Chapter 5 Trigonometric Functions

Chapter 5 Prerequisite Skills

Chapter 5 Prerequisite Skills Question 1 Page 250

a) 0.5878

b) 0.9659

c) –5.6713

d) –0.4142

Chapter 5 Prerequisite Skills Question 2 Page 250

a) 5.9108

b) 32.4765

c) 0.3773

d) –1.4479

Chapter 5 Prerequisite Skills Question 3 Page 250

a)
$$\begin{aligned}\sin\frac{5\pi}{4} &= \sin\left(\pi+\frac{\pi}{4}\right)\\ &= -\sin\frac{\pi}{4}\\ &= -\frac{1}{\sqrt{2}}\end{aligned}$$

b)
$$\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$$

c)
$$\begin{aligned}\tan\frac{3\pi}{4} &= \tan\left(\frac{\pi}{2}+\frac{\pi}{4}\right)\\ &= -\cot\frac{\pi}{4}\\ &= -1\end{aligned}$$

d)
$$\begin{aligned}\sin\frac{5\pi}{6} &= \sin\left(\frac{3\pi}{6}+\frac{2\pi}{6}\right)\\ &= \sin\left(\frac{\pi}{2}+\frac{\pi}{3}\right)\\ &= \cos\frac{\pi}{3}\\ &= \frac{1}{2}\end{aligned}$$

e)
$$\begin{aligned}\cos\frac{5\pi}{3} &= \cos\left(\frac{6\pi}{3}-\frac{\pi}{3}\right)\\ &= \cos\left(2\pi-\frac{\pi}{3}\right)\\ &= \cos\frac{\pi}{3}\\ &= \frac{1}{2}\end{aligned}$$

f)
$$\begin{aligned}\tan\frac{4\pi}{3} &= \tan\left(\pi+\frac{\pi}{3}\right)\\ &= \tan\frac{\pi}{3}\\ &= \sqrt{3}\end{aligned}$$

Chapter 5 Prerequisite Skills **Question 4 Page 250**

a) $\csc\frac{5\pi}{3} = \csc\left(2\pi - \frac{\pi}{3}\right)$
$= -\csc\frac{\pi}{3}$
$= -\frac{2}{\sqrt{3}}$

b) $\sec\frac{7\pi}{6} = \sec\left(\pi + \frac{\pi}{6}\right)$
$= -\sec\frac{\pi}{6}$
$= -\frac{2}{\sqrt{3}}$

c) $\cot\frac{7\pi}{4} = \cot\left(2\pi - \frac{\pi}{4}\right)$
$= -\cot\frac{\pi}{4}$
$= -1$

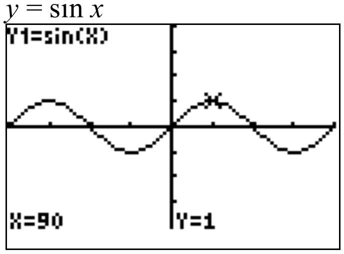
d) $\sec 2\pi = \sec 0$
$= 1$

e) $\cot\frac{3\pi}{2} = \cot\frac{\pi}{2}$
$= 0$

f) $\csc\frac{\pi}{4} = \sqrt{2}$

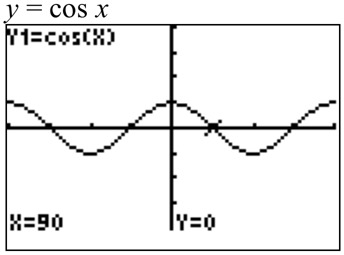
Chapter 5 Prerequisite Skills **Question 5 Page 250**

$y = \sin x$



Chapter 5 Prerequisite Skills **Question 6 Page 250**

$y = \cos x$



Chapter 5 Prerequisite Skills **Question 7 Page 250**

The graphs of the sine and cosine functions are periodic because they repeat a pattern of y-values at regular intervals of their domain.

Chapter 5 Prerequisite Skills **Question 8 Page 250**

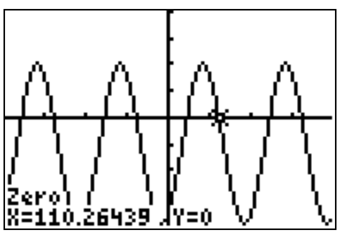
a) Since the equation is in the form: $f(x) = a\sin\left[k(x-d)\right] - c$, a is the amplitude, $\dfrac{360°}{k}$ is the period, d is the phase shift, and c is the vertical translation.

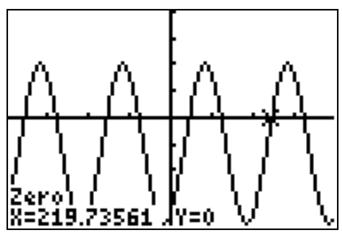
Amplitude is 3; period is 180°; phase shift of 30° right; vertical translation of 1 unit down.

b) Since the graph has amplitude of 3 and is shifted down 1 unit,
Maximum: 3 – 1 = 2; minimum: –3 – 1 = –4

c)

$$\begin{aligned}
0 &= 3\sin\left[2(x-30°)\right]-1 \\
1 &= 3\sin\left[2(x-30°)\right] \\
\frac{1}{3} &= \sin\left[2(x-30°)\right] \\
\sin^{-1}\frac{1}{3} &= 2x-60° \\
19.47° &\doteq 2x-60° \\
79.47° &\doteq 2x \\
x &\doteq 39.7°
\end{aligned}$$





The first three x-intercepts are 39.7°, 110.3°, and 219.7°.

d)

$$\begin{aligned}
y &= 3\sin\left[2(0-30°)\right]-1 \\
y &= 3\sin(-60°)-1 \\
y &\doteq -3.6
\end{aligned}$$

The y-intercept is approximately –3.6.

Chapter 5 Prerequisite Skills **Question 9 Page 250**

a) Amplitude is 2; period is 360°; phase shift of 90° left; vertical translation of 1 unit up.

b) Maximum 2 + 1 = 3, Minimum –2 + 1 = –1

c)

$$\begin{aligned} 0 &= 2\cos(x+90°)+1 \\ -1 &= 2\cos(x+90°) \\ -\frac{1}{2} &= \cos(x+90°) \\ \cos^{-1}\left(-\frac{1}{2}\right) &= x+90° \\ 120°-90° &= x \\ x &= 30° \\ 180°-x &= 150° \\ 360°+x &= 390° \end{aligned}$$

The first three x-intercepts are 30°, 150°, and 390°.

d)

$$\begin{aligned} y &= 2\cos(0+90°)+1 \\ y &= 2(0)+1 \\ y &= 1 \end{aligned}$$

The y-intercept is 1.

Chapter 5 Prerequisite Skills **Question 10 Page 250**

a) $x = 31.3°$

b) $x = 141.3°$

c) $x = 74.3°$

d) $x = 27.9°$

Chapter 5 Prerequisite Skills **Question 11 Page 250**

a) $x = 0.2$

b) $x = 2.3$

c) $x = 0.9$

d) $x = 0.2$

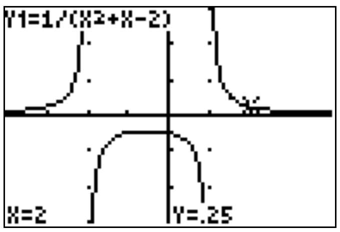
Chapter 5 Prerequisite Skills **Question 12 Page 251**

a) $$\begin{aligned} x^2 + x - 2 &= 0 \\ (x+2)(x-1) &= 0 \end{aligned}$$

The equations of the vertical asymptotes are $x = -2$ and $x = 1$.

b) $y = 0$

c)



Chapter 5 Prerequisite Skills **Question 13 Page 251**

a) The instantaneous rate of change at $x = 2$ is 3. The function is linear so the rate of change is the slope of the line.

b) The instantaneous rate of change is the same as the average rate of change.

Chapter 5 Prerequisite Skills **Question 14 Page 251**

a) $6 \times \frac{60}{25} = 14.4$

Justine rode at a rate of 14.4 km/h.

Chapter 5 Prerequisite Skills **Question 15 Page 251**

a) $h(0.1) = 20(0.1) - 5(0.1)^2$
$= 2 - 0.05$
$= 1.95$

$h(0.5) = 20(0.5) - 5(0.5)^2$
$= 10 - 1.25$
$= 8.75$

$$\begin{aligned}\text{Average rate of change} &= \frac{8.75 - 1.95}{0.5 - 0.1} \\ &= \frac{6.8}{0.4} \\ &= 17\end{aligned}$$

The average rate of change from 0.1 s to 0.5 s is 17 m/s.

b) $h(0.499) = 20(0.499) - 5(0.499)^2$
$\doteq 9.98 - 1.245\,005$
$\doteq 8.734\,995$

$h(0.4999) = 20(0.4999) - 5(0.4999)^2$
$\doteq 9.998 - 1.249\,500\,05$
$\doteq -8.748\,499\,95$

$$\begin{aligned}\text{Average rate of change} &= \frac{8.75 - 8.734\,995}{0.5 - 0.499} \\ &= \frac{0.015\,005}{0.001} \\ &\doteq 15.005\end{aligned}$$

$$\begin{aligned}\text{Average rate of change} &= \frac{8.75 - 8.748\,499\,95}{0.5 - 0.4999} \\ &= \frac{0.001\,500\,05}{0.0001} \\ &\doteq 15.0005\end{aligned}$$

The instantaneous rate of change of height at 0.5 s is approximately 15 m/s.

c) The speed at 0.5 s is represented by the instantaneous rate of change.

Chapter 5 Section 1 **Graphs of Sine, Cosine, and Tangent Functions**

Chapter 5 Section 1 **Question 1 Page 258**

a) The maximum value is $y = 4 + 1 = 5$; the values of x where it occurs are $x = -\frac{3\pi}{2}$ and $\frac{\pi}{2}$.

Maxima: $\left(-\frac{3\pi}{2}, 5\right), \left(\frac{\pi}{2}, 5\right)$

The minimum value is $4 + (-1) = 3$; the values of x where it occurs are $x = -\frac{\pi}{2}$ and $\frac{3\pi}{2}$.

Minima: $\left(-\frac{\pi}{2}, 3\right), \left(\frac{3\pi}{2}, 3\right)$

b) The maximum value is $y = -5 + 1$ or -4.
The values of x where it occurs are $x = -2\pi$, 0, and 2π.
Maxima: $(-2\pi, -4)$, $(0, -4)$, $(2\pi, -4)$

The minimum value is $y = -5 + (-1)$ or -6.
The values of x where it occurs are $x = -\pi$ and π.
Minima: $(-\pi, -6)$, $(\pi, -6)$

c) The maximum value is $y = 1 - 2$ or -1.

The values of x where it occurs are $x = -\frac{3\pi}{2}$ and $\frac{\pi}{2}$.

Maxima: $\left(-\frac{3\pi}{2}, -1\right), \left(\frac{\pi}{2}, -1\right)$

The minimum value is $y = -1 - 2$ or -3.

The values of x where it occurs is $x = -\frac{\pi}{2}$ and $\frac{3\pi}{2}$.

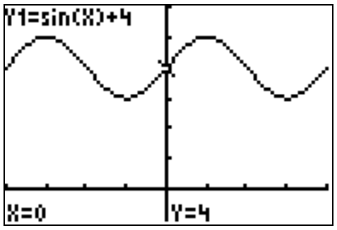
Minima: $\left(-\frac{\pi}{2}, -3\right), \left(\frac{3\pi}{2}, -3\right)$

d) The maximum value is $y = 1 + 1$ or 2.
The values of x where it occurs are $x = -2\pi$, 0, and 2π.
Maxima: $(-2\pi, 2)$, $(0, 2)$, $(2\pi, 2)$

The minimum value is $y = -1 + 1$ or 0.
The values of x where it occurs are $x = -\pi$ and π.
Minima: $(-\pi, 0)$, $(\pi, 0)$

Chapter 5 Section 1 **Question 2 Page 258**

a)



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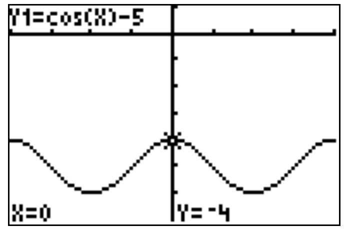
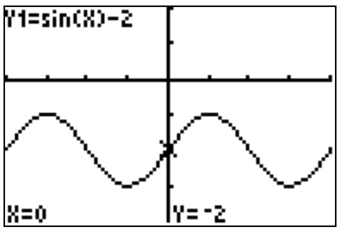
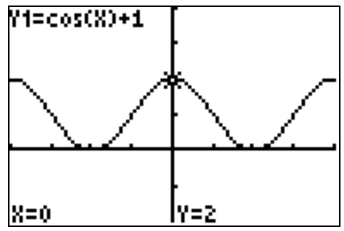


c)



d)



Chapter 5 Section 1 **Question 3 Page 258**

a) $y = 3\sin x$

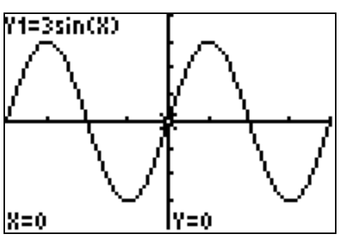
b) $y = 5\cos x$

c) $y = -4\sin x$

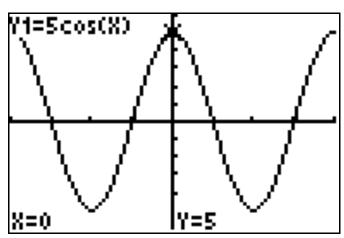
d) $y = -2\cos x$

Chapter 5 Section 1 **Question 4 Page 258**

a)



b)



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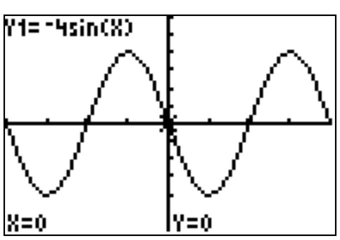
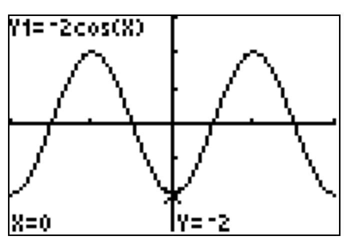


d)



Chapter 5 Section 1 **Question 5 Page 258**

a) $y = \sin\left(x + \frac{\pi}{3}\right)$

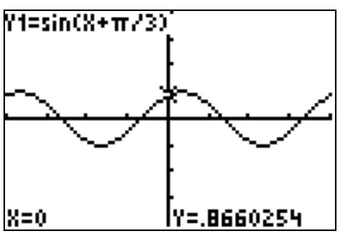
b) $y = \cos\left(x - \frac{5\pi}{6}\right)$

c) $y = \sin\left(x + \frac{3\pi}{4}\right)$

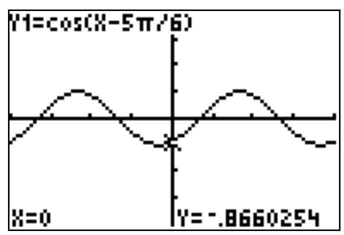
d) $y = \cos\left(x - \frac{4\pi}{3}\right)$

Chapter 5 Section 1 **Question 6 Page 258**

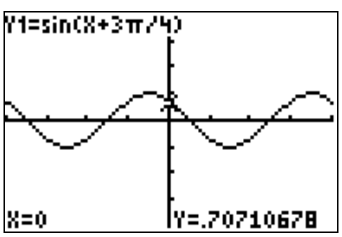
a)



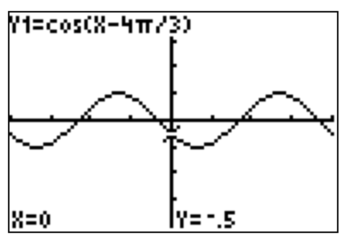
b)



c)



d)



Chapter 5 Section 1 **Question 7 Page 258**

a) $k = \dfrac{2\pi}{\frac{\pi}{2}} = 4$

So, $y = \sin 4x$.

b) $k = \dfrac{2\pi}{\frac{3\pi}{2}} = \dfrac{4}{3}$

So, $y = \cos\dfrac{4}{3}x$.

c) $k = \dfrac{2\pi}{6\pi} = \dfrac{1}{3}$

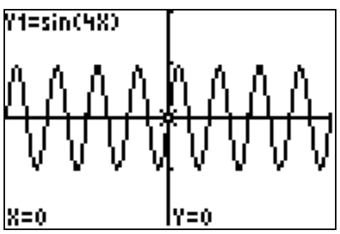
So, $y = \sin\dfrac{1}{3}x$

d) $k = \dfrac{2\pi}{\pi} = 2$

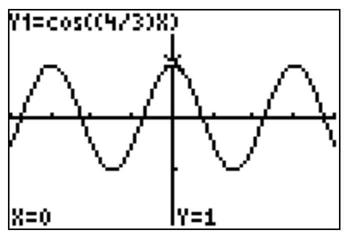
So, $y = \cos 2x$

Chapter 5 Section 1 **Question 8 Page 258**

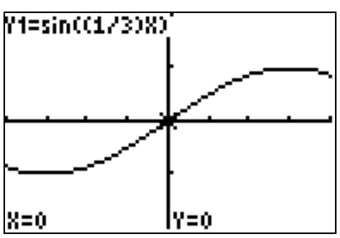
a)



