1 Trigonometry

How do pilots navigate a plane safely to their destination? How do surveyors determine the distance through an inaccessible area to make way for a road? How do carpenters determine the angle to cut for the rafters to support a roof truss? These careers require a knowledge of trigonometry. To do their jobs, these individuals need to understand the relationship between ratios of sides and their related angles. In this chapter, you will investigate and practise the use of trigonometry as it relates to real-world situations.

In this chapter, you will

- solve problems, including those that arise from real-world applications, by determining the measures of the sides and angles of right triangles using the primary trigonometric ratios
- verify, through investigation using technology, the sine law and the cosine law
- describe conditions that guide when it is appropriate to use the sine law or the cosine law, and use these laws to calculate sides and angles in acute triangles
- solve problems that arise from real-world applications involving metric and imperial measurements and that require the use of the sine law or the cosine law in acute triangles

Key Terms

adjacent
angle of depression
angle of elevation
angle of inclination
complementary
cosine law
hypotenuse
opposite
sine law

Demar completed a two-year geomatics diploma at Fleming College. Now, he is training to be a licensed surveyor. Demar works under the supervision of an experienced surveyor and uses trigonometry and computerized instruments to take exact measurements of the land and its features. He uses these measurements to establish official land, air, and water boundaries.

Prerequisite Skills

Solve Equations

1. Solve.

a) $x^2 = 36$

b) $x^2 - 6 = 19$

c) $x^2 = 64 + 36$

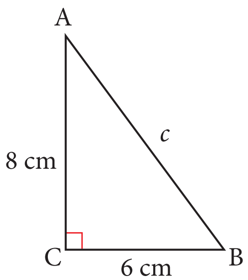
d) $x^2 = 5^2 + 12^2$

e) $7^2 + x^2 = 25^2$

The Pythagorean Theorem

2. Find the measure of the unknown side.

a)



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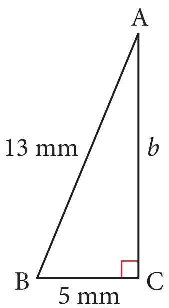

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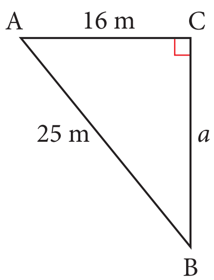


3. A 12-m ladder leans against the wall of a house. The top of the ladder reaches a window 10.5 m above the ground. Calculate the distance from the base of the ladder to the wall of the house.

Ratios

4. Express each ratio in lowest terms.

a) 4:8

b) 15:35

c) 20:50

5. The ratio between the selling price and the purchase price of a computer chip is 18:7. If the computer chip sells for \$27, determine the purchase price of the chip.

Proportions

6. Solve for each unknown.

a) $\dfrac{x}{13} = \dfrac{9}{39}$

b) $15 = \dfrac{45}{x}$

c) $\dfrac{x}{25} = \dfrac{y}{5} = \dfrac{8}{10}$

7. The scale on a Canadian road map is 1:700 000.

a) What does this scale represent?

b) Determine the actual distance between two cities, in kilometres, if the distance on the map is 12 cm.

c) Determine the distance on the map, in centimetres, if the actual distance between two cities is 40 km.

Rounding

8. Round each value to two decimal places.

a) 3.4576

b) 19.832

c) 9015.982 36

9. Evaluate each answer to one decimal place.

a) $\sqrt{59}$

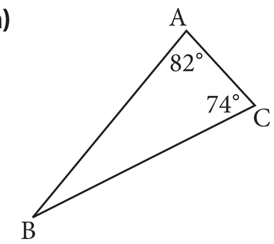
b) $\sqrt{723}$

c) $\sqrt{0.85}$

Angle Sum of a Triangle

10. Determine the measure of the missing angles.

a)



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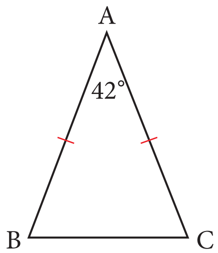

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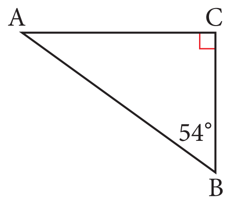
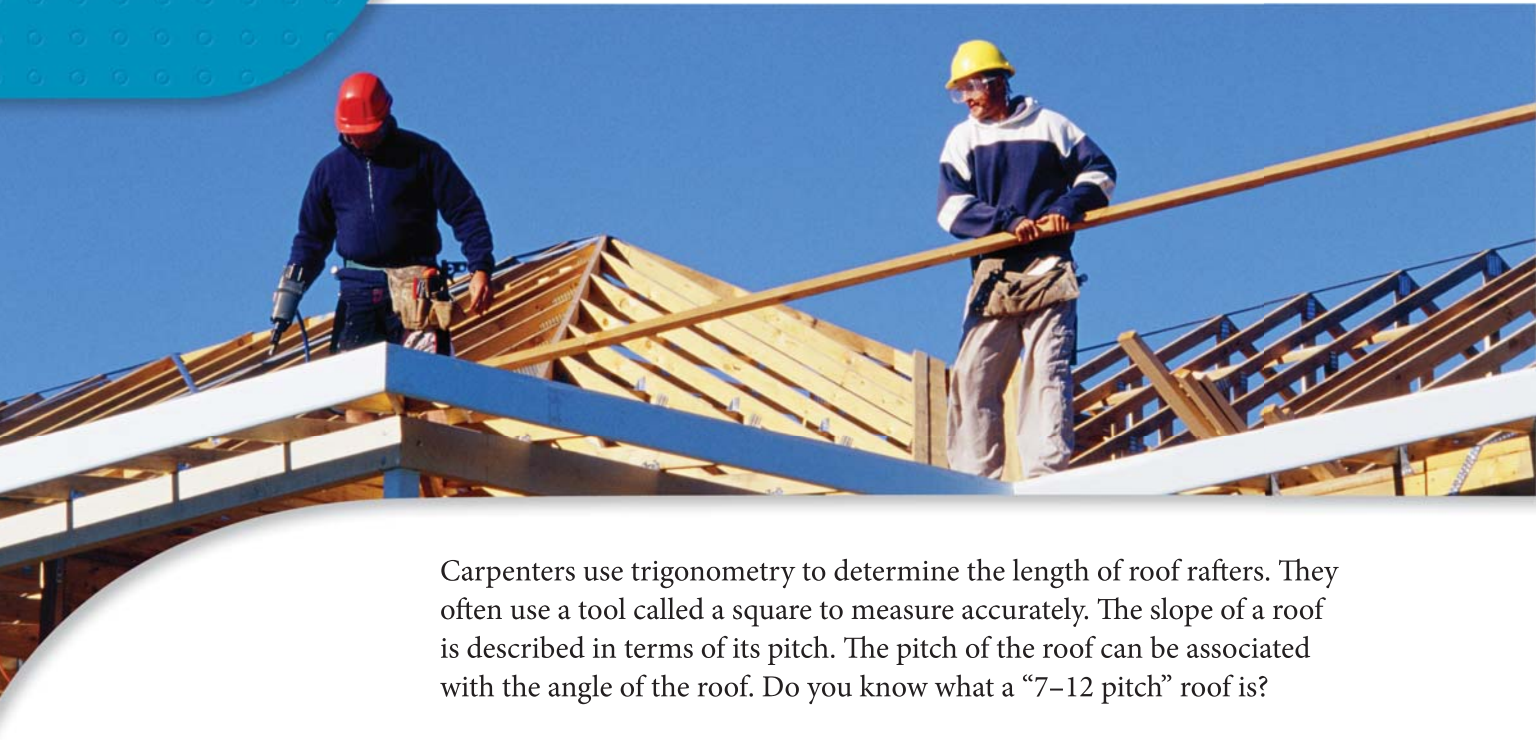


Chapter Problem

An expedition team decides to trek 400 km to the North Pole from a drop point in the Arctic. The team plans to travel 40 km per day. By the end of the third day, they have unknowingly wandered seven degrees off course to the east. At the end of the sixth day, the team navigator makes the discovery and finds that they are now three more degrees off course to the east. Can you determine the correct angle that they must turn to reach the North Pole? Can you calculate how many kilometres they must travel each day for the remainder of their journey?

1.1 Revisit the Primary Trigonometric Ratios



Carpenters use trigonometry to determine the length of roof rafters. They often use a tool called a square to measure accurately. The slope of a roof is described in terms of its pitch. The pitch of the roof can be associated with the angle of the roof. Do you know what a "7–12 pitch" roof is?

Investigate

Tools

- *The Geometer's Sketchpad®*
- computer

Optional

- pencil and paper
- ruler
- protractor

SOH–CAH–TOA?

You will investigate the ratios between pairs of sides associated with an acute angle in a right triangle.

1. Copy the table.

Triangle	AB (cm)	AC (cm)	BC (cm)	∠A (°)	$\frac{BC}{AB}$	sin A	$\frac{AC}{AB}$	cos A	$\frac{BC}{AC}$	tan A
ABC#1										
ABC#2										

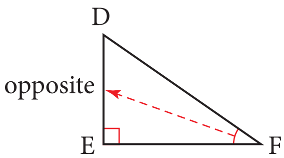
2. Construct a right triangle with sides of 3 cm, 4 cm, and 5 cm, using *The Geometer's Sketchpad®*.

- Open *The Geometer's Sketchpad®*.
- From the **Edit** menu, choose **Preferences**. Click on the **Text** tab, and check the box **For All New Points**. This will label points as you draw them. Click on the **Units** tab. Set the precision for Angle to units, and Distance to tenths.
- From the **Graph** menu, choose **Show Grid**. Drag the origin to the lower left corner of the workspace. From the **Graph** menu, choose **Snap Points**.

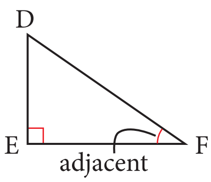
- Use the **Straightedge Tool** to draw a right triangle. Select the three sides of the triangle. From the **Measure** menu, choose **Length**. Move each measurement beside the correct side.
- To measure ∠A, select points B, A, and C, in that order. From the **Measure** menu, choose **Angle**. Move the measurement beside ∠A. Repeat this process for the measures of ∠B and ∠C.

3. Ensure that $\angle C = 90°$.

opposite
- in right △DEF, side DE is the side opposite ∠F

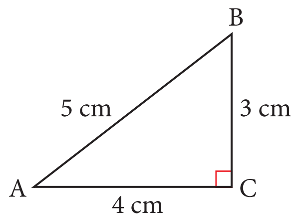


adjacent
- in right △DEF, side EF is the side adjacent to ∠F that is not the hypotenuse



hypotenuse
- the longest side of a right triangle

4. Identify the sides associated with ∠A.



a) Label the sides associated with ∠A. Side BC is **opposite** ∠A. Side AC is **adjacent** to ∠A. Side AB is the **hypotenuse**.

b) Calculate the ratios between the following pair of sides for △ABC.
$\frac{\text{opposite}}{\text{hypotenuse}}, \frac{\text{adjacent}}{\text{hypotenuse}}, \frac{\text{opposite}}{\text{adjacent}}$
Add the information to your table.

c) Calculate the angle measure for ∠A.

d) Calculate the following for ∠A.
sin A, cos A, tan A

e) Construct a triangle that has sides three times longer than △ABC.

f) Repeat steps a) to d) for this new △ABC. Complete the table.

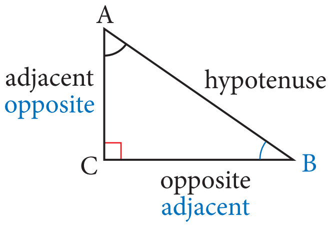
5. Compare ∠A in the two triangles that you constructed. Did the measure of ∠A change? Explain.

6. a) Compare the ratios between the same pairs of sides in both triangles and the trigonometric ratios. What do you notice?

b) Name the sides associated with ∠B. Write the ratios for sin B, cos B, and tan B. Make sure to write the ratio of the side measurements as a fraction.

7. Reflect Recall how to find the slope of a line segment. How is the slope of the hypotenuse AB associated with tan A?

8. Reflect You can associate the sides of a right triangle using the Pythagorean theorem: $c^2 = a^2 + b^2$. How can you associate the sides of a right triangle to a given angle? Use a diagram to help you explain.



$$\sin A = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{BC}{AB} \qquad \sin B = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{AC}{AB}$$

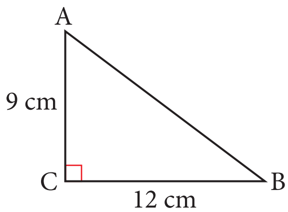
$$\cos A = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{AC}{AB} \qquad \cos B = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{BC}{AB}$$

$$\tan A = \frac{\text{opposite}}{\text{adjacent}} = \frac{BC}{AC} \qquad \tan B = \frac{\text{opposite}}{\text{adjacent}} = \frac{AC}{BC}$$

Example 1

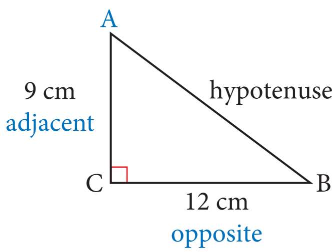
Write the Trigonometric Ratios for an Angle

Write the trigonometric ratios for sin A, cos A, and tan A. Express each answer as a fraction in lowest terms.



