

9 Personal Finance

Good financial sense will help you to achieve your personal goals. In this chapter, you will investigate the world of finance: banks and financial institutions, investing, and credit cards.

For many young adults, buying a vehicle is one of their first major purchases. Later in this chapter, you will look at the financial implications of obtaining and operating a vehicle.

In this chapter, you will

- gather, interpret, and compare information about
 - the various savings alternatives commonly available from financial institutions, the related costs, and possible ways of reducing the costs
 - investment alternatives, and compare the alternatives by considering the risk and the rate of return
 - the costs and incentives associated with various credit cards and debit cards
 - current credit card interest rates and regulations, and determine, through investigation using technology, the effects of delayed payments on a credit card balance
 - procedures and costs involved in insuring a vehicle and the factors affecting insurance rates
 - the procedures and costs involved in buying or leasing a new vehicle or buying a used vehicle
- solve problems involving applications of the compound interest formula to determine the cost of making a purchase on credit and solve problems using technology, that involve the fixed costs and variable costs of owning and operating a vehicle



Key Terms

depreciation

fixed cost

growth rate

Guaranteed Investment
Certificate (GIC)

mutual fund

Registered Education Savings
Plan (RESP)

Registered Retirement
Savings Plan (RRSP)

variable cost

After completing a two-year college course in business management-financial service at Humber College, Sonia took a job as a bank teller. Within a year she was promoted. In her new role as a personal banking officer, Sonia prepares, evaluates, and processes loan applications. Understanding and applying formulas are important skills in her job.



Prerequisite Skills

Decimals

1. Use a calculator to evaluate.

- a) 6.3×2.1
- b) 7×3.04
- c) $10\,000(0.06)(9)$
- d) $500(0.02)(6.5)$
- e) $450 + 450(0.075)(0.25)$
- f) $6750 + 6750(0.035)$

2. Evaluate.

- a) $0.08 \div 2$
- b) $0.055 \div 4$
- c) $0.072 \div 3$
- d) $0.06 \div 4$
- e) $0.085 \div 10$
- f) $0.09 \div 12$

Percents

3. Convert each percent to a decimal.

- a) 16%
- b) 7%
- c) 4.9%
- d) 0.9%
- e) -2.85%
- f) 28.8%

4. Estimate each value. Explain your thinking.

- a) 8% of 750
- b) 8.5% of 5000
- c) 11% of 1900
- d) 2.4% of 1800
- e) 0.9% of 25 000
- f) 2.5% of 10 000

5. Evaluate without the aid of a calculator.

- a) 10% of 3500
- b) 5% of 3500
- c) 1% of 160
- d) 2% of 160
- e) 25% of 10 000
- f) 2.5% of 10 000

6. Evaluate. Express your answer as a decimal.

- a) $8\% \div 4$
- b) $9.6\% \div 3$
- c) $4.8\% \div 12$
- d) $6\% \div 4$
- e) $18\% \div 12$
- f) $11.2\% \div 4$
- g) $17.5\% \div 2$
- h) $15.9\% \div 12$

Exponents

7. Evaluate without the aid of a calculator.

- a) 2^4
- b) 3^3
- c) $(1.2)^2$
- d) 1^{50}
- e) $(0.5)^3$
- f) 0^{365}

8. Evaluate with the aid of a calculator.

- a) $(1.9)^4$
- b) $(2.95)^3$
- c) $(1.25)^2$
- d) $(0.9)^{50}$
- e) $(0.55)^3$
- f) $(0.07)^3$

Compound Interest

9. For each annual rate, r , determine i , the interest rate per compounding period. Round answers to four decimal places, where necessary.

	r (%)	Compounding Frequency	i
a)	9	monthly	
b)	16.9	quarterly	
c)	-4.65	semi-annually	
d)	1.8	quarterly	
e)	0.5	monthly	
f)	28.8	daily	

10. Determine n , the number of compounding periods, for each situation.
- a) quarterly compounding for three years
 - b) monthly compounding for five years
 - c) semi-annual compounding for four years
 - d) daily compounding for six months
 - e) daily compounding for two years
 - f) monthly compounding for 45 years
11. Find the future value of an investment, FV , for each investment, PV . Use the compound interest formula, $FV = PV(1 + i)^n$, where i is the interest rate per compounding period and n is the number of compounding periods over the life of the investment.
- a) \$2000 at 3% compounded annually for seven years
 - b) \$1000 at 9% compounded monthly for three years
 - c) \$500 at 4.8% compounded quarterly for one year
 - d) \$300 at 20% compounded daily for the month of September
12. Use a TVM Solver to evaluate each part of question 11.

Chapter Problem

Rhys is 16 years old and in grade 11. Like many teens, he goes to school, has a part-time job, and has other things he likes to do. He also has goals: to buy a car or a truck, to buy golf clubs, to go skydiving, to travel, to graduate high school. Fortunately, his parents taught him early about money—what it is, how it works, and how to use it to achieve his goals. In this chapter, you will see how some of these goals can be achieved.



9.1

Savings Alternatives



In order to achieve your financial goals, you will need to save money. If you are thinking of buying a car, saving for college, or getting your own place, understanding savings alternatives will help you get closer to meeting your goals.

Investigate

Tools

- computers with Internet access

Optional

- literature from financial institutions about available accounts

Literacy Connect

A service charge is a fee charged by a financial institution for providing services.

A transaction is any activity that occurs within an account. For example, paying a bill or withdrawing cash.

Savings Accounts From Financial Institutions

You can save your money in many places.

- Copy and complete the table to compare the interest rate earned on \$1000 deposited in savings accounts from three financial institutions (for example, a bank, a credit union, and an Internet bank).

Financial Institution	Name of Savings Account	Interest Rate the Account Pays	Interest Earned in One Month on \$1000

- Some institutions charge fees to customers for providing different services while some do not. Research the fees, if any, associated with each account chosen in question 1. Then copy and complete the table.

Financial Institution	Name of Savings Account	Fees for Routine Transactions	Fees for Making 10 Regular Transactions During One Month

- Which account, from which institution, do you prefer? Explain why.

Example 1

Interest Earned on Accounts

Jodie received \$530 from family members for her birthday. She plans to buy a car in the near future and she is putting all of the birthday money toward the purchase. On June 1, she opened a savings account and deposited the \$530. The account pays an annual interest rate of 0.5% compounded daily.

- How much interest will Jodie earn in one month (30 days)?
- How much interest will Jodie earn in six months (183 days)?

Solution

Method 1: Use the Compound Interest Formula

- a) FV represents the future value of the investment, PV represents the present value of the investment, i represents the interest rate per period, and n represents the number of compounding periods.

$$PV = 530, n = 30, i = \frac{0.005}{365}$$

$$\begin{aligned} FV &= PV(1 + i)^n \\ &= 530\left(1 + \frac{0.005}{365}\right)^{30} \\ &= 530.22 \end{aligned}$$

Remember, i is the annual interest rate, r , divided by the number of compounding periods in a year.

$$\begin{aligned} \text{Interest earned} &= 530.22 - 530.00 \\ &= 0.22 \end{aligned}$$

In one month (30 days), Jodie earned 22¢ in interest.

b) $FV = PV(1 + i)^n$

$$\begin{aligned} &= 530\left(1 + \frac{0.005}{365}\right)^{183} \\ &= 531.33 \end{aligned}$$

$$\begin{aligned} \text{Interest earned} &= 531.33 - 530.00 \\ &= 1.33 \end{aligned}$$

In six months, Jodie earned \$1.33 in interest.

Method 2: Use a TVM Solver

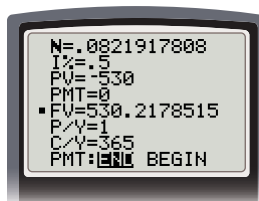
- a) Determine the value of each TVM Solver variable.

$$PV = 530, n = 30, i = \frac{0.005}{365}$$



Remember that N is measured in years. 30 days is $\frac{30}{365}$ of a year. $I\%$ is the **annual** interest rate. PV is a negative value because the investment is money that you cannot use right now.

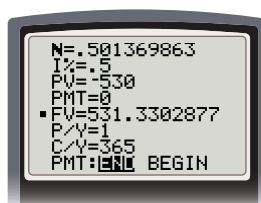
Move the cursor to FV and press **ALPHA** then **ENTER** to determine the future value of Jodie's investment.



Interest earned = $\$530.22 - \530.00 Interest earned is the difference between the future and present values.

Jodie earned 22¢ in interest in one month.

- b) To determine the value of her investment in six months, change N to $\frac{183}{365}$ and determine FV.



There are 183 days in a six-month period.

Interest earned = $\$531.33 - \530.00
In six months, Jodie earned \$1.33 in interest.

Example 1



Determine Service Charges

Hun's bank charges \$5.95 for up to 10 transactions per month plus 75¢ for each additional transaction. In November, he made eight transactions; in December, he made 23 transactions. Determine the service charges deducted from Hun's account balance each month.

Solution

In November, Hun is charged \$5.95, even though he made only eight transactions.

For December, Hun is charged \$5.95 for the first 10 transactions and 75¢ for each of the remaining 13 transactions.

$$\begin{aligned} \text{Service charge} &= 5.95 + (13 \times 0.75) \\ &= 15.70 \end{aligned}$$

Hun is charged \$15.70 for December's transactions.

Key Concepts

- Financial institutions pay interest when you deposit your money into one of their accounts.
- Often, financial institutions charge fees or service charges for providing you with certain services.

Discuss the Concepts

- D1.** How could having your money in a bank account actually cost you money?
- D2.** What are some ways to keep service charges to a minimum?

Practise

A

For help with questions 1 and 2, refer to Example 1.

1. Use a TVM Solver to determine the future value of each amount deposited into a daily interest savings account.
 - a) \$2000 in an account that pays 1% interest per year for the month of April
 - b) \$3000 in an account that pays 1.5% interest per year for the months of July and August
 - c) \$1500 in an account that pays 0.25% interest per year for the months of October, November, and December
 - d) \$400 in an account for one year that pays 2.5% interest per year
 - e) \$500 in an account for one day that pays 1.75% interest per year
 - f) \$2500 in an account for one week that pays 1% interest per year
2. Evaluate each part of question 1 using the compound interest formula.

For help with questions 3 and 4, refer to Example 2.

3. A bank charges \$6.95 for up to 12 transactions per month plus 75¢ for each additional transaction. Determine the fee for each number of transactions during the month of March.

a) 14	b) 9	c) 21
d) 0	e) 18	f) 26
4. Another financial institution charges \$9.95 for the first 20 transactions per month plus \$0.95 for each additional transaction. Determine the service charge for each number of transactions from question 3.

5. Use your answers from questions 3 and 4. Given your current banking needs, which account would serve you better? Explain.

Apply B

6. Sabbi has \$600 in a savings account. This account pays 3.25% interest per year, compounded daily. His financial institution does not charge a fee for transactions on his account. Use a TVM Solver.
- How much interest will Sabbi earn in the month of April?
 - How much interest will he earn in one year?
 - Explain why the answer to part b) is not equal to the answer to part a) multiplied by 12.

For questions 7 to 9, refer to the table.

Banking Option 1	Banking Option 2	Banking Option 3
\$9.75 per month for the first 10 transactions; \$1.25 for each additional transaction	\$14.75 per month for the first 25 transactions; \$1.25 for each additional transaction	\$24.95 per month for an unlimited number of transactions

Literacy Connect

An insurance premium is the amount charged by an insurance company to insure a driver and vehicle.

7. In a typical month, Jack uses an automated bank machine (ABM) twice a week to withdraw cash from his chequing account. Each month, his car payment and his car insurance premium are automatically deducted from his account.
- How many transactions does Jack make in a typical month?
 - Which banking option might be best for Jack?
 - Calculate the total cost and the cost per transaction for each option.
 - Suggest a way that Jack could reduce the banking fees that he pays each month.

