Section 1.4: Comparing Graphs of Linear Motion

Tutorial 1 Practice, page 34

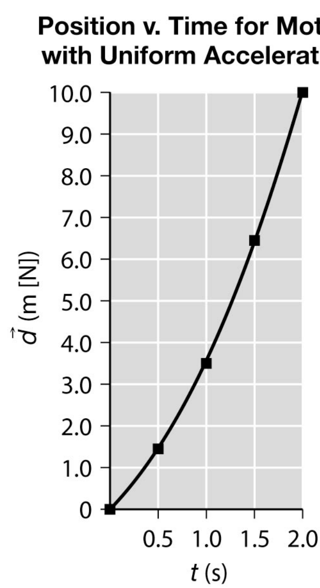
1. Step 1: The data plotted on the velocity–time graph in Figure 8 form an increasing straight-line graph. You can determine acceleration from a velocity–time graph by calculating its slope. Since the velocity–time graph in Figure 8 is a straight line, its slope does not change. So we can calculate the slope or acceleration over any time interval.

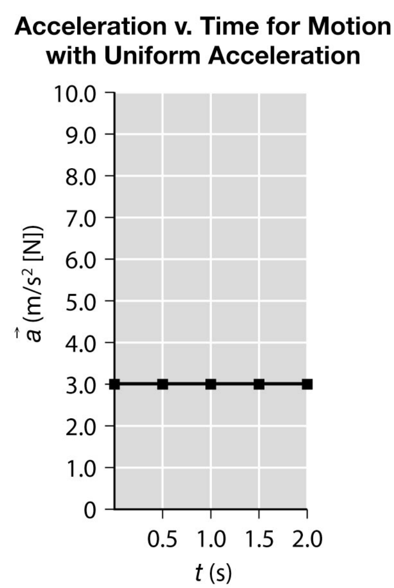
$$\text{slope} = \frac{\text{rise}}{\text{run}}$$
$$\vec{a}_{\text{av}} = \frac{\Delta\vec{v}}{\Delta t} = \frac{6.0 \text{ m/s [N]}}{2.0 \text{ s}}$$
$$\vec{a}_{\text{av}} = 3.0 \text{ m/s}^2 \text{ [N]}$$

Step 2: The table shows the calculations for the area under the curve in Figure 8 at 0.5 s intervals from $t = 0$ s to $t = 2.0$ s.

Time (s)	**Equation** $\Delta\vec{d} = \vec{v}_i\Delta t + \frac{1}{2}\Delta\vec{v}\Delta t$	**Displacement** (m [N])	**Acceleration** (m/s^2 [N])
0.0		0.0	3.0
0.5	$\Delta\vec{d} = \left(2.0\ \frac{\text{m}}{\cancel{\text{s}}}\ \text{[N]}\right)(0.5\ \cancel{\text{s}}) + \frac{1}{2}\left(1.5\ \frac{\text{m}}{\cancel{\text{s}}}\ \text{[N]}\right)(0.5\ \cancel{\text{s}})$	1.4	3.0
1.0	$\Delta\vec{d} = \left(2.0\ \frac{\text{m}}{\cancel{\text{s}}}\ \text{[N]}\right)(1.0\ \cancel{\text{s}}) + \frac{1}{2}\left(3.0\ \frac{\text{m}}{\cancel{\text{s}}}\ \text{[N]}\right)(1.0\ \cancel{\text{s}})$	3.5	3.0
1.5	$\Delta\vec{d} = \left(2.0\ \frac{\text{m}}{\cancel{\text{s}}}\ \text{[N]}\right)(1.5\ \cancel{\text{s}}) + \frac{1}{2}\left(4.5\ \frac{\text{m}}{\cancel{\text{s}}}\ \text{[N]}\right)(1.5\ \cancel{\text{s}})$	6.4	3.0
2.0	$\Delta\vec{d} = \left(2.0\ \frac{\text{m}}{\cancel{\text{s}}}\ \text{[N]}\right)(2.0\ \cancel{\text{s}}) + \frac{1}{2}\left(6.0\ \frac{\text{m}}{\cancel{\text{s}}}\ \text{[N]}\right)(2.0\ \cancel{\text{s}})$	10.0	3.0

Step 3: Use these values to create position–time and acceleration–time graphs.