Solution

Write the names of the sides associated with ∠A.



To find sin A, you need to know the length of the hypotenuse.

Use the Pythagorean theorem.

$$\begin{aligned} c^2 &= 9^2 + 12^2 \\ c^2 &= 81 + 144 \\ c &= \sqrt{225} \\ c &= 15 \end{aligned}$$

$$\sin A = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{12}{15} = \frac{4}{5} \qquad \cos A = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{9}{15} = \frac{3}{5} \qquad \tan A = \frac{\text{opposite}}{\text{adjacent}} = \frac{12}{9} = \frac{4}{3}$$

Example 2

Determine Trigonometric Ratios Using a Calculator

Evaluate. Round your answers to four decimal places.

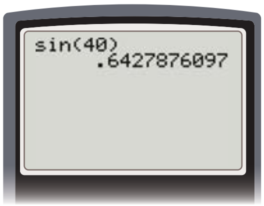
a) $\sin 40°$

b) $\tan 60°$

Solution

a) $\sin 40° = 0.6428$ SIN 40) ENTER

Be sure your calculator is in degree mode first.



b) $\tan 60° = 1.7321$ TAN 60) ENTER

Technology Tip

Remember: not all calculators are the same. Read the manual for the proper key sequence.

For a scientific calculator, you might use this key sequence:
40 SIN =

For a DAL (direct algebraic logic) calculator, you might use this key sequence:
SIN 40 =

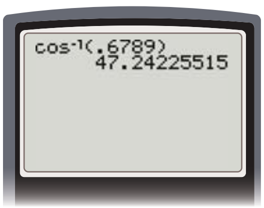
Example 3

Find the Measure of an Angle Given Its Trigonometric Ratio

Find the measure of $\angle A$ if $\cos A = 0.6789$. Round your answer to the nearest tenth of a degree.

Solution

$\cos A = 0.6789$ 2nd COS .6789) ENTER



Use pencil and paper, and a calculator.

$\cos A = 0.6789$
$\angle A = \cos^{-1}(0.6789)$
$\angle A \doteq 47.2°$

$\angle A$ is approximately $47.2°$.

Technology Tip

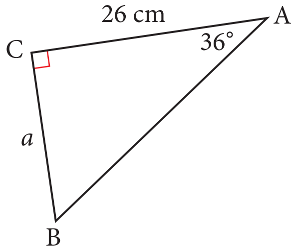
For a scientific calculator, you might use this key sequence:
.6789 2nd COS =

For a DAL calculator, you might use this key sequence:
2nd COS .6789 =

Example 4

Find the Length of a Side

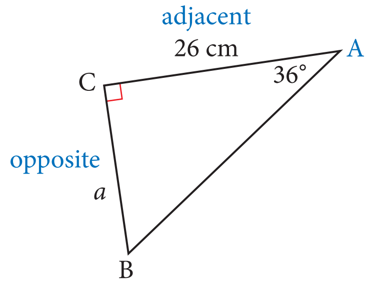
Find the length of side a. Round your answer to one decimal place.



Sides opposite an angle can be labelled with a lower case letter associated with the angle. Side a is opposite $\angle$A.

Solution

The measure of $\angle$A and the side adjacent to the angle are known.



Side a is opposite $\angle$A. AC = 26 cm and is adjacent to $\angle$A.

With the given information, you can use the tangent ratio to find the measure of side a.

$$\tan A = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan 36° = \frac{a}{26}$$

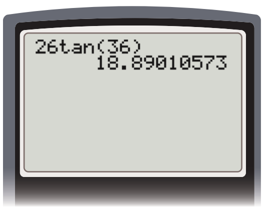
$$26 (\tan 36°) = a$$

$$a = 26 (\tan 36°)$$

$$a \doteq 18.9$$

How would the solution change if the unknown side were represented in the denominator?

Side a is approximately 18.9 cm.



26 [TAN] 36 [ )] [ENTER]

Technology Tip

For a scientific calculator, you might use this key sequence:

26 [×] 36 [TAN] [=]

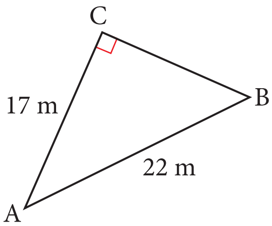
For a DAL calculator, you might use this key sequence:

26 [TAN] 36 [=]

Example 5

Find an Angle Given the Length of Two Sides

Find the measure of ∠A to the nearest tenth of a degree.



Solution

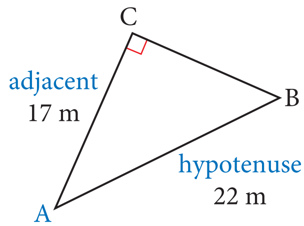
Identify the known sides of the triangle with respect to ∠A.

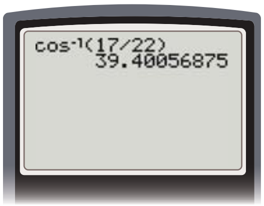
$$\cos A = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos A = \frac{17}{22}$$

$$\angle A = \cos^{-1}\left(\frac{17}{22}\right)$$

$$\angle A \doteq 39.4°$$





2nd COS 17 ÷ 22) ENTER

∠A is approximately 39.4°.

Technology Tip

For a scientific calculator, you might use this key sequence:

(17 ÷ 22) 2nd COS =

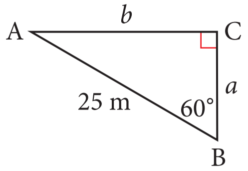
For a DAL calculator, you might use this key sequence:

2nd COS (17 ÷ 22) =

Example 6

Solve a Right Triangle

Solve the triangle. Round your answers to two decimal places, where necessary.



"Solve the triangle." means "Determine the measures of any angles and sides of the triangle that were not given."

Solution

Find the measure of $\angle A$.

$$\angle A = 180° - 90° - 60°$$
$$= 30°$$

complementary
- angles whose sum is 90°

The sum of the interior angles of a triangle is 180°. One angle in a right triangle is always 90°. The sum of the other two angles must equal 90°. These angles are **complementary**.

Check: $60° + 30° = 90°$

Find the measure of side a.

Method 1: Use cos 60°

$$\frac{a}{25} = \cos 60°$$
$$a = 25 \times \cos 60°$$
$$a = 12.5$$

Method 2: Use sin 30°

$$\frac{a}{25} = \sin 30°$$
$$a = 25 \times \sin 30°$$
$$a = 12.5$$

Side a is 12.5 m.

Find the measure of side b.

Method 1: Use sin 60°

$$\frac{b}{25} = \sin 60°$$
$$b = 25 \times \sin 60°$$
$$b \doteq 21.65$$

Method 2: Use cos 30°

$$\frac{b}{25} = \cos 30°$$
$$b = 25 \times \cos 30°$$
$$b \doteq 21.65$$

Method 3: Use the Pythagorean Theorem

Once you have found the measure of side a, you can find the measure of side b.

$$b^2 = 25^2 - 12.5^2$$
$$b^2 = 468.75$$
$$b = \sqrt{468.75}$$
$$b \doteq 21.65$$

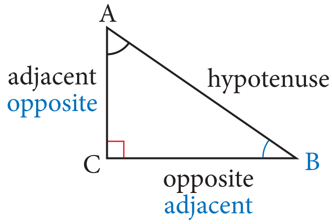
You can also solve for a using the Pythagorean theorem, if you solve for b first.

$$a^2 = 25^2 - 21.65^2$$
$$a^2 = 156.2775$$
$$a = \sqrt{156.2775}$$
$$a \doteq 12.50$$

Side b is approximately 21.65 m.

Key Concepts

- You can use the primary trigonometric ratios to find the measures of sides and angles of a right triangle.



$\sin A = \dfrac{\text{opposite}}{\text{hypotenuse}} = \dfrac{BC}{AB}$ $\qquad$ $\sin B = \dfrac{\text{opposite}}{\text{hypotenuse}} = \dfrac{AC}{AB}$

$\cos A = \dfrac{\text{adjacent}}{\text{hypotenuse}} = \dfrac{AC}{AB}$ $\qquad$ $\cos B = \dfrac{\text{adjacent}}{\text{hypotenuse}} = \dfrac{BC}{AB}$

$\tan A = \dfrac{\text{opposite}}{\text{adjacent}} = \dfrac{BC}{AC}$ $\qquad$ $\tan B = \dfrac{\text{opposite}}{\text{adjacent}} = \dfrac{AC}{BC}$

- Use the acronym **SOH–CAH–TOA** to help you remember the trigonometric ratios.

Discuss the Concepts

D1. Explain to a classmate what SOH–CAH–TOA stands for.

D2. Is it possible to solve a right triangle given just one side measure? Explain.

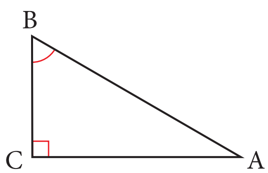
D3. In Example 6, you were shown two methods for finding the measure of side *a*. Look closely at the two methods. What change was made? Is the following true: $\sin 40° = \cos 50°$? Explain.

Practise A

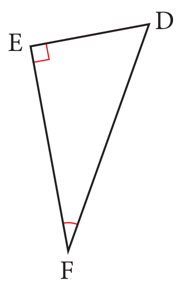
For help with question 1, refer to Example 1.

1. Name the opposite, adjacent, and hypotenuse sides associated with ∠B, ∠F, and ∠Z.

a)



b)



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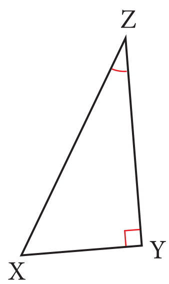


For help with question 2, refer to Example 2.

2. Evaluate. Round your answers to four decimal places.

a) sin 30° **b)** cos 45° **c)** tan 60°

For help with question 3, refer to Example 3.

3. Find the measure of each angle to the nearest tenth of a degree.

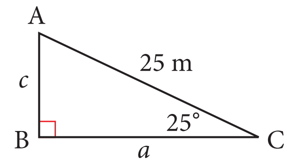
a) sin A = 0.2345 **b)** cos B = 0.8765 **c)** tan C = 1.2345

For help with question 4, refer to Example 4.

4. a) Find the measure of side *a* to the nearest metre.

b) Find the measure of side *c* to the nearest metre.

c) Find the measure of ∠A.

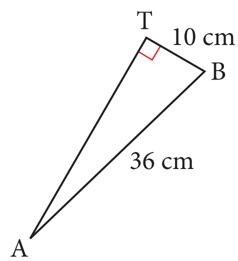


For help with question 5, refer to Example 5.

5. a) Find the measure of ∠A to the nearest tenth of a degree.

b) Find the measure of ∠B to the nearest tenth of a degree.

c) Find the measure of side *b* to the nearest centimetre.



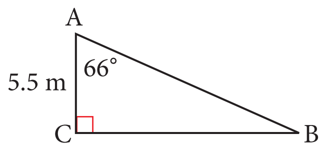
Apply B

Where necessary, round answers to the nearest tenth.

For help with questions 6 to 13, refer to Example 6.

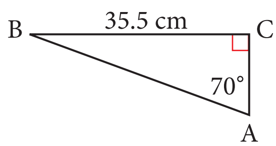
6. a) Find the measure of the hypotenuse.

b) Find the measure of side *a*.



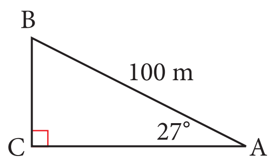
7. a) Find the measure of side *b*.

b) Find the measure of side *c*.

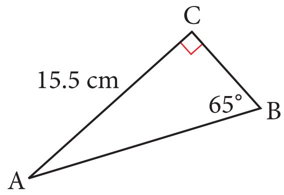


8. a) Find the measure of side *a*.

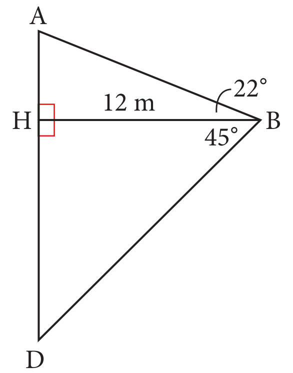
b) Find the measure of side *b*.



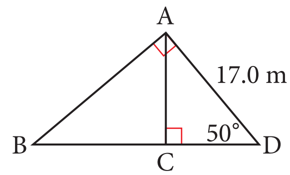
9. Solve $\triangle ABC$.



