8 Compound Interest

Throughout this chapter, you will learn how to calculate simple and compound interest, and apply these skills to financial situations. These situations include money invested for a period of time and loans made by individuals and businesses.

In this chapter, you will

- determine, through investigation using technology, the compound interest for a given investment, using repeated calculations of simple interest, and compare, using a table of values and graphs, the simple and compound interest earned for a given principal and a fixed interest rate over time
- determine, through investigation, and describe the relationship between compound interest and exponential growth
- solve, using a scientific calculator, problems that involve the calculation of the amount, A, and the principal, P, using the compound interest formula $A = P(1 + i)^n$
- calculate the total interest earned on an investment or paid on a loan by determining the difference between the amount and the principal
- solve problems using a TVM Solver, that involve the calculation of the interest rate per compounding period, i, or the number of compounding periods, n, in the compound interest formula $A = P(1 + i)^n$
- determine, through investigation using technology, the effect on the future value of a compound interest investment or loan of changing the total length of time, the interest rate, or the compounding period

Key Terms

amount
compounding period
compound interest
creditor
discount
future value
growth factor
present value
principal
simple interest

As a self-employed financial planner, Tanisha helps clients achieve their financial goals—starting a business, planning for retirement, or paying for a child's education. To become qualified, Tanisha took four continuing education courses in the certified financial planning program at Centennial College.

Prerequisite Skills

Decimals

1. Use a calculator to evaluate.

a) 1.2×4.3

b) 2×1.05

c) $1000(0.04)(7)$

d) $350(0.035)(2.5)$

e) $500 + 500(0.09)(0.5)$

f) $950 + 950(0.04)\left(\frac{7}{12}\right)$

950 [+] 950 [×] 0.04 [×] [(] 7 [÷] 12 [)] [=]

g) $675[1 + (0.025)(4)]$

h) $1000\left[1 + (0.038)\left(\frac{1}{2}\right)\right]$

2. Evaluate.

a) $0.04 \div 2$

b) $0.05 \div 2$

c) $0.064 \div 2$

d) $0.064 \div 4$

e) $0.06 \div 12$

f) $0.085 \div 4$

g) $0.03 \div 12$

h) $0.095 \div 2$

Percents

3. Convert to a decimal.

a) 6%

b) 4%

c) 2.5%

d) 18%

e) 18.5%

f) 12.25%

g) 0.5%

h) 2.33%

4. Evaluate.

a) 5% of \$400

b) 3% of \$1000

c) 5.5% of \$2000

d) 7% of \$350

e) 6% of \$10 000

f) 4.5% of \$2500

g) 1.1225% of \$200 000

h) 6.64% of \$3500

5. Evaluate. Express your answer as a decimal.

a) $6\% \div 2$

b) $8.4\% \div 2$

c) $9.3\% \div 12$

d) $5.2\% \div 4$

e) $16\% \div 4$

f) $21.6\% \div 12$

g) $7.5\% \div 4$

h) $3.3\% \div 12$

6. Estimate each product.

a) 4.1% of \$1000

b) 9.9% of \$5000

c) 3.8% of \$200

d) 5.1% of \$690

e) 4% of \$329.17

f) 5% of \$236 712

Exponents

7. Use a calculator to evaluate. Round your answers to the nearest hundredth.

a) 1.03^2

b) 1.06^8

c) $200(1.03)^6$

200 [×] 1.03 [y^x] 6 [=]

d) $5000(1.0225)^{10}$

e) 2^{-1}

f) 5^{-2}

g) 1.03^{-6}

1.03 [y^x] [±] 6 [=] or

1.03 [y^x] 6 [±] [=]

h) 1.005^{-12}

Simple Interest

8. Use the formula $I = Prt$ to calculate the simple interest earned on each investment. Recall, t represents the time in years and r is the annual interest rate.

a) $P = \$500$, $r = 5\%$, $t = 2$ years

b) $P = \$1200$, $r = 8\%$, $t = \frac{1}{2}$ year

c) $P = \$4000$, $r = 7.5\%$, $t = 9$ months

d) $P = \$4000$, $r = 7.5\%$, $t = 3$ years, 8 months

9. Calculate the interest earned on each investment.

a) \$1000 invested at 6% per year, simple interest, for 3 years

b) \$800 invested at 7.2% per year, simple interest, for 18 months

c) \$1200 invested at 4.8% per year, simple interest, for 315 days

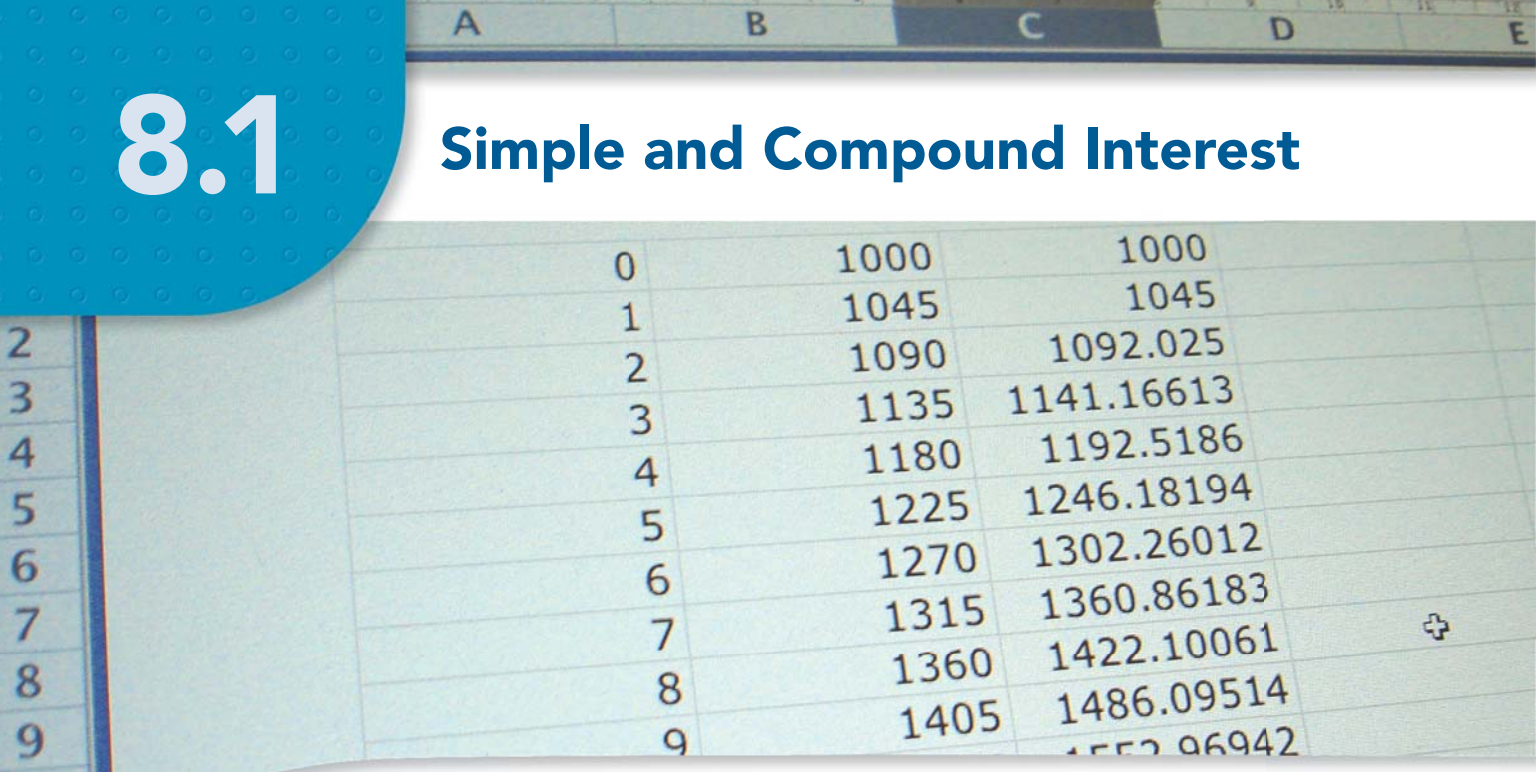
d) \$4000 invested at 5% per year, simple interest, for 2 years, 3 months

Chapter Problem

An accounting technician may have a career in many business sectors, including manufacturing, merchandising, finance, and government. The work involves managing business-to-business accounts, payroll accounts, companies' assets, and internal auditing. A two-year accounting diploma from a community college is required. Further education is needed to become a certified general accountant or a financial planner, among other careers.

