\Delta t} = \frac{115\text{ m [E]}}{25.1\text{ s}} = 4.58\text{ m/s [E]}$$

b. Calculate the jogger's average speed.

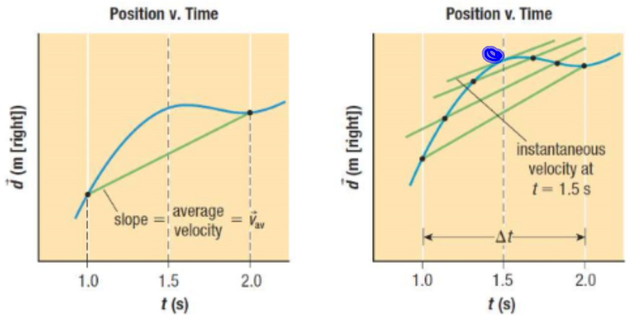
$$v_{av} = \frac{\Delta d}{\Delta t} = \frac{165\text{ m}}{25.1\text{ s}} = 6.57\text{ m/s}$$

2. Graphical interpretation of velocity



(a) (b) (c) (d)

Average velocity:	can be found as the slope of a secant on a position-time graph.
secant	a straight line between 2 points on a graph.

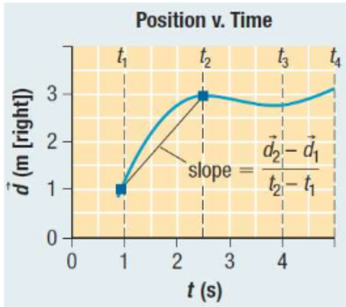


(a) (b)

Instantaneous velocity: $\vec{v}$	can be approximated with secants. but the actual value is the slope of the tangent
tangent	a straight line. that intersects at one point and has the same slope as the graph.
Instantaneous speed:	the magnitude of the slope of the tangent.

The position-time graph to the right shows the details of how an object moved.

a. Calculate the average velocity during the time interval $t_1 = 1.0$ s to $t_2 = 2.5$ s.

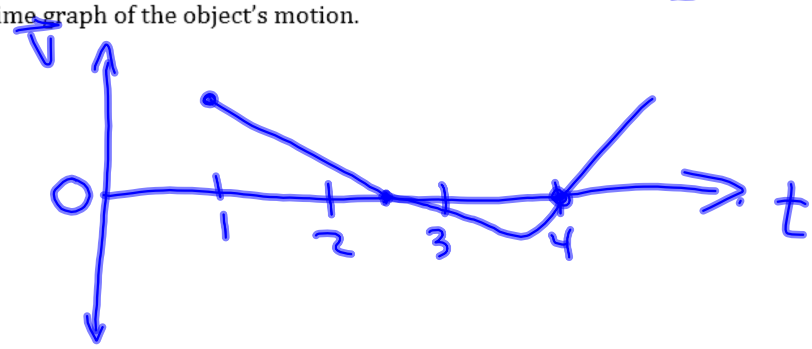


$\vec{v}_{av}$ = slope of the secant

$= \frac{rise}{run} = \frac{2.0 \text{ m[right]}}{1.5 \text{ s}}$

1.3 m/s[right].

b. Analyze the position-time graph. Use your analysis to sketch a qualitative velocity-time graph of the object's motion.

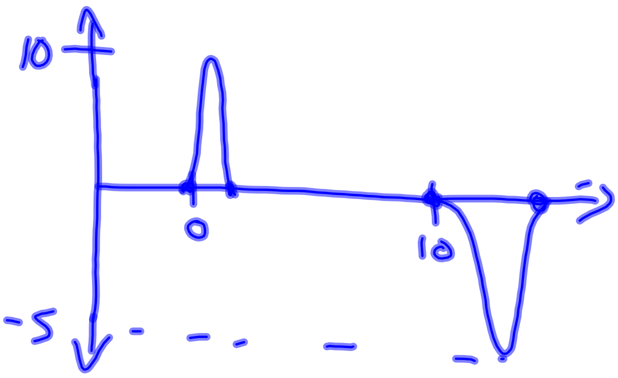


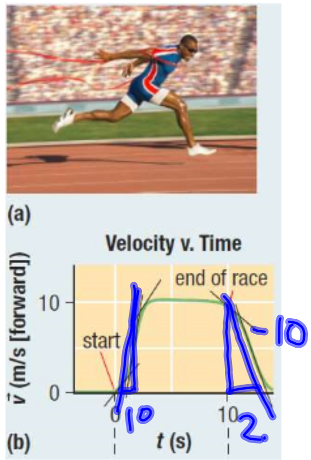
3. Acceleration

Average acceleration:	the change in velocity over time $\vec{a}_{av} = \frac{\Delta \vec{v}}{\Delta t}$ → slope of a secant.
Instantaneous acceleration:	the acceleration at a certain instant in time. → slope of the tangent

Suppose the sprinter to the right is running a 100 m dash. The sprinter's time and distance data have been recorded and used to make the velocity-time graph to the right.