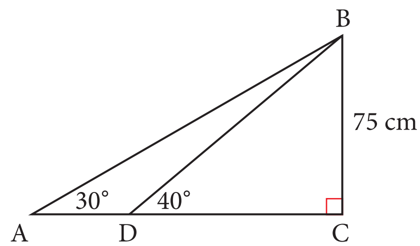
10. Find the measure of side AD.



11. Find the measure of side BC.

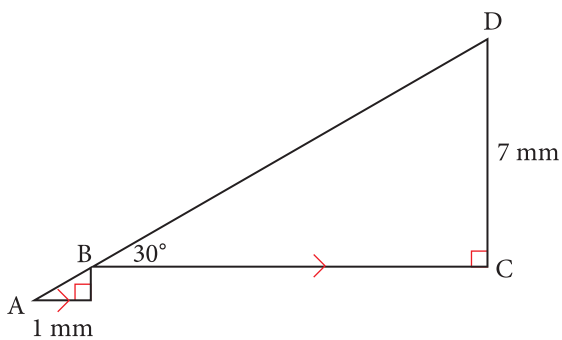


12. Find the measure of side AD.

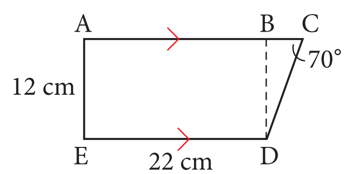


Extend C

13. Find the measure of side AD to the nearest millimetre.



14. Find the area of the trapezoid to the nearest square centimetre.



1.2 Solve Problems Using Trigonometric Ratios

The heights of some structures are best determined using trigonometry rather than a measuring tape. For example, a person could stand a certain distance from the base of the CN Tower and use a clinometer to determine the angle of elevation to the top of the tower. Then, using trigonometry, she could find the height of the tower.

A clinometer measures the angle of a line of sight above or below the horizontal. It is used by construction workers to measure grade angles, by forestry workers to measure the heights of trees, and by movie directors to measure the sun's elevation. It is also used by satellite antenna installers to locate satellites.

Investigate

Tools

- BLM Make Your Own Clinometer
- metre stick
- measuring tape

Angle of Elevation

Work with a partner.

Part A: Vary the Adjacent Side

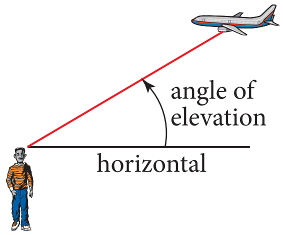
1. Copy the table. Use it to record your results.

Distance From Wall (m)	Height to Eye Level (m)	Angle of Elevation, θ (°)	tan θ	Height of Wall (m)
2.0	0.9			
2.5	0.9			
3.0	0.9			

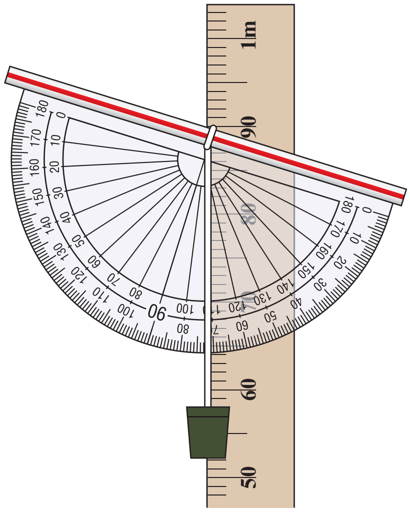
2. Use a hand-made clinometer and stand 2.0 m from the wall of your classroom.

angle of elevation

- the angle between the horizontal and the sight line from the observer's eye to some object above eye level

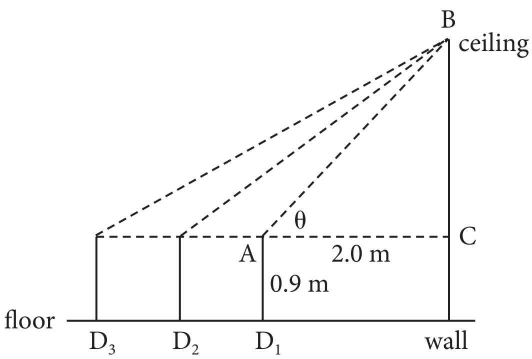


3. Hold the clinometer along the edge of a metre stick at the 90 cm (0.9 m) mark. Look through the straw of the clinometer to where the ceiling meets the wall. Have your partner record the **angle of elevation** (or incline) to where the ceiling meets the wall.



4. Repeat steps 1 to 3 at distances of 2.5 m and 3.0 m from the wall.

5. Copy the diagram that models the situation. Use the data from your table to complete the diagram with additional labels.



The symbol θ represents the angle of elevation.

Part B: Vary the Opposite Side

6. Copy the table.

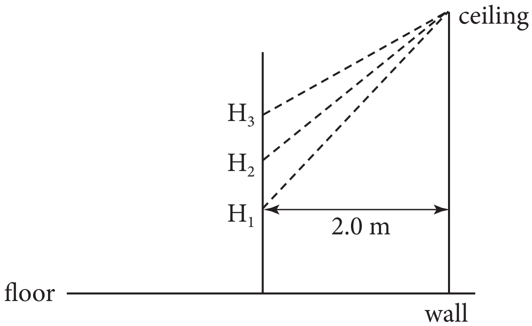
Distance From Wall (m)	Height to Eye Level (m)	Angle of Elevation, θ (°)	tan θ	Height of the Wall (m)
2.0	0.8			
2.0	0.9			
2.0	1.0			

7. Stand 2.0 m from the wall. Hold the clinometer along an edge of a metre stick at the 80 cm (0.8 m) mark. Look through the straw of the clinometer to where the ceiling meets the wall. Have your partner record the angle of elevation to where the ceiling meets the wall.

8. Raise the clinometer 10 cm along the edge of the metre stick to 90 cm (0.9 m). Record the angle of elevation. Make sure your clinometer is level.

9. Repeat step 8 at the 1 m mark.

10. Copy the diagram. Use the data from your table to complete the diagram with additional labels.



H_1, H_2, and H_3 are different heights of the clinometer.

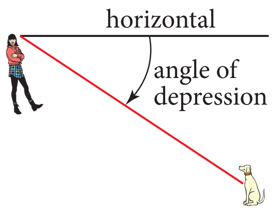
11. How did the angle of elevation change in Part A?

12. Use a measuring tape to find the exact height of the wall.

13. Did your results appear to be close to the height in both parts of the Investigate? How close? If there are discrepancies between the heights, what are possible reasons for the differences?

Example 1

angle of depression
- the angle between the horizontal and the sight line from the observer's eye to a point below eye level



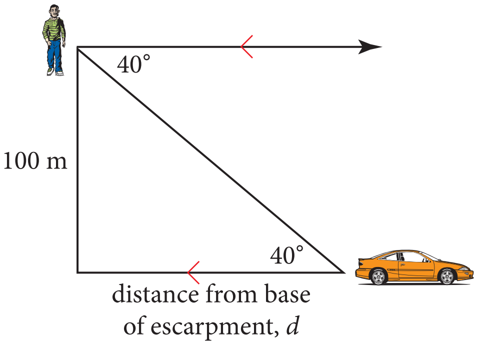
Calculate Distances

From the top of the Niagara Escarpment, Juan sees a car below at an **angle of depression** (or descent) of 40°. Juan is approximately 100 m above the car. How far is the car from the base of the escarpment? Round your answer to the nearest metre.

Solution

What do I know?	What do I need to know?
• angle of depression: 40° • height: 100 m	• distance from the base of the escarpment

Obtain information from the problem.
Use a right triangle to model the problem.



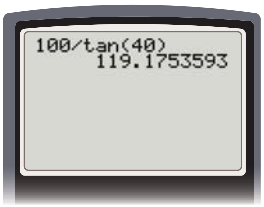
Draw a horizontal line from Juan so that it is parallel to the base of the triangle. The angle made with the line of sight and the distance from the base of the escarpment is equal to the angle of depression. These are alternate angles.

Draw and label the triangle with the information provided.

Solve for the unknown side or angle. In this case, find side *d*.

Use the given information. Write an appropriate trigonometric ratio.

$$\tan 40° = \frac{100}{d}$$
$$d \times \tan 40° = 100$$
$$\frac{d \times \tan 40°}{\tan 40°} = \frac{100}{\tan 40°}$$
$$d \doteq 119$$



100 ÷ TAN 40) ENTER

Technology Tip

For a scientific calculator, you might use this key sequence:

100 ÷ 40 TAN =

For a DAL calculator, you might use this key sequence:

100 ÷ TAN 40 =

Write a concluding sentence.

The car is approximately 119 m from the base of the escarpment.

Example 2

Calculate the Measure of an Angle

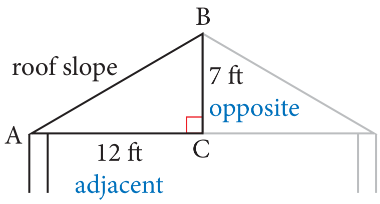
In construction, the pitch of a roof may be given as "7–12" in feet. This means the maximum height of the roof is 7 ft and the distance from the midpoint of the base of the roof to the outer wall is 12 ft.

Calculate the roof's **angle of inclination**. Round your answer to the nearest degree.

angle of inclination
- another name for the angle of elevation

Solution

Model the problem.



∠A is the angle of inclination.

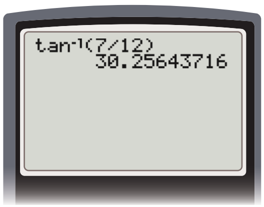
$$\tan A = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan A = \frac{7}{12}$$

$$\angle A = \tan^{-1}\left(\frac{7}{12}\right)$$

$$\angle A \doteq 30°$$

2nd TAN 7 ÷ 12) ENTER



The roof's angle of inclination is approximately 30°.

Technology Tip

For a scientific calculator, you might use this key sequence:

(7 ÷ 12) 2nd TAN =

For a DAL calculator, you might use this key sequence:

2nd TAN (7 ÷ 12) =

Key Concepts

- The angle of elevation, or angle of inclination, is the angle made between the horizontal and the upward line of sight.
- The angle of depression is the angle made between the horizontal and the downward line of sight.
- Use these steps to solve problems using trigonometric ratios.
  – Identify what needs to be calculated. Is it a side or an angle?
  – Model the problem using a right triangle. Label the sides associated with the unknown angle (i.e., opposite, adjacent, hypotenuse).
  – Write an equation using a trigonometric ratio. Substitute the given values.
  – Solve for the unknown measure.
  – Write a concluding sentence that answers the question.

Discuss the Concepts

D1. Is it possible to solve a problem involving right triangles given only the measure of one side? Explain.

D2. Work in pairs. One partner explains to the other how to solve the following problem using trigonometric ratios. (Do *not* actually solve the problem.)

A 10-m ladder is leaning against a wall. The top of the ladder rests against the wall 9.3 m above the ground. What angle does the base of the ladder make with the ground?

Practise A

For questions 1 to 15, round your answers to the nearest unit of measurement.

1. A wheelchair ramp is needed at the entrance of a restaurant. The ramp is to be 6.10 m long and have a rise of 0.45 m. Calculate the angle of inclination of the ramp.

For help with question 2, refer to Example 1.

2. From the top of a bridge over the Burlington Canal, Maria looks down at a sailboat at an angle of depression of 15°. The bridge is 18 m above the water. Calculate the horizontal distance from the bridge to the sailboat.

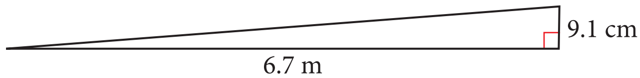
For help with question 3, refer to Example 2.

3. A 7.6-m flagpole is 4.6 m away from a pedestrian. What is the angle of elevation from where the pedestrian is standing to the top of the flagpole?

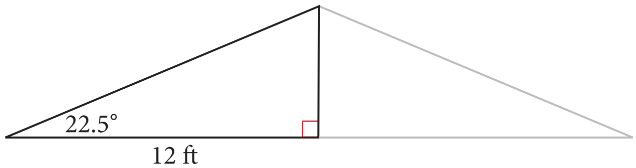
Apply B

For help with question 4, refer to Example 2.

4. A garage floor is made of poured concrete. The length of the garage is 6.7 m and the grade (the rise of the floor from the front to the back) is 9.1 cm. Calculate the angle of inclination of the garage floor.



5. A rafter makes an angle of 22.5° with the roof joist, as shown. How tall is the board supporting the middle of the roof?



6. Safety by-laws state that for a ladder to be stable, the angle the base of the ladder makes with the ground should be between 70° and 80°. A safety inspector at a construction site notices a painter on a 10-m ladder that is leaning against a wall. The base of the ladder is 1.5 m away from the wall. Does the inspector have cause to be concerned? Explain.