As people go through their lives, they invest money in different ways, such as guaranteed investment certificates (GICs), term deposits, savings bonds, registered retirement savings plans (RRSPs), and registered education savings plans (RESPs). People also take out loans to buy expensive items (such as a car, furniture, or a house), to pay for education, or to allow their businesses to grow. In this chapter, you will help the Kwan family manage some of their finances as their small business grows and they save money for the expansion of their business.

8.1 Simple and Compound Interest



simple interest
- the money paid on a loan or investment
- a percent of the principal

principal
- the value of the initial investment or loan

amount
- the final or future value of an investment, including the principal and the accumulated interest

compound interest
- the interest paid on the principal and its accumulated interest

Banks pay you interest for the use of your money. When you deposit money in a bank account, the bank reinvests your money to make a profit.

Simple interest is calculated on the initial value invested (**principal**), P, at an annual interest rate, r, expressed as a decimal for a period of time, t. The interest is added to the principal at the end of the period.

Interest, $I = Prt$

Amount, $A = P + Prt$

Or in factored form, $A = P(1 + rt)$

Compound interest is calculated on the accumulated value of the investment, which includes the principal and the accumulated interest of prior periods.

Investigate

Compound Interest

Compare the growth of a $1000 investment at 7% per year, simple interest, with another $1000 investment at 7% per year, compounded annually.

Tools

- spreadsheet software
- computer

Method 1: Use a Spreadsheet

1. a) Set up the column headings as shown in cells A1, B1, and C1. Adjust the column widths as required.

YEAR	SIMPLE INTEREST	AMOUNT WITH SIMPLE INTEREST ($)

b) Highlight columns B and C. Under the **Format** menu, choose **Cells…**. Under the **Number** tab, choose **Currency**.

c) In cells A2 to A14, enter the numbers from 0 to 12.

d) In cell B2, enter the formula =1000*0.07*A2, and in cell C2, enter =1000+B2.

e) Highlight cells B2 down to C14. From the **Edit** menu, choose **Fill ▸ Down**. You will see the value of the $1000 investment for each year over the 12 years, at 7% per year, simple interest.

2. a) Use your spreadsheet from question 1. Set up the column headings as shown in cells D1, E1, and F1.

AMOUNT AT START OF YEAR ($)	INTEREST EARNED THIS YEAR	AMOUNT WITH COMPOUND INTEREST ($)

b) Highlight columns D to F. Under the **Format** menu, choose **Cells…**. Under the **Number** tab, choose **Currency**.

c) In cell D2, enter 1000. In cell D3, enter the formula =F2. **Fill ▸ Down** to cell D14.

d) In cell E2, enter the formula =D2*0.07, and in cell F2, enter =E2+D2. Highlight cells E2 down to F14. From the **Edit** menu, choose **Fill ▸ Down**.

3. Look at the entries in the lists. Compare the amounts for simple interest to the amounts for compound interest. Explain the differences in the growth of the two amounts.

4. Graph the amount with simple interest and with compound interest.

- Select the graph icon.
- In the popup window, select **XY (scatter)**. Then **Next >**.
- Click on the **Series** tab. Then click **Add** twice.
- Click on **Series 1**, click in the **X-Values** box. Then highlight cells A2 to A14. Click in the **Y-Values** box and clear any data in the box. Then highlight cells C2 to C14. Name the chart Simple Interest.
- Click on **Series 2**, click in the **X-Values** box. Then highlight cells A2 to A14. Click in the **Y-Values** box and clear any data in the box. Then highlight cells F2 to F14. Name the chart Compound Interest.
- Click **Next > Next > Finish**.

5. How do the graphs compare? Explain the differences in the graphs.

6. **Reflect** Describe how compound interest grows relative to simple interest.

7. Identify the type of growth (linear, quadratic, or exponential) demonstrated by simple interest and by compound interest. Justify your choices.

Tools

- scientific calculator
- grid paper

Method 2: Use Pencil and Paper

1. Set up a table as shown. Complete the table for 0 to 12 years. Calculate the simple interest on the amount at the start of the year using the formula $I = Prt$.

Year	Simple Interest ($)	Amount ($)
0		

2. Set up a chart as shown. Complete the chart for 0 to 12 years. Calculate the interest, at 7% per year, on the previous value (amount) and add it to the amount before calculating interest for the next year.

Year	Amount at Start of Year ($)	Compound Interest ($)	Amount at End of Year ($)
0	1000	70	1070
1	1070		

3. Look at the entries in the tables. Compare the amounts for simple interest to the amounts for compound interest. Explain the differences in the growth for the two amounts.

4. Sketch a scatter plot of both sets of data on the same set of axes.

5. How do the graphs compare? Explain the differences in the graphs.

6. **Reflect** Describe how compound interest grows relative to simple interest.

7. Identify the type of growth (linear, quadratic, or exponential) demonstrated by simple interest and by compound interest. Justify your choices.

Tools

- graphing calculator

Technology Tip

Before beginning this investigation, remember to clear all lists and the Y = editor, and make sure all plots have been turned off.

Method 3: Use a Graphing Calculator

1. Enter the numbers from 0 to 12 in list L1 of your graphing calculator. These values represent the times in years.

2. **a)** To generate the values of the $1000 investment each year at 7% per year, simple interest, scroll up to the column heading of list L2. Enter the formula $1000 + 1000 \times 0.07 \times$ L1.

   b) Explain what each part of the formula represents.

3. **a)** To generate the value of the $1000 each year at 7% per year, compounded annually, in the first cell of list L3, enter 1000. In each of the subsequent cells of L3, multiply the previous value by 1.07.

   b) Why is each value in list L3 multiplied by 1.07 to generate the next value? Explain.

4. Look at the entries in lists L2 and L3. Compare the results of simple interest to the amounts for compound interest.

5. To graph the results, press [2nd] [STAT PLOT]. In Plot 1, draw a scatter plot comparing L1 and L2. Set up your screen as shown.

   In Plot 2, draw a scatter plot comparing L1 and L3. Set up your screen as shown.

   Press [ZOOM] **9:ZoomStat** to fit the window to the data in the graph.

6. How do the graphs compare? Explain the differences in the graphs.

7. **Reflect** Describe how compound interest grows relative to simple interest.

8. Identify the type of growth (linear, quadratic, or exponential) demonstrated by simple interest and by compound interest. Justify your choices.

growth factor
- the number that is multiplied by the principal when calculating its accumulated value

compounding period
- the length of time for which interest is calculated before being accumulated

At the end of each time interval, the simple interest formula is used to calculate the interest, which is then added to the principal or previous amount.

The **growth factor** is $1 + i$, where i is the interest rate per **compounding period**, n.

Example

Compare Simple and Compound Interest

Larry wants to invest $700 for five years. Compare the growth of his investment at 4% per year, simple interest, to the same investment at 4% per year, compounded annually.

Solution

Simple Interest

$P = 700$
$r = 4\%$
$\quad = 0.04$
$t = 5$

$A = P(1 + rt)$
$\quad = 700(1 + (0.04)(5))$
$\quad = 840$

Interest earned $= 840 - 700$
$\qquad\qquad = 140$

If Larry invested $700 at 4% per year, simple interest, the amount would be $840 after five years.

The total interest earned is $140.

Compound Interest

The yearly growth factor is $1 + 0.04 = 1.04$.

Use a chart to show the growth of the investment.

Year	$A = P(1.04)$	Amount ($)
0		700.00
1	700.00(1.04)	728.00
2	728.00(1.04)	757.12
3	757.12(1.04)	787.4048
4	787.4048(1.04)	818.90099
5	818.90099(1.04)	851.65703

If Larry invested $700 at 4% per year, compounded annually, the amount would be $851.66 after five years.

The total interest earned is $151.66, which is $11.66 more than with simple interest.

Key Concepts

- Simple interest is calculated using the formula $I = Prt$.
- Money invested with simple interest grows by adding the interest to the principal, $A = P + Prt$ or $A = P(1 + rt)$.
- The rate of change for simple interest is the interest being added to the principal, so it is linear growth.
- Compound interest occurs when the interest is added to the principal at the end of each compounding period, and is included in further calculations of interest.
- Money invested with compound interest grows by multiplying by the growth factor $1 + i$, so it is exponential growth.