Section 1.4 Questions, page 35

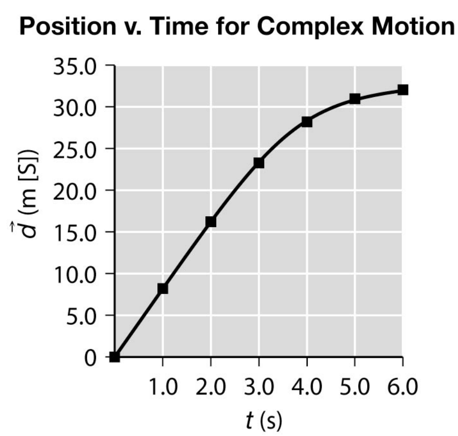
1.

How do you determine ...	Given a ...	Read information from graph	Take the slope	Find the area
position	position–time graph	√		
velocity	position–time graph		√	
velocity	velocity–time graph	√		
velocity	acceleration–time graph			√
acceleration	velocity–time graph		√	
acceleration	acceleration–time graph	√		

2. Step 1: Use the area under the graph to determine the position at each time. Since each rectangle on the grid is 1.0 s by 2.0 m/s [S], they each represent 2.0 m [S]. You can count the grid rectangles to determine the area.

Time (s)	Grid Rectangles	Position (m [S])
0.0	0	0
1.0	4	8
2.0	8	16
3.0	11.5	23
4.0	14	28
5.0	15.5	31
6.0	16	32

Step 2: Use these values to create a position–time graph.



3. (a) Reading from the graph, at $t = 5.0$ s, $\vec{d} = 45.0$ m [S].

(b) Given: $t = 3.0$ s; position–time graph

Required: $\vec{v}_{inst}$

Analysis: $\vec{v}_{inst}$ is equal to the slope, m, of the tangent to the curve at $t = 3.0$ s; $m = \frac{\Delta \vec{d}}{\Delta t}$.

By placing a ruler along the curve in Figure 10 at $t = 3.0$ s, I can picture the tangent. The tangent has a rise of about 45.0 m [S] over a run of 4.0 s.

Solution: $m = \frac{\Delta \vec{d}}{\Delta t}$

$$m = \frac{45.0 \text{ m [S]}}{4.0 \text{ s}}$$

$$\vec{v}_{inst} = 11 \text{ m/s [S]}$$

Statement: The instantaneous velocity of the object at 3.0 s is 11 m/s [S].

(c) Given: $\Delta \vec{d} = 65$ m [S]; $\Delta t = 6.0$ s

Required: $\vec{v}_{av}$

Analysis: $\vec{v}_{av} = \frac{\Delta \vec{d}}{\Delta t}$

Solution: $\vec{v}_{av} = \frac{\Delta \vec{d}}{\Delta t}$

$$= \frac{65 \text{ m [S]}}{6.0 \text{ s}}$$

$$\vec{v}_{av} = 11 \text{ m/s [S]}$$

Statement: The average velocity of the object over the time interval from 0.0 s to 6.0 s is 11 m/s [S].

4. Step 1: The data plotted on the velocity–time graph in Figure 11 form a decreasing straight-line graph. You can determine acceleration from a velocity–time graph by calculating its slope. Since the velocity–time graph in Figure 11 is a straight line, its slope does not change. So we can calculate the slope or acceleration over any time interval.

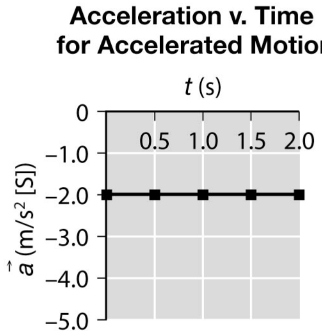
$$\text{slope} = \frac{\text{rise}}{\text{run}}$$

$$\vec{a}_{av} = \frac{\Delta \vec{v}}{\Delta t}$$

$$= \frac{-12 \text{ m/s [S]}}{6.0 \text{ s}}$$

$$\vec{a}_{av} = -2.0 \text{ m/s}^2 \text{ [S]}$$

Step 2: Use this value to create an acceleration–time graph.



Section 1.5: Five Key Equations for Motion with Uniform Acceleration

Tutorial 1 Practice, page 39

1. Given: $\vec{v}_i = 0$ m/s; $\Delta\vec{d} = 17$ m [E]; $\Delta t = 3.8$ s

Required: $\vec{v}_f$

Analysis: $\Delta\vec{d} = \left(\dfrac{\vec{v}_f + \vec{v}_i}{2}\right)\Delta t$

$$\vec{v}_f = 2\frac{\Delta\vec{d}}{\Delta t} - \vec{v}_i$$

Solution: $\vec{v}_f = 2\dfrac{\Delta\vec{d}}{\Delta t} - \vec{v}_i$

$$= 2\left(\frac{17 \text{ m [E]}}{3.8 \text{ s}}\right) - 0 \text{ m/s}$$

$$\vec{v}_f = 8.9 \text{ m/s [E]}$$

Statement: Her final velocity is 8.9 m/s [E].