Literacy Connect

Bi-weekly means occurring every two weeks.
Bi-monthly means occurring every two months.
Semi-monthly means occurring twice a month.

8. Alexa is paid bi-weekly by cheque. She usually uses her debit card two or three times per week.
- Which banking option might be best for Alexa? Explain.
 - On October 1, when she went on-line to do her banking, Alexa noticed bank charge debits from her account by her bank for \$9.75 and \$7.50. Which banking option does Alexa currently use? Explain.
 - How many transactions were made in Alexa's account last month? Explain.
 - Calculate the total cost and the cost per transaction for Alexa last month.



9. Ling rarely carries cash. She prefers to use her debit card. She is paid weekly. Her pay is automatically deposited into her chequing account. Ling uses her debit card for everything from buying groceries to eating in restaurants. Ling estimates that she uses her debit card 15 times per week. She writes cheques occasionally.
- In a typical month, estimate the number of transactions Ling makes.
 - Which banking option do you think that she uses? Why?
 - Calculate the cost per transaction using your answer to part a).
 - A friend suggests that Ling use a credit card for all her purchases instead of her debit card. Explain why you think her friend made this suggestion.

Chapter Problem

10. When Rhys was 15 years old, he got his first part-time job at a coffee shop. At that time he opened two savings accounts. He learned to “pay himself first.” That is, after his pay was deposited into his first savings account, Rhys transferred 10% of his pay to his second account. This account was for long-term savings and investments.
- His first paycheque amount was \$312.73. How much money did he transfer to his second account?
 - How much remained in his first account?
 - The second account paid 3.5% annual interest with daily compounding and had no service charges. Use a TVM Solver to calculate how much interest he would earn in 14 days.
 - After two weeks, Rhys received his second paycheque for \$286.91. How much did he transfer to his second account?
 - How much money was now in his second account? (Assume that the interest earned in part c) had NOT yet been deposited into his account by the bank.)
 - How much interest would he earn in the next 14 days?
 - Estimate the total amount Rhys would have saved in this account in one year.

Extend C

11. Rhys’s father says that calling a savings account a “savings account” is misleading. It should really be called a “spending account.” He says that if you really want to save you should get your money out of a bank account and invest it. What does Rhys’s father mean by this statement? Use what you have learned in this section and your own research to help you explain.

9.2

Investment Alternatives



Mr. and Mrs. Johnson are concerned with their children's future. They initiate this discussion:

“OK, you're going to school, you're working, and you've got some money now. One day—I know, it's a long time from now—you may want to buy a house, go on a cruise, or retire with a million dollars, no, two million dollars. How can you create financial independence for yourself and your kids? Yes, I said your kids!”

How can learning about investments now help you in the future?

Investigate

Tools

- computers with Internet access

Optional

- printed materials from investment companies

mutual fund

- type of investment where people pool their money together to buy stocks, bonds, and other assets
- managed by an investment company that charges a fee

Mutual Funds

A **mutual fund** is one type of investment opportunity. To find information on a variety of mutual funds, go to www.mcgrawhill.ca/links/foundations11 and follow the links.

1. Choose four to six mutual funds. Look at the one-year rate of return for each fund.
 - a) Which fund has the greatest **growth rate**? What is the growth rate?
 - b) Which fund has the least growth rate? What is the growth rate?
 - c) Explain why investing in a mutual fund might be considered risky.
2. How do the five-year or 10-year rates of return compare for the same funds?
3. Use the compound interest formula or a TVM Solver to calculate the approximate value of \$1000 invested 10 years ago in these funds.

Example 1

growth rate

- the percent by which an investment increases (or decreases) in value over a given time

A One-Time Investment

A mutual fund has an average annual rate of return of 12.45%. The investment company charges 2% per year as a fee for managing the account. Suppose \$1000 is invested for three years. Calculate the approximate value of the investment, assuming annual compounding. The future value of the investment will be an approximation since all conditions of the investment may not be known.

Solution

Method 1: Use the Compound Interest Formula

$$PV = 1000, i = 0.1045, n = 3$$

$$\begin{aligned} FV &= PV(1 + i)^n \\ &= 1000(1 + 0.1045)^3 \\ &= 1000(1.1045)^3 \\ &= 1347.40 \end{aligned}$$

The actual rate of interest earned is 12.45% – 2%, or 10.45%. So $i = 0.1045$.

The value of the investment is approximately \$1347.40 after three years.

Method 2: Use a TVM Solver

$$PV = 1000, i = 0.1045, n = 3$$



I% is determined by the growth rate 12.45% minus the management fee of 2%.

Find the future value of the investment, FV .



The value of the investment is approximately \$1347.40 after three years.

Example 2

Literacy Connect

Risk is the uncertainty or the likelihood that an investment will decrease in value.

An Investment That Decreases in Value

Investing often carries an element of risk. Some investments increase in value while some decrease in value. A mutual fund has an average annual rate of return of -5.29% . If the investment company's fees for managing the account are 2% per year, calculate the approximate value of a $\$1000$ investment after two years, assuming annual compounding. The value of the investment will be an approximation since all conditions of the investment may not be known.



Solution

Method 1: Use the Compound Interest Formula

$$PV = 1000, i = 0.0729, n = 2$$

$$\begin{aligned} FV &= PV(1 + i)^n \\ &= 1000(1 - 0.0729)^2 \\ &= 1000(0.9271)^2 \\ &= 859.51 \end{aligned}$$

i is determined by converting $-5.29\% - 2\% = -7.29\%$ to a decimal, which is -0.0729 .

The value of the investment is approximately $\$859.51$ after two years.

Method 2: Use a TVM Solver

$$PV = 1000, i = 0.0729, n = 2$$



i is determined by the rate of return of -5.29% minus the management fee of 2% .

Find the future value of the investment, FV .



The value of the investment is approximately $\$859.51$ after two years.

Example 3

Registered Retirement Savings Plan (RRSP)

- an investment that is set up to provide income after retirement. Generally, you are allowed to put money into an RRSP and claim a deduction on your income tax in that year. Contributions accumulate interest tax-free. When the money is taken out of the RRSP, it is taxed as income.

Regular Investments

Many people set up an investment, such as a **Registered Retirement Savings Plan (RRSP)**, as a series of small, regular investments. Suppose you invest \$200 per month from age 16 until your retirement at age 65. Assume the investment averages a 7% annual rate of return, compounded monthly. How much money will you have upon retirement?

Solution

N is now the number of payments. Monthly payments for 49 years is $12 \times 49 = 588$.

PV is the starting value of the investment, which is zero.

PMT is the value of each payment (i.e., investment), which is \$200. It is negative since it is, for the time being, money out of your pocket.

FV is the future value of the investment. This is the variable that you solve for.

P/Y is the number of payments per year, which, in this case, is 12.

C/Y is the number of compounding periods per year, which, in this case, is 12.

PMT : END/BEGIN Set the payment to the END of each month.



Find the future value of the investment, FV .



The future value is \$1 013 844.75. By age 65, you will have an investment worth more than one million dollars!

Key Concepts

- All investments carry some level of risk. Generally, the greater the risk, the greater the potential return (or loss). Some investments increase in value while others lose value.
- One way to accumulate wealth is to invest regularly over a long period of time. This takes advantage of the power of compound interest.

Discuss the Concepts

- D1.** How can starting your investments when you are young benefit you when you are much older?
- D2.** Explain why some investments carry a degree of risk or uncertainty.
- D3.** How comfortable are you with financial risk? What risks do you take?

Practise

A

1. Express each percent as a decimal.

- | | | |
|----------|----------|----------|
| a) 6% | b) 8% | c) 10% |
| d) 0.5% | e) 3.25% | f) 4.9% |
| g) -2.6% | h) 5.95% | i) 5.06% |

2. Copy and complete the table.

	r (%)	Compounding Frequency	i
a)	9.0	monthly	
b)		quarterly	0.04
c)	-4.6	semi-annually	
d)	1.8		0.0045
e)	0.5	monthly	
f)	12.8		0.032

For help with questions 3 and 4, refer to Example 1.

3. Use the compound interest formula to determine the future value of each three-year investment. Assume interest is compounded annually and that each investment has a 2% management fee.
- \$1000 in a fund that averages 6.08% growth per year.
 - \$5000 in an investment that averages 18.42% growth per year.
 - \$2000 in a mutual fund that averages 2.27% growth per year.
4. Evaluate each part of question 3 using a TVM Solver.
5. Calculate the interest earned for each part of question 3.

For help with question 6, refer to Example 2.

6. One year ago, Jozef invested \$2500 in a mutual fund that decreased in value by 4.92%. The fund has a 1.5% management fee. Determine the value of Jozef's investment at the end of one year.

For help with question 7, refer to Example 3.

7. When Meghdad was 17, he began investing \$2000 per year in a no-fee investment that paid 3.8% interest per year, compounded monthly. Determine the value of Meghdad's investment after five years.

Apply

B

8. a) Hafeeza invested \$2000 in a mutual fund that increased in value in its first year by 1.92%. If there was a 2.5% management fee, determine the value of her investment after one year.
b) Hafeeza decided to leave her money in the same fund. The next year, the fund had a rate of return of 8.83%. Determine the value of her investment at the end of the second year.
c) Over a 10-year period, Hafeeza's original \$2000 investment averaged 7.3% growth. After subtracting the annual management fees, what was the value of her investment?

Literacy Connect

Guaranteed Investment Certificate (GIC)

- a type of investment sold to individuals by banks or trust companies
- usually, GICs pay interest at a fixed rate and cannot be cashed before a specified date

9. To learn about **GICs**, go to www.mcgrawhill.ca/links/foundations11 and follow the links.
- a) What does "GIC" mean? What is a GIC?
b) Is a GIC a high-risk or a low-risk investment? Explain.
c) Find the current annual interest rate paid for a 30-day GIC and calculate how much interest would be paid on a \$1000 investment.
10. Pietra invested \$1000 in a seven-year GIC that pays 4.10% annual interest compounded annually.
- a) Determine the value of the investment after one year.
b) Determine the value of the GIC after two years.
c) Express the future value of this investment as an exponential relation.
d) Use the relation in part c) to determine the value of the GIC at the end of seven years.
e) Graph the relation for the seven years.

Registered Education Savings Plan (RESP)

- an investment set up to save for a child's education. The income from the plan grows tax-free.

Chapter Problem



11. a) Discuss with a partner. In your opinion, is each of the following investments low-risk, medium-risk, or high-risk? Explain your thinking.
- opening a savings account
 - buying units of a mutual fund
 - buying shares in an oil company
 - buying a GIC from a bank
 - buying a hectare of land
 - investing in a friend's invention
 - buying shares in a bank
- b) From the list in part a), which investment might provide the greatest return in the shortest time? Which investment might provide the greatest loss in the shortest time?
12. Kyoko just turned 30 and gave birth to a baby girl. She knows that when her daughter finishes high school, a post-secondary education will cost much more than it does today. Kyoko plans to put \$10 per week into her daughter's **Registered Education Savings Plan (RESP)**. In addition, the federal government will contribute 20% of the investor's RESP contribution each year up to a maximum of \$400 per year.
- How much will Kyoko have invested by her daughter's first birthday?
 - If Kyoko's investment earns 3.85% interest compounded annually in the first year, how much interest will it earn?
 - How much will the federal government contribute to the fund?
 - How much money will be in the fund after one year?
13. After working at a coffee shop for 10 months, Rhys quit to accept a job at a grocery store that pays \$2/h more. His new job pays weekly and he is now able to save about \$25 per week. He buys his first GIC and is planning to make his first RRSP contribution in the near future.
- If Rhys does no other investing in his life other than \$25 per week from now until he is 65 years of age (a total of 49 years), how much money will he have if his investments average a 7% annual rate of return? Assume monthly compounding.
 - How much money will Rhys have invested over the 49 years?
 - How much interest will he have earned?
 - Rhys says that when he retires he will have "ten times more" than your answer to part a). If he continues to save 10% of his pay, explain why he might be correct.

