2 Probability

What would happen if people could predict details of the future? Farmers would know exactly when to plant and harvest their crops. Doctors could identify those at risk of contracting a disease. Businesses would know which products to develop and how to market them. Fortunes would be made in the stock market.

In this chapter, you will learn how probability is used to predict the likelihood that an event will happen.

In this chapter, you will

- identify examples of the use of probability in the media and various ways in which probability is represented
- determine the theoretical probability of an event, and represent the probability in a variety of ways
- perform a probability experiment, represent the results using a frequency distribution, and use the distribution to determine the experimental probability of an event
- compare, through investigation, the theoretical probability of an event with the experimental probability, and explain why they might differ
- determine, through investigation using class-generated data and technology-based simulation models, the tendency of experimental probability to approach theoretical probability as the number of trials in an experiment increases
- interpret information involving the use of probability and statistics in the media, and make connections between probability and statistics

Key Terms

data	statistics
event	theoretical probability
experimental probability	trial
outcome	

Rabia completed the two-year fish and wildlife technician course at Sault College. She works as a park conservation officer, protecting wildlife and the environment. Rabia applies her skills in probability and statistics to provide reliable data on fish and wildlife populations.

Prerequisite Skills

Fractions, Decimals, and Percents

1. Express each fraction as a decimal without the use of a calculator.

a) $\frac{97}{100}$ **b)** $\frac{2}{5}$

c) $\frac{3}{20}$ **d)** $\frac{5}{8}$

2. Use a calculator to express each fraction as a decimal. Round your answers to four decimal places, if necessary.

a) $\frac{17}{40}$ **b)** $\frac{4}{13}$

c) $\frac{5}{6}$ **d)** $\frac{4}{9}$

3. Express each decimal as a fraction in lowest terms.

a) 0.75 **b)** 0.16

c) 0.65 **d)** 0.125

e) 0.3333… **f)** 0.001

4. Express each percent as a fraction in lowest terms.

a) 30% **b)** 25%

c) 80% **d)** 45%

e) $66.\overline{6}\%$ **f)** 100%

5. Evaluate without the use of a calculator. Express answers as a fraction in lowest terms.

a) $1 - \frac{1}{4}$ **b)** $\frac{1}{2} - \frac{1}{6}$

c) $\frac{1}{5}$ of 80 **d)** $\frac{3}{13} \times \frac{1}{6}$

6. Use a calculator to evaluate each expression in question 5. If your calculator has a fraction button, answer as a fraction.

Interpreting Data

7. The table shows the results of rolling a six-sided die several times.

Result	Frequency
1	3
2	4
3	3
4	5
5	2
6	1

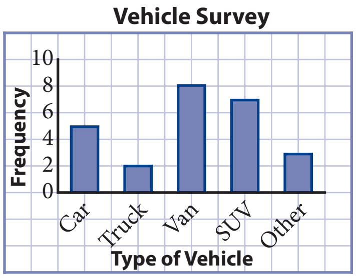
a) What was the total number of rolls?

b) What percent of the total number of rolls resulted in a 4?

c) What fraction of the total number of rolls resulted in an even number?

d) For the number of rolls that resulted in an even number, what percent resulted in a 2?

8. Consider the following graph.



a) What type of graph is this?

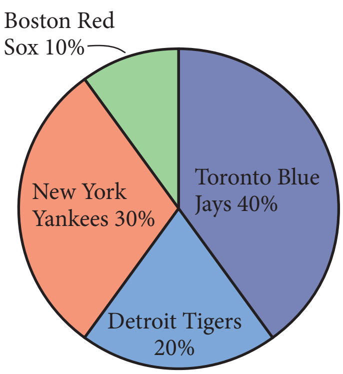
b) How many vehicles were seen?

c) What was the most popular vehicle?

d) What fraction of the vehicles were cars?

e) What percent of the vehicles were trucks?

9. Two hundred people were surveyed. The results are shown in the graph.

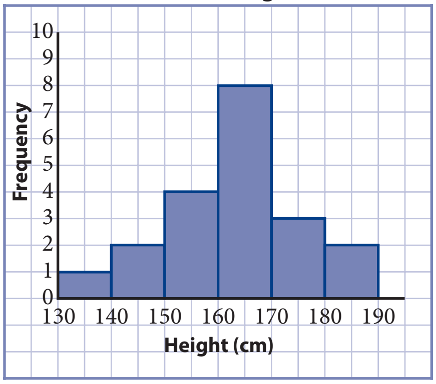


a) Of the people surveyed, how many prefer the Boston Red Sox?

b) What fraction of the people surveyed prefer the Toronto Blue Jays?

c) What percent of the people surveyed prefer the Blue Jays or the New York Yankees?

10. The histogram shows the heights of the students in Mr. Lee's math class.



a) How many students are in the class?

b) How many students are between 160 cm and 170 cm tall?

c) What percent of students are shorter than 160 cm?

d) What fraction of students are taller than 150 cm?

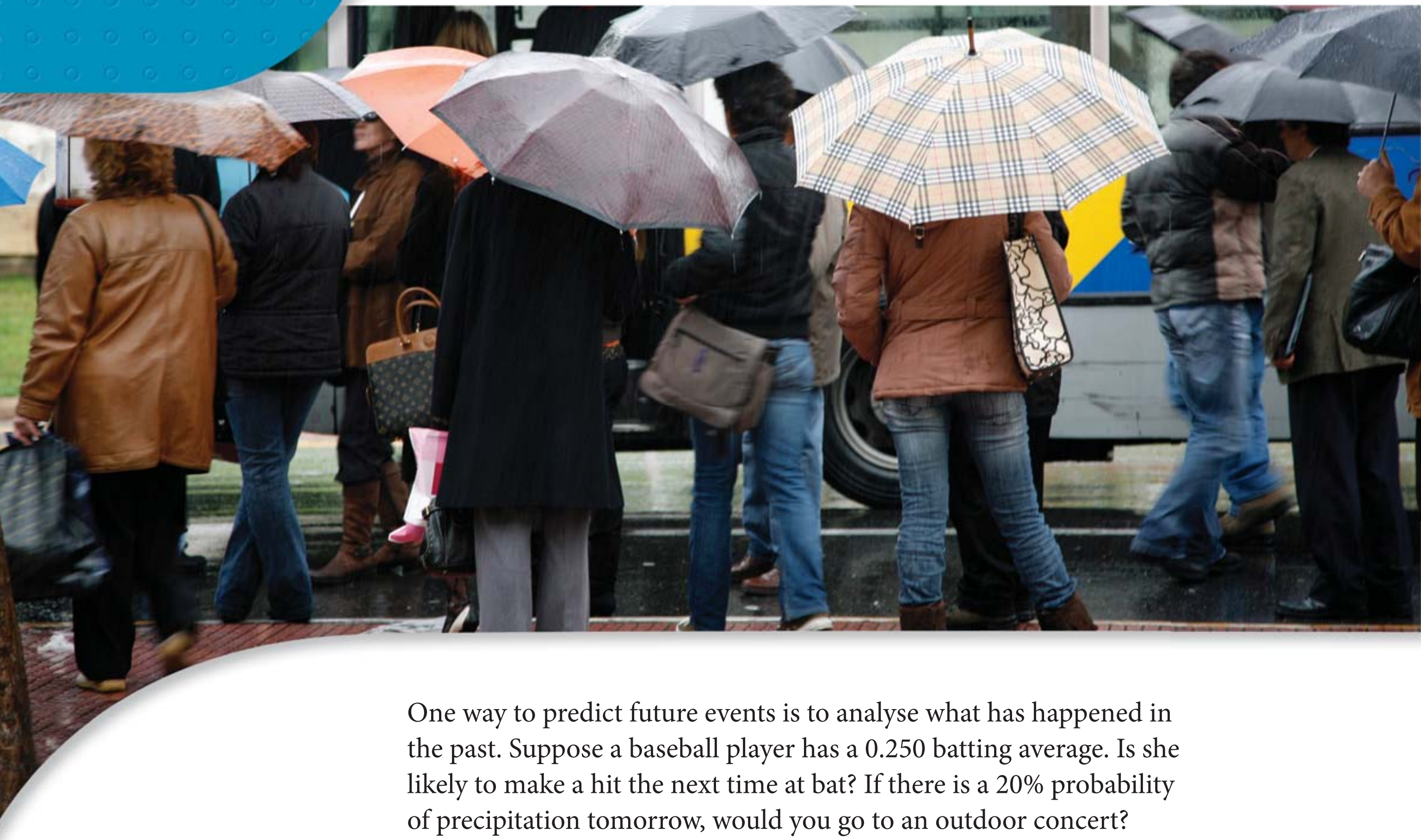
Chapter Problem

Every five years, Statistics Canada conducts a census to collect information about the residents of Canada.

Suggest reasons why it is not possible to conduct a census on wildlife populations. What groups of people would be interested in this kind of information?

Many people rely directly or indirectly on the fishing industry. Sport fishing is very popular throughout Ontario and is important to tourism. How can probability and statistics be used to provide reliable data on fish populations?

2.1 Probability Experiments



One way to predict future events is to analyse what has happened in the past. Suppose a baseball player has a 0.250 batting average. Is she likely to make a hit the next time at bat? If there is a 20% probability of precipitation tomorrow, would you go to an outdoor concert?

Investigate 1

Tools

- 10 coloured tiles; five red and five blue
- opaque bag or box to hold tiles

outcome
- a possible result of an experiment

event
- a set of outcomes with the same result

trial
- one round of a probability experiment

Odd or Even?

