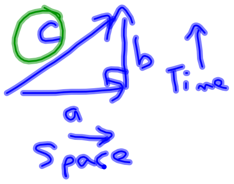


$a^2+b^2=c^2$

SPH4U 11.0 Intro to Relativity

1. **Spacetime**

From reddit user /u/corpuscle634:

Everything, by nature of simply existing, is "moving" at the speed of light (which really has nothing to do with light: more on that later). Yes, that does include you.

Our understanding of the universe is that the way that we perceive space and time as separate things is, to be frank, wrong. They aren't separate: the universe is made of "spacetime," all one word. A year and a lightyear describe different things in our day to day lives, but from a physicist's point of view, they're actually the exact same thing (depending on what kind of physics you're doing).

In our day to day lives, we define motion as a distance traveled over some amount of time. However, if distances and intervals of time are the *exact same thing*, that suddenly becomes completely meaningless. "I traveled one foot for every foot that I traveled" is an absolutely absurd statement!

The way it works is that everything in the universe travels through *spacetime* at some speed which I'll call "c" for the sake of brevity. Remember, motion in spacetime is meaningless, so it makes sense that nothing could be "faster" or "slower" through spacetime than anything else. Everybody and everything travels at one foot per foot, that's just... how it works.

Obviously, though, things do seem to have different speeds. The reason that happens is that time and space are *orthogonal*, which is sort of a fancy term for "at right angles to each other." North and east, for example, are orthogonal: you can travel as far as you want directly to the north, but it's not going to affect where you are in terms of east/west at all.

Just like how you can travel north without traveling east, you can travel through time without it affecting where you are in space. Conversely, you can travel through space without it affecting where you are in time.

You're (presumably) sitting in your chair right now, which means you're not traveling through space at all. Since you have to travel through spacetime at c (speed of light), though, that means *all of your motion* is through time.

By the way, this is why time dilation happens: something that's moving very fast relative to you is moving through space, but since they can only travel through spacetime at c, they have to be moving more slowly through time to compensate (from your point of view).

Light, on the other hand, doesn't travel through time at all. The reason it doesn't is somewhat complicated, but it has to do with the fact that it has no mass.

Something that isn't moving that has mass can have energy: that's what $E = mc^2$ means. Light has no mass, but it*does* have energy. If we plug the mass of light into $E=mc^2$, we get 0, which makes no sense because light has energy. Hence, light can never be stationary.

Not only that, but light can never be stationary from *anybody's* perspective. Since, like everything else, it travels at c through spacetime, that means all of its "spacetime speed" *must* be through space, and none of it is through time.

So, light travels at c. Not at all by coincidence, you'll often hear c referred to as the "speed of light in a vacuum." Really, though, it's the speed that *everything* travels at, and it happens to be the speed that light travels through space at because it has no mass.

2. Velocity-addition formula

Galilean addition of velocities:	$\vec{S} = \vec{V} + \vec{U}$
problem	if my car is moving at speed v and the light from the headlights is moving at c relative to the car, doesn't the light move faster than c?
Relativistic velocity addition:	$S = \dfrac{V+U}{1+\left(\frac{VU}{c^2}\right)}$

Mr. Koopmans' spaceship has a max speed of 0.8*c*. As part of a demonstration, he flies it past the school so that all the students can see.

a. Mr. Koopmans fires a spatial torpedo forward at 0.5*c* relative to the spaceship. How fast does the torpedo move relative to the students on the ground?

$$S = \frac{V+U}{1+\left(\frac{VU}{c^2}\right)} = \frac{0.8c + 0.5c}{1+\left(\frac{0.8 \cdot 0.5 \cdot c^2}{c^2}\right)}$$

$$= \frac{1.3c}{1+0.4}$$

$$= \frac{1.3}{1.4}c = \underline{\underline{0.9286c}}$$

b. Light leaves the spaceship's headlights at *c* relative to the spaceship. How fast does the light move relative to the students on the ground?