14. Johanna bought an \$800, three-year GIC with a variable rate. In the first year, the GIC pays 3.85% annual interest. In the second year, it pays 4.05% annual interest, and in the third year it pays 4.2% annual interest. All interest is compounded monthly. Calculate the value of the GIC at the end of the three years.

Achievement Check

15. A simple method for calculating the percent that should be invested in moderate- to high-risk investments is the Age Balance Indicator (ABI).
$$\text{ABI} = 90 - \text{investor's age}$$
For example, a 20-year-old investor should invest no more than 70% ($90 - 20 = 70$) of the investment amount in riskier investments. A 50-year-old should invest no more than 40%.
- a) Using this method, the younger you are, the more risk you should take. Is this always true?
 - b) The ABI does not consider your current financial situation. What other factors are not considered?
 - c) Produce a scale for considering how risky the following investments are: blue chip stock, GICs, savings account at a bank, volatile stock, mutual funds, Canada Savings Bonds
 - d) “Generally, the higher the potential rate of return, the more risk an investor takes.” This statement, taken with the ABI, says that the younger you are, the higher your potential rate of return. Do you agree or disagree?

Extend

C

16. Keisha has \$1200 in a savings account. She is in grade 11. She is saving for her first year of college, which is two years away. While she wants her money to grow in value, she is not willing to risk having her savings lose value.
- a) Given the investment alternatives explored in this section, what investments would you suggest Keisha choose? Research current interest rates to support your decision.
 - b) Under your plan, determine the value of Keisha’s \$1200 after two years.
 - c) Keisha’s part-time job allows her to save \$250 per month. Determine the value of 24 months of Keisha’s savings if she uses the same investment plan that you chose in part a).
 - d) What is the total amount of money that Keisha will have after two years?

9.3

Manage Credit Cards



Good, bad, convenient, dangerous, safer than cash, scary, and expensive are common words that people use when describing their relationship with credit cards. All of the adjectives are right—depending on how the cards are used. In this section, you will begin to discover the good, the bad, and the ugly of using a credit card.

Investigate

Tools

- computers with Internet access

Optional

- printed materials about credit cards

Literacy Connect

An overdue balance is an amount that remains unpaid after the due date has passed.

Credit Cards

Choose two credit cards: one issued by a bank or other financial institution and one offered by a retailer (for example, an electronics store, a furniture retailer, or a gasoline retailer). Gather the following information about each card. Record your findings in a table or chart.

To learn about companies, their credit cards, and their policies, go to www.mcgrawhill.ca/links/foundations11 and follow the links.

1. Is there an annual fee for holding the card? If so, how much is the annual fee?
2. What annual interest rate is charged on an overdue balance?
3. How often is the interest compounded?
4. How many days after the monthly statement is issued is the payment due?
5. How much interest is charged if the balance is paid in full by the due date?
6. Are there any incentives or rewards associated with being a cardholder?


Example 1

Read the Fine Print

The information found on the back of Ahmad's monthly statement for a credit card issued by his bank is shown.

Literacy Connect

A grace period is a length of time, often days, during which no interest is charged.

 Crownbank	Credit Card STATEMENT																
Ahmad Mustafa 32 Cherry Lane Hometown, ON M7K 2B1	<table><tr><td>Account Number:</td><td>5446 xxxx xxxx 3230</td></tr><tr><td>Credit Limit:</td><td>\$5700</td></tr><tr><td>Daily ABM Cash Advance Limit:</td><td>\$1000</td></tr><tr><td>Annual Interest Rate:</td><td>14.9%</td></tr><tr><td>Annual Fee:</td><td>0</td></tr><tr><td>Grace Period:</td><td>21 days</td></tr><tr><td>Cards Issued:</td><td>1</td></tr><tr><td>Minimum Payment:</td><td>greater of \$10 or 3% of balance</td></tr></table>	Account Number:	5446 xxxx xxxx 3230	Credit Limit:	\$5700	Daily ABM Cash Advance Limit:	\$1000	Annual Interest Rate:	14.9%	Annual Fee:	0	Grace Period:	21 days	Cards Issued:	1	Minimum Payment:	greater of \$10 or 3% of balance
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Annual Interest Rate:	14.9%																
Annual Fee:	0																
Grace Period:	21 days																
Cards Issued:	1																
Minimum Payment:	greater of \$10 or 3% of balance																

- A statement is issued to Ahmad on the 8th of each month. On what date will the payment be due?
- On September's statement, Ahmad has a balance of \$86.36. Determine his minimum payment.
- On October's statement, Ahmad has a balance of \$462.18. Determine his minimum payment.
- If it takes three days to process his payment, what is the latest date that Ahmad can pay October's bill and not be overdue?
- If interest is calculated and compounded daily, determine the daily interest rate. Round your answer to 4 decimal places.
- Calculate the interest charged on October's bill if it is paid in full five days after the due date. Ahmad paid his September bill in full. He made one new purchase for \$462.18 on September 15.

Solution

- The payment will be due on the 29th of each month.
- 3% of \$86.36 is \$2.59, so the minimum payment due is \$10.00.
- 3% of \$462.18 is \$13.87, so the minimum payment due is \$13.87.
- The bill should be paid no later than the 26th of the month.

If the balance is paid in full on or before the due date, no interest is charged.

e) $\frac{14.9\%}{365} = 0.0408\%$.

Remember that i equals the annual rate, 14.9%, divided by the number of compounding periods in one year, 365.

f) Unpaid balances are charged interest from the date of the purchase.

The interest charged during the month may exceed the minimum payment, which means that it will take a very, very long time to pay off the debt if only the minimum payment is made each month.

Method 1: Use the Compound Interest Formula

$$PV = 462.18, i = 0.000408, n = 49$$

$$\begin{aligned} FV &= PV(1 + i)^n \\ &= 462.18(1 + 0.000408)^{49} \\ &= 471.51 \end{aligned}$$

The bill was paid on November 3. This is five days after October 29 and 49 days after the purchase date.

$$\begin{aligned} \text{Interest charged} &= 471.51 - 462.18 \\ &= 9.33 \end{aligned}$$

Ahmad was charged \$9.33 in interest.

Method 2: Use a TVM Solver

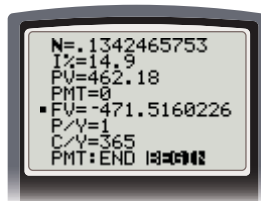


The interest is charged for 49 days, which is $\frac{49}{365}$ of a year.

$$\text{So, } N = \frac{49}{365}, \text{ or } 0.1342465753.$$

You are solving for the value of FV.

Find the value of FV.



Move the cursor to the line FV=0 and press **ALPHA** then **ENTER**.

$$\begin{aligned} \text{Interest charged} &= 471.52 - 462.18 \\ &= 9.34 \end{aligned}$$

Ahmad was charged \$9.34 in interest.

Key Concepts

- Interest rates charged on credit card accounts are often much greater than the interest rates paid on savings accounts.
- Most credit cards compound interest daily on overdue accounts.
- If the balance is paid in full, then no interest is charged during the grace period.
- Some credit card companies offer incentives or rewards to customers for using the card to make purchases.

Discuss the Concepts

- D1.** Why do you think credit card companies charge high interest rates?
- D2.** How can the smart use of a credit card give someone “free credit” for life?

Practise


A

For help with questions 1 to 9, refer to Example 1.

1. Calculate the daily interest rate for each credit card annual interest rate. Express each answer as a percent rounded to 4 decimal places.
 - a) 17.9%
 - b) 28.8%
 - c) 13.9%
 - d) 3.9%
2. Use each of the daily interest rates from question 1 and the compound interest formula to determine the future value of an overdue credit card balance of \$1500 if interest is charged for 55 days.
3. Refer to question 2. Use a TVM Solver to answer each part of the question.
4. A statement for Kendra’s credit card was issued on April 15. Her account offers a grace period of 14 days. It usually takes 3 days for transactions to be processed. Kendra paid her bill on May 19. For how many days will she be charged interest?

Apply B

To answer questions 5 to 8, refer to Mia's credit card statement.

	Credit Card STATEMENT	
	Mia Bailey 17 Seaside Road Hometown, ON M3R 4C2	Credit Limit: \$8000 Annual Interest Rate: 12.9% Daily ABM Cash Advance Limit: \$500 Annual Interest Rate on Cash Advances: 18.9%
	Annual Fee: \$12 Grace Period: 21 days Minimum Payment: greater of \$10 or 3% of balance	
Interest is compounded daily		


5. A statement is issued to Mia on the 18th of each month.
 - a) What is the due date for the January 18 statement?
 - b) What is the due date for the February 18 statement?
6. Determine the minimum monthly payment for each.
 - a) Mia's December statement has a balance of \$289.40.
 - b) After using her credit card for all her holiday shopping, Mia's January statement has a balance of \$1220.74.

Literacy Connect

7. Explain one advantage and one disadvantage of Mia using her credit card to make all her holiday purchases.
8. Determine each daily interest rate charged on Mia's credit card. Express each answer as a percent and as a decimal rounded to six decimal places.
 - a) on cash advances
 - b) on credit card purchases

Chapter Problem

9. Rhys decides to apply for a credit card. His parents have to co-sign his application—that is, they agree to pay the outstanding balance if Rhys cannot or will not pay. On the application, they indicate that they want a spending limit of \$250. A portion of Rhys's first credit card statement is shown.

	Credit Card STATEMENT	
	STATEMENT FROM June 25 to July 26	
7/2	The Jeans Factory	48.00
7/7	T-shirt Haus	22.75
7/22	Soccer Unlimited	28.49
Previous Balance: Payments: Overdue Balance: Interest Charged: New Purchases:	0.00 0.00 0.00 0.00 0.00	New Balance: Minimum Due: Statement Date: July 26 Annual Interest Rate: 16.9% Available Credit:

- a) Calculate the amount of the new purchases made this month, and Rhys's new balance.
- b) If the minimum payment is \$10 or 3% of the balance, whichever is greater, determine Rhys's minimum payment.
- c) Determine the due date if payment is due 21 days after the statement date.
- d) What are the likely reasons that Rhys's parents asked for a \$250 credit limit?
- e) Calculate the daily interest rate. Express the answer as a percent and as a decimal rounded to five decimal places.
- f) Four days after he received the statement, Rhys went on-line and paid the balance. How much interest was he charged?

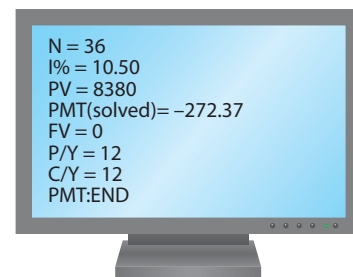
Extend



10. Terrell has four credit cards: two issued by banks, one from a gasoline retailer, and one from a furniture store. The table shows his current credit situation.

Credit Card	Balance as of July 1	Annual Interest Rate	Minimum Payment Due
Bank 1	\$2527	13.9%	3% of balance
Bank 2	\$4318	14.9%	3% of balance
Gasoline Retailer	\$227	18.9%	\$20
Furniture Retailer	\$1308	28.8%	\$100

- a) Calculate Terrell's current debt.
- b) Calculate the total of the minimum payments that he must make this month.
- c) Assuming no other purchases are made, calculate the total interest charged on each account for 30 days. All the credit cards compound interest daily.
- d) Which credit card should Terrell pay in full first? Why?
- e) A credit counsellor advises Terrell to get a personal loan from his bank, pay off all of the credit cards, and then destroy the cards. Using his TVM Solver, he shows the results of his analysis to Terrell.