Discuss the Concepts

D1. When investing money with compound interest, when is the growth greater: at the beginning or the end of the term? Why?

D2. Given the same interest rate, does an investment always earn more interest with compound interest than with simple interest? If yes, explain why. If no, give an example to show it is not true.

Practise A

For help with questions 1 to 3, refer to the Investigate or the Example.

1. Show the growth of a $500 investment, at 6% per year, simple interest, and at 6% per year, compounded annually, for five years, using a table and a graph.

2. Show the growth of an $800 investment, at 8% per year, simple interest, and at 8% per year, compounded annually, for 10 years, using a table and a graph.

3. Shu Ying invested $750 at 5% per year, simple interest. Her sister, Shu Jin, invested $750 at 5% per year, compounded annually. Compare the values of their investments after each year for five years.

Apply B

4. Determine the value of a $1000 investment after six years, at

a) 6.5% per year, simple interest

b) 6.5% per year, compounded annually

5. Using tables and graphs, compare the amounts at the end of each year for three years for $2000 invested at

a) 4% per year, compounded annually

b) 5% per year, compounded annually

c) 6% per year, compounded annually

6. To save for a motorcycle, Abdul deposited $2000 into an account that earned simple interest at 5.4% per year. How much more would the investment earn in three years if it were invested at 5.4% per year, compounded annually? Illustrate your answer using tables and graphs.

7. A bank is offering a new three-year investment. You decide to invest $200.

a) How much interest would you earn each year if the bank pays 3.8% per year, simple interest?

b) How much interest would you earn each year if the bank pays 3.8% per year, compounded annually?

c) Which amount is easier to calculate? Explain your answer.

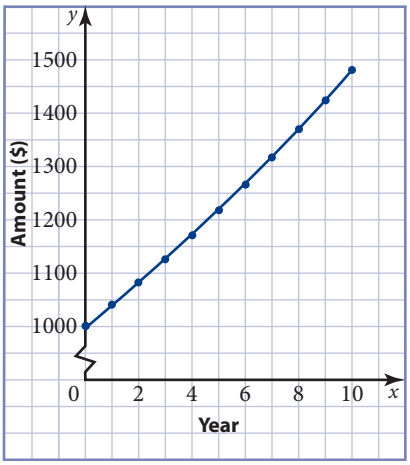
Chapter Problem

8. Five years ago, the Kwans invested $20 000 at 4% per year, compounded annually, so they could open a business this year.

a) Illustrate the growth of their money over the five years using a table and a graph.

b) Describe how their money grows compared to the same investment at 4% per year, simple interest.

9. The graph shows the growth of a $1000 investment at 4% per year, compounded annually.



a) How much is the investment worth after three years?

b) Estimate the time required for the investment to grow to $1500.

c) How would the graph change if the interest rate were 5% per year, compounded annually? Justify your response.

Extend C

10. A city has a population of 75 600 people. The population is expected to grow by 1.3% annually for the next 10 years.

a) Use charts and graphs to illustrate the growth of the population over the next 10 years.

b) Describe how the graph would change if the growth rate were 2% annually.

11. Marcy kept $200 in a savings account paying simple interest at 4% per year for 10 years. Use technology to determine what rate of compound interest would result in the same amount at the end of 10 years.

8.2 Compound Interest

Math Connect

The interest rate per compounding period, i, is the annual or yearly interest rate, r, divided by the number of compounding periods in one year.
The number of compounding periods, n, is the number of compounding periods in one year times the number of years.

Rather than using a table or a graph to see how the value of an investment grows, you can use a formula.

Compound Interest Formula

$A = P(1 + i)^n$,
P is the principal, or initial value.
A is the accumulated amount or future value.
i is the interest rate per compounding period.
n is the number of compounding periods.

The table shows how to convert the yearly interest rate and term for various compounding periods.

Compounding Period	Meaning	Interest Rate, i	Term, n
annually	once per year	unchanged	unchanged
semi-annually	twice per year	divide by 2	multiply by 2
quarterly	four times per year	divide by 4	multiply by 4
monthly	twelve times per year	divide by 12	multiply by 12

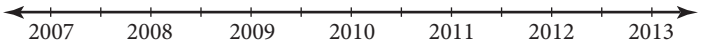
Example 1

Interest Compounded Semi-Annually

Determine how much money you will have if $500 is invested for six years, at 4% per year, compounded semi-annually.

Solution

Interest is compounded semi-annually, meaning twice a year, for six years. There are 2×6, or 12, compounding periods. This can be illustrated on a timeline.



$i = 0.04 \div 2$
$= 0.02$ Divide the yearly interest rate by 2.
$n = 6 \times 2$
$= 12$ Multiply the number of years by 2.
$P = 500$
$A =$ ■
$A = P(1 + i)^n$ Substitute for P, i, and n.
$= 500(1 + 0.02)^{12}$
$= 634.12$

After six years, you will have \$634.12.

Example 2

Interest Compounded Monthly

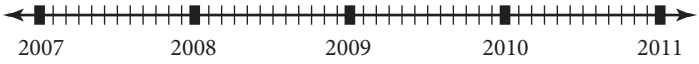
Alice borrowed \$5000 to start a small business. The interest rate on the loan was 9% per year, compounded monthly. She is expected to repay the loan in full after four years.

a) How much must Alice repay?

b) How much of the amount Alice repays will be interest?

Solution

a) Interest is compounded monthly, meaning 12 times a year, for four years. There are 12×4, or 48, compounding periods.



$i = 0.09 \div 12$
$= 0.0075$ Divide the yearly interest rate by 12.
$n = 4 \times 12$
$= 48$ Multiply the number of years by 12.
$P = 5000$
$A =$ ■
$= P(1 + i)^n$
$= 5000(1 + 0.0075)^{48}$
$= 7157.03$

Alice must repay \$7157.03 after four years.

b) Calculate the total interest by subtracting the principal from the amount.

$7157.03 - 5000 = 2157.03$

Alice will pay \$2157.03 in interest.

Key Concepts

- Compound interest can be accumulated at various intervals, such as annually, semi-annually, quarterly, and monthly.
- The compound interest formula $A = P(1 + i)^n$ can be used to calculate the future value, or amount.
  - A is the future value or accumulated amount of an investment or loan.
  - P is the principal.
  - i is the interest rate, in decimal form, per interest period.
  - n is the number of compounding periods.

Discuss the Concepts

D1. Draw a timeline to illustrate each situation.

a) an investment of $1000 at 5% per year, compounded semi-annually, for three years

b) an investment of $550 at 4% per year, compounded quarterly, for two years

c) an investment of $700 at 3% per year, compounded monthly, for two years

D2. You can use the same compound interest formula, $A = P(1 + i)^n$, for both debts and investments. Explain why.

Practise A

For help with questions 1 to 3, refer to Example 1.

1. Evaluate. Use a scientific calculator and round to two decimal places.

a) $500(1.02)^3$

b) $200(1.03)^7$

c) $1000(1.06)^4$

d) $3500(1.0025)^8$

e) $1350(1.0375)^{12}$

f) $12\,500(1.041)^5$

2. Substitute the values into the formula $A = P(1 + i)^n$. Do not evaluate.

a) a $2000 investment at 5% per year, compounded annually, for three years

b) a $1000 loan at 8% per year, compounded semi-annually, for two years

c) a $50 000 loan at 12% per year, compounded quarterly, for five years

d) a $750 investment at 6% per year, compounded monthly, for one year

3. Determine the amount of, and total interest earned on, a $1000 investment at

a) 4% per year, compounded annually, for five years

b) 8% per year, compounded semi-annually, for three years

c) 6.5% per year, compounded quarterly, for two years

d) 3.6% per year, compounded monthly, for four years

For help with question 4, refer to Example 2.

4. To pay for a vacation, Ming Mei borrowed $900, at 6% per year, compounded quarterly. The loan must be paid in full after two years.

a) How much must Ming Mei repay?

b) How much of that amount is interest?