$$S = \frac{V+U}{1+\left(\frac{VU}{c^2}\right)} = \frac{0.8c + c}{1+\left(\frac{0.8c^2}{c^2}\right)}$$

$$= \frac{1.8c}{1+0.8}$$

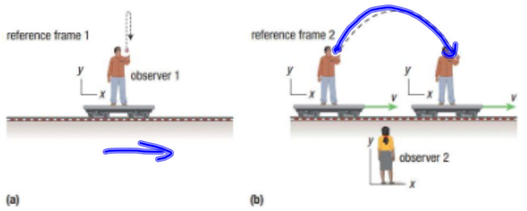
$$= \frac{1.8}{1.8}c$$

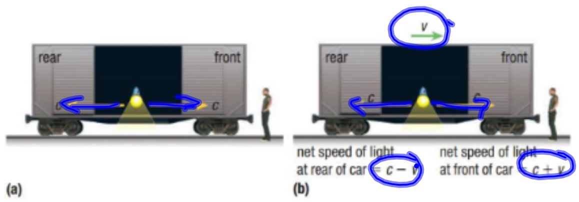
$$= \underline{\underline{c}}$$

SPH4U 11.1 The Special Theory of Relativity

1. Special relativity

Inertial frame of reference:	a coordinate system that moves at a constant speed (or 0). the laws of inertia hold.
Principle of relativity:	the laws of motion are the same in all inertial frames.





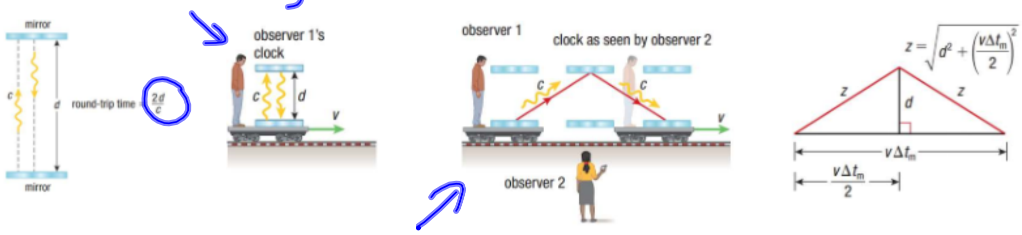
Thought experiment:	an experiment carried out in the imagination, to examine the logic behind a hypothesis.
postulate	a statement that we assume is true in order to develop a theory.
Einstein's theory:	special theory of relativity, published in 1905.
postulate 1	The principle of relativity: the laws of physics are the same in all inertial frames.
postulate 2	The speed of light principle: in at least one inertial frame, the speed of light in a vacuum is independent of the light source's motion.
Special theory of relativity:	all physical laws are the same in all inertial frames, and the speed of light is independent of the motion of the light source or its observer.

Homework: pg. 537 #1, 3, 9-10

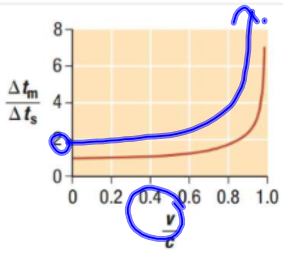
SPH4U 11.2 Time Dilation

1. Time dilation

Time dilation:	the faster something moves relative to us, the slower it experiences time.
Thought experiment:	imagine someone moving horizontally with speed v, measuring time with a "light clock".
light clock	light bounces up and down between two mirrors, taking time $\Delta t = \frac{2d}{c}$ for one cycle.



Observer 1	is stationary relative to the clock. $\Delta t_s = \frac{2d}{c}$.
Observer 2	sees the clock moving horizontally. the light travels the same vertical distance 2d, but also a horizontal distance $v\Delta t$. it still moves at speed c!
equation	$\Delta t_m = \frac{\Delta t_s}{\sqrt{1-\frac{v^2}{c^2}}}$ ← ≤1
Proper time:	Δt_s, time measured by an observer at rest relative to a clock. $\Delta t_s \le \Delta t_m$, always.