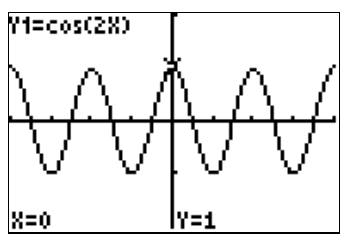
b)



c)



d)

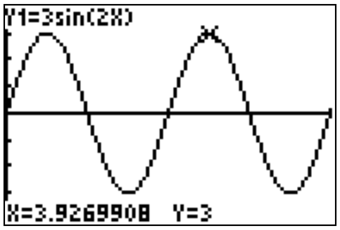


Chapter 5 Section 1 **Question 9 Page 258**

a) $k = \dfrac{2\pi}{\pi} = 2$

So, $y = 3\sin 2x$

b)



Chapter 5 Section 1 **Question 10 Page 258**

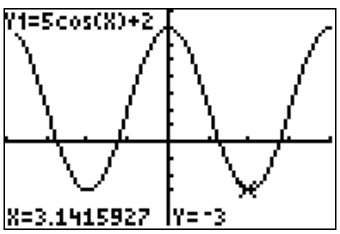
a) $a = \dfrac{7-(-3)}{2} = 5$

The amplitude is 5.

b) $7 - 5 = 2$

The vertical translation is up 2 units.

c)



Chapter 5 Section 1 **Question 11 Page 258**

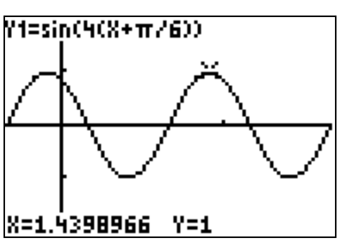
a) Period = $\dfrac{\pi}{3} - \left(-\dfrac{\pi}{6}\right) = \dfrac{\pi}{2}$

b) Since one cycle of $y = \sin x$ begins at $x = 0$, this function has a phase shift of $\dfrac{\pi}{6}$ to the left.

c) $k = \dfrac{2\pi}{\frac{\pi}{2}} = 4$

So, $y = \sin\left[4\left(x + \dfrac{\pi}{6}\right)\right]$

d)



Chapter 5 Section 1 **Question 12 Page 258**

a) The period of the A-note is $\frac{1}{440}$.

b)
$$\begin{aligned}k &= \frac{2\pi}{\frac{1}{440}}\\ &= 880\pi\end{aligned}$$

Chapter 5 Section 1 **Question 13 Page 258**

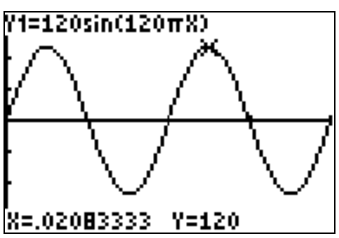
a) The amplitude of the model is 120.

b) The period of the model is $\frac{1}{60}$.

c) $k = \frac{2\pi}{\frac{1}{60}} = 120\pi$

$y = 120\sin(120\pi x)$

d)



Chapter 5 Section 1 **Question 14 Page 259**

a) The function is odd. The graph of $y = \sin(-x)$ is equivalent to the graph of $y = -\sin x$.
Proof

L.S. $= \sin(-x)$
$= \sin(0 - x)$
$= \sin 0 \cos x - \sin x \cos 0$
$= 0 - \sin x$
$= -\sin x$

R.S. $= -\sin x$

L.S. = **R.S.**

b) The function is even. The graph of $y = \cos(-x)$ is equivalent to the graph of $y = \cos x$.
Proof:

L.S. $= \cos(-x)$
$= \cos(0 - x)$
$= \cos 0 \cos x + \sin 0 \sin x$
$= \cos x$

R.S. $= \cos x$

L.S. = R.S.

c) The function is odd. The graph of $y = \tan(-x)$ is equivalent to the graph of $y = -\tan x$.
Proof:

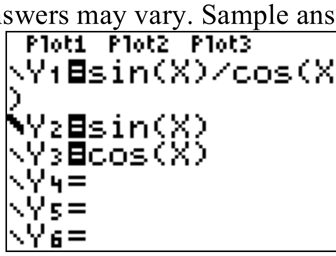
L.S. $= \tan(-x)$
$= \tan(0 - x)$
$= \dfrac{\tan 0 - \tan x}{1 + \tan 0 \tan x}$
$= \dfrac{0 - \tan x}{1 + 0}$
$= -\tan x$

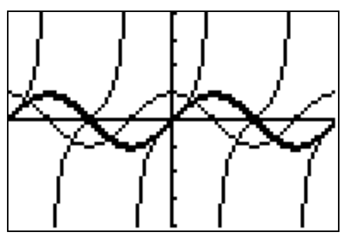
R.S. $= -\tan x$

L.S. = R.S.

Chapter 5 Section 1 **Question 15 Page 259**

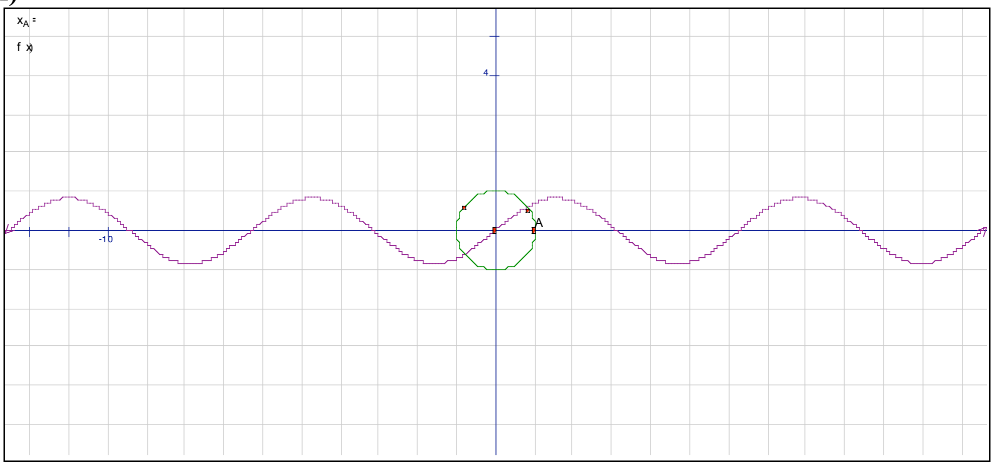
Answers may vary. Sample answer:





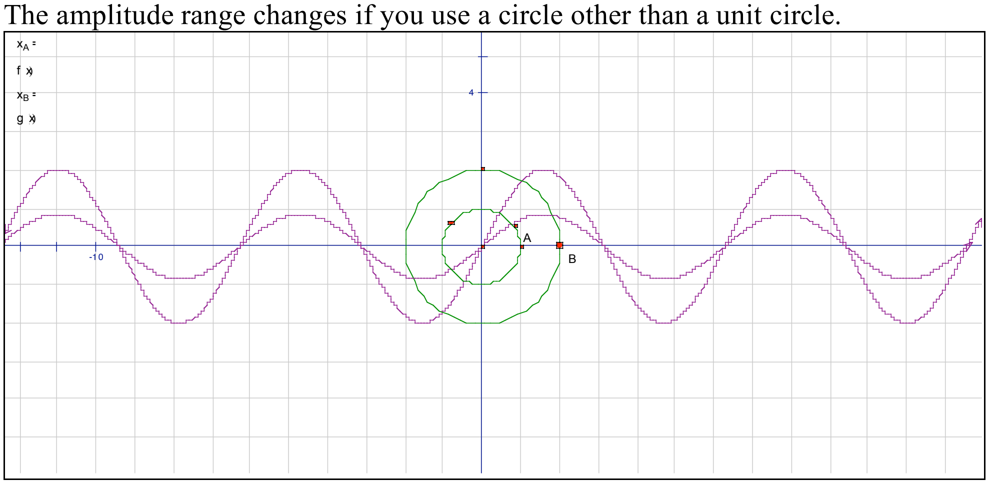
Where $y = \cos x$ and $y = \sin x$ intersect, $y = \tan x$ equals 1.
Where $y = \cos x$ equals zero, $y = \tan x$ is undefined.
Where $y = \sin x$ equals zero, $y = \tan x$ equals 0.

a)-f)

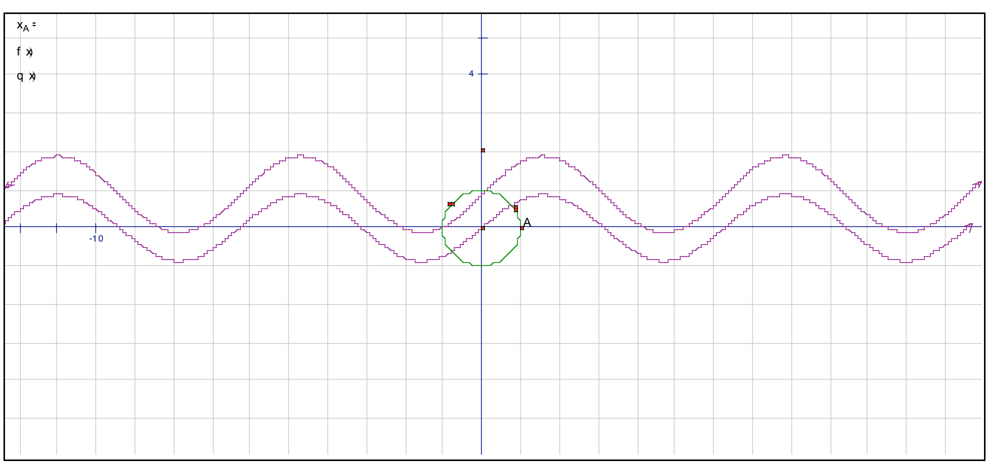


g) For positive x_A, the amplitude gets larger as x_A gets larger.
For negative x_A, the amplitude gets larger as x_A gets larger, but sin x is reflected in the x-axis.

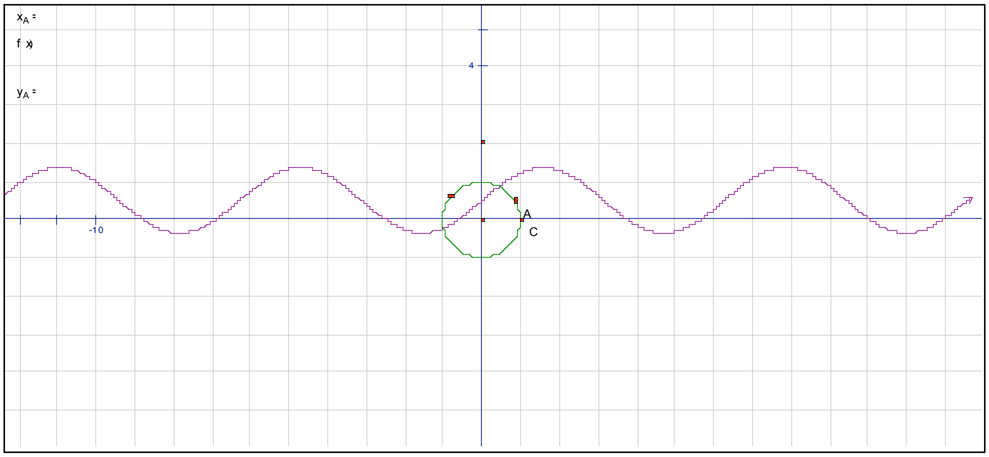
h) The amplitude range changes if you use a circle other than a unit circle.



i)



j)



Chapter 5 Section 1 **Question 17 Page 259**

a) $a = \frac{4-1}{2}$

$= \frac{3}{2}$

The amplitude of the function is $\frac{3}{2}$.

b) $c = 4 - \frac{3}{2}$

$= \frac{5}{2}$

The vertical translation of the function is $\frac{5}{2}$ units up.

c) The desired period is 60 s, since it take 60 s to complete one revolution.

d) $k = \frac{2\pi}{60}$

$= \frac{\pi}{30}$

Chapter 5 Section 1 **Question 18 Page 259**

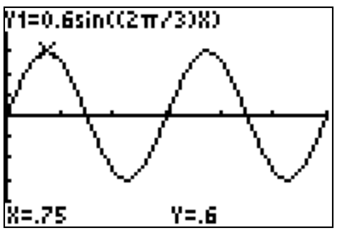
a) period = $\dfrac{60}{20}$ = 3 s/cycle

$$k = \frac{2\pi}{3}$$

$$\begin{aligned} a &= \frac{1.2}{2} \\ &= 0.6 \end{aligned}$$

$$d = 0.6\sin\left(\frac{2\pi}{3}t\right)$$

b)

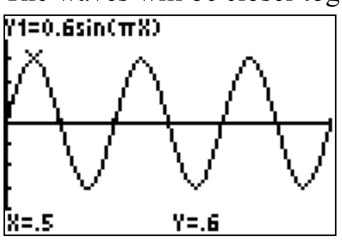


c) period = $\dfrac{60}{30}$ = 2 s/cycle

$$k = \frac{2\pi}{2} = \pi$$

The amplitude remains the same.

The waves will be closer together. The equation becomes $d = 0.6\sin(\pi t)$.



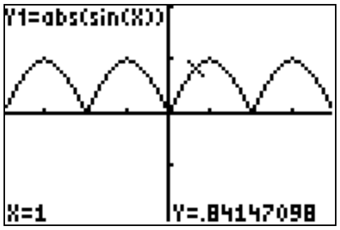
Chapter 5 Section 1 **Question 19 Page 260**

The solutions for Achievement Check can be found in the Teacher's Resource.

Chapter 5 Section 1 **Question 20 Page 260**

a) Answers may vary. For example, I predict that there will be no values of the function below the x-axis.

b)



c) The relation is a function since it satisfies the vertical line test.

d) The relation is even; it is symmetrical about the y–axis.
Prove that $|\sin(-x)| = |\sin x|$.

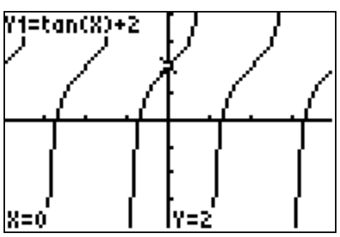
L.S. $= |\sin(-x)|$
$= |\sin(0 - x)|$
$= |\sin 0 \cos x - \sin x \cos 0|$
$= |-\sin x|$
$= |\sin x|$

R.S. $= |\sin x|$

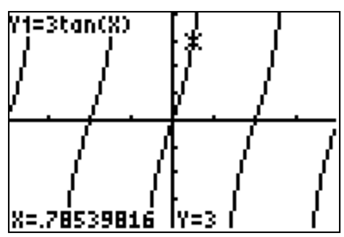
Since **L.S.** = **R.S.**, $y = |\sin x|$ is an even function.

Chapter 5 Section 1 **Question 21 Page 261**

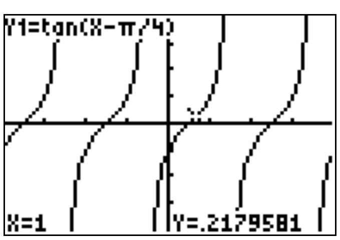
a)



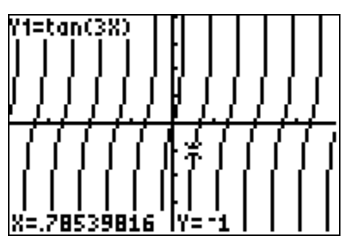
b)



c)

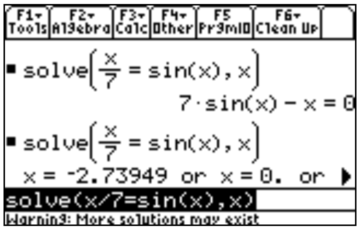


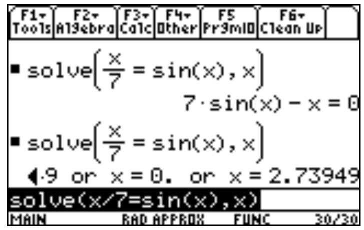
d)



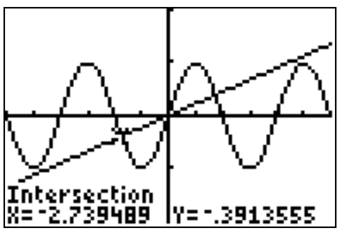
Chapter 5 Section 1 **Question 22 Page 260**

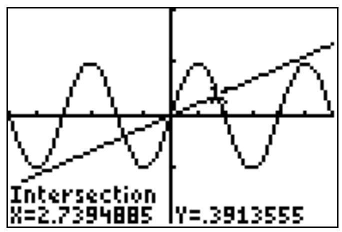
a) The graph of $y = \dfrac{x}{7}$ will intersect the graph of y = sin x 3 times: at the origin and 1 point either side of the origin.