2. Given: $\vec{v}_i = 0$ m/s, $\Delta\vec{d} = 70.0$ m [downhill]; $\Delta t = 5.3$ s;

Required: $\vec{a}_{av}$

Analysis: $\Delta\vec{d} = \vec{v}_i\,\Delta t + \dfrac{1}{2}\vec{a}_{av}\,\Delta t^2$

$$\vec{a}_{av} = 2\frac{\Delta\vec{d} - \vec{v}_i\,\Delta t}{\Delta t^2}$$

Solution: $\vec{a}_{av} = 2\dfrac{\Delta\vec{d} - \vec{v}_i\,\Delta t}{\Delta t^2}$

$$= 2\left(\frac{70 \text{ m [downhill]} - \left(0\ \frac{\text{m}}{\cancel{\text{s}}}\right)(5.3\ \cancel{\text{s}})}{(5.3 \text{ s})^2}\right)$$

$$\vec{a}_{av} = 5.0 \text{ m/s}^2 \text{ [downhill]}$$

Statement: The uniform acceleration experienced by the child is 5.0 m/s^2 [downhill].

Section 1.5 Questions, page 39

1. Given: $\vec{v}_i = 0$ m/s; $\vec{a}_{av} = 2.0$ m/s^2 [N]; $\Delta t = 15$ s

Required: $\Delta\vec{d}$

Analysis: $\Delta\vec{d} = \vec{v}_i\,\Delta t + \dfrac{1}{2}\vec{a}_{av}\,\Delta t^2$

Solution:

$$\Delta\vec{d} = \vec{v}_i\,\Delta t + \frac{1}{2}\vec{a}_{av}\,\Delta t^2$$

$$= \left(0\ \frac{\text{m}}{\cancel{\text{s}}}\right)(15\ \cancel{\text{s}}) + \frac{1}{2}\left(2.0\ \frac{\text{m}}{\cancel{\text{s}^2}}\ \text{[N]}\right)(15\ \cancel{\text{s}})^2$$

$$\Delta\vec{d} = 2.3 \times 10^2 \text{ m [N]}$$

Statement: The displacement of the car is 230 m [N] or 2.3×10^2 m [N].

2. (a) Given: $\vec{v}_i = 20.0$ m/s [E]; $\vec{v}_f = 0$ m/s; $\Delta t = 12$ s

Required: $\vec{a}_{av}$

Analysis: $\vec{v}_f = \vec{v}_i + \vec{a}_{av}\,\Delta t$

$$\vec{a}_{av} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t}$$

Solution: $\vec{a}_{av} = \dfrac{\vec{v}_f - \vec{v}_i}{\Delta t}$

$$= \frac{0\ \frac{\text{m}}{\text{s}} - 20.0\ \frac{\text{m}}{\text{s}}\ \text{[E]}}{12 \text{ s}}$$

$$= \frac{0\ \frac{\text{m}}{\text{s}} + 20.0\ \frac{\text{m}}{\text{s}}\ \text{[W]}}{12 \text{ s}}$$

$$\vec{a}_{av} = 1.7 \text{ m/s}^2 \text{ [W]}$$

Statement: The uniform acceleration of the spacecraft is 1.7 m/s^2 [W].

(b) Given: $\vec{v}_i = 20.0$ m/s [E]; $\vec{v}_f = 0$ m/s; $\Delta t = 12$ s

Required: $\Delta\vec{d}$

Analysis: $\Delta\vec{d} = \left(\dfrac{\vec{v}_f + \vec{v}_i}{2}\right)\Delta t$

Solution: $\Delta\vec{d} = \left(\dfrac{\vec{v}_f + \vec{v}_i}{2}\right)\Delta t$

$$= \left(\frac{0.0\ \frac{\text{m}}{\cancel{\text{s}}} + 20.0\ \frac{\text{m}}{\cancel{\text{s}}}\ \text{[E]}}{2}\right)(12\ \cancel{\text{s}})$$

$$\Delta\vec{d} = 1.2 \times 10^2 \text{ m [E]}$$

Statement: The displacement of the spacecraft is 120 m [E] or 1.2×10^2 m [E].