5. Keisha plans to invest $5000 at 6% per year for five years. Calculate the amounts Keisha would have at the end of the five years if the interest is compounded

a) annually **b)** semi-annually **c)** quarterly

d) monthly **e)** daily

Apply B

6. When Tonya was born, her grandparents invested $10 000 at 5% per year, compounded semi-annually, to pay for her education.

a) What was the investment worth on Tonya's twelfth birthday?

b) What was the investment worth on Tonya's eighteenth birthday?

7. A certain investment fund has grown by an average of 13.6% per year, compounded annually, over the past eight years. How much would an investment of $2000 made eight years ago be worth today?

8. A $5000 investment earns interest at 4% per year, compounded quarterly, for 10 years.

a) What is the value of the investment after one year? two years?

b) What is the interest earned in the second year?

c) What is the interest earned in the tenth year?

d) Explain any differences between your answers in parts b) and c).

9. Wayne invested $2000 at 4.5% per year, compounded semi-annually, and $2500 at 4.2% per year, compounded quarterly. Both investments were for three years.

a) Which investment earned Wayne more money?

b) What is the total interest earned on Wayne's investments?

10. To buy a car, Sangar borrowed $8000 at 4.8% per year, compounded monthly, for one year. His brother, Sanjiv, borrowed $8000 for his car, at 3.2% per year, compounded monthly, for one year. How much more interest did Sangar have to pay than Sanjiv?

11. To find current interest rates for car loans at financial institutions across Canada, go to *www.mcgrawhill.ca/links/foundations11* and follow the links.

a) Which institution charges the lowest interest rate on a 60-month loan? Which institution charges the greatest interest rate?

b) If interest is compounded monthly on a $15 000 loan, compare the total interest paid on the loan using the two interest rates from part a)?

c) Select one institution's 48-month loan interest rate. Compare the total interest payable on a $15 000 loan

i) with simple interest

ii) with interest compounded monthly

12. Warren needs to borrow $3000. Which loan should he take? Justify your choice.

A $3000 for five years at 9% per year, compounded semi-annually

B $3000 for five years at 8.6% per year, compounded quarterly

Literacy Connect

A bond is issued by a government or company to raise large sums of money to be repaid after many years. Simple or compound interest is paid to the investors.

13. The town council voted to issue a bond of $3 000 000 to build a new swimming pool. The bond matures in 10 years, with an interest rate of 5% per year, compounded semi-annually. Principal and accumulated interest are due at the end of the term.

a) Calculate the total amount that the town must pay at the end of the term.

b) Calculate the total interest paid.

14. The city of Melville has a population of 102 000 and a projected growth rate of 2.3% per year, for the next 10 years. The city of Markton has a population of 97 000 and a projected growth rate of 3.7% per year for the next 10 years. Which city is expected to have the greater population in 10 years? By how many people?

15. The Stereo Warehouse is advertising "No money down and pay no interest for one year!" Peter read the fine print and discovered that, although you pay no interest for one year, interest is calculated at 12% per year, compounded monthly, on the price of the merchandise. What would Peter have to pay for a $1150 stereo after one year?

Achievement Check

16. Danielle received an inheritance of $30 000. She wants to split the amount equally among her three children, Robert, David, and Donna.

- Robert plans to buy a house in the near future so he will need the money available. He deposits his portion into a bank account paying interest at 4% per year, compounded quarterly.
- David plans to go to university in a few years. He invests his money in a registered education savings plan (RESP) that pays at 5.5% per year, compounded semi-annually.
- Donna will not need her money for many years. She puts her portion into a trust fund. The fund pays interest at 8% per year, compounded monthly.

a) Find the amounts available to each child after two years. What are the differences between the amounts?

b) What will happen to the differences between the amounts if the money is invested for a longer time?

Extend C

17. Sarah deposited $2000 in an investment fund that earned 12.6% per year, compounded annually. After five years, the proceeds were reinvested in a second investment fund that earned 15.8% per year, compounded semi-annually. If the second fund continues earning at the same rate, how much will Sarah's investment be worth after an additional five years?

18. Determine the yearly interest for each investment.

a) After six months, a $500 investment with interest compounded semi-annually is worth $512.50.

b) After two months, a $2000 investment with interest compounded monthly is worth $2020.05.

c) After six months, a $1000 investment with interest compounded quarterly is worth $1025.16.

19. Bryce bought a savings bond during a recent Ontario Savings Bond campaign. The interest rate increases every year according to the table.

Year	Annual Interest Rate (%)
1	3.7
2	3.8
3	3.9
4	4.0
5	4.25

a) How much will Bryce's investment be worth in five years if he invests $500?

b) How much will Bryce's investment be worth in five years if he invests $1500? Use your answer to part a) to determine the value.

8.3 Present Value

Suppose you want to buy the TV shown. Interest is 4.2% per year, compounded quarterly. Should you pay the full price for the TV today or make the down payment, leave the rest of the money in the bank, and pay the balance after one year?

Investigate Growth Factors

1. Consider the sequence 1, 2, 4, 8, ….

a) Continue the sequence for three more terms.

b) What is the growth factor?

2. Consider the sequence 8, 12, 18, 27, 40.5, ….

a) Continue the sequence for three more terms.

b) What is the growth factor?

3. Consider the sequence 2, 6, 18, ….

a) What is the growth factor?

b) One of the later terms in the sequence is 4374. Determine the previous two terms.

c) Describe how you found your answer in part b).

4. A sequence has a growth factor of 5. One of the terms is 9375.

a) Describe a method for finding the previous terms.

b) Determine the previous two terms.

5. An investment paid interest at 4% per year, compounded annually, and so has a growth factor of 1.04. Its value at the end of the term is $4326.75.
   a) Explain why this investment is like a sequence with a growth factor of 1.04.
   b) Describe a method for finding the value of the investment at the end of the previous year.
   c) Calculate the value of the investment at the end of each of the previous two years.

6. **Reflect** If the final amount of an investment is known, describe a method that could be used to determine all the previous values, including the principal.

To find the amount of an investment, you can use the formula $A = P(1 + i)^n$. The formula can be rearranged to find the principal.

$$A = P(1 + i)^n$$

$$\frac{A}{(1 + i)^n} = \frac{P(1 + i)^n}{(1 + i)^n}$$

To solve for P, divide both sides by $(1 + i)^n$.

$$P = \frac{A}{(1 + i)^n}$$

This formula can also be expressed using a negative exponent.

$$\text{or } P = A(1 + i)^{-n}$$

present value
- the value of an investment or loan on a date before the end of the term

future value
- the value of an investment or loan at the end of the term

P represents the principal value of a loan or investment. It is also known as the **present value**.

A represents the amount of a loan or investment is worth after a period of time. It is also known as the **future value**.

Example 1

Investment

Sam wants to invest enough money today to have $3200 for tuition when he goes to college in two years. If he invests his money at 6% per year, compounded monthly, how much does he need to invest?

Solution

$A = 3200$ The amount needed in two years.

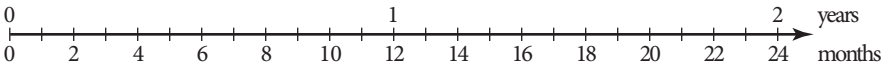
$n = 2 \times 12$

$= 24$

$i = 0.06 \div 12$

$= 0.005$

$P =$ ■ P is the unknown quantity.



$P = A(1 + i)^{-n}$
$= 3200(1 + 0.005)^{-24}$ Substitute for A, i, and n.
$= 3200(1.005)^{-24}$
$= 2838.99$

Sam needs to invest \$2838.99 today to have \$3200 in two years.

creditor
- a person or organization that lends money

discount
- to sell an investment at a value less than its usual price

Just as you can find the principal value of an investment, you can find the present value of a loan. **Creditors** often sell their loans to other creditors. They **discount** the value of the loan by calculating the present value at current interest rates.

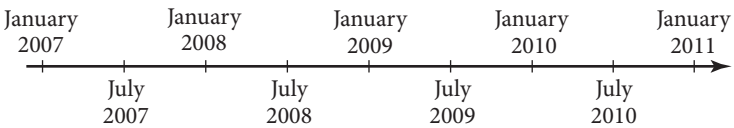
Example 2

Credit Debt

John has a loan for \$5000 that is due in four years. He wants to pay off his debt early. The creditor is willing to discount the loan at an interest rate of 8% per year, compounded semi-annually. How much would the creditor be willing to accept today?