Analyze the velocity-time graph. Use your analysis to make a sketch of an acceleration-time graph of the sprinter's motion.





$\frac{10}{2} = 5.$

From your acceleration-time graph, determine the maximum acceleration and the time at which it occurs.

Max accel: $10\ m/s^2$

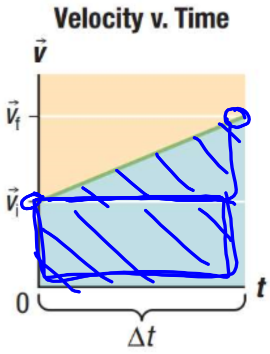
Min accel: $-5\ m/s^2$

Homework: page 16: #1-2, 5-6, 8-9

SPH4U 1.2 Equations of Motion

1. One-dimensional motion with constant acceleration

Average acceleration:	$\vec{a}_{av} = \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t}$ → slope of a V-T graph.
Velocity-time graph	velocity vs. time.
area under graph	$\Delta \vec{d} = \frac{1}{2}(\vec{v}_f + \vec{v}_i)\Delta t$



Solve the average acceleration equation for v_f. Then substitute that value into the displacement equation above and simplify.

$$\vec{a}_{av} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t} \rightarrow \vec{v}_f = \vec{a}_{av}\Delta t + \vec{v}_i$$

$$\Delta \vec{d} = \frac{1}{2}(\vec{v}_f + \vec{v}_i)\Delta t$$

$$= \frac{1}{2}([\vec{a}_{av}\Delta t + \vec{v}_i] + \vec{v}_i)\Delta t$$

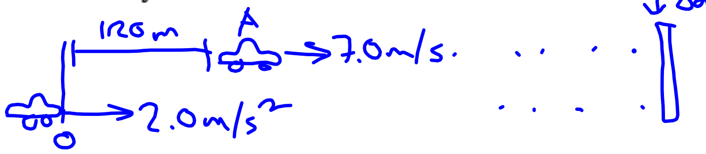
$$= \left(\frac{1}{2}\vec{a}_{av}\Delta t + \vec{v}_i\right)\Delta t$$

$$\boxed{\Delta \vec{d} = \frac{1}{2}\vec{a}_{av}\Delta t^2 + \vec{v}_i\Delta t}$$

	Equation	Variables in the equation	Variables not in the equation
Equation 1	$\Delta\vec{d} = \left(\frac{\vec{v}_f + \vec{v}_i}{2}\right)\Delta t$	$\Delta \vec{d}$, $\vec{v}_f$, $\vec{v}_i$, Δt	$\vec{a}$
Equation 2	$\vec{v}_f = \vec{v}_i + \vec{a}\Delta t$	$\vec{v}_f$, $\vec{v}_i$, $\vec{a}$, Δt	$\Delta \vec{d}$
Equation 3	$\Delta\vec{d} = \vec{v}_i\Delta t + \frac{1}{2}\vec{a}\Delta t^2$	. . .	$\vec{v}_f$
Equation 4	$\Delta\vec{d} = \vec{v}_f\Delta t - \frac{1}{2}\vec{a}\Delta t^2$	. . .	$\vec{v}_i$
Equation 5	$\vec{v}_f^{\,2} = \vec{v}_i^{\,2} + 2\vec{a}\Delta d$	. . .	Δt.

Two cars are at rest on a straight road. Car A starts 120 m ahead of car B, and both begin moving in the same direction at the same time. Car A moves at a constant velocity of 7.0 m/s [forward]. Car B moves at a constant acceleration of 2.0 m/s² [forward]. Calculate how long it will take for car B to catch up with car A, and calculate the velocities of the two cars when they meet.

$\downarrow \Delta \vec{d}_A = \Delta d_B$



$$\Delta \vec{d}_A = 120\text{m} + 7.0\Delta t$$

$$\Delta \vec{d}_B = \cancel{\vec{v}_i}^{\,0}\Delta t + \tfrac{1}{2}\vec{a}\Delta t^2 = \tfrac{1}{2}(2.0)\Delta t^2$$

$$120 + 7.0\Delta t = \Delta t^2$$

$$\Delta t^2 - 7.0\Delta t - 120 = 0.$$

$$(\Delta t + 8)(\Delta t - 15) = 0.$$

$\Delta t = -8\text{s} \leftarrow$ Impossible.

$$\therefore \underline{\Delta t = 15\text{s}}$$

A motorcyclist drives along a straight road with a velocity of 30.0 m/s [forward]. The driver applies the brakes and slows down at 5.0 m/s² [backward].

a. Calculate the braking time.

G	iven
R	equired
E	quations
S	olution
S	tatement.