3. Given: $\vec{v}_i = 15$ m/s [W]; $\vec{a}_{av} = 7.0$ m/s^2 [E]; $\Delta t = 4.0$ s

Required: $\vec{v}_f$

Analysis: $\vec{v}_f = \vec{v}_i + \vec{a}_{av}\,\Delta t$

Solution: $\vec{v}_f = \vec{v}_i + \vec{a}_{av}\,\Delta t$

$$= 15\ \frac{\text{m}}{\text{s}}\ \text{[W]} + \left(7.0\ \frac{\text{m}}{\text{s}^{\cancel{2}}}\ \text{[E]}\right)(4.0\ \cancel{\text{s}})$$

$$= -15\ \frac{\text{m}}{\text{s}}\ \text{[E]} + 28\ \frac{\text{m}}{\text{s}}\ \text{[E]}$$

$$\vec{v}_f = 13 \text{ m/s [E]}$$

Statement: The final velocity of the helicopter is 13 m/s [E].

4. For go-cart A:
Given: $v = 20.0$ m/s; $\Delta d = 1.0$ km
Required: Δt

Analysis: $v = \dfrac{\Delta d}{\Delta t}$

$$\Delta t = \frac{\Delta d}{v}$$

Solution: $\Delta t = \dfrac{\Delta d}{v}$

$$= \frac{1.0\ \cancel{\text{km}}}{20.0\ \frac{\cancel{\text{m}}}{\text{s}}}\left(\frac{1000\ \cancel{\text{m}}}{1\ \cancel{\text{km}}}\right)$$

$$\Delta t = 50\ \text{s}$$

Statement: Go-cart A takes 50 s to go around the track.

For go-cart B:
Given: $v_i = 0$ m/s; $a = 0.333$ m/s^2; $\Delta d = 1.0$ km
Required: Δt

Analysis: $\Delta d = v_i \Delta t + \dfrac{1}{2} a_{av} \Delta t^2$

$$\Delta d = (0\ \text{m/s})\Delta t + \frac{1}{2} a_{av} \Delta t^2$$

$$\Delta t^2 = 2\frac{\Delta d}{a_{av}}$$

Solution: $\Delta t^2 = 2\dfrac{\Delta d}{a_{av}}$

$$= 2\frac{1.0\ \cancel{\text{km}}}{0.333\ \frac{\cancel{\text{m}}}{\text{s}^2}}\left(\frac{1000\ \cancel{\text{m}}}{1.0\ \cancel{\text{km}}}\right)$$

$$\Delta t^2 = 6000\ \text{s}^2$$

$$\Delta t = 77\ \text{s}$$

Statement: Go-cart B takes 77 s to go around the track. This time is greater than Go-cart A, which took 50 s. Go-cart A wins the race by 27 s.

5. Given: $v_i = 5.0$ m/s; $v_f = 7.5$ m/s; $\Delta d = 50.0$ m
Required: a_{av}

Analysis: $v_f^{\,2} = v_i^{\,2} + 2a_{av}\Delta d$

$$a_{av} = \frac{v_f^{\,2} - v_i^{\,2}}{2\Delta d}$$

Solution: $a_{av} = \dfrac{v_f^{\,2} - v_i^{\,2}}{2\Delta d}$

$$= \frac{\left(7.5\ \frac{\text{m}}{\text{s}}\right)^2 - \left(5.0\ \frac{\text{m}}{\text{s}}\right)^2}{2(50.0\ \text{m})}$$

$$= \frac{56.25\ \frac{\text{m}^{\cancel{2}}}{\text{s}^2} - 25.00\ \frac{\text{m}^{\cancel{2}}}{\text{s}^2}}{100\ \cancel{\text{m}}}$$

$$a_{av} = 0.31\ \text{m/s}^2$$

Statement: The boat's average acceleration is 0.31 m/s^2.

6. (a) Given: $\Delta\vec{d} = 4.50 \times 10^2$ m [up]; $\Delta t = 4.0$ s; $\vec{v}_i = 0$ m/s

Required: $\vec{a}_{av}$

Analysis: $\Delta\vec{d} = \vec{v}_i \Delta t + \dfrac{1}{2}\vec{a}_{av}\Delta t^2$

$$\vec{a}_{av} = 2\frac{\Delta\vec{d} - \vec{v}_i \Delta t}{\Delta t^2}$$

Solution: $\vec{a}_{av} = 2\dfrac{\Delta\vec{d} - \vec{v}_i \Delta t}{\Delta t^2}$

$$= 2\frac{4.5\times 10^2\ \text{m [up]} - \left(0\ \frac{\text{m}}{\cancel{\text{s}}}\right)(4.0\ \cancel{\text{s}})}{(4.0\ \text{s})^2}$$

$$\vec{a}_{av} = 56\ \text{m/s}^2\ \text{[up]}$$

Statement: The spacecraft's acceleration is 56 m/s^2 [E].