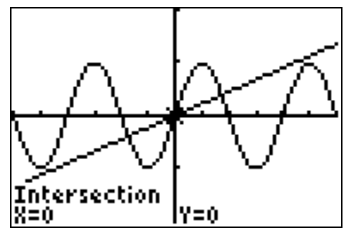




b)







Chapter 5 Section 1 **Question 23 Page 260**

a) $\sin^{-1}\left(\dfrac{\sqrt{3}}{2}\right) = \dfrac{\pi}{3}$

b) $\cos^{-1}\left(\dfrac{\sqrt{3}}{2}\right) = \dfrac{\pi}{6}$

c) $\tan^{-1}(1) = \dfrac{\pi}{4}$

d) $\cos^{-1}\left(-\dfrac{1}{\sqrt{2}}\right) = \dfrac{3\pi}{4}$

Chapter 5 Section 1 **Question 24 Page 260**

a) i)
$$\begin{aligned} x^2 + y^2 &= r^2 \\ r^2 &= 4 \\ r &= 2 \end{aligned}$$

ii) $x = r\cos\theta$
$y = r\sin\theta$

$$\begin{aligned} 3x + 4y &= 5 \\ 3r\cos\theta + 4r\sin\theta &= 5 \\ r(3\cos\theta + 4\sin\theta) &= 5 \\ r &= \frac{5}{3\cos\theta + 4\sin\theta} \end{aligned}$$

iii) $x^2 + y^2 - 4y = \sqrt{x^2 + y^2}$

$$\begin{aligned} r^2 - 4r\sin\theta &= \sqrt{r^2} \qquad (x^2 + y^2 = r^2, y = r\sin\theta) \\ r^2 &= r + 4r\sin\theta \\ r^2 &= r(1 + 4\sin\theta) \\ r &= 1 + 4\sin\theta \end{aligned}$$

b) i) $x^2 + y^2 = 36 \qquad (r^2 = x^2 + y^2)$

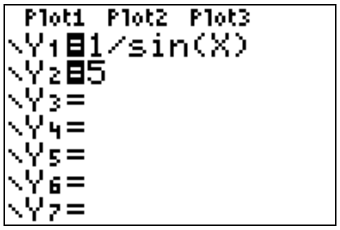
ii)
$$\begin{aligned} x &= r\cos\theta \\ \cos\theta &= \frac{x}{r} \\ r &= 3\cos\theta \\ r &= 3\left(\frac{x}{r}\right) \\ r^2 &= 3x \\ x^2 + y^2 &= 3x \qquad (r^2 = x^2 + y^2) \end{aligned}$$

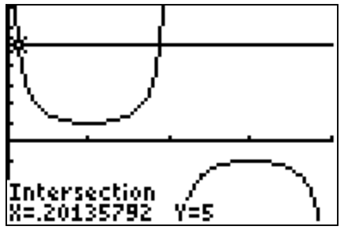
iii) $y = r\sin\theta \qquad x = r\cos\theta$

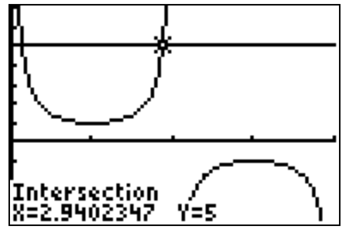
$\frac{y}{r} = \sin\theta \qquad x = r\cos\theta$

$$\begin{aligned} r &= 2\sin\theta + 2\cos\theta \\ r &= 2\left(\frac{y}{r}\right) + 2\left(\frac{x}{r}\right) \\ r &= \frac{2y + 2x}{r} \\ r^2 &= 2y + 2x \qquad (r^2 = x^2 + y^2) \\ x^2 + y^2 &= 2x + 2y \end{aligned}$$

Chapter 5 Section 2 **Question 1 Page 267**

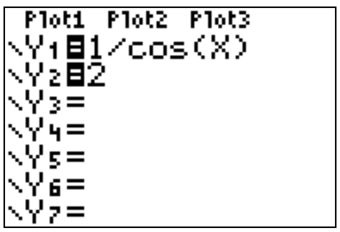


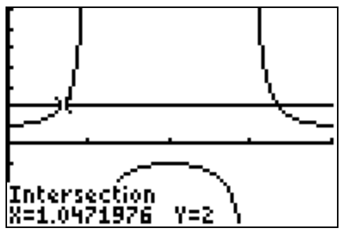


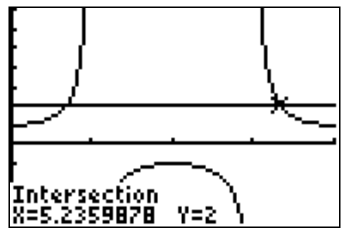


The values of x such that $\csc x = 5$ are $x \doteq 0.20$ and $x \doteq 2.94$.

Chapter 5 Section 2 **Question 2 Page 267**

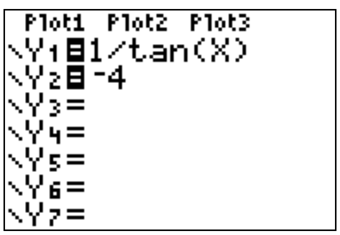


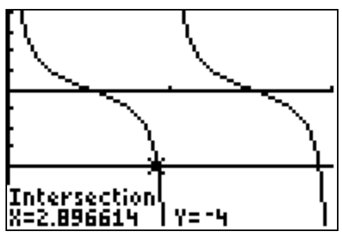


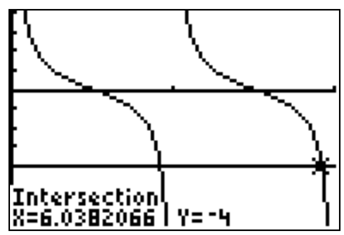


The values of x such that $\sec x = 2$ are $x \doteq 1.05$ and $x \doteq 5.24$.

Chapter 5 Section 2 **Question 3 Page 267**







The values of x such that $\cot x = -4$ are $x \doteq 2.90$ and $x \doteq 6.04$.

Chapter 5 Section 2 **Question 4 Page 267**

a) Answers may vary. For example, the cosecant function is the reciprocal of the sine function and $\sin^{-1}$ is the opposite operation of sine.

b) $\csc \frac{1}{\sqrt{2}} \doteq 1.5393$

$\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}$ or about 0.7854

Chapter 5 Section 2 **Question 5 Page 267**

a) Answers may vary. For example, the secant function is the reciprocal of the cosine function and $\cos^{-1}$ is the opposite operation of cosine.

b) $\sec\frac{\sqrt{3}}{2} \doteq 1.5425$

$\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{6}$ or about 0.5236

Chapter 5 Section 2 **Question 6 Page 267**

a) Answers may vary. For example, the cotangent function is the reciprocal of the tangent function and $\tan^{-1}$ is the opposite operation of tangent.

b) $\cot 1 \doteq 0.6421$

$\tan^{-1}(1) = \frac{\pi}{4}$ or about 0.7854

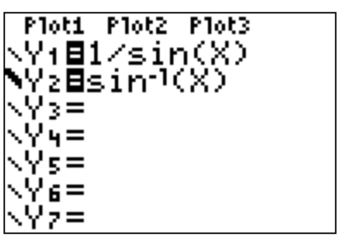
Chapter 5 Section 2 **Question 7 Page 267**

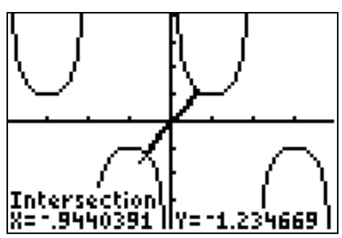
a) $\sec x = \csc\left(x + \frac{\pi}{2}\right)$ or $\sec x = \csc\left(x - \frac{3\pi}{2}\right)$

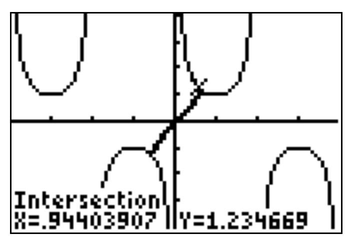
b) Answers may vary. Yes, the phase shift can be increased or decreased by one period, 2π.

Chapter 5 Section 2 **Question 8 Page 267**

a)







b) The values of x where $\csc x = \sin^{-1} x$ are $x \doteq -0.9440$ and $x \doteq 0.9440$.

c) $\csc(0.9440) = 1.235$ and $\csc(-0.9440) = 1.235$

$\sin^{-1}(0.9440) = 1.235$ $\qquad$ $\sin^{-1}(-0.9440) = 1.235$

Chapter 5 Section 2 **Question 9 Page 268**

a) $\sec x = \dfrac{d}{w}$

$d = w\sec x$

b) The minimum value of x is 0.

$$\begin{aligned}\tan x &= \frac{2w}{w}\\ &= 2\end{aligned}$$

$\tan^{-1}(2) \doteq 1.1071$

The maximum value of x is approximately 1.1071.

The range is approximately $0 \le x \le 1.1071$.

c) The minimum distance is *w/*

$d^2 = w^2 + (2w)^2$

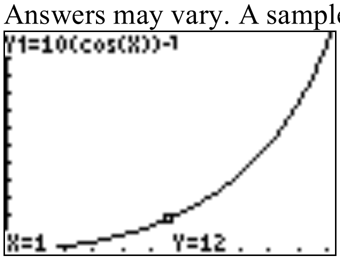
$d^2 = 5w^2$

$d = \sqrt{5}w$

The maximum distance in terms of w is $\sqrt{5}w$.

Assuming the lifeguard swims a portion of the distance, then $w \le d \le \sqrt{5}w$.

d) Answers may vary. A sample solution is shown.



e) Answers may vary. A sample solution is shown.
The total distance will be shorter.
It is dangerous to run on the deck; it would be easy to fall and get hurt and thus take longer to get to the troubled swimmer.
The lifeguard can swim faster, than she can run.

Chapter 5 Section 2 **Question 10 Page 268**

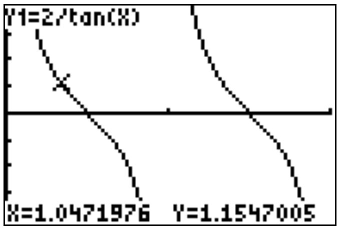
a) $\cot x = \dfrac{d}{2}$

$d = 2\cot x$

b) $d = 2\cot\frac{\pi}{3}$
$\doteq 1.15$

The awning must project approximately 1.15 m from the wall.

c)



d) As x approaches 0, d approaches infinity. This means that the angle of elevation on the summer solstice approaches the horizon and so the length of the awning approaches infinity.

As x approaches $\frac{\pi}{2}$, d approaches 0. This means that the angle of elevation on the summer solstice approaches an overhead location and the length of the awning approaches 0.

Chapter 5 Section 2 **Question 11 Page 268**

a)
$$\cot x = \frac{rg}{v^2}$$
$$\cot x = \frac{3.5(9.8)}{(5.4)^2}$$
$$\cot x \doteq 1.1763$$
$$\frac{1}{\tan x} \doteq 1.1763$$
$$\tan x \doteq \frac{1}{1.1763}$$
$$\tan x \doteq 0.8501$$
$$x \doteq \tan^{-1}(0.8501)$$
$$x \doteq 0.70$$

b) No, the angle does not double.

c) No, the angle is not halved.

Proof: $\cot x = \dfrac{rg}{v^2}$

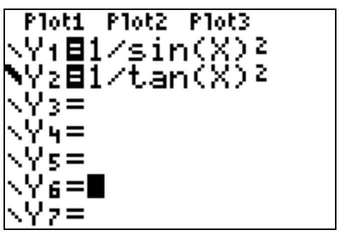
$$\cot x = \frac{(3.5\times 2)(9.8)}{5.4^2}$$
$$\cot x \doteq 2.3525$$
$$\frac{1}{\tan x} \doteq 2.3525$$
$$\tan x \doteq \frac{1}{2.3525}$$
$$\tan x \doteq 0.4251$$
$$x \doteq \tan^{-1}(0.4251)$$
$$x \doteq 0.40$$

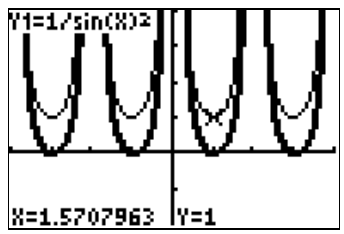
Proof: $\cot x = \dfrac{rg}{v^2}$

$$\cot x = \frac{3.5(9.8)}{(5.4\times 2)^2}$$
$$\cot x \doteq 0.2941$$
$$\frac{1}{\tan x} \doteq 0.2941$$
$$\tan x \doteq \frac{1}{0.2941}$$
$$\tan x \doteq 3.4006$$
$$x \doteq \tan^{-1}(3.4006)$$
$$x \doteq 1.28$$

Chapter 5 Section 2 **Question 12 Page 268**

a)





So, by looking at the graphs, $\csc^2 x \neq \cot^2 x$.

Answers may vary. For example, $\csc^2 x - 1 = \cot^2 x$.

b) **L.S.** $= \csc^2 x - 1$

$$= \frac{1}{\sin^2 x} - \frac{\sin^2 x}{\sin^2 x}$$
$$= \frac{1-\sin^2 x}{\sin^2 x}$$
$$= \frac{\cos^2 x}{\sin^2 x} \quad \text{(Pythagorean identity)}$$

R.S. $= \cot^2 x$

$$= \frac{1}{\tan^2 x}$$
$$= \frac{\cos^2 x}{\sin^2 x}$$

Since **L.S.** = **R.S.**, $\csc^2 x - 1 = \cot^2 x$ is an identity.

Chapter 5 Section 2 **Question 13 Page 269**

a) $\sec x = \dfrac{d}{500}$

$$d = 500\sec x$$

b) $d = 500\sec\dfrac{5\pi}{12}$

$d = 500\sec\left(\dfrac{3\pi}{12} + \dfrac{2\pi}{12}\right)$

$d = 500\sec\left(\dfrac{\pi}{4} + \dfrac{\pi}{6}\right)$

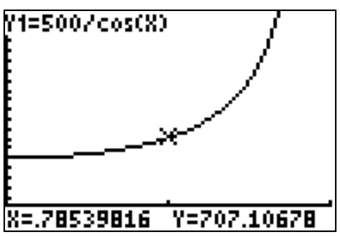
$d = \dfrac{500}{\left(\cos\dfrac{\pi}{4}\cos\dfrac{\pi}{6}\right) - \left(\sin\dfrac{\pi}{4}\sin\dfrac{\pi}{6}\right)}$

$d = \dfrac{500}{\left(\dfrac{1}{\sqrt{2}}\right)\left(\dfrac{\sqrt{3}}{2}\right) - \left(\dfrac{1}{\sqrt{2}}\right)\left(\dfrac{1}{2}\right)}$

$d = \dfrac{500}{\dfrac{\sqrt{3}-1}{2\sqrt{2}}}$

$d = \dfrac{1000\sqrt{2}}{\sqrt{3}-1}$

c)

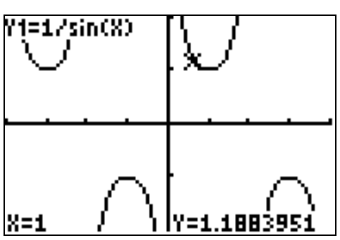


Chapter 5 Section 2 **Question 14 Page 269**

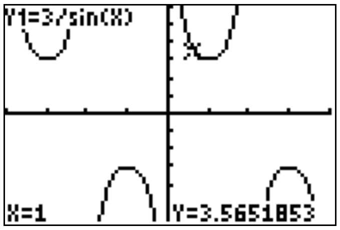
Solutions for the Achievement Check questions are shown in the Teacher's Resource.

Chapter 5 Section 2 **Question 15 Page 269**

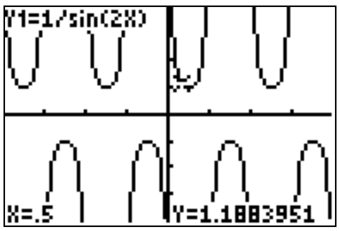
a)



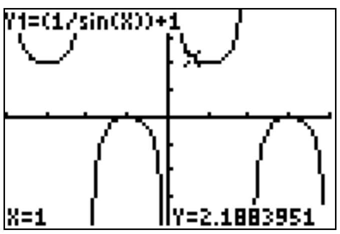
b) i) For each x value, the y value will be multiplied by 3.



