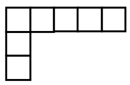
## Chapter 8 Combining Functions

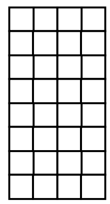
### Chapter 8 Prerequisite Skills

**Chapter 8 Prerequisite Skills** **Question 1 Page 414**

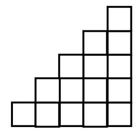
**a)** Pattern A



Pattern B



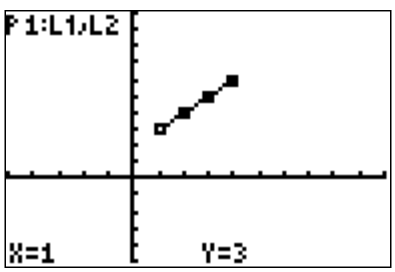
Pattern C

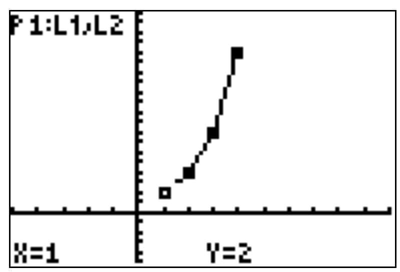


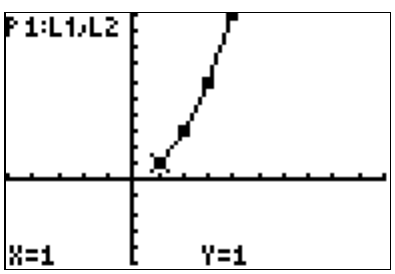
**b)** A: linear increasing one step each time.
B: exponential increasing by a multiple of 2.
C: quadratic increasing by an increasing amount each time.

**Chapter 8 Prerequisite Skills** **Question 2 Page 414**

**a)**



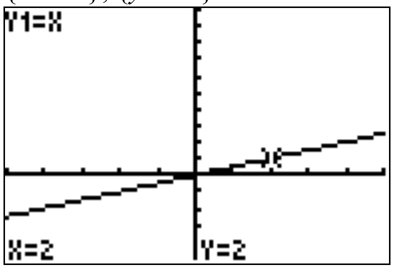

**b)** Yes, A is a line, B is increasing exponentially, C is increasing in the shape of a quadratic.

**Chapter 8 Prerequisite Skills** **Question 3 Page 414**

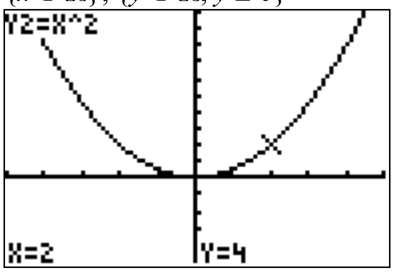
A: $C = n + 2$
B: $C = 2^n$
C: $C = \frac{1}{2}n^2 + \frac{1}{2}n$ or $\frac{n(n+1)}{2}$

**Chapter 8 Prerequisite Skills** **Question 4 Page 414**

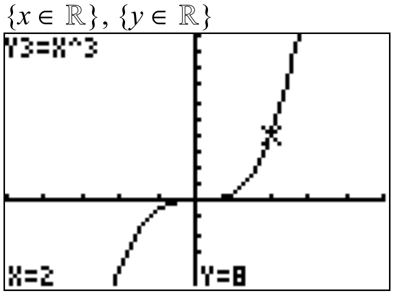
**a)** $\{x \in \mathbb{R}\}, \{y \in \mathbb{R}\}$



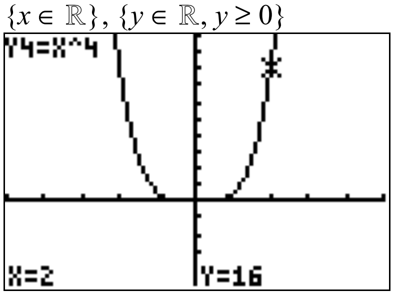

**b)** $\{x \in \mathbb{R}\}, \{y \in \mathbb{R}, y \geq 0\}$




**c)** $\{x \in \mathbb{R}\}, \{y \in \mathbb{R}\}$




**d)** $\{x \in \mathbb{R}\}, \{y \in \mathbb{R}, y \geq 0\}$




**Chapter 8 Prerequisite Skills** **Question 5 Page 414**

**a)** odd, symmetric about the origin

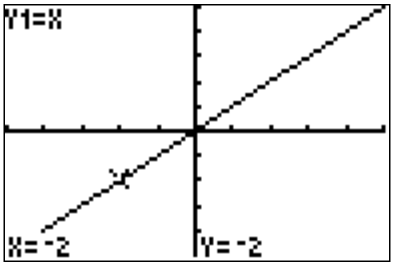
**b)** even, symmetric about the $y$-axis

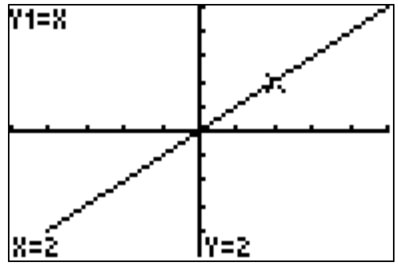
**c)** odd, symmetric about the origin

**d)** even, symmetric about the $y$-axis

**Chapter 8 Prerequisite Skills** **Question 6 Page 414**

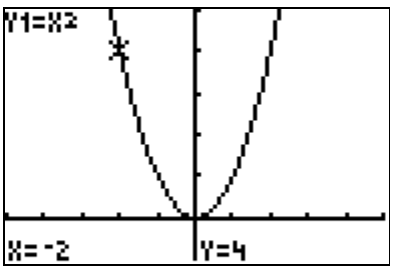
**a)**

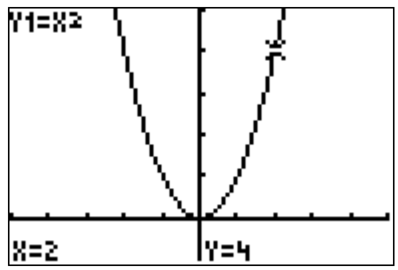



$f(-x) = -f(x)$, so the graph is odd.

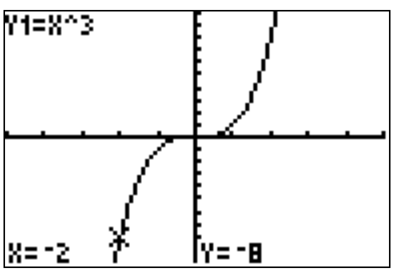
**b)**

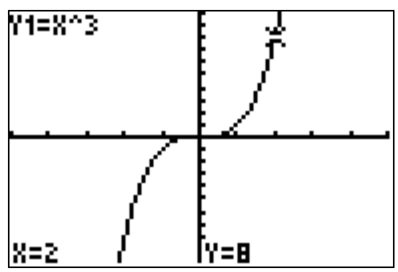



$f(-x) = f(x)$, so the graph is even.

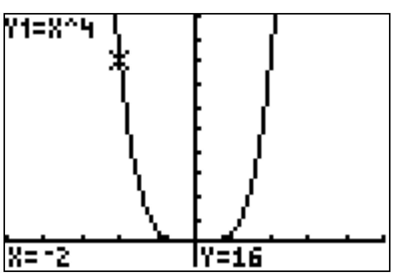
**c)**

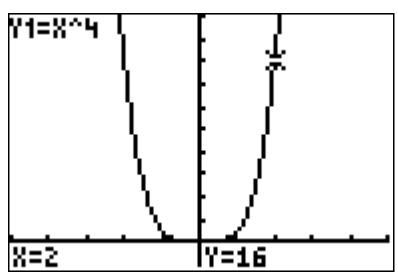



$f(-x) = -f(x)$, so the graph is odd.

**d)**

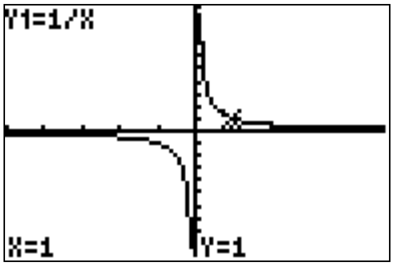


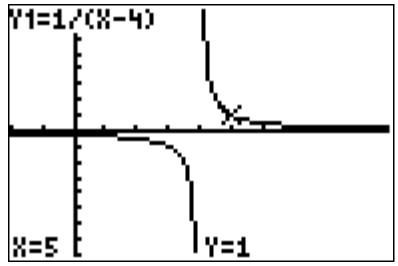

$f(-x) = f(x)$, so the graph is even.

**Chapter 8 Prerequisite Skills** **Question 7 Page 414**

**a)** $\{x \in \mathbb{R}, x \neq 0\}$, $\{y \in \mathbb{R}, y \neq 0\}$




**b)** $\{x \in \mathbb{R}, x \neq 4\}$, $\{y \in \mathbb{R}, y \neq 0\}$




**Chapter 8 Prerequisite Skills** **Question 8 Page 414**

**a)** $u(x) = \dfrac{x-2}{(x-2)(x+2)}$

$u(x) = \dfrac{1}{x+2}, \; x \neq 2, \; x \neq -2$

**b)** $v(x) = \dfrac{(x-3)(x+2)}{x+2}$

$v(x) = x - 3, \; x \neq -2$

**a) i)** $\{x \in \mathbb{R}, x \neq 2, x \neq -2\}$, $\{y \in \mathbb{R}, y \neq 0, y \neq \frac{1}{4}\}$

The domain can be found using the restrictions.
To find the range, find domain of the inverse function and the point where the function is discontinuous.

Inverse

$$x = \frac{1}{y+2}$$

$$y+2 = \frac{1}{x}$$

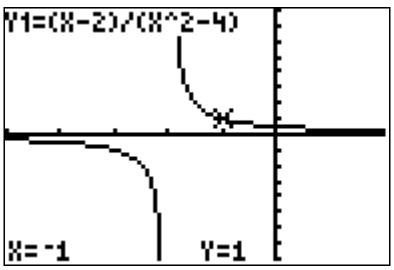
$$y^{-1} = \frac{1}{x} - 2,\ x \neq 0$$

Discontinuity

$$u(2) = \frac{1}{2+2}$$

$$= \frac{1}{4}$$

**ii)**




**iii)** Vertical asymptote $x = -2$; horizontal asymptote $y = 0$

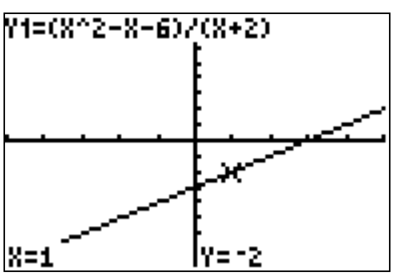
hole: $\left(2, \frac{1}{4}\right)$

**b) i)** $\{x \in \mathbb{R}, x \neq -2\}$, $\{y \in \mathbb{R}, y \neq -5\}$

The domain can be found using the restrictions.
To find the range, find the point where the function is discontinuous.

$$v(-2) = -2 - 3$$
$$= -5$$

**ii)**




**iii)** hole: (–2, –5)

**Chapter 8 Prerequisite Skills** **Question 10 Page 414**

See question 9 part ii).

**Chapter 8 Prerequisite Skills** **Question 11 Page 415**

Reasons may vary. A sample solution is shown.

**a)** iv; a glider gradually loses altitude and may level off at some points

**b)** iii; after one push, the distance of the swing gradually decreases

**c)** ii; the rocket would go up and then come down after reaching its maximum height.

**d)** i; the distance of the tip never changes

**Chapter 8 Prerequisite Skills** **Question 12 Page 415**

Answers may vary. A sample solution is shown.

**i)** $0 \text{ s} \le t \le 8 \text{ s}$, $-5 \text{ cm} \le d \le 5 \text{ cm}$
There are 8 local minima and maxima, so the time is 8 s.
The maximum distance the tip of the metronome can travel is 10 cm.

**ii)** $0 \text{ s} \le t \le 3 \text{ s}$, $0 \text{ m} \le d \le 5 \text{ m}$
A toy rocket is launched from a platform that is approximately 4 m high and takes approximately 3 s to hit the ground.

**iii)** $0 \text{ s} \le t \le 10 \text{ s}$, $-2 \text{ m} \le d \le 2 \text{ m}$
The first push is 2 m forward and the distance decreases each swing. It takes approximately 10 s for the swing to go 4 full swings back and forth.

**iv)** $0 \text{ s} \le t \le 10 \text{ s}$, $0 \text{ m} \le d \le 3 \text{ m}$
A toy glider launched from 3 m takes approximately 10 s to reach the ground.

**Chapter 8 Prerequisite Skills** **Question 13 Page 415**

**a)**

$$\begin{aligned} x &= y - 2 \\ y &= x + 2 \\ f^{-1}(x) &= x + 2 \end{aligned}$$

**b)**

$$\begin{aligned} x &= 4y + 3 \\ 4y &= x - 3 \\ y &= \frac{x-3}{4} \\ g^{-1}(x) &= \frac{x-3}{4} \end{aligned}$$

**c)**

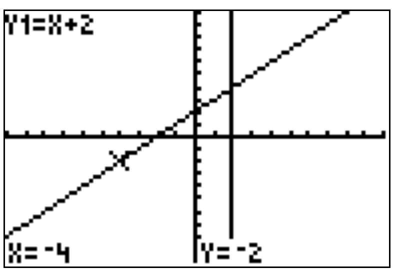
$$\begin{aligned} x &= y^2 - 5 \\ y^2 &= x + 5 \\ y &= \pm\sqrt{x+5} \\ h^{-1}(x) &= \pm\sqrt{x+5},\ x \ge -5 \end{aligned}$$

**d)**

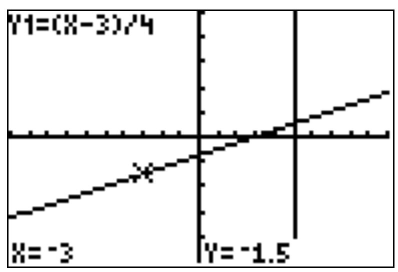
$$\begin{aligned} x &= \frac{1}{y+1} \\ y + 1 &= \frac{1}{x} \\ y &= \frac{1}{x} - 1 \\ k^{-1}(x) &= \frac{1}{x} - 1,\ x \ne 0 \end{aligned}$$

**Chapter 8 Prerequisite Skills** **Question 14 Page 415**

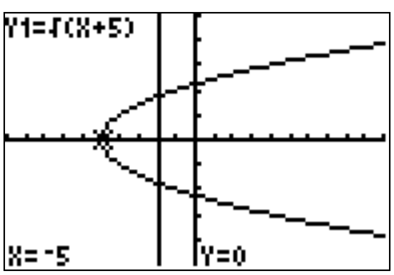
The inverses of parts a), b), and d) are functions, since they pass the vertical line test.



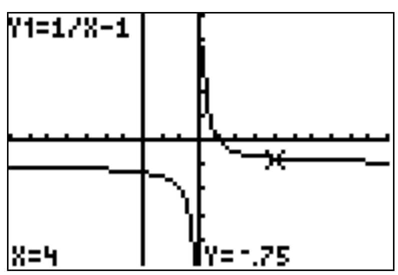
function



function



not a function



function

**Section 1** **Sums and Differences of Functions**

**Chapter 8 Section 1** **Question 1 Page 424**

**a) i)** Blue **ii)** Red **iii)** Yellow

**b) i)** $y = 3x$

| $x$ | $y$ |
|---|---|
| 1 | 3 |
| 2 | 6 |
| 3 | 9 |
| 4 | 12 |

**ii)** $y = x^2 - 1$

| $x$ | $y$ |
|---|---|
| 1 | 0 |
| 2 | 3 |
| 3 | 8 |
| 4 | 15 |

**iii)** $y = 2^x$

| $x$ | $y$ |
|---|---|
| 1 | 2 |
| 2 | 4 |
| 3 | 8 |
| 4 | 16 |

The tables match the number of tiles shown in each stage.

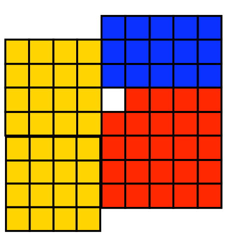
**Chapter 8 Section 1** **Question 2 Page 424**

**a)** $y = 3x + x^2 - 1 + 2^x$

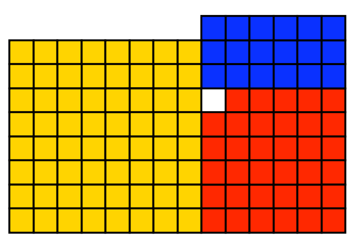
**b) i)**
$$\begin{aligned} f(x) &= 3x + x^2 - 1 + 2^x \\ f(5) &= 3(5) + (5)^2 - 1 + 2^5 \\ &= 15 + 25 - 1 + 32 \\ &= 71 \end{aligned}$$

**ii)**
$$\begin{aligned} f(x) &= 3x + x^2 - 1 + 2^x \\ f(6) &= 3(6) + (6)^2 - 1 + 2^6 \\ &= 18 + 36 - 1 + 64 \\ &= 117 \end{aligned}$$

**c)**



Stage 5
71 tiles



Stage 6
117 tiles

**Chapter 8 Section 1** **Question 3 Page 424**

**a) i)**
$y = 5x + x + 7$
$y = 6x + 7$

**ii)**
$y = 5x - (x + 7)$
$y = 5x - x - 7$
$y = 4x - 7$

**iii)**
$y = x + 7 - 5x$
$y = -4x + 7$

**b) i)**
$y = -2x + 5 + (-x + 9)$
$y = -3x + 14$

**ii)**
$y = -2x + 5 - (-x + 9)$
$y = -2x + 5 + x - 9$
$y = -x - 4$

**iii)**
$y = -x + 9 - (-2x + 5)$
$y = -x + 9 + 2x - 5$
$y = x + 4$

**c) i)**
$y = x^2 + 4 + 1$
$y = x^2 + 5$

**ii)**
$y = x^2 + 4 - 1$
$y = x^2 + 3$

**iii)**
$y = 1 - (x^2 + 4)$
$y = 1 - x^2 - 4$
$y = -x^2 - 3$

**d) i)**
$y = -3x^2 + 4x + 3x - 7$
$y = -3x^2 + 7x - 7$

**ii)**
$y = -3x^2 + 4x - (3x - 7)$
$y = -3x^2 + 4x - 3x + 7$
$y = -3x^2 + x + 7$

**iii)**
$y = 3x - 7 - (-3x^2 + 4x)$
$y = 3x - 7 + 3x^2 - 4x$
$y = 3x^2 - x - 7$

**Chapter 8 Section 1** **Question 4 Page 424**

**a)**
$h(x) = 4x + 3 + 3x - 2$
$h(x) = 7x + 1$
$h(2) = 7(2) + 1$
$h(2) = 15$

**b)**
$j(x) = 4x + 3 - (3x - 2)$
$j(x) = 4x + 3 - 3x + 2$
$j(x) = x + 5$
$j(-1) = -1 + 5$
$j(-1) = 4$

**c)**
$k(x) = 3x - 2 - (4x + 3)$
$k(x) = 3x - 2 - 4x - 3$
$k(x) = -x - 5$
$k(0) = -0 - 5$
$k(0) = -5$

**Chapter 8 Section 1** **Question 5 Page 424**

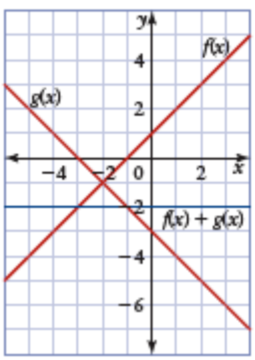
**a)**
$$h(x) = -4x^2 + 5 + 2x - 3$$
$$h(x) = -4x^2 + 2x + 2$$
$$h(-3) = -4(-3)^2 + 2(-3) + 2$$
$$h(-3) = -4(9) - 6 + 2$$
$$h(-3) = -40$$

**b)**
$$j(x) = -4x^2 + 5 - (2x - 3)$$
$$j(x) = -4x^2 + 5 - 2x + 3$$
$$j(x) = -4x^2 - 2x + 8$$
$$j(0) = -4(0)^2 - 2(0) + 8$$
$$j(0) = 8$$

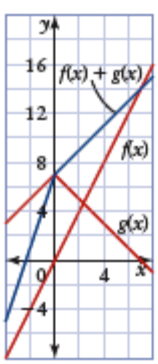
**c)**
$$k(x) = 2x - 3 - (-4x^2 + 5)$$
$$k(x) = 2x - 3 + 4x^2 - 5$$
$$k(x) = 4x^2 + 2x - 8$$
$$k(3) = 4(3)^2 + 2(3) - 8$$
$$k(3) = 4(9) + 6 - 8$$
$$k(3) = 34$$

**Chapter 8 Section 1** **Question 6 Page 424**

**a)** $\{x \in \mathbb{R}\}$, $\{y \in \mathbb{R}, y = -2\}$

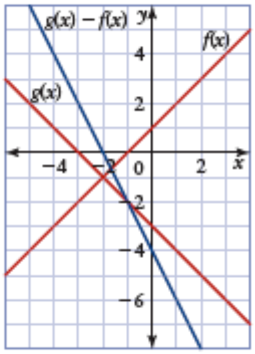



**b)** $\{x \in \mathbb{R}\}$, $\{y \in \mathbb{R}\}$

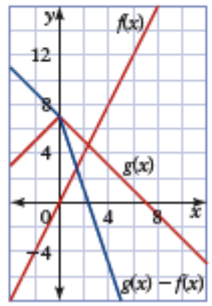


**Chapter 8 Section 1** **Question 7 Page 424**

**a)**

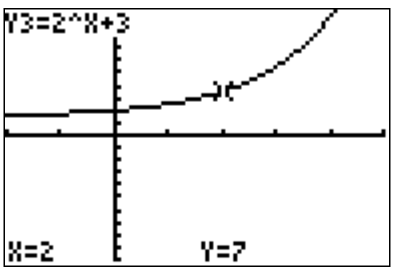


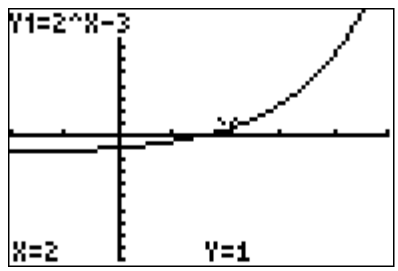

**b)**

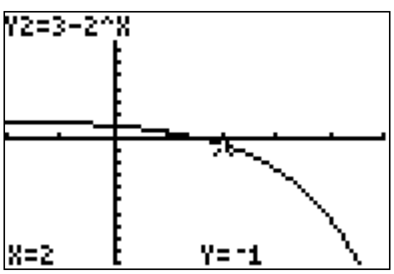



**Chapter 8 Section 1** **Question 8 Page 425**

**a)**




**b)** **i)** $f(x) = 2^x$ translated 3 units up

**ii)** $f(x) = 2^x$ translated 3 units down

**iii)** reflection of $f(x) = 2^x$ in the $x$-axis and a translation of 3 units up

**c) i)** $\{x \in \mathbb{R}\}, \{y \in \mathbb{R}, y > 3\}$

The range is the domain of the inverse.

$$x = 2^y + 3$$
$$x - 3 = 2^y$$
$$\log(x-3) = y\log 2$$
$$y = \frac{\log(x-3)}{\log 2},\ x > 3$$

**ii)** $\{x \in \mathbb{R}\}, \{y \in \mathbb{R}, y > -3\}$

The range is the domain of the inverse.

$$x = 2^y - 3$$
$$x + 3 = 2^y$$
$$\log(x+3) = y\log 2$$
$$y = \frac{\log(x+3)}{\log 2},\ x > -3$$

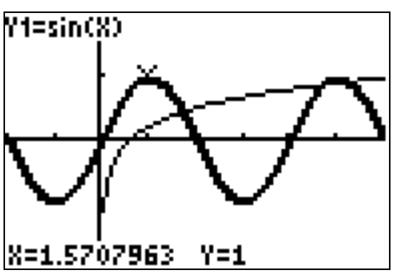
**iii)** $\{x \in \mathbb{R}\}, \{y \in \mathbb{R}, y < 3\}$

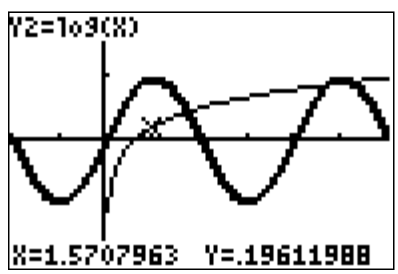
The range is the domain of the inverse.

$$x = 3 - 2^y$$
$$2^y = 3 - x$$
$$y\log 2 = \log(3-x)$$
$$y = \frac{\log(3-x)}{\log 2},\ x < 3$$

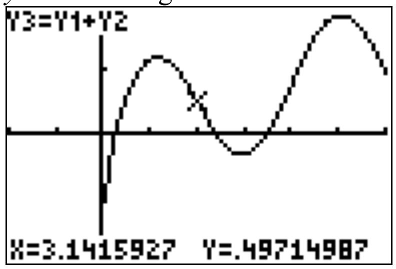
**Chapter 8 Section 1** **Question 9 Page 425**

**a)**



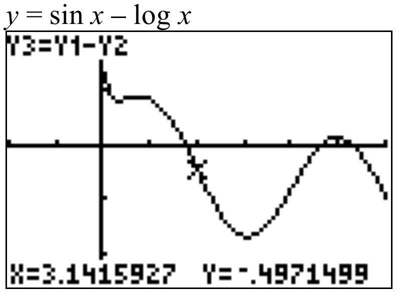



**b)** $y = \sin x + \log x$




| $x$ | $f(x) = \sin x$ | $g(x) = \log x$ | $y = \sin x + \log x$ |
|---|---|---|---|
| 0 | 0 | undefined | undefined |
| $\frac{\pi}{2}$ | 1 | 0.1961 | 1.1961 |
| $\pi$ | 0 | 0.4971 | 0.4971 |
| $\frac{3\pi}{2}$ | –1 | 0.6732 | –0.3268 |
| $2\pi$ | 0 | 0.7982 | 0.7982 |

**c)** $y = \sin x - \log x$



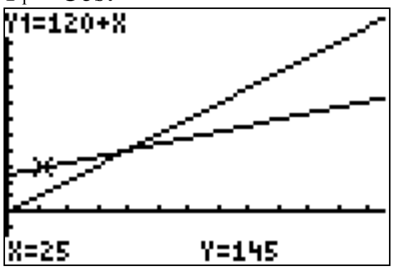

| $x$ | $f(x) = \sin x$ | $g(x) = \log x$ | $y = \sin x - \log x$ |
|---|---|---|---|
| 0 | 0 | undefined | undefined |
| $\frac{\pi}{2}$ | 1 | 0.1961 | 0.8039 |
| $\pi$ | 0 | 0.4971 | –0.4971 |
| $\frac{3\pi}{2}$ | –1 | 0.6732 | –1.6732 |
| $2\pi$ | 0 | 0.7982 | –0.7982 |

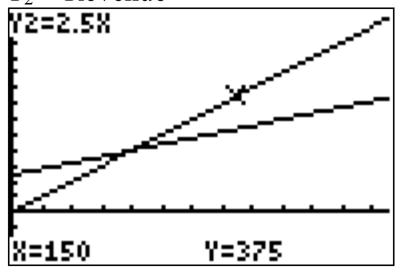
**d)** see parts b) and c)

**Chapter 8 Section 1** **Question 10 Page 425**

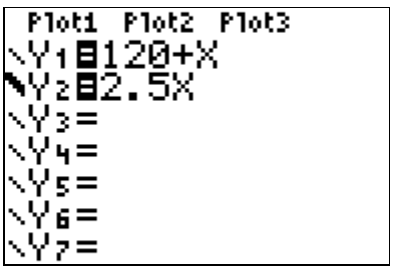
**a)** **i)** $C = 120 + h$ **ii)** $R = 2.5h$

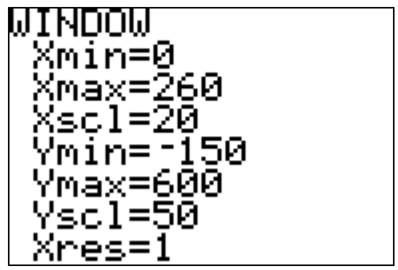
**b)** $Y_1$ = Cost $Y_2$ = Revenue

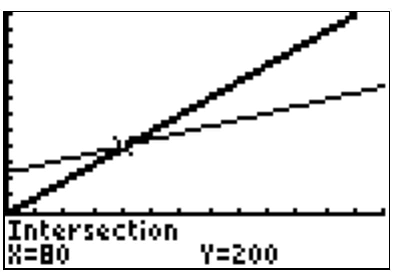


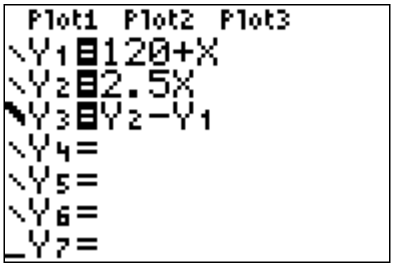

**c)** The break-even point is the point at which the revenue and cost are equal. When the vendor has sold 80 hotdogs, the cost and the revenue are both equal to $200.00.