On Earth, an astronaut has a pulse of 75.0 beats/min. He travels into space in a spacecraft capable of reaching very high speeds.

a. Determine the astronaut's pulse with respect to a clock on Earth when the spacecraft travels at a speed of 0.10c.

$$\Delta t_s = \frac{1}{75} = 1.33\times10^{-2}\ \text{min.}$$

$$\Delta t_m = \frac{\Delta t_s}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{1.33\times10^{-2}\ \text{min}}{\sqrt{1-\frac{(0.10c)^2}{c^2}}} = \frac{1.33\times10^{-2}}{\sqrt{1-0.01}}$$

$$= \underline{1.34\times10^{-2}\ \text{min}}$$

$$\text{Pulse: } \frac{1}{\Delta t_m} = \frac{1}{1.34\times10^{-2}} = 74.6\ \text{bpm} \doteq \underline{75\ \text{bpm}}$$

b. Determine the astronaut's pulse with respect to a clock on Earth when the spacecraft travels at a speed of 0.90c.

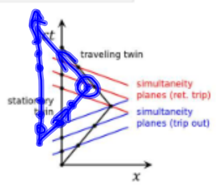
$$\Delta t_m = \frac{\Delta t_s}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{1.33\times10^{-2}\ \text{min}}{\sqrt{1-\frac{(0.90c)^2}{c^2}}} = \frac{1.33\times10^{-2}}{\sqrt{1-0.81}}$$

$$= \underline{3.05\times10^{-2}\ \text{min}}$$

$$\text{Pulse: } \frac{1}{\Delta t_m} = \frac{1}{3.05\times10^{-2}} = \underline{33\ \text{bpm}}$$

2. Twin paradox

Twin paradox:	imagine two twins. Twin 1 leaves Earth in a high-speed rocket, then returns home, finding Twin 2 has aged more than Twin 1.
relativity	why? For both twins, time should seem slowed down for the other twin.
solution 1	for paths from A to B, the shortest path through space is also the longest path through time.
solution 2	when Twin 1 changes direction, he switches between two inertial frames that line up differently with Twin 2.
verification	in the 1970s, time dilation was verified using atomic clocks on passenger jets.

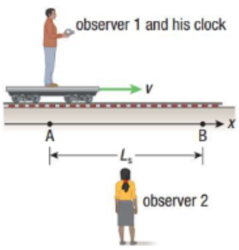


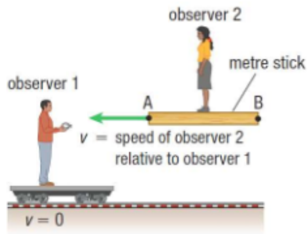
Homework: pg. 587 #2-4, 6-7

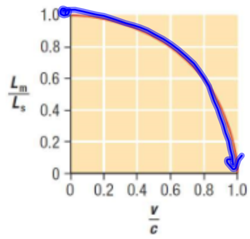
SPH4U 11.3 Length Contraction and Relativistic Momentum

1. Length contraction

Proper length:	L_s, length measured by a stationary observer.
Relativistic length:	Observer 2 measures time Δt_m for observer 1 to travel from point A to B (distance L_s) at speed v. $L_s = v\Delta t_m$. For Observer 1, $L_m = v\Delta t_s$.
equation	$L_m = L_s\sqrt{1-\frac{v^2}{c^2}}$







An observer on Earth measures the length of a spacecraft travelling at a speed of 0.700c to be 78.0 m long. Determine the proper length of the spacecraft.

$L_m = 78.0\text{m}, v = 0.700c$.