Use the words *monthly payment*, *debt*, and *interest* to explain the meaning of the numbers on the screen in terms of Terrell's monthly payment and how long it will take him to get out of debt.

9.4

Obtain a Vehicle



Used cars are less expensive to buy and less expensive to insure. New cars can be bought or leased. Depending on your situation, there are advantages to buying or leasing. The initial costs for obtaining a new vehicle are usually greater than those for obtaining a used vehicle.

Investigate

Tools

- classified advertisements in a newspaper
- used vehicle magazines
- telephone
- telephone book

Optional

- computers with Internet access

Literacy Connect

A certified vehicle is one that has passed a safety inspection by a mechanic.

A lease is a long-term rental agreement.

Research Vehicles for Sale

Choose one or two vehicles that interest you. Your choices should be realistic for your current financial situation.

1. Use advertisements, phone a local car dealership, or use the Internet to obtain the cost of buying and/or leasing a new vehicle.

Ask:

- What features are included? (For example, air conditioning or a CD player.)
- Are other options available?
- Is the price quoted with or without taxes?
- Are there other costs involved in obtaining the vehicle?

2. Use classified advertisements in your local newspaper, local used vehicle magazines, or the Internet to find used models of the vehicles you chose.

Ask:

- Is the vehicle certified?
- Has the vehicle ever been involved in a collision? If so, ask for details.
- How many kilometres has the car been driven?
- Is the price negotiable?

Example 1

Buy a New Vehicle

A local dealership is selling a new compact car for \$17 995 plus taxes. The dealership offers financing at 4.9% annual interest, compounded monthly, over four years. You have saved \$3000 for a down payment. You will finance the rest. What will be your monthly payment?

Literacy Connect

A down payment is the initial payment due upon the purchase or lease of an item.

Solution

First, you need to calculate the after-tax cost of the car. Car dealerships charge PST and GST.

On some calculators, you can enter
17 995 $+$ 14 $\%$ \times 17 995 $=$.

On all calculators, you can enter 17 995 \times 1.14 $=$.

The total cost of the vehicle is \$20 514.30.

The amount to be financed is the total cost less the down payment.

$$\begin{aligned}\text{Financed amount} &= \$20\,514.30 - \$3000 \\ &= \$17\,514.30\end{aligned}$$

Use a TVM Solver or an on-line calculator to determine the monthly payment.



N is the number of payments. Monthly payments for four years is 48 payments.

PV is the amount that was borrowed.

PMT is the value of each payment. It is a negative amount since it is money you cannot use right now.

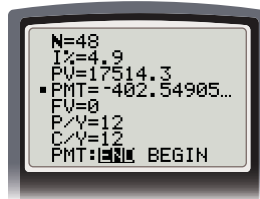
FV is the future value of the loan after four years have passed, i.e., 0.

P/Y is the number of payments per year. In this case it is 12.

C/Y is the number of compounding periods per year. In this case it is 12.

PMT: END/BEGIN Set the payment to the END of each month.

Find the value of PMT.



The monthly payment will be \$402.55.

Example 2

Total Cost of a Vehicle

Refer to Example 1.

- Determine the total amount paid for the vehicle.
- Calculate the total interest paid.

Solution

- Forty-eight payments of \$402.55 need to be made to repay the loan of \$17 514.30.

$$\begin{aligned}\text{Loan repayment} &= 402.55 \times 48 \\ &= 19\,322.40\end{aligned}$$

$$\begin{aligned}\text{Total cost of car} &= \text{loan repayment} + \text{down payment} \\ &= 19\,322.40 + 3000 \\ &= 22\,322.40\end{aligned}$$

The total amount paid for the vehicle is \$22 322.40.

- Interest paid = total amount paid for the car – original price of car
 $= 22\,322.40 - 20\,514.30$
 $= 1808.10$

The total interest paid is \$1808.10.

Example 3

Lease a New Vehicle

Leasing a vehicle is basically entering into a long-term rental agreement. You drive the car but you do not own it. To lease a new car selling for \$24 000, a customer agrees to pay a \$1000 down payment and to make 48 monthly payments of \$369.

- Calculate the total cost of leasing the vehicle.
- Calculate the average cost per month over the life of the lease.

Solution

- Total cost = down payment + monthly payments
 $= 1000 + (48 \times 369)$
 $= 1000 + 17\,712$
 $= 18\,712$

The total cost of leasing the car for four years will be \$18 712.

- $18\,712 \div 48 = 389.83$

The average cost per month, over the life of the lease, is \$389.83.

Example 4

Buy a Used Vehicle

A used car will cost much less than a new model of the same car. Sometimes, a used car loan will have a shorter payback period than a loan for a new car. A car is advertised for sale in a local newspaper for \$4500.

- Determine the total cost of the vehicle with 8% PST.
- Use a TVM Solver or an on-line calculator to determine the monthly payment for a \$5000 loan at 8% interest, compounded monthly, for two years.

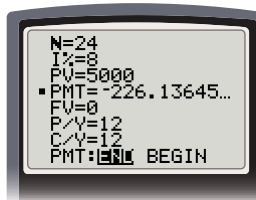
Solution

a) Cost with tax = 4500×1.08
= 4860

The total cost of the vehicle, including PST, is \$4860.

Most used vehicles sold privately are subject only to PST.

b)



N is the number of payments.
Monthly payments for two years equals 24 payments.

The monthly payment will be \$226.14.

Key Concepts

- Buying or leasing a new vehicle is a big expenditure.
- Often, buying or leasing a new vehicle involves making a down payment.
- A good used car will cost a lot less than a new model of the same car.
- Leasing a new vehicle is basically entering into a long-term rental agreement.

Discuss the Concepts

- D1.** Talk to 10 students who own vehicles. Where did they get the vehicle (for example, from a parent, a friend, or through a private sale)? Is there a pattern in their answers?
- D2.** Explain why buying a car is not an investment.
- D3.** Rolly says, "I think I'll get a motorcycle. It's way cheaper, and I mean more than just the payments." What do you think Rolly means by his last statement?

For help with questions 1 and 2, refer to Example 1.

1. Calculate the after-tax cost of the following new and used vehicles available at a local car dealership.
 - a) a two-year-old minivan selling for \$22 995
 - b) a new sports car selling for \$36 250
 - c) a five-year-old sports utility vehicle (SUV) selling for \$17 999
 - d) a new compact car selling for \$12 995

Use this information for questions 2, 3, and 4.

Three cases of financing a used car are shown. Assume interest is compounded monthly.

- a) \$4000 borrowed for three years at 9% interest
- b) \$8500 borrowed for four years at 8.5% interest
- c) \$15 000 borrowed for five years at 9.25% interest

2. Use a TVM Solver or an on-line calculator to determine the monthly payments for each case.

For help with questions 3 and 4, refer to Example 2.

3. Calculate the total amount paid to the financial institution for each loan.
4. Calculate the total amount of interest paid over the life of each loan.

For help with question 5, refer to Example 3.

5. Calculate the total cost of each new car lease.
 - a) a car worth \$18 000 leasing for \$1000 down plus 36 payments of \$299
 - b) a minivan worth \$23 500 leasing for 48 payments of \$399
 - c) a luxury sedan worth \$72 000 leasing for a \$7500 down payment and 39 monthly payments of \$899

For help with question 6, refer to Example 4.

6. Vehicles purchased from a private seller (e.g., your neighbour) are not subject to the goods and services tax (GST). Only the provincial sales tax (PST) is charged when you change the vehicle's ownership papers. Calculate the PST due on each used car purchase.
 - a) a nine-year-old compact car bought from a friend for \$2500
 - b) a 12-year-old mid-size diesel car sold for \$4200
 - c) a 33-year-old camper van with an appraised value of \$300

Apply**B**

7. Explain why three identical model vehicles of the same age, found in the classified advertisements of a local newspaper, could have three very different prices.

Literacy Connect

8. These words or abbreviations are commonly seen in advertisements for vehicles.

5-spd	auto	obo	PS	170K
AWD	e-test	PB	cert	PW
A/C	FWD	loaded	PL or PDL	'00

Work in a small group. Discuss the meaning of each term. Research the meanings of any terms you do not recognize.

9. To lease a new car worth \$30 000, a customer agrees to pay a \$1000 down payment and 48 payments of \$525.
- Calculate the total cost of leasing the vehicle.
 - Calculate the average cost per month over the life of the lease.
 - After 48 months, the customer returns the vehicle to the dealership. What options do you think that the customer has at this point?

Chapter Problem

10. Rhys cannot believe the cost for a young male driver to insure his own car, even with a driver-training certificate. Rhys would like to buy a five- or six-year-old pickup truck in two years when he graduates from high school. He has seen the model he wants on a used-vehicle Web site for \$6500.
- Calculate the after-tax cost of a vehicle worth \$6500 purchased from a private seller.
 - Rhys decides to open a third savings account. This account has no user fees and pays 3.25% annual interest, compounded daily. How much money will Rhys need to save each week starting now to pay cash for the truck in two years?
 - Until Rhys buys his truck, he agrees to pay his parents the increase in the insurance premium when he is added to the family policy. He will also pay for all of the fuel that he uses. The increase to the family's insurance premium is \$760 per year. Rhys estimates that he will spend about \$20 per week on fuel. Approximately how much will it cost him to drive his parents' car each month?

11. Leasing an imported sports car requires a \$5000 down payment and monthly payments of \$695 for four years.
 - a) Determine the total amount spent to lease the car.
 - b) Calculate the average cost per month over the life of the lease.

Achievement Check

12. A band decides to buy a mini-van to transport their equipment. They find a van at a used vehicle dealership. The dealer paid \$5000 for the van and made some needed repairs: he spent \$185 for new brakes, replaced all four tires at \$125 per tire, replaced a window for \$600, and cleaned the van.
 - a) If the dealer wants to make a 10% profit on the sale, how much will he charge for the van?
 - b) A parent of a band member agrees to lend them money to pay for the van. If she asks for no interest and wants the loan paid back in one year, how much is the monthly payment?
 - c) If you are advising the group about the cost of buying the van, what other operating expenses should you point out?



Extend

C

13. Research the costs of leasing and buying a new vehicle of your choice.
 - a) Compare the initial costs, including the down payments.
 - b) Compare the monthly payments.
 - c) Compare the total cost over the life of the lease or loan.
 - d) Describe two advantages to leasing a new car over buying the car.
 - e) Describe two advantages to buying a new car over leasing the car.
14. A local car dealer wants to sell all his current new vehicles before the next year's models are available. He is offering 0.9% financing for 48 months. A small pickup truck is selling for a pre-tax price of \$22 180 plus transport and PDI charges of \$1100.
 - a) Research the meaning of PDI. Explain what it means.
 - b) Determine the cost of the truck, after taxes, with transport and PDI charges included.
 - c) Determine the monthly payment for a customer who pays a \$5000 down payment and finances the rest.
 - d) Determine the total amount spent to purchase the vehicle in part c).
 - e) As vehicles get older, they lose their value. If the truck loses 15% of its value each year, how much will the truck be worth once the loan is paid in full?

9.5

Operate a Vehicle



Once you obtain a vehicle of your own, the expenses really start to mount. You are now the principal driver so your insurance costs go up—sometimes they go way up! You drive more so your fuel costs go up. In addition, you are making payments on something that is losing value every day that you drive it.

Investigate 1

fixed cost

- an expense that remains the same from one month to the next

variable costs

- an expense that varies in amount or frequency

Operating Expenses

Brianna is in her first year of college. She lives about 10 km from the campus. She just purchased her first car: a five-year-old compact for \$5500. Brianna had some money saved for a down payment and borrowed \$4000 from her credit union.