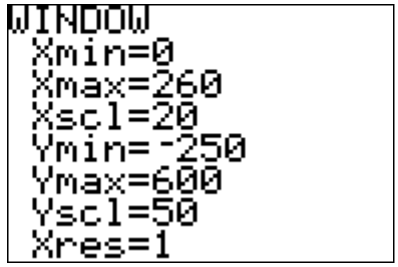


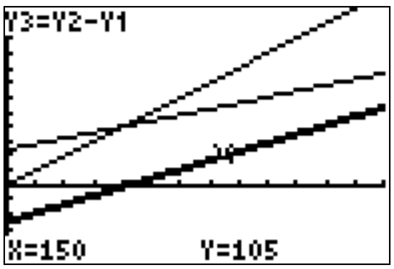



**d)** $P(h) = 2.5h - (120 + h)$
$P(h) = 2.5h - 120 - h$
$P(h) = 1.5h - 120$



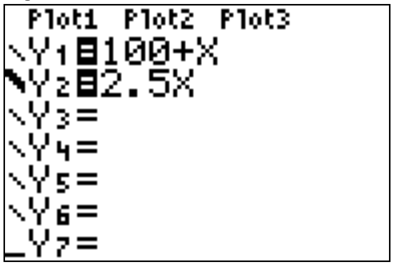



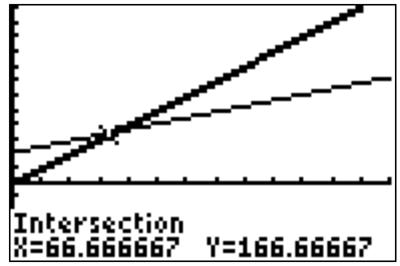

**e)** $C(h)$: $\{h \in \mathbb{Z}, 0 \le h \le 250\}$, $\{C \in \mathbb{R}, 120 \le C \le 370\}$
$R(h)$: $\{h \in \mathbb{Z}, 0 \le h \le 250\}$, $\{R \in \mathbb{R}, 0 \le R \le 625\}$
$P(h)$: $\{h \in \mathbb{Z}, 0 \le h \le 250\}$, $\{P \in \mathbb{R}, -120 \le P \le 255\}$

**f)** $P(250) = 1.5(250) - 120$
$= 255$
The maximum daily profit is \$255.

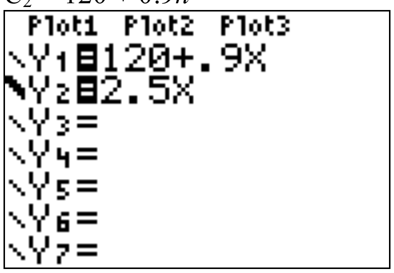
**Chapter 8 Section 1** **Question 11 Page 425**

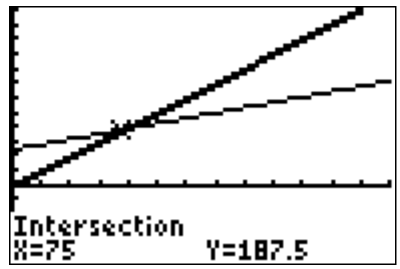
**a) i)** $C_1 = 100 + h$




$C_2 = 120 + 0.9h$

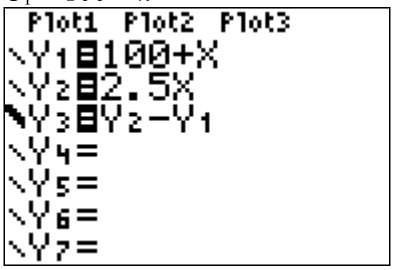


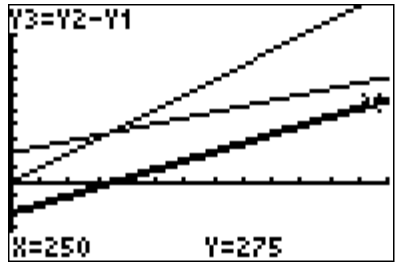




$C_1 = 100 + h$ has the most favourable effect on the break-even point since the vendor will break-even after selling less hotdogs.
When the vendor has sold approximately 67 hotdogs, the cost and the revenue are both equal to approximately \$166.67.

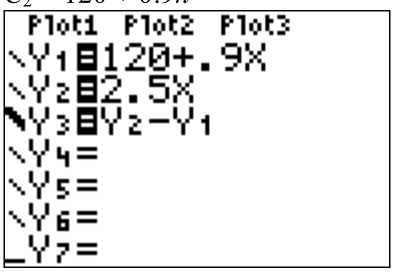
**ii)** *Method 1:*

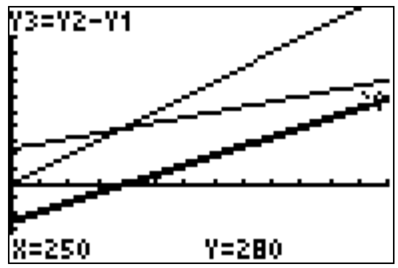
$C_1 = 100 + h$




$C_2 = 120 + 0.9h$




$C_2 = 120 + 0.9h$ has the most favourable effect on the maximum profit. The potential daily profit becomes \$280.

*Method 2:*

$C_1$

$$\begin{aligned}\text{Maximum daily profit} &= 2.5h - (100 + h)\\ &= 2.5(250) - (100 + 250)\\ &= 625 - 350\\ &= 275\end{aligned}$$

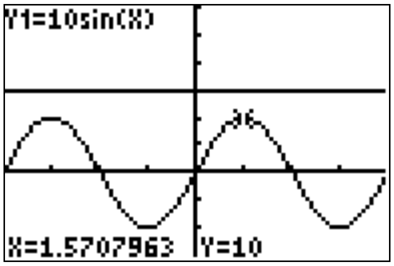
$C_2$

$$\begin{aligned}\text{Maximum daily profit} &= 2.5h - (120 + 0.9h)\\ &= 2.5(250) - [120 + 0.9(250)]\\ &= 625 - (120 + 225)\\ &= 280\end{aligned}$$

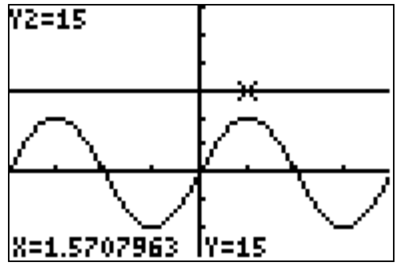
**b)** Answers may vary. A sample solution is shown.
If you always sell a lot, choose $C_2$ (reduce the variable cost) since it has a higher maximum value, but if you often sell around 70, choose $C_1$ (reduce the fixed cost) because it has a lower break-even point.

**Chapter 8 Section 1** **Question 12 Page 425**

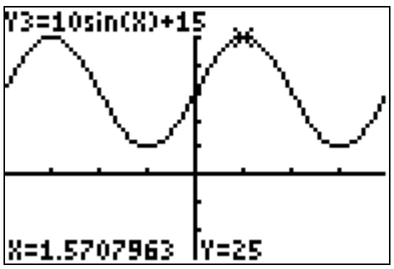
**a)** AC




DC




**b)**



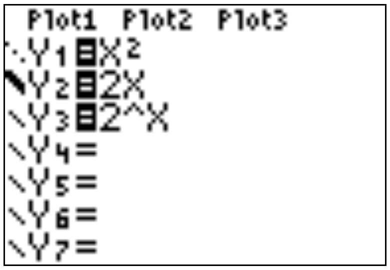

**c)** From the graph, the domain and range are: $\{t \in \mathbb{R}\}$, $\{y \in \mathbb{R}, 5 \le y \le 25\}$

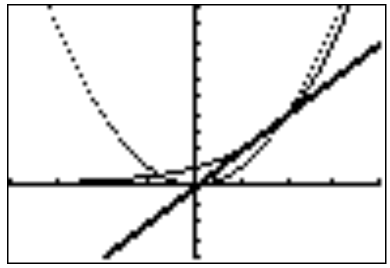
**d) i)** 5 **ii)** 25 **iii)** $\dfrac{5+25}{2} = 15$

**Chapter 8 Section 1** **Question 13 Page 426**

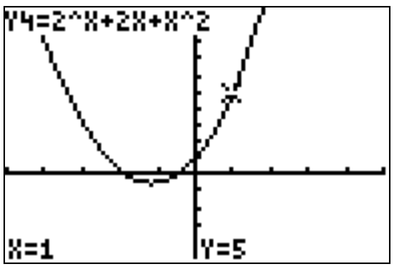
**a)** The function and the total tiles pattern are the same.

**b)**

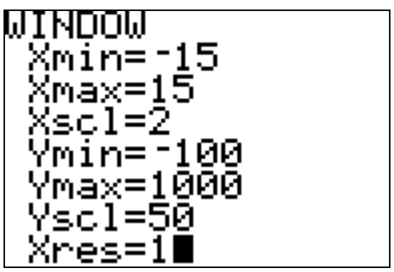


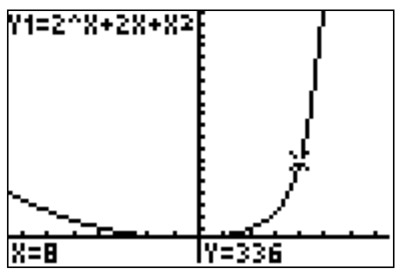



**c)**




**d)**




$T(x)$ seems to converge with $f(x) = 2^x$.

**e)** Answers may vary. A sample solution is shown.
The rate of change of the exponential function is continuously increasing at a greater rate than the other component functions.

**Chapter 8 Section 1** **Question 14 Page 426**

**a)** Yes, $f(x) + g(x) = g(x) + f(x)$ is true for all functions $f(x)$ and $g(x)$ using the commutative property.
Examples may vary. A sample solution is shown.

Example 1:
$f(x) = x + 7$, $g(x) = 5x - 3$

$$\begin{aligned} f(x)+g(x) &= x+7+5x-3 \\ &= 6x+4 \\ g(x)+f(x) &= 5x-3+x+7 \\ &= 6x+4 \end{aligned}$$

Example 2:
$f(x) = a$, $g(x) = b$

$$\begin{aligned} f(x)+g(x) &= a+b \\ g(x)+f(x) &= b+a \\ &= a+b \end{aligned}$$

Example 3:

$$f(x) = x^2 - 6x + 2,\ g(x) = \frac{1}{x+1} - 5$$

$$\begin{aligned} f(x)+g(x) &= x^2-6x+2+\frac{1}{x+1}-5 \\ &= x^2-6x+\frac{1}{x+1}-3 \\ g(x)+f(x) &= \frac{1}{x+1}-5+x^2-6x+2 \\ &= x^2-6x+\frac{1}{x+1}-3 \end{aligned}$$

**b)** No, $f(x) - g(x) = g(x) - f(x)$ is not true for all functions $f(x)$ and $g(x)$.
Examples may vary. A sample solution is shown.

Example 1:
$f(x) = -2x + 3$, $g(x) = 5x - 2$

$$\begin{aligned} f(x)-g(x) &= -2x+3-(5x-2) \\ &= -2x+3-5x+2 \\ &= -7x+5 \\ g(x)-f(x) &= 5x-2-(-2x+3) \\ &= 5x-2+2x-3 \\ &= 7x-5 \end{aligned}$$

Example 2:
$f(x) = a$, $g(x) = b$

$$\begin{aligned} f(x)-g(x) &= a-b \\ g(x)-f(x) &= b-a \\ &= -a+b \end{aligned}$$

Example 3:
$f(x) = x^3 - 2x^2 + 6$, $g(x) = 2x^3 - 5x - 3$

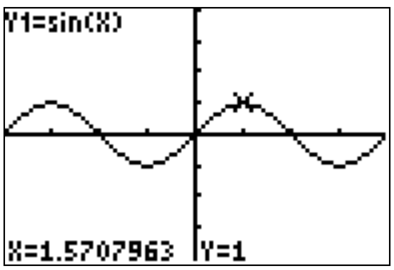
$$\begin{aligned} f(x) - g(x) &= x^3 - 2x^2 + 6 - (2x^3 - 5x - 3) \\ &= x^3 - 2x^2 + 6 - 2x^3 + 5x + 3 \\ &= -x^3 - 2x^2 + 5x + 9 \end{aligned}$$

$$\begin{aligned} g(x) - f(x) &= 2x^3 - 5x - 3 - (x^3 - 2x^2 + 6) \\ &= 2x^3 - 5x - 3 - x^3 + 2x^2 - 6 \\ &= x^3 + 2x^2 - 5x - 9 \end{aligned}$$

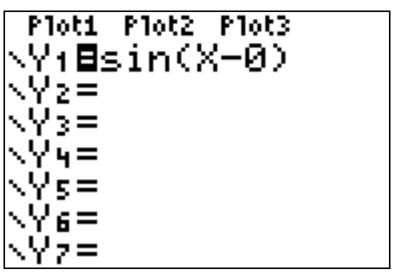
**c)** The commutative property holds true for the sum of two functions, but not the difference of two functions.

**Chapter 8 Section 1** **Question 15 Page 426**

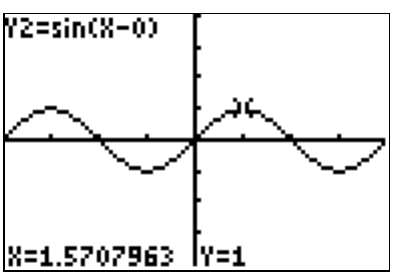
**a)**




**b)**

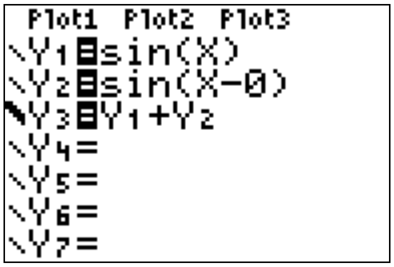


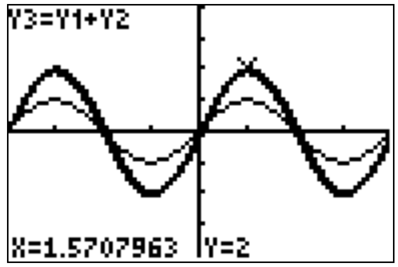

**c)**




$f(x) = g(x)$, when $c = 0$

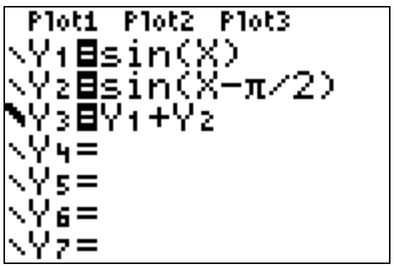
**d)**

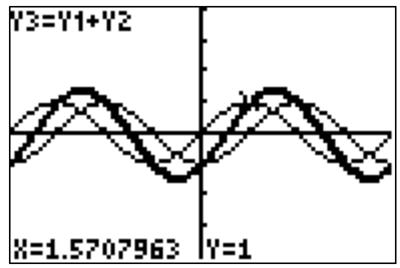


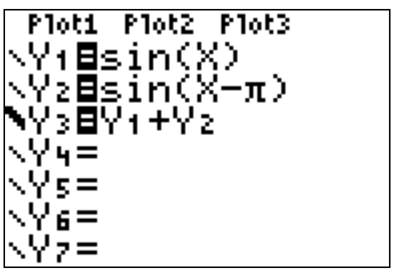

**e)** $\{x \in \mathbb{R}\}$, $\{y \in \mathbb{R}, -2 \le y \le 2\}$

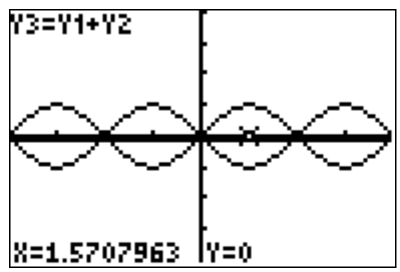
**f)** **i)** shifted to the right and the amplitude is multiplied by $\sqrt{2}$



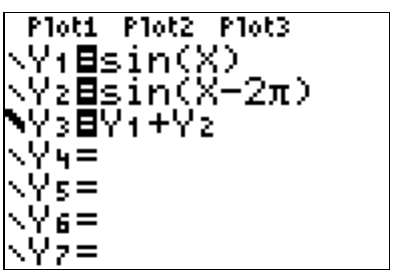



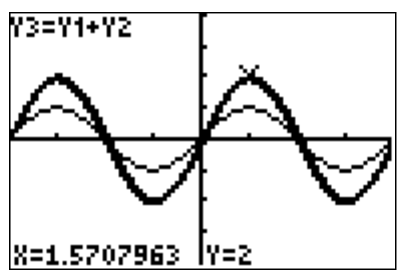

**ii)** horizontal line




**iii)** same as when $c = 0$



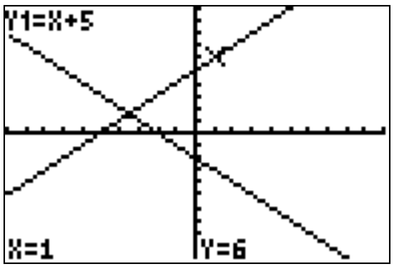

**g)** **i)** $\{x \in \mathbb{R}\}$, $\{y \in \mathbb{R}, -\sqrt{2} \le y \le \sqrt{2}\}$
**ii)** $\{x \in \mathbb{R}\}$, $\{y = 0\}$
**iii)** $\{x \in \mathbb{R}\}$, $\{y \in \mathbb{R}, -2 \le y \le 2\}$

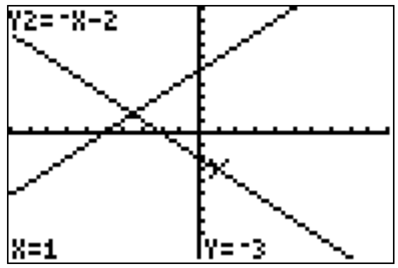
**h)** Answers may vary. A sample solution is shown.
When two equal waves meet crest to crest or trough to trough, the magnitude is doubled.
When two equal waves meet crest to trough, the waves cancel each other out.

**Chapter 8 Section 1** **Question 16 Page 427**

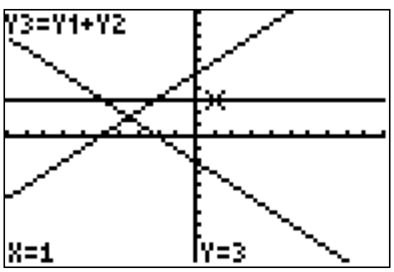
**a)** 
$$\begin{aligned} y &= -g(x) \\ &= -(x+2) \\ &= -x-2 \end{aligned}$$

**b)**

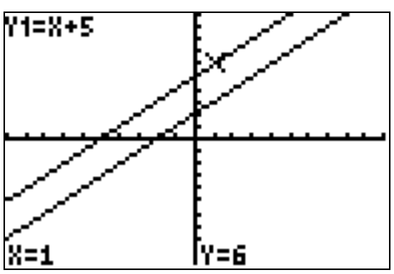


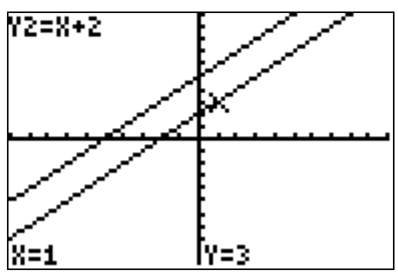




**c)** 
$$y = f(x) + \left[-g(x)\right]$$
$$y = x+5+[-(x+2)]$$
$$y = 3$$

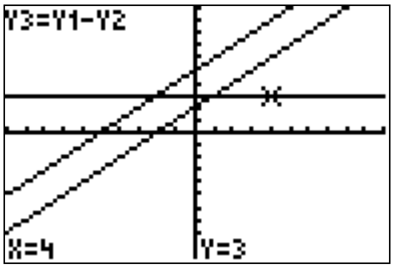



**d)**



**e)** $y = f(x) - g(x)$
$y = (x+5) - (x+2)$
$y = x + 5 - x - 2$
$y = 3$




The results are the same.

**f)** Answers may vary. A sample solution is shown.
Subtracting the functions is the same as adding the opposite.

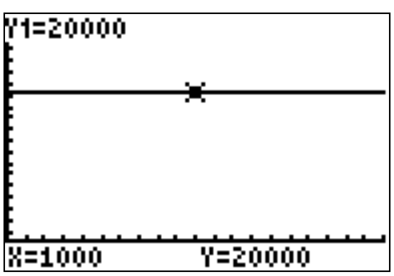
**Chapter 8 Section 1** **Question 17 Page 427**

$f(x) = a$, $g(x) = b$

$$\begin{aligned} f(x) - g(x) &= a - b \\ f(x) + \left[-g(x)\right] &= a + (-b) \\ &= a - b \\ f(x) - g(x) &= f(x) + \left[-g(x)\right] \end{aligned}$$

**Chapter 8 Section 1** **Question 18 Page 427**

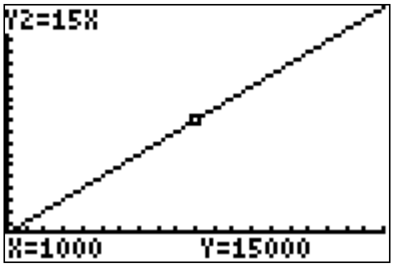
**a)** Fixed costs = 12 000 + 3000 + 5000
= 20 000
The fixed costs are \$20 000. They are not affected by the number of games.



**b)** Variable costs = $6x + 9x$

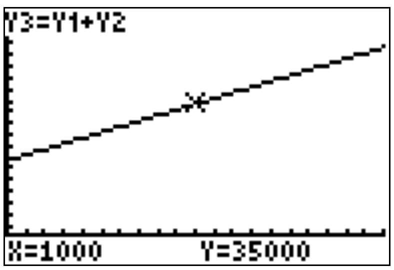
$= 15x$

The total variable costs are \$15/game. The cost increases per game at a constant rate.



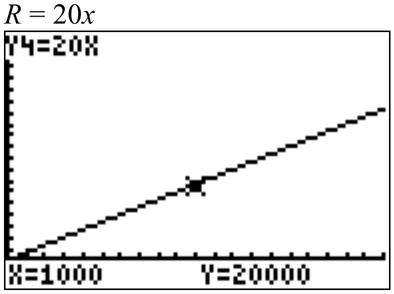

**c)** $C = 20\ 000 + 15x$

This function represents the total operating costs.

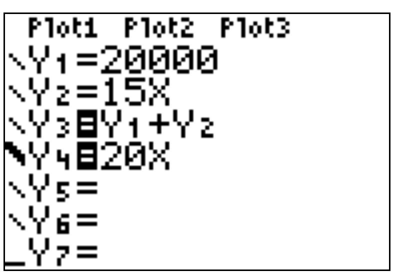


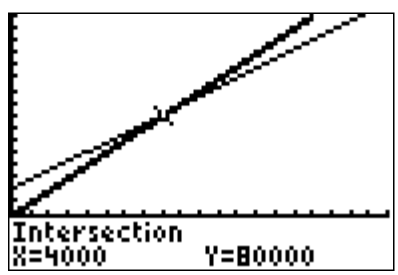

**d)** The revenue is increasing at a constant rate.

$R = 20x$




**e)**

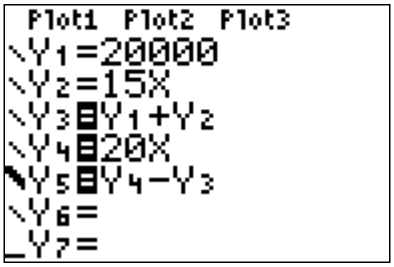


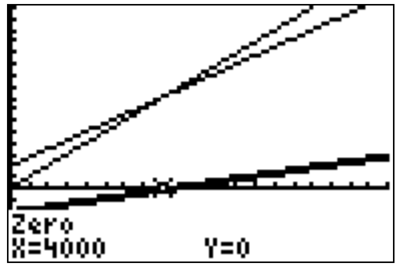




The break-even point is the point at which the revenue and cost are equal (the profit equals zero).

When 4000 games are sold, the cost and revenue equal \$80 000.

**f)** $Y_5 = 20x - (20\ 000 + 15x)$
$= 20x - 20\ 000 - 15x$
$= 5x - 20\ 000$




This function represents the profit.

**g)** **i)** To the left of the $x$-intercept is a loss in profit.
**ii)** At the $x$-intercept is the break-even point.
**iii)** To the right of the $x$-intercept is a profit.

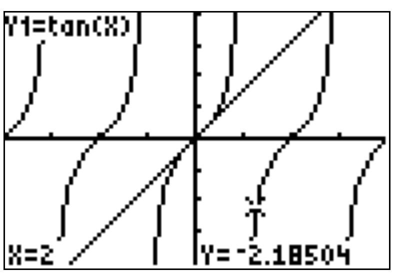
**h)** Answers may vary. A sample solution is shown.
**i)** The break-even point would move to the left and thus be reached sooner.
**ii)** The break-even point would move to the right and thus be reached later. Assuming fixed costs stay the same.

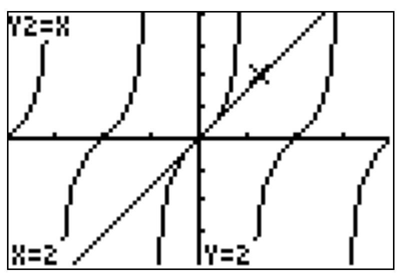
**Chapter 8 Section 1** **Question 19 Page 427**

Solutions to Achievement Check questions are provided in the Teacher's Resource.

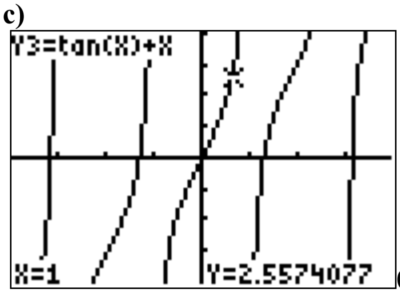
**Chapter 8 Section 1** **Question 20 Page 428**

**a)**



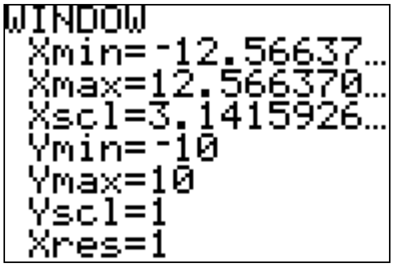



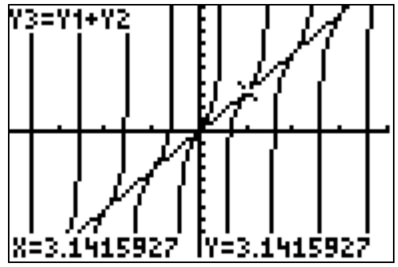

**b), c)**



0

**d)**



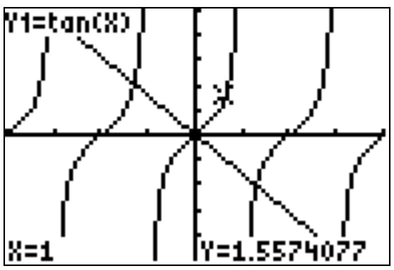



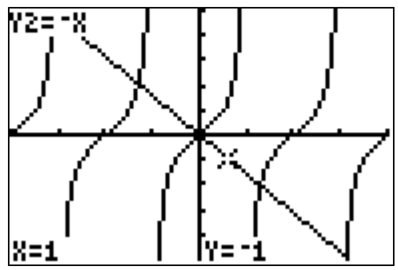

There appear to be an infinite number of intersection points.

**e)** $g(x)$ intersects $h(x)$ at the point of inflection due to the variable vertical translation.

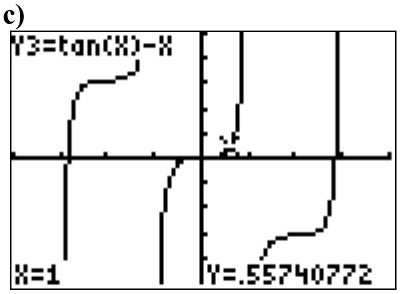
**Chapter 8 Section 1** **Question 21 Page 428**

**a)**

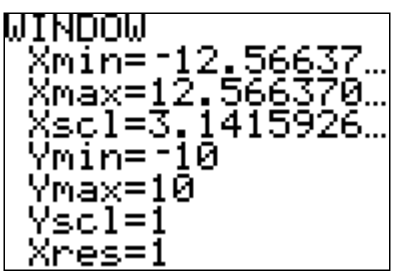


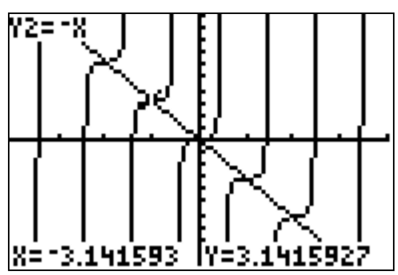




**b), c)**




**d)**




There appear to be an infinite number of intersection points.

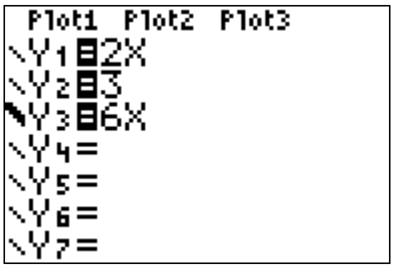
**e)** $g(x)$ intersects $h(x)$ at the point of inflection due to the variable vertical translation.

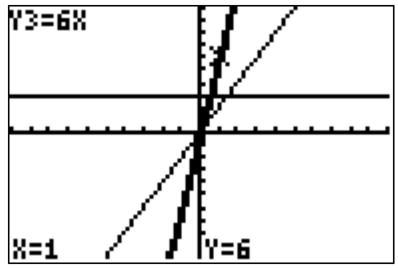
**Chapter 8 Section 1** **Question 22 Page 428**

Yes, the superposition principle can be extended to multiplication or division of two functions.
Answers may vary. A sample solution is shown.
Example 1:

$f(x) = 2x,\ g(x) = 3$

$$\begin{aligned} h(x) &= f(x) \times g(x) \\ &= 6x \end{aligned}$$



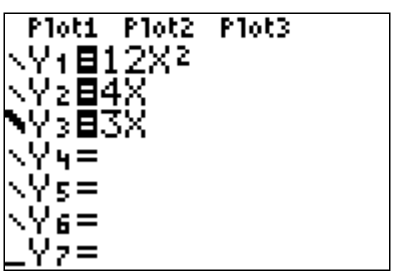

| $x$ | $f(x) = 2x$ | $g(x) = 3$ | $h(x) = 6x$ |
|---|---|---|---|
| –2 | –4 | 3 | –12 |
| –1 | –2 | 3 | –6 |
| 0 | 0 | 3 | 0 |
| 1 | 2 | 3 | 6 |
| 2 | 4 | 3 | 12 |

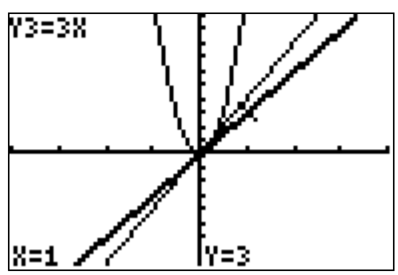
Example 2:

$f(x) = 12x^2,\ g(x) = 4x$

$$h(x) = \frac{f(x)}{g(x)},\ g(x) \neq 0$$

$$\begin{aligned} h(x) &= \frac{12x^2}{4x} \\ &= 3x,\ x \neq 0 \end{aligned}$$



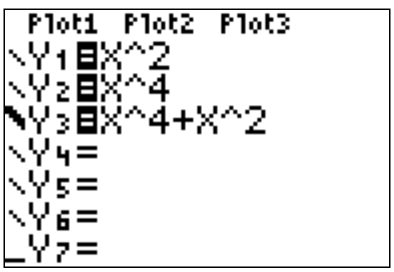



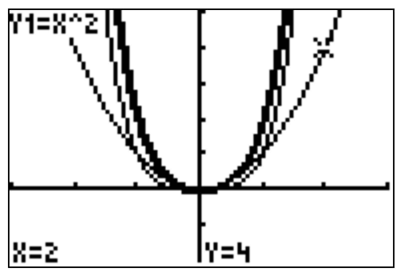

| $x$ | $f(x) = 12x^2$ | $g(x) = 4x$ | $h(x) = 3x$ |
|---|---|---|---|
| –2 | 28 | –8 | –6 |
| –1 | 12 | –4 | –3 |
| 0 | 0 | 0 | 0 |
| 1 | 12 | 4 | 3 |
| 2 | 48 | 8 | 6 |

The sum of two even functions is even.
Examples will vary. A sample solution is shown.