(b) Given: $\Delta\vec{d} = 4.50 \times 10^2$ m [up]; $\Delta t = 4.0$ s; $\vec{v}_i = 0$ m/s; $\vec{a}_{av} = 56$ m/s^2 [up]

Required: $\vec{v}_f$

Analysis: $\vec{v}_f = \vec{v}_i + \vec{a}_{av}\Delta t$

Solution: $\vec{v}_f = \vec{v}_i + \vec{a}_{av}\Delta t$

$$= 0\ \frac{\text{m}}{\text{s}} + \left(56\ \frac{\text{m}}{\text{s}^{\cancel{2}}}\ \text{[up]}\right)(4.0\ \cancel{\text{s}})$$

$$\vec{v}_f = 2.2\times 10^2\ \text{m/s [up]}$$

Statement: After 4.0 s, the velocity of the spacecraft is 220 m/s [up] or 2.2×10^2 m/s [up].

7. Answers may vary. Sample answer:
Since Equation 4 does not include Δt, isolate Δt in Equations 1 and 2, then set them equal to each other.

Equation 1:

$$\Delta \vec{d} = \left(\frac{\vec{v}_{\text{f}} + \vec{v}_{\text{i}}}{2}\right)\Delta t$$

$$\Delta t = \frac{2\Delta \vec{d}}{\vec{v}_{\text{f}} + \vec{v}_{\text{i}}}$$

Equation 2:

$$\vec{v}_{\text{f}} = \vec{v}_{\text{i}} + \vec{a}_{\text{av}}\,\Delta t$$

$$\Delta t = \frac{\vec{v}_{\text{f}} - \vec{v}_{\text{i}}}{\vec{a}_{\text{av}}}$$

$$\Delta t = \Delta t$$

$$\frac{2\Delta \vec{d}}{\vec{v}_{\text{f}} + \vec{v}_{\text{i}}} = \frac{\vec{v}_{\text{f}} - \vec{v}_{\text{i}}}{\vec{a}_{\text{av}}}$$

$$2\vec{a}_{\text{av}}\,\Delta \vec{d} = \vec{v}_{\text{f}}^{\,2} - \vec{v}_{\text{i}}^{\,2}$$

$$\vec{v}_{\text{f}}^{\,2} = \vec{v}_{\text{i}}^{\,2} + 2\vec{a}_{\text{av}}\,\Delta \vec{d}$$

Since Equation 5 does not include $\vec{v}_{\text{i}}$, isolate $\vec{v}_{\text{i}}$ in Equations 1 and 2, then set them equal to each other.

Equation 1:

$$\Delta \vec{d} = \left(\frac{\vec{v}_{\text{f}} + \vec{v}_{\text{i}}}{2}\right)\Delta t$$

$$\vec{v}_{\text{f}} + \vec{v}_{\text{i}} = \frac{2\Delta \vec{d}}{\Delta t}$$

$$\vec{v}_{\text{i}} = \frac{2\Delta \vec{d}}{\Delta t} - \vec{v}_{\text{f}}$$

Equation 2:

$$\vec{v}_{\text{f}} = \vec{v}_{\text{i}} + \vec{a}_{\text{av}}\,\Delta t$$

$$\vec{v}_{\text{i}} = \vec{v}_{\text{f}} - \vec{a}_{\text{av}}\,\Delta t$$

$$\vec{v}_{\text{i}} = \vec{v}_{\text{i}}$$

$$\frac{2\Delta \vec{d}}{\Delta t} - \vec{v}_{\text{f}} = \vec{v}_{\text{f}} - \vec{a}_{\text{av}}\,\Delta t$$

$$\frac{2\Delta \vec{d}}{\Delta t} = 2\vec{v}_{\text{f}} - \vec{a}_{\text{av}}\,\Delta t$$

$$2\Delta \vec{d} = \left(2\vec{v}_{\text{f}} - \vec{a}_{\text{av}}\,\Delta t\right)\Delta t$$

$$2\Delta \vec{d} = 2\vec{v}_{\text{f}}\,\Delta t - \vec{a}_{\text{av}}\,\Delta t^2$$

$$\Delta \vec{d} = \vec{v}_{\text{f}}\,\Delta t - \frac{1}{2}\vec{a}_{\text{av}}\,\Delta t^2$$