Consider this experiment.

Place five red and five blue coloured tiles in a bag. Without looking, take out two tiles. The two selected tiles are called an **outcome**. If the tiles are the same colour, the **event** is considered even. If the tiles are not the same colour, the event is considered odd. Then replace the tiles.

Work with a partner.

1. Suppose you conducted 20 **trials** of the experiment. How many odd results would you expect to get? How many even results?
2. Conduct 20 trials of the experiment. Record the results in a tally chart.
3. Compare your answers to questions 1 and 2. How did the actual results compare to your expectations?

4. Combine your results with those of nine other pairs of students. How do the combined results compare to your expectations?

5. **Reflect** Suppose you were to predict the number of odd events from five trials of the experiment. Which results would you use to make your prediction, your results from question 2, or the combined results from question 4? Explain your choice.

In the Investigate you conducted a probability experiment. The experiment consisted of 20 trials. In each trial, two tiles were drawn. Each trial had two possible events: odd or even.

Investigate 2

Tools

- 15 coloured tiles: 7 yellow, 5 red, and 3 blue
- opaque bag or box to hold tiles

Gone Fishing

The Ministry of Natural Resources wants to know the ratio of bass to carp to catfish in a lake. The ministry has asked registered anglers to keep track of the number of each type of fish they catch. You will use coloured tiles to simulate this method of analysing fish populations.

1. Put the 15 tiles in a bag. The yellow tiles will represent the number of bass in the lake, the red tiles will represent the number of carp, and the blue tiles will represent the number of catfish.

2. Without looking, draw one tile from the bag. Record the results in a tally chart. Replace the tile.

3. Suppose you repeated this experiment 30 times. What fraction of the events do you expect to be bass? What fraction do you expect to be carp?

4. Conduct 30 trials of this experiment. Record the results in a tally chart.

5. Use the data from your tally chart. Draw a bar graph showing the number of each type of fish.

6. a) Write your experimental results as a ratio, bass : carp : catfish.
   b) How does this ratio compare to the ratio of the colours of tiles in the bag?
   c) Does the simulation give a reasonable estimate of the ratio of the colours of tiles in the bag? Explain.

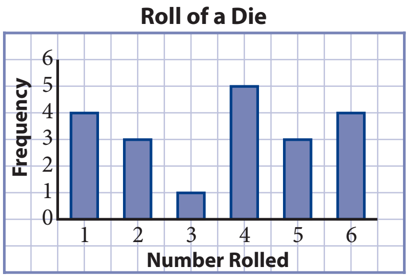
7. **Reflect** If you performed 100 trials instead of 30, would the estimate likely be more or less accurate? Explain.

8. **Reflect** Do you think the number of each type of fish caught is an accurate indicator of the actual ratio of fish in the lake? Explain.

Example 1

Roll a Single Die

The results of rolling a six-sided die are displayed in the graph.



experimental probability
- determined using the results of an experiment or simulation
- $\frac{\text{number of successful trials}}{\text{total number of trials}}$

a) How many times was a 5 rolled?

b) Find the **experimental probability** of rolling a 6. Express your answer as a fraction in lowest terms, as a decimal, and as a percent.

c) Find the experimental probability of **not** rolling a 6. How is this related to the probability of rolling a 6?

d) How would you expect the heights of the bars to relate to each other? Explain.

Solution

a) From the graph, a 5 was rolled three times.

b) Add the frequencies to determine the total number of rolls.
$4 + 3 + 1 + 5 + 3 + 4 = 20$

The number 6 was rolled four times.

$$\begin{aligned} P(\text{rolling } 6) &= \frac{\text{number of successful trials}}{\text{total number of trials}} \\ &= \frac{\text{number of times 6 was rolled}}{\text{total number of trials}} \\ &= \frac{4}{20} \\ &= \frac{1}{5} \end{aligned}$$

Express $\frac{1}{5}$ as a decimal.

$\frac{1}{5} = 0.2$

Express $\frac{1}{5}$ as a percent.

$\frac{1}{5} = 20\%$

The probability of rolling a 6 is $\frac{1}{5}$, or 0.2, or 20%.

c) From part b), the total number of rolls is 20. The number 6 was rolled four times. So, a different number turned up 16 times.

$$\begin{aligned} P(\text{not rolling } 6) &= \frac{\text{number of successful trials}}{\text{total number of trials}} \\ &= \frac{\text{number of times a number other than 6 was rolled}}{\text{total number of trials}} \\ &= \frac{16}{20} \\ &= \frac{4}{5} \end{aligned}$$

$$\begin{aligned} P(\text{rolling a } 6) + P(\text{not rolling a } 6) &= \frac{1}{5} + \frac{4}{5} \\ &= 1 \end{aligned}$$

It is certain the number rolled will be either a 6 or a number other than 6. So, the sum of the probabilities is 1.

d) Because each number is equally likely, you would expect 1, 2, 3, 4, 5, and 6 to turn up an equal number of times. The bar graph would have all bars of equal height. Since one roll of the die does not depend on the previous rolls, the likelihood of getting a particular number on each roll is the same. In 20 rolls of the die, it is possible to get all 6s. Experimental probability is a measure of what actually happened, not what is expected to happen.

It is possible to have zero successful trials, which gives a probability of 0. For example, the probability of rolling a 7 with a single die is 0.

The number of successful trials can equal the total number of trials. In this case, the probability is 1. For example, the probability of rolling a 1, 2, 3, 4, 5, or 6 is 1.

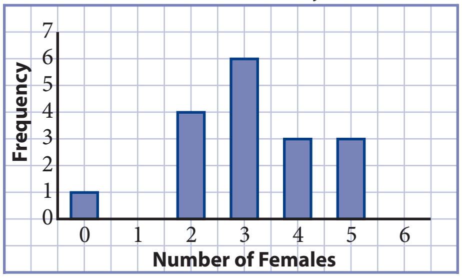
Probability always has a value between 0 (certain not to happen) and 1 (certain to happen).

Example 2

How Many Females?

A probability experiment was designed to find the expected number of females in a family of six children. To simulate the genders of the six children, a coin was tossed six times. Heads represented a male; tails represented a female. The experiment was repeated a number of times. The results are shown in the graph.

Six Children: How Many Females



a) How many trials were performed?

b) What is the experimental probability of having two females in a family of six children?

c) According to this experiment, what is the average number of females in a family with six children? How can you tell?

Solution

a) The height of each bar indicates the number of times a trial predicted a given number of children. For example, the event "zero females" occurred one time. The numbers of occurrences were 1, 0, 4, 6, 3, 3, and 0 respectively.

Their sum is: $1 + 0 + 4 + 6 + 3 + 3 + 0 = 17$.
There were 17 trials.

b) From the graph, the event "two females" occurred four times.

$$
\begin{aligned}
P(2\text{ females}) &= \frac{\text{number of successful trials}}{\text{total number of trials}} \\
&= \frac{\text{number of trials with 2 females}}{\text{total number of trials}} \\
&= \frac{4}{17}
\end{aligned}
$$

c) The average number of females per family of six will be the total number of females from the experiment divided by the total number of trials.

The total number of females can be shown in a table.

Number of Females Per Family	Occurrences	Total Number of Females
0	1	$0 \times 1 = 0$
1	0	$1 \times 0 = 0$
2	4	$2 \times 4 = 8$
3	6	$3 \times 6 = 18$
4	3	$4 \times 3 = 12$
5	3	$5 \times 3 = 15$
6	0	$6 \times 0 = 0$

$$\begin{aligned}\text{Total number of females} &= 8 + 18 + 12 + 15 \\ &= 53\end{aligned}$$

$$\begin{aligned}\text{Average number of females per family} &= \frac{\text{total number of females}}{\text{total number of trials}} \\ &= \frac{53}{17} \\ &\doteq 3.1\end{aligned}$$

From this experiment, the average number of females in a family with six children is 3.1.

Key Concepts

- Probability is a measure of the likelihood that a specific event will occur.
- Probability experiments can be used to estimate the probability of an event.
- Experimental probability $= \dfrac{\text{number of successful trials}}{\text{total number of trials}}$.
- Probability is always a value between 0 and 1.

Discuss the Concepts

D1. Does experimental probability always give an accurate prediction of the likelihood that an event will occur? Explain.

D2. Which would you consider to be more accurate: a probability experiment with five trials, or one with 100 trials? Explain.

D3. In a probability experiment, you toss a fair coin 10 times. Is it possible that heads will turn up 10 times? Explain.

Practise A

For help with question 1, refer to Example 1.

1. In a probability experiment, 15 out of 50 trials were successful.

a) Determine the experimental probability of a successful trial. Express your answer as a fraction in lowest terms, as a percent, and as a decimal.

b) Write the probability of an unsuccessful trial as a fraction in lowest terms. Explain how you got your answer.

2. Two six-sided dice were rolled 20 times. Doubles were rolled four times. Determine the experimental probability of rolling doubles. Express your answer as a fraction in lowest terms, as a percent, and as a decimal.

3. A coin was tossed 30 times. The experimental probability of turning up heads was $\frac{2}{5}$.

a) How many times did the coin turn up heads?

b) How many times did it turn up tails?

c) What was the experimental probability of it turning up tails? Describe two different methods of finding the answer.

For help with question 4, refer to Example 2.