Solution

$A = 5000$
$n = 4 \times 2$
$= 8$
$i = 0.08 \div 2$
$= 0.04$
$P = \blacksquare$



$P = A(1 + i)^{-n}$
$= 5000(1 + 0.04)^{-8}$
$= 5000(1.04)^{-8}$
$= 3653.45$

The creditor would be willing to accept \$3653.45 today to pay off the loan.

Key Concepts

- To calculate the amount of an investment or loan, use the formula $A = P(1 + i)^n$.
- To calculate the principal value (present value) of an investment or loan, use the formula $P = A(1 + i)^{-n}$.

Discuss the Concepts

D1. Explain the difference between *amount* and *principal*. Use words, numbers, and/or a diagram.

D2. When calculating the principal value of an amount, which will be the smaller value: the principal or the amount? Why?

D3. Kerry wants to invest enough money today to have $4000 in two years, at 6% per year, compounded quarterly. Describe the appropriate steps and scientific calculator keystrokes needed to solve this problem.

D4. Look at the opening illustration on page 436. What factors would you need to consider when determining which payment plan would be a better deal?

Practise A

For help with questions 1 and 2, refer to Example 1.

1. Evaluate. Round to two decimal places.

a) $2000(1.04)^{-6}$

b) $750(1.005)^{-12}$

c) $500(1.01)^{-10}$

d) $10\,000(1.03)^{-8}$

e) $2450(1.0075)^{-18}$

f) $1500(1.1)^{-3}$

2. Calculate the present value of each amount.

a) $5000 needed in four years, invested at 6% per year, compounded annually

b) $2000 needed in two years, invested at 4% per year, compounded semi-annually

c) $1000 needed in three years, invested at 4.5% per year, compounded monthly

d) $10 000 needed in five years, invested at 8% per year, compounded quarterly

For help with question 3, refer to Example 2.

3. Calculate the discounted value of each loan today.

a) a $2000 debt due in three years, discounted at 6% per year, compounded semi-annually

b) a $5000 debt due in four years, discounted at 5% per year, compounded quarterly

c) a $100 000 debt due in five years, discounted at 7.5% per year, compounded monthly

d) a $1000 debt due in three years, discounted at 5% per year, compounded semi-annually

Apply B

4. Steve borrowed some money at 9% per year, compounded semi-annually. After three years, he paid $2604.52 to pay off the loan. What sum of money did Steve borrow?

5. What principal should be invested today to have $1000 after two years if interest is paid at 4% per year, compounded quarterly?

6. At the birth of their child, how much should a couple invest to have $20 000 on the child's eighteenth birthday, if interest is paid at 8% per year, compounded quarterly? How much interest would be earned?

7. An investment fund pays 6.3% per year, compounded monthly. How much should a 25-year-old woman invest in the fund to have $50 000 by age 35?

8. Jamie took out a $3000 loan, due in four years. If interest is 5.7% per year, compounded semi-annually, how much should Jamie's creditor be willing to accept to pay off the loan today?

Chapter Problem

9. The Kwans will need $50 000 in three years to expand their business. What is the equivalent value today, if inflation is projected to be approximately 3.5% per year, compounded annually?

10. At Giant TV Sales, you can choose between two payment options for a new plasma TV. The first option is to pay $2399.99 now. The second option is to pay a $399.99 down payment plus $2200.00 after one year. If interest is 4.2% per year, compounded quarterly, which option is the better deal?

11. Jenay will invest some money on July 3, her sixteenth birthday, at 4.5% per year, compounded monthly. How much should she invest if she wants to have $10 000 on the November 3 following her eighteenth birthday?

12. Mike lent some money to a relative. The relative will pay back $1000 in one year, $2000 in two years, and $3000 in three years. What is the combined value of the loan today, if interest is calculated at 7.5% per year, compounded semi-annually?

13. Andelko will inherit $30 000 when he turns 21 in six months. He will borrow money today to purchase a new car and will pay off the principal plus interest in a lump sum with his inheritance. The bank offers short-term loans at a rate of 8% per year, compounded monthly.

a) How much can Andelko borrow today for the new car?

b) How much of the $30 000 payment will be interest?

Extend C

14. Interest on a $5000 loan is 4.8% per year, compounded monthly. The loan is due in six years. If the creditor were to sell the loan to another creditor, discounted at 4.2% per year, compounded quarterly,

a) how much would the new creditor pay?

b) how much would the original creditor earn on the loan?

15. Emilie borrowed $2700 at 8.6% per year, compounded quarterly. After the first year, she repaid $1000. She is expected to repay the loan in full after three years.

a) How much must Emilie repay?

b) Suppose after the second year, Emilie repaid another $1000. How much must she repay now?

c) Suppose after the first year, Emilie repaid $2000. How much must she repay now?

d) Determine the total amount Emilie repaid in each situation.

16. For each loan, determine the number of years between the initial loan and repayment. You may need to guess and test.

a) $1225.04 was repaid for a loan of $1000 at 7% per year, compounded annually.

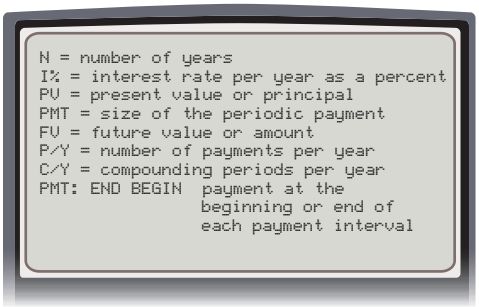
b) $2979.69 was repaid for a loan of $2000 at 8% per year, compounded monthly.

c) $1097.84 was repaid for a loan of $850 at 6.5% per year, compounded semi-annually.

8.4 The TVM Solver

TVM Solver
- a feature of the TI-83 Plus/84 Plus calculators that is used for financial calculations

A graphing calculator can be used to make calculations using the compound interest formula, $FV = PV(1 + i)^n$. The Time–Value–Money **(TVM) Solver** allows you to enter the value of each variable and solve for the remaining unknown value with a simple keystroke.



When entering the interest rate in the TVM Solver, express it as a percent, not as a decimal.

Investigate 1

Future Investment

Tools
- graphing calculator

Samir invested $500 at 6% per year, compounded quarterly. What will the investment be worth after three years?

1. To access the TVM Solver, press APPS **1:Finance**, then **1:TVM Solver…**.

2. Set up the values.
   N = Enter the number of years.
   I% = Enter the annual interest rate as a percent.
   PV = Enter the principal.
   PMT = Enter 0. When there are no regular payments, always set PMT = 0.

FV = Enter 0 (a temporary value).
P/Y = Enter 1. When there are no regular payments, always set P/Y = 1.
C/Y = Enter the number of compounding periods per year.
PMT: Choose END.

3. Use the arrow keys to move the cursor to **FV**. Press [ALPHA] [SOLVE].

4. What was Samir's investment worth after three years?

Technology Tip
The future value, FV, is shown as a negative number because it is money you cannot use right now. When a value is positive, it represents money you are receiving.

Investigate 2 Discount Investment

An investment will be worth $4000 in four years. If the interest rate is 5% per year, compounded monthly, what is the present value of the investment?

1. Access the TVM Solver and set up the values. Use **PV** = 0 and enter the appropriate value for **FV**.

2. Move the cursor to **PV** and press [ALPHA] [SOLVE].

3. What is the present value of this investment?

4. How much interest will be paid at the end of four years?

Key Concepts

- The TVM Solver on a graphing calculator can be used to solve problems involving compound interest. Enter the known values, and enter 0 for the unknown value. With your cursor at the location of the unknown value, press [ALPHA] [SOLVE].
- The TVM Solver uses the compound interest formula $A = P(1 + i)^n$. When PV or FV are displayed as negative numbers, they represent money you cannot use right now.

Discuss the Concepts

D1. What values should you enter in each line of the TVM Solver for each problem? Do not evaluate.

a) $3000 is borrowed for two years at 5% per year, compounded monthly

b) $5000 is due in three years, discounted at 9% per year, compounded semi-annually

Practise A

For help with questions 1 to 3, refer to Investigate 1.

1. Determine the amount of a $2000 investment after five years if interest is 6% per year, compounded semi-annually.

2. Ginny borrowed $1000, at 8.4% per year, compounded monthly. How much must she repay at the end of two years?

3. Chin Lee invests $7500 today, at 5.5% per year, compounded semi-annually. After how many years will he have enough to buy a $9000 motorcycle?