Section 1.6: Acceleration Near Earth's Surface

Tutorial 1 Practice, page 41

1. Given: $\vec{v}_i = 0$ m/s; $\Delta t = 2.6$ s;
$\vec{a} = \vec{g} = 9.8$ m/s^2 [down]

Required: $\Delta\vec{d}$

Analysis: $\Delta\vec{d} = \vec{v}_i\,\Delta t + \frac{1}{2}\vec{a}(\Delta t)^2$

Solution:

$$\Delta\vec{d} = \vec{v}_i\,\Delta t + \frac{1}{2}\vec{a}(\Delta t)^2$$
$$= \left(0\ \frac{\text{m}}{\cancel{\text{s}}}\right)(2.6\ \cancel{\text{s}}) + \frac{1}{2}\left(9.8\ \frac{\text{m}}{\cancel{\text{s}^2}}\ [\text{down}]\right)(2.6\ \cancel{\text{s}})^2$$
$$\Delta\vec{d} = 33\text{ m [down]}$$

Statement: The displacement of the ball is 33 m [down], so the building is 33 m tall.

2. (a) Given: $\vec{v}_i = 0$ m/s; $\Delta\vec{d} = 52$ m [down];
$\vec{a} = \vec{g} = 9.8$ m/s^2 [down]

Required: Δt

Analysis:

$$\Delta\vec{d} = \vec{v}_i\,\Delta t + \frac{1}{2}\vec{a}(\Delta t)^2$$
$$= (0\text{ m/s})\Delta t + \frac{1}{2}\vec{a}(\Delta t)^2$$
$$(\Delta t)^2 = \frac{2\Delta\vec{d}}{\vec{a}}$$
$$\Delta t = \sqrt{\frac{2\Delta\vec{d}}{\vec{a}}}$$

Solution: $\Delta t = \sqrt{\dfrac{2\Delta\vec{d}}{\vec{a}}}$

$$= \sqrt{\frac{2(52\ \cancel{\text{m}})}{\left(9.8\ \frac{\cancel{\text{m}}}{\text{s}^2}\right)}}$$
$$\Delta t = 3.3\text{ s}$$

Statement: The penny takes 3.3 s to fall 52 m.

(b) Given: $\vec{v}_i = 0$ m/s; $\Delta\vec{d} = 52$ m [down];
$\vec{a} = \vec{g} = 9.8$ m/s^2 [down]

Required: $\vec{v}_f$

Analysis: $v_f^2 = v_i^2 + 2a\Delta d$

$$v_f = \sqrt{v_i^2 + 2a\Delta d}$$

Solution: $v_f = \sqrt{v_i^2 + 2a\Delta d}$

$$= \sqrt{\left(0\ \frac{\text{m}}{\text{s}}\right)^2 + 2\left(9.8\ \frac{\text{m}}{\text{s}^2}\right)(52\text{ m})}$$
$$v_f = 32\text{ m/s}$$

Statement: The final velocity of the penny is 32 m/s.

Tutorial 2 Practice, page 42

1. (a) Given: $\vec{v}_i = 8.3$ m/s [up]; $\vec{v}_f = 0.0$ m/s;
$\vec{a} = \vec{g} = 9.8$ m/s^2 [down]

Required: Δd

Analysis: $v_f^2 = v_i^2 + 2a\Delta d$

$$\Delta d = \frac{v_f^2 - v_i^2}{2a}$$

Solution: $\Delta d = \dfrac{v_f^2 - v_i^2}{2a}$

$$= \frac{\left(0\ \frac{\text{m}}{\text{s}}\right)^2 - \left(8.3\ \frac{\text{m}}{\text{s}}\right)^2}{2\left(-9.8\ \frac{\text{m}}{\text{s}^2}\right)}$$
$$= \frac{-68.89\ \frac{\text{m}^{\cancel{2}}}{\cancel{\text{s}^2}}}{-19.6\ \frac{\cancel{\text{m}}}{\cancel{\text{s}^2}}}$$
$$\Delta d = 3.5\text{ m}$$

Statement: The ball will reach a maximum height of 3.5 m.

(b) Given: $\vec{v}_i = 8.3$ m/s [up]; $\vec{v}_f = 0.0$ m/s;
$\vec{a} = \vec{g} = 9.8$ m/s^2 [down]

Required: Δt

Analysis: $a = \dfrac{v_f - v_i}{\Delta t}$

$$\Delta t = \frac{v_f - v_i}{a}$$

Solution: $\Delta t = \dfrac{v_f - v_i}{a}$

$$= \frac{0\ \frac{\cancel{\text{m}}}{\cancel{\text{s}}} - 8.3\ \frac{\cancel{\text{m}}}{\cancel{\text{s}}}}{\left(-9.8\ \frac{\cancel{\text{m}}}{\text{s}^{\cancel{2}}}\right)}$$
$$\Delta t = 0.85\text{ s}$$

Statement: It will take the ball 0.85 s to reach its maximum height.