G: $\vec{v}_i = 30.0\text{ m/s [f]}$, $\vec{v}_f = 0\text{ m/s}$, $\vec{a} = 5.0\text{ m/s}^2\text{ [b]}$, $= -5.0\text{ m/s}^2\text{ [f]}$.

R: Δt

E: $\vec{v}_f = \vec{v}_i + \vec{a}\Delta t$

S: $\Delta t = \dfrac{\vec{v}_f - \vec{v}_i}{a} = \dfrac{0 - 30}{-5}$

$$= \underline{6.0\text{s}}.$$

S: $\therefore$ the braking time is 6.0s.

b. Determine the braking distance (displacement).

G: $\vec{v}_i = 30.0\ m/s\ [F]$, $\vec{v}_f = 0\ m/s$, $\vec{a} = -5.0\ m/s^2\ [f]$.

R: $\vec{\Delta d}$. E: $v_f^2 = v_i^2 + 2a\Delta d$

S: $\Delta d = \dfrac{v_f^2 - v_i^2}{2a} = \dfrac{0^2 - 30^2}{2(-5)}$

$= \dfrac{-900}{-10} = 90\ m\ [f]$

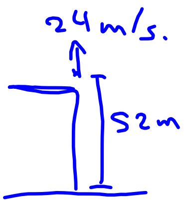
S: $\therefore$ the braking distance is $90\ m\ [f]$.

2. Freely falling objects

Free fall:	the motion of an object where the only force is gravity $\rightarrow$ $F_g = mg$, $g = 9.8\ m/s^2$.

A ball is thrown from a height of 52 m from the top of a building with a velocity of 24 m/s straight up.

a. Determine the velocity of the ball at ground level.



G: $\vec{v}_i = 24\ m/s\ [u]$, $\vec{\Delta d} = 52\ m\ [d] = -52\ m\ [u]$, $\vec{a} = -9.8\ m/s^2\ [u]$.

R: $\vec{v}_f$ E: $v_f^2 = v_i^2 + 2a\Delta d$

S: $v_f = \sqrt{24^2 + 2(-9.8)(-52)} = 40\ m/s\ [d]$

S: $\therefore$ $v_f = 40\ m/s\ [d]$.

b. How long does it take for the ball to reach the ground?

G

R

E

S

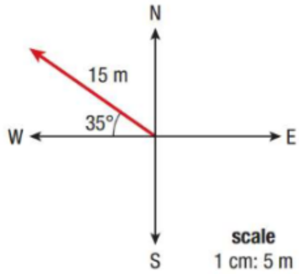
S.

Homework: page 21: #1-5, 8

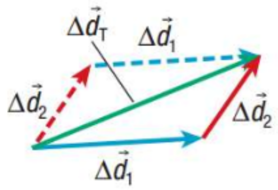
SPH4U 1.3 Displacement in Two Dimensions

1. Determining total displacement

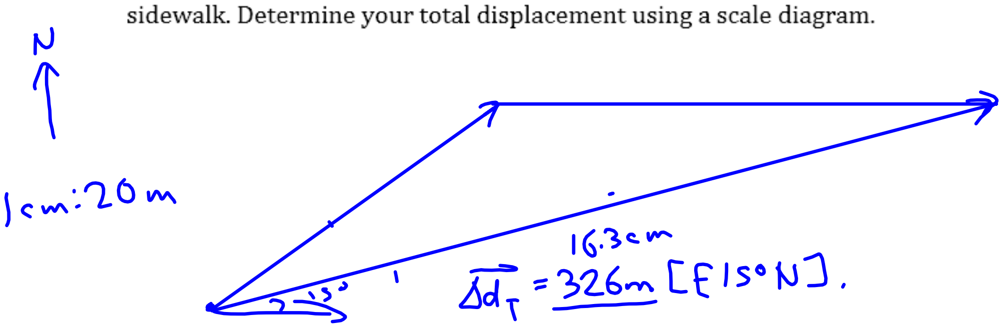
Displacement vector:	magnitude and direction.
direction	write [W35°N], etc.
Three methods for total displacement:	① Scale diagram ② Cosine/sine laws ③ Perpendicular components.



① Scale diagram method:	draw the vectors using a ruler, a protractor, and a scale. → connect vectors tip-to-tail.



Suppose you walk to a friend's house, taking a shortcut across an open field. Your first displacement is 140 m [E35°N] across the field. Then you walk 200.0 m [E] along the sidewalk. Determine your total displacement using a scale diagram.

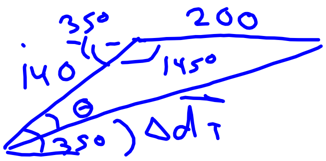


$\Delta\vec{d}_T = 326\text{m}$ [E15°N].

② Cosine and sine laws method:	sketch the problem, then use trigonometry to solve the triangle.

→ only works for 2 vectors.