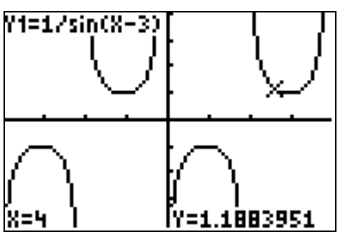
ii) The period is π.



iii) The graph is shifted up 1.

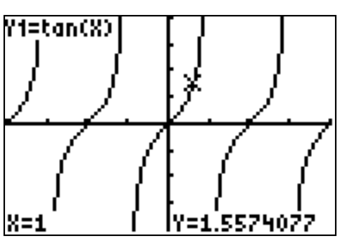


iv) The graph has a phase shift of 3 to the right.

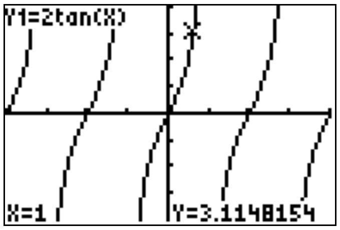


Chapter 5 Section 2 **Question 16 Page 269**

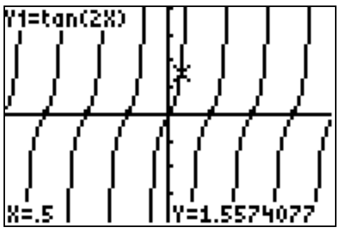
a) $y = \tan x$



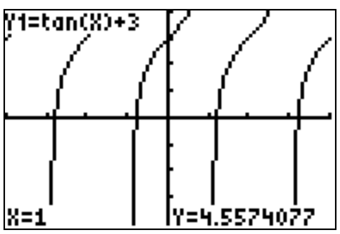
b) i) For each x value, the y value is multiplied by 2.



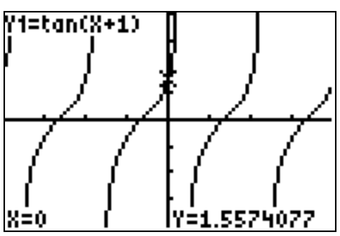
ii) The period is π.



iii) The graph is shifted up 3 units.



iv) The graph has a phase shift of 1 unit left.



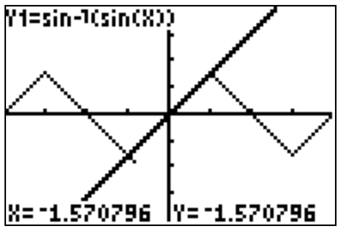
Chapter 5 Section 2 **Question 17 Page 269**

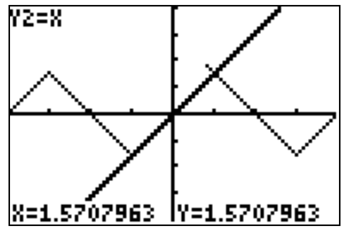
a) Proof:

$$\begin{aligned}\textbf{L.S.} &= \sin^{-1}(\sin 0.5) \qquad \textbf{R.S.} = 0.5\\ &\doteq \sin^{-1}(0.4794)\\ &= 0.5\end{aligned}$$

L.S. = R.S.

b) No, the equation is only true for $x \in \left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right]$.

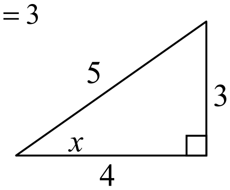




Chapter 5 Section 2 **Question 18 Page 269**

Using the Pythagorean theorem, the measure of the unknown side is 3 units.

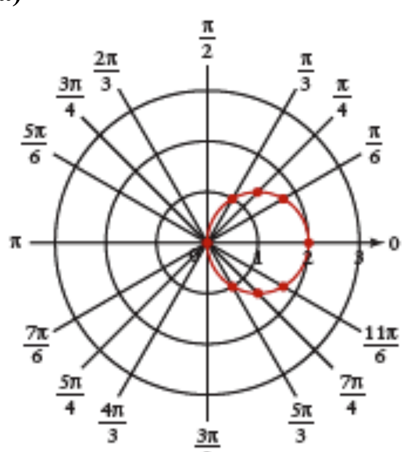
$$
\begin{aligned}
a^2 &= 25 - 16 \\
a &= \sqrt{9} \\
a &= 3
\end{aligned}
$$



$$\sin(x) = \frac{3}{5}$$

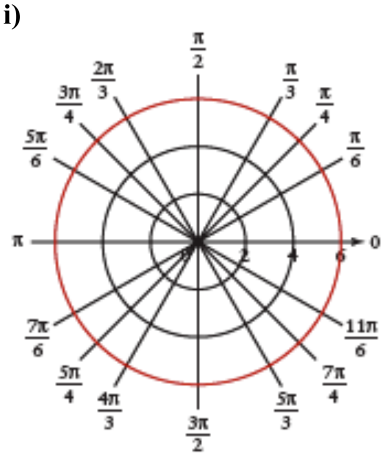
Chapter 5 Section 2 **Question 19 Page 269**

a)

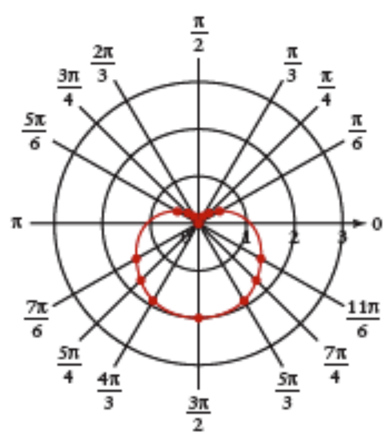


θ	$r = 2\cos\theta$	(r, θ)
0	**2**	**(2, 0)**
$\frac{\pi}{6}$	$\sqrt{3}$	$\left(\sqrt{3}, \frac{\pi}{6}\right)$
$\frac{\pi}{4}$	$\sqrt{2}$	$\left(\sqrt{2}, \frac{\pi}{4}\right)$
$\frac{\pi}{3}$	**1**	$\left(1, \frac{\pi}{3}\right)$
$\frac{\pi}{2}$	**0**	$\left(0, \frac{\pi}{2}\right)$
$\frac{2\pi}{3}$	**–1**	
$\frac{3\pi}{4}$	$-\sqrt{2}$	$\left(-\sqrt{2}, \frac{3\pi}{4}\right)$
$\frac{5\pi}{6}$	$-\sqrt{3}$	$\left(-\sqrt{3}, \frac{5\pi}{6}\right)$
π	**–2**	$(-2, \pi)$

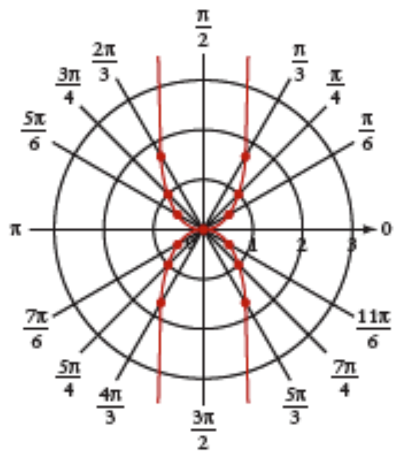
b) i)



ii)



iii)

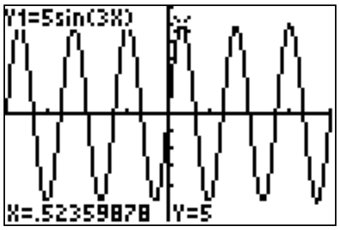


$\frac{\pi}{2}$
$\frac{2\pi}{3}$
$\frac{\pi}{3}$
$\frac{3\pi}{4}$
$\frac{\pi}{4}$
$\frac{5\pi}{6}$
$\frac{\pi}{6}$
π
0
1
2
3
$\frac{7\pi}{6}$
$\frac{11\pi}{6}$
$\frac{5\pi}{4}$
$\frac{7\pi}{4}$
$\frac{4\pi}{3}$
$\frac{5\pi}{3}$
$\frac{3\pi}{2}$

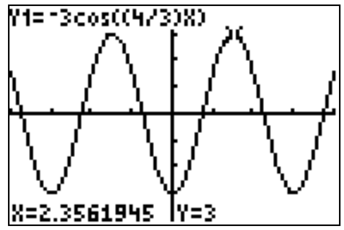
Chapter 5 Section 3 **Sinusodial Functions of the Form $f(x) = a\sin[k(x-d)] + c$ and $f(x) = a\cos[k(x-d)] + c$**

Chapter 5 Section 3 **Question 1 Page 275**

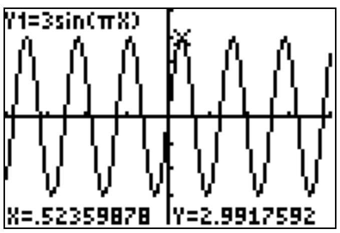
a) Amplitude = 5; Period: $\dfrac{2\pi}{3}$



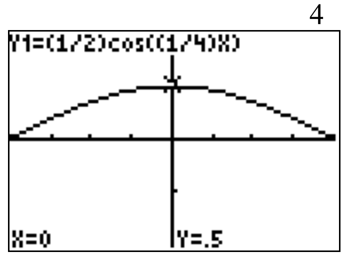
b) Amplitude = 3; Period: $\dfrac{2\pi}{\frac{4}{3}} = \dfrac{3\pi}{2}$



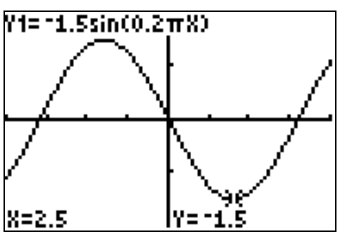
c) Amplitude = 3; period: $\dfrac{2\pi}{\pi} = 2$



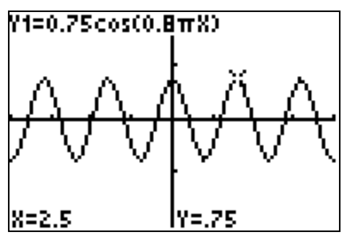
d) Amplitude $= \dfrac{1}{2}$; period: $\dfrac{2\pi}{\frac{1}{4}} = 8\pi$



e) Amplitude = 1.5; period: $\dfrac{2\pi}{0.2\pi} = 10$



f) Amplitude: = 0.75; period: $\dfrac{2\pi}{0.8\pi} = 2.5$



Chapter 5 Section 3 **Question 2 Page 275**

a) From the graph it is a sine function.

The amplitude is 3 and the period is $\dfrac{\pi}{2}$.

$$k = \frac{2\pi}{\frac{\pi}{2}}$$
$$= 4$$

Equation: $y = 3\sin 4x$

b) From the graph it is a cosine function.

The amplitude is $\frac{1}{2}$ and the period is 1.

$$k = \frac{2\pi}{1}$$
$$= 2\pi$$

Equation: $y = \frac{1}{2}\cos 2\pi x$

Chapter 5 Section 3 **Question 3 Page 276**

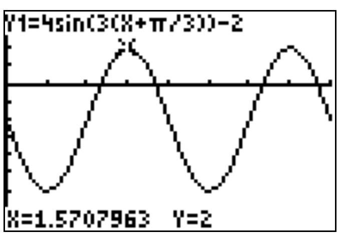
a) The amplitude is 4.

b) The period is $\frac{2\pi}{3}$.

c) The phase shift is $\frac{\pi}{3}$ rad to the left.

d) The vertical translation is 2 units down.

e)



Chapter 5 Section 3 **Question 4 Page 276**

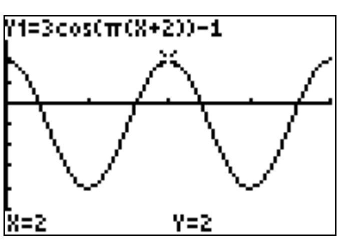
a) The amplitude is 3.

b) The period is $\frac{2\pi}{\pi} = 2$.

c) The phase shift is 2 rad to the left.

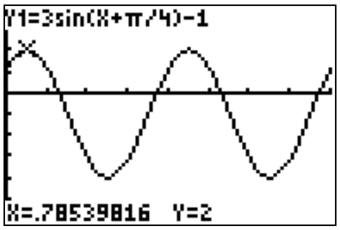
d) The vertical translation is 1 unit down.

e)

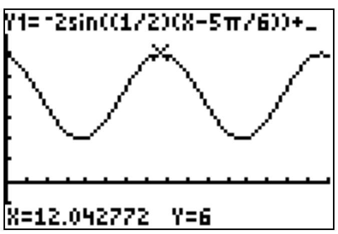


Chapter 5 Section 3 **Question 5 Page 276**

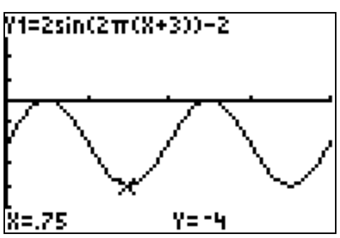
a) $y = 3\sin\left(x + \frac{\pi}{4}\right) - 1$: amplitude is 3; period is 2π; phase shift is $\frac{\pi}{4}$ rad left; vertical translation is 1 unit down.



b) $y = -2\sin\left[\frac{1}{2}\left(x - \frac{5\pi}{6}\right)\right] + 4$: amplitude is 2; period: $\frac{2\pi}{\frac{1}{2}}$ = 4π; phase shift is $\frac{5\pi}{6}$ rad to the right; vertical translation of 4 units up.

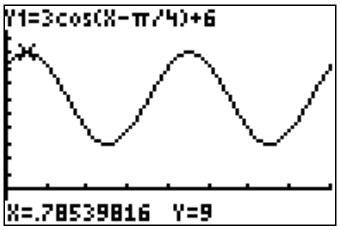


c) $y = 2\sin\left[2\pi\left(x + 3\right)\right] - 2$: amplitude is 2; period: $\frac{2\pi}{2\pi}$ = 1; phase shift is 3 rad left; vertical translation of 2 units down.

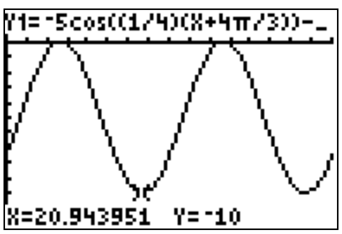


Chapter 5 Section 3 **Question 6 Page 276**

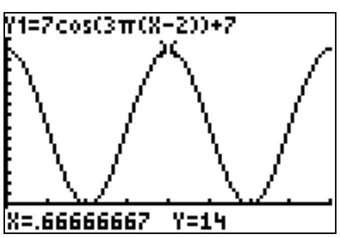
a) $y = 3\cos\left(x - \frac{\pi}{4}\right) + 6$: amplitude is 3; period: 2π; phase shift is $\frac{\pi}{4}$ rad to the right; vertical translation of 6 units up.



b) $y = -5\cos\left[\frac{1}{4}\left(x + \frac{4\pi}{3}\right)\right] - 5$: amplitude is 5; period: $\dfrac{2\pi}{\frac{1}{4}}$ = 8π; phase shift $\frac{4\pi}{3}$ rad to the left, vertical translation 5 units down.



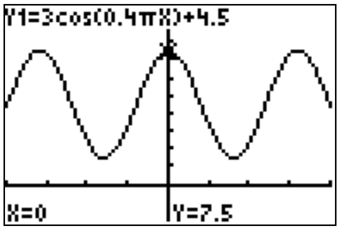
c) $y = 7\cos\left[3\pi\left(x - 2\right)\right] + 7$: amplitude is 7; period: $\frac{2\pi}{3\pi} = \frac{2}{3}$; phase shift 2 rad to the right; vertical translation 7 units up.



Chapter 5 Section 3 **Question 7 Page 276**

a) The amplitude, period and vertical translation will be the same. The phase shift will be different. Since the maximum value of the cosine function normally occurs at the beginning (end) of the period. There is no phase shift.
Equation: $h = 3\cos(0.4\pi t) + 4.5$

b)

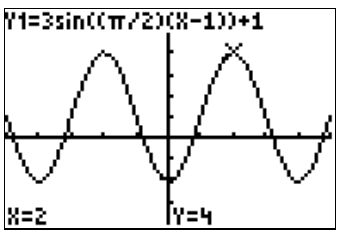


Chapter 5 Section 3 **Question 8 Page 276**

a) The amplitude is 3, the period is 4, the phase shift is 1 right, and the vertical translation is up 1 unit.

b) Equation: $y = 3\sin\left[\frac{\pi}{2}(x-1)\right]+1$

c)



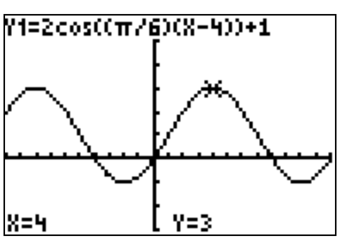
d) Yes, the graphs match.