Example 1:

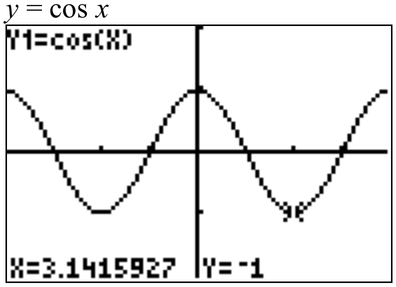
$$f(x)=x^2,\ g(x)=x^4$$
$$\begin{aligned}h(x)&=f(x)+g(x)\\&=x^4+x^2\end{aligned}$$
$$\begin{aligned}f(-x)&=(-x)^2 & g(-x)&=(-x)^4\\&=x^2 & &=x^4\\f(-x)&=f(x)\text{ even} & g(-x)&=g(x)\text{ even}\end{aligned}$$
$$\begin{aligned}h(-x)&=(-x)^4+(-x)^2\\&=x^4+x^2\\h(-x)&=h(x)\text{ even}\end{aligned}$$



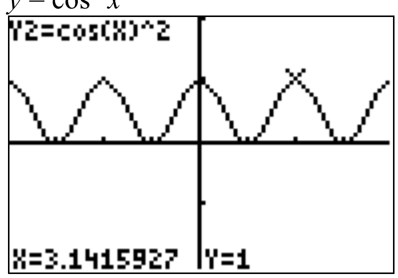




Example 2:

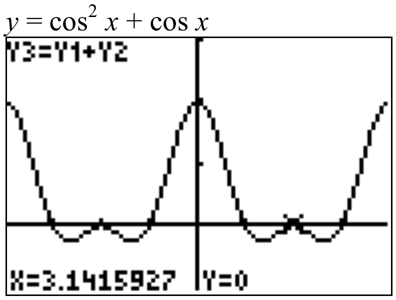
$y = \cos x$




$y = \cos^2 x$



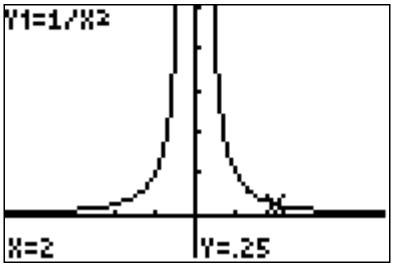

$y = \cos^2 x + \cos x$



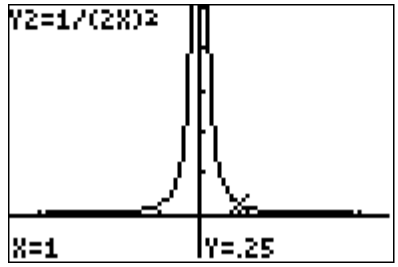

The functions are all even.

Example 3:

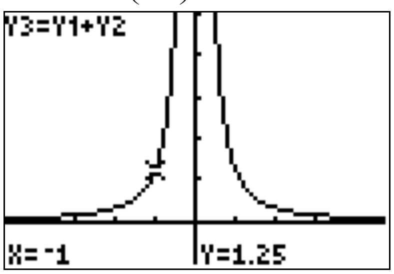
$y = \dfrac{1}{x^2}$




$y = \left(\dfrac{1}{2x}\right)^2$




$y = \dfrac{1}{x^2} + \left(\dfrac{1}{2x}\right)^2$




The functions are all even.

**Chapter 8 Section 1** **Question 24 Page 428**

B

$$
\begin{aligned}
f(x+1) - f(x) &= 6x - 8 \\
&\text{let } x = 1 \\
f(1+1) - f(1) &= 6(1) - 8 \\
f(2) - f(1) &= -2 \\
f(2) - 26 &= -2 \quad \text{substitute } f(1) = 26 \\
f(2) &= 26 - 2 \\
f(2) &= 24
\end{aligned}
$$

**Chapter 8 Section 1** **Question 25 Page 428**

D

*Method 1:*

find the limit as $x \to \pm\infty$

$$\frac{\frac{2x}{x}+\frac{5}{x}}{\frac{3x}{x}+\frac{8}{x}} = \frac{2+\frac{5}{x}}{3+\frac{8}{x}}, \text{ as } x \to \pm\infty, \frac{5}{x} \text{ and } \frac{8}{x} \text{ approach } 0$$

$$= \frac{2}{3}$$

*Method 2:*

Suppose $\frac{2x+5}{3x+8} = \frac{2}{3}$

$$3(2x+5) = 2(3x+8)$$

$$6x+15 = 6x+16$$

$$\text{but } 15 \neq 16$$

There is no solution.

**Chapter 8 Section 1** **Question 26 Page 428**

$$x = \frac{1}{4-y}, \frac{1}{x} = 4-y$$

$$\begin{aligned}
\frac{1}{x}+4x+y-yx-1 &= (4-y)+4\left(\frac{1}{4-y}\right)+y-y\left(\frac{1}{4-y}\right)-1 \\
&= 4-1-y+y+\frac{4}{4-y}-\frac{y}{4-y} \\
&= 3+\frac{4-y}{4-y} \\
&= 3+1 \\
&= 4
\end{aligned}$$

$$x = \frac{1}{A} \text{ or } x = -A$$

$$(Ax - 1)(x + A) = 0$$

$$Ax^2 + Ax - x - A = 0$$

$$-A = C$$

$$A = -C$$

If the roots are negative reciprocals, then $A = -C$.

$$x = \frac{1}{A} \text{ or } x = A$$

$$(Ax + 1)(x + A) = 0$$

$$Ax^2 + Ax + x + A = 0$$

$$A = C$$

If the roots are reciprocals, then $A = C$.

**Section 2** **Products and Quotients of Functions**

**Chapter 8 Section 2** **Question 1 Page 435**

**a)** even: A and B since they are both even functions;
odd: A and C since A is even and C is odd, B and C since B is even and C is odd.

**b)** Yes, two combinations multiply to form an odd function.

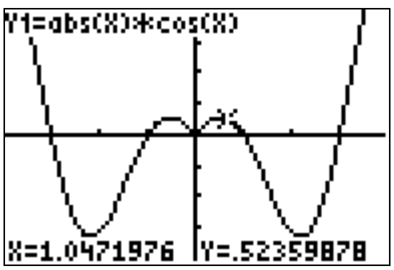
**Chapter 8 Section 2** **Question 2 Page 435**

The product of all three functions is odd since there are two even functions and one odd function.

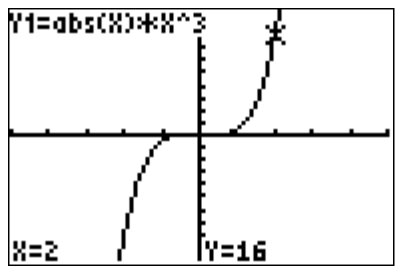
**Chapter 8 Section 2** **Question 3 Page 435**

A: $y = |x|$; B: $y = \cos x$; C: $y = x^3$

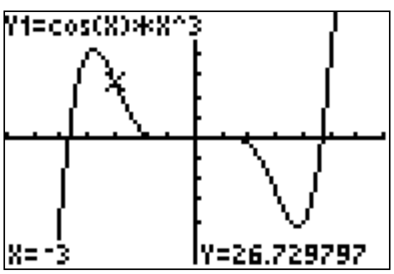
A × B = even



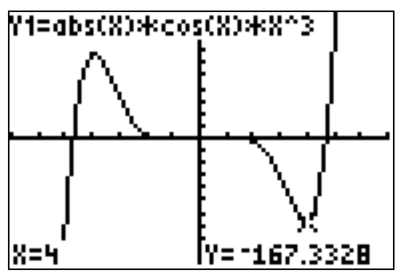

A × C = odd




B × C = odd



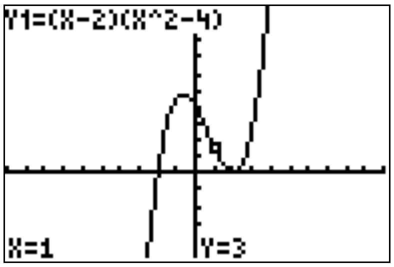

A × B × C



**a)** $y=(x-2)(x^2-4)$

$y=x^3-2x^2-4x+8$

$\{x\in\mathbb{R}\}$, $\{y\in\mathbb{R}\}$




**b)** $y=\dfrac{x-2}{x^2-4}$

$y=\dfrac{x-2}{(x-2)(x+2)}$

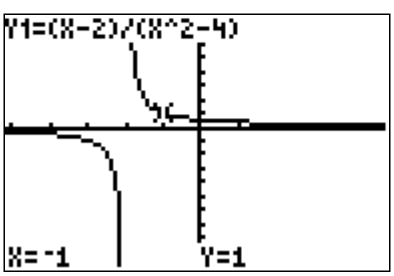
$y=\dfrac{1}{x+2}$, $x\neq 2$, $x\neq -2$

There is a hole when $x = 2$.

$$y=\frac{1}{2+2}$$
$$=\frac{1}{4}$$
$$\left(2,\frac{1}{4}\right)$$

There is a vertical asymptote at $x = -2$ and a horizontal asymptote at $y = 0$.

The domain and range; $\{x\in\mathbb{R}, x\neq 2, x\neq -2\}$, $\{y\in\mathbb{R}, y\neq\frac{1}{4}, y\neq 0\}$



**c)** $y = \dfrac{x^2 - 4}{x - 2}$

$y = \dfrac{(x-2)(x+2)}{(x-2)}$

$y = x + 2,\ x \neq 2$

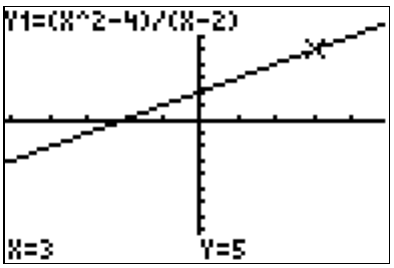
There is a hole when $x = 2$.

$y = 2 + 2$

$\quad = 4$

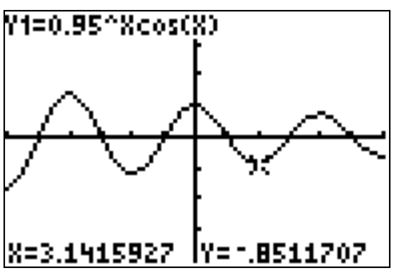
(2, 4)

The domain and range; $\{x \in \mathbb{R}, x \neq 2\}$, $\{y \in \mathbb{R}, y \neq 4\}$

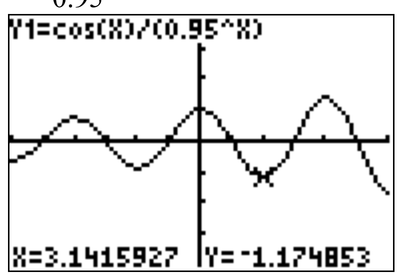



**Chapter 8 Section 2** **Question 5 Page 436**

**a)** $y = 0.95^x \cos x$, $\{x \in \mathbb{R}\}$, $\{y \in \mathbb{R}\}$



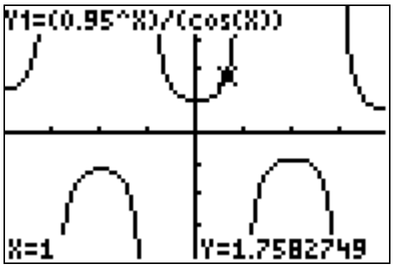

**b)** $y = \dfrac{\cos x}{0.95^x}$, $\{x \in \mathbb{R}\}$, $\{y \in \mathbb{R}\}$



**c)** $y = \dfrac{0.95^x}{\cos x}$, $\cos x \neq 0$

The vertical asymptotes are $x = \dfrac{(2n+1)\pi}{2}$, $n \in \mathbb{Z}$ and the horizontal asymptote is $y = 0$.

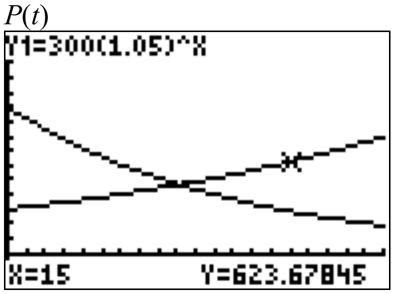
The domain and range; , $\{x \in \mathbb{R}, x \neq \dfrac{(2n+1)\pi}{2}, n \in \mathbb{Z}\}$, $\{y \in \mathbb{R}, y \neq 0\}$



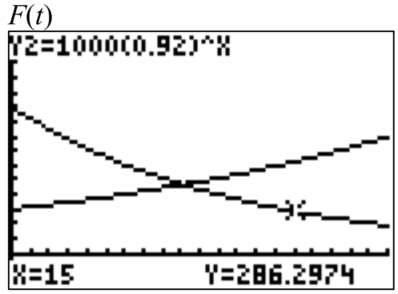

**Chapter 8 Section 2** **Question 6 Page 436**

**a)** Both functions are exponential, with fish increasing $P(t)$ and food decreasing $F(t)$.

$P(t)$

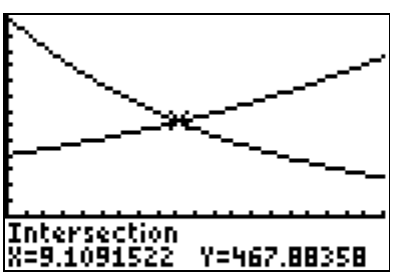



$F(t)$



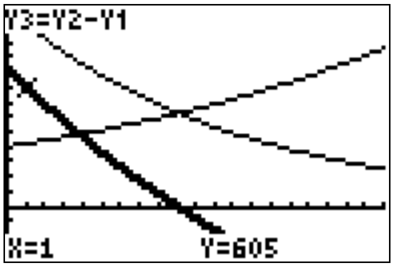

**b)** $P(t)$: $\{t \in \mathbb{R}\}$, $\{P \in \mathbb{R}, P \geq 0\}$
$F(t)$: $\{t \in \mathbb{R}\}$, $\{F \in \mathbb{R}, F \geq 0\}$

**c)**

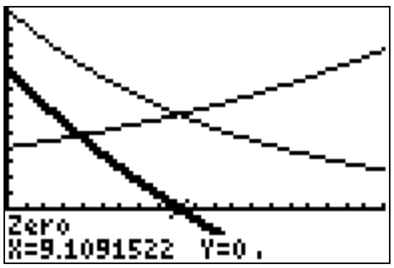



The point of intersection is approximately (9.11, 467.88).
In 9.11 years, the number of fish and the amount fish food both equal 467.88.

**d)** Answers may vary. A sample solution is shown.
This function is the amount of fish food minus the number of fish. When the function is positive, there is a surplus of food. When the function is negative, there is not enough food.




**e)**

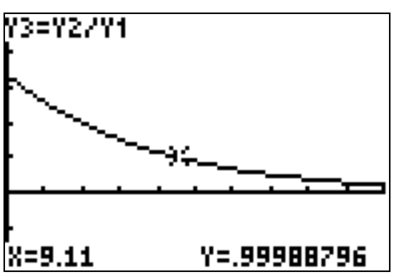



The $t$-intercept is approximately 9.11. It is the same as the crisis point.

**f)** Answers may vary. A sample solution is shown.
$P(t)$ should start to decrease since the amount of food is decreasing.
The graphs might level off at the crisis point.

**Chapter 8 Section 2** **Question 7 Page 436**

**a)** Answers may vary. A sample solution is shown.
This function represents the ratio of food to fish. If the function is greater than one, there is more than enough food. The graph is decreasing.

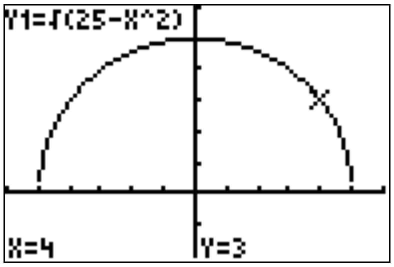



**b)** The coordinates at the crisis point are approximately (9.11, 1). This means that after 9.11 years, the amount of food is equal to the number of the fish.

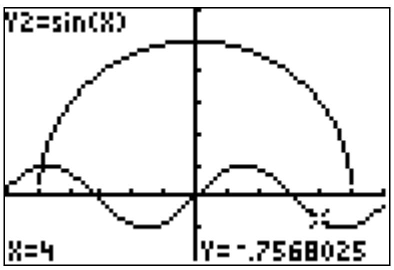
**c)** Before the crisis point there is plenty of food. At the crisis point there is exactly enough food. After the crisis point there is not enough food.

**Chapter 8 Section 2** **Question 8 Page 436**

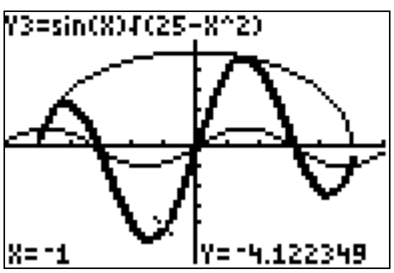
**a)** The shape of the graph is a semi-circle. The graph is even since it is symmetric about the $y$-axis.



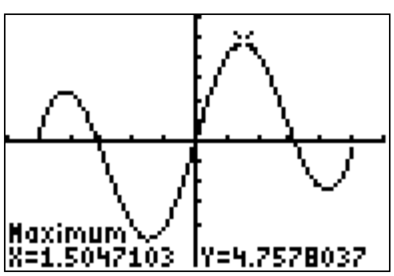

**b)** The graph of $g(x)$ is odd since it is symmetric about the origin.




**c)** Predictions will vary. Sample prediction: The graph will be sinusoidal and have limited domain and range. It will be odd.




**d)**




Domain: $\{x \in \mathbb{R}, -5 \le x \le 5\}$; Range: $\{y \in \mathbb{R}, -4.76 \le x \le 4.76\}$
Estimate since the maximum occurs at an unknown $x$-value.

**Chapter 8 Section 2** **Question 9 Page 436**

**a)** $y = \dfrac{\sin x}{\sqrt{25 - x^2}}$

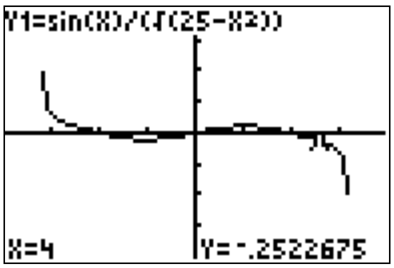
$25 - x^2 > 0$

$25 > x^2$

$-5 < x < 5$

The function is odd since it is symmetric about the origin.

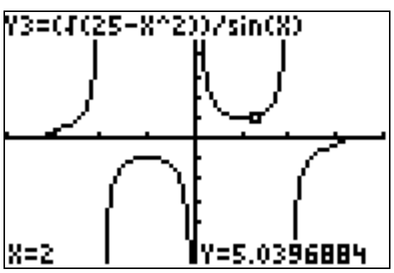
Domain: $\{x \in \mathbb{R}, -5 < x < 5\}$; Range: $\{y \in \mathbb{R}\}$




**b)** $y = \dfrac{\sqrt{25 - x^2}}{\sin x}$, $\sin x \neq 0$, $x \neq -\pi, x \neq 0, x \neq \pi$

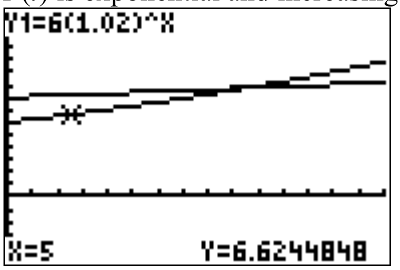
The function is odd since it is symmetric about the origin.

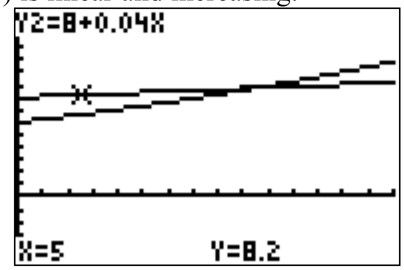
Domain: $\{x \in \mathbb{R}, -5 \le x \le 5, x \neq -\pi, x \neq 0, x \neq \pi\}$; Range: $\{y \in \mathbb{R}\}$



**Chapter 8 Section 2** **Question 10 Page 437**

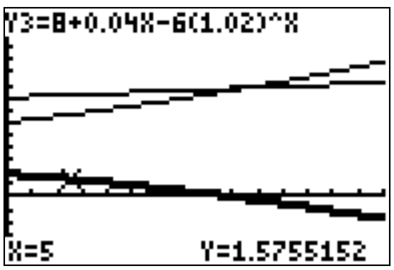
**a)** $P(t)$ is exponential and increasing, while $F(t)$ is linear and increasing.



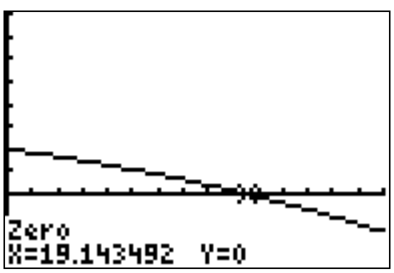




**b)** $y = 8 + 0.04t - 6(1.02)^t$

Answers may vary. A sample solution is shown.
In 2008, there is a surplus, since the function is positive.
After 2019, there will be a food shortage.




Answers may vary. A sample solution is shown.

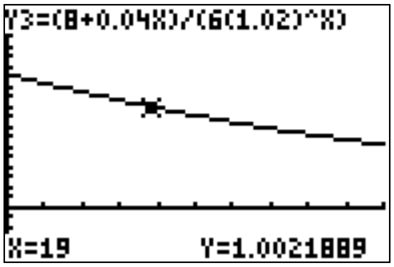



After about 19 years there will be a food shortage.

**c)** (0, 2); In 2000, the maximum is 2, which is the amount of the surplus of food.

**Chapter 8 Section 2** **Question 11 Page 437**

**a)** The function is decreasing.




**b)** The food production is a maximum in 2000. This is the same as in 10c).

**c)** When $\frac{F(t)}{P(t)} > 1$, there is a surplus of food. When $\frac{F(t)}{P(t)} < 1$, there is a shortage of food. For Terra, there is a surplus of food before 2019 and a shortage of food after 2019.

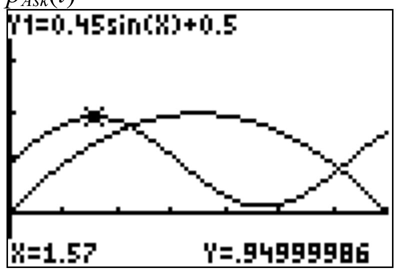
**Chapter 8 Section 2** **Question 12 Page 437**

Answers may vary. A sample solution is shown.
Trade the surplus now to save for the future shortage or to improve the food production plan.

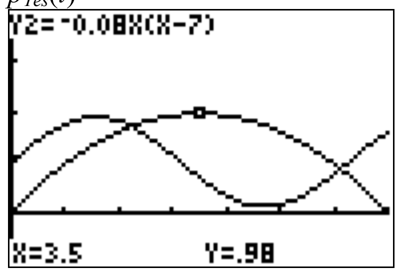
**Chapter 8 Section 2** **Question 13 Page 437**

**a)** $p_{Ask}(t)$ is periodic; $\{t \in \mathbb{R}, t \geq 0\}, \{p_{Ask} \in \mathbb{R}, 0.05 \leq p_{Ask} \leq 0.95\}$
$p_{Yes}(t)$ is quadratic, increasing and then decreasing;
$\{t \in \mathbb{R}, 0 \leq t \leq 7\}, \{p_{Yes} \in \mathbb{R}, 0 \leq p_{Yes} \leq 0.98\}$

$p_{Ask}(t)$




$p_{Yes}(t)$



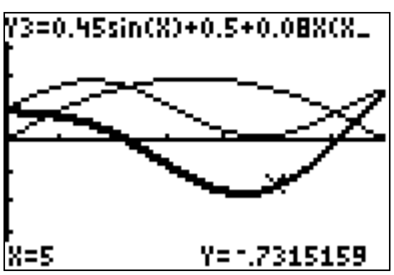

**b)** $\dfrac{(4n-3)\pi}{2}$ days after the dance

At the maximum values of $p_{Ask}(t)$, which occur at $\dfrac{\pi}{2}, \dfrac{5\pi}{2}, \dfrac{9\pi}{2}, ....$ days after the dance.

Carlos is most likely to ask Keiko to dance approximately 1.6, 7.9, 14.1, …. days after the dance.

**c)** At the maximum value of $p_{Yes}(t)$, which occurs approximately 3.5 days after the dance.

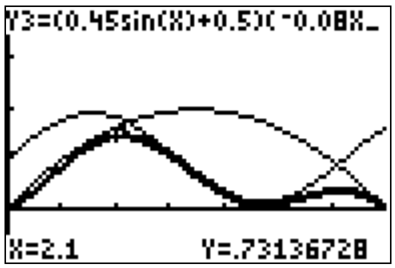
**d)** Answers may vary. A sample solution is shown.
The zeros of this graph are the points of intersection of the graphs $p_{Yes}(t)$ and $p_{Ask}(t)$
These are the points at which the probabilities are equally likely.
Above the $x$-axis, Carlos is more likely to ask than Keiko is likely to say yes.
Below the $x$-axis, Carlos is less likely to ask than Keiko is likely to say yes.



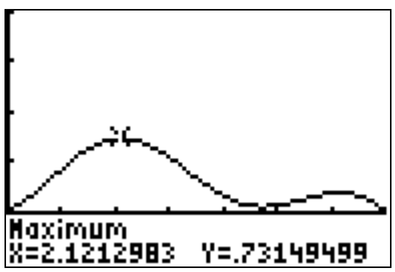
**a)** $y = (0.45\sin t + 0.5)\left[-0.08t(t-7)\right]$

$y = (0.45\sin t + 0.5)(-0.08t^2 + 0.56t)$

$y = -0.036t^2 \sin t + 0.252t\sin t - 0.04t^2 + 0.28t$

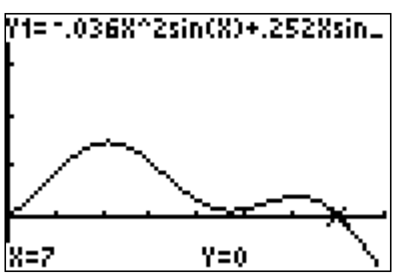



**b)**




There is approximately a 73% chance of Carlos and Keiko agreeing to date 2.1 days after the dance.

**c)**




Carlos has no chance of dating Keiko after 7 days.

**d)** Answers may vary. A sample solution is shown.
Force Carlos to ask Keiko out after 3.5 days, when she is most likely to say yes.

**Chapter 8 Section 2** **Question 15 Page 438**

Yes, the product of functions does commute.
Examples may vary. A sample solution is shown.

$$f(x)=5x-2,\ g(x)=-3x+1$$
$$\begin{aligned} f(x)g(x)&=(5x-2)(-3x+1)\\ &=-15x^2+11x-2\\ g(x)f(x)&=(-3x+1)(5x-2)\\ &=-15x^2+11x-2\\ f(x)g(x)&=g(x)f(x) \end{aligned}$$

$$f(x)=a,\ g(x)=b$$
$$f(x)g(x)=a\times b$$
$$g(x)f(x)=b\times a$$
$$a\times b=b\times a,\ a,\ b\in\mathbb{R}$$

**Chapter 8 Section 2** **Question 16 Page 438**

No, the division of two functions does not commute.
Examples may vary. A sample solution is shown.

$$f(x)=x^2-9,\ g(x)=x+3$$
$$\begin{aligned} \frac{f(x)}{g(x)}&=\frac{x^2-9}{x+3}\\ &=\frac{(x-3)(x+3)}{x+3}\\ &=x-3,\ x\neq-3\\ \frac{g(x)}{f(x)}&=\frac{x+3}{x^2-9}\\ &=\frac{x+3}{(x-3)(x+3)}\\ &=\frac{1}{x-3},\ x\neq-3,\ x\neq3\\ \frac{f(x)}{g(x)}&\neq\frac{g(x)}{f(x)} \end{aligned}$$

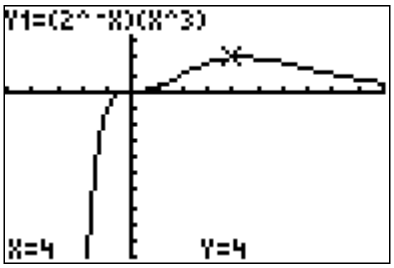
**Chapter 8 Section 2** **Question 17 Page 438**

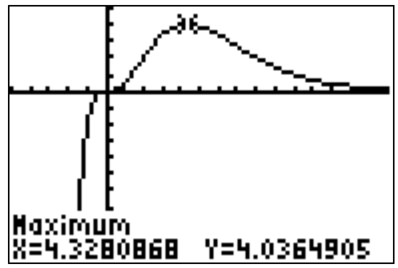
$x$-intercept 0, $y$-intercept 0
horizontal asymptote $y = 0$
For $x < 0$ the function is negative and increasing.
For $0 < x < 4.33$ the function is positive and increasing.
For $x > 4.33$ the function is positive and decreasing.

as $x \to \infty, y \to 0$

as $x \to -\infty, y \to -\infty$

Domain: $\{x \in \mathbb{R}\}$; Range: $\{y \in \mathbb{R}, y \leq 4.04\}$




**Chapter 8 Section 2** **Question 18 Page 438**

Solutions to Achievement Check questions are provided in the Teacher's Resource.

**Chapter 8 Section 2** **Question 19 Page 438**

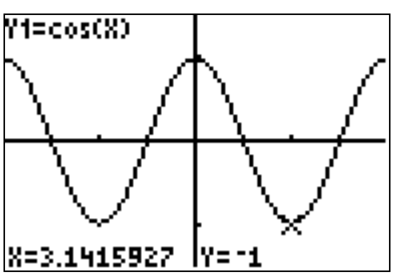
**a)**, **c)** **i)** The square of $f(x)$ is even, since the product of two even functions is even. Examples may vary. A sample solution is shown.

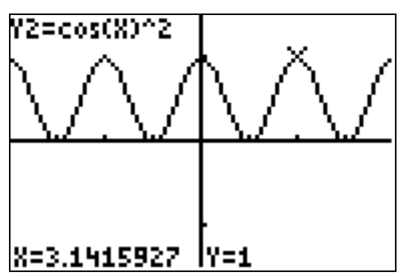
$f(x) = x^{2k}$, $k \in \mathbb{Z}$ is an even function

$$\left[f(x)\right]^2 = \left[x^{2k}\right]^2$$
$$= x^{4k}, k \in \mathbb{Z} \text{ is an even function}$$

$f(x) = \cos x$ is an even function

$\left[f(x)\right]^2 = \cos^2 x$ is an even function



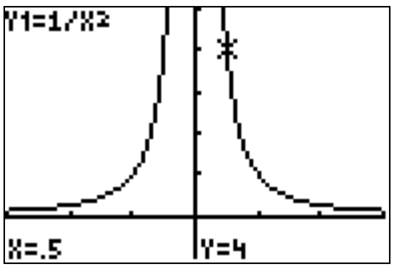
**ii)** The cube of $f(x)$ is even, since the product of three even functions is even. Examples may vary. A sample solution is shown.