Using your solution diagram from the previous problem, determine the total displacement using the cosine and sine laws.



$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$c^2 = 140^2 + 200^2 - 2(140)(200)\cos 145°$$

$$c = \sqrt{105\,473}$$

$$\Delta\vec{d}_T = 324.8\text{ m}$$ ← Magnitude.

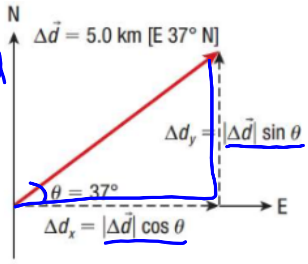
Cosine Law.

Sine Law: $\dfrac{\sin\theta}{200} = \dfrac{\sin 145}{324.8} \rightarrow \theta = \sin^{-1}\left(\dfrac{200\sin 145}{324.8}\right)$

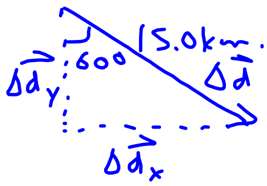
$= 20.7°$ → $35° - 20.7 = 14.3°$

$\therefore\ \Delta\vec{d}_T = 320\text{ m}$ [E14°N].

Perpendicular components method:	break each vector into 2 perpendicular components, then add
x-component	$\Delta \vec{d}_x = \lvert\Delta\vec{d}\rvert \cos\theta$ — will swap if the angle is measured from vertical
y-component	$\Delta \vec{d}_y = \lvert\Delta\vec{d}\rvert \sin\theta$



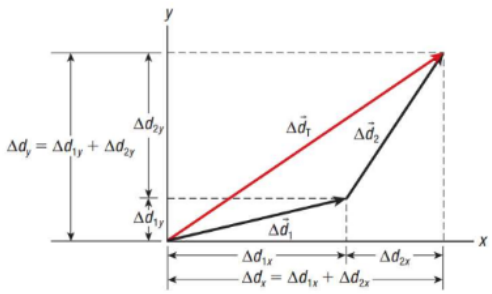
A polar bear walks toward Churchill, Manitoba. The polar bear's displacement is 15.0 km [S60.0°E]. Determine the components of the displacement.



$\cos 60° = \frac{\Delta d_x}{\Delta d} \rightarrow \Delta d_x = \Delta d \cos 60° = 7.5 \text{ m [S]}$

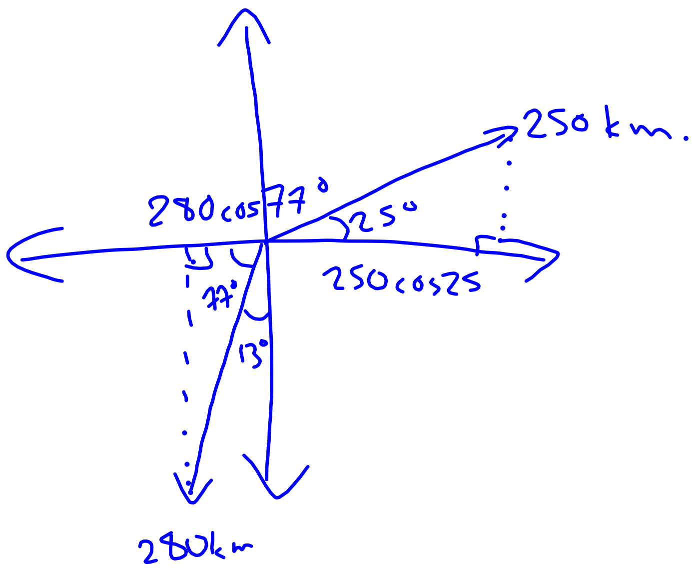
$\sin 60° = \frac{\Delta d_y}{\Delta d} \rightarrow \Delta d_y = \Delta d \sin 60° = 13 \text{ m [E]}.$

2. Adding vectors algebraically



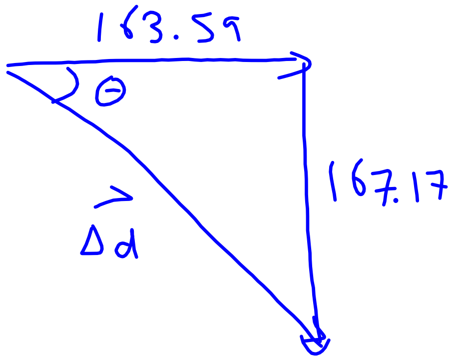
An airplane flies 250 km [E25°N], and then flies 280 km [S13°W]. Using components, calculate the airplane's total displacement.