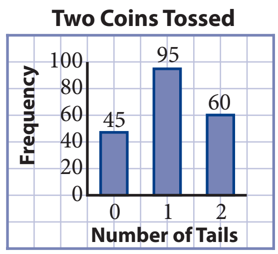
4. Two coins were tossed a total of 200 times. The results are shown in the graph.

Find the experimental probability for each event.

a) two heads

b) one head

c) two tails



Apply B

Literacy Connect

5. Refer to the graph in question 4.

a) How do you think the bars for two heads and two tails should compare? Explain your reasoning.

b) What are the two different outcomes that result in the event "one head"?

c) Based on the information in the graph, what fraction of the time would you expect the event "one head"?

d) Use your answers to parts a) and c) to determine what fraction of the time you would expect to get HH and TT.

6. Sam tossed a coin 10 times and heads turned up nine times.

a) Express the experimental probability of turning up heads as a percent.

b) Is the experimental probability what you expected? Explain.

7. At a light bulb factory, a batch of bulbs is rejected if more than 5% of the bulbs in a sample taken from the batch are defective.

a) If 240 bulbs are tested and eight are defective, will the batch be rejected? Show your calculations.

b) In a batch of 1000 bulbs, exactly 100 are defective. A sample of 200 bulbs from the batch of 1000 is tested. Do you expect the batch will be rejected? Explain your reasoning.

c) In a large batch of bulbs, 1% of the bulbs are defective. A sample of the bulbs is tested. Is it possible that the entire batch will be rejected? Explain your reasoning.

8. In Investigate 1, two tiles were taken from a bag containing 10 tiles. There were five tiles each of two different colours. Sandy drew *evens* 12 times out of 20. Is each statement true or false? Explain.

a) The experimental probability of drawing *evens* is $\frac{3}{5}$.

b) The event *evens* was more likely than the event *odds*.

Chapter Problem

9. A Ministry of Natural Resources employee caught and tagged 100 fish in a small lake. Two weeks later, 100 fish were caught, 10 of which had previously been tagged.

a) Estimate the number of fish in the lake. Explain your reasoning.

b) Suggest some factors that could account for differences between your estimate and a classmate's.

Extend C

10. Refer to Investigate 1. Collect the results from all students in your class and find the total number of odds and evens based on the class results.

a) Does there seem to be a pattern? Is the percent of evens 50%, greater than 50%, or less than 50%?

b) Simulate this problem using technology. Compare the class results with the results you found from your simulation.

Literacy Connect

11. Refer to question 10. Explain why the probability of evens is less than 50%. In your explanation, give the probability as a fraction.

2.2 Theoretical Probability

Consider the following situation.

In a TV game show based on a popular board game, Janna is trying to win money for a charity of her choice. She has to choose between two options:
- automatically win \$500 for her charity
- win \$5000 for her charity if she rolls doubles with two six-sided dice

What would you do?

Investigate 1 Building on Probability

1. Copy and complete each statement. Express the probability as a fraction.

a) When tossing a coin, the probability of it turning up heads is ■.

b) When rolling a single die, the probability of turning up 4 is ■.

c) When drawing a card from a regular deck of playing cards, the probability of it being an ace is ■.

d) A radio station held a contest; 100 people qualified. There will be 10 winners. For the 100 contestants, the probability of winning is ■.

2. Reflect Refer to question 1. For each situation, explain the meaning of the numerator and the denominator in your answer.

3. Reflect Refer to your answer to question 2. Describe how you can calculate the probability of an event.

$$\text{Experimental probability} = \frac{\text{number of successful trials}}{\text{total number of trials}}$$

theoretical probability

- the number of successful outcomes as a fraction of the total number of possible outcomes

Theoretical probability is another measure of the likelihood of an event. It is the ratio of the number of successful outcomes and the total number of possible outcomes.

$$\text{Theoretical probability} = \frac{\text{number of successful outcomes}}{\text{total number of possible outcomes}}$$

To calculate the theoretical probability, all outcomes must be equally likely.

"Equally likely" means the same chance of occurring because the conditions are fair.

For example, in the toss of a fair coin, the chances of getting heads or tails are equally likely.

Investigate 2

Who Gets What?

Jason, Tony, and Lisa have each won a video game from Ace Video Games. Inc. as part of an advertising promotion. The video games, Xtreme Racing, Golf Legends, and Hoops will be randomly assigned to the three winners. What is the probability Jason will receive Golf Legends?

1. Draw a table with three columns, as shown. Label them with the names Jason, Tony, and Lisa.

Jason	Tony	Lisa

2. Use X, G, and H to represent the games. In your table record all the different possible ways the games can be given to the three people.

3. **Reflect**

a) In how many different ways can the three games be arranged?

b) In how many of these arrangements does Jason receive Golf Legends?

c) Assume that all the arrangements are equally likely. What is the theoretical probability that Jason receives Golf Legends?

4. **Reflect** Another way to approach the problem is to consider the number of different games and the fact that there is an equal chance of getting each of them. Explain how this approach will lead to the same result as when we considered the arrangements.

Example 1

Draw!

A standard deck of playing cards has 52 cards, 13 of each suit.

If one card is drawn from the deck, find the probability of each event.

a) a heart

b) an ace

c) a heart, a club, or a jack

d) a black diamond

e) a heart, a club, a spade, or a diamond

Solution

a)
$$\begin{aligned} P(\text{heart}) &= \frac{\text{number of successful outcomes}}{\text{total number of possible outcomes}} \\ &= \frac{\text{number of hearts}}{\text{total number of cards}} \\ &= \frac{13}{52} \\ &= \frac{1}{4} \end{aligned}$$

b) $P(\text{ace}) = \dfrac{\text{number of successful outcomes}}{\text{total number of possible outcomes}}$

$= \dfrac{\text{number of aces}}{\text{total number of cards}}$

$= \dfrac{4}{52}$

$= \dfrac{1}{13}$

c) $P(\text{heart, club, or jack}) = \dfrac{\text{number of successful outcomes}}{\text{total number of possible outcomes}}$

$= \dfrac{\text{number of hearts, clubs, or jacks}}{\text{total number of cards}}$

$= \dfrac{13 + 13 + 2}{52}$

Be careful not to count the jacks twice; two are already included in the hearts and clubs.

$= \dfrac{28}{52}$

$= \dfrac{7}{13}$

d) $P(\text{black diamond}) = \dfrac{\text{number of successful outcomes}}{\text{total number of possible outcomes}}$

$= \dfrac{\text{number of black diamonds}}{\text{total number of cards}}$

$= \dfrac{0}{52}$

There are no black diamonds, so it is impossible to draw.

$= 0$

e) $P(\text{heart, club, spade, or diamond})$

$= \dfrac{\text{number of successful outcomes}}{\text{total number of possible outcomes}}$

$= \dfrac{\text{number of hearts, clubs, spades, diamonds}}{\text{total number of cards}}$

$= \dfrac{13 + 13 + 13 + 13}{52}$

$= \dfrac{52}{52}$

You are certain to draw a heart, club, spade, or diamond.

$= 1$

Example 2

Roll the Dice

Recall Janna's dilemma from the introduction.

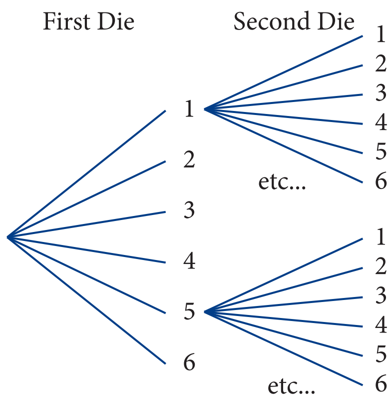
What is the probability of rolling doubles with a pair of dice?

Solution

Method 1: Draw a Tree Diagram or Table

Draw a tree with six branches representing the six outcomes from rolling the first die.

From each branch, draw six more branches representing the six outcomes from rolling the second die. The tree will have a total of 36 branches, representing all possible rolls of the two dice.



There are six outcomes that result in doubles.

$$P(\text{rolling doubles}) = \frac{\text{number of ways of rolling doubles}}{\text{total number of possible rolls}}$$
$$= \frac{6}{36}$$
$$= \frac{1}{6}$$

Another way to show all possible outcomes is to use a table.

	1	**2**	**3**	**4**	**5**	**6**
1	1, 1	1, 2	1, 3	1, 4	1, 5	1, 6
2	2, 1	2, 2	2, 3	2, 4	2, 5	2, 6
3	3, 1	3, 2	3, 3	3, 4	3, 5	3, 6
4	4, 1	4, 2	4, 3	4, 4	4, 5	4, 6
5	5, 1	5, 2	5, 3	5, 4	5, 5	5, 6
6	6, 1	6, 2	6, 3	6, 4	6, 5	6, 6

Method 2: Work With a Simpler Problem

Assume one die has been rolled. There are six possible outcomes for the second roll, one of which will match the results of rolling the first die, giving doubles.

$$P(\text{rolling doubles}) = P(\text{second die will match the first die})$$
$$= \frac{1}{6}$$

The probability of rolling the same number as on the first die is $\frac{1}{6}$.

Key Concepts

- Theoretical probability $= \dfrac{\text{number of successful outcomes}}{\text{total possible number of outcomes}}$, where all outcomes are equally likely.
- The probability of an event is a value between 0 and 1.

Discuss the Concepts

D1. Use the formula for theoretical probability to explain why the value must be between 0 and 1.

D2. Give an example of an event that has a 0 probability of happening. Explain why the probability is 0.