Chapter 5 Section 3 **Question 9 Page 276**

a) The amplitude is 2, the period is 12, the phase shift is 4 right, the vertical translation is up 1 unit.

b) Equation: $y = 2\cos\left[\frac{\pi}{6}(x-4)\right]+1$

c)



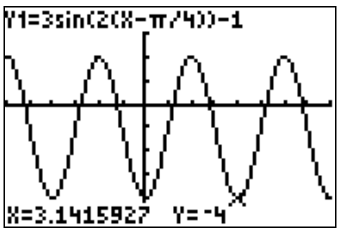
Chapter 5 Section 3 **Question 10 Page 277**

a) The amplitude is 3, the period is π, the phase shift is $\frac{\pi}{4}$ to the right, and the vertical translation is 1 unit down.

$k = \frac{2\pi}{\pi} = 2$

Equation: $y = 3\sin\left[2\left(x - \frac{\pi}{4}\right)\right] - 1$

Check:

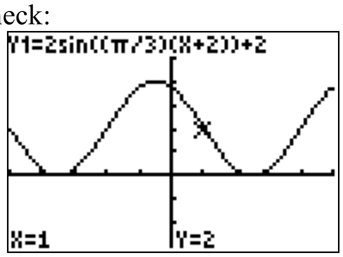


b) The amplitude is 2, the period is 6, the phase shift is 2 to the left, and the vertical translation is 2 units up.

$k = \frac{2\pi}{6} = \frac{\pi}{3}$

Equation: $y = 2\sin\left[\frac{\pi}{3}(x+2)\right] + 2$

Check:



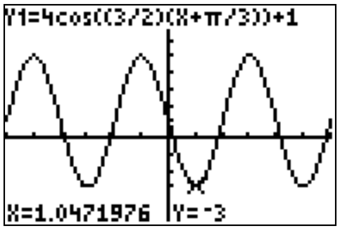
Chapter 5 Section 3 **Question 11 Page 277**

a) The amplitude is 4, the period is $\frac{4\pi}{3}$, the phase shift is $\frac{\pi}{3}$ to the left, and the vertical translation is up 1 unit.

$k = \frac{2\pi}{\frac{4\pi}{3}} = \frac{3}{2}$

Equation: $y = 4\cos\left[\frac{3}{2}\left(x + \frac{\pi}{3}\right)\right] + 1$

Check:

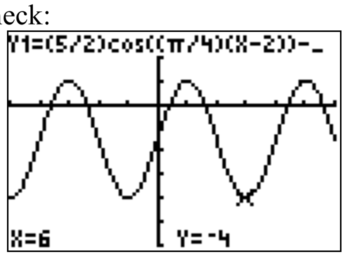


b) The amplitude is $\frac{5}{2}$, the period is 8, the phase shift is 2 to the right, and the vertical translation is $\frac{3}{2}$ units down.

$$k = \frac{2\pi}{8} = \frac{\pi}{4}$$

Equation: $y = \frac{5}{2}\cos\left[\frac{\pi}{4}(x-2)\right] - \frac{3}{2}$

Check:



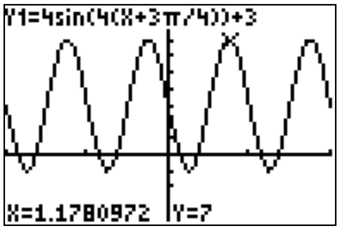
Chapter 5 Section 3 **Question 12 Page 277**

a) Amplitude: $\frac{7+1}{2} = 4$; period = $\frac{\pi}{2}$; phase shift is $\frac{3\pi}{4}$ rad to the left; vertical translation is 3 units up (7 – 4 = 3).

$$k = \frac{2\pi}{\frac{\pi}{2}} = 4$$

Equation: $y = 4\sin\left[4\left(x + \frac{3\pi}{4}\right)\right] + 3$

b)



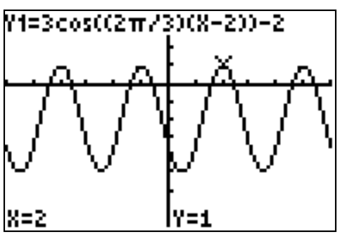
Chapter 5 Section 3 **Question 13 Page 277**

a) The amplitude: $\frac{1+5}{2}=3$, period is 3, phase shift is 2 rad to the right, vertical translation is 2 units down (3 – 1).

$$k=\frac{2\pi}{3}$$

Equation: $y=3\cos\left[\frac{2\pi}{3}\left(x-2\right)\right]-2$

b)



Chapter 5 Section 3 **Question 14 Page 277**

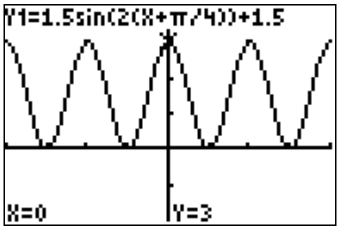
Answers may vary. Sample answer:

a) period is π

$$\begin{aligned}k&=\frac{2\pi}{\pi}\\&=2\end{aligned}$$

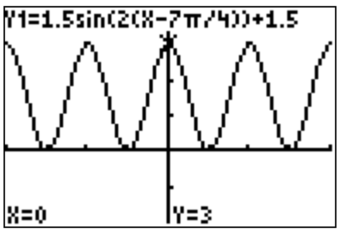
$$y=1.5\sin\left[2\left(x+\frac{\pi}{4}\right)\right]+1.5$$

b)



c) Yes, there are other sine functions have match the given properties. For example,

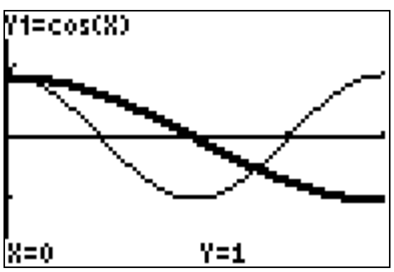
$$y = 1.5\sin\left[2(x - \frac{7\pi}{4})\right] + 1.5$$

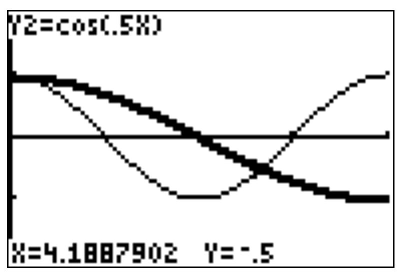


Chapter 5 Section 3 **Question 15 Page 277**

a) The number of points of intersection is 2, one at the beginning of the period and one other point.

b)





The points of intersection are (0, 1), and $\left(\frac{4\pi}{3}, -\frac{1}{2}\right)$.

Chapter 5 Section 3 **Question 16 Page 277**

Answers may vary.

Chapter 5 Section 3 **Question 17 Page 278**

Answers may vary.

Chapter 5 Section 3 **Question 18 Page 278**

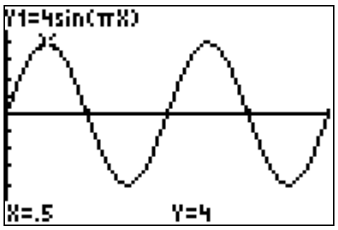
Answers may vary. A sample solution is shown.

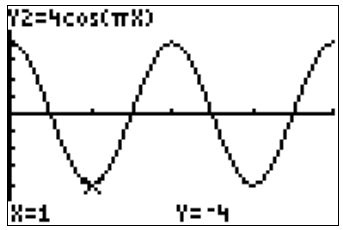
a) Suppose the rotation starts at (4, 0) and goes counter clockwise.

amplitude is 4, the period is $\frac{2\pi}{2} = \pi$; $x = 4\cos \pi t$

b) $y = 4\cos \pi t$ $y = 4\sin \pi t$

c)





d) $\textbf{L.S.} = (4\sin \pi t)^2 + (4\cos \pi t)^2$ $\textbf{R.S.} = 16$

$$= 16\sin^2 \pi t + 16\cos^2 \pi t$$
$$= 16(\sin^2 \pi t + \cos^2 \pi t)$$
$$= 16 \quad \text{(Pythagorean identity)}$$

Since **L.S.** = **R.S.**, $x^2 + y^2 = 16$ is true for all values of t.

e) The relation in part d) is always true since it is an identity.

Chapter 5 Section 3 **Question 19 Page 278**

a) $\text{amplitude} = \frac{1.5}{2}$

$$= \frac{3}{4}$$

period: 6

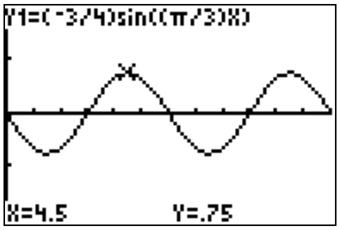
$k: = \frac{2\pi}{6}$

$$= \frac{\pi}{3}$$

Since the buoy is on its way down instead of up, it is reflected in the x-axis.

Equation: $v = -\frac{3}{4}\sin\frac{\pi}{3}t$

b)

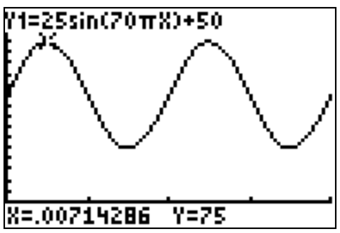


Chapter 5 Section 3 **Question 20 Page 278**

a) Amplitude = 25; period: $\frac{60}{2100}=\frac{1}{35}$; *k:* $\frac{2\pi}{\frac{1}{35}}=70\pi$; vertical translation: 75 – 25 = 50 units up

Equation: $h=25\sin(70\pi t)+50$

b)



c) If the engine speed increases to 2400 rpm, only the period changes.

period: $\frac{60}{2400}=\frac{1}{40}$

k: $\frac{2\pi}{\frac{1}{40}}=80\pi$

Equation: $h=25\sin(80\pi t)+50$

Chapter 5 Section 3 **Question 21 Page 278**

The solutions for Achievement Check can be found in the Teacher's Resource.

Chapter 5 Section 3 **Question 22 Page 279**

$y=a\csc\left[k(x-d)\right]+c$

a: multiply the y-value by a

k: changes the period to $\frac{2\pi}{k}$

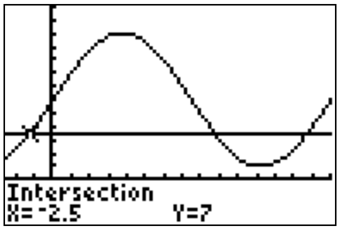
d: phase shifts work the same as for sinusoidal function

c: vertical translations work the same as for sinusoidal functions

Chapter 5 Section 3 **Question 23 Page 279**

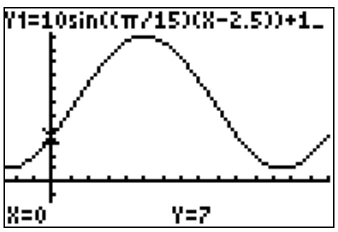
a) The phase shift required is 2.5 to the right.

Graph $h = 10\sin\frac{\pi}{15}t + 12$ and $h = 7$ to find at which t value, the height is 7 m.



When $t = -2.5$ s , the height is 7 m, so shift the graph 2.5 units to the right.

$$h = 10\sin\left[\frac{\pi}{15}(t - 2.5)\right] + 12$$



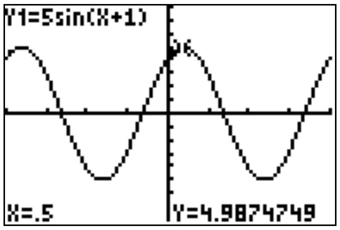
b) Another method to determine the phase shift is to solve for d when $h = 7$ and $t = 0$.

$$\begin{aligned}
h &= 10\sin\left[\frac{\pi}{15}(t - d)\right] + 12 \\
7 &= 10\sin\left[\frac{\pi}{15}(0 - d)\right] + 12 \\
-5 &= 10\sin\left[\frac{\pi}{15}(-d)\right] \\
-\frac{1}{2} &= \sin\left[\frac{\pi}{15}(-d)\right] \\
-0.5236 &= \frac{\pi}{15}(-d) \\
-2.5 &= -d \\
2.5 &= d
\end{aligned}$$

Therefore, the phase shift is 2.5 to the right.

Chapter 5 Section 3 **Question 24 Page 279**

a)



b) $y = 5\sin(x+1)$

$y = 5\left[\sin x\cos 1 + \sin 1\cos x\right]$

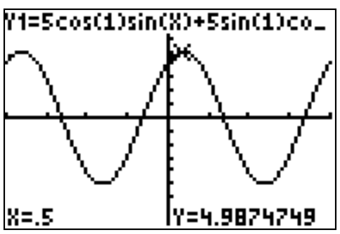
$y = 5\cos 1\sin x + 5\sin 1\cos x$

Since $y = a\sin x + b\cos x$,

$a\sin x = 5\cos 1\sin x$ $\qquad$ $b\cos x = 5\sin 1\cos x$

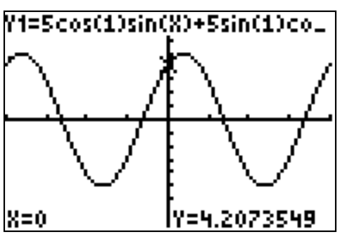
$a = 5\cos 1$ $\qquad$ $b = 5\sin 1$

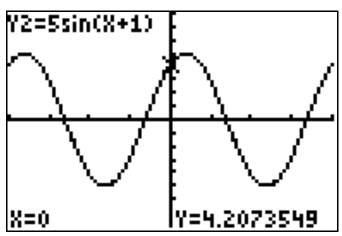
$a \doteq 2.7$ $\qquad$ $b \doteq 4.2$



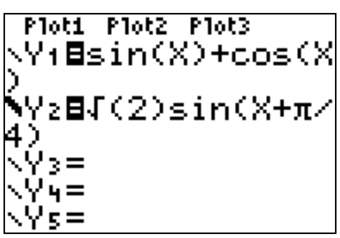
c) $a \doteq 2.7, b \doteq 4.2$

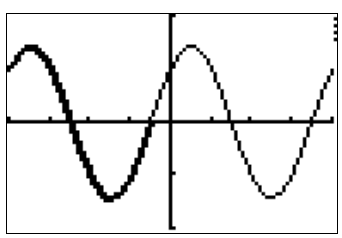
d)



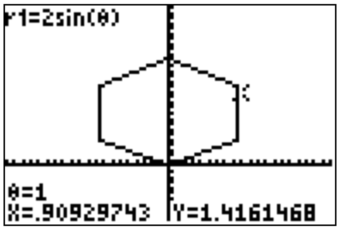


e)

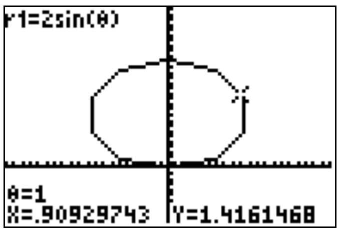




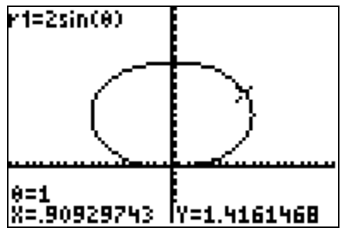
$r = 2\sin\theta$:



a) $\theta\text{step} = \dfrac{\pi}{10}$

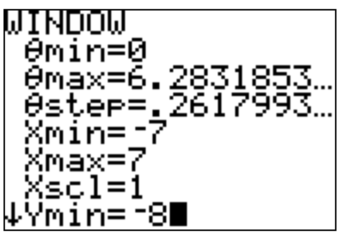


$\theta\text{step} = \dfrac{\pi}{100}$

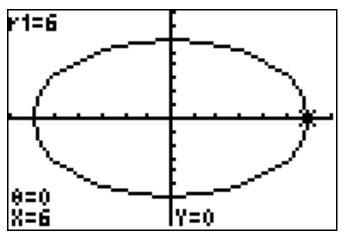


Smaller increments of θstep make the graph smoother and more circular.