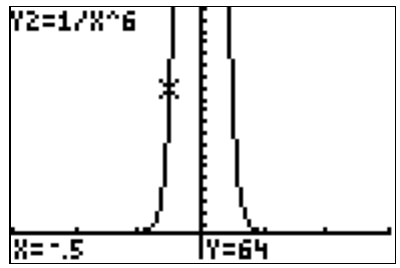
$$f(x) = x^{2k},\ k \in \mathbb{Z} \text{ is an even function}$$

$$\begin{aligned}\left[f(x)\right]^3 &= \left[x^{2k}\right]^3 \\ &= x^{6k},\ k \in \mathbb{Z} \text{ is an even function}\end{aligned}$$

$$f(x) = \frac{1}{x^2}, \text{ is an even function}$$

$$\left[f(x)\right]^3 = \frac{1}{x^6}, \text{ is an even function}$$




**a), c)** **iii)** The $n$th power of $f(x)$ is even, since the product of $n$ even functions is even. Examples may vary. A sample solution is shown.

$$f(x) = x^{2k},\ k \in \mathbb{Z} \text{ is an even function}$$

$$\begin{aligned}\left[f(x)\right]^n &= \left[x^{2k}\right]^n \\ &= \left(x^{kn}\right)^2,\ n,\ k \in \mathbb{Z} \text{ is an even function}\end{aligned}$$

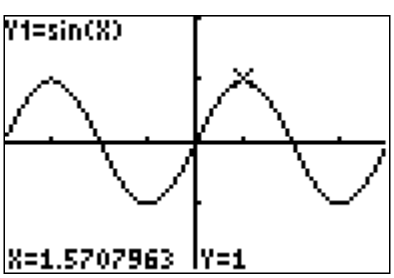
**b), c)** **i)** The square of $f(x)$ is even, since the product of two odd functions is even. Examples may vary. A sample solution is shown.

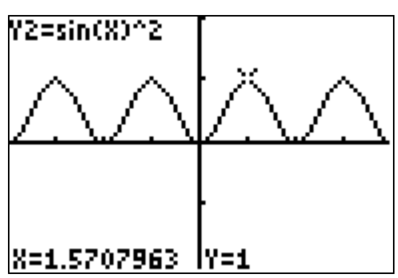
$$f(x) = x^{2k+1},\ k \in \mathbb{Z} \text{ is an odd function}$$

$$\begin{aligned}\left[f(x)\right]^2 &= \left[x^{2k+1}\right]^2 \\ &= x^{4k+2},\ k \in \mathbb{Z} \text{ is an even function}\end{aligned}$$

$$f(x) = \sin x \text{ is and odd function}$$

$$\left[f(x)\right]^2 = \sin^2 x \text{ is an even function}$$



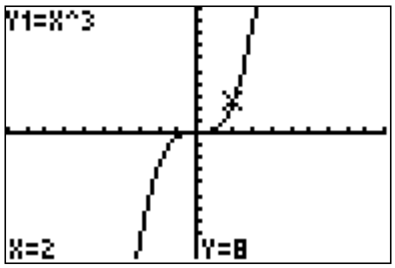
**b), c)** **ii)** The cube of $f(x)$ is odd, since the product of three odd functions is odd.
Examples may vary. A sample solution is shown.

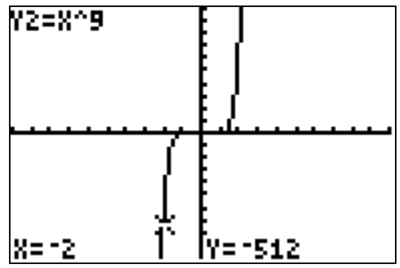
$$f(x) = x^{2k+1},\ k \in \mathbb{Z} \text{ is an odd function}$$

$$\begin{aligned}\left[f(x)\right]^3 &= \left[x^{2k+1}\right]^3 \\ &= x^{6k+3},\ k \in \mathbb{Z} \text{ is an odd function}\end{aligned}$$

$f(x) = x^3$ is an odd function

$\left[f(x)\right]^3 = x^9$ is and odd function




**b)** **iii)** $[f(x)]^n$ is even if $n$ is even and $[f(x)]^n$ is odd when $n$ is odd.
Examples may vary. A sample solution is shown.

$$f(x) = x^{2k+1},\ k \in \mathbb{Z} \text{ is an odd function}$$

$$\begin{aligned}\left[f(x)\right]^n &= \left[x^{2k+1}\right]^n \\ &= x^{2nk+n},\ k,\ n \in \mathbb{Z}\end{aligned}$$

If $n$ is odd, then $f(x)$ is odd

If $n$ is even, then $f(x)$ is even

**Chapter 8 Section 2** **Question 20 Page 438**

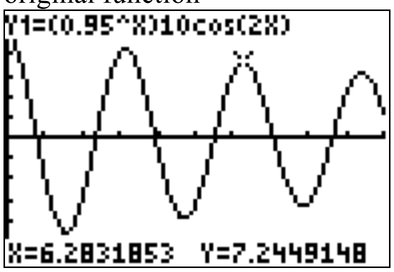
**a)** period = $\dfrac{2\pi}{2}$ or simply $\pi$

$L = \dfrac{g}{4\pi^2}T^2$, where $L$ is the length of the pendulum, $g = 9.8$ is the force of gravity,
T is the period

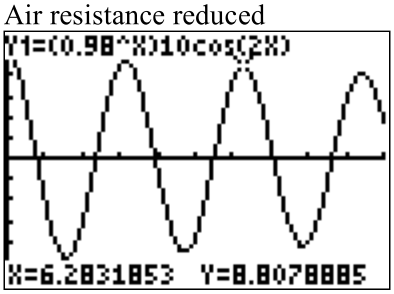
$L = \dfrac{9.8}{4\pi^2}(\pi^2)$
$= 2.45$

The length of the pendulum is 2.45 m.

**b)** original function

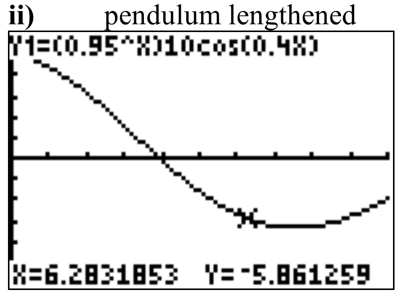



**i)** Air resistance reduced




The amplitude is greater.

**ii)** pendulum lengthened




The period is longer.

**Chapter 8 Section 2** **Question 21 Page 438**

$x + \dfrac{xy}{100} = 12$ $\qquad x - \dfrac{xy}{100} = 8$

$100x + xy = 1200$ $\qquad 100x - xy = 800$

$$\begin{aligned}100x + xy &= 1200\\ 100x - xy &= 800\\ \hline 200x \quad &= 2000\\ x &= 10\end{aligned}$$

**Chapter 8 Section 2** **Question 22 Page 438**

| | | |
|---|---|---|
| $l_1$ | 7 | 15 |
| $l_2$ | 4 | $x$ |
| | $l_3$ | $l_4$ |

$$l_1 \times l_3 = 7$$

$$l_1 \times l_4 = 15 \Rightarrow l_4 = \frac{15}{l_1}$$

$$l_2 \times l_3 = 4 \Rightarrow l_2 = \frac{4}{l_3}$$

$$l_2 \times l_4 = x$$

$$x = \frac{4}{l_3} \times \frac{15}{l_1} \quad \text{substitute } l_2 = \frac{4}{l_3} \text{ and } l_4 = \frac{15}{l_1}$$

$$x = \frac{60}{l_3 \times l_1}$$

$$x = \frac{60}{7}$$

**Chapter 8 Section 2** **Question 23 Page 438**

$$f(xy) = \frac{f(x)}{y}$$

$$\text{let } x = 500, y = \frac{6}{5}$$

$$\begin{aligned} f(600) &= f\left(500 \times \frac{6}{5}\right) \\ &= \frac{f(500)}{\frac{6}{5}} \\ &= \frac{3}{\frac{6}{5}} \\ &= 3 \times \frac{5}{6} \\ &= \frac{5}{2} \end{aligned}$$

$$
\begin{aligned}
V_c &= 9\times 11\times 38.5 \quad \text{Volume of the container}\\
&= 3811.5\\
3811.5 &= 1.1V_w\\
V_w &= 3465 \qquad \text{Volume of the water}\\
h_w &= \frac{3465}{9\times 11}\\
h_w &= 35 \qquad \text{Height of the water}
\end{aligned}
$$

The water should be filled to a depth of 35 cm so the ice does not rise above the top of the container.

**Chapter 8 Section 3** **Question 1 Page 447**

**a)**
$$y=-(x+3)^2+2$$
$$y=-(x^2+6x+9)+2$$
$$y=-x^2-6x-7$$

**b)**
$$y=\left[(-x+2)+3\right]^2$$
$$y=(-x+5)^2$$
$$y=x^2-10x+25$$

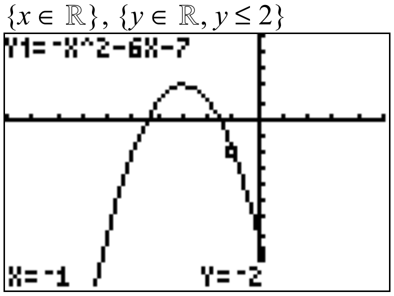
**c)**
$$y=-(-x+2)+2$$
$$y=x$$

**d)**
$$y=\left[(x+3)^2+3\right]^2$$
$$y=(x^2+6x+9+3)^2$$
$$y=(x^2+6x+12)^2$$
$$y=x^4+6x^3+12x^2+6x^3+36x^2+72x+12x^2+72x+144$$
$$y=x^4+12x^3+60x^2+144x+144$$

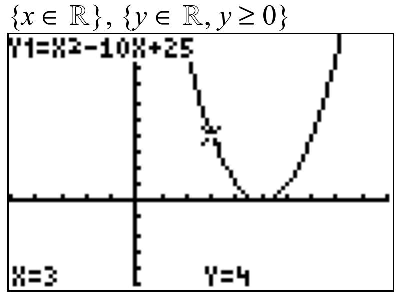
**e)**
$$x=-y+2$$
$$y=2-x$$
$$f^{-1}(x)=2-x$$
$$f^{-1}\left(f(x)\right)=2-(-x+2)$$
$$=x$$

**Chapter 8 Section 3** **Questions 2 and 3 Page 447**

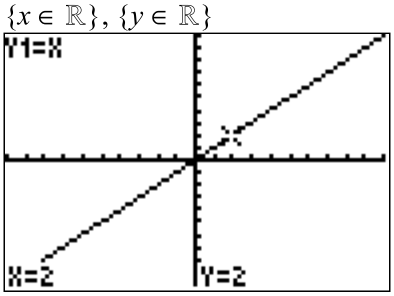
**a)** $\{x\in\mathbb{R}\}, \{y\in\mathbb{R}, y\le 2\}$



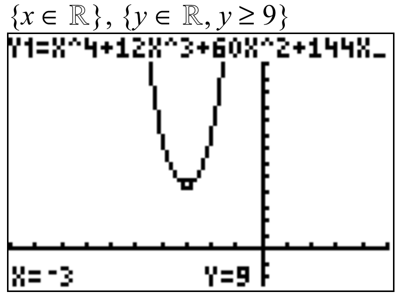

**b)** $\{x\in\mathbb{R}\}, \{y\in\mathbb{R}, y\ge 0\}$



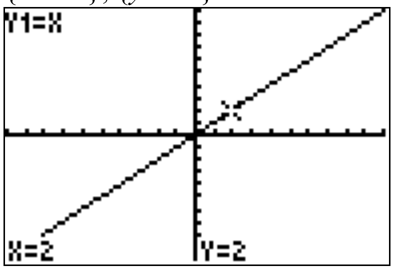

**c)** $\{x\in\mathbb{R}\}, \{y\in\mathbb{R}\}$




**d)** $\{x\in\mathbb{R}\}, \{y\in\mathbb{R}, y\ge 9\}$



**e)** $\{x \in \mathbb{R}\}$, $\{y \in \mathbb{R}\}$




**Chapter 8 Section 3** **Question 4 Page 447**

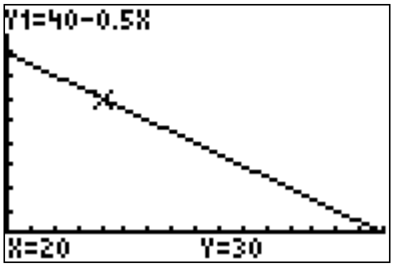
**a)**
$$g(f(x)) = \frac{1}{\left(x^2+2x-4\right)+1}$$
$$= \frac{1}{x^2+2x-3}$$
$$g(f(0)) = \frac{1}{0^2+2(0)-3}$$
$$= -\frac{1}{3}$$

**b)**
$$g(-2) = \frac{1}{-2+1}$$
$$= -1$$
$$f(g(-2)) = (-1)^2+2(-1)-4$$
$$= 1-2-4$$
$$= -5$$

**c)**
$$g(f(x)) = \frac{1}{x^2+2x-3}$$
$$g(f(1)) = \frac{1}{1^2+2(1)-3}$$
$$= \frac{1}{0}, \text{ undefined}$$
$$g(f(-3)) = \frac{1}{(-3)^2+2(-3)-3}$$
$$= \frac{1}{0}, \text{ undefined}$$

**Chapter 8 Section 3** **Question 5 Page 447**

**a)** $B(t)$ is decreasing at a constant rate.



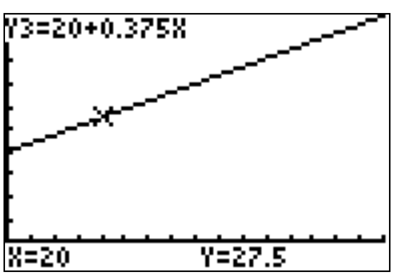

**b)** $B(0) = 40 - 0.5(0)$
$= 40$
The Blue party has 40% popularity at the beginning of the campaign.

**c)** From the equation $B(t) = 40 - 0.5t$, the slope is –0.5.
The popularity of the Blue party is decreasing by 0.5%/day.

**d)** Answers may vary. A sample solution is shown.
No, you cannot tell if these are the only two parties in the election. The Blue party starts off with 40% and the Red party with 20%, there is a good possibility that there are more than two parties. Knowing the percent of undecided voters would help.

**Chapter 8 Section 3** **Question 6 Page 447**

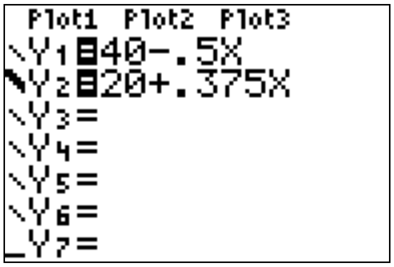
**a)** $R(t) = 20 + 0.75\left[40 - (40 - 0.5t)\right]$
$R(t) = 20 + 0.75(0.5t)$
$R(t) = 20 + 0.375t$
The graph is increasing at a constant rate.

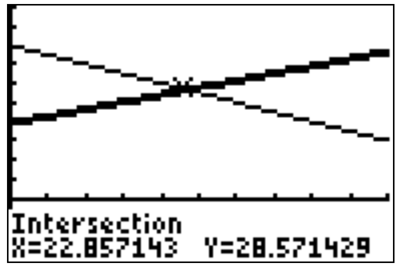



**b)** $R(0) = 20 + 0.375(0)$
$= 20$
The Red party has 20% popularity at the beginning of the campaign.

**c)** From the equation $R(t) = 20 + 0.375t$, the slope is 0.375.
The popularity of the Red party is increasing by 0.375%/day.

**d)**

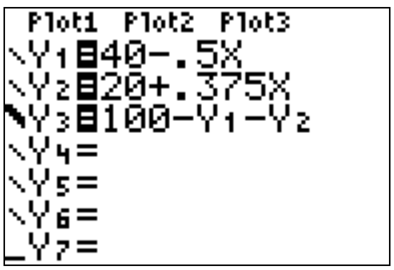


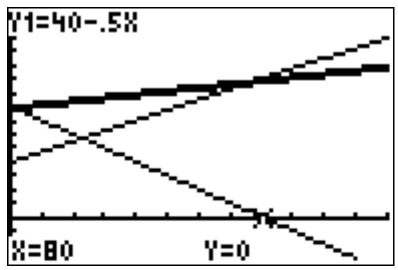



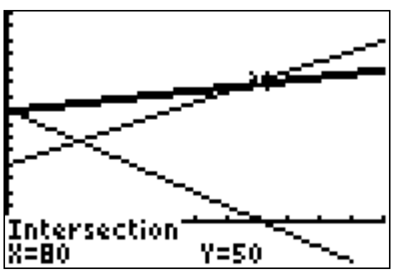

Answers may vary. A sample solution is shown.
If the election is held before 22.9 days, the Blue party will win. If the election is held after 22.9 days, the Red party will win. 22.9 days is approximately when the two functions intersect.

**Chapter 8 Section 3** **Question 7 Page 447**

**a)**




$V(t)$ represents the voters not decided on the Red or Blue parties. The graph is increasing at a constant rate.

**b)** See graphs in part a).
Answers may vary. A sample solution is shown.
The third party will win if the election is held before 80 days. At 80 days, both the red party and the third party have 50% of the vote, while the Blue Party has 0% of the vote.

**c)** Answers may vary. A sample solution is shown.
If there were a fourth party, it would split the popularity in $V(t)$, and we could not tell who will win at any point.

**Chapter 8 Section 3** **Question 8 Page 447**

No, $f(g(x)) = g(f(x))$ is not true for all functions $f(x)$ and $g(x)$.
Examples may vary. A sample solution is shown.
$f(x) = x + 1$, $g(x) = 2x$

$$\begin{aligned} f(g(x)) &= 2x+1 \\ g(f(x)) &= 2(x+1) \\ &= 2x+2 \\ f(g(x)) &\neq g(f(x)) \end{aligned}$$

**Chapter 8 Section 3** **Question 9 Page 447**

**a)** $$\begin{aligned} x &= y^3 \\ y &= x^{\frac{1}{3}} \end{aligned}$$

**b)** $$\begin{aligned} f(f^{-1}(x)) &= \left(x^{\frac{1}{3}}\right)^3 \\ &= x \end{aligned}$$

**c)** $$\begin{aligned} f^{-1}(f(x)) &= \left(x^3\right)^{\frac{1}{3}} \\ &= x \end{aligned}$$

**d)** The answers are the same.

**e)** $f(f^{-1}(3)) = 3$ $f(f^{-1}(5)) = 5$ $f(f^{-1}(-1)) = -1$

$f(f^{-1}(x)) = x$ for all values of $x$.

**Chapter 8 Section 3** **Question 10 Page 448**

Answers may vary. A sample solution is shown.

Let $f(x) = \frac{1}{x+2}$ $\qquad g(x) = x - 9$

$$x = \frac{1}{y+2} \qquad x = y - 9$$

$$y + 2 = \frac{1}{x} \qquad y = x + 9$$

$$y = \frac{1}{x} - 2 \qquad g^{-1}(x) = x + 9$$

$$f^{-1}(x) = \frac{1}{x} - 2$$

$$\begin{aligned} f\left(f^{-1}(x)\right) &= \frac{1}{\left(\frac{1}{x} - 2\right) + 2} \\ &= \frac{1}{\frac{1}{x}} \\ &= x \end{aligned} \qquad \begin{aligned} g\left(g^{-1}(x)\right) &= x + 9 - 9 \\ &= x \end{aligned}$$

$$\begin{aligned} f^{-1}\left(f(x)\right) &= \frac{1}{\frac{1}{x+2}} - 2 \\ &= x + 2 - 2 \\ &= x \end{aligned} \qquad \begin{aligned} g^{-1}\left(g(x)\right) &= x - 9 + 9 \\ &= x \end{aligned}$$

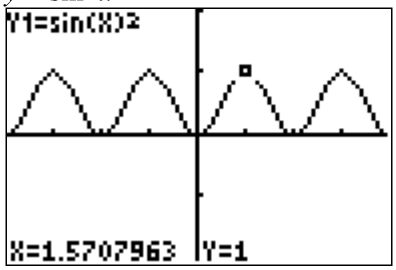
Yes.

$f(f^{-1}(x)) = x$ for all functions whose inverses are functions

$f^{-1}(f(x)) = x$ for all functions whose inverses are functions

**Chapter 8 Section 3** **Question 11 Page 448**

**a), b)** $y = \sin^2 x$

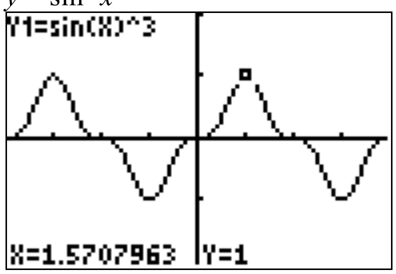



**c)** Yes, the function in part a) is periodic, since it repeats itself from left to right.

**d)** $\{x \in \mathbb{R}\}$, $\{y \in \mathbb{R}, 0 \le y \le 1\}$

**Chapter 8 Section 3** **Question 12 Page 448**

**a)** $y = \sin^3 x$




The function is periodic since it repeats itself.
$\{x \in \mathbb{R}\}$, $\{y \in \mathbb{R}, -1 \le y \le 1\}$

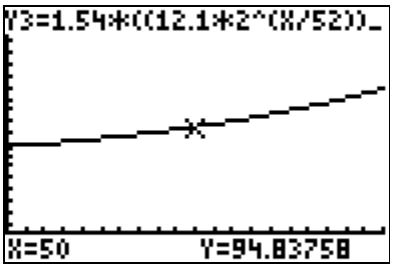
**b)** Answers may vary. A sample solution is shown.
The functions are both periodic and have a maximum value of 1. The functions differ in their minimum values (0 versus –1) and period ($\pi$ versus $2\pi$). One function is even, while the other is odd.

**Chapter 8 Section 3** **Question 13 Page 448**

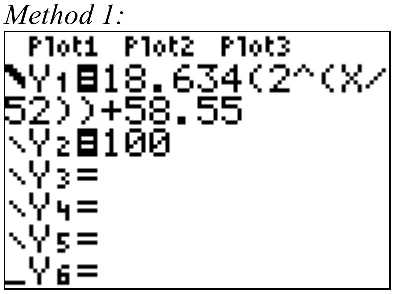
**a)** $C(t) = 1.54\left(12.1 \times 2^{\frac{t}{52}}\right) + 58.55$

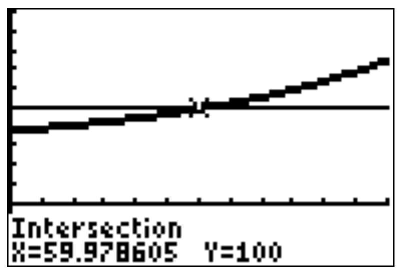
$C(t) = 18.634 \times 2^{\frac{t}{52}} + 58.55$

**b)**




**c)** *Method 1:*




It will take approximately 60 years to reach 100 ppm.

*Method 2:*

$$
\begin{aligned}
100 &= 18.634 \times 2^{\frac{t}{52}} + 58.55 \\
41.45 &= 18.634 \times 2^{\frac{t}{52}} \\
\log \frac{41.45}{18.634} &= \log 2^{\frac{t}{52}} \\
\log \frac{41.45}{18.634} &= \frac{t}{52} \log 2 \\
\frac{t}{52} &= \frac{\log \frac{41.45}{18.634}}{\log 2} \\
t &= \frac{52 \log \frac{41.45}{18.634}}{\log 2} \\
&\doteq 60.0
\end{aligned}
$$

Answers may vary. A sample solution is shown.

Let $f(x) = x + 3$ and $g(x) = 2x - 5$

$$x = y + 3 \qquad\qquad x = 2y - 5$$
$$y = x - 3 \qquad\qquad 2y = x + 5$$
$$f^{-1}(x) = x - 3 \qquad\qquad y = \frac{x+5}{2}$$
$$g^{-1}(x) = \frac{x+5}{2}$$

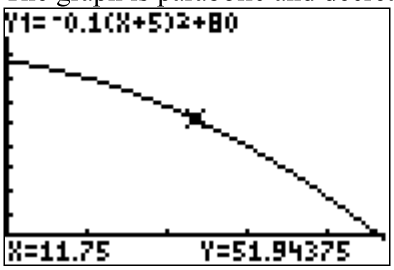
$$\begin{aligned}
\left(f \circ g\right)^{-1}(x) &= \left[f(g(x))\right]^{-1} \\
&= \left[2x - 5 + 3\right]^{-1} \\
&= \left[2x - 2\right]^{-1} \\
x &= 2y - 2 \\
2y &= x + 2 \\
y &= \frac{x+2}{2}
\end{aligned}$$

$$\begin{aligned}
g^{-1}\left(f^{-1}(x)\right) &= \frac{(x-3)+5}{2} \\
&= \frac{x+2}{2} \\
f^{-1}\left(g^{-1}(x)\right) &= \left(\frac{x+5}{2}\right) - 3 \\
&= \frac{x+5}{2} - \frac{6}{2} \\
&= \frac{x-1}{2}
\end{aligned}$$

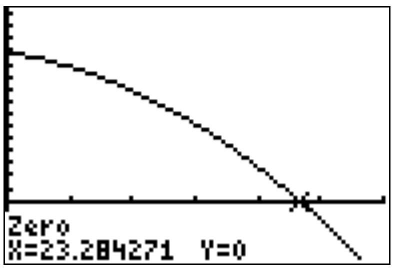
Therefore, $\left(f \circ g\right)^{-1}(x) = (g^{-1} \circ f^{-1})(x)$.

**Chapter 8 Section 3** **Question 15 Page 448**

**a)** The graph is parabolic and decreasing.



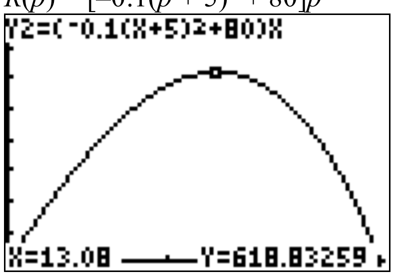

**b)**




$\{p \in \mathbb{R}, 5 \le p \le 23.28\}$

**c)** This graph does not show the optimum price since there is no maximum.

**d)** $R(p) = [-0.1(p + 5)^2 + 80]p$

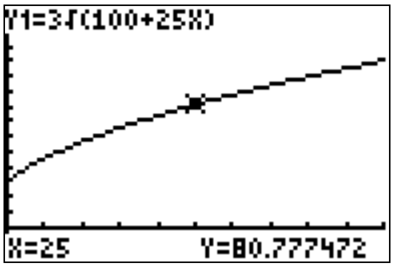



**e)** Funky Stuff should sell Funky Teddy Bears at \$13.08 each, which would bring a revenue of \$618.83.

**Chapter 8 Section 3** **Question 16 Page 448**

**a)** $W(t) = 3\sqrt{100 + 25t}$

**b)** $\{t \in \mathbb{R}, t \geq 0\}$, $\{W \in \mathbb{R}, W \geq 30\}$




**Chapter 8 Section 3** **Question 17 Page 448**

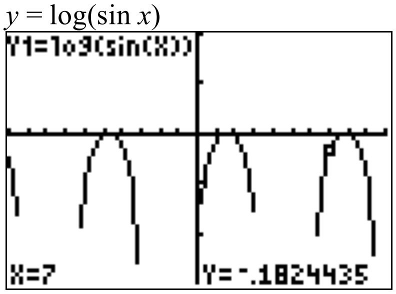
Solutions to Achievement Check questions are provided in the Teacher's Resource.

**Chapter 8 Section 3** **Question 18 Page 449**

**a)** $\{x \in \mathbb{R}, x > 0\}$

**b)** The function can only exist when $\sin x > 0$; it will be periodic and only no-positive $y$-values since $\sin x$ is never greater than 1.

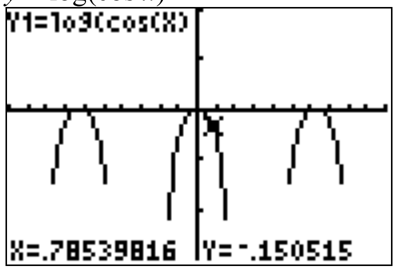
**c)** $y = \log(\sin x)$




$\{x \in \mathbb{R}, 2n\pi < x < (2n + 1)\pi, n \in \mathbb{Z}\}$, $\{y \in \mathbb{R}, y \leq 0\}$

**d)** Similar to part c), but shift $\frac{\pi}{2}$ to the left.

**e)** $y = \log(\cos x)$




$$\left\{x \in \mathbb{R},\ \frac{(4n-5)\pi}{2} < x < \frac{(4n-3)\pi}{2},\ n \in \mathbb{Z}\right\},\ \{y \in \mathbb{R},\ y \le 0\}$$

**Chapter 8 Section 3** **Question 19 Page 449**

**a)** $y = \left(\frac{1}{x}\right)^2 - 9$

$y = \frac{1}{x^2} - 9,\ x \ne 0$

To find the range find the inverse of $y = f(g(x))$.

$x = \frac{1}{y^2} - 9$

$x + 9 = \frac{1}{y^2}$

$y^2 = \frac{1}{x+9}$

$y^{-1} = \pm\sqrt{\frac{1}{x+9}},\ x > -9$

$\{x \in \mathbb{R}, x \ne 0\}, \{y \in \mathbb{R}, y > -9\}$

**b)** $y = \dfrac{1}{x^2 - 9}$, $x \neq -3$, $x \neq 3$

To find the range, find the inverse of $g(f(x))$.

$x = \dfrac{1}{y^2 - 9}$

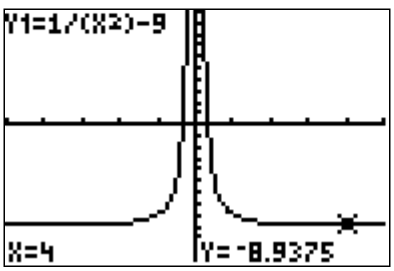
$y^2 - 9 = \dfrac{1}{x}$

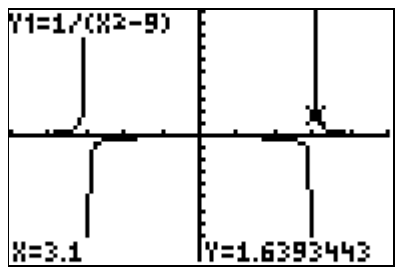
$y^2 = \dfrac{1}{x} + 9$

$y = \pm\sqrt{\dfrac{1}{x} + 9}$, $x > 0$, $x \leq -\dfrac{1}{9}$

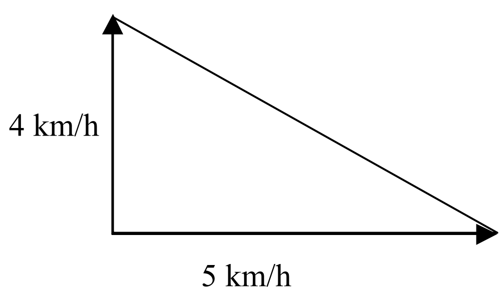
$\{x \in \mathbb{R}, x \neq -3, x \neq 3\}$, $\{y \in \mathbb{R}, y \leq -\dfrac{1}{9}, y > 0\}$

**c)**




**Chapter 8 Section 3** **Question 20 Page 449**




$d$ = speed × time

$$d(t) = \sqrt{(4t)^2 + (5t)^2}$$
$$= \sqrt{16t^2 + 25t^2}$$
$$= \sqrt{41}t$$

**Chapter 8 Section 3** **Question 21 Page 449**

$$\begin{aligned}
x &= \frac{ay+b}{cy+d} \\
cxy+dx &= ay+b \\
cxy-ay &= b-dx \\
y(cx-a) &= b-dx \\
y &= \frac{b-dx}{cx-a} \\
f^{-1}(x) &= \frac{b-dx}{cx-a}
\end{aligned}$$

**Chapter 8 Section 3** **Question 22 Page 449**

(9 + 99 + 999 + 9999 + 99 999 + 999 999 + 9 999 999 + 99 999 999 + 999 999 999) ÷ 9
= 123 456 789
It does not contain the digit 0.