With a partner, brainstorm the expenses that Brianna is likely to incur over the next 12 months and then estimate the costs. Separate the costs into **fixed costs** and **variable costs**.

Investigate 2

Tools

- computers with Internet access

Automobile Insurance

1. Find out about different insurance companies, the cost of car insurance, and the types of coverage you can purchase. Go to www.mcgrawhill.ca/links/foundations11 and follow the links. Complete an on-line quote for the purchase of a six-year-old compact car with a value of \$4000.
2. Make a list of the factors that affect the amount that someone pays to insure himself/herself and the vehicle. Given your situation right now, list the factors that are in your favour for lower insurance costs.

Example 1

Insure a Vehicle

Ralf is 19 and single, and he owns a seven-year-old mid-sized car. He called several insurance agents and the lowest quote he received was \$2620/year. There are two payment options: he can pay the insurance premium in full once a year, or he can make monthly payments of \$230.

- Calculate the annual cost if he chooses the monthly instalments.
- Calculate the difference between the two payment methods.
- Suggest reasons why Ralf might choose each option.

Solution

a) Total monthly payments = 230×12
 $= 2760$

If Ralf pays monthly, then after a year he will have paid \$2760 for insurance.

b) Difference between payments = $2760 - 2620$
 $= 140$

The difference between paying once and paying monthly is \$140.

- c) Choosing the one-time annual payment is less expensive in the long run but choosing the monthly payments allows Ralf to pay smaller amounts, which is more affordable.

Example 2

Literacy Connect

Fuel efficiency is a measure of how far a vehicle travels per unit of fuel. Common units of fuel efficiency are litres per 100 km (L/100 km) and miles per gallon (mpg).

Calculate Fuel Costs

DeVaughan's truck has a 76-L fuel tank and a fuel efficiency rating of 11.8 L/100 km.

- Explain what the fuel efficiency rating on DeVaughan's truck means.
- How far can DeVaughan's truck travel on one tank of fuel?
- How much fuel would his truck use on a 450-km trip?
- Explain how to determine the cost of the fuel for the trip in part c).

Solution

a) Under normal driving conditions, DeVaughan's truck will use approximately 11.8 L of gas to travel 100 km. A vehicle that uses less fuel to travel 100 km is more fuel-efficient.

$$\begin{aligned} \text{b) } \frac{11.8 \text{ L}}{100 \text{ km}} &= \frac{76 \text{ L}}{\blacksquare \text{ km}} && \text{Use a proportion.} \\ \blacksquare &= \frac{76 \times 100}{11.8} \\ \blacksquare &\doteq 644.07 \end{aligned}$$

DeVaughan's truck can travel approximately 644 km on one tank of gas.

$$\begin{aligned} \text{c) } \frac{11.8 \text{ L}}{100 \text{ km}} &= \frac{\blacksquare \text{ L}}{450 \text{ km}} \\ \blacksquare &= \frac{11.8 \times 450}{100} \\ \blacksquare &\doteq 53.1 \end{aligned}$$

DeVaughan's truck will use approximately 53 L of gas for the trip.

d) The cost of the fuel will be 53 times the current price of one litre of gas.

Example 3

Depreciation

A new mid-sized vehicle sells for \$21 135. Marizia researched used cars of the same model and found the following information.

Age of Vehicle (years)	Average Selling Price (\$)
1	16 000
2	12 750
3	11 000
4	9 800

depreciation

- the amount that the value of an item decreases over time

- Calculate the **depreciation** of the vehicle during the first year, in dollars.
- Calculate the depreciation after one year, as a percent of the new vehicle price.
- Calculate the depreciation after four years, as a percent of the new vehicle price.



Solution

$$\begin{aligned}\text{a) Depreciation} &= \text{new car price} - \text{value after one year} \\ &= 21\,135 - 16\,000 \\ &= 5135\end{aligned}$$

The vehicle depreciated by \$5135 in the first year.

$$\begin{aligned}\text{b) Percent depreciation} &= \frac{\text{actual depreciation}}{\text{new car price}} \times 100 \\ &= \frac{5135}{21\,135} \times 100 \\ &= 24.3\end{aligned}$$

$$5135 \div 21\,135 \% = 24.3$$

The vehicle depreciated about 24% after the first year.

$$\begin{aligned}\text{c) Actual depreciation} &= 21\,135 - 9800 \\ &= 11\,335\end{aligned}$$

$$\text{Percent depreciation} = \frac{11\,335}{21\,135} \times 100$$

$$11\,335 \div 21\,135 \% = 53.6$$

The vehicle depreciated about 54% after the fourth year.

Key Concepts

- Fixed costs are expenses that remain the same from one month to the next; variable costs are expenses that vary in their amount or their frequency.
- Depreciation is the amount by which a vehicle loses value over time.
- One of the major expenses for drivers is insurance. This is especially true for young drivers.

Discuss the Concepts

- D1.** Why do you think that young male drivers pay more, on average, for car insurance than young female drivers?
- D2.** Your neighbour owns a 1959 Ford Thunderbird, which is in mint condition and rare. Is this vehicle an expense or an asset? Explain.

For help with questions 1 and 2, refer to Example 1.

Use this information for questions 1 and 2.

Many insurance companies give customers a quote for the cost of insurance for one year. Most companies have payment plans.

Three cases are shown.

- a) Vic's insurance company quotes him an annual insurance cost of \$1948 or a payment plan of \$169 per month.
- b) Faith receives an annual insurance quote of \$466. The company offers her an option of quarterly instalments of \$118.
- c) Ramon is 19 and owns a sports car. His insurance company quotes him an annual insurance fee of \$3870 and offers him weekly instalments of \$74.42.

1. Calculate the difference between the annual fee and the total cost of the instalments in each case.
2. For each case, which payment option would you choose? Explain your choice.

For help with questions 3 and 4, refer to Example 2.

Use this information for questions 3 to 5.

	Vehicle	Tank Size (L)	Fuel Efficiency (L/100 km)
a)	motorcycle	14	1.4
b)	sub-compact car	30	5.9
c)	mid-sized sedan	55	7.8
d)	minivan	75	10.2
e)	full-sized van	92	13.5

3. Use the current price of 1 L of regular gasoline to calculate the cost to fill the tank of each vehicle.
4. Determine the distance that each vehicle can travel on one full tank of gas.
5. In the United States, fuel is sold in gallons. One US gallon is approximately equal to 3.785 L. Determine the number of US gallons required to fill the tank of each vehicle.

Literacy Connect

6. Categorize each automobile expense as either a fixed expense or a variable expense. Explain your choice.
- a) lease payment
 - b) parking fines
 - c) insurance
 - d) gasoline
 - e) depreciation
 - f) loan repayment
 - g) licence plate sticker
 - h) oil changes
 - i) bridge tolls
 - j) monthly parking permits

For help with questions 7 and 8, refer to Example 3.

7. Calculate the depreciation on each vehicle.
- a) A new car worth \$14 595 sells for \$12 259 one year later.
 - b) An SUV worth \$52 999 can be purchased for \$43 000 one year later.
 - c) A one-year-old minivan can be bought for \$18 500. New, it sold for \$22 950.
8. Refer to question 7. Calculate the first-year depreciation for each vehicle as a percent of its original selling price. Round your answer to the nearest percent.

Apply

B

9. Maurice spent about \$1200 on vehicle maintenance last year. This year he expects to pay 10% more on maintenance. How much should Maurice budget for maintenance this year?
10. Rather than measuring their vehicle's fuel efficiency in litres per 100 km, some drivers choose to use kilometres per litre—the distance the vehicle will travel using 1 L of fuel. Calculate the fuel consumption, in kilometres per litre in each case. Round your answers to one decimal place.
- a) When Rado filled his car's tank last week, it took 47.6 L. His trip odometer read 622 km.
 - b) On a weekend vacation, Sharlee used 32.8 L to travel 385 km.
 - c) Steve's diesel sedan travelled 1070 km on 54.6 L of diesel.

Chapter Problem

- 11.** A new model of the pickup truck Rhys wants costs \$24 500. On average, this model of truck depreciates 21% per year. Rhys plans to buy a truck in two years after he graduates from high school.
- Determine the value of a one-year-old model of this truck.
 - In two years, Rhys plans to save approximately \$6000 for a vehicle. What age of pickup truck should he look for? Show calculations to explain your answer.
- 12.** Many students who live in larger cities choose to purchase a scooter rather than a car. Research the costs of purchasing and operating (insurance, fuel consumption, depreciation, regular maintenance, and repairs) a 125-cc scooter.



Achievement Check

- 13.** The value of an SUV worth \$48 000 depreciates by 18% each year.
- Determine the value of the SUV after five years.
 - Express the depreciated value of the vehicle as an exponential relation. Graph this relation for the first five years.
 - Use the equation from part b) to determine the value of the SUV when it is 10 years old.
 - Explain why the purchase of an SUV should be considered to be an expense rather than an investment.

Extend



- 14.** During a vacation to South Carolina, Dylan kept these records.
The currency exchange rate was 1 US\$ = 1.18 CDN\$.

Date	Odometer Reading (km)	Distance (km)	Fuel Use (US gallons)	Fuel Use (L)	Fuel Costs (US\$)	Fuel Costs (CDN\$)	Unit Fuel Cost (CDN\$/L)	Fuel Efficiency (km/L)
Mar. 15	236 083	-----	-----	-----	-----	-----	-----	-----
Mar. 16	236 948		12.7		41.00			
Mar. 17	237 760		12.3		40.00			
Mar. 22	237 897		2.6		8.50			
Mar. 23	238 780		12.5		40.25			
Mar. 24	239 541		-----	42.9	-----	42.85		

- Copy and complete the table.
- How many kilometres did Dylan travel during his vacation?
- Calculate the average fuel consumption for the car in kilometres per litre and in litres per 100 km.
- Calculate the total amount Dylan spent on fuel in Canadian dollars.

**9.1 Savings Alternatives,
pages 462–467**

1. Mykela's bank charges her \$11.50 per month for the first 12 transactions made against her chequing account and \$1.25 for each subsequent transaction. Last month, she made 19 transactions.
 - a) Calculate Mykela's total bank fees last month.
 - b) Describe two things that Mykela could do to reduce her bank fees.
2. A daily interest savings account pays 0.25% interest per year when the balance in the account is under \$5000.
 - a) Use the compound interest formula to calculate the interest earned on a balance of \$425 in a 31-day month.
 - b) Use a TVM Solver to answer part a).
3. Calculate the interest earned on \$425 in a 31-day month in a daily interest savings account that pays 3.50% interest per year.

**9.2 Investment Alternatives,
pages 468–475**

4. Last year, Gail invested \$4000 in a mutual fund. The fund had a one-year rate of return of 14.38%. The investment company charges Gail a 2.25% management fee. Calculate the current value of Gail's investment assuming that interest is paid at the end of the year.

5. Mohammed purchased a five-year GIC that pays 3.65% annual interest compounded annually.
 - a) Express the future value of the GIC as an exponential relation.
 - b) Use this relation to calculate the future value of the GIC at the end of the five-year period if he invests \$2000.
 - c) Use a TVM Solver to answer part b).

**9.3 Manage Credit Cards,
pages 476–481**

6. Describe one advantage and one disadvantage of using a credit card.
7. Aiden had the brakes on his car repaired for \$344. He charged the debt to his credit card on February 10. When he received his statement, he noticed a due date of March 5. He forgot all about the bill until he received the next month's statement. He paid the bill on April 3. The bank charges 16.9% annual interest compounded daily. How much interest did Aiden's mistake cost him?