Practise A

For help with questions 1 and 2, refer to Example 1.

1. A card is randomly selected from a standard deck of cards. Write the theoretical probability of each event as a fraction in lowest terms.

a) a spade

b) a face card (jack, queen, or king)

c) not a face card

d) a black jack

e) a red or black card

f) a red face card

2. Queen's Slipper playing cards are popular in Australia. In addition to the standard 52 cards, a deck contains the 11, 12, and 13 of each suit. When drawing a card from such a deck, find the theoretical probability of drawing

a) an 11

b) an 11, 12, or 13

3. Britt rolls a regular six-sided die. Find the theoretical probability of each event. Express your answer as a fraction in lowest terms.

a) rolling a 6

b) rolling a number greater than 3

c) rolling an 8

d) rolling an even number

4. A pet store has 10 cats, 12 dogs, and 3 turtles. If Bobby randomly selects a pet to take home, find the theoretical probability of getting
   a) a cat
   b) a turtle
   c) a dog or a turtle

For help with question 5, refer to Example 2.

5. Suppose you roll two six-sided dice. Find the theoretical probability of rolling each sum. Express each answer as a fraction in lowest terms.
   a) 2
   b) 11
   c) a sum greater than 5
   d) 7
   e) not 7

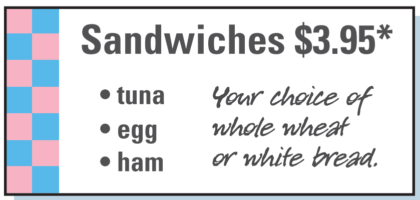
Apply B

6. During a game of musical chairs, 10 people walk around eight chairs waiting for the music to stop. Find the probability of a person not getting a chair.

7. Twelve charms representing the 12 months of the year are attached, in order, onto a chain bracelet. Find the probability that the clasp is between the charms for June and July. Include a diagram with your solution.

8. Suppose you roll two six-sided dice.
   a) Explain why the probability of rolling a sum of 14 is 0.
   b) Explain why the probability of rolling a sum from 2 to 12 is 1.

9. Ronald's Restaurant offers this sandwich menu.



   a) How many different types of sandwiches are possible?
   b) Suppose you like all sandwich fillings and both types of bread. If you randomly selected a sandwich, what is the probability of each sandwich being
      i) tuna on white bread?
      ii) egg salad or ham on whole wheat bread?
      iii) ham?
      iv) egg salad on white or whole wheat bread?

Literacy Connect

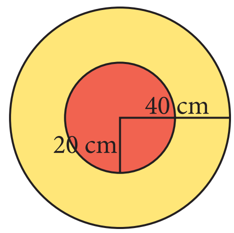
10. The complement of an event is *all other events*. For example, when rolling a die, the complement of rolling a 1 is rolling a 2, 3, 4, 5, or 6; the complement of rolling an odd number is rolling an even number.

a) Use the tree diagram or the table from Example 2 to find the probability of not rolling doubles.

b) Explain how you can use the thinking from Method 2 to find the probability of not rolling doubles.

c) P(rolling doubles) + P(not rolling doubles) = 1. Explain why this makes sense.

11. The radius of the entire dartboard is 40 cm, while the radius of the red circle is 20 cm.



a) If a dart is thrown at random, find the probability that it will land in the red circle. Justify your answer.

b) What assumption have you made?

Chapter Problem

12. A fisheries employee caught a number of bass, carp, and catfish and is preparing to tag them for tracking purposes. There are a total of 60 fish: 20 bass, 25 carp, and 15 catfish. A fish is randomly selected to be tagged.

a) Find the probability that the fish selected is a catfish. Express your answer as a fraction in lowest terms.

b) Find the probability that the fish selected is a bass or a carp. Suggest two possible methods for doing this.

c) If you know for certain that the fish selected is not a bass, find the probability that it is a carp. Express your answer as a decimal.

Extend C

13. Ann, Bob, and Cathy are posing for a group photograph.

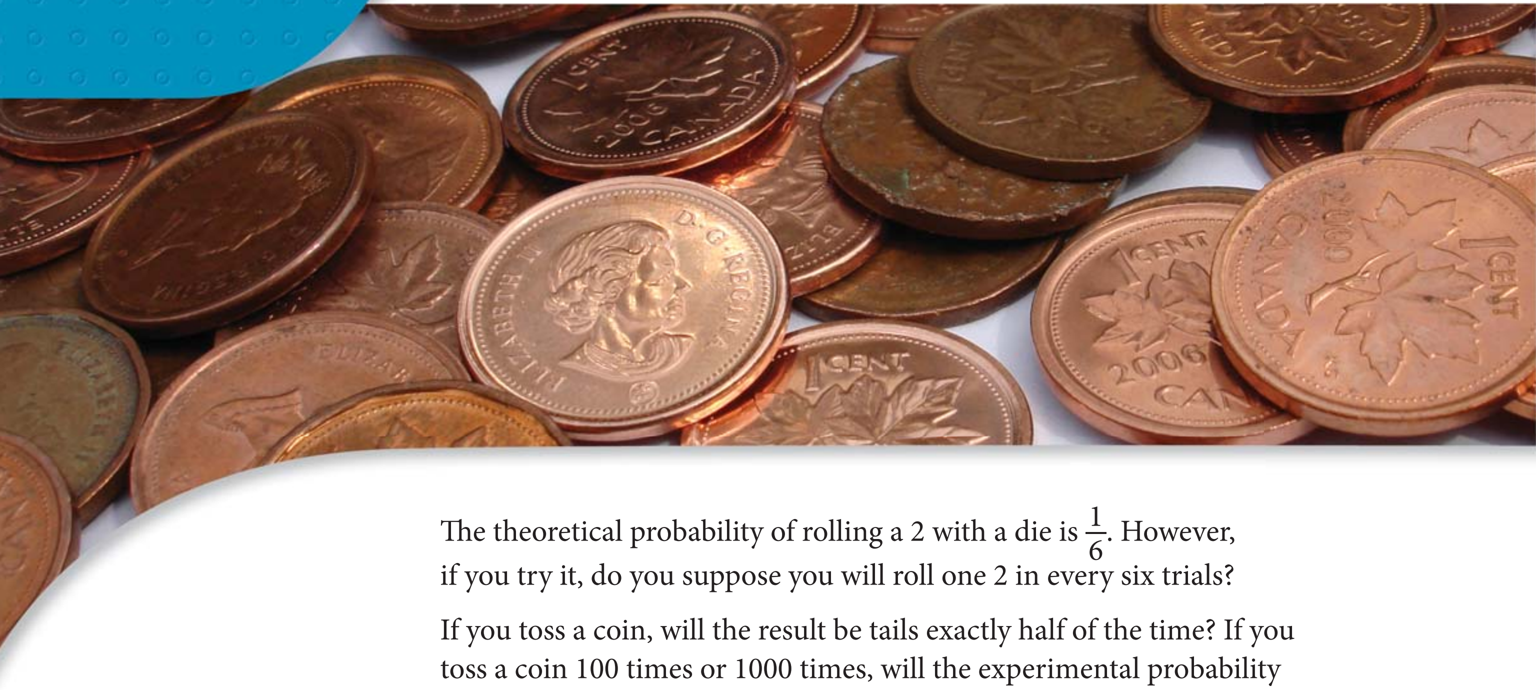
a) What is the probability that Ann will not be standing between Bob and Cathy?

b) What is the probability that Bob and Cathy will be standing beside each other?

c) Are the probabilities for parts a) and b) related? Explain.

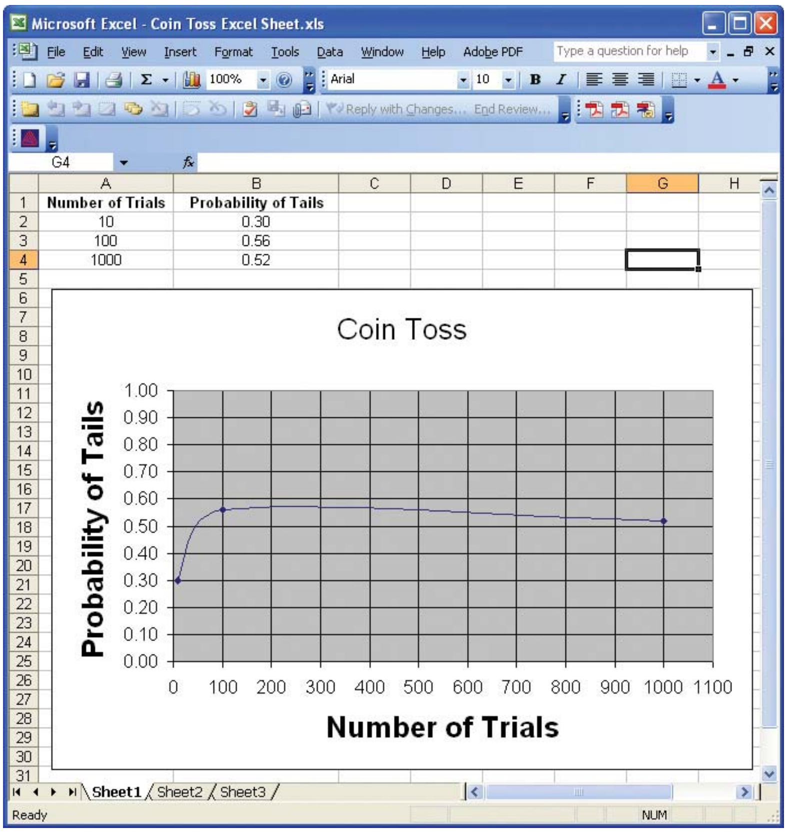
14. A cylindrical drum contains clear plastic pellets to be sent to a factory. By accident, a red pellet has contaminated the container of clear pellets. The pellets from the drum are poured into a cone-shaped container of the same height and radius as the cylinder. What is the probability of the red pellet remaining in the drum? Justify your answer.