**Chapter 8 Section 3** **Question 23 Page 449**

$$
\begin{aligned}
\sqrt{1-\sqrt{1-x}} &= x,\ 0 \le x \le 1 \\
\left(\sqrt{1-\sqrt{1-x}}\right)^2 &= x^2 \\
1-\sqrt{1-x} &= x^2 \\
1-x^2 &= \sqrt{1-x} \\
\left(1-x^2\right)^2 &= \left(\sqrt{1-x}\right)^2 \\
1-2x^2+x^4 &= 1-x \\
x^4-2x^2+x &= 0 \\
x(x^3-2x+1) &= 0 \\
x(x-1)(x^2+x-1) &= 0 \\
x &= 0 \\
&\text{or} \\
x &= 1 \\
&\text{or} \\
x &= \frac{-1 \pm \sqrt{1^2-4(1)(-1)}}{2(1)} \\
x &= \frac{-1 \pm \sqrt{5}}{2},\ \text{since } 0 \le x \le 1,\ x = \frac{-1-\sqrt{5}}{2} \text{ is an extraneous root}
\end{aligned}
$$

The roots are $x = 0$ or $x = 1$ or $x = \dfrac{-1+\sqrt{5}}{2}$

**Chapter 8 Section 3** **Question 24 Page 449**

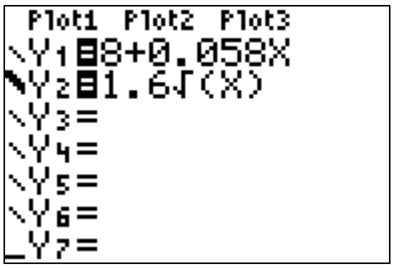
$$
\begin{aligned}
x+y &= 7 \\
xy &= 25 \\
\frac{1}{x}+\frac{1}{y} &= \frac{x}{xy}+\frac{y}{xy} \\
&= \frac{x+y}{xy} \\
&= \frac{7}{25}
\end{aligned}
$$

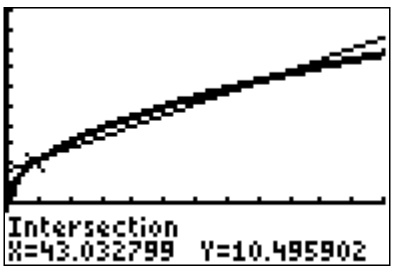
**Chapter 8 Section 3** **Question 25 Page 449**

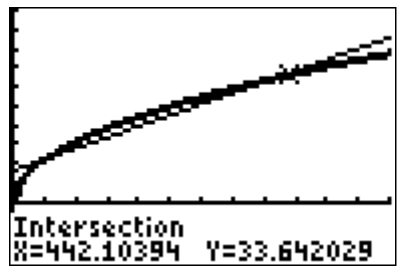
210, 321, 320, 310, 432, 431, 430, 421, 420, 410
There are 10 decreasing three-digit numbers less than 500.

**Chapter 8 Section 4** **Question 1 Page 457**

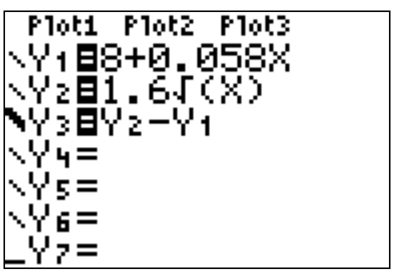
**a), b) i)** The minimum number of homes will decrease and the maximum number of homes will increase. From the intersection points, the approximate number of homes becomes $44 \le n \le 442$, $n \in \mathbb{Z}$.
(Note: This is based on building whole houses, no partial houses.)

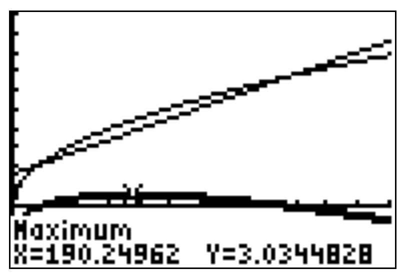



**a), b) ii)** The maximum potential profit will increase. From the maximum of the difference function, the maximum potential profit becomes approximately \$3.0 million.



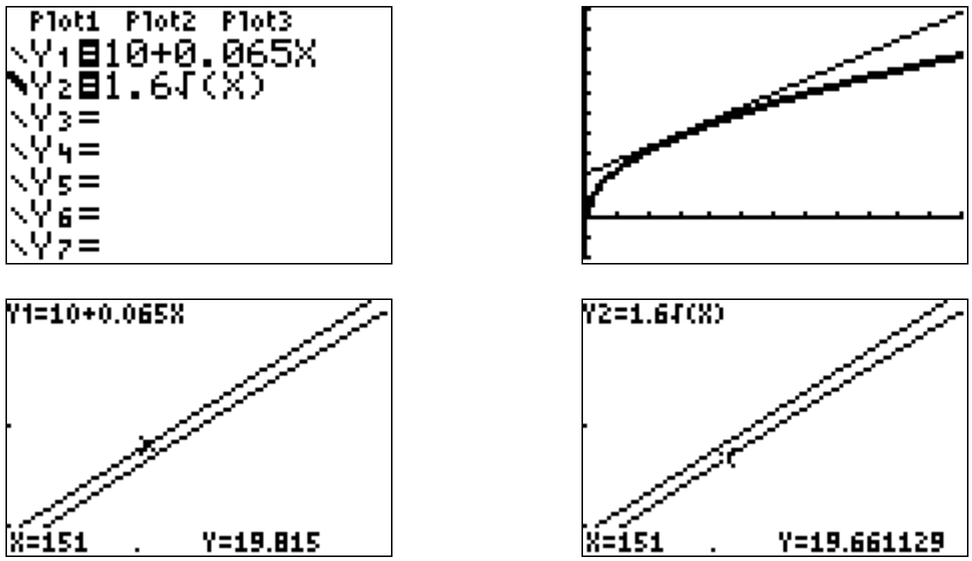



**Chapter 8 Section 4** **Question 2 Page 457**

**a) i)** There will be no minimum or maximum number of homes, since the cost is higher than the revenue.

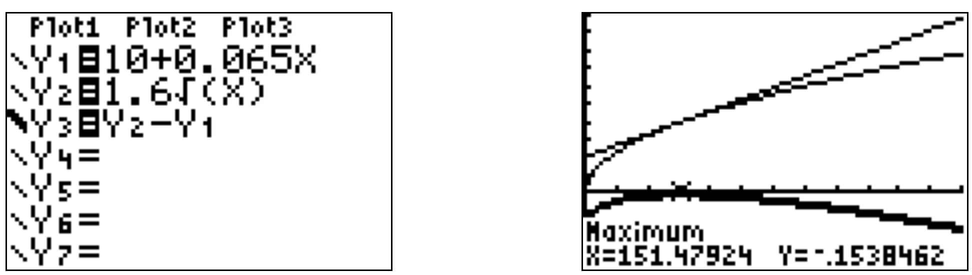
**ii)** There will be no profit, but a minimum loss of \$153 846.

**b) i)**




This is a zoomed view showing that the two graphs do not cross.

**b) ii)**



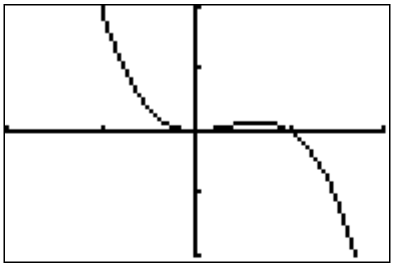

**Chapter 8 Section 4** **Question 3 Page 458**

**a)** From the graph, the points of intersection appear to be (0, 0) and (1, 1).

**i)** $f(x) > g(x)$ for $\left(-\infty, 0\right) \cup \left(0, 1\right)$

**ii)** $f(x) < g(x)$ for $(1, \infty)$

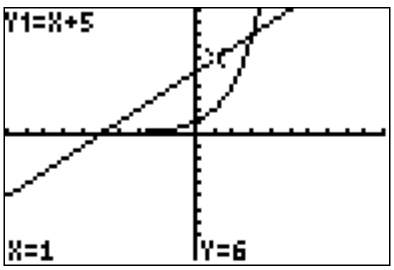
**b)**

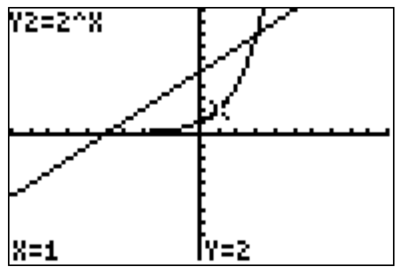


**c)** $f(x)-g(x)>0$ for $(-\infty,0)\cup(0,1)$
This is the same answer as in part a) i).

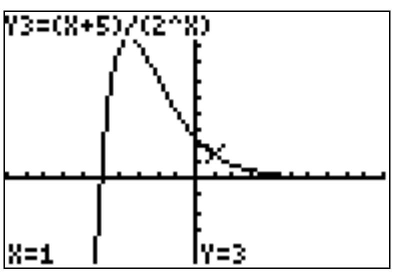
**Chapter 8 Section 4** **Question 4 Page 458**

**a)**

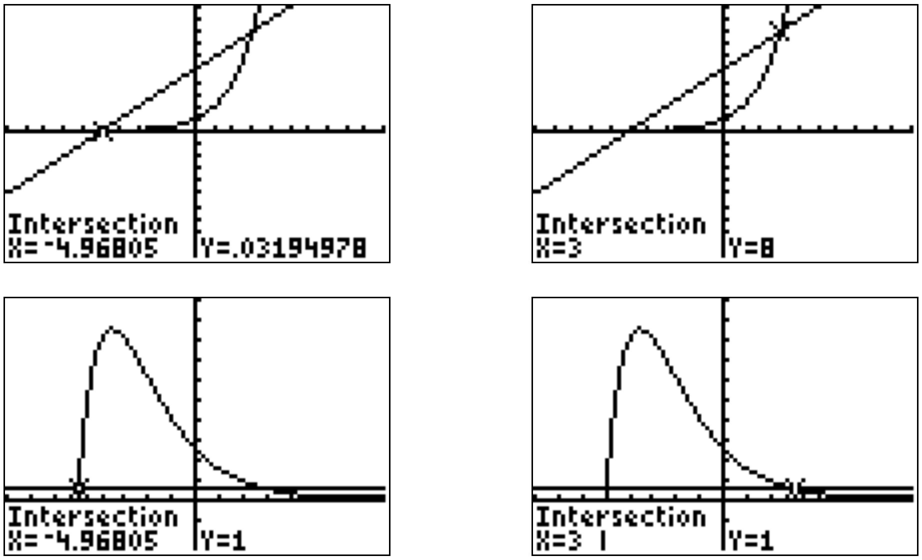



**b)**



**c)**

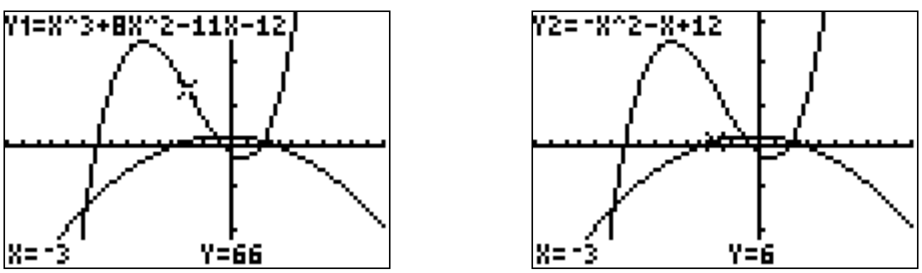



**i)** When $y = \dfrac{u(x)}{v(x)} > 1$, $u(x) > v(x)$ for approximately (–5, 3).

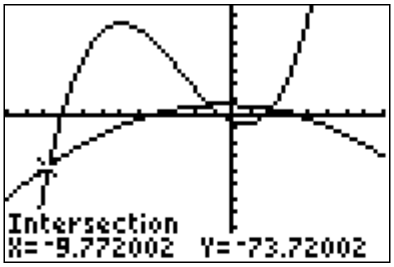
**ii)** When $y = \dfrac{u(x)}{v(x)} < 1$, $u(x) < v(x)$ for approximately $(-\infty, -5) \cup (3, \infty)$.

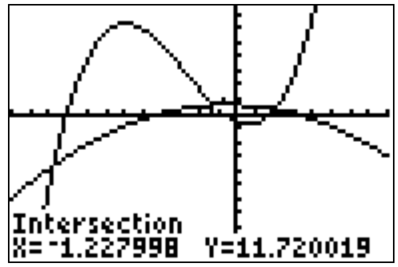
**Chapter 8 Section 4** **Question 5 Page 458**

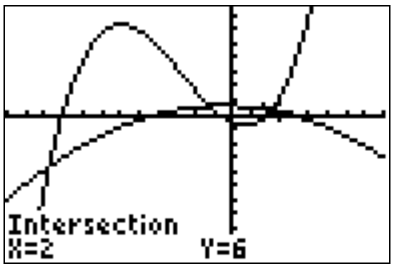
**a)**



**b)**



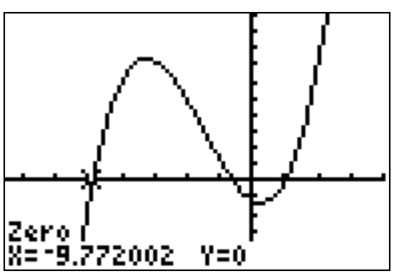



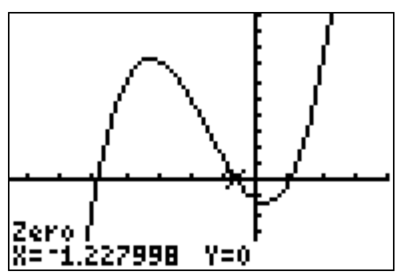

**i)** $f(x) > g(x)$ for approximately $(-9.77, -1.23) \cup (2, \infty)$.

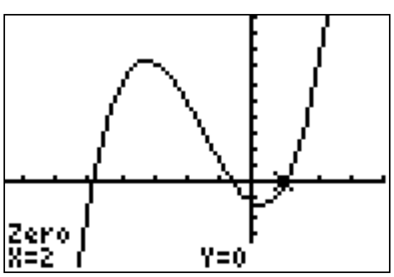
**ii)** $g(x) > f(x)$ for approximately $\left(-\infty, -9.77\right) \cup (-1.23, 2)$.

**Chapter 8 Section 4** **Question 6 Page 458**

Answers may vary. A sample solution is shown.
$y = f(x) - g(x)$
$f(x) - g(x) > 0, f(x) > g(x)$ for approximately $(-9.77, -1.23) \cup (2, \infty)$
$f(x) - g(x) < 0, g(x) > f(x)$ for approximately $\left(-\infty, -9.77\right) \cup (-1.23, 2)$



**Chapter 8 Section 4** **Question 7 Page 458**

**a)** Answers may vary. A sample solution is shown.
Subtract $g(x)$ from $f(x)$.

**b)** $y = x - (x - 2)^2$

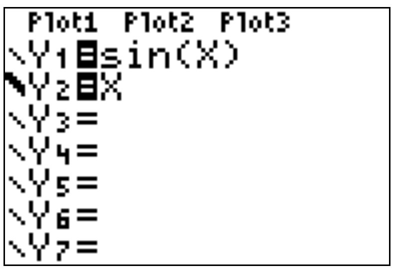
$y = x - (x^2 - 4x + 4)$

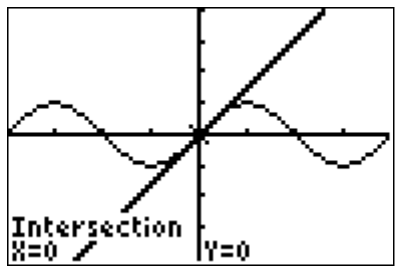
$y = x - x^2 + 4x - 4$

$y = -x^2 + 5x - 4$

**c)** Yes $y = g(x) - f(x)$ could be used to show where $f(x) > g(x)$ and $g(x) > f(x)$.
When the graph is above the $x$-axis, $g(x) > f(x)$.
When the graph is below the $x$-axis, $f(x) > g(x)$.

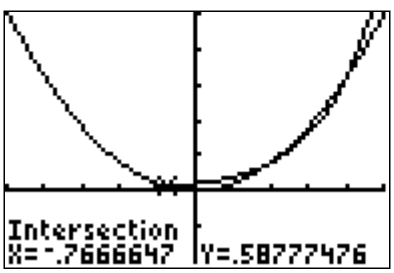
**Chapter 8 Section 4** **Question 8 Page 458**

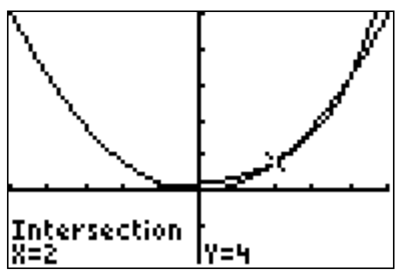


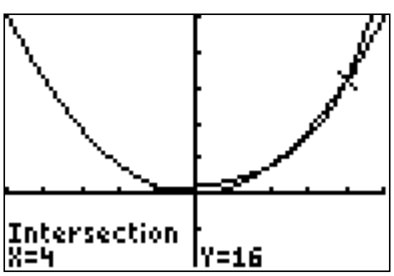




$\sin x < x$ for $(0, \infty)$.

**Chapter 8 Section 4** **Question 9 Page 458**

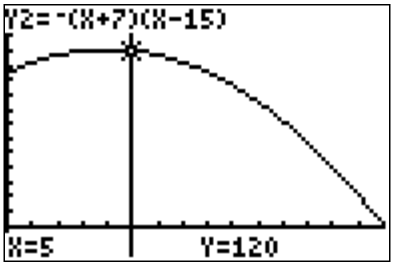


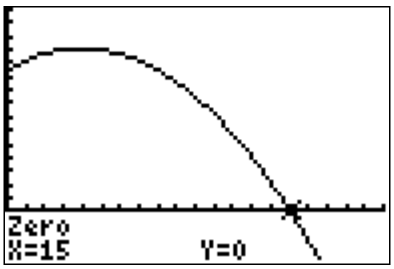




$x^2 > 2^x$, for $(-\infty, -0.77) \cup (2, 4)$

**a)**




**b)**

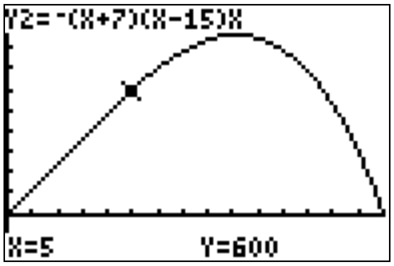



$5 \le p < 15$, note the minimum restriction on $p$

**c)** $\{p \in \mathbb{R}, 5 \le p < 15\}$, $\{N \in \mathbb{R}, 0 < N \le 120\}$

**Chapter 8 Section 4** **Question 11 Page 459**

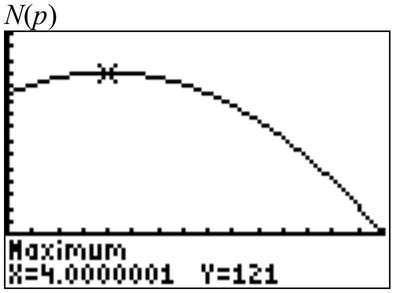
**a)**



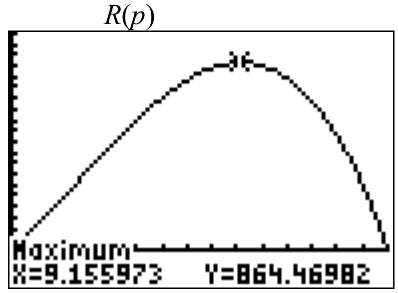

**b)** $5 \le p < 15$

This result is the same as in question 10 part b).

**c)** $N(p)$




$R(p)$



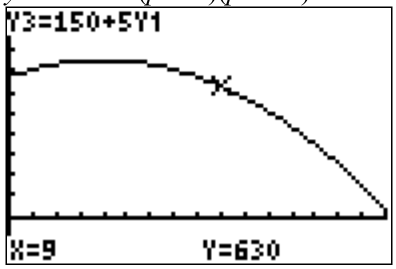

The maximum of $N(p)$ is at (4, 121), while the maximum of $R(p)$ is at approximately (9.16, 864.47). The maximum revenue occurs at a different point then the maximum number of people attending the show.

The price affects where the maximum is.

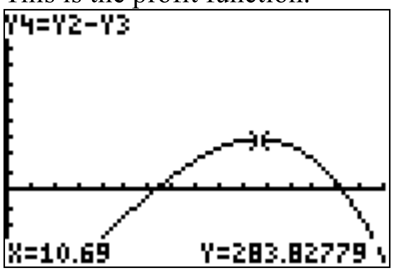
**d)** The optimum ticket price should be \$9.16 to bring in the maximum revenue of \$864.47.

**Chapter 8 Section 4** **Question 12 Page 459**

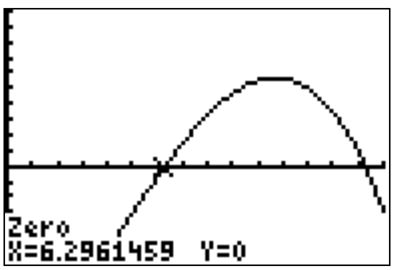
**a)** $y = 150 - 5(p + 7)(p - 15)$

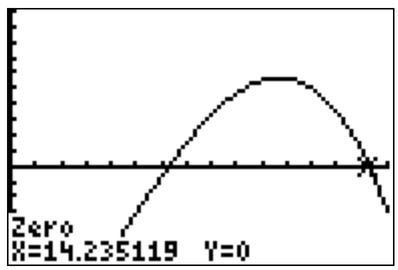



**b)** This is the profit function.




**c)**

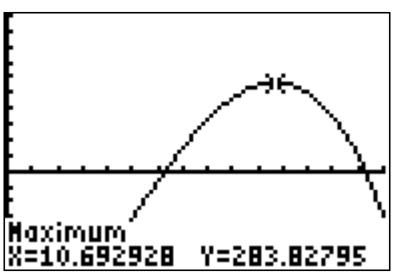



$R(p) - C(p) > 0$, for approximately (6.30, 14.24). This is when there is a profit.

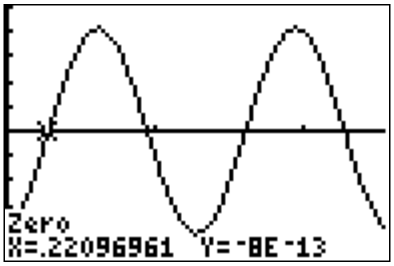
**d)** The maxima of $y = R(p)$ and $y = R(p) - C(p)$ do not occur at the same value of $p$. $C(p)$ changes the maximum in the profit function.

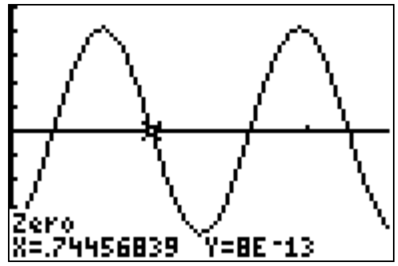
**e)**

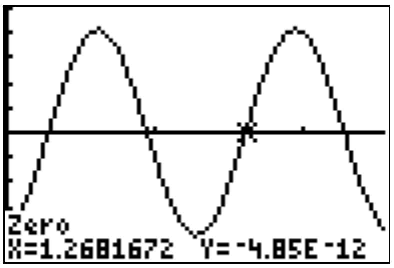


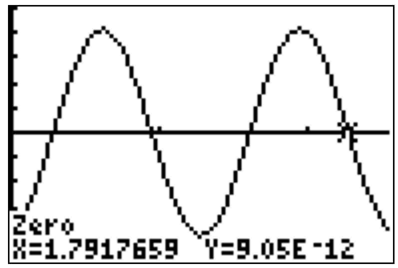

According to the graph, the maximum profit per showing is \$283.83 when the ticket price is \$10.69.

**Chapter 8 Section 4** **Question 13 Page 459**



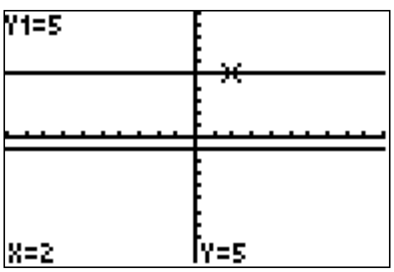




The mass is below the equilibrium position at approximately
$0 \le t < 0.22$, $0.74 < t < 1.27$, and $1.79 < t \le 2$.

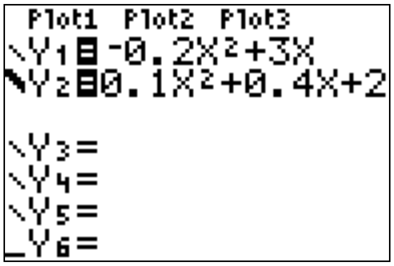
**Chapter 8 Section 4** **Question 14 Page 459**

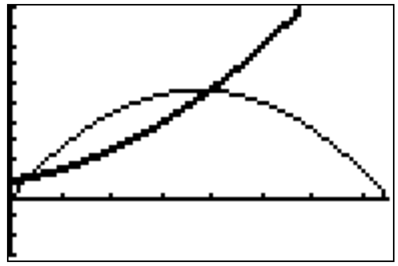
Answers may vary. A sample solution is shown.
$f(x) = 5$, $g(x) = -1$



**Chapter 8 Section 4** **Question 15 Page 459**

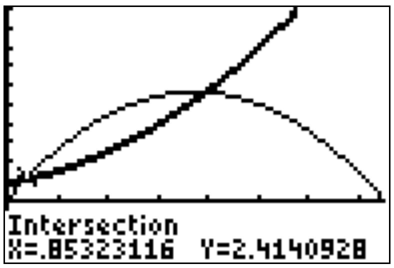
**a)**

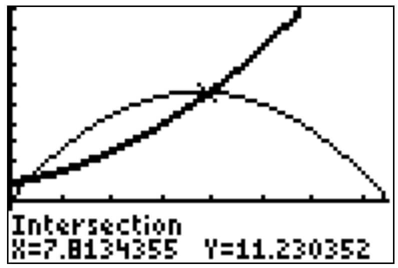


**b)** There are 2 points of intersection. They are the points where revenue equals cost.

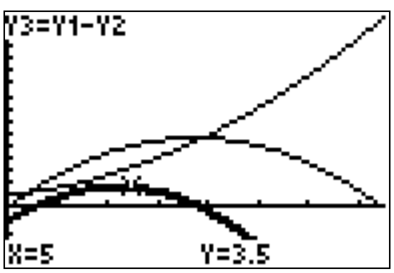
**c)**



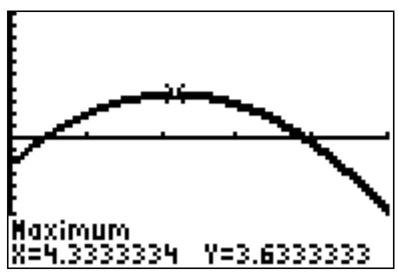




$R > C$, for $0.853 < n < 7.813$
When $R > C$ there is a profit.

**d)**




**e)**




**i)** The optimum number of units sold is approximately 4333.

**ii)** $3633 \div 4333 \doteq 0.84$
The maximum profit per unit sold is approximately \$0.84/unit.

**iii)** The total profit is approximately \$3633.

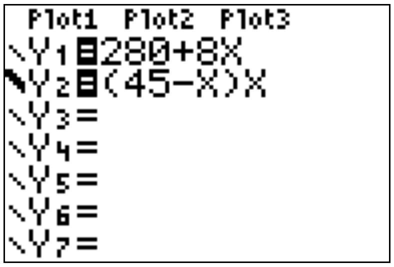
**f)** Answers may vary. A sample solution is shown.
Revenue depends on the price and quantity sold, so the result may not be linear.
Cost depends on materials, employees, and other factors which may not be linear.

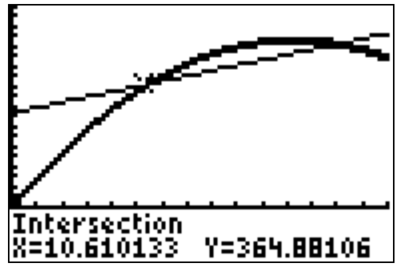
**a)** $C(n) = 280 + 8n$

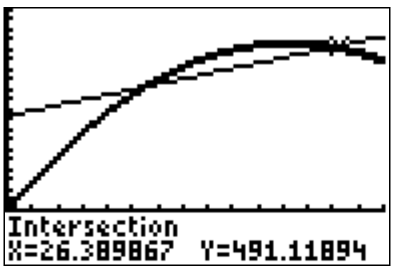
**b)** $R(n) = (45 - n)n$

**c)** $R(n) > C(n)$

**d)**

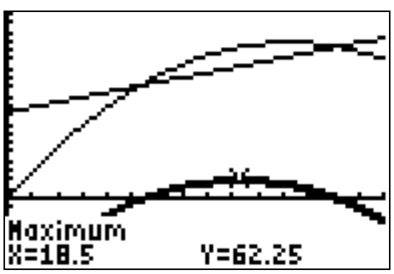


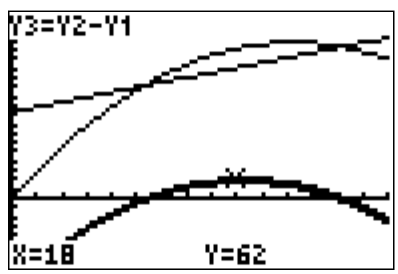



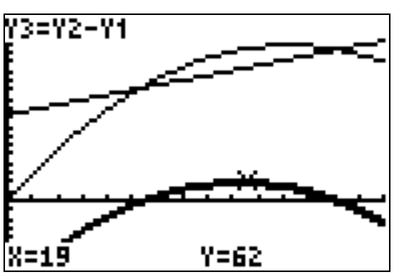




$11 \le n \le 26$
Claire should build from 11 to 26 birdhouses to make a profit.