9.4 Obtain a Vehicle, pages 482–488

8. You purchase a vehicle from your neighbour. Which tax is charged on the purchase?
9. A four-year lease of a hybrid car requires a \$4500 down payment and monthly payments of \$399.
 - a) Determine the total cost of the lease.
 - b) Calculate the average cost per month over the life of the lease.

10. Use a TVM Solver to determine each monthly payment. Assume interest is compounded monthly.
- \$3000 borrowed for two years at 8.5% annual interest
 - \$10 200 borrowed for five years at 9% annual interest

9.5 Operate a Vehicle, pages 489–495

11. A luxury import dealership claims its vehicles retain 92% of their value each year.
- Determine the value of a three-year-old, two-door convertible that sells for \$36 000 when new.
 - Determine the value of a five-year-old, four-door sedan that sells for \$87 000 when new.
12. Use the current cost of one litre of regular gasoline.
- Calculate the cost of filling a 45-L, mid-sized car.
 - Calculate the cost of filling a motorcycle that has an 18-L gas tank.
 - If the motorcycle travels an average of 425 km on one tank of gas, calculate the fuel efficiency in litres per 100 km and in kilometres per litre.
13. Some luxury vehicles have low rates of depreciation: they “hold their value well.” The table shows the value of a new luxury sedan and the values of identical models of used vehicles up to three years old.

Age of Car (years)	Estimated Value (\$)
new	67 000
1	58 960
2	50 705
3	43 600

- Calculate the decrease in value of the car between the first and second years.
- Express your answer to part a) as a percent. Round your answer to the nearest percent.
- What percent of its original value has the car lost over three years? Round your answer to one decimal place.
- Use your answer to part c) to find the average percent rate of depreciation for the three years. Round your answer to one decimal place.
- Use your answer to part d) to express the average depreciation of this vehicle as an exponential relation.
- Use the relation in part e) to predict the value of the sedan after 10 years.

9

Practice Test

For questions 1 to 4, choose the best answer.

- Which annual interest rate will most likely be paid by a bank to customers with money in a savings account?
A 1.0% **B** 7.5%
C 16.9% **D** 28.8%
- Which annual interest rate will most likely be charged by a bank to customers with money owing on a credit card account?
A 1.0% **B** 7.5%
C 16.9% **D** 28.8%
- Which should not be considered an investment?
A buying a GIC
B buying a new car
C buying shares in an oil company
D buying a hectare of land
- Which term is used to describe the amount by which an item loses its value over time?
A asset
B investment
C depreciation
D deflation
- Suppose you invest \$100 per month from age 16 until your retirement at age 65 and that this investment averages a 6% rate of return, compounded monthly.
 - How much of your own money will you have invested over the 49 years?
 - Use a TVM Solver to calculate how much money you will have upon retirement.
 - How much interest will you earn?
- Janis is 30 years old and has been making regular RRSP contributions for five years. She has about \$14 000 in one mutual fund and about \$9000 in another. Last year, the first fund had a 9.72% growth rate, while the second fund experienced a -2.98% rate of return. She pays a yearly 2% management fee for each fund. Assuming that interest is paid at the end of the year, determine the total value of her RRSPs.
- Leasing a two-seater convertible requires a \$2500 down payment and monthly payments of \$339 for four years.
 - Determine the total amount spent to lease the car by the end of the lease.
 - Calculate the average cost per month over the term of the lease.
- An eight-cylinder van has an 80-L fuel tank and a fuel efficiency rating of 12.2 L/100 km.
 - Explain the fuel efficiency rating.
 - How far can the van travel on one tank of fuel?
 - How much fuel would the van use on a 425-km trip?

Chapter Problem Wrap-Up

Rhys wants to purchase a pickup truck sooner rather than later. Rhys still works about 25 h per week for \$10.25/h. His average net pay each week is \$250.

- Each payday, Rhys transfers 10% to his long-term savings account. On average, how much does he transfer to that account each week?
- Rhys transfers \$120 each week to his third account, which is used for saving for the truck. How much more money will he have after one year than if he transferred \$70 each week to this account? This account has no user fees and pays 3.25% annual interest, compounded daily.
- How much of each week's pay is left for Rhys to use as he wishes?



9. Use a chart to organize your work.

- For each category list the advantages and disadvantages of each option.

Savings:

- chequing account
- savings account

Making Purchases:

- credit card
- debit card

Obtaining a Vehicle:

- purchasing a new vehicle
- purchasing a used vehicle
- leasing a vehicle

- List the advantages and risks (low, moderate, high) of each investment option.

- GICs
- mutual funds
- RRSPs
- RESPs

- List the various expenses for obtaining and operating a vehicle. Classify each as a fixed cost or a variable cost. Provide an estimate of each cost.

Chapter 7: Exponents

1. Write as a single power, then evaluate.

- a) $6^3 \times 6^2$ b) $10^8 \div 10^4$
 c) $\left(\frac{1}{4}\right)^2 \times \left(\frac{1}{4}\right)^3$ d) $\frac{7^{11}}{7^9}$
 e) $(2^5)^2$ f) $[(-1)^3]^8$

2. Evaluate. Express your answers as whole numbers or fractions.

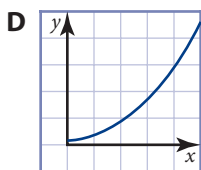
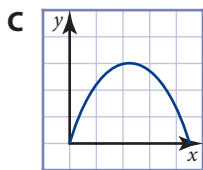
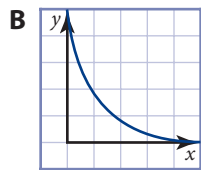
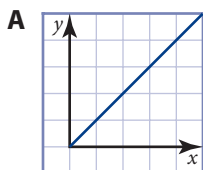
- a) $(-2)^{-2}$ b) 33^0 c) 5^{-3}
 d) 2^0 e) 10^{-3} f) 135^{-1}

3. Show that this relation is exponential.

x	0	1	2	3	4	5
y	$\frac{2}{3}$	2	6	18	54	162

4. Which model (linear, quadratic, or exponential) would best describe each situation? Why? Match each description with a graph.

- a) a runner slowing down by half her speed every 5 s
 b) the speed of an airplane speeding up by 10 km/h each second
 c) the path of a soccer ball when kicked
 d) the population of a town that increases by 2% every year



5. A fox population is declining by 1.8% per year. The population can be modelled using the formula $P = 210(0.982)^n$, where P is the population after n years.

- a) Use technology to graph this relation.
 b) What is the current fox population?
 c) What is the expected fox population in 8 years?

6. The half-life of plutonium-238 is 87 years.

The expression $1000\left(\frac{1}{2}\right)^{\frac{n}{87}}$ is used to calculate the number of milligrams remaining from the original 1 g after n years. Determine the number of milligrams remaining after

- a) 6 months b) 25 years c) 500 years

Chapter 8: Compound Interest

7. Use a table and a graph to determine the growth of a \$1500 investment for 10 years at

- a) 6% per year, simple interest
 b) 6% per year, compounded annually

8. Evaluate each expression. Use a scientific calculator and round your answer to two decimal places.

- a) $25(1.025)^3$ b) $300(1.0175)^{16}$
 c) $25(1.025)^{-3}$ d) $300(1.0175)^{-16}$

9. A \$3000 investment earns interest at 5% per year compounded quarterly for ten years.
- What is the value of the investment after one year? two years?
 - What is the interest earned in the fifth year?
10. Calculate the present value of each amount.
- \$4000 needed in two years, invested at 5.5% per year compounded annually
 - \$2000 needed in two years, invested at 4% per year compounded semi-annually
 - \$12 500 needed in seven years, invested at 7.5% per year compounded quarterly
11. What interest rate, compounded monthly, will triple the value of an investment after
- 12 years? b) 15 years? c) 20 years?
12. Compare the graphs of $A = 500(1.045)^n$ and $A = 500(1.06)^n$. How does changing the compound interest rate from 4.5% to 6% affect the shape of the graph? What does this mean in terms of the value of the investment?
- Chapter 9: Personal Finance**
13. A bank charges \$4.95 for up to ten transactions per month plus 50¢ for each additional transaction. Determine the fee for each number of transactions during the month of April.
- 0 b) 14 c) 2
 - 20 e) 17 f) 24
14. Celia bought a \$750, three-year GIC. In the first year, the GIC pays 3.5% annual interest. In the second year, it pays 3.8% annual interest. In the third year, it pays 5.25% annual interest. All interest is compounded monthly. Calculate the value of Celia's GIC after three years.
15. Use the compound interest formula to determine the future value of an overdue credit card balance of \$652 if interest is compounded daily for 25 days at
- 16.9% annual interest
 - 18.5% annual interest
16. Calculate the total cost of each new car lease.
- a \$16 500 car leasing for a \$1650 down payment and 36 payments of \$349
 - a \$24 990 sedan leasing for monthly payments of \$499 for 4 years
17. Veronica's truck has a 68-L fuel tank and a fuel efficiency rating of 12.5 L/100 km.
- How far can Veronica's truck travel on one tank of fuel?
 - How much fuel would her truck use driving from Ottawa to Sudbury (607 km)?
18. Calculate the depreciation on each vehicle.
- A \$13 895 new car sells for \$11 500 one year later.
 - A one-year-old SUV can be bought for \$44 400. New, it sold for \$51 699.

Task

Organise Your Personal Finances

Suppose you have finished college and find a job. You earn an annual salary of \$32 500.

1. Now that you are working, you have decided

to establish a credit rating. You know one way to do this is to apply for (and use) a credit card. Gather information on the credit cards offered by several financial institutions or retailers. Read the small print on the application forms carefully. Credit card interest rates, annual fees, and rewards or special offers vary. Choose one or two credit cards that offer the best deal for you. Explain your choice. Be sure to address these points:

- What is the minimum annual income needed to apply for the card (if any)?
- What are the initial and annual fees (if any)?
- What special features or rewards are offered with the card?
- What interest rate is charged on an unpaid balance?
- What happens if you pay the entire balance by the due date?
- Can you get a cash advance?
- How is the interest calculated on purchases? on cash advances?
- What are the security features of the card?
- What steps should you take if your card is lost or stolen?



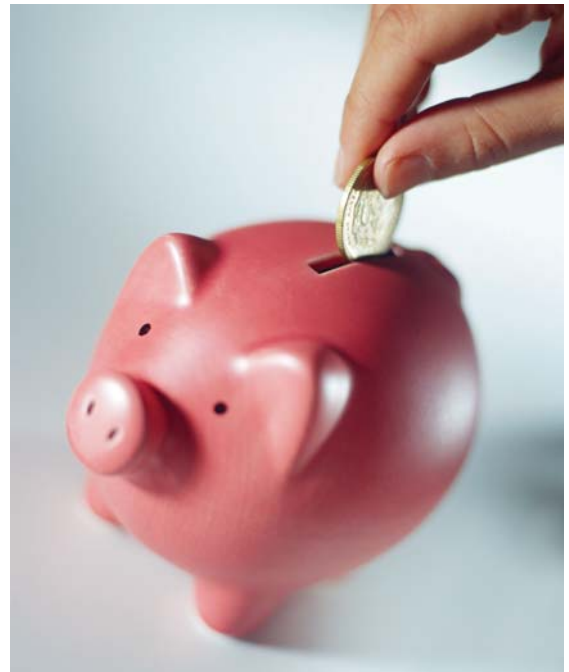
2. You have decided to buy or lease a vehicle. Experts recommend that you spend no more than 15% of your income on transportation. Choose a vehicle you like and could afford to own or lease given your income above.



- a) Did you choose to buy a new vehicle, lease a new vehicle, or buy a used vehicle? Explain.
- b) Calculate the cost of buying the vehicle or the conditions of the lease.
- c) Describe how you will finance the purchase or lease, including a payment schedule.
- d) If you choose leasing, how many kilometres can you drive per year without being charged an additional fee?
- e) Set up a monthly budget for the costs of operating the vehicle. Include your calculations and assumptions.