2.3 Compare Experimental and Theoretical Probabilities



The theoretical probability of rolling a 2 with a die is $\frac{1}{6}$. However, if you try it, do you suppose you will roll one 2 in every six trials?

If you toss a coin, will the result be tails exactly half of the time? If you toss a coin 100 times or 1000 times, will the experimental probability eventually reach 50%?



Number of Trials	Probability of Tails
10	0.30
100	0.56
1000	0.52

Investigate 1

Tools

- coins
- grid paper

Optional

- spreadsheet software
- random number generator

Toss Coins

1. If you toss a coin 10 times, how many times do you expect to turn up heads? Explain your reasoning.

2. Copy the table for one to 10 trials.

Trial Number	Number of Heads	Average Number of Heads
1		
2		
3		
. . .		
10		

3. Toss a coin 10 times. In row 1, record the number of times that heads turns up in the second column. The 10 tosses represent one trial. For this row, the average number of heads will be the same as the number of heads.

4. Repeat the experiment. In row 2, record the number of times that heads turns up in the second column. For this row, the average number of heads will be the sum of the first two values in the Number of Heads column divided by 2, the number of trials.

5. Repeat step 4 until you have completed 10 rows of the table. For each row, the average number of heads will be the sum of the values in the Number of Heads column, divided by the number of trials.

6. Draw a scatter plot of Number of Trials versus Average Number of Heads.

7. **Reflect** What do you notice about the graph as the number of trials increases? Why do you suppose this is the case? Do you think this is always the case? Explain.

Investigate 2

Tools

- graphing calculator
- grid paper

How Many Girls?

When a child is born, there is a 50% chance it will be a girl. You will use a graphing calculator to find the probability that two out of three children in a family are girls.

1. Use a graphing calculator. Enter the month and day of your birthday, followed by the street number of your home, using all digits. Press STO▶. Press MATH. Use the arrow keys to highlight **PRB**. Press ENTER twice.

2. Press MATH. Use the arrow keys to highlight **PRB**. Select **5:randInt**. Type **0** , **1** , **3**).

To simulate rolling two dice, you would use **randInt(1,6,2)**.

The program will generate 0s and 1s, in sets of three to represent the genders of the three children. Assume a 0 represents a male and a 1 represents a female.

3. Press ENTER to simulate one family of three children. Copy and complete the table for 1 to 25 trials.

Trial Number	Number of Girls
1	

Each time you press ENTER, another set of three "children" will be generated.

4. **Reflect** Of the 25 trials, how many times were there two girls? Is this what you expected? Explain.

5. **Reflect** Without calculating, what do you expect is the probability of having two boys out of three children? Explain.

Example 1

Will It Rain?

During the month of April 2006, at least some rainfall was recorded on 12 different days at the Sarnia Airport weather station.

a) Suppose one day in April 2006 is selected at random. What is the probability of choosing a day on which it rained?

b) Refer to the probability of precipitation you calculated in part a). Describe how to make a spinner to simulate the situation for part a).

c) On a graphing calculator, press MATH, select **PRB** and then **1:rand**. Press ENTER several times. Describe what occurs and how this method can be used in place of the spinner in part b).

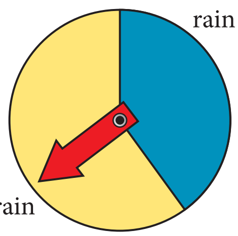
Solution

a) There are 30 days in April and it rained on 12 of those days.

$$
\begin{aligned}
P(\text{rain on selected day}) &= \frac{\text{number of days of rain}}{\text{total number of days}} \\
&= \frac{12}{30} \\
&= \frac{2}{5} \\
&= 0.4 \text{ or } 40\%
\end{aligned}
$$

The probability of choosing a day in April 2006 on which it rained is 40%.

b) To make a spinner to model rain 40% of the time, create a sector angle whose measure represents 40% of the entire circle.



One full revolution is 360°.

$$
\begin{aligned}
40\% \text{ of } 360° &= 0.40 \times 360° \\
&= 144°
\end{aligned}
$$

The sector angle representing rain should have a measure of 144°.

c) The **rand** command on a graphing calculator generates a random decimal between 0 and 1. You can decide that if it gives a result less than or equal to 0.4, then it will represent a rainy day. A result greater than 0.4 will represent a dry day.

Example 2

How Many Girls?

Suppose a couple would like to have three children.

a) Determine the theoretical probability of having two girls and one boy.

b) Explain how your answer in part a) can help determine the theoretical probability of having two boys and one girl.

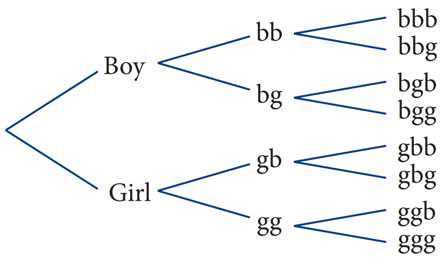
c) Determine the theoretical probability of having at least one girl.

Math Connect

Mark the branches that represent a successful outcome. The total numbers of outcomes is the total number of branches on the right side of the tree.

Solution

a) Draw a tree diagram with three levels, each level representing one child.



There are a total of eight branches. Three branches represent two girls and one boy.

$$P(\text{2 girls, 1 boy}) = \frac{\text{number of outcomes with 2 girls}}{\text{total number of outcomes}}$$
$$= \frac{3}{8}$$

The probability of having two girls and one boy in a family with three children is $\frac{3}{8}$.

b) The theoretical probability of having two boys and one girl would be the same as having two girls and one boy.

c) Method 1: Use a Tree Diagram

Having at least one girl means the couple can have one, two, or three girls. Examine the branches of the tree diagram. There are: three branches with one girl; three branches with two girls; and one branch with three girls.

$$P(\text{at least 1 girl}) = P(\text{1 girl, 2 girls, or 3 girls})$$
$$= \frac{\text{number of branches with at least 1 girl}}{\text{total number of branches}}$$
$$= \frac{3 + 3 + 1}{8}$$
$$= \frac{7}{8}$$

Method 2: Use the Complement

If there are three children, there must be zero, one, two, or three girls. The complement of "at least one girl" is "no girls."

$$P(\text{no girls}) = \frac{\text{number of branches with 0 girls}}{\text{total number of branches}}$$
$$= \frac{1}{8}$$

Finding the probability of the complement

$$P(0\text{ girls}) + P(\text{at least one girl}) = 1$$
$$\frac{1}{8} + P(\text{at least one girl}) = 1$$
$$P(\text{at least one girl}) = 1 - \frac{1}{8}$$
$$P(\text{at least one girl}) = \frac{7}{8}$$

Therefore the theoretical probability of having at least one girl in three children is $\frac{7}{8}$.

Key Concepts

- Real-life situations can be simulated by probability experiments.
- Computers and graphing calculators have random number generators that can simulate probability experiments.
- The theoretical probability and experimental probability of an event are not necessarily the same. As the number of trials increases, the experimental probability usually gets closer to the theoretical probability.

Discuss the Concepts

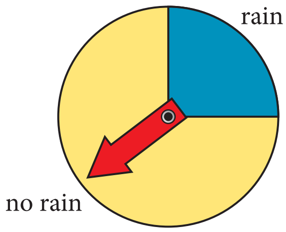
D1. How can a spinner be used to simulate rolling a six-sided die?

D2. If you perform a probability experiment, does the reliability increase or decrease when you increase the number of trials? Explain your reasoning.

D3. If you toss a coin 10 times, is it possible to obtain 10 heads? How many heads do you expect? Would your answer change if you used a graphing calculator to simulate the situation instead of tossing a real coin? Explain.

Practise A

1. Consider the spinner shown.



   a) What is the theoretical probability of the spinner landing on rain?

   b) If the spinner lands on no rain 13 times in 15 trials, what would be the experimental probability of a day having no rain?

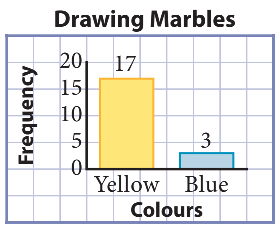
2. You toss a coin 10 times. It turns up heads eight times.

   a) What is the experimental probability of turning up heads?

   b) What is the theoretical probability of turning up heads?

   c) If you tossed the coin several more times, would you expect the experimental probability to increase or decrease? Explain.

3. In a bag, there are 14 yellow marbles and six blue marbles. A marble is removed, the colour is recorded, and then it is put back into the bag. This is repeated for a total of 20 times. The results are displayed on the bar graph.



   a) What is the experimental probability of drawing a yellow marble? Express your answer as a percent.

   b) What is the theoretical probability of drawing a yellow marble? Express your answer as a percent.

4. A random number generator is used to simulate the results of rolling a die 30 times. The results are shown in the table.

Outcome	Frequency
1	4
2	6
3	7
4	5
5	6
6	2

   a) Determine the probability of each outcome.

   b) Is each probability in part a) theoretical or experimental? Explain your reasoning.