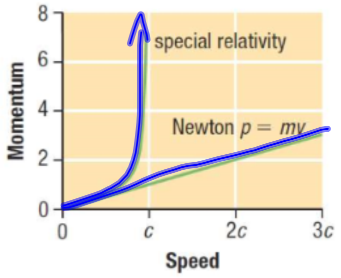
$$L_m = L_s\sqrt{1-\frac{v^2}{c^2}}$$

$$L_s = \frac{L_m}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$= \frac{78.0}{\sqrt{1-\frac{(0.700c)^2}{c^2}}} = \frac{78.0}{\sqrt{1-0.49}} = \underline{\underline{109\text{ m}}}.$$

2. Relativistic momentum

Relativistic momentum:	momentum behaves differently at fast speeds.
equation	$p = \dfrac{mv}{\sqrt{1-\frac{v^2}{c^2}}}$



In experiments to study the properties of subatomic particles, physicists routinely accelerate electrons to speeds close to the speed of light. An electron has a mass of 9.11 x 10^{-31} kg and moves with a speed of 0.99*c*.

a. Calculate the electron's momentum using the non-relativistic equation.

$$p_{classical} = mv$$
$$= (9.11\times10^{-31})(0.99)(3.0\times10^{8})$$
$$= 2.7\times10^{-22}\ \text{kg m/s}.$$

b. Calculate the electron's relativistic momentum. Compare the relativistic momentum and the non-relativistic momentum.

$$p_{relativistic} = \frac{mv}{\sqrt{1-\frac{v^2}{c^2}}}$$
$$= \frac{(9.11\times10^{-31})(0.99)(3.0\times10^{8})}{\sqrt{1-0.99^2}}$$
$$= 1.9\times10^{-21}\ \text{kgm/s}$$

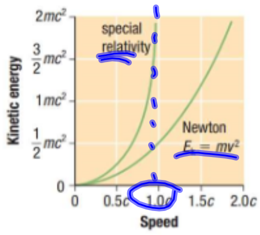
Relativistic > Classical, by about 7x.

Homework: pg. 597 #1-2, 4-6

SPH4U 11.4 Mass-Energy Equivalence

1. Mass-energy equivalence

Relativistic mass:	$m_{relativistic} = \frac{m}{\sqrt{1-\frac{v^2}{c^2}}}$ m: rest mass
Total energy:	$E_{total} = \frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}}$
rest energy	$E_{rest} = mc^2$.
Relativistic kinetic energy:	$E_k = E_{total} - E_{rest}$ $E_k = \frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}} - mc^2$.
Conservation of mass-energy:	rest mass and energy are equivalent



The average home in Canada uses 3.6 x 10^{10} J of energy per day. Imagine that a cabbage with a rest mass of 0.750 kg could be completely converted to another form of energy (although in a nuclear reaction only a fraction of the mass is converted to electrical energy).

a. Calculate how much energy is released by the cabbage.

$E_{rest} = mc^2$
$= (0.750)(3.0\times10^8)^2$
$= 6.75\times10^{16}$ J
$\approx 6.8\times10^{16}$ J.

b. Determine the number of days this cabbage could supply energy for an average home in Canada.

$t = \frac{6.75\times10^{16}}{3.6\times10^{10}}$
$= 1.9\times10^6$ days   > 5000 years.

2. Subatomic particles

Small masses:	we use eV (electron-volts) instead of J and kg to describe the energy and mass of small particles.
conversion	1 eV = 1.602×10^{-19} J (same as e, except in J, not C).

An electron has a speed of 0.900*c* in a laboratory, and the rest mass of an electron is 9.11 x 10^{-31} kg. With respect to the laboratory's frame of reference, calculate the electron's rest energy, total energy, and kinetic energy in electron-volts.

$$E_{rest} = mc^2$$

$$= (9.11\times10^{-31})(3.0\times10^{8})^2\left(\frac{1\ \text{eV}}{1.602\times10^{-19}}\right)$$

$$= \underline{\underline{0.512\ \text{MeV}}}$$

$$E_{total} = \frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$= \frac{0.512\ \text{MeV}}{\sqrt{1-0.90^2}}$$

$$= \underline{\underline{1.17\ \text{MeV}}} \quad (1.17\times10^6\ \text{eV}).$$

$$E_k = E_{total} - E_{rest}$$

$$= 1.17 - 0.512$$

$$= \underline{\underline{0.66\ \text{MeV}}}$$

Homework: pg. 603 #1, 3-5, 7-8, 12