For help with questions 4 and 5, refer to Investigate 2.

4. Eduardo wants to invest enough money today to have $5000 in three years, for a down payment on a car. How much should Eduardo invest today, at 5% per year, compounded quarterly?

5. A no-interest $5000 loan is due in four years. If the creditor were to sell the loan to another creditor, discounted at 9% per year, compounded monthly, how much would the new creditor pay?

Apply B

6. Maria deposited $1000 into an account paying interest at 4.2% per year, compounded monthly. How long will it take for the money to grow to $1500?

7. Keenan invested $2000 in a term deposit that pays 6% per year, compounded semi-annually.

a) How long will it take for Keenan's investment to double in value?

b) Would a $10 000 investment double in value in the same length of time? Explain.

8. **a)** What interest rate, compounded quarterly, is needed for a $2000 investment to increase to $3000 after five years?

b) Would the same interest rate double a $5000 investment after five years? Explain.

9. What interest rate, compounded semi-annually, will double the value of a $3000 investment after

a) three years?

b) four years?

c) five years?

10. Which will reach a value of $5000 faster: $3000 invested at 6% per year, compounded monthly, or $3500 invested at 6.5% per year, compounded semi-annually?

11. Which will double faster: money invested at 8% per year, compounded semi-annually, or at 7.5% per year, compounded quarterly? Justify your answer.

12. You want to be a millionaire by the time you are 55 years old. If you invest $20 000 on your eighteenth birthday at 8% per year, compounded semi-annually, will you meet your goal? If not, what interest rate would you require?

13. How much money would you need to invest on your eighteenth birthday at 8% per year, compounded semi-annually, to be a millionaire by the time you are 60 years old? 65 years old?

14. Rosalind owns a savings bond that will pay her $10 000 when it matures in five years. She needs money now for college tuition, and her cousin is willing to buy the bond at a suitable discount. Current bank rates vary from 3.5% to 5.5% per year, for various savings bonds.

a) What is the minimum fair discount price for the bond?

b) What is the maximum fair discount price for the bond?

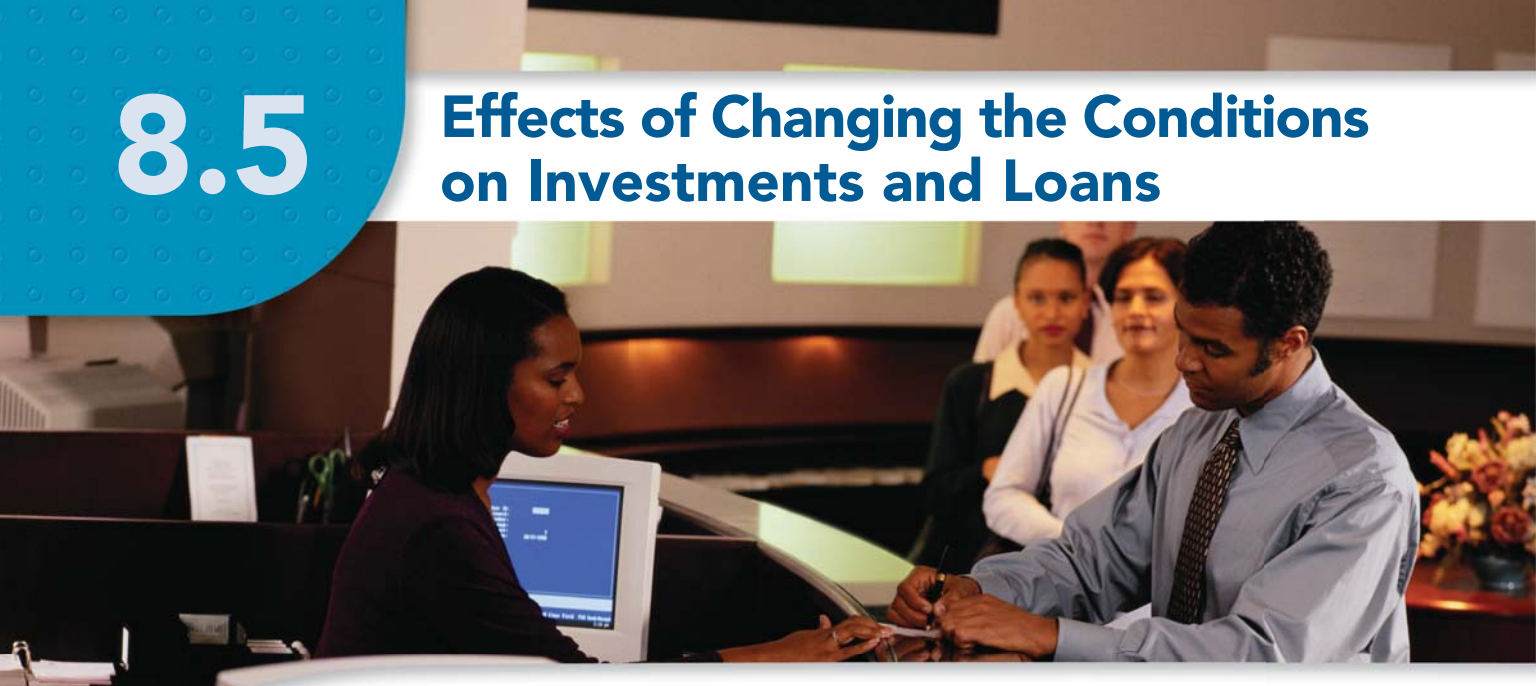
Extend C

15. Use the TVM Solver to compare the amounts of interest paid on a $600 investment, after four years, at different interest rates and different compounding periods. Describe the method you used.

16. The TVM Solver can be used to determine interest rates. Reed owns a bond that will pay $1200 when it matures in four years. Naomi offered to buy the bond today for $1000. Use a TVM Solver to determine the annual interest rate that Naomi is offering, if interest is compounded semi-annually.

17. Use the TVM Solver to determine the interest rates, compounded semi-annually, quarterly, and monthly, that would give the same interest after one year as 10% per year, simple interest. Describe the method you used.

8.5 Effects of Changing the Conditions on Investments and Loans



Banks, paycheque advance companies, loan companies, and stores lend money to their customers at varying interest rates, compounding periods, and terms. It is important for consumers to understand how changing the conditions of a loan will affect the amount they will have to repay. This is also true for changing the conditions of an investment.

Investigate

Tools

- graphing calculator

Investments

Ken deposited $500 into an investment fund that has historically earned 11.3% per year, compounded annually. He intends to leave the money in the fund for at least four years.

1. **a)** Assuming the same rate of return, how much will Ken's investment be worth in four years?
   b) Predict the value of Ken's investment in four years if the rate of return is doubled.
   c) Verify your prediction using technology. Describe the results.
2. **Reflect** Describe how doubling the interest rate affects the value of an investment. Does the amount or the total interest paid double? Explain.

3. **a)** Predict the value of the investment if Ken doubles the amount of time he keeps his money in the fund.

   b) Verify your prediction using technology. Describe the results.

4. **Reflect** Describe how doubling the amount of time money is invested affects the value of an investment. Does the amount or the total interest paid double? Explain.

Example 1

Use Technology to Compare Different Compounding Periods

Anna has $2000 available to invest at 12% per year, compounded annually. She will need the money in six to eight years to finance her children's education.

a) Use a graphing calculator, graph $A = 2000(1.12)^n$.

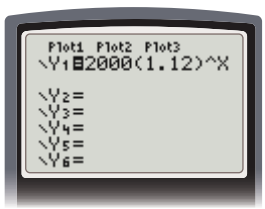
b) Use the **CALC** feature to determine the value of A for

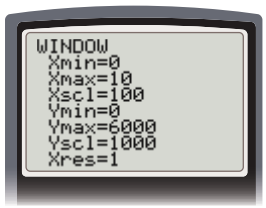
i) $n = 6$ **ii)** $n = 7$ **iii)** $n = 8$

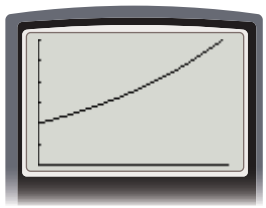
c) Describe the shape of the graph in part a). What happens when you change the total length of time the money is invested?