7. Solve the problem from question D2 in Discuss the Concepts. Is the ladder stable? Explain.

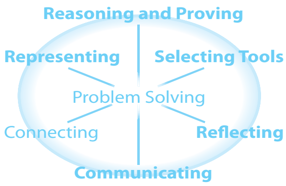
8. A rescue helicopter sights a boat in distress at an angle of 40° from the horizontal. The helicopter is hovering 400 m above the water. What is the horizontal distance between the helicopter and the boat?

Chapter Problem

9. The expedition team decided to have a practice run prior to their North Pole trek. One team member started to walk due north. The other three travelled 65° east of north at a pace of 3 km/h. How far off the first team member's course were they after 2 h?

Literacy Connect

10. Wheelchair ramps must have a 1:12 ratio of vertical height to horizontal length to meet safety standards. The safety standards for other types of ramps are different. What are the safety standards for building a skateboard ramp at a skateboard park? Investigate the ramps at a skateboard park and provide measurements of one of the ramps. Determine the angle of inclination. Include a diagram.



11. The CN Tower is 553.33 m high. Lina looks up at the top of the tower at a 15° angle of elevation. She calculates the distance, d, from the base of the tower as follows:

$$\frac{d}{553.33} = \tan 15°$$

$$d = 553.33 \times \tan 15°$$

$$d \doteq 149$$

Explain why Lina's solution is incorrect. Write a correct solution.

12. Two buildings are 20 m apart. The angle from the top of the shorter building to the top of the taller building is 20°. The angle from the top of the shorter building to the base of the taller building is 45°. What is the height of the taller building?

13. The shuttle *Enterprise* lifts off from Cape Canaveral. Calculate the angle of elevation of the shuttle, from an observer located 8 km away, when the shuttle reaches a height of 3500 m.

14. The Instrument Landing System (ILS) common to most major airports uses radio beams to bring an aircraft down a 3° glide slope. A pilot noted that his height above the ground was 200 m. How far would the pilot have to travel before landing on the runway?

Extend C

15. From the top of a 200-m high cliff, the angles of depression of two boats on the water are 20° and 25°. How far apart are the boats? What assumptions must you make?

16. The high end of a 22-ft-long roof rafter is nailed to a 7-ft vertical support, which is located at the middle of the roof. Calculate the height of the first support piece to be nailed 16 in. from the middle support piece to the nearest tenth of a foot.

1.3 The Sine Law

sine law
- the relationship between the length of the sides and their opposite angles in any triangle

Some situations are modelled by non-right triangles. The Leaning Tower of Pisa, for example, leans from its vertical and does not form a right angle with the ground. The height of the tower must be determined using other tools of trigonometry, such as the **sine law**. How can you use the sine law to calculate the height of the tower?

Investigate

Tools

- *The Geometer's Sketchpad*®
- computer

The Sine Law

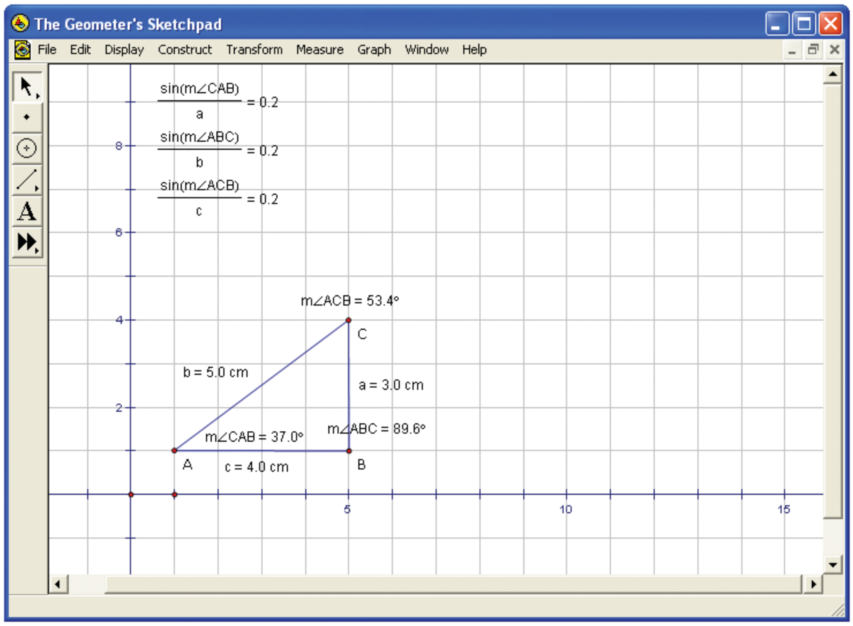
1. Construct a right triangle using *The Geometer's Sketchpad*®. Measure the lengths of the sides and the angles.
   - Open *The Geometer's Sketchpad*®.
   - From the **Edit** menu, choose **Preferences**. Click on the **Text** tab, and check the box **For All New Points**. This will label points as you draw them.
   - From the **Graph** menu, choose **Show Grid**. Drag the origin to the lower left corner of the workspace. From the **Graph** menu, choose **Snap Points**.

Technology Tip
Hold the Shift key down while drawing a vertical or horizontal line.

- Use the **Straightedge Tool** to draw a right triangle. Select the three sides of the triangle. From the **Measure** menu, choose **Length**. Drag each measurement beside the correct side.
- Right-click on each measurement in turn. Choose **Label Distance Measurement**. Enter the label *a* for BC, *b* for AC, and *c* for AB.
- Select the points B, A, and C, in that order. From the **Measure** menu, choose **Angle**. Move the measurement beside ∠A. Repeat this process for the measures of ∠B and ∠C.

2. Calculate the ratio of the sine of an angle and the length of the opposite side.

From the **Measure** menu, choose **Calculate**. A calculator box will appear. Click on the **Functions** button, and choose **sin**. Click on the measure of ∠A. Press the ÷ button. Click on the measurement of the side opposite ∠A. Click on **OK**. Repeat this process to calculate the corresponding ratios for ∠B and ∠C.



3. **Reflect** What do you notice about the three ratios?

4. Drag point B so that the triangle is no longer a right triangle. How are the ratios affected?

5. Drag point A, and then point C. How are the ratios affected?

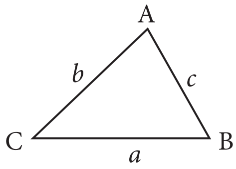
6. **Reflect** Are these ratios equal for any triangle? Explain.

7. **Reflect** Evaluate sin 90°. What is its value? Is there a relation between using the sine law and using the sine ratio for a right triangle?

The Sine Law $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

or

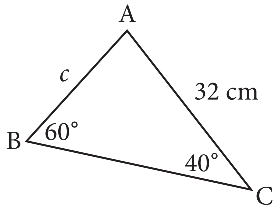
$\dfrac{\sin A}{a} = \dfrac{\sin B}{b} = \dfrac{\sin C}{c}$



Example 1

Find the Measure of a Side

Find the measure of side c to the nearest centimetre.



Solution

Solve for c.

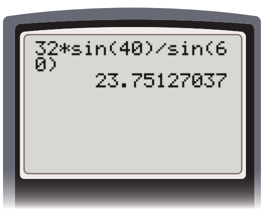
$$\frac{c}{\sin 40^\circ} = \frac{32}{\sin 60^\circ}$$

$$c = \frac{32 \times \sin 40^\circ}{\sin 60^\circ}$$

$$c \doteq 24$$

Use the proportion in the form $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$ to find the measure of a side. Note that any two ratios can be used at one time.

To make your calculation easier, start with the unknown as the numerator on the left.



32 × SIN 40) ÷ sin 60) ENTER

Side c is approximately 24 cm.

Technology Tip

For a scientific calculator, you might use this key sequence:

40 SIN × 32 ÷ 60 SIN =

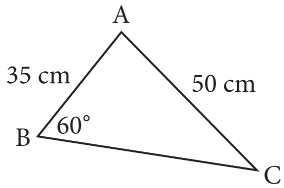
For a DAL calculator, you might use this key sequence:

32 × SIN 40 ÷ SIN 60 =

Example 2

Find the Measure of an Angle

Find the measure of ∠C to the nearest tenth of a degree.



Solution

$$\frac{\sin C}{c} = \frac{\sin B}{b}$$

$$\frac{\sin C}{35} = \frac{\sin 60°}{50}$$

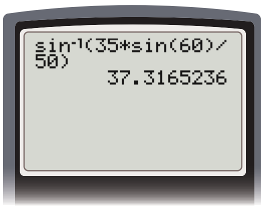
$$\sin C = \frac{35 \times \sin 60°}{50}$$

$$\angle C = \sin^{-1}\left(\frac{35 \times \sin 60°}{50}\right)$$

$$\angle C \doteq 37.3°$$

Use the proportion in the form $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$ to find the measure of an angle.

Check that, for all triangles, the largest angle is opposite the longest side. The smallest angle is opposite the shortest side.



2nd SIN 35 × SIN 60) ÷ 50) ENTER

The measure of ∠C is approximately 37.3°.

Technology Tip

For a scientific calculator, you might use this key sequence:

60 SIN × 35 ÷ 50 2nd SIN =

For a DAL calculator, you might use this key sequence:

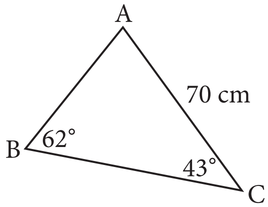
2nd SIN (35 × SIN 60 ÷ 50) =

Example 3

Solve Triangles

Solve the following triangles. Round your answers to the nearest unit of measurement.

a)



b) For a triangle XYZ, $\angle X = 65°$, $x = 14$ cm, and $y = 9$ cm.

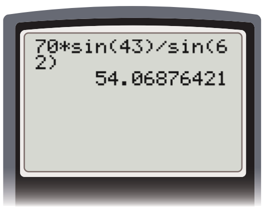
Solution

a) $\angle A = 180° - 62° - 43°$
$= 75°$

Find the measure of ∠A first.

$$\frac{c}{\sin 43°} = \frac{70}{\sin 62°}$$
$$c = \frac{70 \times \sin 43°}{\sin 62°}$$
$$c \doteq 54$$

Next find the measure of the shortest side. In this case, find *c*. The shortest side is opposite the smallest angle.

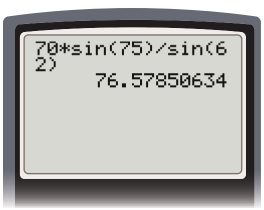


70 × SIN 43) ÷ SIN 62) ENTER

Side c is approximately 54 cm.

$$\frac{a}{\sin 75°} = \frac{70}{\sin 62°}$$
$$a = \frac{70 \times \sin 75°}{\sin 62°}$$
$$a \doteq 77$$

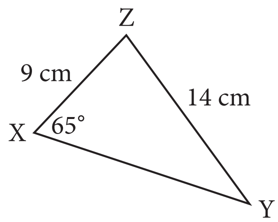
Now find the measure of the longest side. The longest side is opposite the largest angle.



70 × SIN 75) ÷ SIN 62) ENTER

Side *a* is approximately 77 cm.

b) Sketch the triangle.



$$\frac{\sin Y}{9} = \frac{\sin 65°}{14}$$

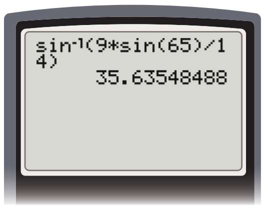
$$\sin Y = \frac{9 \times \sin 65°}{14}$$

$$Y = \sin^{-1}\left(\frac{9 \times \sin 65°}{14}\right)$$

$$\angle Y \doteq 36°$$

Find the smaller angle measure first. In this case, it is $\angle Y$.

Use the proportion in the form $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$ to find the measure of an angle.



2nd SIN 9 × SIN 65) ÷ 14) ENTER

$$\angle Z = 180° - 65° - 36°$$
$$= 79°$$

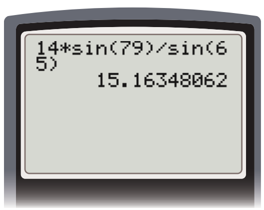
Next find the measure of the third angle.

$$\frac{z}{\sin 79°} = \frac{14}{\sin 65°}$$

$$z = \frac{14 \times \sin 79°}{\sin 65°}$$

$$z \doteq 15$$

Use the proportion in the form $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ to find the unknown side, z.



14 × SIN 79) ÷ SIN 65) ENTER

Side z is approximately 15 cm.