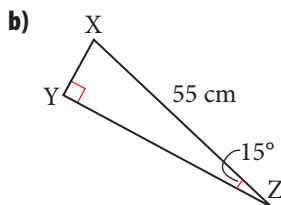
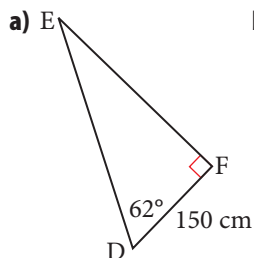
3. Now that you have a regular income, you have decided to “pay yourself first” by saving some of your money. Experts recommend saving a minimum of 10% of your income. Research a variety of investment options. Decide what goals you have for your savings. Choose three investments that would help you reach your goals. Explain your choices. Be sure to address these points:

- Are your goals long term or short term?
- For how many years will you be investing your money?
- What are the interest rates offered by the investments you have chosen?
- What is the doubling time for your investment choices?



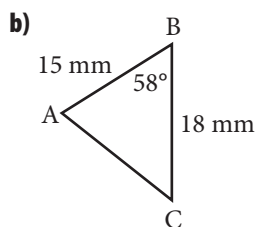
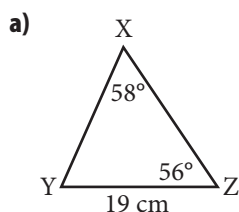
Chapter 1: Trigonometry

1. Solve each right triangle.

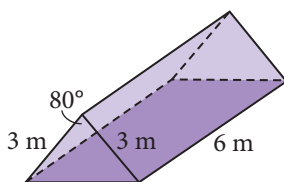


2. A fisherman wants to make sure his boat is anchored safely. He knows that the angle of depression the anchor cable makes with the horizontal when the boat is anchored should be less than 12° to be safe. The boat is 100 m above the seabed and the anchor cable is 440 m long. Is the fisherman safely anchored? Explain.

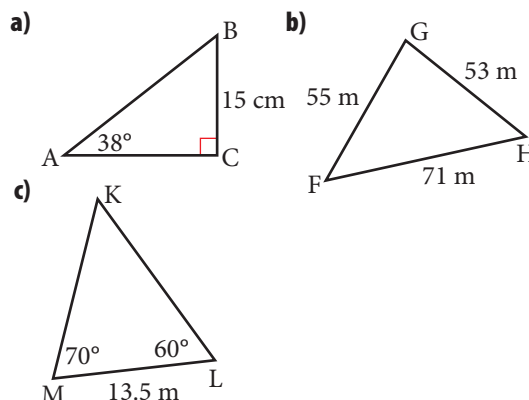
3. Solve each triangle.



4. A square tarpaulin with a side length of 6 m is secured by rope to create a makeshift tent. The sides of the tarpaulin meet at an angle of 80° as shown. If the sun is directly overhead, what area of shade does the tarpaulin provide, to the nearest square metre?



5. Which formula should you use to solve each triangle?



6. Refer to question 5. Solve the triangles.

Chapter 2: Probability

7. A customer service call centre manager decides to start a new training program for her employees if more than 2% of their callers in one day are dissatisfied with their service.
- a) The manager surveys 340 callers and finds that none are dissatisfied. She expects 20 more callers. Is it possible that the employees will have to take the training program? Explain your reasoning.
- b) If 360 callers were surveyed and 12 were dissatisfied, would the manager start the training program? Show your calculations to justify your answer.
- c) The manager decides to repeat the survey the next day. If 33 out of 1000 callers are unsatisfied, and the manager only surveys 100, do you expect she will find the same results as in part b)? Explain your reasoning.

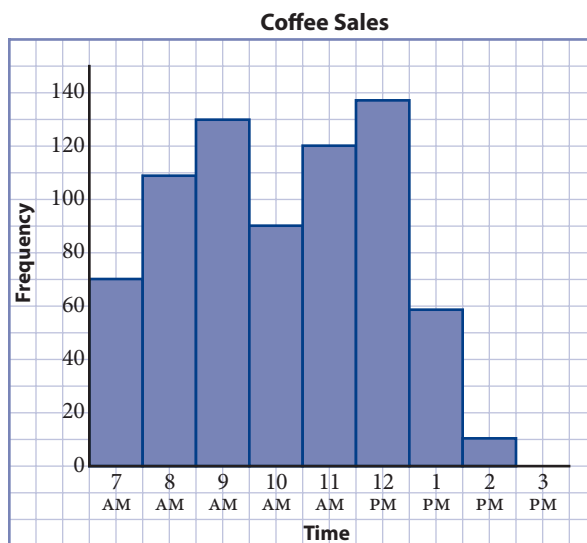
8. A hockey team is about to accept an award at a banquet. There are 18 players on the team; 2 goalies, 6 defencemen, and 10 forwards. A player is randomly selected to accept the award on behalf of the team.
- Find the probability that the player selected is a defenceman. Express your answer as a fraction in lowest terms.
 - Find the probability that the player selected is a goalie or a forward. Express your answer as a fraction in lowest terms. Suggest two possible methods for finding the probability.
 - If you know for certain that the player selected is not a goalie, find the probability that the player is a defenceman. Express your answer as a decimal.
9. When rolling two dice 40 times and adding the results, an even sum was obtained 10 times.
- What is the experimental probability of an even sum? Express your answer as a fraction in lowest terms.
 - How does this compare to the theoretical probability?
 - If the dice were rolled 40 more times, would an even sum turn up more frequently to “make up” for the previous rolls? Explain.
 - Explain how you could use the **randInt(1,2,40)** command to simulate this experiment. Why does this work?
10. A government study has shown that 8 out of 10 collisions involve drivers who are drowsy, using a cell phone, or distracted. In one year, 11% of licensed drivers are involved in a collision.
- What is the probability that a driver involved in a collision was not drowsy, using a cell phone, or distracted? Express your answer as a fraction in lowest terms.
 - What percent of drivers are not involved in a collision in one year?
 - In one year, what percent of all drivers are involved in a collision and are drowsy, using a cell phone, or distracted at the time?
 - In one year, what percent of all drivers will be involved in a collision and will not be drowsy, using a cell phone, or distracted at the time?

Chapter 3: One Variable Statistics

11. Choose the best sampling technique for each survey. Explain your choice.
- Ms. Donnelly wants to know what speakers would be the most interesting for career day.
 - A librarian wants to know who is the most popular author.
 - Byung wants to know how much Ontario college students pay for first-year tuition.
 - Lynn wants to know what her classmates did over the summer holidays.

- 12.** Identify the bias in each survey. Suggest how it might be removed.
- A ski resort asks skiers: *What is your favourite winter activity?*
 - A TV show invites the studio audience to ask questions of the guests.
- 13.** Classify each set of data as either discrete or continuous. Which type of graph would best suit each situation? Explain your choice.
- the number of pets students have at home
 - the time it takes students to complete a 2-km run
- 14.** Give an example of when the median is the best measure of central tendency of a set of data.
- 15.** Find the mean, the median, and the mode of each set of data. Which measure of central tendency best describes the data? Explain.
- the value of prizes, in dollars:
5, 5, 5, 5, 5, 5, 5, 5, 20, 20, 120
 - the masses of rabbits in a warren, in kilograms: 2.2, 3.7, 3.4, 2.4, 3.0, 3.7, 2.6, 3.5, 2.9, 3.8, 2.7
- 16.** A set of data has a range of 55. The least value in the set of data is 168. What is the greatest value in the set of data?
- 17.** Calculate the variance and the standard deviation for each set of data.
- 4, 3, 5, 1, 17, 5, 1, 4, 2, 9, 2
 - 35, 44, 37, 41, 41, 36, 37, 40, 29, 38

- 18.** Examine the histogram.
- Could a circle graph have been used to display the data in the histogram? Explain.
 - What type of distribution is shown?
 - What does the shape of the curve tell you about the sales?
 - List three other facts you can gather from the histogram.



- 19.** Create a set of data that would be modelled using a skewed distribution.

Chapter 4: Quadratic Relations I

- 20.** Write a relation that models each table of values.

a)

x	y
-2	35.0
-1	27.5
0	25.0
1	27.5
2	35.0
3	47.5

b)

x	y
-2	-4
0	-2
2	-4
4	-10
6	-20
8	-34

21. A rock is thrown off a cliff. The path of the rock is modelled by the relation $h = -4.9t^2 + 1.5t + 115$, where h is the rock's vertical distance from the ground, in metres, and t is the time, in seconds.

a) Copy and complete the table of values.

Time (s)	Distance (m)
0	
1	
2	
3	
4	
5	

- b) How can you tell that this relationship is quadratic? Give two reasons.
- c) Graph this relation.
- d) After how many seconds will the rock land?
22. Graph the data given in each table of values. Write a relation in the form of $y = a(x - h)^2$ that models each graph.

a)

x	y
-6	32
-5	18
-4	8
-3	2
-2	0
-1	2

b)

x	y
-1.0	-8.0
0.0	-4.5
1.0	-2.0
2.0	-0.5
3.0	0.0
4.0	-0.5

23. Graph each relation by plotting the vertex and two other points. Then draw a smooth curve through the points.

a) $y = (x + 1)^2 - 1$

b) $y = -0.8(x + 8)^2 - 3$

c) $y = -2(x - 6)^2 + 2$

d) $y = 3(x + 3)^2 - 2$

24. The manager of a local restaurant is trying to decide how much money to spend on advertising. She knows that an increase in advertising spending will increase her profit, up to a point. The situation is modelled by the relation $R = -0.0005(A - 5000)^2 + 12\,500$, where R is the extra revenue and A is the amount spent on advertising, both in dollars.

a) Create a table of values and graph the relation.

b) What is the vertex of the parabola? What do the coordinates of the vertex represent in this situation?

25. Some types of powerful microphones use a parabolic reflector to direct sound waves into the receiver. One such microphone has a reflector that is 60 mm wide and 27 mm deep.

a) Write a relation to model the parabolic shape of the reflector.

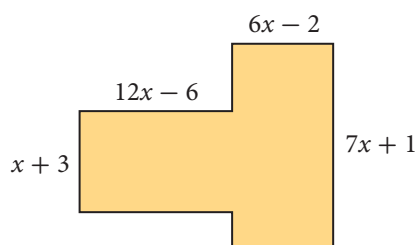
b) Graph your relation.

Chapter 5: Quadratic Relations II

26. Expand and simplify.

- a) $(3x + 7)(8x + 2)$
- b) $(50 - x)(5x + 3)$
- c) $(x + 11)(100x - 10)$
- d) $(8x - 5)(8x + 5)$

27. Write an expression, in simplified form, for the area of this shape.



28. For each parabola, write a relation in standard form, $y = ax^2 + bx + c$.

- a) $a = 2$, $b = 6$, y -intercept is 11
- b) $y = 5x^2 + bx + c$, vertex at $(1, 4)$
- c) $a = -3$, passes through $(2, 11)$ and $(0, -5)$
- d) $y = 5x^2 + bx + c$, minimum of 0 when $x = -3$

29. Determine the y -intercept of each relation.

- a) $y = x(x + 12)$
- b) $y = (x - 2.4)(x + 5)$

30. Factor each polynomial.

- a) $x^2 - 17x + 66$
- b) $x^2 + 8x + 7$
- c) $x^2 - 13x + 40$
- d) $x^2 - 3x - 18$
- e) $x^2 + 13x$
- f) $x^2 - 9$

31. Factor each trinomial fully. Expand to check.

- a) $2x^2 - 2x - 4$
- b) $-6x^2 - 12x + 144$
- c) $3x^2 + 3x - 126$
- d) $7x^2 + 42x - 49$

32. Which pairs of expressions are equivalent? How do you know?

- a) $3x^2 - 12x - 63$ $3(3 + x)(x + 7)$
- b) $7x^2 + 42x - 49$ $7(x + 7)(x - 1)$
- c) $4x^2 + 18x + 18$ $4(3 + x)(x + 1.5)$
- d) $-x^2 - x + 2$ $-(x + 1)(x - 2)$

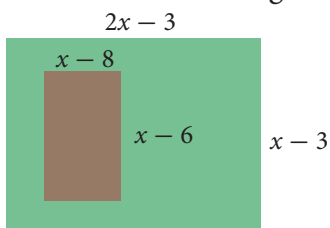
33. Find the zeros of each quadratic relation.