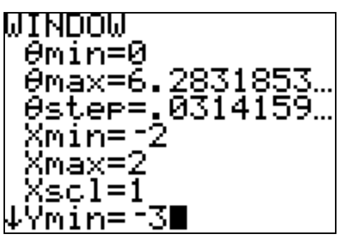
b) i)



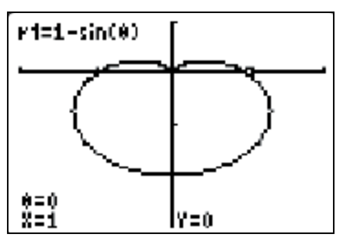
Ymax = 8



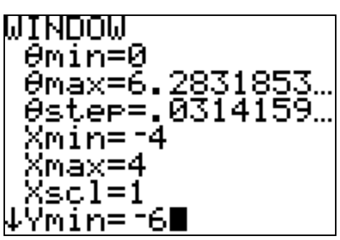
ii)



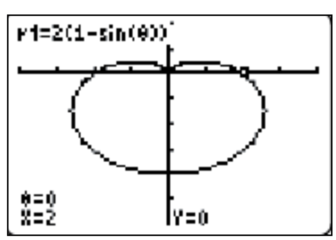
Ymax = 1



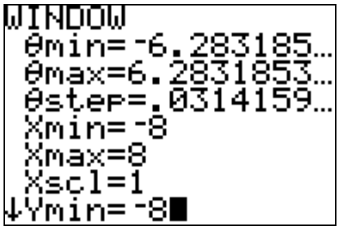
iii)

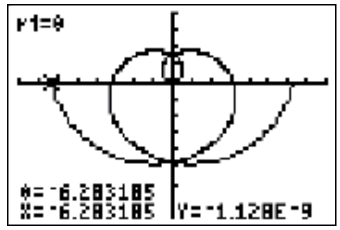


Ymax = 2



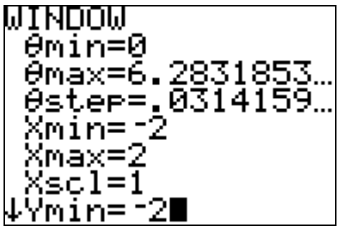
iv)

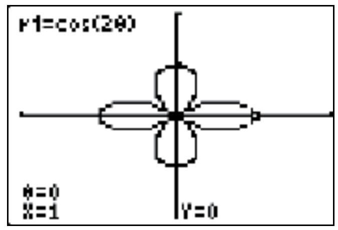




Ymax = 4

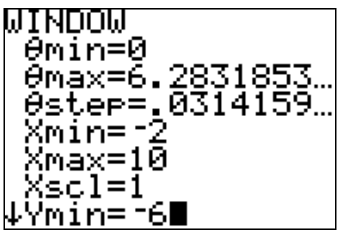
v)

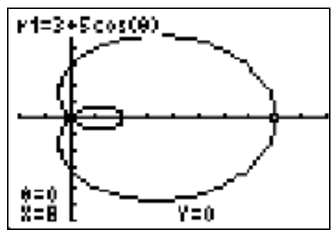




Ymax = 2

vi)





Ymax = 6

Chapter 5 Extension

The solutions for the Extension can be found in the Teacher’s Resource.

Chapter 5 Section 4 **Question 1 Page 287**

a)
$$\begin{aligned}\sin x &= \frac{1}{4}\\ \sin^{-1}\frac{1}{4} &= x\\ x &\doteq 0.25\\ &\text{or}\\ x &\doteq \pi - 0.25\\ x &\doteq 2.89\\ x &\doteq 0.25 \text{ or } x \doteq 2.89\end{aligned}$$

b)
$$\begin{aligned}\cos x &= -0.75\\ \cos^{-1}(-0.75) &= x\\ x &\doteq 2.42\\ &\text{or}\\ x &\doteq 2\pi - 2.42\\ x &\doteq 3.86\\ x &\doteq 2.42 \text{ or } x \doteq 3.86\end{aligned}$$

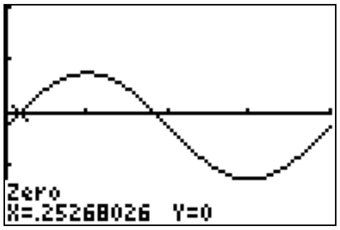
c)
$$\begin{aligned}\tan x &= 5\\ \tan^{-1}5 &= x\\ x &\doteq 1.37\\ &\text{or}\\ x &\doteq \pi + 1.37\\ x &\doteq 4.51\\ x &\doteq 1.37 \text{ or } x \doteq 4.51\end{aligned}$$

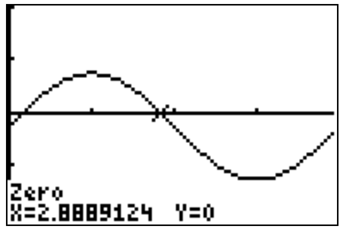
d)
$$\begin{aligned}\sec x &= 4\\ \frac{1}{\cos x} &= 4\\ \cos x &= \frac{1}{4}\\ \cos^{-1}\left(\frac{1}{4}\right) &= x\\ x &\doteq 1.32\\ &\text{or}\\ x &\doteq 2\pi - 1.32\\ x &\doteq 4.97\\ x &\doteq 1.32 \text{ or } x \doteq 4.97\end{aligned}$$

e)
$$\begin{aligned}3\cot x &= -2\\ \frac{1}{\tan x} &= -\frac{2}{3}\\ \tan x &= -\frac{3}{2}\\ \tan^{-1}\left(-\frac{3}{2}\right) &= x\\ x &\doteq 5.30\\ &\text{or}\\ x &\doteq 5.30 - \pi\\ x &\doteq 2.16\\ x &\doteq 2.16 \text{ or } x \doteq 5.30\end{aligned}$$

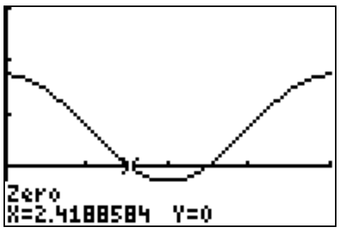
f)
$$\begin{aligned}2\csc x &= -5\\ \frac{1}{\sin x} &= -\frac{5}{2}\\ \sin x &= -\frac{2}{5}\\ \sin^{-1}\left(-\frac{2}{5}\right) &= x\\ x &\doteq -0.41 \Rightarrow x \doteq 5.87\\ &\text{or}\\ x &\doteq \pi + 0.41\\ x &\doteq 3.55\\ x &\doteq 3.55 \text{ or } x \doteq 5.87\end{aligned}$$

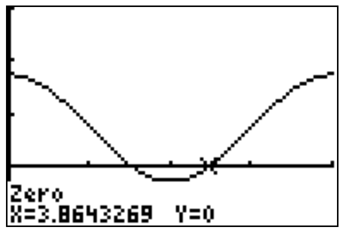
a) $x \doteq 0.25$ or $x \doteq 2.89$



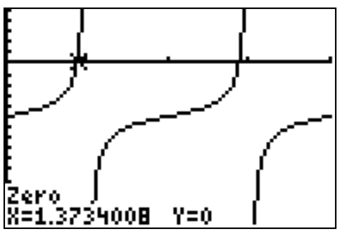


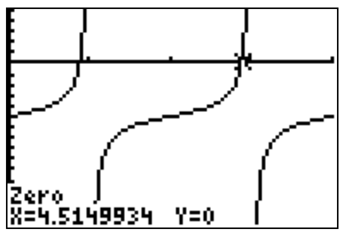
b) $x \doteq 2.42$ or $x \doteq 3.86$



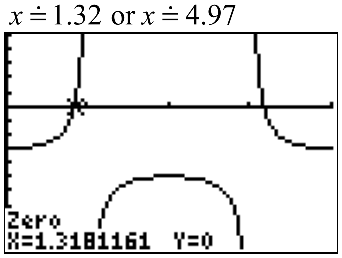


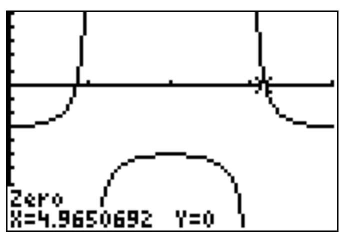
c) $x \doteq 1.37$ or $x \doteq 4.51$



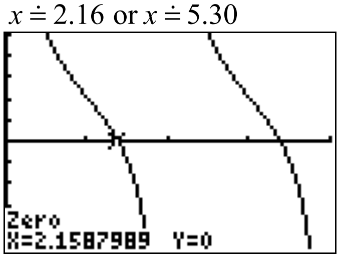


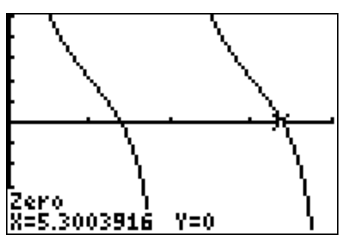
d) $x \doteq 1.32$ or $x \doteq 4.97$



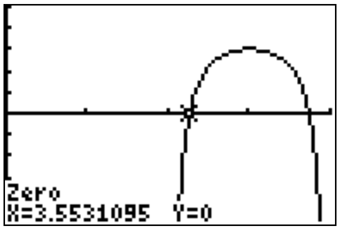


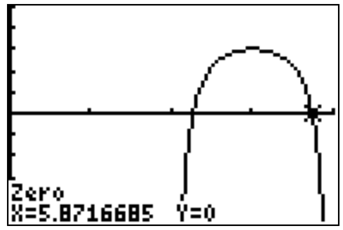
e) $x \doteq 2.16$ or $x \doteq 5.30$





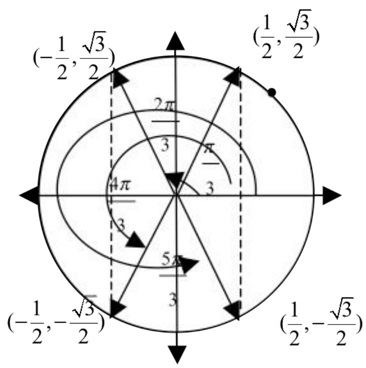
f) $x \doteq 3.55$ or $x \doteq 5.87$





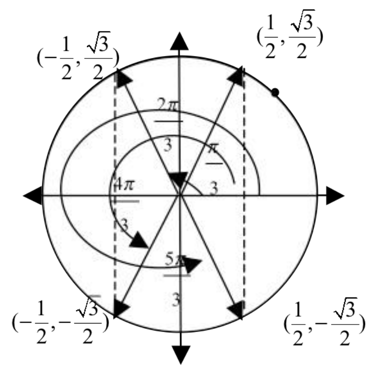
Chapter 5 Section 4 **Question 3 Page 287**

a) $\sin x = -\frac{\sqrt{3}}{2}$



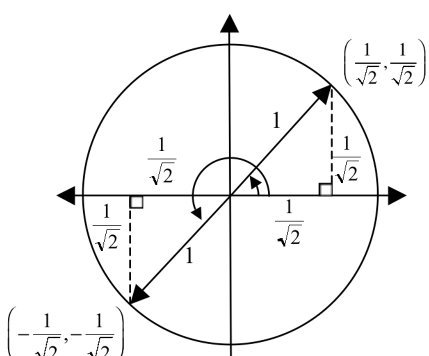
$x = \frac{4\pi}{3}$ or $x = \frac{5\pi}{3}$

b) $\cos x = \frac{1}{2}$



$x = \frac{\pi}{3}$ or $x = \frac{5\pi}{3}$

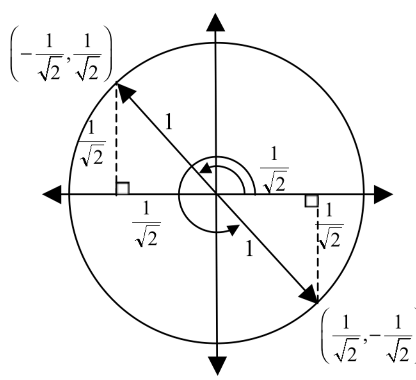
c) $\tan x = 1$



$x = \frac{\pi}{4}$ or $x = \frac{5\pi}{4}$

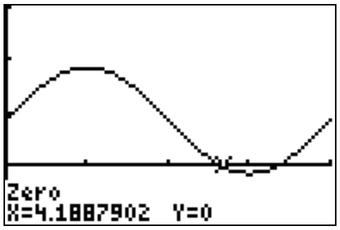
d) $\cot x = -1$

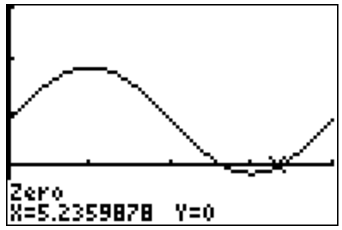
$$\frac{1}{\tan x} = -1$$
$$\tan x = -1$$



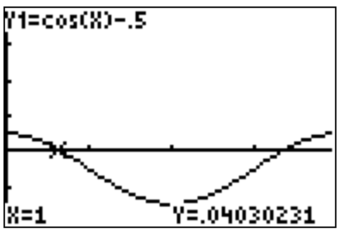
$x = \frac{3\pi}{4}$ or $x = \frac{7\pi}{4}$

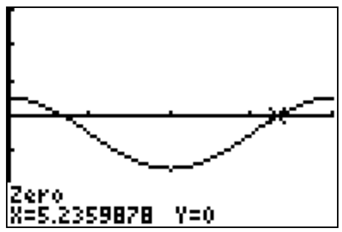
a) $\sin x + \frac{\sqrt{3}}{2} = 0$



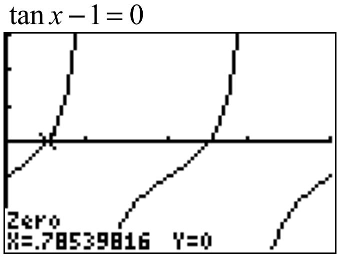


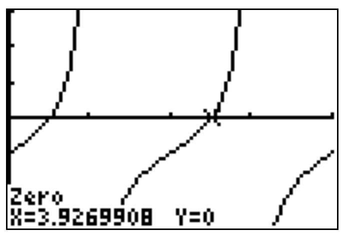
b) $\cos x - 0.5 = 0$



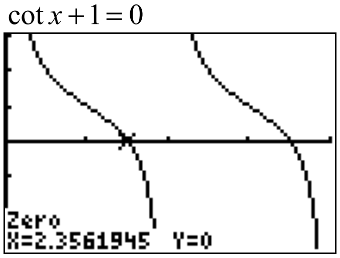


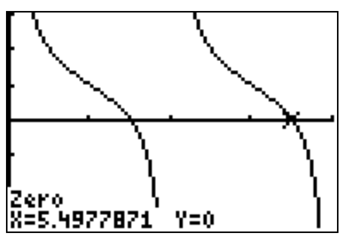
c) $\tan x - 1 = 0$





d) $\cot x + 1 = 0$





Chapter 5 Section 4 **Question 5 Page 287**

a) $\sin^2 x = 0.64$
$\sin x = \pm\sqrt{0.64}$
$\sin x = \pm 0.8$

Case 1:
$\sin x = 0.8$
$x \doteq 0.93$ or $x \doteq 2.21$

Case 2:
$\sin x = -0.8$
$x \doteq 5.36$ or $x \doteq 4.07$

b) $\cos^2 x = \frac{4}{9}$
$\cos x = \pm\sqrt{\frac{4}{9}}$
$\cos x = \pm\frac{2}{3}$

Case 1:
$\cos x = \frac{2}{3}$
$x \doteq 0.84$ or $x \doteq 5.44$

Case 2:
$\cos x = -\frac{2}{3}$
$x \doteq 2.30$ or $x \doteq 3.98$

c) $\tan^2 x = 1.44$
$\tan x = \pm\sqrt{1.44}$
$\tan x = \pm 1.2$

Case 1:
$\tan x = 1.2$
$x \doteq 0.88$ or $x \doteq 4.02$

Case 2:
$\tan x = -1.2$
$x \doteq 5.41$ or $x \doteq 2.27$

d) $\sec^2 x = 2.5$
$\frac{1}{\cos^2 x} = 2.5$
$\cos^2 x = \frac{4}{10}$
$\cos x = \pm\frac{2}{\sqrt{10}}$