Solution

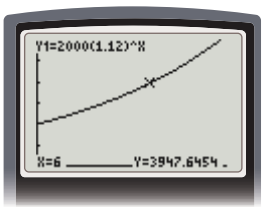
a)





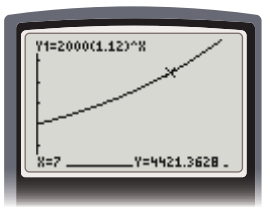


b) i)



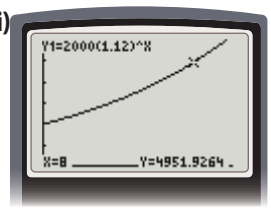
When $n = 6$, the amount is $3947.65.

ii)



When $n = 7$, the amount is $4421.36.

iii)



When $n = 8$, the amount is $4951.93.

c) The graph gets steeper from left to right.
Growth between the sixth and seventh years:
$\$4421.36 - \$3947.65 = \$473.72$
Growth between the seventh and eighth years:
$\$4951.93 - \$4421.36 = \$530.56$
As the total length of time increases, the investment grows by an increasing amount each year.

Example 2

Future Investment

Tyler wants to have $5000 in four years time. How much would Tyler need to invest today at 4% per year, compounded quarterly? at 4.5% per year, compounded quarterly?

Solution

Technology Tip
For the TVM Solver:
A = FV
P = PV
n = N

Method 1: Use a TVM Solver

For 4%, enter the following information in the TVM Solver.

Move the cursor to **PV** and press ALPHA [SOLVE].

PV = 4264.11

For 4.5%, enter the following information in the TVM Solver.

Move the cursor to **PV** and press ALPHA [SOLVE].

PV = 4180.55

Tyler would have to invest $4264.11 at 4%, or $4180.55 at 4.5%.

Method 2: Use a Scientific Calculator

For 4%:

$i = 0.04 \div 4$
$= 0.01$
$n = 4 \times 4$
$= 16$
$A = 5000$
$P = 5000(1 + 0.01)^{-16}$
$= 4264.11$ 5000 [×] 1.01 [y^x] [±] 16 [=]

For 4.5%:

$i = 0.045 \div 4$
$= 0.011\,25$
$n = 4 \times 4$
$= 16$
$A = 5000$
$P = 5000(1 + 0.011\,25)^{-16}$
$= 4180.55$ 5000 [×] 1.01125 [y^x] [±] 16 [=]

Tyler would have to invest $4264.11 at 4%, or $4180.55 at 4.5%.

Technology Tip
Calculators vary. You may need to enter [y^x] 16 [±] to get the negative exponent.

Key Concepts

- When changing any conditions of an investment or loan, the amount or principal will also change.
- Doubling an interest rate or term more than doubles the total interest. This is due to the effects of compounding.
- The more frequent the compounding period, the greater the effects of any changes to the investment or loan.

Discuss the Concepts

D1. Erin has $1200 to invest. Determine, without calculating, which rate would provide a greater amount of interest after three years:
- 5% per year, compounded quarterly
- 5% per year, compounded semi-annually

Explain your answer.

D2. A $3000 debt is due to be repaid in four years. When attempting to sell the debt, the creditor received a smaller principal value with a higher interest rate. Should the creditor have been surprised? Explain.

Practise A

For help with questions 1 and 2, refer to Example 1.

1. For a $1500 investment, at 7% per year, compounded semi-annually, use a graph to compare the final amounts and total interest after

a) three years

b) four years

c) five years

2. Compare the graphs of $A = 1000(1.03)^n$ and $A = 1000(1.05)^n$. How does changing the compound interest rate from 3% per year to 5% per year affect the shape of the graph? What does this mean in terms of the value of the investment?

For help with question 3, refer to the Investigate.

3. A $675 investment earns interest at 3.4% per year, compounded semi-annually, for five years. How will the interest and the amount be affected if you double

a) the interest rate?

b) the total length of time?

For help with question 4, refer to Example 2.

4. Nobuko hopes to have $3000 in two years to buy a home theatre system. Use technology to find the amounts she would need to invest to reach her goal at

a) 4% per year, compounded semi-annually

b) 5% per year, compounded semi-annually

Apply B

5. Renée has $3400 to invest for three years at 6% per year. She is wondering how much the compounding period affects the amount. Investigate what happens to Renée's investment for compounding periods that are annual, semi-annual, quarterly, and monthly. Describe the results.

6. Terry is confused about the various compounding periods offered by his bank. If he deposits $6000 into an investment account for one year at 5% per year, how much more interest will he earn by compounding

a) semi-annually instead of annually?

b) quarterly instead of annually?

c) monthly instead of annually?

7. Raheela plans to purchase a new car in three years and hopes to have $18 000 at that time.

a) Determine the principal that Raheela needs to invest today to have $18 000 after three years

i) at 4.5% per year, compounded monthly

ii) at 4.5% per year, compounded semi-annually

b) Which principal is less? Why?

8. Barb plans to invest $10 000 in a term deposit for two years. She has three choices.

A 6.8% per year, simple interest

B 6.2% per year, compounded semi-annually

C 6.0% per year, compounded quarterly

Which plan should she choose? Why?

9. Jayeed recently inherited $8000. He plans to use half the money now and invest the other half for at least three years. He has narrowed the investment down to three choices.

A 3.25% per year, simple interest, cashable any time

B 3.0% per year, compounded monthly, cashable after two years

C 3.5% per year, compounded semi-annually, cashable after four years

a) Which plan earns the most interest after four years? Does that mean it is the best option? Justify your response.

b) Jayeed decides he will use the investment to make a down payment on a car in two and a half years. Which plan should he choose? Why?

10. A paycheque advance company will give you money for your paycheque two weeks before you receive it, provided you show evidence of a regular paycheque from your employer.

a) For a $1200 paycheque, how much money would Jasper receive?

b) What annual interest rate, compounded weekly, is Jasper paying for this loan?

Chapter Problem

11. The Kwans' business has been successful. They wish to invest $25 000 at 5% per year, compounded semi-annually, and a further $25 000 at 4.8% per year, compounded monthly. Each investment will be for 10 years.

a) Calculate the difference in interest earned by the two investments.

Literacy Connect

b) Write a short paragraph explaining the difference in the interest earned.

12. **a)** How much needs to be invested today to have $10 000 after five years

i) at 6% per year, compounded monthly?

ii) at 6% per year, compounded semi-annually?

b) Which principal is greater? Why?

13. Interest rates vary depending on the size of the investment and the term. Visit your local bank and find out its current interest rates for different principal investments in guaranteed investment certificates (GICs) for specific periods of time. Copy the table, then use your research to complete the table.

Principal ($)	Term (years)	Interest Rate (%)	Compounding Period	Amount ($)
500	1		monthly	
500	3		quarterly	
1 000	5		semi-annually	
2 000	1		annually	
5 000	5		monthly	
10 000	2		quarterly	
10 000	5		semi-annually	
10 000	5		annually	

Achievement Check

14. **a)** Suppose you need $10 000 in five years to repay some college expenses. Compare the amounts you would need to invest in these three options to reach your goal.

A A bond that pays interest at 7% per year, compounded monthly.

B An investment fund that pays interest at 8% per year, compounded semi-annually.

C A term deposit account that pays interest at 6.5% per year, compounded daily.

b) Suppose you invest $7000 now. What yearly interest rate, compounded annually, is necessary to have enough to repay the expenses from part a)?

Extend C

15. On the birth of their son Jamison, the Turleys invested $5000 at 4.9% per year, compounded annually. On Jamison's fifth birthday, the investment matured and his parents reinvested the amount at 5.1% per year, compounded semi-annually, for five years. On his tenth birthday, his parents reinvested the amount at 5.3% per year, compounded quarterly, for five more years. How much more interest did the Turleys earn than if they had invested the $5000 for 15 years at 4.8% per year, compounded monthly?

16. Executives at a company have decided to acquire a corporate jet for company use. They can purchase the aircraft for $2 000 000, and resell it after 10 years for $1 500 000. Interest lost on the money used for the purchase is estimated at 5% per year, compounded semi-annually. Alternatively, they can lease the aircraft for $200 000 per year.

a) Calculate the cost of purchasing the aircraft, keeping it for 10 years, and then reselling it. Include the cost of lost interest on the money used for the purchase.

b) Calculate the cost of leasing the aircraft for 10 years.