Apply B

5. A die was rolled six times. The number 3 was rolled twice, and the number 2 was rolled four times.

a) Find the experimental probability of the following:
i) rolling a 3 **ii)** rolling a 2 **iii)** rolling a 6

b) What is wrong with this experiment as a predictor of experimental probability?

6. When rolling two dice 60 times, doubles turned up a total of 15 times.

a) What is the experimental probability of doubles? Answer as a fraction in lowest terms.

b) What is the theoretical probability of rolling doubles?

c) Do you think the dice are fair? Explain your reasoning.

d) If the dice were rolled 60 more times, would doubles turn up less frequently to "make up" for the previous rolls? Explain.

7. Refer to question 1. Describe how you could use a graphing calculator to simulate the same situation

a) by using the **rand** command

b) by using the **randInt(** command

8. A multiple-choice test has 10 questions. Each question has four possible answers.

a) If a student randomly guesses at each of the 10 questions, how many would you expect to be correct? Explain your reasoning.

b) Explain how you can use the **randInt(1,4,10)** command on a graphing calculator to simulate guessing at 10 questions on a multiple-choice test.

9. Refer to question 8. Chris used the **randInt(** command in his simulation, 20 times. The results are shown in the following table.

Trial Number	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Number Correct	4	3	2	4	2	2	2	4	3	2	6	2	6	1	1	3	3	1	1	2
Average Number Correct																				

a) According to these results, what is the experimental probability of passing a 10-question test by guessing? Assume each question is multiple choice with four possible answers.

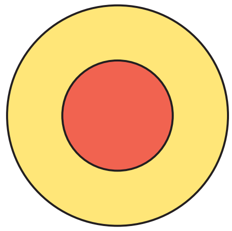
b) Is the result from part a) greater than you expected? Explain. Try this yourself and compare your results to the ones in the table.

10. Enter the data from the table in question 9 into a spreadsheet.
   - Enter the titles in cells A1 to A3.
   - In B3, enter the following formula: =AVERAGE(B2:B2).
   - Copy the formula and fill the remaining 19 cells in row 3. Hint: Click and drag the lower right corner of cell B3 to extend it to the cell U3.
   - Create a scatter plot of Average Number Correct versus Trial Number.

   a) Examine the formula in row 3. What is it calculating? In the formula, why does the first B2 stay the same while the second cell reference changes to C2, D2, and so on?

   b) Describe what you see in the scatter plot. Do a rough sketch of it on paper. If you performed more than 20 trials, how would the graph change? Explain your reasoning.

11. The area of a red circle in the centre of the dart board has one-quarter of the area of the entire dart board. This means there is a 25% theoretical probability of a dart hitting the red circle. During a game of darts, a shot landing in the red is worth two points, while a shot landing in the yellow scores only one point. In one game, 16 shots landed in red, while 24 landed in yellow.



   a) Find the experimental probability of landing in red. Express your answer as a decimal.

   b) How does your answer in part a) compare to the theoretical probability?

   c) What factor might explain this difference?

Chapter Problem

12. In a small lake, it has been determined that bass, carp, and catfish are present in the ratio 3:5:2. However, in one sample, the three species have been caught in the ratio 3:3:2.

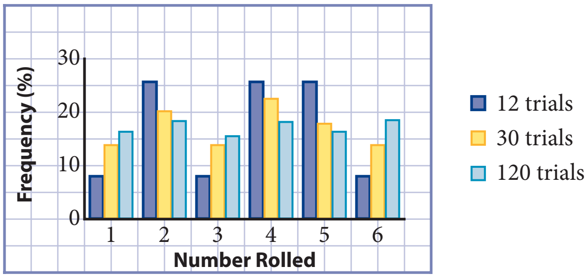
   a) What is the theoretical probability of catching a bass?

   b) What is the experimental probability of catching a bass?

   c) Suggest reasons why there is a difference between the experimental and theoretical probabilities in this situation.

Achievement Check

13. The graph shows the percent frequency for the possible outcomes when a die was rolled. The results are shown for 12, 30, and 120 trials.



a) For 12 trials, in approximately what percent of the results was a 2 rolled?

b) For 12 trials, exactly how many times was a 3 rolled? Justify your answer.

c) What is happening to the heights of the bars as you progress from 12 to 30 then to 120 trials?

d) If you had data for 1000 trials, and included this on a multiple bar graph, describe what you would expect to see. Explain your reasoning.

Extend C

14. A game of chance involves the rolling of two dice. You win \$5 if you roll doubles, otherwise you have to pay \$1. If you played this game repeatedly, would you expect to win or lose money? Justify your answer.

15. You and a partner each toss a coin. If both are heads, your partner gives you three marbles. If both are tails, your partner gives you one marble. If one is heads and the other is tails, you give your partner two marbles.

a) Is this a fair game, or does the game favour one of the players? Explain.

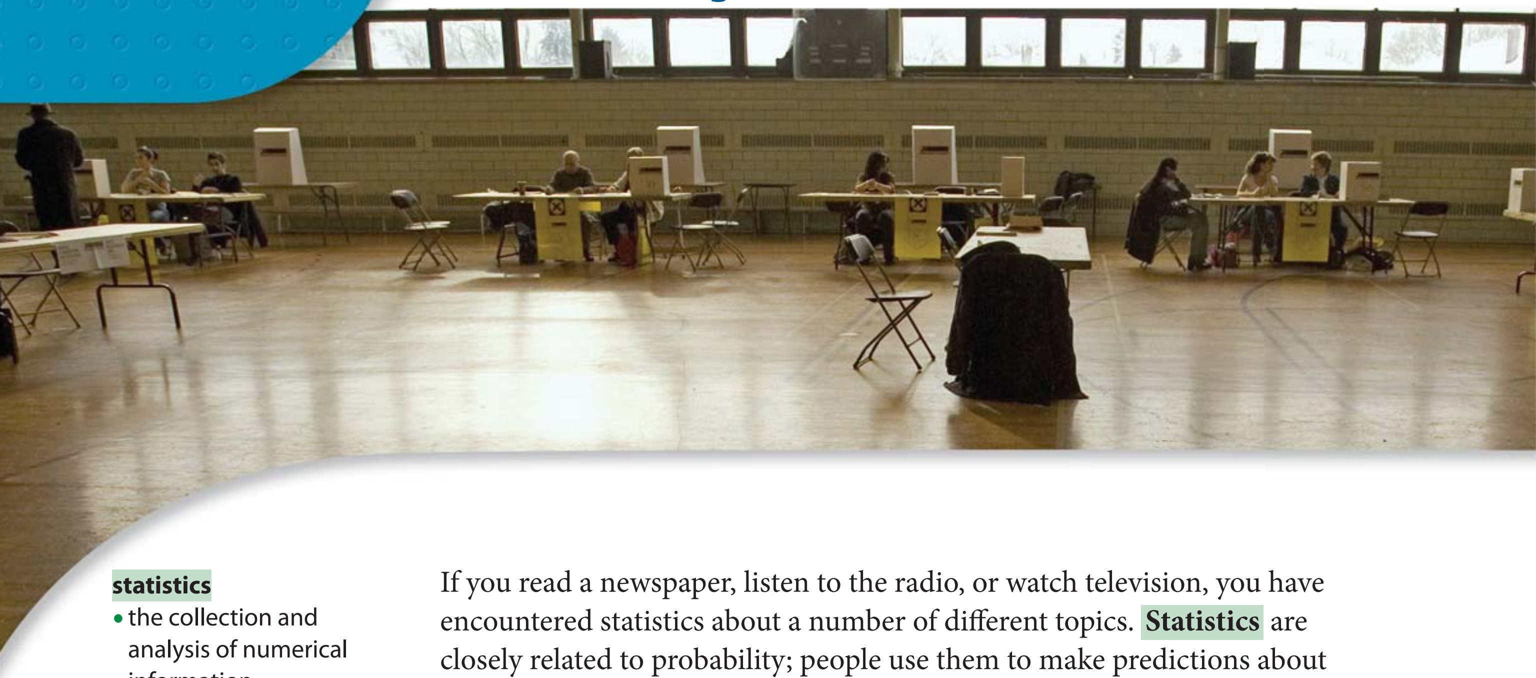
b) Work with a partner. Record the results from 10 plays.

c) Use technology to simulate the results from 1000 plays.

d) Was your answer to part a) correct? Explain.

16. Sandor used a graphing calculator to find **randInt(2,12,10)**. He claimed that this will model the rolling of sums from 2 to 12 with a pair of dice 10 times. Marucia says that this procedure is not correct. Conduct several trials and record the results. Who is right? Explain.

2.4 Interpret Information Involving Probability



statistics
- the collection and analysis of numerical information

data
- facts or pieces of information

If you read a newspaper, listen to the radio, or watch television, you have encountered statistics about a number of different topics. **Statistics** are closely related to probability; people use them to make predictions about future events. Probability experiments involve simulating real-life events and using the results to make predictions. Statistics involve gathering **data** from real-life events in order to make predictions about future events.