Case 1:
$\cos x = \frac{2}{\sqrt{10}}$
$x \doteq 0.89$ or $x \doteq 5.40$

Case 2:
$\cos x = -\frac{2}{\sqrt{10}}$
$x \doteq 2.26$ or $x \doteq 4.03$

e) $\cot^2 x = 1.21$

$$\frac{1}{\tan^2 x} = 1.21$$

$$\tan^2 x = \frac{1}{1.21}$$

$$\tan x = \pm\frac{1}{1.1}$$

$$\tan x = \pm\frac{10}{11}$$

Case 1:

$$\tan x = \frac{10}{11}$$

$x \doteq 0.74$ or $x \doteq 3.88$

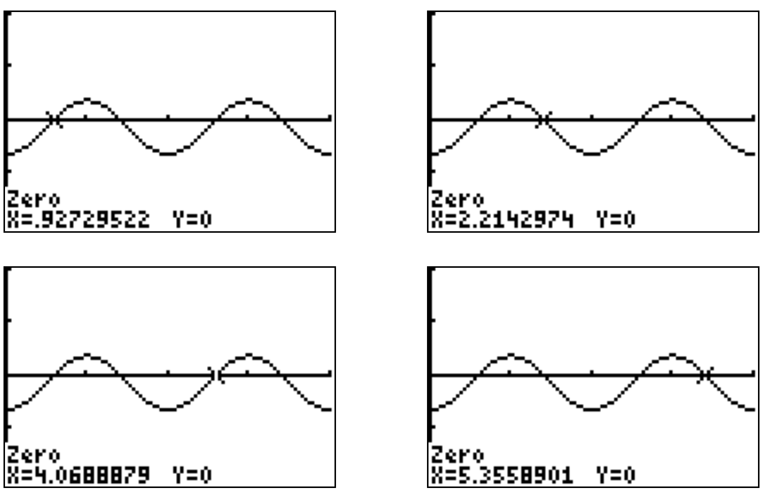
Case 2:

$$\tan x = -\frac{10}{11}$$

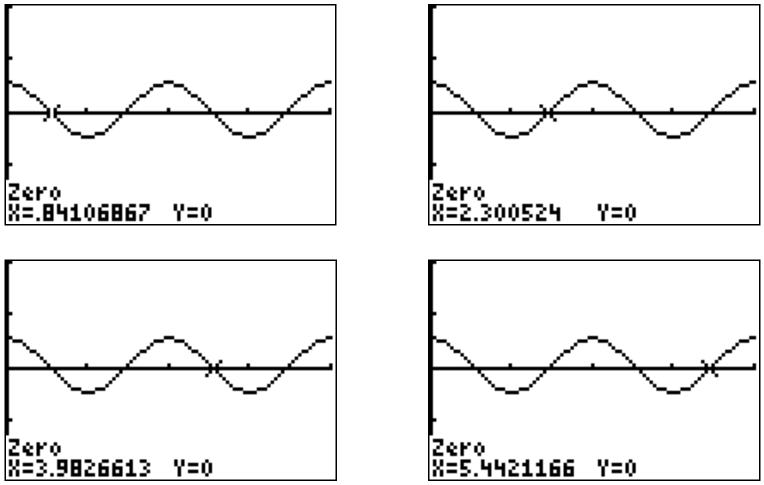
$x \doteq 5.55$ or $x \doteq 2.40$

Chapter 5 Section 4 **Question 6 Page 287**

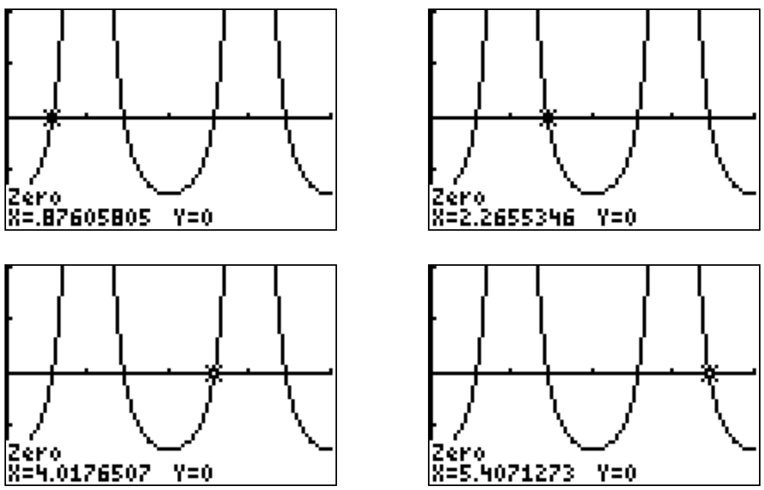
a)



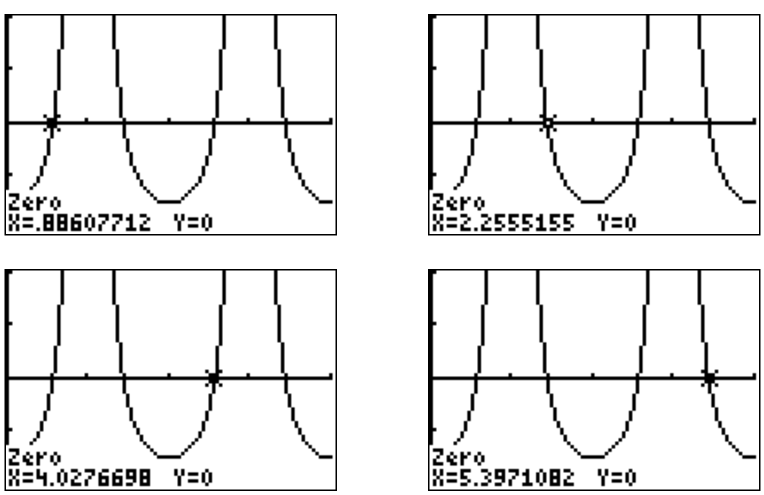
b)


Zero
X=.84106867 Y=0
Zero
X=2.300524 Y=0
Zero
X=3.9826613 Y=0
Zero
X=5.4421166 Y=0

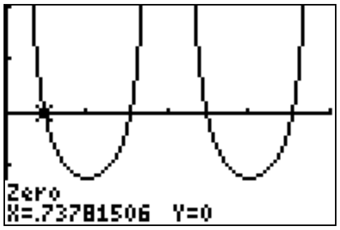
c)

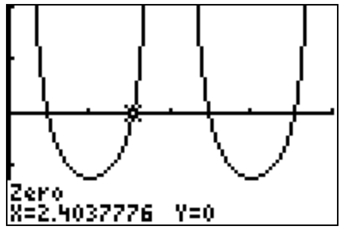

Zero
X=.87605805 Y=0
Zero
X=2.2655346 Y=0
Zero
X=4.0176507 Y=0
Zero
X=5.4071273 Y=0

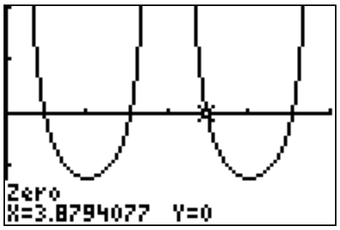
d)

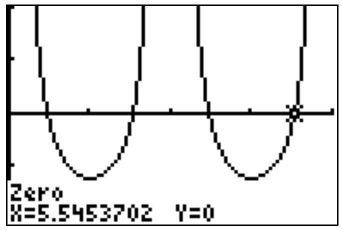

Zero
X=.88607712 Y=0
Zero
X=2.2555155 Y=0
Zero
X=4.0276698 Y=0
Zero
X=5.3971082 Y=0

e)









Chapter 5 Section 4 **Question 7 Page 287**

a) $\sin^2 x = \frac{1}{4}$

$\sin x = \pm\frac{1}{2}$

Case 1:

$\sin x = \frac{1}{2}$

$x = \frac{\pi}{6}$ or $x = \frac{5\pi}{6}$

Case 2:

$\sin x = -\frac{1}{2}$

$x = \frac{11\pi}{6}$ or $x = \frac{7\pi}{6}$

b) $\cos^2 x = \frac{3}{4}$

$\cos x = \pm\frac{\sqrt{3}}{2}$

Case 1:

$\cos x = \frac{\sqrt{3}}{2}$

$x = \frac{\pi}{6}$ or $x = \frac{11\pi}{6}$

Case 2:

$\cos x = -\frac{\sqrt{3}}{2}$

$x = \frac{5\pi}{6}$ or $x = \frac{7\pi}{6}$

c) $\tan^2 x = 3$

$\tan x = \pm\sqrt{3}$

Case 1:

$\tan x = \sqrt{3}$

$x = \dfrac{\pi}{3}$ or $x = \dfrac{4\pi}{3}$

Case 2:

$\tan x = -\sqrt{3}$

$x = \dfrac{5\pi}{3}$ or $x = \dfrac{2\pi}{3}$

d) $3\csc^2 x = 4$

$\dfrac{1}{\sin^2 x} = \dfrac{4}{3}$

$\sin^2 x = \dfrac{3}{4}$

$\sin x = \pm\dfrac{\sqrt{3}}{2}$

Case 1:

$\sin x = \dfrac{\sqrt{3}}{2}$

$x = \dfrac{\pi}{3}$ or $x = \dfrac{2\pi}{3}$

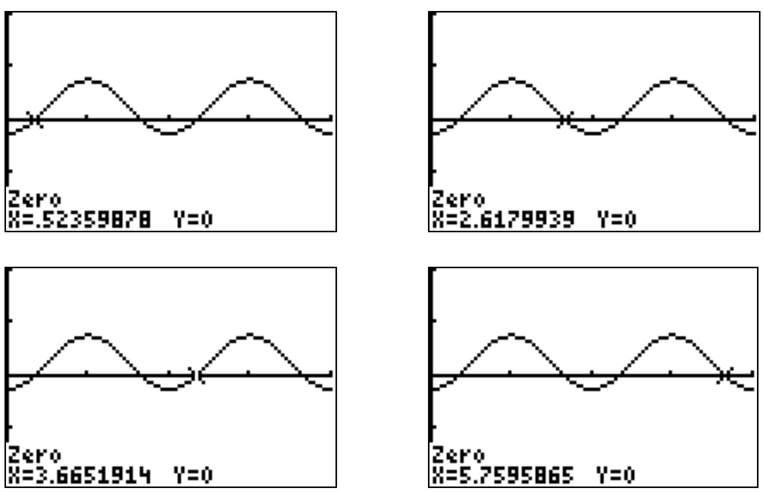
Case 2:

$\sin x = -\dfrac{\sqrt{3}}{2}$

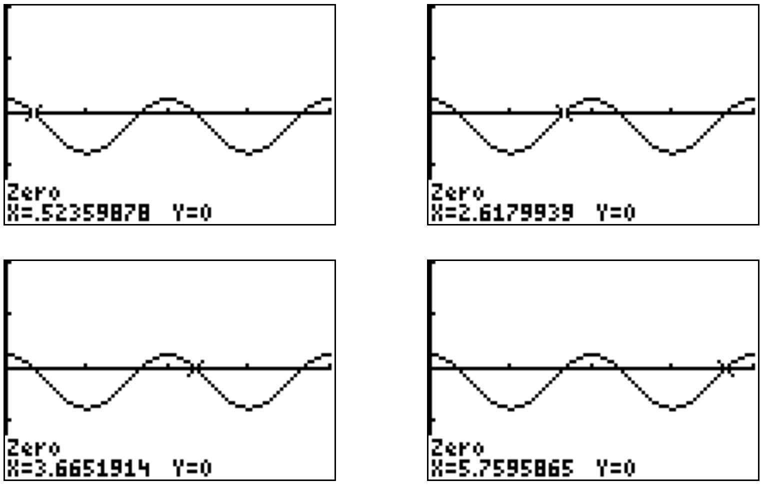
$x = \dfrac{4\pi}{3}$ or $x = \dfrac{5\pi}{3}$

Chapter 5 Section 4 **Question 8 Page 288**

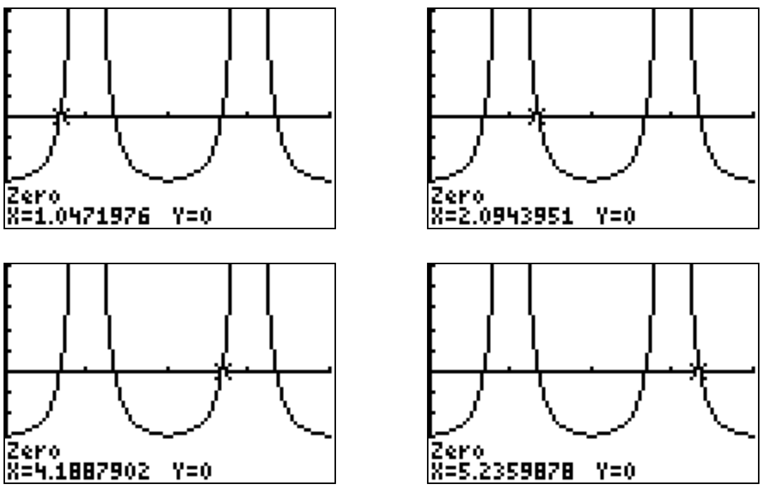
a)



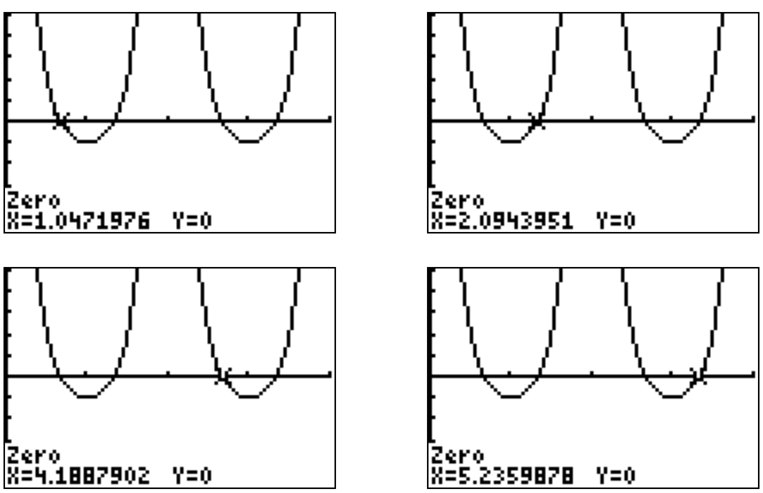
b)


Zero
X=.52359878 Y=0
Zero
X=2.6179939 Y=0
Zero
X=3.6651914 Y=0
Zero
X=5.7595865 Y=0

c)


Zero
X=1.0471976 Y=0
Zero
X=2.0943951 Y=0
Zero
X=4.1887902 Y=0
Zero
X=5.2359878 Y=0

d)


Zero
X=1.0471976 Y=0
Zero
X=2.0943951 Y=0
Zero
X=4.1887902 Y=0
Zero
X=5.2359878 Y=0

Chapter 5 Section 4 **Question 9 Page 288**

$$\sin^2 x - 2\sin x - 3 = 0$$
$$(\sin x - 3)(\sin x + 1) = 0$$

$\sin x = 3$ or $\sin x = -1$

Case 1:
$\sin x = 3$

Case 2:
$\sin x = -1$
$x = \frac{3\pi}{2}$

Since the maximum value possible for sin x is 1, there are no solutions.

The only solution on the given domain is $x = \frac{3\pi}{2}$.

Chapter 5 Section 4 **Question 10 Page 288**

$$\csc^2 x - \csc x - 2 = 0$$
$$(\csc x - 2)(\csc x + 1) = 0$$

$\csc x = 2$ or $\csc x = -1$

Case 1:
$\csc x = 2$
$\frac{1}{\sin x} = 2$
$\sin x = \frac{1}{2}$
$x = \frac{\pi}{6}$ or $x = \frac{5\pi}{6}$

Case 2:
$\csc x = -1$
$\sin x = -1$
$x = \frac{3\pi}{2}$

Chapter 5 Section 4 **Question 11 Page 288**

$$2\sec^2 x + \sec x - 1 = 0$$
$$(2\sec x - 1)(\sec x + 1) = 0$$

$2\sec x = 1$ or $\sec x = -1$

Case 1:

$$\begin{aligned}2\sec x &= 1\\ \sec x &= \frac{1}{2}\\ \frac{1}{\cos x} &= \frac{1}{2}\\ \cos x &= 2\end{aligned}$$

Case 2:

$$\begin{aligned}\sec x &= -1\\ \cos x &= -1\\ x &= \pi\end{aligned}$$

Since the maximum value for cos x is 1, there are no solutions.
The solution is $x = \pi$.

Chapter 5 Section 4 **Question 12 Page 288**

$$\begin{aligned}\tan^2 x + \tan x - 6 &= 0\\ (\tan x + 3)(\tan x - 2) &= 0\end{aligned}$$

$\tan x = -3$ or $\tan x = 2$

Case 1:

$\tan x = -3$

$x \doteq 1.89$ or $x \doteq 5.03$

Case 2:

$\tan x = 2$

$x \doteq 1.11$ or $x \doteq 4.25$

The solution is $x \doteq 1.11$ or $x \doteq 1.89$ or $x \doteq 4.25$ or $x \doteq 5.03$.

Chapter 5 Section 4 **Question 13 Page 288**

a)

$$\begin{aligned}\sin 2x &= 0.8\\ 2x &= \sin^{-1}(0.8)\\ 2x &\doteq 0.93\\ x &\doteq 0.46\end{aligned}$$

or

$$\begin{aligned}x &\doteq \frac{\pi}{2} - 0.46\\ x &\doteq 1.11\end{aligned}$$

The solution is $x \doteq 0.46$ or $x \doteq 1.11$.

b)

$$
\begin{aligned}
5\sin 2x &= 3\\
\sin 2x &= \frac{3}{5}\\
2x &= \sin^{-1}\left(\frac{3}{5}\right)\\
2x &\doteq 0.64\\
x &\doteq 0.32\\
&\text{or}\\
x &\doteq \frac{\pi}{2} - 0.32\\
x &\doteq 1.25
\end{aligned}
$$

The solution is $x \doteq 0.32$ or $x \doteq 1.25$.

c)

$$
\begin{aligned}
-4\sin 2x &= -3\\
\sin 2x &= \frac{3}{4}\\
2x &= \sin^{-1}\left(\frac{3}{4}\right)\\
2x &\doteq 0.85\\
x &\doteq 0.42\\
&\text{or}\\
x &\doteq \frac{\pi}{2} - 0.42\\
x &\doteq 1.15
\end{aligned}
$$

The solution is $x \doteq 0.42$ or $x \doteq 1.15$.