(c) Given: $\vec{v}_i = 0.0$ m/s; $\Delta\vec{d} = 3.5$ m [down]; $\vec{a} = \vec{g} = 9.8 \text{ m/s}^2$ [down]

Required: Δt

Analysis: $\Delta d = v_i \Delta t + \frac{1}{2}a(\Delta t)^2$

$$= (0 \text{ m/s})\Delta t + \frac{1}{2}a(\Delta t)^2$$

$$(\Delta t)^2 = \frac{2\Delta d}{a}$$

$$\Delta t = \sqrt{\frac{2\Delta d}{a}}$$

Solution: $\Delta t = \sqrt{\frac{2\Delta d}{a}}$

$$= \sqrt{\frac{2(3.5 \cancel{\text{m}})}{\left(9.8 \frac{\cancel{\text{m}}}{\text{s}^2}\right)}}$$

$$\Delta t = 0.85 \text{ s}$$

Statement: It will take the ball 0.85 s to reach its initial height from its maximum height.

2. Given: $\vec{v}_i = 3.0$ m/s [down]; $\Delta\vec{d} = 12$ m [down]; $\vec{a} = \vec{g} = 9.8 \text{ m/s}^2$ [down]

Required: $\vec{v}_f$

Analysis: $v_f^2 = v_i^2 + 2a\Delta d$

$$v_f = \sqrt{v_i^2 + 2a\Delta d}$$

Solution: $v_f = \sqrt{v_i^2 + 2a\Delta d}$

$$= \sqrt{\left(3.0 \frac{\text{m}}{\text{s}}\right)^2 + 2\left(9.8 \frac{\text{m}}{\text{s}^2}\right)(12 \text{ m})}$$

$$v_f = 16 \text{ m/s}$$

Statement: The velocity of the rock when it hits the water is 16 m/s [down].

Section 1.6 Questions, page 43

1. An object that is dropped close to Earth's surface will accelerate toward Earth at 9.8 m/s^2.

2. Answers may vary. Sample answer:
When a basketball player appears to "hang" in mid-air, he is being affected by gravity. Due to gravity, his vertical speed is slowing down from the moment he jumps until he reaches the maximum height of the jump. So, there is a point when the player appears to "hang" in mid-air because he is moving up so slowly, then not moving up, then slowly starting to move down again.

3. (a) The only acceleration the ball experiences is due to gravity: 9.8 m/s^2 [down].

(b) The only acceleration the ball experiences is due to gravity: 9.8 m/s^2 [down].

(c) The only acceleration the ball experiences is due to gravity: 9.8 m/s^2 [down].

4. (a) Given: $\vec{v}_i = 0.0$ m/s; $\Delta\vec{d} = 1.5$ m [down]; $\vec{a} = \vec{g} = 9.8 \text{ m/s}^2$ [down]

Required: Δt

Analysis: $\Delta d = v_i \Delta t + \frac{1}{2}a(\Delta t)^2$

$$= (0 \text{ m/s})\Delta t + \frac{1}{2}a(\Delta t)^2$$

$$(\Delta t)^2 = \frac{2\Delta d}{a}$$

$$\Delta t = \sqrt{\frac{2\Delta d}{a}}$$

Solution: $\Delta t = \sqrt{\frac{2\Delta d}{a}}$

$$= \sqrt{\frac{2(1.5 \cancel{\text{m}})}{\left(9.8 \frac{\cancel{\text{m}}}{\text{s}^2}\right)}}$$

$$\Delta t = 0.55 \text{ s}$$

Statement: It will take the ball 0.55 s to hit the ground.

(b) Given: $\vec{v}_i = 0.0$ m/s; $\Delta\vec{d} = 0.75$ m [down]; $\vec{a} = \vec{g} = 9.8 \text{ m/s}^2$ [down]

Required: $\vec{v}_f$

Analysis: $v_f^2 = v_i^2 + 2a\Delta d$

$$v_f = \sqrt{v_i^2 + 2a\Delta d}$$

Solution: $v_f = \sqrt{v_i^2 + 2a\Delta d}$

$$= \sqrt{\left(0.0 \frac{\text{m}}{\text{s}}\right)^2 + 2\left(9.8 \frac{\text{m}}{\text{s}^2}\right)(0.75 \text{ m})}$$

$$v_f = 3.8 \text{ m/s}$$

Statement: The velocity of the ball when it is halfway to the ground is 3.8 m/s [down].