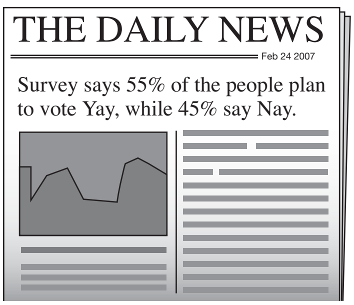
Investigate 1

Tools
- graphing calculator
- grid paper

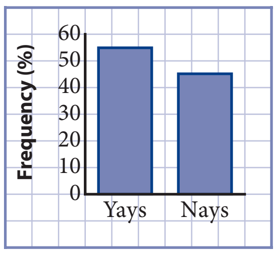
Optional
- spreadsheet software
- random number generator

Opinion Polls

A public opinion poll is a survey asking people about issues. Before an election, people were asked whom they planned to vote for: the Yays or the Nays. The results are shown.



Poll Results



1. Use a graphing calculator. Press MATH. Use the arrow keys to highlight **PRB**.

2. Select **1:rand** and press ENTER.

3. On the screen, you should now see **rand**. Press ENTER a few times and see what happens.

4. For this activity, a random number less than 0.55 will be considered a vote for the Yays. A number greater than or equal to 0.55 will be considered a vote for the Nays.

5. **Reflect** Explain how the statements in step 4 are related to the results of the poll.

6. On paper, create a tally chart with columns for the Yays and Nays.

7. Perform the **rand** command by pressing ENTER. Record the result in your tally chart.

8. Repeat step 7 until you have performed 100 trials.

9. Find the totals for each of the two parties and display the data using a bar graph.

10. **Reflect** How do the results of your 100 trials compare to the original 55% and 45% values?

11. **Reflect** Is it possible for the Nays to win your "mini-election"? Explain your reasoning.

12. Compare your results to those of classmates. Did the Nays win for any of your classmates?

Investigate 2

Tools

- graphing calculator

Optional

- spreadsheet software with random number generator application
- appropriate spinner to simulate the problem

Key Saves

After 813 regular season games, goalie Martin Brodeur of the New Jersey Devils had compiled a 0.912 save percentage (SPCT). This means that out of every 1000 shots, he made 912 saves. SPCT is calculated by dividing the total number of saves by the total number of shots.

Brodeur and the New Jersey Devils have faced the Toronto Maple Leafs several times in the playoffs. If the Leafs average 25 shots per game, how many goals can they expect to score in a game?

1. Three different options are suggested to simulate 25 shots on goal.

   Option 1: Perform the **rand** command 25 times on a graphing calculator. Describe which results will represent a goal.

   Option 2: Perform the command **randInt(0,1000,25)** on a graphing calculator. Describe what this command does and what determines a goal.

   Option 3: Create a spinner for which 0.912 of the spinner is shaded to represent a save, while the remainder represents a goal.

2. Use one of the three methods to simulate 25 shots on goal. Record the number of goals. Assume the Leafs allow two goals each game. State whether this would be a win for the Leafs or for the Devils.

3. Repeat step 2 until one of the teams has won four games.

4. **Reflect** How realistic is this simulation? What assumptions have been made? Explain why this might not make an accurate prediction about the result of a playoff series.

5. **Reflect** The term *save percentage* is somewhat incorrect. Explain why.

Example

Favourite Music

A local radio station surveyed 200 students from one high school to determine their favourite music.

The results are shown in the table.

Music	Percent of Students
rock	45
rap	35
country	20

a) Express each percent as a decimal, and as a fraction in lowest terms.

b) If there are 4000 high school students in the city, how many of them would you expect to like rock? rap? country?

c) Is it possible that the poll might not be accurate? What factors could have influenced the responses?

Solution

a) Rock

$$45\% = \frac{45}{100}$$
$$= 0.45$$
$$\frac{45}{100} = \frac{9}{20}$$

Rap

$$35\% = \frac{35}{100}$$
$$= 0.35$$
$$\frac{35}{100} = \frac{7}{20}$$

Country

$$20\% = \frac{20}{100}$$
$$= 0.20$$
$$\frac{20}{100} = \frac{1}{5}$$

Check: The total number of students should be 4000, the original number of students.

b) Rock

45% of 4000
$= 0.45 \times 4000$
$= 1800$

Rap

35% of 4000
$= 0.35 \times 4000$
$= 1400$

Country

20% of 4000
$= 0.20 \times 4000$
$= 800$

c) Yes, it is possible that the results are not accurate.
Some factors that could have influenced the responses are
- the type of music normally played by the radio station conducting the survey
- popular music teachers or bands within the school

Key Concepts

- Probability and statistics are presented through the media in a variety of contexts.
- Statistics are collected from real-life events or studies.
- Statistics, like probability, help predict the result of future events.

Discuss the Concepts

D1. Explain the similarities and differences between statistics and experimental probability.

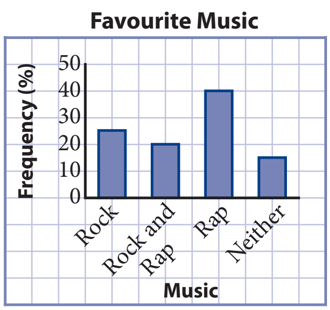
D2. Why might you question some of the statistics you see in the media?

D3. Describe how you use statistics in your life.

Practise A

For help with question 1, refer to the Example.

1. All students in a high school were asked if they like rock, rap, both rock and rap, or neither. The results are displayed on the graph. A student from this same high school was chosen as the winner of a contest. Determine the probability that this student likes



a) rock but not rap

b) either rock or rap, but not both

c) rock or rap or both

2. A football quarterback has completed 125 passes in 200 attempts so far this season.

a) What percent of his passes has he completed?

b) If he attempts 30 passes in the next game, how many would you expect him to complete?

c) Suggest some factors that might affect your estimate.

3. After 12 games, the Toronto Maple Leafs have five wins, four losses, and three overtime losses. Teams are awarded two points for a win, one point for an overtime loss, and no points for a loss.

a) How many points do the Leafs have after 12 games?

b) Predict how many points the Leafs will have if the regular season has 82 games.

4. Your high school's girls' volleyball team has the following record: seven wins, four losses, and two ties. A win is worth three points, a loss is worth zero points, and tie is worth one point.

a) How many points does the team have after 13 games?

b) Predict how many points the volleyball team will have if the regular season has 20 games.

Apply B

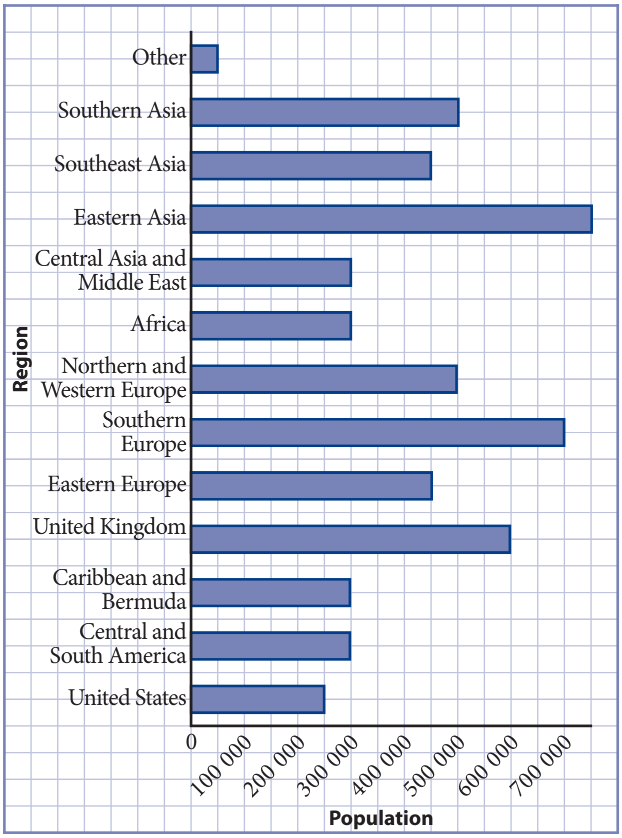
5. After half a season, a major league baseball player has a 0.300 batting average.
   a) In your own words, describe what this means. If necessary, work with a partner who is knowledgeable about baseball.
   b) If this player gets to bat 40 times in the next 10 games, how many hits would you expect him to get? For simplicity, assume he either gets a hit or makes an out.
   c) Is it possible that your prediction in part b) is not accurate? Explain.

6. A study by Health Canada has shown that, by age 14, one in four teenagers have tried smoking. Of those who try smoking, 36% become smokers.
   a) What is the probability that a person has not tried smoking by age 14?
   b) What percent of people who have tried smoking do not become smokers?
   c) What percent of the population have tried smoking by age 14, but not become smokers?
   d) How many Canadians do you estimate have tried or will try smoking by age 14, and become smokers? Assume the Canadian population is approximately 33 million people.

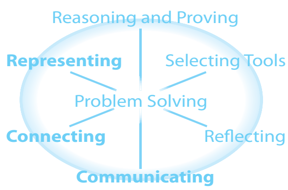
7. Canadian researchers have found that the number of women with multiple sclerosis (MS) has risen compared to the number of men who have the disease. For those born in the 1930s, there were 1.9 females with MS for every male. However, for those born in the 1980s, there are now 3.2 women who have MS for every man who has it.
   a) For Canadians born in the 1930s, what percent of those who have MS are female?
   b) For Canadians born in the 1980s, what percent of those who have MS are female?
   c) Divide your answer in part b) by your answer from part a). Now divide 3.2 by 1.9. Which of these calculations appears to indicate a greater increase? Justify your response.
   d) Why do you suppose the statistics were presented using the numbers 1.9 and 3.2 instead of using percents?