Homework: page 29: #2, 4-6, 10-11



$$\vec{\Delta d}_{Tx} = \vec{\Delta d}_{1x} + \vec{\Delta d}_{2x}$$
$$= 250\cos 25° - 280\cos 77°.$$
$$= 163.59 \text{ km [E]}.$$

$$\vec{\Delta d}_{Ty} = \vec{\Delta d}_{1y} + \vec{\Delta d}_{2y}$$
$$= 250\sin 25° - 280\sin 77°$$
$$= -167.17 \text{ km [N]}.$$
$$= 167.17 \text{ km [S]}.$$



$$|\vec{\Delta d}| = \sqrt{\Delta d_x^2 + \Delta d_y^2}$$
$$= \sqrt{163.59^2 + 167.17^2}$$
$$= 233.9 \text{ km}$$
$$\theta = \tan^{-1}\left(\frac{167.17}{163.59}\right)$$
$$= 45.6°$$

$\therefore \vec{\Delta d} = 230 \text{ km [E } 46° \text{ S]}.$

SPH4U 1.4 Velocity and Acceleration in Two Dimensions

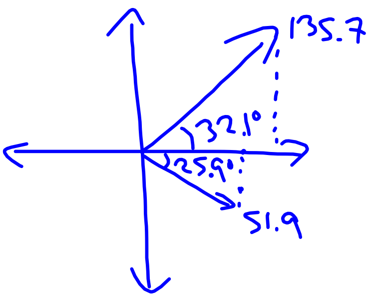
1. Velocity and speed in two dimensions

Change in velocity:	change in size or direction. any change in velocity causes acceleration.
average velocity	$\vec{v}_{av} = \frac{\Delta \vec{d}}{\Delta t}$, where $\Delta \vec{d}$ is the total displacement.
total displacement	$\Delta \vec{d}_T = \Delta \vec{d}_1 + \Delta \vec{d}_2 + \ldots$ Use components method.
total distance	$\Delta d_T = \Delta d_1 + \Delta d_2 + \ldots \rightarrow$ for average speed.

A family drives from Saint John, New Brunswick, to Moncton. Assuming a straight highway, this part of the drive has a displacement of 135.7 km [E32.1°N], and takes 1.5 h. From Moncton they drive to Amherst, Nova Scotia. The second displacement is 51.9 km [E25.9°S], and takes 1.0 h.

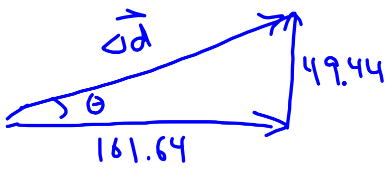
a. Calculate the average velocity of the family's vehicle.

$\vec{v}_{av} = \frac{\Delta \vec{d}}{\Delta t}$ $\quad \Delta t = 2.5\,h \quad \Delta \vec{d} = \Delta \vec{d}_1 + \Delta \vec{d}_2$



$\Delta \vec{d}_x = \Delta \vec{d}_{1x} + \Delta \vec{d}_{2x}$
$= 135.7\cos 32.1° + 51.9\cos 25.9°$
$= 161.64$ km [E].

$\Delta \vec{d}_y = \Delta \vec{d}_{1y} + \Delta \vec{d}_{2y}$
$= 135.7\sin 32.1° - 51.9\sin 25.9°$
$= 49.44$ km [N].



$|\Delta \vec{d}| = \sqrt{161.64^2 + 49.44^2}$
$= \underline{169\text{ km}}$

$\theta = \tan^{-1}\left(\frac{49.44}{161.64}\right)$
$= \underline{17.0°}$

$\therefore \vec{d} = 169\text{km [E17.0N]}$

$\vec{v}_{av} = \frac{\Delta \vec{d}}{\Delta t} = \frac{169\text{ km}}{2.5\text{ h}} = \underline{67.6\text{ km/h [E17.0°N]}}.$

b. Calculate the average speed of the family's vehicle.

$v_{av} = \frac{\Delta d}{\Delta t}$ $\quad \Delta d = 135.7 + 51.9 = 187.6$ km.

$\Delta t = 2.5$ h.

$v_{av} = \frac{187.6}{2.5} = \underline{75.0 \text{ km/h}}$.

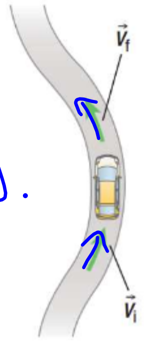
Could you have solved this problem differently? Could you have calculated the velocity of the first part of the drive, the velocity of the second part, and added them together?

No. Need $\Delta \vec{d}_T$ first.

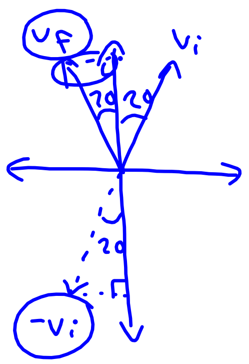
2. Multiplying vectors by scalars and subtracting vectors

Scalar multiplication:	can multiply a vector by a scalar to change its magnitude (length). $\vec{B} = 2\vec{A}$, $\vec{B} = k\vec{A}$. If $0<k<1$, it shrinks $\vec{A}$.
subtracting vectors	$\vec{A} - \vec{B} = \vec{A} + (-\vec{B}) = \vec{A} + (-1)\vec{B}$ → flip the second vector backwards and add.