- a) $y = (x + 3)(x - 3)$
- b) $y = 3(x - 14)(x - 1)$
- c) $y = -10x^2$
- d) $y = (x + 4)(x + 4)$
- e) $y = 8(x - 8)(x + 0.5)$
- f) $y = 20x(x - 9)$

34. A toy rocket is pumped full of air and released upward. Its height can be approximated using the relation $h = -10t^2 + 49.8t + 1$, where h is the height above the ground, in metres, and t is the time, in seconds.

- a) One of the zeros of the relation is -0.02 . Does this zero mean anything in terms of the situation? Explain.
- b) What is the other zero? What does this zero mean in terms of the situation?

35. a) Write an expression, in simplified form, for the area of grass in this yard.



- b) The measurements are in metres. For what value of x will the area of grass be 111 m^2 ?

Chapter 6: Geometry in Design

36. Sunflowers have many tiny flowers called florets arranged in a pattern on their heads. These florets create the appearance of many spirals, some opening clockwise and some counter-clockwise. Often the number of spirals in each direction exactly match the data shown in the table.

Sunflower Size	Number of Clockwise Spirals	Number of Counter-Clockwise Spirals
small	21	34
average	34	55
above average	55	89
very large	89	144

- a) Calculate the ratio of counter-clockwise spirals to clockwise spirals for each sunflower size. Express each answer as a decimal to five decimal places.
- b) What do you notice about these ratios?
- c) An underdeveloped sunflower has 13 clockwise spirals. Use your answer in part b) to determine the number of counter-clockwise spirals. Round your answer to the nearest whole number.
37. Patterns are used in the design and manufacture of clothing. What other everyday objects are created from patterns?
38. A new tennis ball package is being designed to have a volume of 1200 cm^3 . Its height must be three times its diameter. The height and diameter must each be a whole number of centimetres.
- a) Determine the minimum height and diameter of the can.
- b) Select a suitable scale. Draw a net that can be used to make a scale model of the package.
- c) Cut out your net and fold it to make a scale model of the package.
39. A garden designer is creating a layout for a backyard garden. The design must fit into a rectangle measuring 30 ft by 50 ft and each feature must be in the shape of a rectangle. The client would like each feature to be at least 5 ft away from any other feature. Each feature must have the area shown.

Garden Feature	Area (ft ²)
flowerbed #1	50
flowerbed #2	150
garden shed	150
fountain	150
garden furniture area	150
pond	100

Design a garden that meets the specifications.

40. An architect wants to build a scale model of a rectangular palace that is to have a length of 310 m, a width of 200 m, and a height of 30 m. Suggest a suitable scale and calculate the dimensions for the model.

Chapter 7: Exponents

41. Write each expression as a single power. Then evaluate the single power.

a) $4^1 \times 4^4$ b) $9^{31} \div 9^{28}$
 c) $\left(\frac{1}{2}\right)^4 \times \left(\frac{1}{2}\right)^3$ d) $\frac{11^6}{11^3}$
 e) $(3^3)^3$ f) $[(-2)^5]^2$

42. In 1957 an earthquake in Alaska measured about 9.0 on the Richter scale. An earthquake with magnitude 6.0 occurred in Japan in early 2007. How many times more intense was the earthquake in Alaska than the one in Japan?

43. Write each expression as a single power, then evaluate. Express your answers as whole numbers or fractions.

a) $10^{-4} \times 10^3$ b) $(4^3)^{-1}$
 c) $\frac{5^{-1}}{5^{-3}}$ d) $\frac{1}{(3^{-1})^5}$
 e) $(-7)^{-4}(-7)^5$ f) $\left(\frac{1}{2}\right)^9 \times \left(-\frac{1}{2}\right)^{-10}$

44. a) Write $\frac{1}{20^7}$ as a power with a base of 20.
 b) Write 3^{11} as a power with a base of $\frac{1}{3}$.

45. Which relations are exponential? Explain how you know.

A

x	0	1	2	3	4	5
y	6	12	24	48	96	192

B

x	-3	-2	-1	0	1	2
y	1	2	4	8	9	15

C

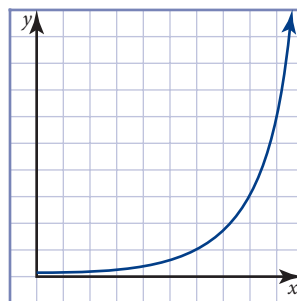
x	1	2	3	4	5	6
y	125	25	5	1	0.2	0.04

46. a) Sketch the graph of each relation on the same set of axes. Check using a graphing calculator.

A $y = 3x$ **B** $y = 2(3x)$
C $y = 4(3x)$ **D** $y = 0.5(3x)$

- b) Describe the role of a in $y = a(3x)$.

47. Which statement would best describe the graph?



- A** the number of pages of a 300-page document printed
B the population of trout decreases by 5% every 10 years
C the number of bacteria triples every 2 h
D the height of a parachutist jumping out of a plane

48. Which model (linear, quadratic, or exponential) would best describe each situation in question 47?
49. A photocopier is set to enlarge an image to 150% of its original size.
- If you make a copy of the enlarged image and enlarge it to 150%, what percent of the original is the second image?
 - How many times would you have to enlarge the image to 150% for it to be at least three times as large as the original? Explain.
50. Cells in a culture are growing by a factor of 2.7 per day. The number of cells in the culture can be estimated using the relation $N = 200(2.7)^d$, where d is the number of days.
- Use technology to graph the relation.
 - How many cells does this culture begin with?
 - How many cells would there be after 1 day? After 5 days?
51. The half-life of carbon-14 is 5730 years. The relation $C = \left(\frac{1}{2}\right)^{\frac{n}{5730}}$ is used to calculate the concentration, C , in parts per trillion, remaining n years after death. Determine the carbon-14 concentration in each item.
- a 1000-year-old wooden cup
 - a 7500-year-old fly frozen in ice
 - a 30 000-year-old fossil

Chapter 8: Compound Interest

52. Show the growth of a \$4000 investment, at both 7.5% per year, simple interest and 7.5% per year, compounded annually, for five years, using a table and a graph.
53. Determine the value of a \$500 investment after three years, at 7% per year, simple interest and 6.75% per year, compounded annually.
54. Evaluate. Use a scientific calculator and round to two decimal places.
- $100(1.06)^6$
 - $750(1.085)^{10}$
 - $100(1.06)^{-6}$
 - $750(1.085)^{-10}$
55. To make a down payment, Orton borrowed \$2400, at 7.5% per year, compounded semi-annually.
- How much must he repay after two years?
 - How much must he repay after two years if he paid \$1000 back after the first year?
56. A certain mutual fund has grown by an average of 10.4% per year, compounded annually, over the past eight years. How much would an initial investment of \$3500 be worth today?
57. Evaluate. Round to two decimal places.
- $2500(1.02)^{-8}$
 - $8000(1.03)^{-5}$
58. What principal should be invested today to have \$1000 after four years if interest is paid at 5.5% per year, compounded quarterly?

59. Calculate the discounted value of each loan.
- a) a \$700 debt due in one year, discounted at 6.5% per year, compounded semi-annually
 - b) a \$4000 debt due in three years, discounted at 5.2% per year, compounded quarterly
 - c) a \$2500 debt due in two years, discounted at 6% per year, compounded monthly
 - d) a \$1000 debt due in 18 months, discounted at 8% per year, compounded semi-annually
60. Sandro needs to invest enough money today to have \$7000 in three years, for a down payment on a condominium. How much should Sandro invest today, at 6.5% per year, compounded monthly?
61. Kai invests \$950 today, at 9.5% per year, compounded semi-annually. After how many years will he have enough money to buy a \$1400 scooter?
62. Rose borrowed \$2000, at 7.6% per year, compounded quarterly. How much must she repay at the end of one year?
63. Harumi hopes to have \$2500 in three years to visit New Zealand. Use technology to compare the amounts she would need to invest at 3.5% per year, compounded semi-annually and at 4% per year, compounded quarterly, to reach her goal.
64. Bethany plans to invest \$4000 for two years. She has three choices.
- A 5.45% per year simple interest
 - B 5.3% per year, compounded semi-annually
 - C 5.2% per year, compounded quarterly
- How much interest would Bethany earn with each investment plan?

Chapter 9: Personal Finance

65. Determine the interest earned on each amount deposited in a daily interest savings account.
- a) \$1600 in an account for May that pays 0.5% per year
 - b) \$350 in an account for May and June that pays 1% per year
 - c) \$2200 in an account for 100 days that pays 0.65% per year
 - d) \$3000 in an account for one year that pays 2.25% per year
 - e) \$4000 in an account for one day that pays 1.75% per year
 - f) \$1250 in an account for one week that pays 1.25% per year
66. Mustafa's credit card charges 17.5% annual interest on regular purchases and 19.5% annual interest on cash advances. Determine the interest due for each situation.
- a) a balance of \$244.85 for 21 days
 - b) a \$2500 cash advance for 45 days
 - c) a balance of \$642.11 for 3 days

- 67.** When Behrooz started working full-time, he began investing \$1500 per year in a no-fee investment that has paid 5.2% per year, compounded quarterly. Determine the value of Behrooz's investment after ten years (before his eleventh deposit).
- 68.** One year ago, Terrant invested \$3200 in a mutual fund that decreased in value by 2.74%. Given an annual 1.2% management fee, determine the current value of Terrant's investment.
- 69.** Lisa's credit card requires a minimum payment of the greater of \$25 or 5% of the balance. What is her minimum payment for each balance?
- a)** \$1439.19 **b)** \$844.70
c) \$383.68 **d)** \$1052.58
- 70.** Bradley's credit card gives a refund of 0.5% on the first \$3000 spent on the card, then 1% on further purchases. How much must Bradley spend in one year to recover the \$29 annual fee for the credit card?
- 71.** Three cases of financing a used car are shown. Use a TVM Solver or an on-line calculator to determine the monthly payments for each case, assuming monthly compounding.
- a)** \$5600 borrowed for three years at 8.75% interest
b) \$9000 borrowed for four years at 9% interest
c) \$14 750 borrowed for five years at 9.4% interest
- 72.** Calculate the total amount paid to the financial institution for each loan in question 71.
- 73.** Lang called several insurance agents for quotes on car insurance. The best offers were \$2250 annually, monthly payments of \$195, and payments of \$1150 every 6 months.
- a)** Calculate the annual cost of the monthly and semi-annually instalments.
b) List the options from least to most expensive.
c) Suggest reasons why Lang might choose each option.
- 74.** Calculate the fuel consumption, in kilometres per litre, for each situation. Round your answers to one decimal place.
- a)** Leo kept track of his mileage as a business expense. He travelled 1584 km and used 117.2 L of fuel.
b) On a road trip, Sheila used 48 L of fuel to travel 600 km.
c) A fuel-efficient car travelled 1230 km on one tank of fuel. The car used 62 L of gas.
- 75.** Calculate the depreciation on the following vehicles.
- a)** A new car worth \$16 299 sells for \$14 759 one year later.
b) An SUV worth \$56 850 can be purchased for \$48 290 one year later.