Literacy Connect

c) Write a short paragraph advising the president of the company on which plan should be adopted.

17. Jessica plans to invest $4000 for five years, at which time she wants to have at least $6000. What interest rate, rounded to two decimal places, does she require, if the investment is compounded

a) annually?

b) semi-annually?

c) quarterly?

d) monthly?

Math Connect

The effective annual interest rate is the equivalent annual interest rate when compounding occurs more often than once a year.

18. Susan received a credit card bill for $2122.67 at the end of December. She decided to wait until the end of January to pay the bill. At the end of January she received a bill for $2159.82 even though she had made no further purchases. Use a TVM Solver to determine the effective annual interest rate, compounded monthly.

8 Review

8.1 Simple and Compound Interest, pages 422–429

1. Show the growth of a $2000 investment, at both 5% per year simple interest and 5% per year, compounded annually, for six years, using a table and a graph.

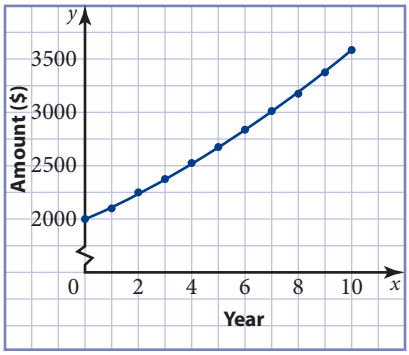
2. Use tables and graphs to compare the amounts after four years for a $1500 investment at

a) 3% per year, compounded annually

b) 3.5% per year, compounded annually

c) 4% per year, compounded annually

3. The graph shows the growth of a $2000 investment at 6% per year, compounded annually.



a) How much is the investment worth after five years?

b) Estimate the time it would take for the investment to double in value.

c) How would the graph change if the interest rate were 4% per year, compounded annually? Justify your response.

8.2 Compound Interest, pages 430–435

4. Substitute the values into the formula $A = P(1 + i)^n$. Do not evaluate.

a) a $600 investment at 7% per year, compounded semi-annually, for three years

b) a $4000 loan at 9% per year, compounded quarterly, for five years

c) a $6000 loan at 8.4% per year, compounded monthly, for three years

d) a $1200 investment at 4.5% per year, compounded semi-annually, for two years

5. Bill made two investments.

A $5000 at 5.5% per year, compounded quarterly, for four years

B $2500 at 5.8% per year, compounded semi-annually, for four years

a) Which investment earned him more money?

b) What is the total interest earned on his investments?

6. Barbara borrowed $2300 at 10% per year, compounded quarterly.

a) How much must she repay after five years?

b) How much of the amount she repays is interest?

8.3 Present Value, pages 436–441

7. Neaz wants to invest enough money today so that his son will have $4800 toward his expenses when he goes to college in five years. If the annual interest rate is 5.7% per year, compounded monthly, how much should Neaz invest?

8. Suppose you owe a sum of $10 000 due in six years. Your creditor is willing to accept early payment of the loan by discounting it at 9.6% per year, compounded monthly. How much should your creditor be willing to accept to pay off the loan today?

9. Jai wants to buy a car. Which is the better deal, with interest rates at 5% per year, compounded semi-annually?

- **Plan A**: $16 250 cash now
- **Plan B**: $1000 down plus $15 500 in one year
- **Plan C**: $500 down plus $16 000 in one year

8.4 The TVM Solver, pages 442–445

10. Copy and complete the table. Use a TVM Solver.

Present Value ($)	Future Value ($)	Term (years)	Compounding Period	Annual Interest Rate (%)
8 000	12 000	5	monthly	
6 000	13 000	10	semi-annually	
1 340	2 000		quarterly	6
100 000	1 000 000		semi-annually	8
4 000		3	monthly	3
	25 000	8	quarterly	5.5

11. A $1000 investment earns interest at 4% per year, compounded quarterly.

a) How long will it take to double the value of the investment?

b) Would other investments double in the same length of time? Explain.

8.5 Effects of Changing the Conditions on Investments and Loans, pages 446–453

12. For a $3000 investment, at 6% per year, compounded quarterly, use a graph to compare the final amounts after each time period.

a) one year **b)** two years

c) three years

13. You deposit $2000 into an investment account for two years at 7% per year. How much will you earn if interest is compounded

a) annually?

b) semi-annually?

c) quarterly?

d) monthly?

14. Marlon can purchase a company car for his real estate business for $30 000. He expects to sell the car for $12 000 after five years. Interest lost on the money used for the purchase is estimated at 6% per year, compounded quarterly. Alternatively, Marlon can lease the car for $4000 per year.

a) Calculate the cost of purchasing the car, keeping it for five years, and then selling it. Include the cost of lost interest on the money used for the purchase.

b) Calculate the cost of leasing the car for five years.

c) Which plan is a better deal for Marlon? How much more would he pay with the other plan?

8 Practice Test

For questions 1 to 4, choose the best answer.

1. Which statement is false when comparing simple and compound interest?

A Simple interest is paid on the principal value but compound interest is paid on an accumulating value.

B Compound interest is always greater than simple interest.

C Simple interest is an example of linear growth and compound interest is an example of exponential growth.

D Compound interest grows faster than simple interest after the first interest period, if the yearly interest rates are equal.

2. When money is invested at 5% per year, compounded semi-annually, for five years,

A $n = 5$ and $i = 0.05$

B $n = 5$ and $i = 0.025$

C $n = 10$ and $i = 0.025$

D $n = 10$ and $i = 0.05$

3. When changing the compounding period on an investment, which statement is true?

A More frequent compounding results in a greater amount of interest.

B More frequent compounding results in the same amount of interest.

C More frequent compounding results in a lesser amount of interest.

D More frequent compounding changes both the principal and amount, so there is no consistent result.

4. Which formulas are not correct?

A $P = A(1 + i)^n$

B $A = P(1 + i)^n$

C $P = A(1 + i)^{-n}$

D $P = A(1 - i)^n$

5. Show the growth of an $1000 investment, at 7% per year, simple interest, and at 7% per year, compounded annually, for 10 years. Use a table and a graph.

6. A credit card company charges interest at 18.5% per year, compounded monthly. Andrea has an unpaid balance of $768.42. If she does not pay off her balance and makes no further purchases, how much will she owe after

a) one month?

b) three months?

7. Brenda and Al have $5000 to invest at 6% per year, compounded quarterly. How long would it take for their investment to grow to $8000?

8. Erik needs to borrow $2000. Which loan should he take?

A $2000 for three years at 10% per year, compounded semi-annually

B $2000 for three years at 9.2% per year, compounded quarterly

Justify your response.

9. If $4000 is invested for 10 years, what annual interest rate, compounded semi-annually, would double the money?

Chapter Problem Wrap-Up

In Section 8.1, you illustrated the growth of money the Kwans set aside to begin a business. In Section 8.3, you determined how much the Kwans needed to set aside to have money for business growth. In Section 8.5, you explained the effects of two compounding periods on their investments.

The Kwans are ready to sell their business for $250 000. They will invest the proceeds from the sale, along with the amounts of their investments in Section 8.5. Research current interest rates and provide a plan for the Kwans to invest their money in six different ways. Each investment should mature in a different year, over the next 10 years. Write a report for the Kwans and include the details of their investments, as well as showing the growth of their investments each year until they mature.

10. Jeeva is 10 years old. His parents have decided to invest some money for his education, so that he will have $15 000 at age 18 when he goes to college. If the investment can earn 6.6% per year, compounded monthly, how much will his parents need to invest?

11. Use words, numbers, and graphs to illustrate the differences among investing $1000 for five years, at 4.5% per year, compounded

a) annually **b)** semi-annually

c) quarterly **d)** monthly

12. A perpetuity is an investment that continues forever, paying out the interest but leaving the principal untouched. The interest rate depends on the economy at the time. A school sets up a scholarship perpetuity with a donation of $50 000. The recipient receives a scholarship worth the amount of the interest earned in that year. The interest rates for the last five years are shown in the table.

Annual Interest Rate (%)	Compounding Period	Scholarship Amount ($)
8	semi-annually	
7.5	quarterly	
5.5	semi-annually	
7	semi-annually	
9	annually	

a) Find the amount of each scholarship.

b) What yearly interest rate, compounded annually, is necessary to guarantee a scholarship of at least $5000?