**e)**



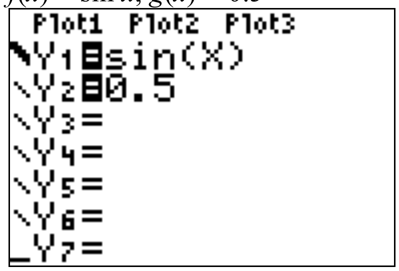

Since Claire cannot sell half of a birdhouse; she should build 18 or 19 birdhouses at a profit of $62.

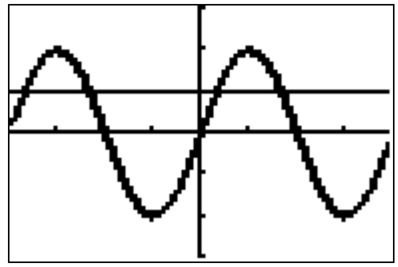
**Chapter 8 Section 4** **Question 17 Page 460**

Solutions to Achievement Check questions are provided in the Teacher's Resource.

**Chapter 8 Section 4** **Question 18 Page 460**

Answers may vary. A sample solution is shown.
$f(x) = \sin x$, $g(x) = 0.5$

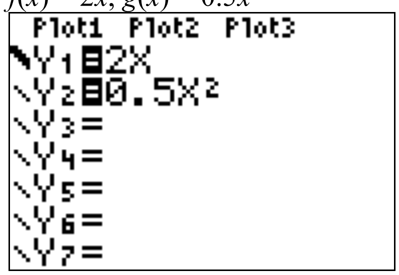


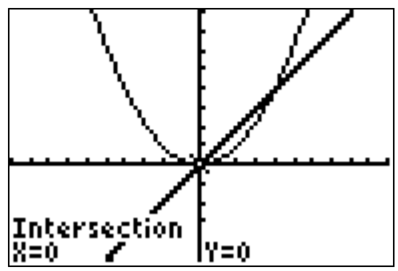



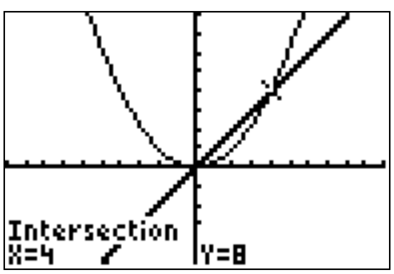
**Chapter 8 Section 4** **Question 19 Page 460**

Answers may vary. A sample solution is shown.

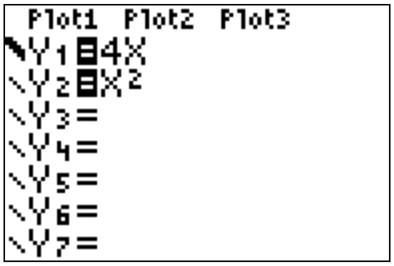
**a)** $f(x) = 2x$, $g(x) = 0.5x^2$

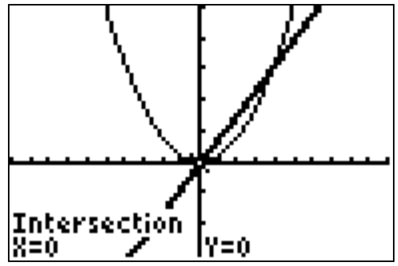


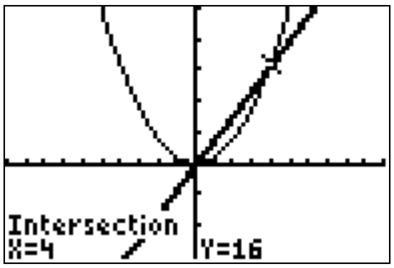



**b)** Yes. For example, $f(x) = 4x$, $g(x) = x^2$

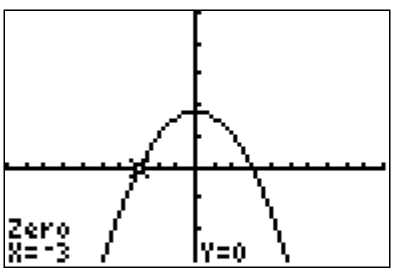


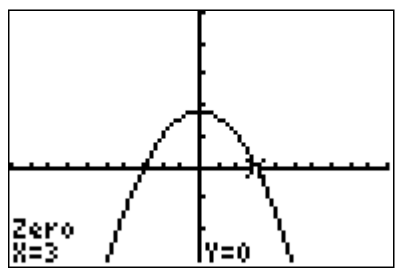



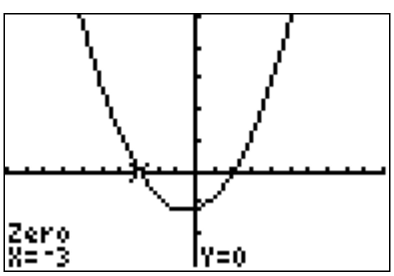

**Chapter 8 Section 4** **Question 20 Page 460**

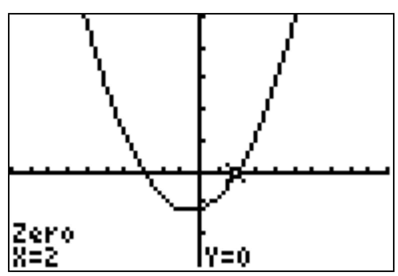
**a)** Answers may vary. A sample solution is shown.
$f(x) = -(x - 3)(x + 3)$, $g(x) = x + 3$
$f(x) > 0$




$g(x) - f(x) > 0, \text{ for } (-\infty, -3) \cup (2, \infty)$

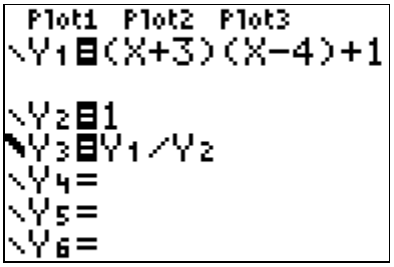


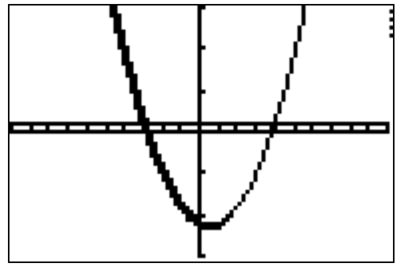



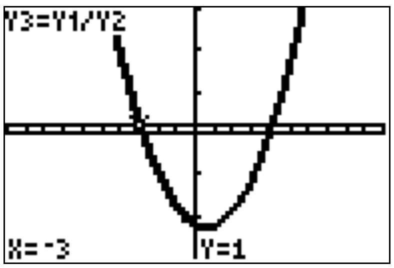

**b)** Yes, there is more than one solution to part a) .
$f(x) = -2(x - 3)(x + 3)$, $g(x) = 2x + 6$ will give the same solution.

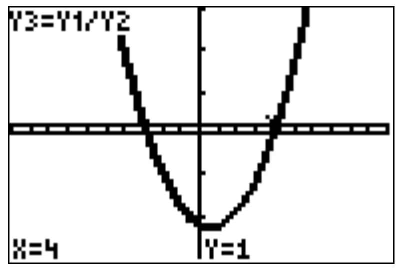
**Chapter 8 Section 4** **Question 21 Page 460**

Answers may vary. A sample solution is shown.
$f(x) = (x + 3)(x - 4) + 1$ and $g(x) = 1$






**Chapter 8 Section 4** **Question 22 Page 460**

$7 + 6 + 5 + 4 + 3 + 2 + 1 + 0 = 28$
The first person shakes hands with 7 people besides themselves, the second person shakes hands with 6 people besides themselves and the first person, the third person shakes hands with 5 people besides themselves and the first and second person, and so on.
There are 8 people at the party.

**Chapter 8 Section 4** **Question 23 Page 460**

$$\begin{aligned}(x+y+z)^2 &= x^2+xy+xz+xy+y^2+yz+xz+yz+z^2\\ &= (x^2+y^2+z^2)+(2xy+2xz+2yz)\\ &= (x^2+y^2+z^2)+2(xy+xz+yz)\end{aligned}$$

substitute $x^2+y^2+z^2=3$ and $xy+xz+yz=3$

$$\begin{aligned}(x+y+z)^2 &= 3+2(3)\\ &= 9\\ x+y+z &= \pm 3\end{aligned}$$

**Chapter 8 Section 4** **Question 24 Page 460**

Let $x$ = pennies, $5x$ = nickels, $10x$ = dimes, $25x$ = quarters

$164 = x + 5x + 10x + 25x$

$164 = 41x$

$x = 4$

4 + 4 + 4 + 4 = 16

Bill has 16 coins.

**Chapter 8 Section 4** **Question 25 Page 460**

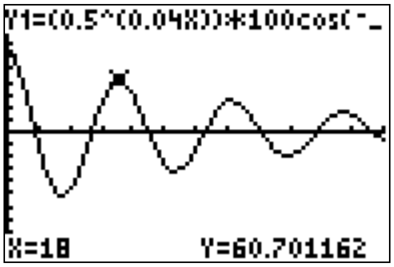
Since every third number is divisible by 3, one of the 3 consecutive numbers will have to be divisible by 3.

Examples:

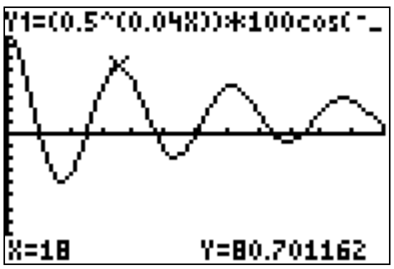
| | |
|---|---|
| 3, 4, 5 | 3 is divisible by 3 |
| 4, 5, 6 | 6 is divisible by 3 |
| 5, 6, 7 | 6 is divisible by 3 |
| 6, 7, 8 | 6 is divisible by 3 |
| 7, 8, 9 | 9 is divisible by 3 |

**Section 5** **Making Connections: Modelling With Combined Functions**

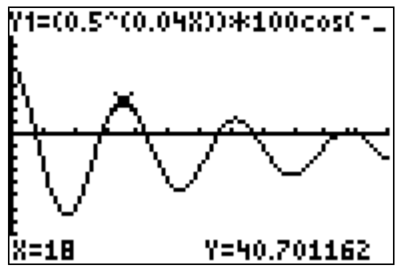
**Chapter 8 Section 5** **Question 1 Page 469**




**a)**

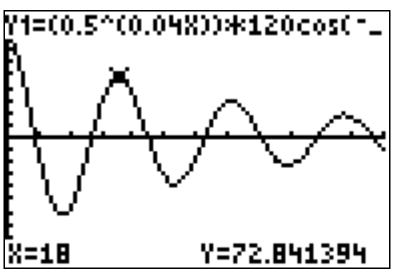



**b)**

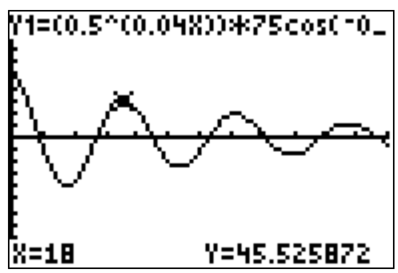



**Chapter 8 Section 5** **Question 2 Page 469**

**a)**

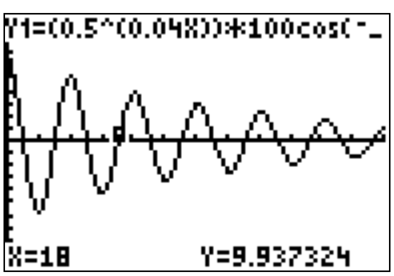



**b)**

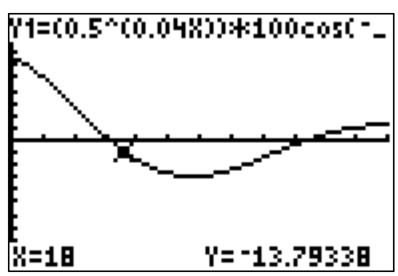



**Chapter 8 Section 5** **Question 3 Page 469**

**a)**

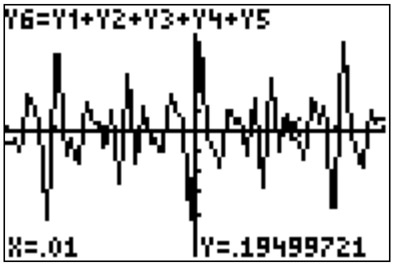



**b)**



**Chapter 8 Section 5** **Question 4 Page 469**

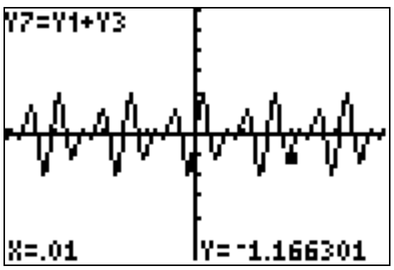
**a)**

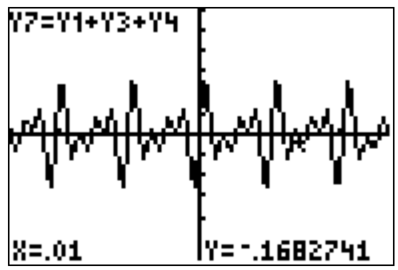



**b)** Answers may vary. A sample solution is shown.
They look similar.

**Chapter 8 Section 5** **Question 5 Page 469**

**a)**

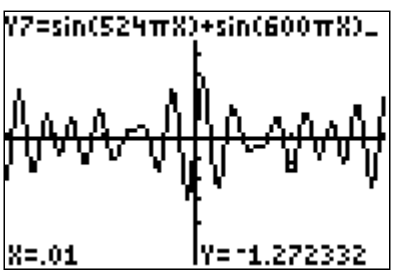



**b)** Answers may vary. A sample solution is shown.
They look similar, but the amplitude is smaller.

**Chapter 8 Section 5** **Question 6 Page 469**

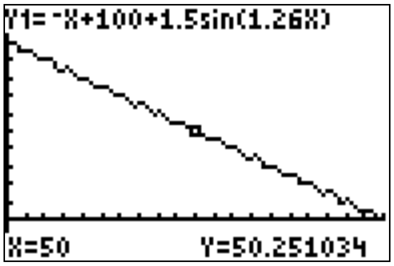
**a)**



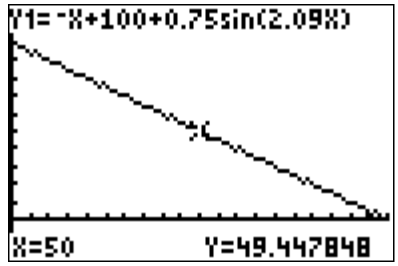

**b)** Answers may vary. A sample solution is shown.
This is not a pattern like the others.

**Chapter 8 Section 5** **Question 7 Page 469**

**a)**

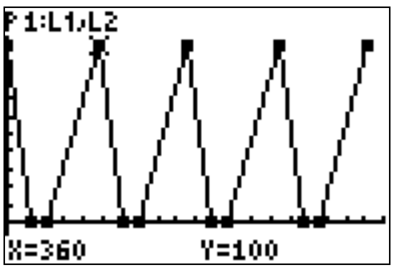



**b)**




**Chapter 8 Section 5** **Question 8 Page 470**

**a)**




**b)** Answers may vary. A sample solution is shown.
$0 < x < 100$; Skier going down the hill, –1 m/s
$100 < x < 160$; Skier in line, 0 m/s
$160 < x < 360$; Skier on lift, 0.5 m/s

**c)** Answers may vary. A sample solution is shown.
The crest to crest distance is 100 m and they are 360 m apart.

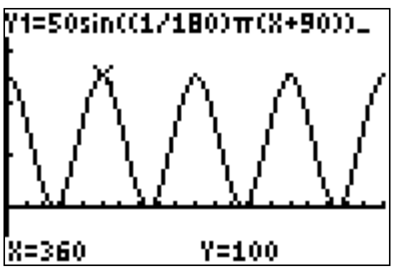
$$A = \frac{100}{2} = 50$$

$$f = \frac{1}{360}$$

$$\begin{aligned} M(t) &= A\sin(2\pi ft) \\ &= 50\sin\left[2\pi\left(\frac{1}{360}\right)t\right] \\ &= 50\sin\left(\frac{1}{180}\pi t\right) \end{aligned}$$

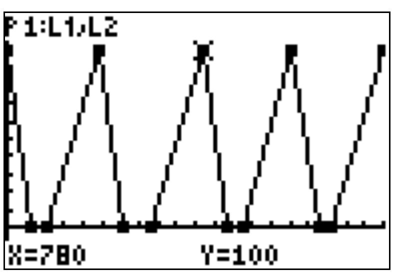
Looking at the graph, the function needs to be translated vertically by 50 units up and 90 units to the left.

$$= 50\sin\left[\frac{1}{180}\pi(t+90)\right]+50$$

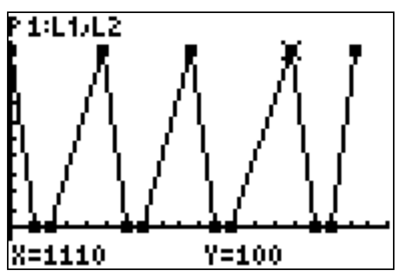



**Chapter 8 Section 5** **Question 9 Page 470**

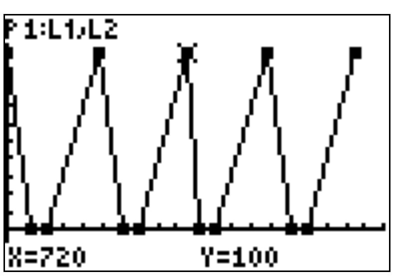
**a)**




**b)**

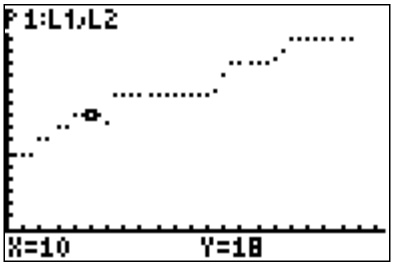



**c)**

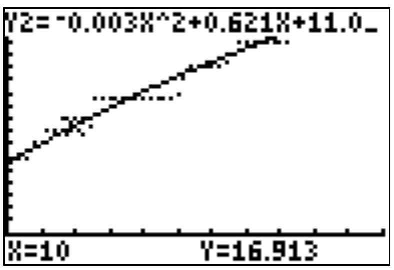


**Chapter 8 Section 5** **Question 10 Page 470**

**a)**

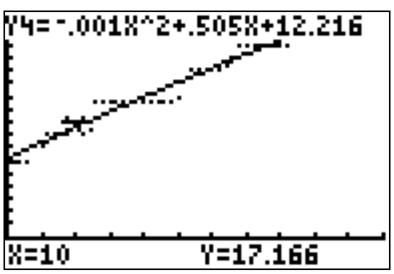



**b)** Answers may vary. A sample solution is shown.
Using regression on the graphing calculator, a curve of best fit that is quadratic has a greater $R^2$-value.



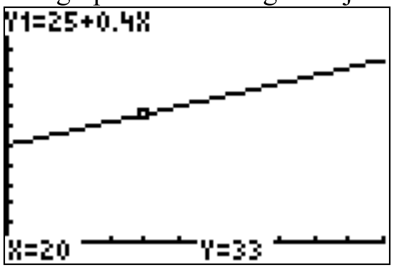

**c)** $N(t) \doteq -0.003x^2 + 0.621x + 11.003$

**d)** $N(t) \doteq -0.001x^2 + 0.505x + 12.216$
Removing the outlier changes the model for $N(t)$. Yes, the effect is significant; it gives a better approximation for each value.

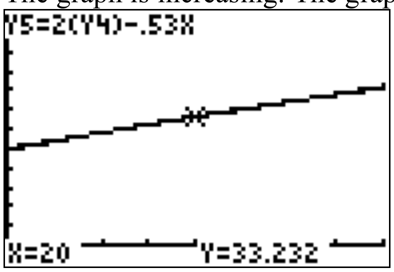


**Chapter 8 Section 5** **Question 11 Page 471**

**a)** The graph is increasing. The junior leagues are producing more high caliber players.

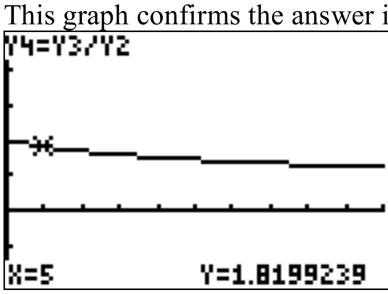



**b)** The graph is increasing. The graphs are almost equal.

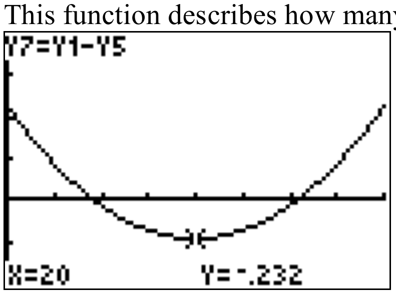



**c)** Answers may vary. A sample solution is shown.
The NHL players' careers have stayed relatively constant over the years.

**d)** This graph confirms the answer in part c) since this graph is relatively constant.



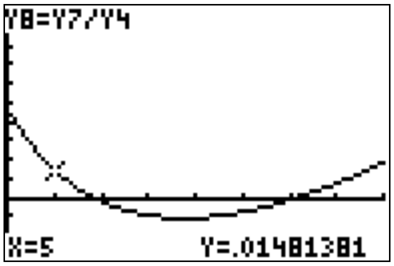

**e)** This function describes how many more draftees than retirees there are.




**f)** The graph is greater than zero, less than zero, then greater than zero. At first there is a surplus of draftees, then not enough, then a surplus again.

**Chapter 8 Section 5** **Question 12 Page 471**

**a)** This function describes the number of extra players per team, over time.




**b)** Answers may vary. A sample solution is shown.
The $t$-intercepts are the years when there are no extra players. The quality of hockey may decrease at and below the $t$-intercepts.

**c)** Answers may vary. A sample solution is shown.
$P(t)$ increases; $y = \dfrac{P(t)}{N(t)}$ increases so that it no longer crosses the $t$-axis; There is a surplus of players to draft. The quality of hockey should increase.

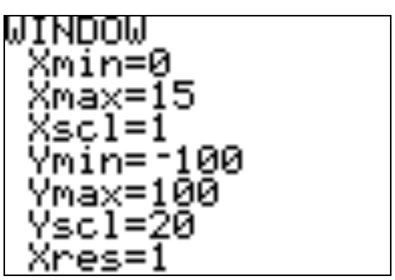
**Chapter 8 Section 5** **Question 13 Page 471**

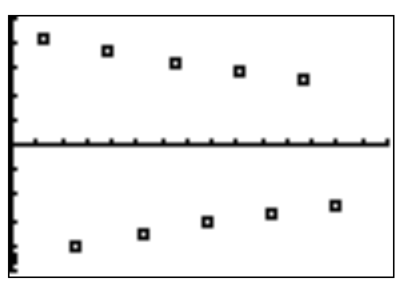
Answers may vary. A sample solution is shown.

**a)**

| Time (s) | Horizontal displacement (cm) |
|---|---|
| 0.0 | –90 |
| 1.3 | 85 |
| 2.6 | –80 |
| 3.9 | 75 |
| 5.2 | –70 |
| 6.5 | 65 |
| 7.8 | –62 |
| 9.1 | 58 |
| 10.4 | –55 |
| 11.7 | 52 |
| 13.0 | –48 |

**b)**



**c)** Since the period is 2.6 s and each swing loses about 6%: $-\cos\left(\dfrac{2\pi t}{2.6}\right) \times 90(0.94)^{\frac{t}{1.3}}$

**Chapter 8 Section 5** **Question 14 Page 471**

Answers may vary. A sample solution is shown.
The hill is 120 m high, and the rate of change is decreasing exponentially

$$y = 120\left(\frac{1}{2}\right)^{nx}$$

$$60 = 120\left(\frac{1}{2}\right)^{10n}$$

$$\frac{1}{2}^{1} = \frac{1}{2}^{10n}$$

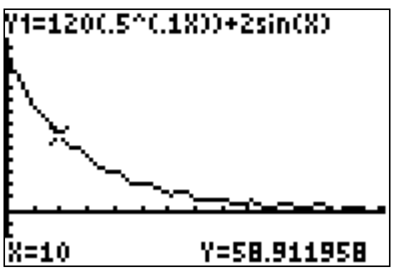
$$1 = 10n$$

$$n = \frac{1}{10}$$

$$n = 0.1$$

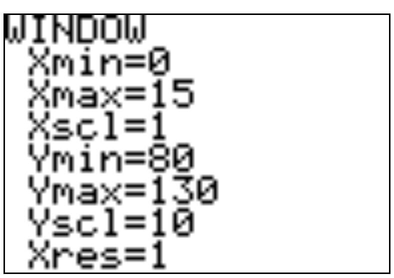
$$y = 120(0.5^{0.1x})$$

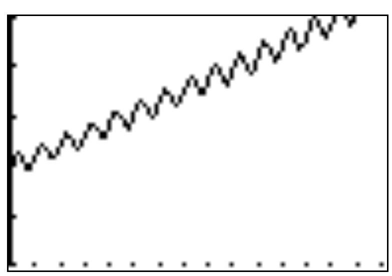
To model the height through the moguls, generate a sinusoidal function to reflect the correct amplitude and wavelength using parameters in *The Geometer's Sketchpad*® or investigate using a graphing calculator. $A = 2$, $k = 1$
$y = 120(0.5^{0.1x}) + 2\sin x$




**Chapter 8 Section 5** **Question 15 Page 471**

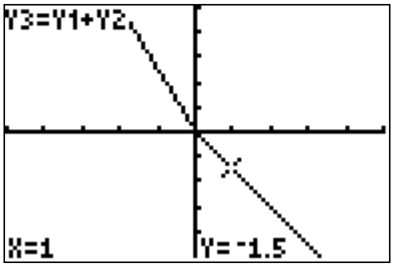
Answers may vary. A sample solution is shown.
Some animal populations increase by approximately the same percent every year; however, they vary throughout the year in an almost sinusoidal pattern.
For example, $100(1.02)^x \times (0.02\sin(2\pi x) + 1)$ represents a population growing at a rate of approximately 2%/year, sinusoidal during the year.



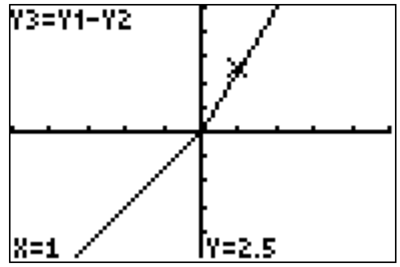



**Chapter 8 Review** **Question 1 Page 472**

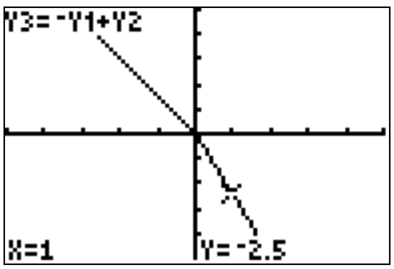
**b) i)**




**ii)**



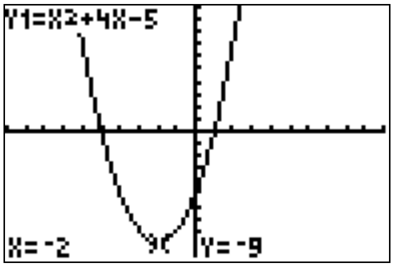

**iii)**




| $x$ | $f(x)$ | $g(x)$ | $f(x) + g(x)$ | $f(x) - g(x)$ | $g(x) - f(x)$ |
|---|---|---|---|---|---|
| –2 | 1 | 4 | 5 | –3 | 3 |
| –1 | 0.5 | 2 | 2.5 | –1.5 | 1.5 |
| 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0.5 | –2 | –1.5 | 2.5 | –2.5 |
| 2 | 1 | –4 | –3 | 5 | –5 |

**a)** $y = x - 2 + x^2 + 3x - 3$

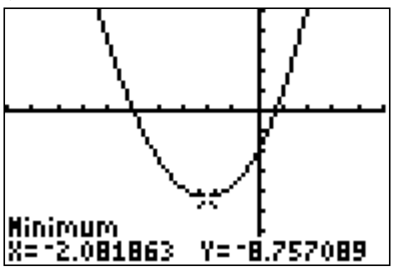
$y = x^2 + 4x - 5$




Determine the domain and range from the graph; $\{x \in \mathbb{R}\}$, $\{y \in \mathbb{R}, y \geq -9\}$

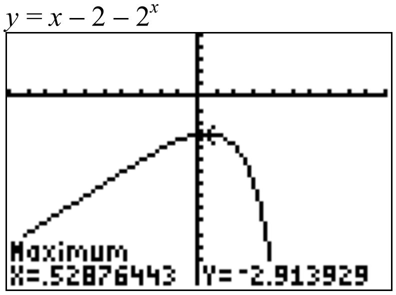
**b)** $y = x - 2 + x^2 + 3x - 3 + 2^x$

$y = x^2 + 4x - 5 + 2^x$




Determine the domain and range from the graph;
approximately $\{x \in \mathbb{R}\}$, $\{y \in \mathbb{R}, y \geq -8.76\}$

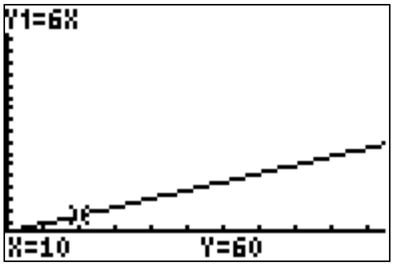
**c)** $y = x - 2 - 2^x$



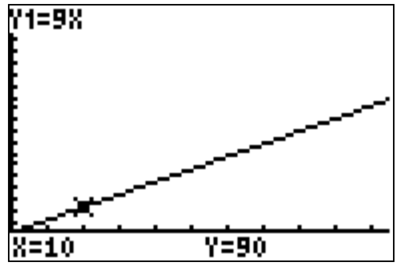

Determine the domain and range from the graph;
approximately $\{x \in \mathbb{R}\}$, $\{y \in \mathbb{R}, y \leq -2.9\}$

**Chapter 8 Review** **Question 4 Page 472**

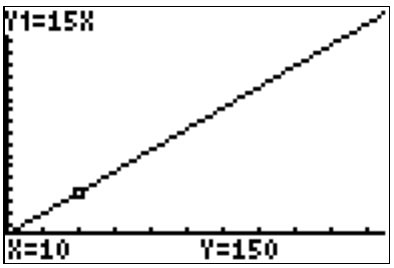
**a)** $W = 6x$




**b)** $T = 9x$




**c)** $E = 6x + 9x$
$= 15x$

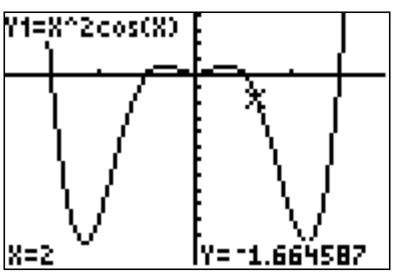



**d)** $E = 15(52)$
$= 780$
Max can earn \$780.00 in one week.

**Chapter 8 Review** **Question 5 Page 472**

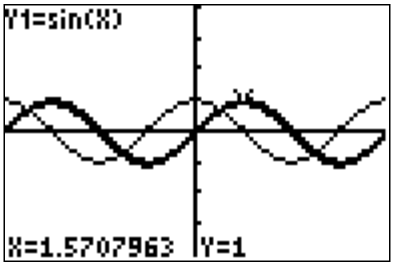
**a)** $y = u(x)v(x)$ should have line symmetry about the $y$-axis since $u(x)$ and $v(x)$ are both even functions.