In the diagram to the right, the car's initial velocity is 60 km/h [N20°E] and its final velocity is 60 km/h [N20°W]. Find the change in the car's velocity.



$\Delta\vec{v} = \vec{v}_f - \vec{v}_i = v_f + (-\vec{v}_i)$.



$= 60$ km/h [N20°W] + 60 km/h [S20°W].

$\Delta\vec{v}_x = 60\sin20° + 60\sin20°$

$= 41.04$ km/h [W].

$\Delta\vec{v}_y = 60\cos20° - 60\cos20°$

$= 0$ km/h.

$\therefore \underline{\Delta\vec{v} = 41 \text{ km/h [W]}}$.

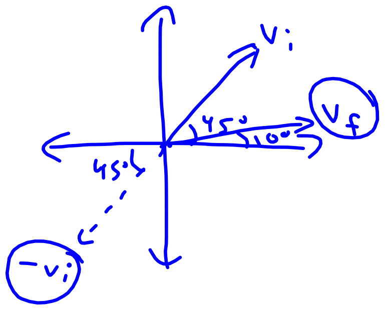
3. Acceleration in two dimensions

Average acceleration:	$\vec{a}_{av} = \frac{\Delta\vec{v}}{\Delta t}$
net velocity	$\Delta\vec{v}_x = \vec{v}_{fx} - \vec{v}_{ix}$, $\Delta\vec{v}_y = \vec{v}_{fy} - \vec{v}_{iy}$ $\lvert\Delta\vec{v}\rvert = \sqrt{\Delta v_x^2 + \Delta v_y^2}$, $\theta = \tan^{-1}\left(\frac{\Delta v_y}{\Delta v_x}\right)$.

A car turns from a road into a parking lot and into an available parking space. The car's initial velocity is 4.0 m/s [E45.0°N]. The car's velocity just before the driver decreases speed is 4.0 m/s [E10.0°N]. The turn takes 3.0 s. Calculate the average acceleration of the car during the turn.

$\Delta t = 3.0\,s.$ $\vec{a}_{av} = \frac{\Delta\vec{v}}{\Delta t}$ $\Delta\vec{v} = \vec{v}_f - \vec{v}_i$

$= \vec{v}_f + (-\vec{v}_i).$

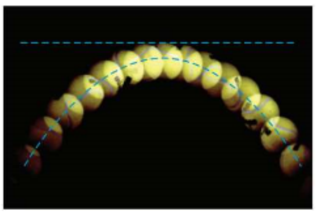


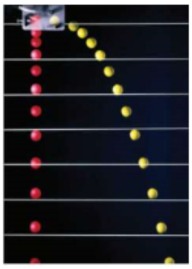
Homework: page 35: #1-2, 6-7, 10

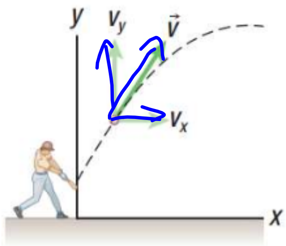
SPH4U 1.5 Projectile Motion

1. Projectile motion

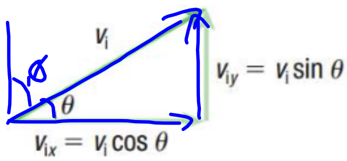
Projectile:	an object launched through the air falls to the ground in a parabolic arc.
range $\Delta \vec{d}_x$	horizontal displacement.
projectile motion	horizontal velocity is constant. $(\vec{a}_x = 0)$. vertical acceleration is constant $(\vec{a}_y = g)$. solve for x and y separately (share Δt).





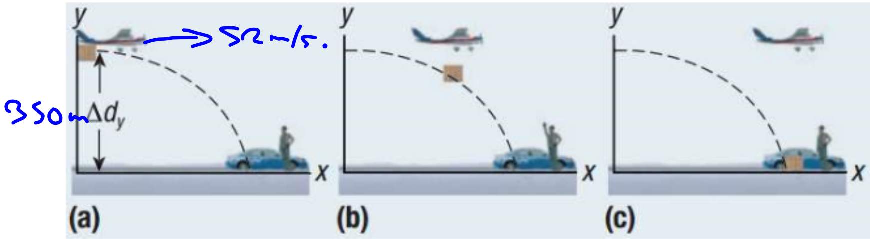


Initial velocity:	$v_{ix} = v_i \cos\theta$ $v_{iy} = v_i \sin\theta$



Direction of Motion	Description	Equations of Motion
horizontal motion (x)	constant $\vec{v}$	$v_{ix} = v_i \cos\theta$ $\Delta d_x = v_{ix} \Delta t$
vertical motion (y)	constant $\vec{a}$	$v_{iy} = v_i \sin\theta$ $\Delta d_y = v_{iy} \Delta t - \frac{1}{2} g \Delta t^2$ $v_{fy} = v_{iy} - g\Delta t$ $v_{fy}^2 = v_{iy}^2 - 2g\Delta d_y$.