Example 4

Solve Problems Using the Sine Law

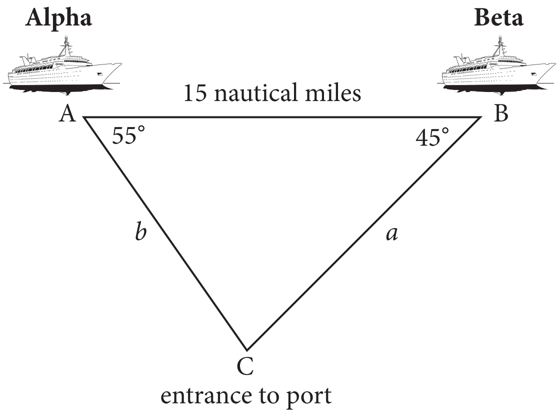
Two ships are located 15 nautical miles apart. *Alpha*'s angle to the entrance of the port is 55° with respect to *Beta*. *Beta*'s angle to the entrance to the port is 45° with respect to *Alpha*. Which ship is closer to the port entrance? How far is the ship from port? Round your answer to the nearest tenth.

Solution

Literacy Connect

The nautical mile is used around the world for maritime and aviation measures.
1 nautical mile = 1.852 km

Draw the triangle to model the problem.

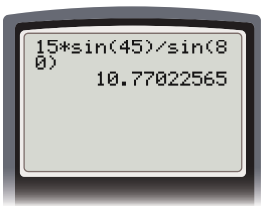


Remember that the shortest side is opposite the smallest angle.

$$\angle C = 180° - 55° - 45°$$
$$= 80°$$

Side b is the shortest side. Find its measure.

$$\frac{b}{\sin B} = \frac{c}{\sin C}$$
$$\frac{b}{\sin 45°} = \frac{15}{\sin 80°}$$
$$b = \frac{15 \times \sin 45°}{\sin 80°}$$
$$b \doteq 10.8$$



15 [×] [SIN] 45 [)] [÷] [SIN] 80 [)] [ENTER]

Alpha is closer to port. It is approximately 10.8 nautical miles from the port entrance.

Key Concepts

- An acute triangle, ABC, can be solved using the sine law if you know:
  – two angle measures and one side measure
  – an angle measure and two side measures, provided one of the sides is opposite the given angle
- The measure of a side of a triangle can be calculated using a proportion made of two of the ratios from the sine law: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$.
- The measure of an angle of a triangle can be calculated using a proportion made of two of the ratios from the sine law:
$\dfrac{\sin A}{a} = \dfrac{\sin B}{b} = \dfrac{\sin C}{c}$.

Discuss the Concepts

D1. Is it possible to solve △ABC given the measures of all three sides using the sine law? Explain.

D2. Work in pairs. Explain to your partner how to solve △ABC, where ∠A = 65°, ∠B = 75°, and a = 8 cm.

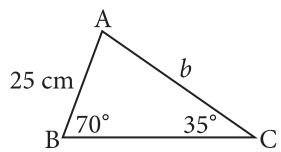
Practise A

In questions 1 to 4, round your answers to the nearest tenth.

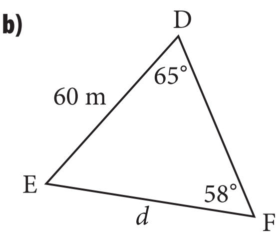
For help with question 1, refer to Example 1.

1. Find the measure of the indicated side in each triangle.

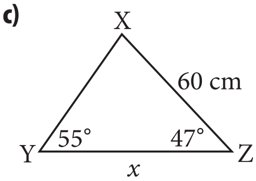
a)



b)



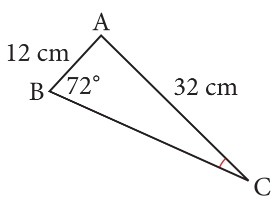
c)



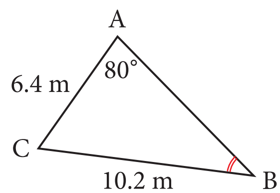
For help with question 2, refer to Example 2.

2. Find the measure of the unknown angle as indicated.

a)



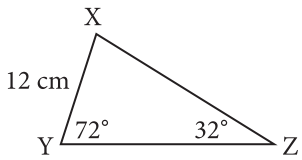
b)



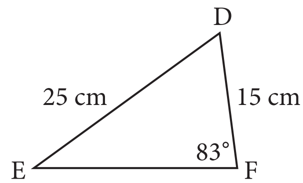
For help with questions 3 and 4, refer to Example 3.

3. Solve each triangle.

a)



b)



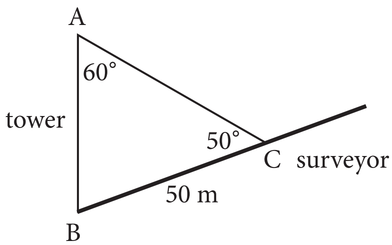
4. **a)** Solve $\triangle ABC$, given $\angle B = 39°$, $\angle C = 79°$, and $a = 24$ cm.

b) Solve $\triangle DEF$, given $\angle D = 75°$, $d = 25$ m, and $e = 10$ m.

Apply B

For help with question 5, refer to Example 4.

5. A communication tower is built on the slope of a hill. A surveyor, 50 m uphill from the base of the tower, measures an angle of 50° between the ground and the top of the tower. The angle from the top of the tower to the surveyor is 60°. Calculate the height of the tower to the nearest metre.



Chapter Problem

6. The expedition team decides to have another practice run. Two team members head due north at a pace of 4 km/h. The second pair decide to head 60° west of north travelling at the same pace. How far from the first pair is the second pair after 2 h?

Literacy Connect

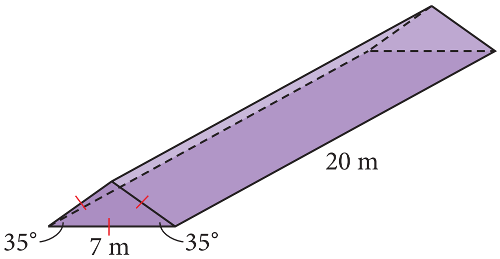
7. You are asked to solve a triangle with two known sides using the sine law. Explain to a classmate what additional information you need to know about the triangle.

8. A shed is 8 ft wide. One rafter makes an angle of 30° with the horizontal on one side of the roof. A rafter on the other side makes an angle of 70° with the horizontal. Calculate the length of the shorter rafter to the nearest foot.

9. Three islands—Fogo, Twillingate, and Moreton's Harbour—form a triangular pattern in the ocean. Fogo and Twillingate are 15 nautical miles apart. The angle between Twillingate and Moreton's Harbour from Fogo is 45°. The angle between Moreton's Harbour and Fogo from Twillingate is 65°. How far is Moreton's Harbour from the other two islands to the nearest nautical mile?

Achievement Check

10. A house is 7 m wide and 20 m long. The roof slopes at an angle of 35° as shown. The two rectangular parts on top of the roof are to be shingled at a cost of $\$25/m^2$. Calculate the total cost, to the nearest dollar.

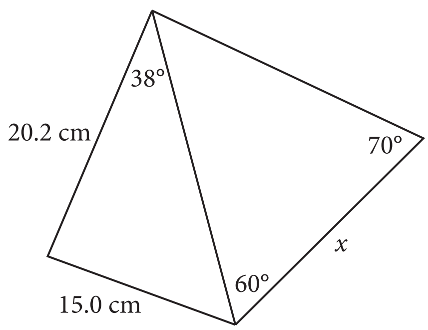


Extend C

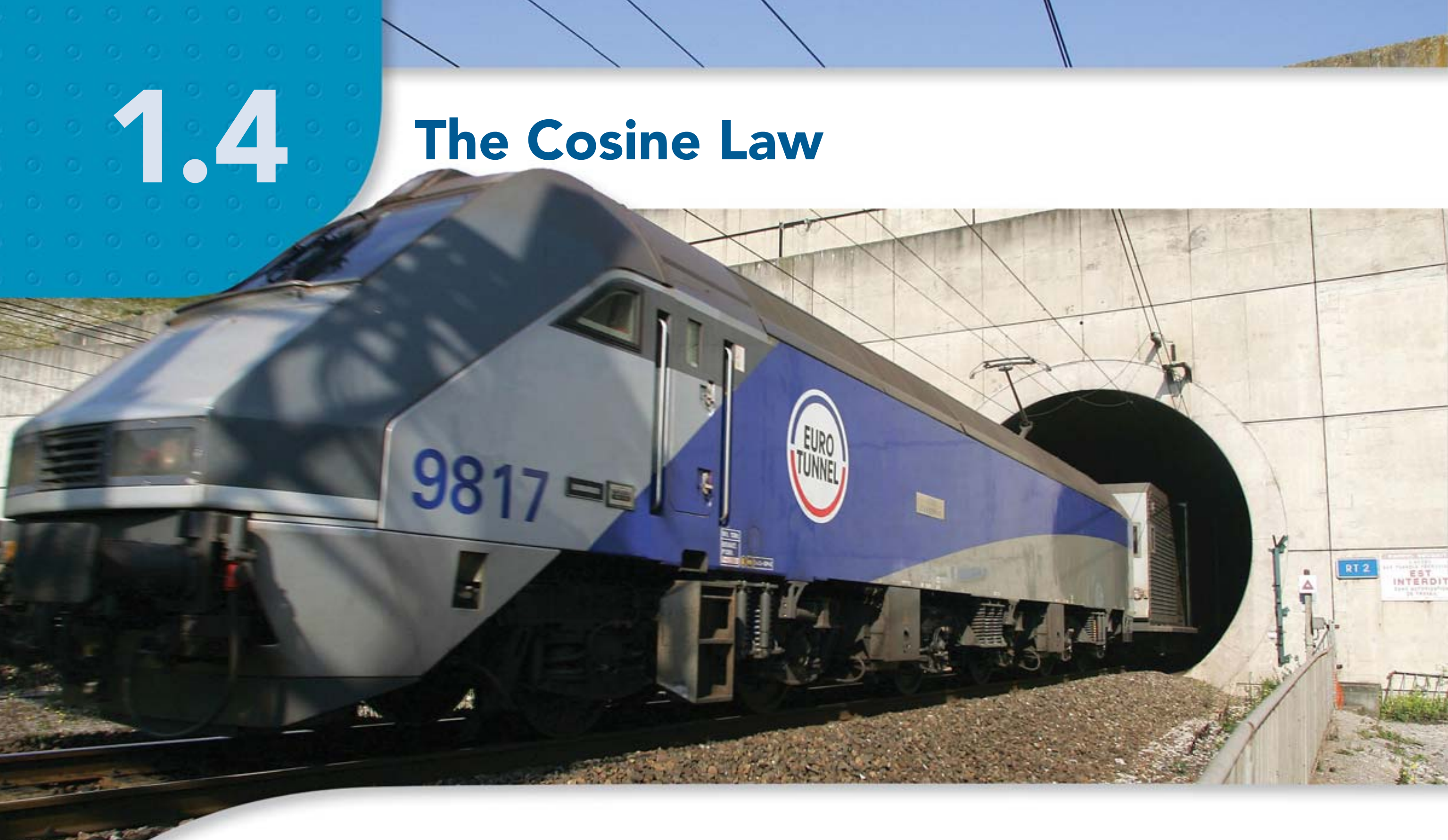
11. The Leaning Tower of Pisa leans 5.5° from its vertical. Suppose that the sun is directly overhead. A surveyor notices that the distance from the base of the tower to the tip of its shadow is 5.35 m. What is the height of the tower on the lower side to the nearest tenth of a metre?

12. Search the Internet for an image of the Leaning Tower of Pisa. Cut and paste the image into *The Geometer's Sketchpad*®. Determine the angle that the tower makes with the vertical.

13. Find the length of side x to the nearest tenth of a centimetre.



1.4 The Cosine Law



cosine law
- the relationship between the lengths of the three sides and the cosine of an angle in any triangle

The Channel Tunnel that links England and France, is 51.5 km long with 37.5 km under the English Channel. Construction started on both sides of the Channel. When the two crews met, the walls of the two tunnels were only a few millimetres off. Trigonometry had something to do with this amazing feat! In this lesson you learn about the **cosine law**.