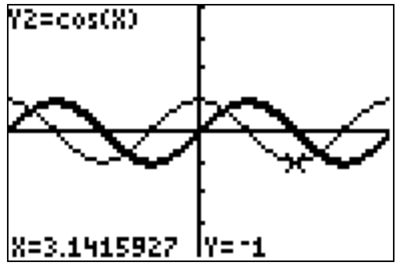
**b)**



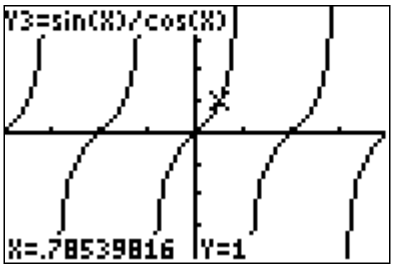
**Chapter 8 Review** **Question 6 Page 472**

**a)**




**b)**

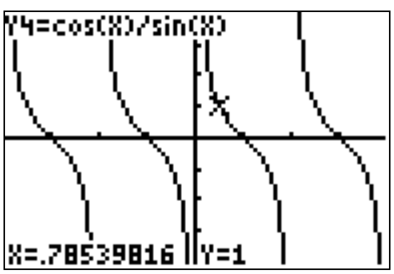



**c)** $\{x \in \mathbb{R}, x \neq \dfrac{(2n-1)\pi}{2}, n \in \mathbb{Z}\}$, $\{y \in \mathbb{R}\}$

**d)** $y = \tan x$ (ratio identity)

**Chapter 8 Review** **Question 7 Page 472**

**a)**




**b)** $\{x \in \mathbb{R}, x \neq n\pi, n \in \mathbb{Z}\}$, $\{y \in \mathbb{R}\}$

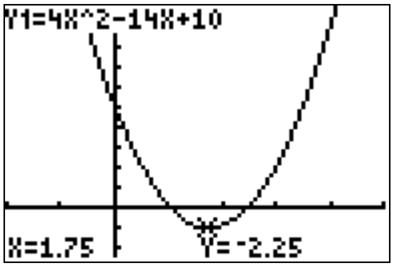
**c)** Answers may vary. A sample solution is shown.

$y = \dfrac{g(x)}{f(x)}$ is a reflection in the $y$-axis (or $x$-axis) and a translation right $\dfrac{\pi}{2}$ units of the function $y = \dfrac{f(x)}{g(x)}$.

**a)** $y = (2x-5)^2 + 3(2x-5)$

$y = 4x^2 - 20x + 25 + 6x - 15$

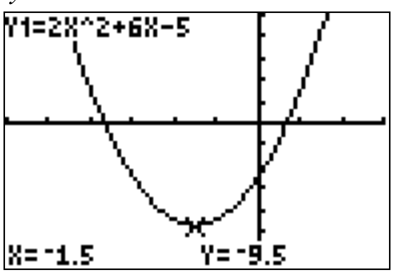
$y = 4x^2 - 14x + 10$




$\{x \in \mathbb{R}\}$, $\{y \in \mathbb{R}, y \geq -2.25\}$

**b)** $y = 2(x^2 + 3x) - 5$

$y = 2x^2 + 6x - 5$

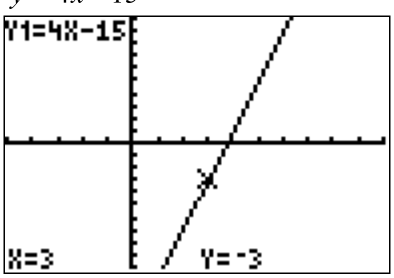



$\{x \in \mathbb{R}\}$, $\{y \in \mathbb{R}, y \geq -9.5\}$

**c)** $y = 2(2x-5) - 5$

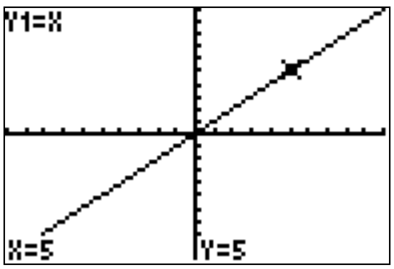
$y = 4x - 10 - 5$

$y = 4x - 15$



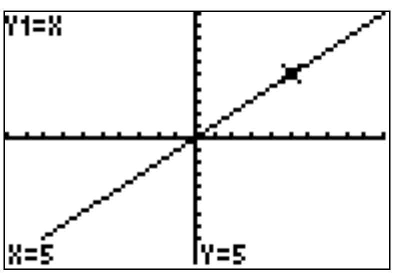

$\{x \in \mathbb{R}\}$, $\{y \in \mathbb{R}\}$

**d)**
$$\begin{aligned} x &= 2y-5 \\ 2y &= x+5 \\ y &= \frac{x+5}{2} \\ g^{-1}(x) &= \frac{x+5}{2} \\ g^{-1}(g(x)) &= \frac{(2x-5)+5}{2} \\ &= \frac{2x}{2} \\ &= x \end{aligned}$$
$\{x \in \mathbb{R}\}$, $\{y \in \mathbb{R}\}$




**Chapter 8 Review** **Question 9 Page 472**

**a)**

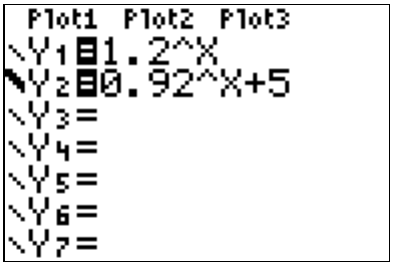


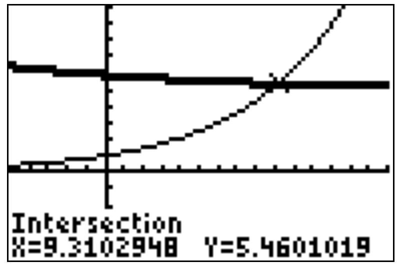

**b)** Answers may vary. A sample solution is shown.
$$\begin{aligned} f(x) &= 2x-3 \\ x &= 2y-3 \\ 2y &= x+3 \\ y &= \frac{x+3}{2} \\ f^{-1}(x) &= \frac{x+3}{2} \\ f(f^{-1}(x)) &= 2\left(\frac{x+3}{2}\right)-3 \\ &= x+3-3 \\ &= x \end{aligned}$$

$$\begin{aligned} f(x) &= x^2 \\ x &= y^2 \\ y &= \pm\sqrt{x} \\ f^{-1}(x) &= \pm\sqrt{x} \\ f(f^{-1}(x)) &= \left(\pm\sqrt{x}\right)^2 \\ &= x \end{aligned}$$

**Chapter 8 Review** **Question 10 Page 473**

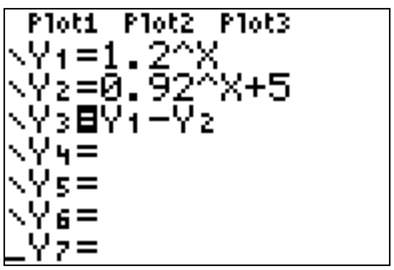
**a)**

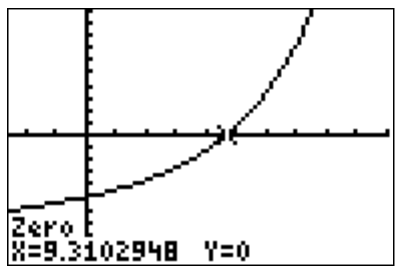



**i)** $f(x) > g(x)$ for approximately $x > 9.31$
**ii)** $g(x) > f(x)$ for approximately $x < 9.31$

**b)** One way is shown in part a).
Graph $f(x) - g(x)$
When $f(x) - g(x) > 0$, $f(x) > g(x)$
When $f(x) - g(x) < 0$, $f(x) < g(x)$




**Chapter 8 Review** **Question 11 Page 473**

Answers may vary. A sample solution is shown.
Two plant populations over time. One established but shrinking, the other new and taking over.

**Chapter 8 Review** **Question 12 Page 473**

**a)** **i)** $C > R$ for all values of $x$, except at point A, where $C = R$.

**ii)** There are no values for which $R > C$.

**b)** This business venture is not profitable since the cost is greater or equal to the revenue.

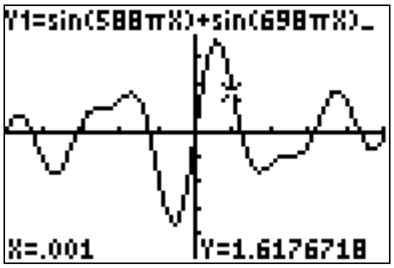
**c)** Answers may vary. A sample solution is shown.
The business owner should reduce costs or charge more without losing sales.

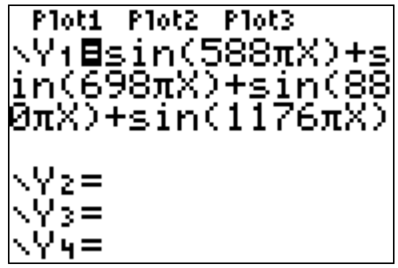
**Chapter 8 Review** **Question 13 Page 473**

**a)** High D = $294 \times 2$
$= 588$

The frequency of high D is 588 Hz.

**b)**



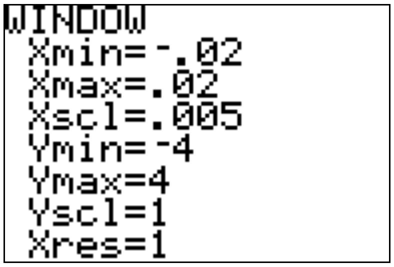

The waveform is symmetric about the origin, so it is an odd function.

Answers may vary. A sample solution is shown.

**a)** $B^b\ D\ F \qquad E^b\ G\ B^b$

**b)** $C\ D\ G \qquad C\ E^b\ F\#$

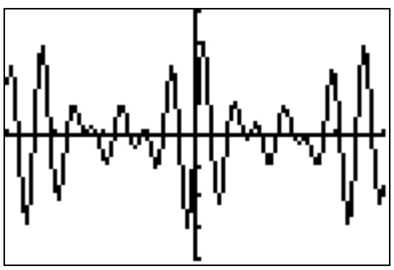
**c)**




$B^b\ D\ F$

$y = \sin\left[2\pi\left(\frac{466}{2}\right)x\right] + \sin[2\pi(294)x] + \sin[2\pi(349)x]$

$y = \sin 466\pi x + \sin 588\pi x + \sin 698\pi x$

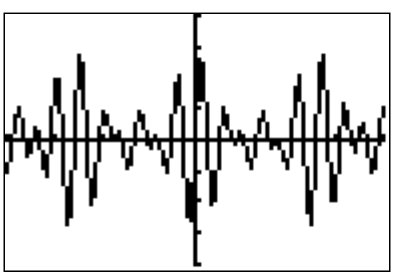


Symmetric about the origin and a repeating pattern.

$E^b\ G\ B^b$

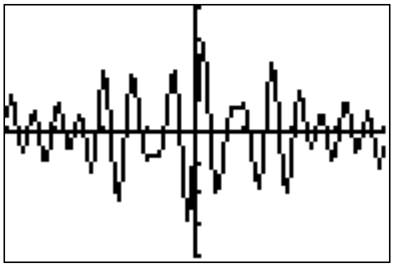
$y = \sin[2\pi(311)] + \sin[2\pi(392)] + \sin[2\pi(466)]$

$y = \sin 622\pi + \sin 784\pi + \sin 932\pi$



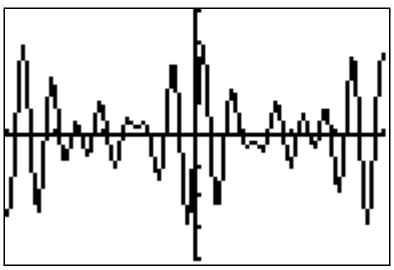
Symmetric about the origin and a repeating pattern.

*C D G*
$y = \sin[2\pi(262)] + \sin[2\pi(294)] + \sin[2\pi(392)]$
$y = \sin 524\pi + \sin 588\pi + \sin 784\pi$



Symmetric about the origin, but the pattern is not repeating

*C E$^b$ F#*
$y = \sin[2\pi(262)] + \sin[2\pi(311)] + \sin[2\pi(370)]$
$y = \sin 524\pi + \sin 622\pi + \sin 740\pi$



Symmetric about the origin, but the pattern is not repeating.

**Chapter 8 Problem Wrap-Up**

Solutions to the Chapter Problem Wrap-Up are provided in the Teacher's Resource.

## Chapter 8 Practise Test

**Chapter 8 Practise Test** **Question 1 Page 474**

The correct solution is **A**.

$y = (x^2) + 2$

**Chapter 8 Practise Test** **Question 2 Page 474**

The correct solution is **D**.

$$y = \frac{x-1}{x^2-1}$$
$$y = \frac{x-1}{(x-1)(x+1)}$$
$$y = \frac{1}{x+1},\ x \neq -1,\ x \neq 1$$

**Chapter 8 Practise Test** **Question 3 Page 474**

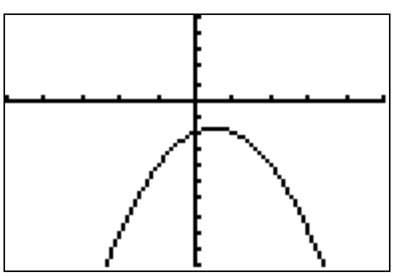
The correct solution is **D**.

$$y = (x-2)^2 + (-x+3)$$
$$y = x^2 - 4x + 4 - x + 3$$
$$y = x^2 - 5x + 7$$

**Chapter 8 Practise Test** **Question 4 Page 474**

The correct solution is **A**.

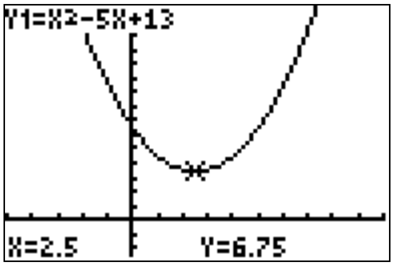
$g(x) - f(x) < 0$



**a)** $y = (x-3)^2 + x + 4$

$y = x^2 - 6x + 9 + x + 4$

$y = x^2 - 5x + 13$

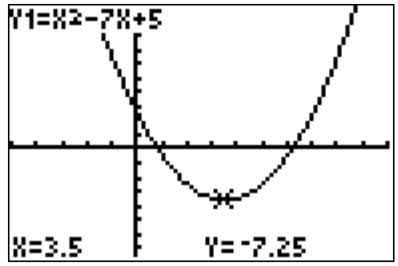



$\{x \in \mathbb{R}\}$, $\{y \in \mathbb{R}, y \geq 6.75\}$

**b)** $y = (x-3)^2 - (x+4)$

$y = x^2 - 6x + 9 - x - 4$

$y = x^2 - 7x + 5$

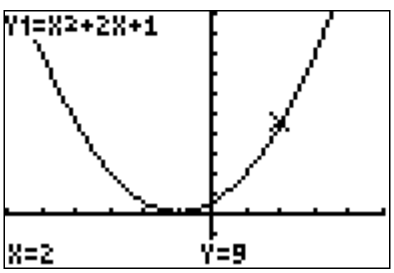



$\{x \in \mathbb{R}\}$, $\{y \in \mathbb{R}, y \geq -7.25\}$

**c)** $y = \left[(x+4) - 3\right]^2$

$y = (x+1)^2$

$y = x^2 + 2x + 1$




$\{x \in \mathbb{R}\}$, $\{y \in \mathbb{R}, y \geq 0\}$

**d)** Find $g^{-1}(x)$

$x = y + 4$

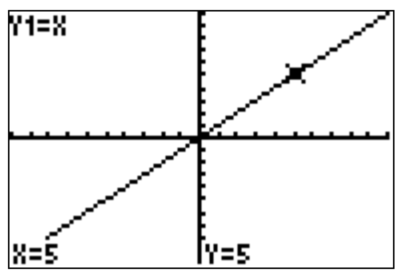
$y = x - 4$

$g^{-1}(x) = x - 4$

$y = g^{-1}(g(x))$

$y = (x+4) - 4$

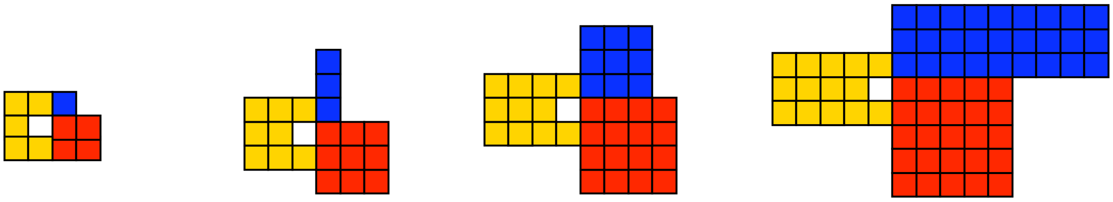
$y = x$




$\{x \in \mathbb{R}\}$, $\{y \in \mathbb{R}\}$

Answers may vary. A sample solution is shown.

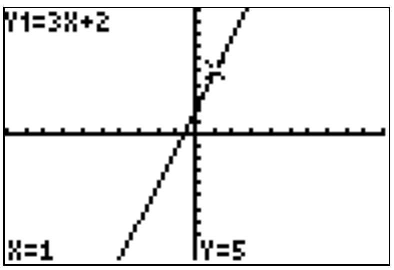
**a)**

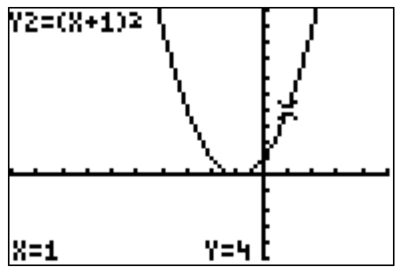


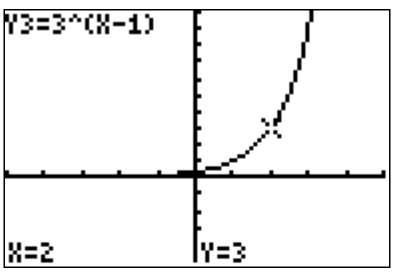
Stage 1 Stage 2 Stage 3 Stage 4

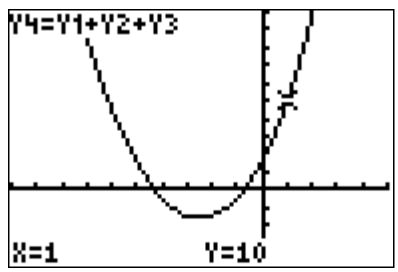
**b)** yellow: linear $y = 3x + 2$
red: quadratic $y = (x + 1)^2$
blue: exponential $y = 3^{x-1}$
total: $y = x^2 + 5x + 3 + 3^{x-1}$

**c)**



**a)** $y = \dfrac{x+5}{x^2+9x+20}$

$y = \dfrac{x+5}{(x+4)(x+5)}$

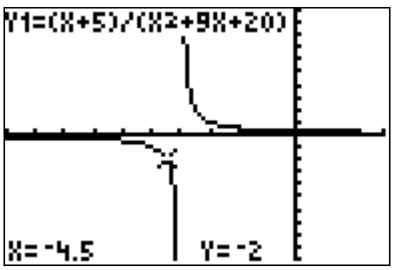
$y = \dfrac{1}{x+4},\ x \neq -5,\ x \neq -4$

vertical asymptote: $x = -4$, horizontal asymptote: $y = 0$
hole when $x = -5$

$y = \dfrac{1}{-5+4}$

$y = -1$

hole at (–5, –1)



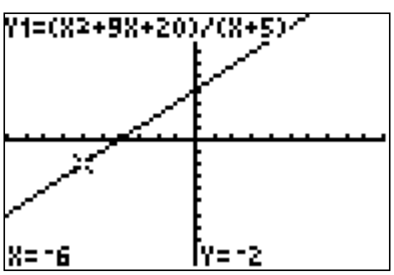

$\{x \in \mathbb{R}, x \neq -5, x \neq -4\}$, $\{y \in \mathbb{R}, y \neq 0, y \neq -1\}$

**b)** $y = \dfrac{x^2+9x+20}{x+5}$

$y = \dfrac{(x+4)(x+5)}{x+5}$

$y = x+4,\ x \neq -5$

hole when $x = -5$
$y = -5 + 4$
$y = -1$
hole at (–5, –1)




$\{x \in \mathbb{R}, x \neq -5\}$, $\{y \in \mathbb{R}, y \neq -1\}$

**a)**

$$C = 2\pi r$$

$$r = \frac{C}{2\pi}$$

$$V = \frac{4\pi r^3}{3}$$

$$V(C) = \frac{4\pi\left(\frac{C}{2\pi}\right)^3}{3}$$

$$V(C) = \frac{1}{3} \times 4\pi\left(\frac{C^3}{8\pi^3}\right)$$

$$V(C) = \frac{1}{3} \times \frac{C^3}{2\pi^2}$$

$$V(C) = \frac{C^3}{6\pi^2}$$

**b)**

$$SA = 4\pi r^2$$

$$r^2 = \frac{SA}{4\pi}$$

$$r = \sqrt{\frac{SA}{4\pi}}$$

$$V = \frac{4\pi r^3}{3}$$

$$V(SA) = \frac{1}{3} \times 4\pi\left(\sqrt{\frac{SA}{4\pi}}\right)^3$$

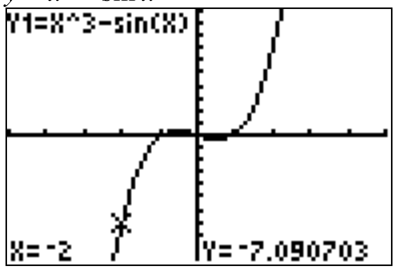
$$V(SA) = \frac{1}{3} \times 4\pi\left(\frac{SA}{4\pi}\sqrt{\frac{SA}{4\pi}}\right)$$

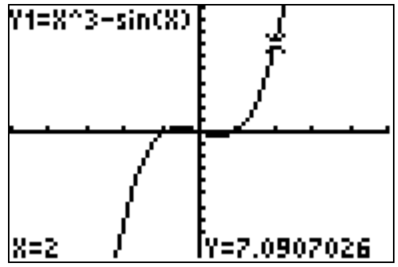
$$V(SA) = \frac{1}{3} \times \left(\frac{SA}{2}\sqrt{\frac{SA}{\pi}}\right)$$

$$V(SA) = \frac{SA}{6}\sqrt{\frac{SA}{\pi}}$$

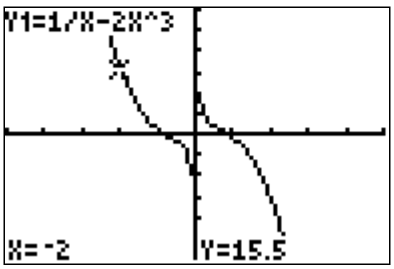
**Chapter 8 Practise Test** **Question 10 Page 474**

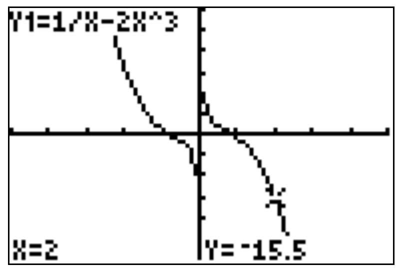
The difference of two odd functions is odd.
Examples may vary. A sample solution is shown.
$y = x^3 - \sin x$



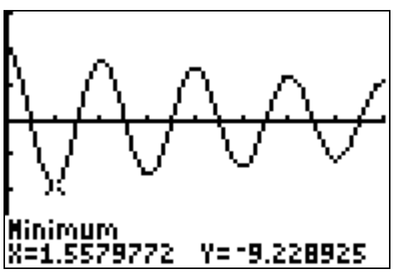

$y = \frac{1}{x} - 2x^3$

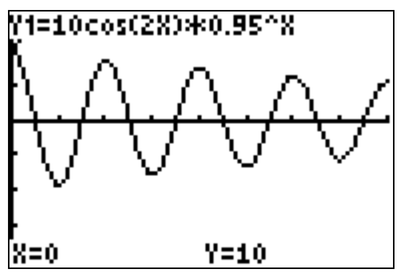



**Chapter 8 Practise Test** **Question 11 Page 475**

**a)**




This is damped harmonic oscillation.
$\{t \in \mathbb{R}, t \geq 0\}$, $\{x \in \mathbb{R}, -9.23 \leq x \leq 10\}$

**b) i)** $y = 10\cos(2t)$

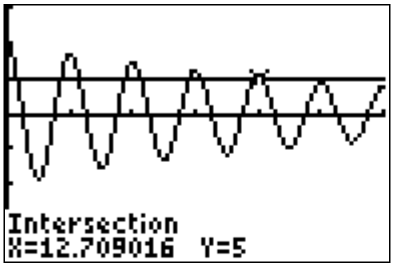
**ii)** $y = 0.95^t$

**c)** From the graph: When $t = 0$, $x(t) = 10$.
The bob was released from a horizontal distance of 10 cm from the rest position.

**d)** The rate of change is the greatest at $x(t) = 0$ when the pendulum bob crosses the rest position the first time.

**e)** The rate of change is zero at the crests and troughs. This occurs when the pendulum bob changes direction.

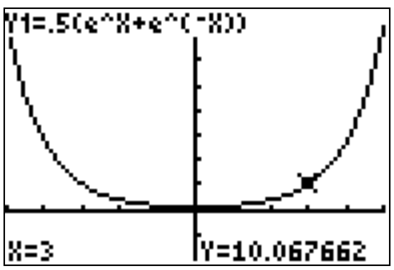
**f)** The amplitude will diminish to 50% of its initial value when $5 = 10\cos(2t) \times 0.95^t$
Find the point of intersection on the graph.




After approximately 12.7 s, the amplitude will diminish to 50% of its initial value. This is the last time the magnitude is greater than 5.

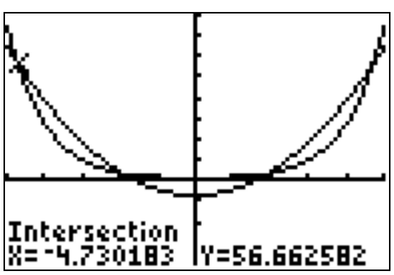
**Chapter 8 Practise Test** **Question 12 Page 475**

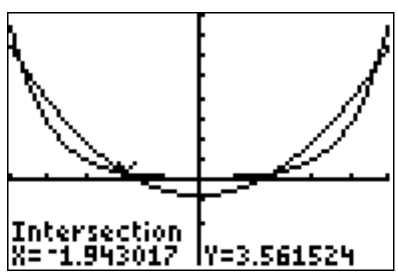
**a)**

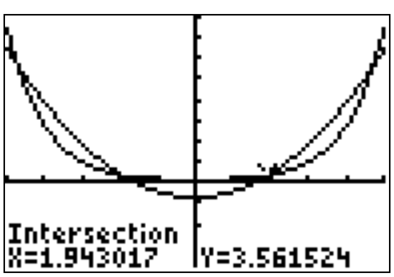


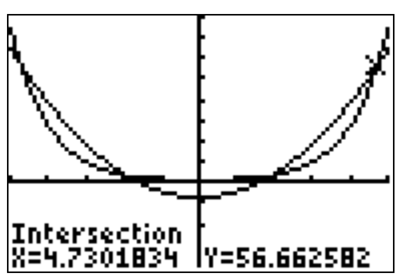

**b)** Answers may vary. A sample solution is shown.
$y \doteq 2.855x^2 - 7.217$

**c)** Point of intersection: (–4.73, 56.67), (–1.94, 3.56), (1.94, 3.56), (4.73, 56.67)



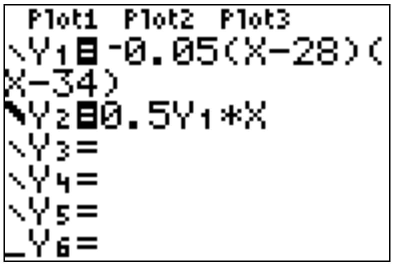



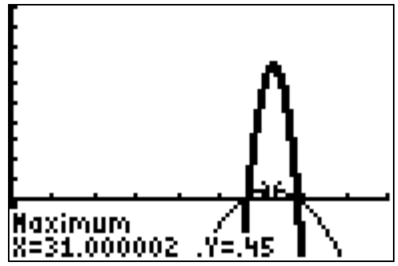



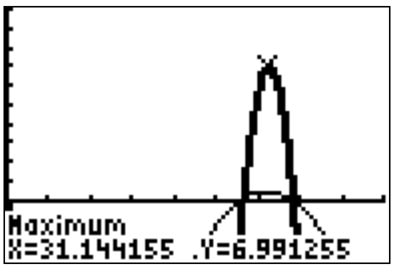



**Chapter 8 Practise Test** **Question 13 Page 475**

**a)**



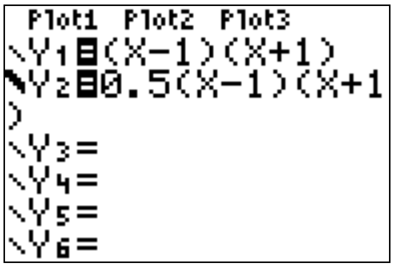

$S(T)$ is a parabolic function with maximum value at (31, 0.45). $I(T)$ is a function with maximum value at approximately (31.1, 6.99)

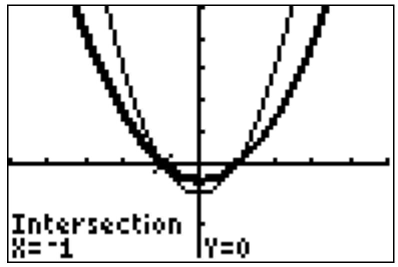
**b)** Boulder beach will attract the greatest amount of swimmers, which is 45, when the temperature is 31°C.