An airplane pilot drops a package of supplies to a motorist as he flies horizontally at a height of 350 m over the highway. The speed of the airplane is a constant 52 m/s. The figure below shows the package (a) as it leaves the airplane, (b) in mid-drop, and (c) when it lands on the highway.



a. Calculate how long it takes for the package to reach the highway.

G: $\Delta d_y = -350\,m$, $v_x = 52\,m/s$, $g = 9.8\,m/s^2$, $v_{iy} = 0\,m/s$.

R: Δt E: $\Delta d_y = \cancel{v_{iy}}^{0}\Delta t - \frac{1}{2} g \Delta t^2$

S: $\Delta d_y = -\frac{1}{2} g \Delta t^2 \rightarrow \Delta t = \sqrt{\frac{-2\Delta d_y}{g}} = \sqrt{\frac{-2(-350)}{9.8}}$

$= 8.5\,s$.

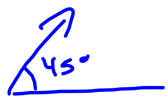
b. Determine the range of the package.

R: Δd_x E: $\Delta d_x = v_x \Delta t$

S: $\Delta d_x = (52)(8.45\,s) = 439\,m$

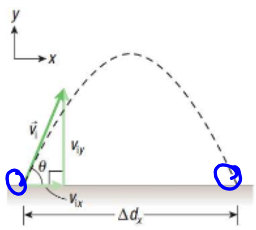
$= 440\,m$

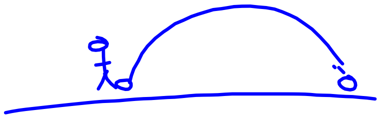
Maximum range.



2. The range equation

Range equation:	$\Delta d_x = \frac{v_i^2}{g}\sin(2\theta)$ → only use this when $\Delta d_y = 0$. → only need to know v_i and θ!
elapsed time	$\Delta t = 2 v_i \sin\theta / g$.
maximum range	$\frac{v_i^2}{g}$, this happens when $\sin(2\theta) = 1$ → when $\theta = 45°$
air resistance	we assume it is 0.





$\Delta d_y = 0$. ✓

Suppose you kick a soccer ball at 28 m/s toward the goal at a launch angle of 21°.

a. How long does the soccer ball stay in the air?

$$\Delta t = \frac{2 v_i \sin\theta}{g} = \frac{2(28)\sin 21}{9.8} = \underline{2.0\,s}.$$

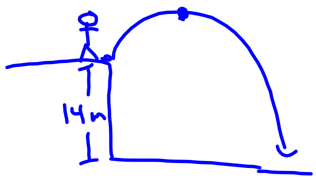
b. Determine the distance the soccer ball would need to cover to score a goal (the range).

$$\Delta d_x = \frac{v_i^2}{g}\sin 2\theta$$

$$= \frac{(28)^2}{9.8}\sin(2 \times 21) = \underline{53\,m}.$$

A golfer hits a golf ball with an initial velocity of 25 m/s at an angle of 30.0° above the horizontal. The golfer is at an initial height of 14 m above the point where the ball lands.

a. Calculate the maximum height of the ball.



$v_{fy} = 0\,m/s$.

b. Determine the ball's velocity on landing.

$\vec{v_f} = \vec{v_{fx}} + \vec{v_{fy}}$.

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SPH4U 1.6 Relative Motion

1. Relative motion

Frame of reference:	a coordinate system from which we measure motion.
relative velocity	velocity relative to some frame of reference.

$\vec{v}_{AC}$: velocity of A relative to C.

$\vec{v}_{AC} = \vec{v}_{AB} + \vec{v}_{BC}$

$\vec{v}_{AF} = \vec{v}_{AB} + \vec{v}_{BC} + \vec{v}_{CD} + \vec{v}_{DE} + \vec{v}_{EF}$

Passengers on a cruise ship are playing shuffleboard. The shuffleboard disc's velocity relative to the ship is 4.2 m/s [forward], and the ship is traveling in the same direction as the disc at 4.6 km/h relative to Earth when the water is stationary.

a. What is the disc's velocity relative to Earth?

$\vec{v}_{DE} = \vec{v}_{DS} + \vec{v}_{SE}$ $\vec{v}_{DS} = 4.2\text{ m/s [f]}.$

$v_{SE} = 4.6\text{ km/h [f]} \times \frac{1000\text{ m}}{3600\text{ s}} = 1.28\text{ m/s [f]}.$

$\vec{v}_{DE} = 4.2 + 1.28 = 5.5\text{ m/s [f]}.$

b. What is the disc's velocity relative to Earth, when the disc is moving in the opposite direction?