Investigate

Tools

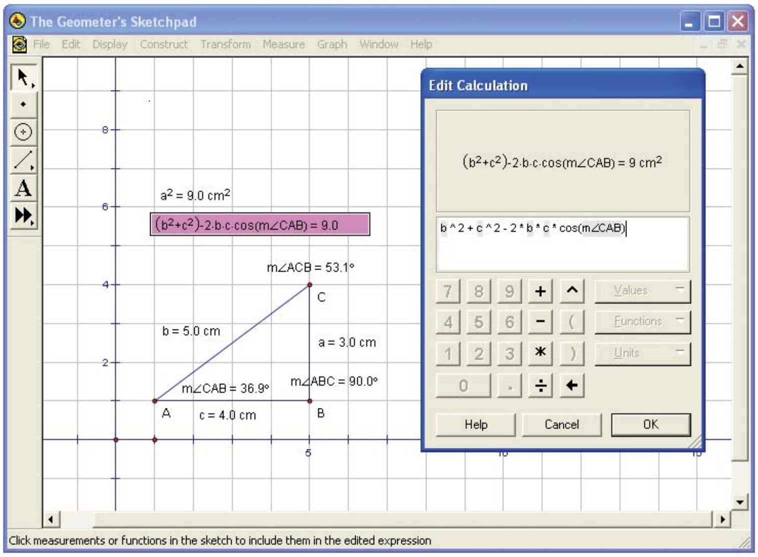
- *The Geometer's Sketchpad®*
- computer

The Cosine Law

1. Construct a right triangle using *The Geometer's Sketchpad®*. Measure the lengths of the sides, and the angles.
- Open *The Geometer's Sketchpad®*.
- From the **Edit** menu, choose **Preferences**. Click on the **Text** tab, and check the box **For All New Points**. This will label points as you draw them.
- From the **Graph** menu, choose **Show Grid**. Drag the origin to the lower left corner of the workspace. From the **Graph** menu, choose **Snap Points**.
- Use the **Straightedge Tool** to draw a right triangle. Select the three sides of the triangle. From the **Measure** menu, choose **Length**. Drag each measurement beside the correct side.

- Right-click on each measurement in turn. Choose **Label Distance Measurement**. Enter the label a for BC, b for AC, and c for AB.
- To measure $\angle$A, select points B, A, and C, in that order. From the **Measure** menu, choose **Angle**. Move the measurement beside $\angle$A. Repeat this process for the measures of $\angle$B and $\angle$C.

2. Calculate the value of a^2 and of the expression $b^2 + c^2 - 2bc \cos A$.
- Choose **Calculate** from the **Measure** menu. A calculator box will appear. Click on the measure of length a, then click the ^ button and the 2 button. Click on **OK**.
- Click on the measure of length b, then click the ^ button and the 2 button. Click on the + button. Enter the remaining terms. Click on **OK**.



3. Reflect How does the value of a^2 compare to the value of the expression $b^2 + c^2 - 2bc \cos A$?

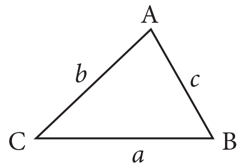
4. Drag point B so that the triangle is no longer a right triangle. How does the value of a^2 compare to the value of the expression?

5. Drag point A, and then point C. How does the value of a^2 compare to the value of the expression?

6. Reflect The relationship $a^2 = b^2 + c^2 - 2bc \cos A$ in $\triangle$ABC is known as the cosine law. What is the value of cos 90°? What do you call the equation in this case?

7. Reflect How can you rearrange the cosine law to get an equation for cos A?

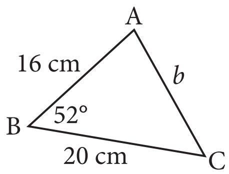
The Cosine Law $a^2 = b^2 + c^2 - 2bc \cos A$



Example 1

Find the Measure of a Side Given Two Sides and the Contained Angle

Find the measure of the unknown side to the nearest tenth of a centimetre.



Solution

Use the cosine law.

The sine law cannot be used here. Why?

Write the equation to find the measure of the unknown side.

$b^2 = a^2 + c^2 - 2ac \cos B$

$b^2 = 20^2 + 16^2 - 2(20)(16) \cos 52°$

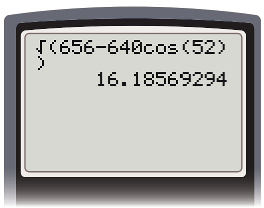
$b^2 = 400 + 256 - 640 \cos 52°$

$b^2 = 656 - 640 \cos 52°$

$b = \sqrt{656 - 640 \cos 52°}$

$b \doteq 16.2$

Length is a positive value. Find the positive square root.



2nd x^2 656 − 640 COS 52)) ENTER

Side b is approximately 16.2 cm.

Technology Tip

For a scientific calculator, you might use this key sequence:

656 − 52 COS × 640 = √ =

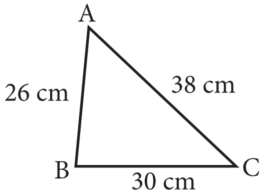
For a DAL calculator, you might use this key sequence:

√ (656 − 640 COS 52) =

Example 2

Find the Measure of an Angle Given Three Side Lengths

Find the measure of ∠A to the nearest degree.



Solution

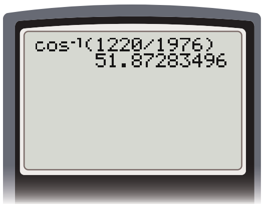
Use the cosine law.

Write the equation to find the measure of ∠A.

$$\begin{aligned} a^2 &= b^2 + c^2 - 2bc\cos A \\ 2bc\cos A &= b^2 + c^2 - a^2 \\ 2(38)(26)\cos A &= 38^2 + 26^2 - 30^2 \\ 1976\cos A &= 1444 + 676 - 900 \\ 1976\cos A &= 1220 \\ \cos A &= \frac{1220}{1976} \\ \angle A &= \cos^{-1}\left(\frac{1220}{1976}\right) \\ \angle A &\doteq 52^\circ \end{aligned}$$

Rearrange the equation to isolate the term containing A.

2nd COS 1220 ÷ 1976) ENTER



The measure of ∠A is 52°, to the nearest degree.

Technology Tip

For a scientific calculator, you might use this key sequence:

1220 ÷ 1976
2nd COS =

For a DAL (direct algebraic logic) calculator, you might use this key sequence:

2nd COS (
1220 ÷ 1976
) =

Example 3

Math Connect

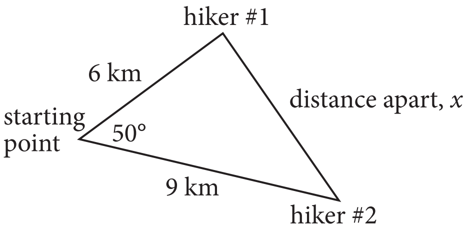
The Bruce Trail is the oldest and longest continuous footpath in Canada. It runs along the Niagara Escarpment from Niagara to Tobermory, spanning more than 850 km of main trail and 250 km of side trails.

Solve a Problem Using the Cosine Law

Two hikers set out in different directions from a marked tree on the Bruce Trail. The angle formed between their paths measures 50°. After 2 h, one hiker is 6 km from the starting point and the other is 9 km from the starting point. How far apart are the hikers, to the nearest tenth of a kilometre?

Solution

Draw a diagram to model the problem.

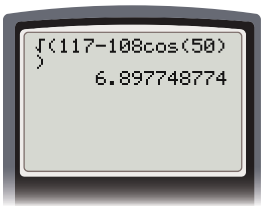


Let x represent the distance apart, in kilometres.

Use the cosine law.

You are given the measures of two sides and a contained angle.

$$x^2 = 6^2 + 9^2 - 2(6)(9)\cos 50°$$
$$x^2 = 36 + 81 - 108\cos 50°$$
$$x^2 = 117 - 108\cos 50°$$
$$x = \sqrt{117 - 108\cos 50°}$$
$$x \doteq 6.9$$



[2nd] [x^2] 117 [−] 108 [COS] 50 [)] [)] [ENTER]

The hikers are approximately 6.9 km apart.

Technology Tip

For a scientific calculator, you might use this key sequence:

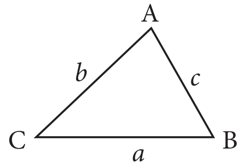
117 [−] 108 [×] 50 [COS] [=] [√] [=]

For a DAL calculator, you might use this key sequence:

[√] [(] 117 [−] 108 [COS] 50 [)] [=]

Key Concepts

- For $\triangle ABC$, to find the measure of any side, given two sides and the contained angle, the cosine law can be written as follows:



$a^2 = b^2 + c^2 - 2bc \cos A$
$b^2 = a^2 + c^2 - 2ac \cos B$
$c^2 = a^2 + b^2 - 2ab \cos C$

- The cosine law can also be used to find the measure of an unknown angle, given three sides.

$\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$, $\cos B = \dfrac{a^2 + c^2 - b^2}{2ac}$, $\cos C = \dfrac{a^2 + b^2 - c^2}{2ab}$

Discuss the Concepts

D1. Explain to a classmate what information about a triangle is necessary to use the cosine law to find the measure of an unknown side.

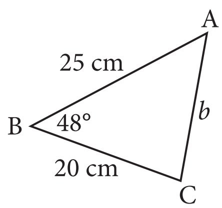
D2. Explain when you would use the sine law and when you would use the cosine law.

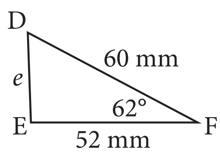
Practise A

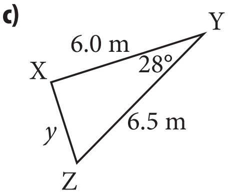
In questions 1 and 2, round your answers to the nearest tenth.

For help with question 1, refer to Example 1.

1. Find the measure of the unknown side.

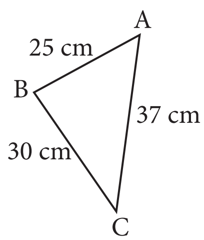
a)


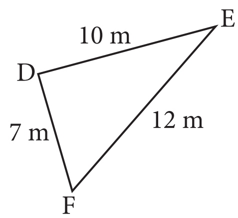
b)


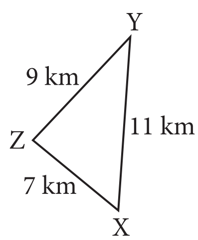
c)


For help with question 2, refer to Example 2.

2. Find the measure of the unknown angle as indicated.

a) Find $\angle$A.


b) Find $\angle$D.


c) Find $\angle$X.


Apply B

3. Find the measures of the unknown sides and angles in △ABC, given ∠A = 32°, b = 25.5 m, and c = 22.5 m. Round side lengths to the nearest tenth of a metre and angles to the nearest degree.

4. Find the measures of the unknown angles in △ABC, given a = 14 m, b = 15 m, and c = 8 m. Round angles to the nearest degree.

Chapter Problem

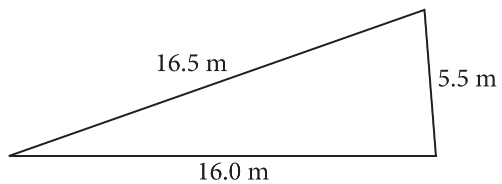
5. The expedition team plan a final practice run to test the range of their communication equipment. One member travels a distance of 12 km due north. Another team member heads 50° east of north and travels a distance of 10 km. How far apart are the two team members? Round your answer to the nearest tenth of a kilometre.

Literacy Connect

6. What information about a triangle do you need to know to use the cosine law? Provide examples to help you explain.

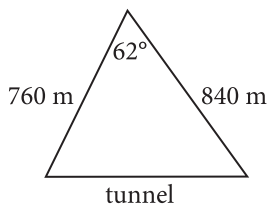
For help with question 7, refer to Example 2.

7. A motocross ramp is to be built for an upcoming race. The measures for the sides of the ramp are as shown. Calculate the angle of inclination of the ramp to the nearest degree.

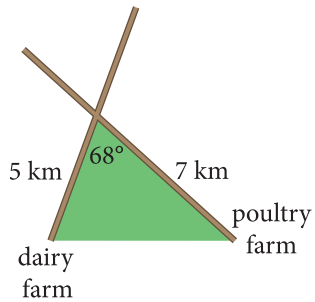


For help with question 8, refer to Example 3.

8. Dahliwal is an engineer. For his latest contract, he has to determine the length of a tunnel that is to be built through a mountain. He chooses a point facing the mountain. He measures a distance of 840 m from one end of the tunnel to the point and a distance of 760 m from the other end of the tunnel to the point. The angle at the point to both ends of the tunnel is 62°. Calculate the length of the proposed tunnel to the nearest metre.



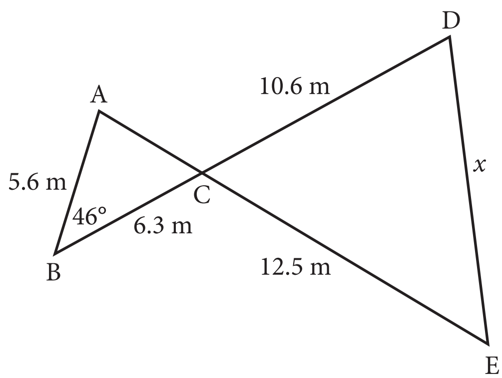
9. An intersection between two country roads makes an angle of 68°. Along one road, 5 km from the intersection, is a dairy farm. Along the other road, 7 km from the intersection, is a poultry farm. How far apart are the two farms? Round your answer to the nearest tenth of a kilometre.



10. A flock of Canada geese are flying in a V-formation that forms an angle of 68°. The lead goose is 12.8 m from the last goose on the left and 13.5 m from the last goose on the right. How far apart are the last two geese in the V-formation? Round your answer to the nearest tenth of a metre.