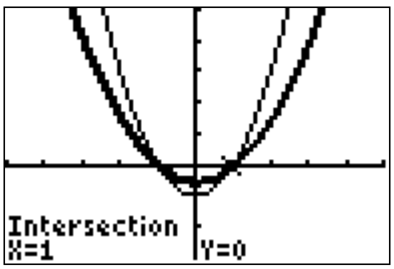
**c)** The Boulder Beach ice-cream vendor will earn the maximum profit when the temperature is approximately 31°C. This is approximately the same temperature as in part b), which makes sense, since that is when the most swimmers (customers) will be at Boulder Beach.

**a)** Answers may vary. A sample solution is shown.
$f(x) = (x - 1)(x + 1)$ and $g(x) = 0.5(x - 1)(x + 1)$

**b)** Answers may vary. A sample solution is shown.
*Method 1:*
Graph $f(x)$ and $g(x)$ on the same axis. Find the points of intersection, then visually find when $f(x) > g(x)$ and $f(x) < g(x)$

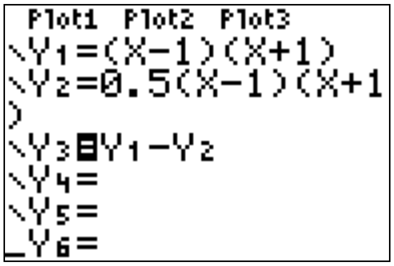


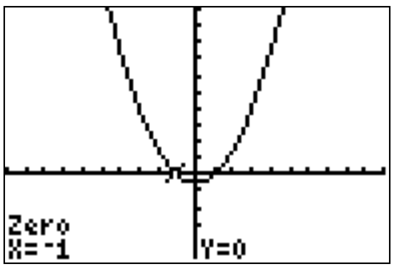



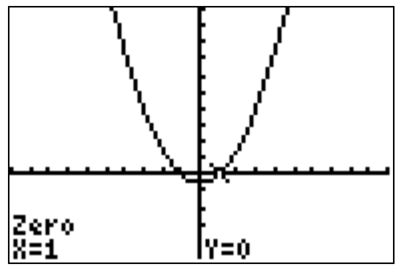




$f(x) < g(x)$ for $(-1, 1)$
$f(x) > g(x)$ for $(-\infty, -1) \cup (1, \infty)$

*Method 2:*
$f(x) - g(x)$

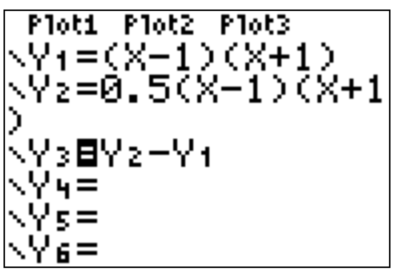


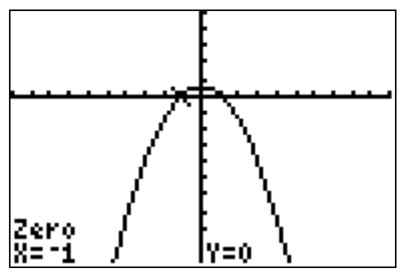



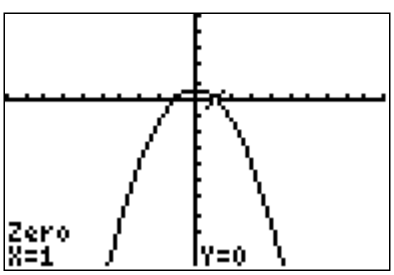




When $f(x) - g(x) < 0$, $f(x) < g(x)$ for $(-1, 1)$
When $f(x) - g(x) > 0$, $f(x) > g(x)$ for $(-\infty, -1) \cup (1, \infty)$

*Method 3:*
$g(x) - f(x)$

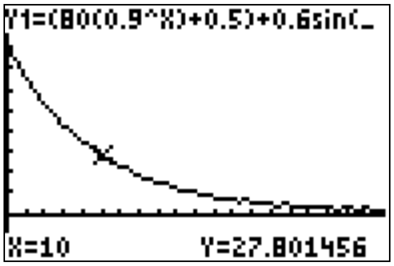


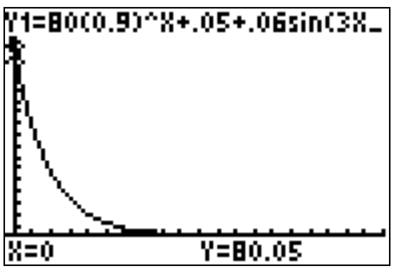

When $g(x) - f(x) > 0$, $f(x) < g(x)$ for $(-1, 1)$
When $g(x) - f(x) < 0$, $f(x) > g(x)$ for $(-\infty, -1) \cup (1, \infty)$

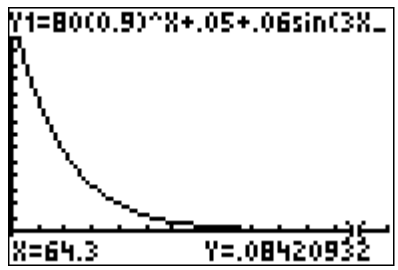
**a)**




**b)** Answers may vary. A sample solution is shown.
The skier encounters moguls at 10 s, where the bumps start on the graph.

**c)**




$\{t \in \mathbb{R}, 0 \le t \le 64.3\}$, $\{h \in \mathbb{R}, 0 \le h \le 80\}$

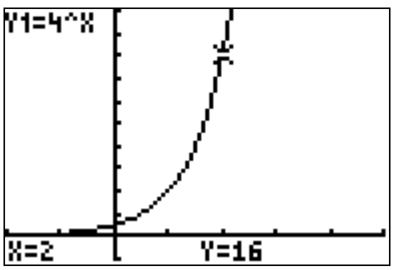
## Chapter 6 to 8 Review

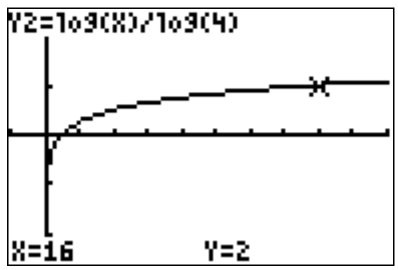
**Chapter 6 to 8 Review** **Question 1 Page 476**

**a)**

| $x$ | $y$ |
|---|---|
| $-2$ | $\frac{1}{16}$ |
| $-1$ | $\frac{1}{4}$ |
| 0 | 1 |
| $\frac{1}{2}$ | 2 |
| 1 | 4 |
| 2 | 16 |
| 3 | 64 |

**b)**




**Chapter 6 to 8 Review** **Question 2 Page 476**

**a)** $5^x = 25$
$5^x = 5^2$
$x = 2$

**b)** $2^x = 128$
$2^x = 2^7$
$x = 7$

**c)** $3^x = \frac{1}{27}$
$3^x = 27^{-1}$
$3^x = (3^3)^{-1}$
$3^x = 3^{-3}$
$x = -3$

**d)** $10^x = 1000$
$10^x = 10^3$
$x = 3$

**e)** $7^x = 7^4$
$x = 4$

**f)** $10^x = 10^{-5}$
$x = -5$

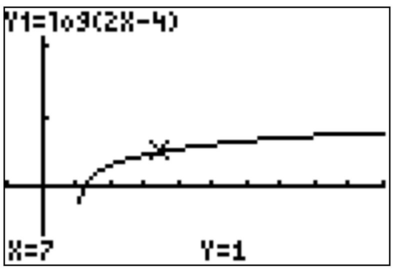
**Chapter 6 to 8 Review** **Question 3 Page 476**

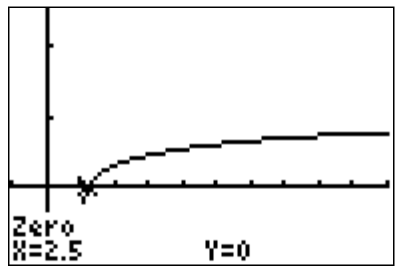
**a)** $I(3) = 100(0.39)^3$
$\doteq 5.9$
The intensity at a depth of 3 m is approximately 5.9 units.

**b)** $I(5) = 100(0.39)^5$
$\doteq 0.9$
The intensity at a depth of 5 m is approximately 0.9 units.

**Chapter 6 to 8 Review** **Question 4 Page 476**

**a)**




The restriction on the domain:

$$\begin{aligned} 2x - 4 &> 0 \\ 2x &> 4 \\ x &> 2 \end{aligned}$$

Domain: $\{x \in \mathbb{R}, x > 2\}$, Range: $\{y \in \mathbb{R}\}$
From the graph, the $x$-intercept is 2.5.
There is no $y$-intercept.
The vertical asymptote is $x = 2$.

**b)** $y = -\log(2x - 4)$ is a reflection in the $x$-axis of the function $y = \log(2x - 4)$.

**Chapter 6 to 8 Review** **Question 5 Page 476**

**a)**
$$\begin{aligned} \log_6 36^5 &= \log_6 (6^2)^5 \\ &= \log_6 6^{10} \\ &= 10 \end{aligned}$$

**b)**
$$\begin{aligned} \log_4 (32 \times 2) &= \log_4 64 \\ &= 3 \end{aligned}$$

**Chapter 6 to 8 Review** **Question 6 Page 476**

**a)**
$$\begin{aligned} \log 4^x &= \log 15 \\ x \log 4 &= \log 15 \\ x &= \frac{\log 15}{\log 4} \\ x &\doteq 1.95 \end{aligned}$$

**b)**
$$\begin{aligned} x &= \frac{\log 18}{\log 2} \\ x &\doteq 4.17 \end{aligned}$$

**Chapter 6 to 8 Review** **Question 7 Page 476**

**a)** $$\begin{aligned} V(0) &= 28\ 000(0.75)^0 \\ &= 28\ 000 \end{aligned}$$
The initial value of the vehicle (when $t = 0$) was $28 000.

**b)** $$\begin{aligned} 14\ 000 &= 28\ 000(0.75)^t \\ \frac{1}{2} &= 0.75^t \\ \log\frac{1}{2} &= t\log 0.75 \\ t &= \frac{\log\frac{1}{2}}{\log 0.75} \\ t &\doteq 2.4 \end{aligned}$$
It will take approximately 2.4 years for the vehicle to depreciate to half its initial value.

**Chapter 6 to 8 Review** **Question 8 Page 476**

**a)** $$\begin{aligned} \beta &= 10\log\left(\frac{8.75\times10^{-3}}{10^{-12}}\right) \\ &\doteq 99 \end{aligned}$$
The decibel rating of a rock concert is approximately 99 dB.

**b)** $$\begin{aligned} 20 &= 10\log\left(\frac{I}{10^{-12}}\right) \\ 2 &= \log\left(\frac{I}{10^{-12}}\right) \\ 10^2 &= \frac{I}{10^{-12}} \\ I &= 10^2\times10^{-12} \\ I &= 10^{-10} \end{aligned}$$
The intensity of sound from the radio is $10^{-10}$ W/m$^2$.

**Chapter 6 to 8 Review** **Question 9 Page 476**

**a)**
$$\begin{aligned}\left(3^3\right)^{x+2} &= \left(3^2\right)^{5-2x}\\ 3^{3x+6} &= 3^{10-4x}\\ 3x+6 &= 10-4x\\ 3x+4x &= 10-6\\ 7x &= 4\\ x &= \frac{4}{7}\end{aligned}$$

**b)**
$$\begin{aligned}10^{5x+4} &= (10^3)^{3x}\\ 10^{5x+4} &= 10^{9x}\\ 5x+4 &= 9x\\ 9x-5x &= 4\\ 4x &= 4\\ x &= 1\end{aligned}$$

**c)**
$$\begin{aligned}\left(4^3\right)^{x+5} &= \left(4^2\right)^{2x-1}\\ 4^{3x+15} &= 4^{4x-2}\\ 3x+15 &= 4x-2\\ 4x-3x &= 15+2\\ x &= 17\end{aligned}$$

**Chapter 6 to 8 Review** **Question 10 Page 476**

**a)**
$$\begin{aligned}\log 8 &= n\log 3\\ n &= \frac{\log 8}{\log 3}\\ n &\doteq 1.89\end{aligned}$$

**b)**
$$\begin{aligned}n\log 5.8 &= \log 100\\ n &= \frac{2}{\log 5.8}\\ n &\doteq 2.62\end{aligned}$$

**c)**
$$\begin{aligned}\log 10^{n+5} &= \log 7\\ (n+5)\log 10 &= \log 7\\ n+5 &= \log 7\\ n &= \log 7 - 5\\ n &\doteq -4.15\end{aligned}$$

**d)**
$$\begin{aligned}\log 2^{-n} &= \log 6\\ -n\log 2 &= \log 6\\ -n &= \frac{\log 6}{\log 2}\\ -n &\doteq 2.58\\ n &\doteq -2.58\end{aligned}$$

**e)**
$$\begin{aligned}\log 278^{3n-7} &= \log 21^{2n+5}\\ (3n-7)\log 278 &= (2n+5)\log 21\\ 3n\log 278 - 7\log 278 &= 2n\log 21 + 5\log 21\\ 3n\log 278 - 2n\log 21 &= 5\log 21 + 7\log 278\\ n(3\log 278 - 2\log 21) &= 5\log 21 + 7\log 278\\ n &= \frac{5\log 21 + 7\log 278}{3\log 278 - 2\log 21}\\ n &\doteq 5.06\end{aligned}$$

**f)**
$$\begin{aligned}\log 5^{2n} &= \log 0.75^{n-4}\\ 2n\log 5 &= (n-4)\log 0.75\\ 2n\log 5 &= n\log 0.75 - 4\log 0.75\\ 2n\log 5 - n\log 0.75 &= -4\log 0.75\\ n(2\log 5 - \log 0.75) &= -4\log 0.75\\ n &= \frac{-4\log 0.75}{2\log 5 - \log 0.75}\\ n &\doteq 0.33\end{aligned}$$

**a)**

$$A(t) = A_o\left(\frac{1}{2}\right)^{\frac{t}{h}}$$

$$16.85 = 20\left(\frac{1}{2}\right)^{\frac{48}{h}}$$

$$0.8425 = \frac{1}{2}^{\frac{48}{h}}$$

$$\log 0.8425 = \frac{48}{h}\log\frac{1}{2}$$

$$h = \frac{48\log\frac{1}{2}}{\log 0.8425}$$

$$h \doteq 194$$

The half-life is approximately 194 h.

**b)**

$$A(t) = A_o\left(\frac{1}{2}\right)^{\frac{t}{h}}$$

$$78.1 = 100\left(\frac{1}{2}\right)^{\frac{1}{h}}$$

$$0.781 = \frac{1}{2}^{\frac{1}{h}}$$

$$\log 0.781 = \frac{1}{h}\log\frac{1}{2}$$

$$h = \frac{\log\frac{1}{2}}{\log 0.781}$$

$$h \doteq 2.8$$

The half-life is approximately 2.8 h.

**Chapter 6 to 8 Review** **Question 12 Page 477**

**a)** 
$$\log\left(\frac{x^2-1}{x-1}\right) = \log\frac{(x-1)(x+1)}{(x-1)}$$
$$= \log(x+1)$$

Restrictions:

$x+1>0,\ x\neq 1$

$x>-1,\ x\neq 1$

**b)** 
$$\log x^{\frac{1}{2}} + 3\log x - 2\log x = \frac{1}{2}\log x + \log x$$
$$= \frac{3}{2}\log x,\ x>0$$

**c)** 
$$\log\left(1+\frac{2y}{x}+\frac{y^2}{x^2}\right) + \log x^2 = \log\left(x^2+2xy+y^2\right)$$
$$= \log(x+y)^2$$
$$= 2\log(x+y),\ x>-y,\ x\neq 0$$

**a)**
$$3^2 = -3x + 5$$
$$9 - 5 = -3x$$
$$4 = -3x$$
$$x = -\frac{4}{3}$$

**b)**
$$\log\left(\frac{x+5}{x+1}\right) = \log(3x)$$
$$\frac{x+5}{x+1} = 3x$$
$$x + 5 = 3x(x+1)$$
$$x + 5 = 3x^2 + 3x$$
$$3x^2 + 3x - x - 5 = 0$$
$$3x^2 + 2x - 5 = 0$$
$$(x-1)(3x+5) = 0$$
$$x = 1$$

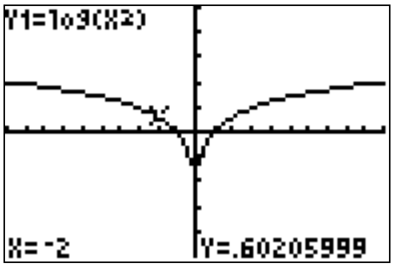
$x = -\frac{5}{3}$ is an extraneous root since $\log(x+1)$ and $\log(3x)$ would be undefined

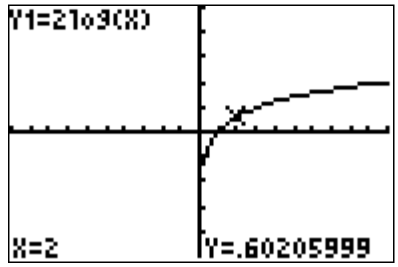
**c)**
$$\log_5(x-6) + \log_5(x-2) = 1$$
$$\log_5\left[(x-6)(x-2)\right] = 1$$
$$5^1 = x^2 - 8x + 12$$
$$x^2 - 8x + 12 - 5 = 0$$
$$x^2 - 8x + 7 = 0$$
$$(x-7)(x-1) = 0$$
$$x = 7$$

$x = 1$ is an extraneous root since $\log_5(x-6)$ and $\log_5(x-2)$ would be undefined.

**Chapter 6 to 8 Review** **Question 14 Page 477**

They are different because for 2 log $x$, $x > 0$, for log $x^2$, $x \in \mathbb{R}$.
The graphs of $f(x) = \log x^2$ and $g(x) = 2\log x$ are different because the domain of the graph of $g(x)$ is $\{x > 0, x \in \mathbb{R}\}$, while the domain of $f(x)$ is $\{x \in \mathbb{R}\}$.




**Chapter 6 to 8 Review** **Question 15 Page 477**

**a)** $P(t) = 38\ 000 \times (1.12)^t$

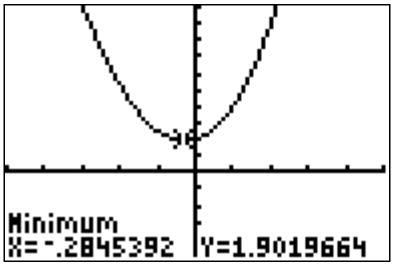
**b)**
$$\begin{aligned} P(t) &= 38\ 000 \times (1.12)^t \\ 2 &= 1.12^t \\ \log 2 &= t \log 1.12 \\ t &= \frac{\log 2}{\log 1.12} \\ t &\doteq 6.1 \end{aligned}$$
It will take approximately 6.1 years for the population to double.

**c)**
$$\begin{aligned} P(t) &= 38\ 000 \times (1.12)^t \\ 100\ 000 &= 38\ 000 \times 1.12^t \\ \frac{100}{38} &= 1.12^t \\ \log\left(\frac{50}{19}\right) &= t \log 1.12 \\ t &= \frac{\log\left(\frac{50}{19}\right)}{\log 1.12} \\ t &\doteq 8.5 \end{aligned}$$
It will take approximately 8.5 years for the population to reach 100 000.

**Chapter 6 to 8 Review** **Question 16 Page 477**

**a)** $y = 2^x + 2 + x^2 - 1$

$y = 2^x + x^2 + 1$

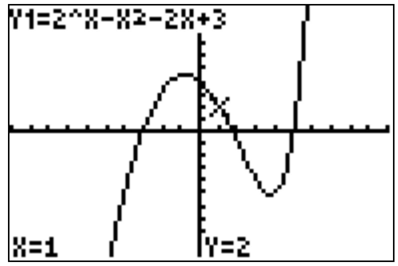



Domain: $\{x \in \mathbb{R}\}$, Range: $\{y \in \mathbb{R}, y \geq 1.9\}$

**b)** $y = 2^x + 2 - (x^2 - 1) - 2x$

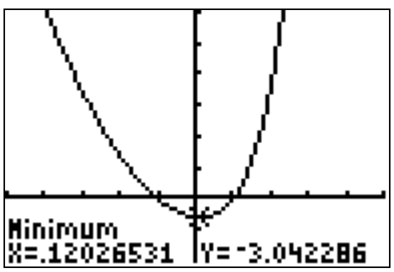
$y = 2^x - x^2 - 2x + 2 + 1$

$y = 2^x - x^2 - 2x + 3$



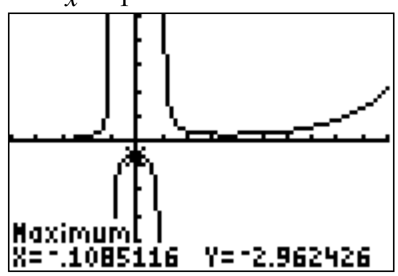

Domain: $\{x \in \mathbb{R}\}$, Range: $\{y \in \mathbb{R}\}$

**c)** $y = (2^x + 2)(x^2 - 1)$




Domain: $\{x \in \mathbb{R}\}$,
Range: $\{y \in \mathbb{R}, y \geq -3.04\}$

**d)** $y = \dfrac{2^x + 2}{x^2 - 1}$, $x \neq -1$, $x \neq 1$



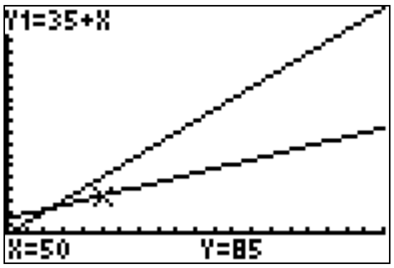

Domain: $\{x \in \mathbb{R}, x \neq -1, x \neq 1\}$,
Range: $\{y \in \mathbb{R}, y \leq -2.96, y > 0\}$

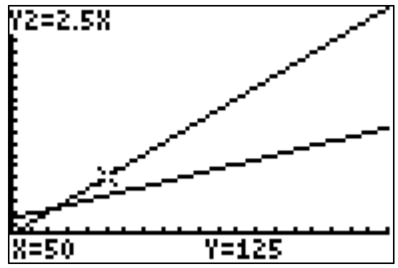
**Chapter 6 to 8 Review** **Question 17 Page 477**

**a)** **i)** $C(n) = 35 + n$, $0 \leq n \leq 200$

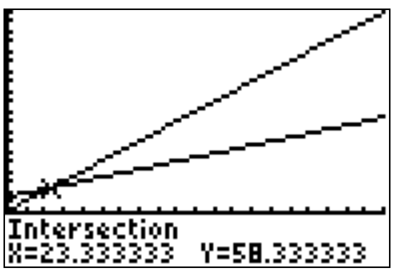
**ii)** $R(n) = 2.5n$, $0 \leq n \leq 200$

**b)**




**c)**

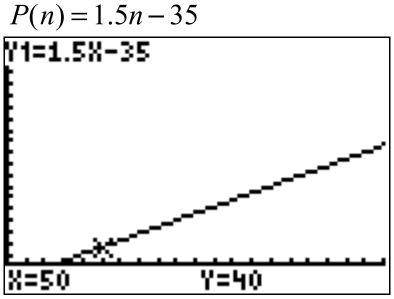



The point of intersection of the two graphs, approximately (23.33, 58.33), is the break-even point. This is where the total cost equals Kathy's revenue.
Kathy makes a profit if she sells 24 or more cups of apple cider.
Kathy loses money if she sells 23 or fewer cups of apple cider.

**d)** $P(n) = R(n) - C(n)$

$P(n) = 2.5n - (35 + n)$

$P(n) = 1.5n - 35$

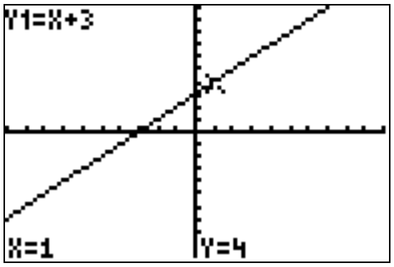



**e)** $P(200) = 1.5(200) - 35$

$= 265$

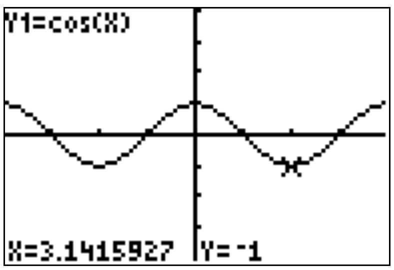
Kathy's maximum daily profit is $265.

**a)** $f(x) = x + 3$ is a linear function.



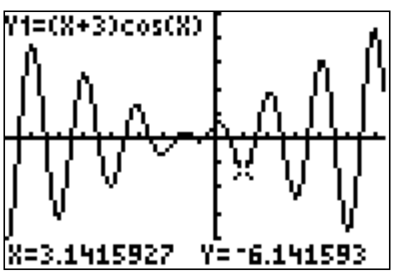

Since the function is not symmetric about the $y$-axis or the origin, it is neither even nor odd.

**b)** $g(x) = \cos(x)$ is a periodic function.




The function is even since it is symmetric about the $y$-axis.

**c)** $y = (x + 3) \cos x$
Damped harmonic motion: a wave that gets stretched vertically the farther it gets from $x = -3$.




**d)** See the graphs in parts a), b), and c).

**e)** Domain: $\{x \in \mathbb{R}\}$, Range: $\{y \in \mathbb{R}\}$

**Chapter 6 to 8 Review** **Question 19 Page 477**

**a)**
$$y=\sqrt{\frac{1}{x^2}-9}$$
$$y=\sqrt{\frac{1}{x^2}-\frac{9x^2}{x^2}}$$
$$y=\sqrt{\frac{1-9x^2}{x^2}}$$
$$y=\frac{\sqrt{1-9x^2}}{x},\ x\neq 0$$

Restrictions:
$$1-9x^2\geq 0$$
$$1\geq 9x^2$$
$$x^2\leq\frac{1}{9}$$
$$x\geq-\frac{1}{3}\text{ or }x\leq\frac{1}{3}$$
$$-\frac{1}{3}\leq x\leq\frac{1}{3},\ x\neq 0$$

Domain: $\left\{x\in\mathbb{R},-\frac{1}{3}\leq x<0,\ 0<x\leq\frac{1}{3}\right\}$, Range: $\{y\in\mathbb{R}\}$

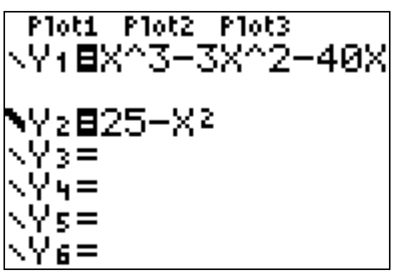
**b)**
$$y=\frac{1}{\left(\sqrt{x-9}\right)^2}$$
$$y=\frac{1}{x-9},\ x>9$$

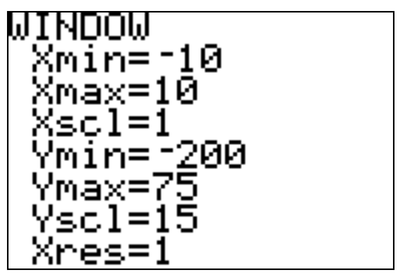
Restriction:
$x-9>0$ since it is a square root in the denominator.
$x>9$

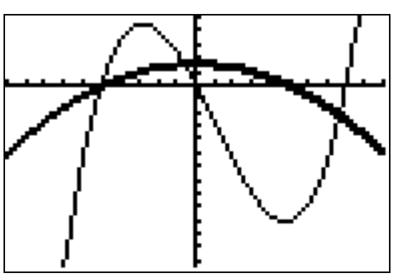
Domain: $\{x\in\mathbb{R}, x>9\}$, Range: $\{y\in\mathbb{R}, y>0\}$

**Chapter 6 to 8 Review** **Question 20 Page 477**

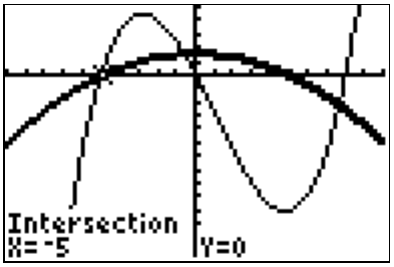
**a)**

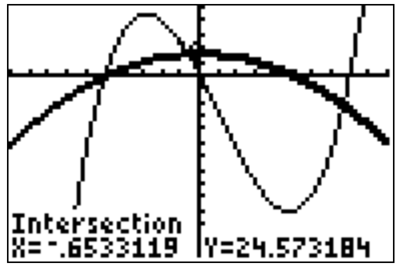


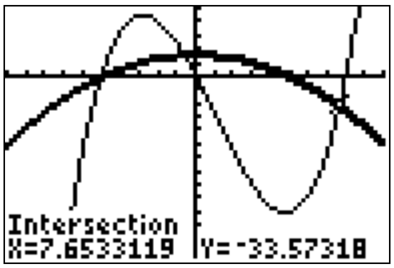



**b)**

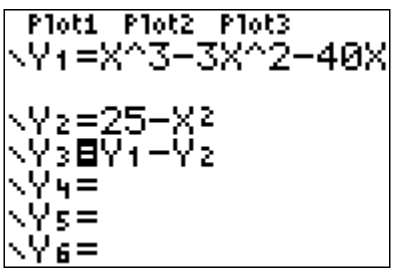


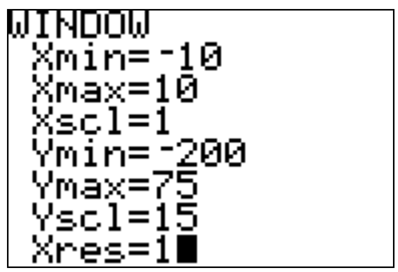



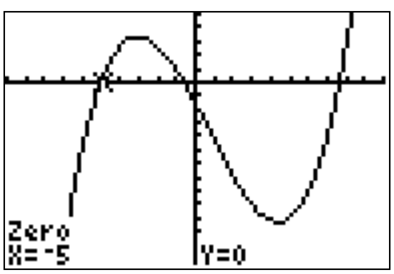



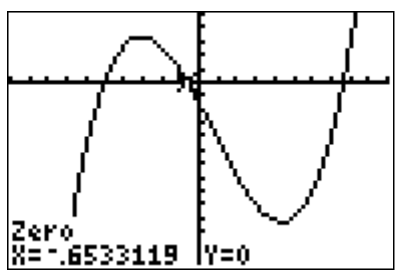

$f(x) > g(x)$ for $(-5, -0.65) \cup (7.65, \infty)$

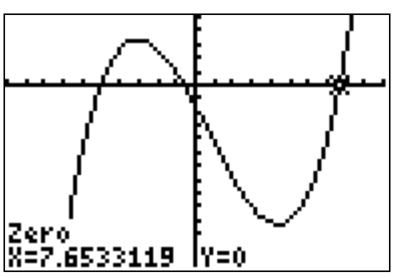
**c)** Graph $f(x) - g(x)$ when $f(x) - g(x) > 0$, $f(x) > g(x)$.




$f(x) > g(x)$ for $(-5, -0.65) \cup (7.65, \infty)$