8. The horizontal bar graph shows the countries of origin for immigrants in Canada in 2001.

Immigrant Population by Place of Birth



Adapted from Statistics Canada, http://www40.statcan.ca/l01/cst01/demo34a.htm, Feb 14, 2007.



a) What percent of immigrants were from Eastern Asia (mainly from China) or Southern Asia (mainly from India)?

b) Why do you suppose China and India are two large sources of immigrants to Canada?

c) You know that an immigrant is from the United States, Eastern Europe, or the United Kingdom. What is the probability the person is from the United States?

Achievement Check

Literacy Connect

9. In a recent article about the effects of playing loud music on MP3 players, it was suggested that one in eight students who have MP3 players may have already suffered irreparable hearing loss.

a) In a survey of 30 students who have MP3 players, how many would you expect to have hearing loss?

b) How do you suppose the researcher arrived at the ratio of one in eight?

c) Search the Internet to find more information about the research on this topic. Create a question related to probability or statistics based on your findings.

Extend C

10. The Canadian Radio-television and Telecommunications Commission (CRTC) controls the amount of Canadian content on both radio and television in Canada. Find its Web site and answer the following questions.

a) How much Canadian content is required for radio and television? Is it the same for both media?

b) Can radio and TV stations play the Canadian shows during the night when people are asleep? Explain.

c) If a TV show hires a Canadian actor to stand in the background, will this make the CRTC consider the show to be Canadian? Give details.

d) Create a question involving probability using information from the CRTC Web site.

11. A local radio station has predicted a 30% probability of precipitation (P.O.P.) for each of the next two days.

a) Describe how you can model a 30% P.O.P. using the **rand** command on a graphing calculator.

b) Perform an experiment that will allow you to predict the probability of having no rain for the next two days.

c) Find the theoretical probability of having two days with no rain, given a P.O.P. of 30%.

2 Review

2.1 Probability Experiments, pages 60–67

1. In Tim's coffee shop, a study was done to see how many people buy coffee and a doughnut. Of 160 people who came in one day, 60 bought coffee *and* a doughnut. The rest bought coffee *or* a doughnut. Find the experimental probability that the next person will buy both coffee and a doughnut.

a) as a fraction in lowest terms

b) as a percent

c) as a decimal

2. Complaints were made to a manufacturer about malfunctioning computer chips. The company promptly tested 10 different chips from the production line and found them all to be working properly.

a) Does this mean the chips are likely all working properly? Explain.

b) How could the company do better quality control?

2.2 Theoretical Probability, pages 68–75

3. From a standard deck of 52 playing cards plus two jokers, find the probability of each event. Express each answer as a fraction in lowest terms.

a) a red card

b) a black face card

c) an ace, 2, or 3

d) a red card that is not a face card

4. Two dice are rolled. Find the probability that the sum of the numbers is

a) 11 **b)** not 11

c) 2, 3, or 4 **d)** a multiple of 3

e) greater than 1 **f)** greater than 3

5. Matthew has black socks, white socks, blue jeans, dress pants, a red shirt, a green shirt, and a T-shirt.

a) Draw a tree diagram showing his choices for socks, pants, and shirt.

b) Find the probability that Matthew selects at random

i) blue jeans and the T-shirt

ii) white socks

iii) black socks, dress pants, and a red shirt

iv) white socks and not the T-shirt

2.3 Compare Experimental and Theoretical Probabilities, pages 76–85

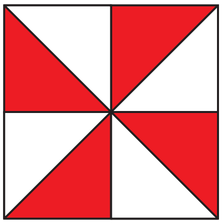
6. Two dice were rolled 20 times. Doubles were rolled five times.

a) Find the experimental probability of rolling doubles. Express your answer as a percent.

b) If you were to roll the dice 20 more times, would you expect five doubles again? Explain.

c) If you were to roll the dice 20 times, how many doubles would you theoretically expect? Justify your answer.

7. The figure shows a unique dartboard.



a) For a randomly thrown dart, what is the theoretical probability of landing on red? Explain your reasoning.

b) During a game of darts, 32 out of 40 landed on red. Determine the experimental probability of landing on red, expressed as a decimal.

c) In the game in part b), two points were awarded for landing on red and one point for landing on white. How might this explain the difference between experimental and theoretical probability?

8. You perform the command **randInt(1,5,10)** on a graphing calculator.

a) Describe what will happen.

b) How many 2s would you expect to be among the results? Explain your reasoning.

9. You perform the **rand** function on a graphing calculator.

a) Describe what will happen.

b) If you performed this command 20 times, how many of the results would you expect between 0.2 and 0.7? Explain your reasoning.

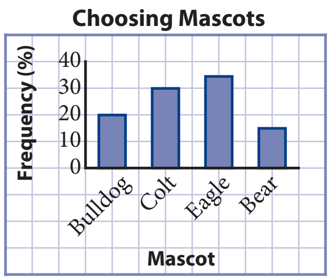
2.4 Interpret Information Involving Probability, pages 86–93

10. A basketball player made 40 out of 50 free throws in last week's games.

a) Find the player's free-throw percentage.

b) If the player averages eight free throws per game, how many of them should she expect to make?

11. The school council at Jackson Secondary School surveyed the students to help select a new football team mascot. The results are shown in the graph.



a) If 80 students were surveyed, how many of them voted for a bulldog?

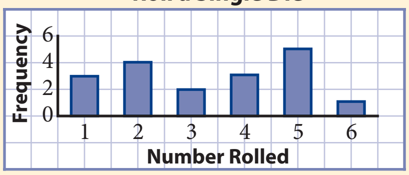
b) Johnson Secondary School has an eagle as their mascot, so those at Jackson Secondary who chose an eagle are asked to vote for another animal instead. What is the probability that a person who originally voted for an eagle will now vote for a bear?

2 Practice Test

For questions 1 to 3, choose the best answer.

1. During a probability experiment, a die was rolled 18 times. The results are shown in the bar graph.

Roll a Single Die



The experimental probability of rolling a 2 is

A $\frac{1}{6}$ **B** $\frac{2}{18}$ **C** $\frac{2}{9}$ **D** 16.7%

2. The theoretical probability of rolling a 6 is

A $\frac{1}{6}$ **B** $\frac{1}{18}$ **C** $\frac{1}{3}$ **D** 0

3. To simulate guessing on 10 multiple-choice questions, each with four possible answers, it would be appropriate to use:

A 10 spins on a spinner divided into quarters, each quarter of a different colour

B **randInt(1,4,10)** on a graphing calculator

C drawing and replacing 10 playing cards from a standard deck of cards

D any of these methods

4. True or false?

When rolling two dice, the sums 2 through 11 are all equally likely.

5. Richie Rich has a limousine, a sports car, and a motorcycle for travelling from his condominium to the airport. There, he has a helicopter and a jet for flying to his favourite golf courses.

a) Draw a tree diagram, showing his choices for the two stages of his trip to play golf.

b) Find the probability that he randomly:

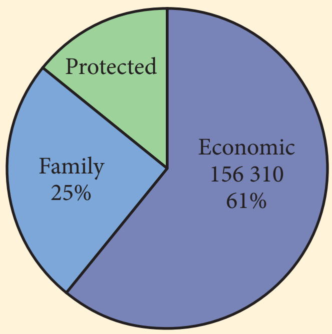
i) takes the sports car then his helicopter

ii) does not take his limousine

iii) takes the sports car or motorcycle and the jet

6. Immigrants come to Canada for a variety of reasons. A report from Citizenship and Immigration Canada divides the reasons into three categories: business/economic, family, and protected persons. In 2005, there were 256 246 new immigrants.

Immigrants to Canada in 2005



a) Find the number of people in the "family" and "protected" categories.

b) If an immigrant to Canada does not fall into the "protected" category, find the probability, as a percent, that he or she came to Canada for business/economic reasons.

Chapter Problem Wrap-Up

The government and anglers have a keen interest in protecting Ontario's waterways and fish stocks. Sometimes, species are endangered due to overfishing or to the introduction of invasive species. Search the media or the Internet to find statistics on declining fish stocks or increasing invasive species in Ontario (or Canadian) waters. Write a brief report on your findings.

Write a concluding statement that uses probability to describe the future of a particular fish stock or effects on the fishing industry.

7. If a farmer waits one week to sell his corn, there is a 50% chance that he will earn an extra \$10 000. However, there is a 10% chance that he will lose \$30 000. Should he sell now, or wait one week?

a) Describe how you could use a spinner to perform a probability experiment to simulate this situation.

b) Describe how you could use the **rand** command on a graphing calculator to perform the same probability experiment.

c) Use either a spinner or a graphing calculator for 25 trials. Each time, record whether the farmer gained \$10 000, lost \$30 000, or neither. Calculate the average amount the farmer will lose or gain if he waits one week to sell his corn.

8. A spinner is designed for three outcomes. The blue outcome is twice as likely as the red, while the yellow is three times as likely as the blue.

a) Find the measures of the three angles required to make this spinner.

b) For this spinner, find the theoretical probability of

i) landing on red

ii) not landing on red

c) Describe how the command **randInt(1,9,1)** can simulate spinning red, yellow, or blue.

d) Perform the **randInt(1,9,1)** command 20 times, each time recording whether it indicates red, blue, or yellow. How many times did red occur twice in a row? How many times did yellow occur twice in a row?

e) Find the theoretical probability of yellow occurring twice in a row. Hint: Part d) will provide an estimate, which can be improved upon if you perform more than 20 trials. You might also try drawing a tree diagram.