Extend C

11. Solve for x to the nearest tenth of a metre.



12. **a)** A triangle is built using three poles with lengths of 170 cm, 152 cm, and 88 cm. What are the angles between adjacent poles?

b) Could you still build a triangle if the 152-cm pole was replaced by a pole half as long?

Math Connect

1 knot = 1 nautical mile per hour

13. Two ships set sail from port. *Alpha* is sailing at 12 knots and *Beta* is sailing at 10 knots. After 3 h, the ships are 24 nautical miles apart. Calculate the angle between the ships at the time they sailed from port. Round your answer to the nearest degree.

1.5 Make Decisions Using Trigonometry

Carpenters have to calculate the length of a lumber support for a roof rafter. Engineers have to determine the angle of elevation or depression when calculating the height of a structure. Pilots use trigonometry to navigate aircraft, and to correct for wind effects. Ships' captains do the same for ocean currents. To calculate unknown measurements, these professionals first need to decide which trigonometric tool or formula to use.

Investigate 1

Tools

- BLM Match Me Up cards
  – 6 triangle cards
  – 3 formula/tool cards

Decision, Decision, Decision: You Be the Judge

Work with a partner.

In the game *Match Me Up*, you start with six triangle cards, each showing a triangle and its associated measures.

You also get three tool cards. The tools are:
- primary trigonometric ratios
- sine law
- cosine law

You have to decide which trigonometric formula or tool you should use to solve each triangle.

Taking turns, one player turns over a triangle card and the other player has to select the correct tool to solve the triangle. If you find the correct tool and answer, you get three points. If you only choose the correct tool, you get one point. If you only answer the question correctly, you get two points.

Play continues until each player's six cards have been played.

The player with the greatest score is the winner.

Investigate 2 Trigonometry Problems

Work in pairs.

Three problems are given below.

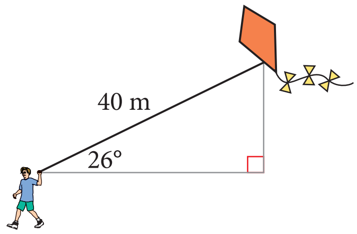
Each partner decides which formula to use to solve the problem.

Each partner takes turns explaining how to solve the problem.

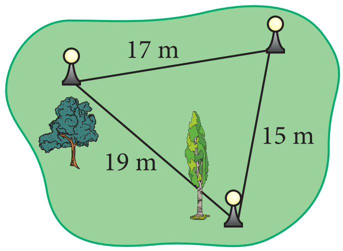
Check your solutions with the teacher.

Problems

1. Chi is flying his kite. The kite string is 40 m long. The angle of elevation the string makes with the horizontal is 26°. What is the height of the kite?



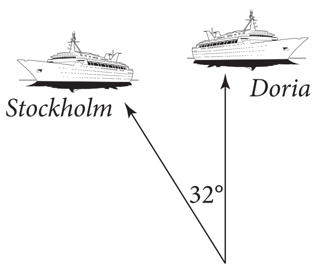
2. Three lights are located in a park along three different paths. The distance between the first light and the second light is 15 m. The distance between the second light and the third light is 19 m. The distance between the first light and the third light is 17 m. Calculate the angles between the lights.



Math Connect

1 knot = 1.852 km/h

3. Two ships leave port at exactly the same time. *Doria* heads north at 10 knots and *Stockholm* heads 32° west of north at 12 knots. Calculate how far apart the two ships are after 2 h.



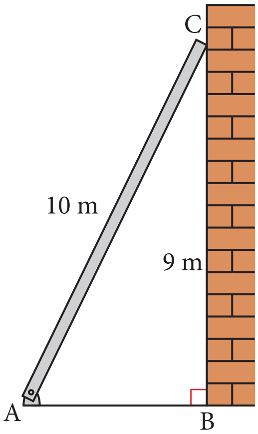
Example 1

Use the Primary Trigonometric Ratios

A 10-m ladder leans against a wall. The top of the ladder is 9 m above the ground. Safety standards call for the angle between the base of the ladder and the ground to be between 70° and 80°. Is the ladder safe to climb?

Solution

Draw a diagram to model the problem. Label it with the given information.



This is a right triangle and you have to find the angle the base of the ladder makes with the ground.

Determine the formula to use.

The problem involves a right triangle. Use the primary trigonometric ratios.

Locate the angle and label it A. The wall is opposite the angle and the ladder forms the hypotenuse of the triangle.

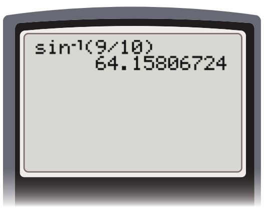
Write a ratio using the sides associated with the unknown angle.

$$\sin A = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\sin A = \frac{9}{10}$$ Substitute the known values into the ratio.

$$\angle A = \sin^{-1}\left(\frac{9}{10}\right)$$ Solve for the angle.

$$\angle A \doteq 64°$$



2nd SIN 9 ÷ 10) ENTER

The base of the ladder makes an angle with the ground that is less than 70°. The ladder is not safe to climb.

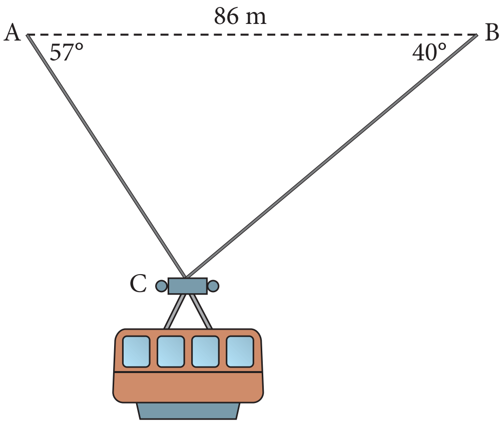
Example 2

Use the Sine Law

A cable car stops part of the way across an 86-m wide gorge. The cable holding the car makes an angle of depression of 57° at one end and an angle of depression of 40° at the other end. How long is the cable that holds the car? Round your answer to the nearest metre.

Solution

Draw a diagram to model the problem. Label the diagram with the given information.



Decide which formula to use. This is not a right triangle, so primary trigonometric ratios cannot be used.

You are given the measures of two angles and one side of the triangle, so use the sine law.

$$\angle C = 180° - 57° - 40°$$
$$= 83°$$

Write a proportion to find an unknown side.

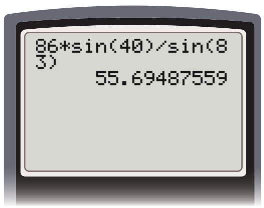
You have to find the lengths of AC and BC, which are the length of the cable.

$$\frac{AC}{\sin B} = \frac{AB}{\sin C}$$

$$\frac{AC}{\sin 40°} = \frac{86}{\sin 83°}$$

$$AC = \frac{86 \times \sin 40°}{\sin 83°}$$

$$AC \doteq 55.7$$



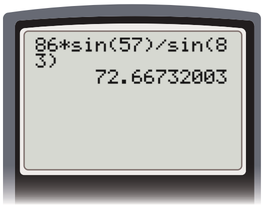
86 × SIN 40) ÷ SIN 83) ENTER

$$\frac{BC}{\sin A} = \frac{86}{\sin C}$$

$$\frac{BC}{\sin 57°} = \frac{86}{\sin 83°}$$

$$BC = \frac{86 \times \sin 57°}{\sin 83°}$$

$$BC \doteq 72.7$$



86 × SIN 57) ÷ SIN 83) ENTER

Cable length = 55.7 + 72.7
= 128.4

The length of the cable is approximately 128 m.

Example 3

Use the Cosine Law

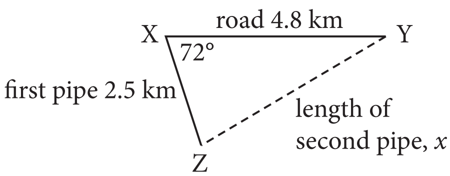
A sewer pipe for a new subdivision has to be laid underground. A connection is made to the main service pipe at either end of the 4.8-km stretch of road. One pipe, 2.5 km long, makes an angle of 72° at one end of the road.

a) Calculate the length of the second pipe that will connect the first pipe to the other end of the road.

b) What is the measure of the angle made by connecting the two pipes?

Solution

a) Draw and label a diagram that models the problem.



Decide which tool or formula to use. This is not a right triangle, so primary trigonometric ratios cannot be used.
The measures of two sides and a contained angle are given. Use the cosine law.

Technology Tip

For a scientific calculator, you might use this key sequence:

[( ] 2.5 [x^2] [+] 4.8 [x^2] [−] 2 [×] 2.5 [×] 4.8 [×] 72 [COS] [ )] [√]

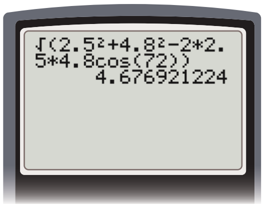
For a DAL calculator, you might use this key sequence:

[√] [( ] 2.5 [x^2] [+] 4.8 [x^2] [−] 2 [×] 2.5 [×] 4.8 [COS] 72 [ )] [=]

$$x^2 = 2.5^2 + 4.8^2 - 2(2.5)(4.8)\cos 72°$$

$$x = \sqrt{2.5^2 + 4.8^2 - 2(2.5)(4.8)\cos 72°}$$

$$x \doteq 4.67$$



[2nd] [x^2] 2.5 [x^2] [+] 4.8 [x^2] [−] 2 [×] 2.5 [×] 4.8 [COS] 72 [ )] [ )] [ENTER]

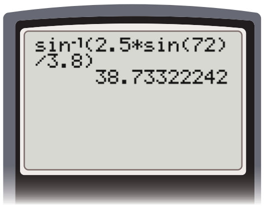
The length of the second pipe will be approximately 4.7 km.

b) $\dfrac{\sin Y}{2.5} = \dfrac{\sin 72°}{3.8}$

Calculate the measure of the smallest angle first using the sine law. The smallest angle is ∠Y.

$$\sin Y = \frac{2.5 \times \sin 72°}{3.8}$$

$$\angle Y \doteq 38.73°$$



[2nd] [SIN] 2.5 [×] [SIN] 72 [ )] [÷] 3.8 [ )] [ENTER]

The angle made between the road and the second pipe is about 39°.

The angle at the connection of the two pipes is:

$$\angle Z \doteq 180° - 52° - 39°$$
$$\doteq 89°$$

The angle made between the connecting pipes is approximately 89°.

Key Concepts

- Decide which formula or tool to use based on the type of triangle the situation presents.
- If the problem is modelled by a right triangle, use the primary trigonometric ratios.
- If the problem is modelled by an acute triangle
  - with two angles and a given side or two sides and an opposite angle, use the sine law
  - with two sides and a contained angle or three sides, use the cosine law

Discuss the Concepts

D1. Is it possible to use any of the formulas to solve a right triangle given only the measure of one side? Explain.

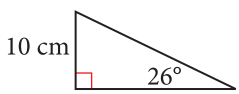
D2. Is it possible to find an angle measure in a triangle given the measures of one angle and one side? Explain.

Practise A

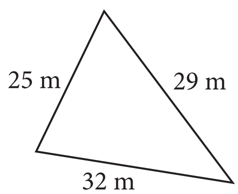
Round your answers to the nearest tenth, where necessary.

1. Work in pairs. One partner chooses a triangle. The other partner decides which formula to use to solve it.

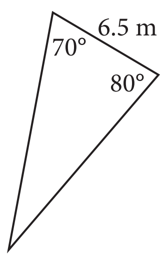
a)



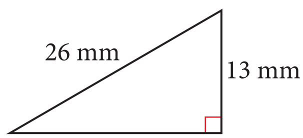
b)



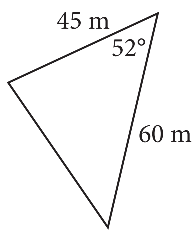
c)



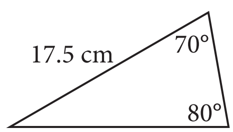
d)



e)



f)

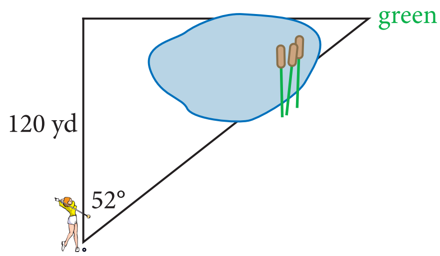


2. Refer to question 1. Solve each triangle.

Apply B

For help with questions 3 and 4, refer to Example 1.

3. Lorie Kane, one of Canada's great female golfers, hits a tee shot short of a water hazard (a pond). A second shot to the centre of the green will give her a chance for an eagle. However, she can lay up directly in front for 120 yd, avoiding the hazard, and then take a third shot to the green. She decides to go for the green on her second shot using a four-iron, which has a maximum distance of 200 yd. She estimates the angle between the fairway and the shot to the green to be 52°. Did she make the right decision? Explain. What assumptions are you making?