$\vec{v}_{DS} = -4.2\text{ m/s [f]}.$

$\vec{v}_{DE} = \vec{v}_{DS} + \vec{v}_{SE} = -4.2 + 1.28$

$= 2.9\text{ m/s [b]}.$

c. What is the disc's velocity relative to Earth, when the water is moving at 1.1 m/s [forward], and the disc is again moving forward?

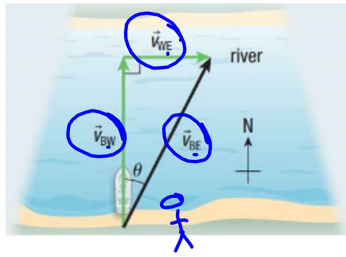
$\vec{v}_{DS} = 4.2\text{ m/s [f]},\ \vec{v}_{SW} = 1.28\text{ m/s [f]},\ \vec{v}_{WE} = 1.1\text{ m/s [f]}.$

$\vec{v}_{DE} = \vec{v}_{DS} + \vec{v}_{SW} + \vec{v}_{WE}$

$= 4.2 + 1.28 + 1.1$

$= 6.6\text{ m/s [f]}.$

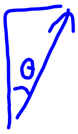
The boat to the right is heading due north as it crosses a wide river. The velocity of the boat is 10.0 km/h relative to the water. The river has a uniform velocity of 5.00 km/h due east.



a. Determine the boat's velocity relative to an observer on the riverbank.

$\vec{v}_{BE} = \vec{v}_{BW} + \vec{v}_{WE}$

$= 10.0\ km/h\ [N] + 5.00\ km/h\ [E].$

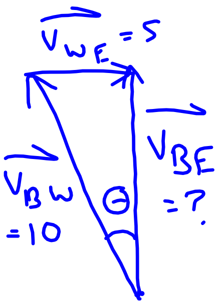


$v_{BE} = \sqrt{10^2+5^2} = 11.18\ km/h.$

$\theta = \tan^{-1}\left(\frac{5}{10}\right) = 26.6^\circ$

$\therefore \vec{v}_{BE} = 11.2\ km/h\ [N26.6^\circ E].$

b. The driver of the boat now wants to arrive across the water at a location that is due north of his present location. He moves at the same speed of 10.0 km/h relative to the water, and the river is flowing east at 5.00 km/h. In which direction should he head, and what is the speed of the boat relative to the shore?



$\sin\theta = \frac{O}{H} = \frac{\vec{v}_{WE}}{\vec{v}_{BW}} = \frac{5}{10} = \frac{1}{2}.$

$\theta = \sin^{-1}\left(\frac{1}{2}\right) = 30.0^\circ$

$\therefore$ direction is $[N30.0^\circ W]$.

$v_{BW}^2 = v_{BE}^2 + v_{WE}^2$

$v_{BE} = \sqrt{v_{BW}^2 - v_{WE}^2}$

$= \sqrt{10^2 - 5^2}$

$= \sqrt{75} = 8.66\ km/h.$

$\therefore$ relative speed is 8.66 km/h.

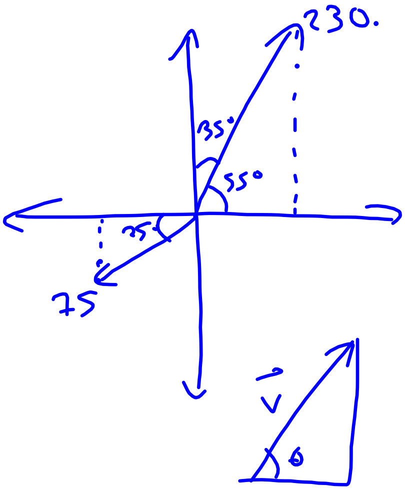
2. Airplanes

velocity of a plane	$\vec{v}_{PE} = \vec{v}_{PA} + \vec{v}_{AE}$ $\vec{v}_{PA}$: airspeed velocity $\vec{v}_{AE}$: wind velocity.
tailwind	wind blows with the plane. (+)
headwind	wind blows against the plane (−)
crosswind	wind blows perpendicular.

The air velocity of a small plane is 230 km/h [N35°E] when the wind is blowing at 75 km/h [W25°S]. Determine the velocity of the plane relative to the ground.

$\underline{\vec{v}_{PE} = \vec{v}_{PA} + \vec{v}_{AE}}$

$\vec{v}_{PA} = 230$ km/h [N35°E],
$\vec{v}_{AE} = 75$ km/h [W25°S].



$\vec{v}_x = 230\cos 55° - 75\cos 25°$

$=$

$\vec{v}_y = 230\sin 55° - 75\sin 25°$

$=$

$\therefore\ \vec{v}_{PE} = 170$ km/h [E68°N].

Homework: page 49: #1-2, 4-5