5. (a) Given: $\vec{v}_i$ = 80.0 m/s [up]; $\vec{v}_f$ = 0.0 m/s;
$\vec{a} = \vec{g}$ = 9.8 m/s^2 [down]

Required: Δd

Analysis: $v_f^2 = v_i^2 + 2a\Delta d$

$$\Delta d = \frac{v_f^2 - v_i^2}{2a}$$

Solution:

$$\Delta d = \frac{v_f^2 - v_i^2}{2a}$$
$$= \frac{\left(0\ \frac{\text{m}}{\text{s}}\right)^2 - \left(80.0\ \frac{\text{m}}{\text{s}}\right)^2}{2\left(-9.8\ \frac{\text{m}}{\text{s}^2}\right)}$$
$$= \frac{-6400\ \frac{\text{m}^{\not 2}}{\not{\text{s}}^{\not 2}}}{-19.6\ \frac{\not{\text{m}}}{\not{\text{s}}^{\not 2}}}$$
$$= 326.5\ \text{m}$$
$$= 330\ \text{m}$$
$$\Delta d = 3.3 \times 10^2\ \text{m}$$

Statement: The arrow will reach a maximum height of 3.3×10^2 m or 330 m.

(b) Given: $\vec{v}_i$ = 80.0 m/s [up]; $\vec{v}_f$ = 0.0 m/s;
$\vec{a} = \vec{g}$ = 9.8 m/s^2 [down]

Required: Δt

Analysis: $a = \frac{v_f - v_i}{\Delta t}$

$$\Delta t = \frac{v_f - v_i}{a}$$

Solution: $\Delta t = \frac{v_f - v_i}{a}$

$$= \frac{0\ \frac{\not{\text{m}}}{\not{\text{s}}} - 80.0\ \frac{\not{\text{m}}}{\not{\text{s}}}}{\left(-9.8\ \frac{\not{\text{m}}}{\text{s}^{\not 2}}\right)}$$
$$\Delta t = 8.2\ \text{s}$$

Statement: It will take the arrow 8.2 s to reach its maximum height.

(c) Determine the time it will take the arrow to fall from its maximum height and add that amount to 8.2 s.

Given: $\vec{v}_i$ = 0.0 m/s; $\Delta\vec{d}$ = 3.3×10^2 m [down];
$\vec{a} = \vec{g}$ = 9.8 m/s^2 [down]

Required: Δt

Analysis: $\Delta d = v_i \Delta t + \frac{1}{2}a(\Delta t)^2$

$$= (0\ \text{m/s})\Delta t + \frac{1}{2}a(\Delta t)^2$$
$$(\Delta t)^2 = \frac{2\Delta d}{a}$$
$$\Delta t = \sqrt{\frac{2\Delta d}{a}}$$

Solution:

$$\Delta t = \sqrt{\frac{2\Delta d}{a}}$$
$$= \sqrt{\frac{2(326.5\ \not{\text{m}})}{\left(9.8\ \frac{\not{\text{m}}}{\text{s}^2}\right)}}\ \text{(two extra digits carried)}$$
$$\Delta t = 8.2\ \text{s}$$

Statement: The total amount of time that the arrow is in the air is double 8.2 s or 16 s.

6. Given: $\vec{v}_i$ = 3.61 m/s [down];
$\Delta\vec{d}$ = 28.4 m [down]; $\vec{a} = \vec{g}$ = 9.8 m/s^2 [down]

Required: $\vec{v}_f$

Analysis: $v_f^2 = v_i^2 + 2a\Delta d$

$$v_f = \sqrt{v_i^2 + 2a\Delta d}$$

Solution:

$$v_f = \sqrt{v_i^2 + 2a\Delta d}$$
$$= \sqrt{\left(3.61\ \frac{\text{m}}{\text{s}}\right)^2 + 2\left(9.8\ \frac{\text{m}}{\text{s}^2}\right)(28.4\ \text{m})}$$
$$v_f = 24\ \text{m/s}$$

Statement: The velocity of the rock when it hits the ground is 24 m/s [down].

7. Answers may vary. Sample answer:
The object begins moving at a velocity of 30.0 m/s [up]. It accelerates down due to gravity. After 3 s, the object reaches its maximum height, then it falls for 4 s. An example would be a ball kicked up from a bridge or building because it falls down farther than it travels up.

8. Answers may vary. Sample answer:
A situation where an object experiences acceleration greater than gravity is when a space shuttle takes off. The space shuttle will accelerate upward at around 20 m/s^2, much faster than gravity, in order to get off the planet.