4. A golfer is faced with a shot that has to pass over some trees. The trees are 33 ft tall. The golfer finds himself 7 yd behind these trees, which obstruct him from the green. He decides to go for the green by using a 60° lob wedge. This club will allow the ball an angle of elevation of 60°. Did he make the right choice? Explain. What assumptions are you making? Hint: 1 yd = 3 ft

Literacy Connect

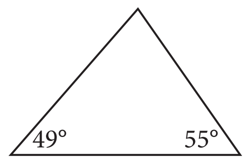
5. Golf is one sport in which skills in trigonometry are useful. What other sports use trigonometry to help a player be more skillful? Provide examples of the types of plays that might use trigonometry.

For help with question 6, refer to Example 2.

6. Two tracking stations, 5 km apart, track a weather balloon floating between them. The tracking station to the west tracks the balloon at an angle of elevation of 52°, and the station to the east tracks the balloon at an angle of elevation of 60°. How far is the balloon from the closest tracking station?

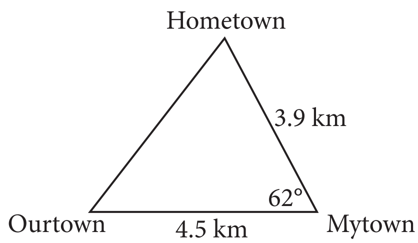
7. Three cell phone towers form a triangle. The distance between the first tower and the second tower is 16 km. The distance between the second tower and the third tower is 19 km. The distance between the first tower and the third tower is 19 km. Calculate the angles between the cell phone towers.

8. A triangular garden is to be enclosed by a fence. How much fencing will be required?



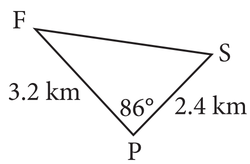
9. Complete the information needed to solve each triangle. Draw the triangle to help you explain.
   a) a right triangle, given one side and ■
   b) an acute triangle, given two sides and ■
   c) an acute triangle, given one angle and ■

10. Three roads join Hometown, Mytown, and Ourtown.
    a) What is the distance from Hometown to Ourtown?
    b) What angles do the roads make at Hometown and at Ourtown?



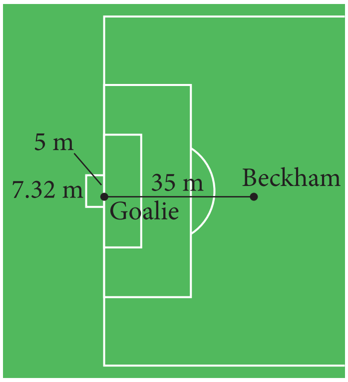
Achievement Check

11. The three stages of a triathlon involve swimming, cycling, and running, in that order. The distances for each stage can vary. For a triathlon held in Hawaii each year, competitors swim across an ocean bay, cycle 180.2 km, and run 42.2 km. In the diagram, S is the start of the swim and F is the finish of the swim. A surveyor, at point P, used the dimensions shown to calculate the length of SF across the ocean bay.
    a) Find the distance the athletes swim in the Hawaiian triathlon.
    b) What is the total distance of the race?
    c) What assumptions have you made?

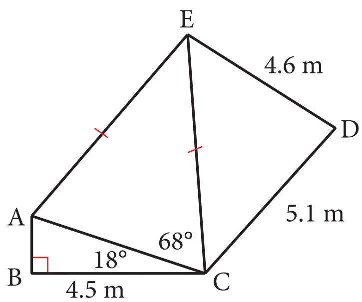


Extend C

12. David Beckham, one of professional soccer's most talented players, can bend his kicks but sometimes misses his target. Beckham gets ready for a free kick from 35 m away. He is located directly in front of the goalkeeper, who is 5 m from the right goal post. The net is 7.32 m wide and 2.44 m high. Once Beckham kicks the ball, the ball's angle of elevation of 4° takes a turn to the right at 9°. The goalie has no chance of making the save because the ball is heading toward the upper right corner of the net. Will Beckham score? Explain.



13. Find the measure of ∠CED to the nearest degree.



14. Murray is a forest ranger in a lookout tower 52 m above the ground. Directly south of him, at an angle of depression of 2.2° is a ranger station. The station has just radioed Murray to ask if he can spot a group of lost hikers. Murray spots the hikers camped out 60° east of south and at an angle of depression of 1.5° from the tower.

a) How far is the lookout tower from the ranger station?

b) How far is the lookout tower from the hikers?

c) How far are the hikers from the ranger station?

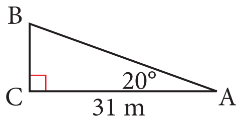
d) In which direction, to the nearest degree, should the rescue team leave the ranger station to reach the hikers?

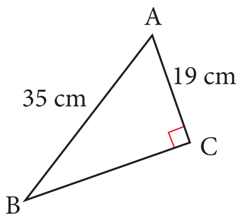
1 Review

1.1 Primary Trigonometric Ratios, pages 6–15

Where necessary, round answers to the nearest tenth.

1. Solve the right triangles.

a) 

b) 

2. Solve $\triangle ABC$ where $\angle C = 90°$, $a = 15$ cm, and $b = 7$ cm.

3. Is it possible to solve $\triangle ABC$, given $\angle C = 90°$ and $c = 35$ cm? Explain. If not, what additional information is necessary?

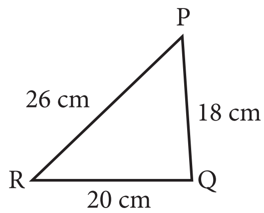
1.2 Solve Problems Using Trigonometric Ratios, pages 16-23

4. A communication tower casts a shadow of 55 m when the sun is at an angle of elevation of 72°. What is the height of the tower to the nearest metre?

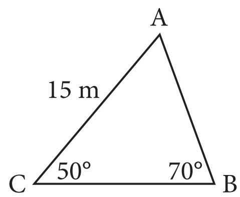
5. A person walks 5 km north, turns east, and then walks another 6 km. At what angle, east of north, did the person stop?

1.3 The Sine Law, pages 24-33

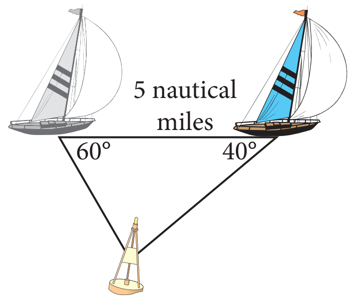
6. Is it possible to solve the triangle using the sine law? Explain. If not, what information is required?



7. Solve the triangle.

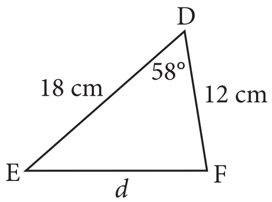


8. A sailboat is 5 nautical miles east of its starting point. At the start of its journey, it made an angle of 60° with a buoy on the right side of its path. After 45 min, it made an angle of 40° with the buoy as shown. How far is the sailboat from the buoy after 45 min?



1.4 The Cosine Law, pages 34-41

9. Find the length of the unknown side, *d*.

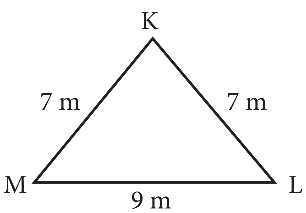


10. What information must be known about a triangle to use the cosine law? Provide examples with diagrams to help you explain.

11. Create a question with a triangle that can be solved using the cosine law. Trade problems with a classmate and solve each other's questions.

12. Two cyclists leave from the same location with an angle of 63° between their paths. Johal cycles at a speed of 35 km/h and Julio at a speed of 40 km/h. How far apart are they after 3 h?

13. Solve △KLM using the cosine law.

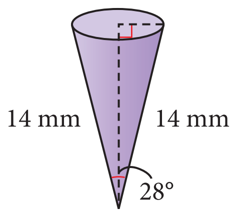


1.5 Make Decisions Using Trigonometry, pages 42-51

14. The pitch of a roof is 45°. The rise of the roof is 12 ft. A carpenter decided to cut a roof rafter 20 ft long to allow for a 1-ft overhang. Did the carpenter cut the correct length for the rafter? Explain. Draw a diagram and show your work. Include the formulas that you use.

15. The posts of a hockey goal are 2.0 m apart. Leah is 3.8 m from one post and 4.2 m from the other post. Within what angle must she shoot the puck to score a goal?

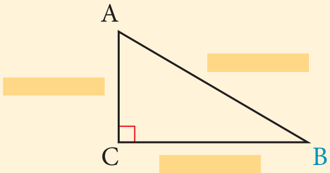
16. Determine the radius of the cone.



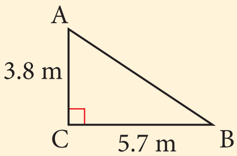
17. How can you solve the triangle in question 6 if it is not possible to solve it using the sine law? Solve it.

1 Practice Test

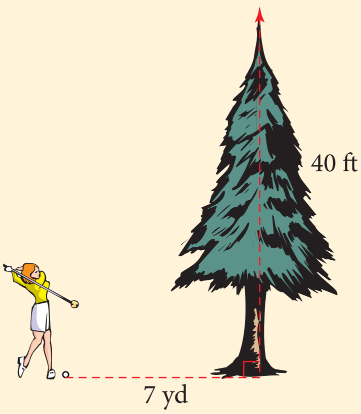
1. Copy and complete the diagram. Name the sides of the right triangle associated with ∠B, as adjacent, opposite, or hypotenuse.



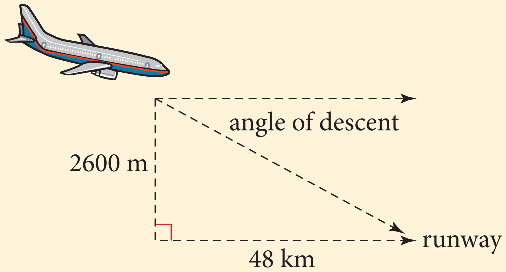
2. Solve △ABC.



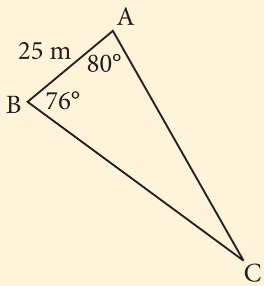
3. A golfer hit her tee shot so that it landed about 7 yd behind a 40-ft tall pine tree as shown. She decided to take her second shot and hoped the ball would make it over the top of the tree. She used her lob wedge and hit the ball, sending it upward at an angle of 60°. Was she able to clear the top of the tree? Show your solution.



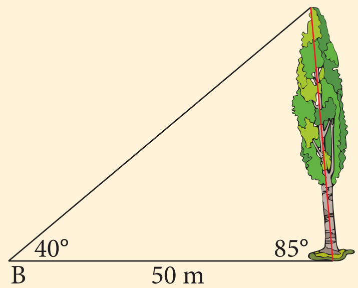
4. An airplane flying at an altitude of 2600 m is approaching an airport runway located 48 km away. Calculate the airplane's angle of descent. Round your answer to the nearest tenth of a degree.



5. Solve △ABC.



6. A wind-swept tree grows at angle of 85°. An environmental scientist wants to know the height of the tree. She walks 50 m from the tree and measures an angle of 40° to the top of the tree. How tall is the tree?



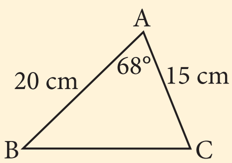
Chapter Problem Wrap-Up

The expedition team set out from the city of Iqaluit on a course 5° east of north and set up camp 15 km from their starting point. The next day, the team set out on a course 25° east of north but encountered a blizzard in the evening. They decided to set up camp until the storm subsided. They estimated that they had travelled at 2 km/h for 8 h. Not knowing their position, they radioed for help.

a) Draw a diagram to show the route travelled by the team. Include distances and angle directions.

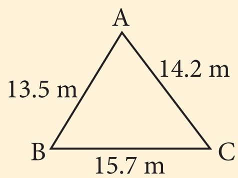
b) Determine the shortest distance and direction a rescue team from Iqaluit would have to travel to reach the team.

7. Solve △ABC.



8. While on a camping trip, Claire hung her food bag up to keep it away from the wildlife. The bag was 6 m above the ground, suspended from the middle of a 6.2-m length of rope between two branches that are at the same height and 4 m apart. What angle did the rope make at the point where the food bag was hung?

9. Determine the measures of ∠A, ∠B, and ∠C.



10. **a)** Explain why it is possible to solve a right triangle using the sine law if the measures of one side and one angle are given. Is this the best method? Why or why not?

b) Is it possible to solve a right triangle using the cosine law? Explain.