Chapter 5 Section 4 **Question 14 Page 288**

$$
\begin{aligned}
2\tan^2 x + 1 &= 0\\
2\tan^2 x &= -1\\
\tan^2 x &= -\frac{1}{2}\\
\tan x &= \sqrt{-\frac{1}{2}}
\end{aligned}
$$

There is no solution since there are no real roots.

Chapter 5 Section 4 **Question 15 Page 288**

$$
\begin{aligned}
3\sin 2x - 1 &= 0 \\
3\sin 2x &= 1 \\
\sin 2x &= \frac{1}{3} \\
2x &= \sin^{-1}\left(\frac{1}{3}\right) \\
2x &\doteq 0.34
\end{aligned}
$$

$$
x \doteq 0.17 \quad \text{or} \quad x \doteq \frac{\pi}{2} - 0.17 \quad \text{or} \quad x \doteq \pi + 0.17 \quad \text{or} \quad x \doteq \frac{3\pi}{2} - 0.17
$$

$$
\doteq 1.40 \qquad \doteq 3.31 \qquad \doteq 4.54
$$

The solution is $x \doteq 0.17$ or $x \doteq 1.40$ or $x \doteq 3.31$ or $x \doteq 4.54$.

Chapter 5 Section 4 **Question 16 Page 288**

$$
\begin{aligned}
6\cos^2 x + 5\cos x - 6 &= 0 \\
(2\cos x + 3)(3\cos x - 2) &= 0
\end{aligned}
$$

$2\cos x = -3$ or $3\cos x = 2$

Case 1:

$$
\begin{aligned}
2\cos x &= -3 \\
\cos x &= -\frac{3}{2}
\end{aligned}
$$

Case 2:

$$
\begin{aligned}
3\cos x &= 2 \\
x &= \cos^{-1}\frac{2}{3} \\
x &\doteq 0.84 \quad \text{or} \quad x \doteq 2\pi - 0.84 \\
&\doteq 5.44
\end{aligned}
$$

Since the minimum value for cos x is –1, there is no solution.

The solution is $x \doteq 0.84$ or $x \doteq 5.44$.

Chapter 5 Section 4 **Question 17 Page 288**

$$
\begin{aligned}
3\csc^2 x - 5\csc x - 2 &= 0 \\
(\csc x - 2)(3\csc x + 1) &= 0
\end{aligned}
$$

$\csc x = 2$ or $3\csc x = -1$

Case 1:

$\csc x = 2$

$\frac{1}{\sin x} = 2$

$\sin x = \frac{1}{2}$

$x = \sin^{-1}\left(\frac{1}{2}\right)$

$x = \frac{\pi}{6}$ or $x = \frac{5\pi}{6}$

Case 2:

$3\csc x = -1$

$\csc x = -\frac{1}{3}$

$\frac{1}{\sin x} = -\frac{1}{3}$

$\sin x = -3$

Since the minimum value for sin x is –1, there is no solution.

The solution is $x = \frac{\pi}{6}$ or $x = \frac{5\pi}{6}$.

Chapter 5 Section 4 **Question 18 Page 288**

$$\sec^2 x + \sec x + 6 = 0$$
$$(\sec x + 3)(\sec x + 2) = 0$$

$\sec x = -3$ or $\sec x = -2$

Case 1:

$\sec x = -3$

$\frac{1}{\cos x} = -3$

$\cos x = -\frac{1}{3}$

$x = \cos^{-1}\left(-\frac{1}{3}\right)$

$x \doteq 1.91$

or

$x \doteq 2\pi - 1.91$

$\doteq 4.37$

Case 2:

$\sec x = -2$

$\frac{1}{\cos x} = -2$

$\cos x = -\frac{1}{2}$

$x = \cos^{-1}\left(-\frac{1}{2}\right)$

$x = \frac{2\pi}{3}$ or $x = \frac{4\pi}{3}$

The solution is $x \doteq 1.91$ or $x \doteq 4.37$ or $x = \frac{2\pi}{3}$ or $x = \frac{4\pi}{3}$.

Chapter 5 Section 4 **Question 19 Page 288**

$$2\tan^2 x - 5\tan x - 3 = 0$$
$$(2\tan x + 1)(\tan x - 3) = 0$$

$2\tan x = -1$ or $\tan x = 3$

Case 1:
$$2\tan x = -1$$
$$\tan x = -\frac{1}{2}$$
$$x = \tan^{-1}\left(-\frac{1}{2}\right)$$
$$x \doteq 5.82 \text{ or } x \doteq 2.68$$

Case 2:
$$\tan x = 3$$
$$x = \tan^{-1} 3$$
$$x \doteq 1.25 \text{ or } x \doteq 4.39$$

The solution is $x \doteq 1.25$ or $x \doteq 2.68$ or $x \doteq 4.39$ or $x \doteq 5.82$.

Chapter 5 Section 4 **Question 20 Page 288**

a) The left side cannot be factored since there are no two integers that have a product of –3 and a sum of 1.

b)
$$\sin x = \frac{-1 \pm \sqrt{1^2 - 4(3)(-1)}}{2(3)}$$
$$\sin x = \frac{-1 \pm \sqrt{13}}{6}$$
$$\sin x \doteq 0.43 \text{ or } \sin x \doteq -0.77$$
$$x \doteq 0.45 \text{ or } x \doteq -0.88$$

c) $x \doteq 0.45$

$$x \doteq 2\pi - 0.88$$
$$\doteq 5.41$$

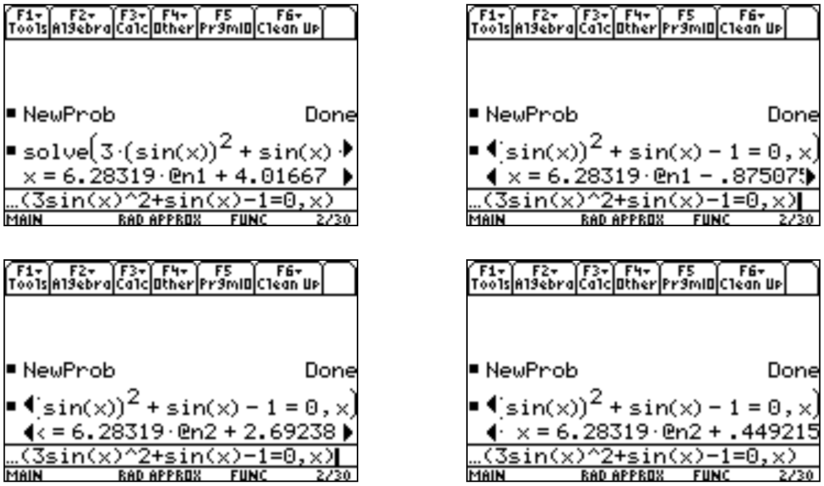
$$x \doteq \pi - 0.45$$
$$\doteq 2.69$$

$$x \doteq \pi + 0.88$$
$$\doteq 4.02$$

The solution is $x \doteq 0.45$ or $x \doteq 2.69$ or $x \doteq 4.02$ or $x \doteq 5.41$.

Chapter 5 Section 4 Question 21 Page 288

a)



b) When checking with a graphing calculator, technology allows you to check all the zeros on the graph within the domain. The CAS gives you the solution within the interval $x \in [0, 2\pi]$.

Chapter 5 Section 4 Question 22 Page 288

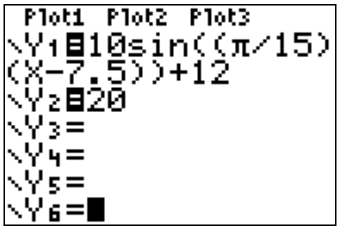
a)

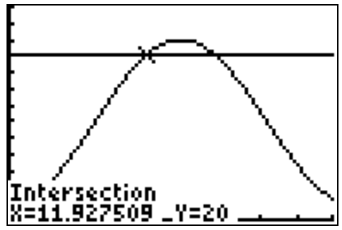
$$\begin{aligned}
10\sin\left[\frac{\pi}{15}(t-7.5)\right]+12 &= 20 \\
10\sin\left[\frac{\pi}{15}(t-7.5)\right] &= 8 \\
\sin\left[\frac{\pi}{15}(t-7.5)\right] &= \frac{4}{5} \\
\sin^{-1}\left(\frac{4}{5}\right) &= \frac{\pi}{15}(t-7.5)
\end{aligned}$$

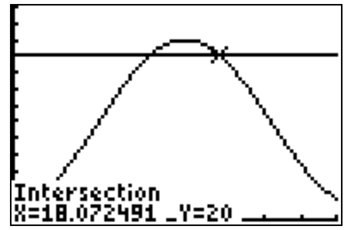
$$\begin{aligned}
\frac{\pi}{15}(t-7.5) &\doteq 0.93 & \text{or} \quad \frac{\pi}{15}(t-7.5) &\doteq 2.21 \\
\frac{\pi}{15}t &\doteq 0.93+\frac{7.5\pi}{15} & \frac{\pi}{15}t &\doteq 2.21+\frac{7.5\pi}{15} \\
\frac{\pi}{15}t &\doteq 2.50 & \frac{\pi}{15}t &\doteq 3.79 \\
t &\doteq 11.93 & t &\doteq 18.07
\end{aligned}$$

The solution is $t \doteq 11.93$ or $t \doteq 18.07$.

b)







Chapter 5 Section 4 **Question 23 Page 288**

$$\begin{aligned}4\sin x\cos 2x+4\cos x\sin 2x-1&=0\\4\sin(x+2x)-1&=0 \quad \text{(compound angle formula)}\\4\sin(3x)&=1\\\sin 3x&=\frac{1}{4}\\3x&=\sin^{-1}\frac{1}{4}\\3x&\doteq 0.25\\x&\doteq 0.08\end{aligned}$$

The smallest possible solution is $x \doteq 0.08$.

Chapter 5 Section 4 **Question 24 Page 288**

$$\begin{aligned}r&=\frac{v^2}{g}\sin 2\theta\\20&=\frac{225}{9.8}\sin 2\theta\\0.87&\doteq\sin 2\theta\\2\theta&\doteq 1.06\\\theta&\doteq 0.53 \quad \text{or} \quad \theta\doteq\frac{\pi}{2}-0.53\\&\qquad\qquad\qquad\;\doteq 1.04\end{aligned}$$

The angles that the cannon can be aimed to hit the target are approximately 0.53 and 1.04.

Chapter 5 Section 4 **Question 25 Page 289**

Solutions to the Achievement Check can be found in the Teacher's Resource.

Chapter 5 Section 4 **Question 26 Page 289**

$$
\begin{aligned}
\tan x \cos^2 x - \tan x &= 0 \\
\tan x(\cos^2 x - 1) &= 0 \\
\tan x(\cos x - 1)(\cos x + 1) &= 0
\end{aligned}
$$

$\tan x = 0$ or $\cos x = 1$ or $\cos x = -1$

Case 1:
$\tan x = 0$
$x = -2\pi, -\pi, 0, \pi, 2\pi$

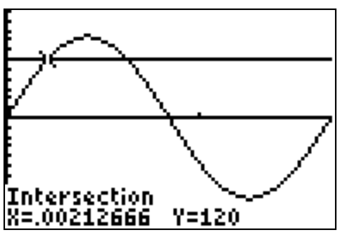
Case 2:
$\cos x = 1$
$x = -2\pi, 0, 2\pi$

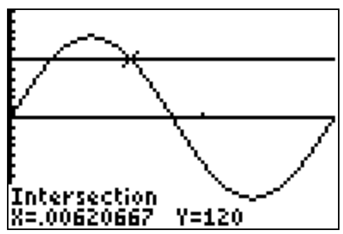
Case 3:
$\cos x = -1$
$x = -\pi, \pi$

The solution is $x = -2\pi$ or $x = -\pi$ or $x = 0$ or $x = \pi$ or $x = 2\pi$.

Chapter 5 Section 4 **Question 27 Page 289**

a)





By looking at the graphs, the voltage is greater than 120 V from approximately 0.002 s to 0.006 s.

$$
\begin{aligned}
t &\doteq 0.006 - 0.002 \\
&\doteq 0.004
\end{aligned}
$$

The voltage is greater than 120 V for approximately 0.004 s.

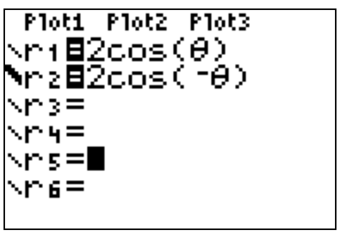
b) No, it is not safe to use this component because the voltage is greater than 120 V for longer than the safety limit.

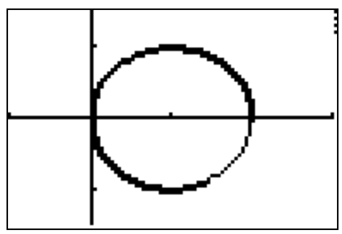
Chapter 5 Section 4 **Question 28 Page 289**

$$\frac{\cos x}{1+\sin x}+\frac{1+\sin x}{\cos x}=2$$
$$\frac{\cos^2 x}{\cos x(1+\sin x)}+\frac{(1+\sin x)^2}{\cos x(1+\sin x)}=2$$
$$\frac{\cos^2 x+1+2\sin x+\sin^2 x}{\cos x(1+\sin x)}=2$$
$$\frac{1+1+2\sin x}{\cos x(1+\sin x)}=2 \qquad \text{(Pythagorean Identity)}$$
$$\frac{2(1+\sin x)}{\cos x(1+\sin x)}=2$$
$$\frac{2}{\cos x}=2$$
$$\cos x=1$$
$$x=-2\pi \text{ or } x=0 \text{ or } x=2\pi$$

Chapter 5 Section 4 **Question 29 Page 289**

a)

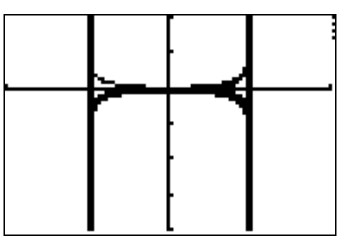




Since $2\cos\theta = 2\cos(-\theta)$, the function is even and thus is symmetric about the polar axis.

b) $r^2=\tan\theta$

Replace r with $-r$ and θ with $\theta+\pi$.
$(-r)^2=\tan(\theta+\pi)$
$r^2=\tan\theta$ (cofunction identity)



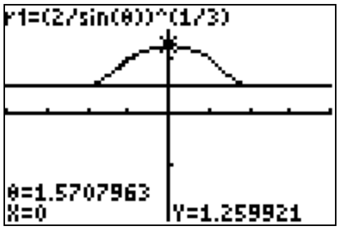
There is symmetry about the pole.

c) $r^3 \sin\theta = 2$

Replace θ with $\pi - \theta$.

$r^3 \sin(\pi - \theta) = 2$

$r^3 \sin\theta = 2$ (cofunction identity)

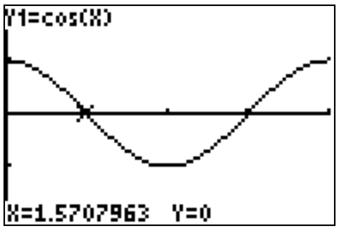


There is symmetry about $\theta = \dfrac{\pi}{2}$ (the y-axis).

Chapter 5 Section 5 **Making Connections and Instantaneous Rate of Change**

Chapter 5 Section 5 **Question 1 Page 296**

a)



b) The instantaneous rate of change appears to be 0 at $x = 0$ or $x = \pi$ or $x = 2\pi$.

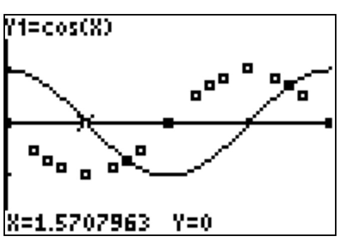
c) The maximum value is $x = \dfrac{3\pi}{2}$ and the minimum value is $x = \dfrac{\pi}{2}$.

Chapter 5 Section 5 **Question 2 Page 296**

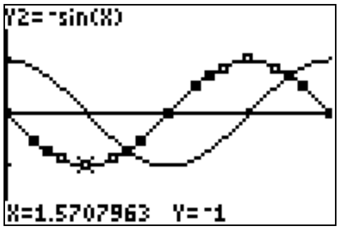
a)

Angle x	$f(x) = \cos x$	Instantaneous Rate of Change
0	1	0
$\frac{\pi}{6}$	0.87	–0.50
$\frac{\pi}{4}$	0.71	–0.71
$\frac{\pi}{3}$	0.50	–0.87
$\frac{\pi}{2}$	0	–1
$\frac{2\pi}{3}$	–0.50	–0.87
$\frac{3\pi}{4}$	–0.71	–0.71
$\frac{5\pi}{6}$	–0.87	–0.50
π	–1	0
$\frac{7\pi}{6}$	–0.87	0.50
$\frac{5\pi}{4}$	–0.71	0.71
$\frac{4\pi}{3}$	–0.50	0.87
$\frac{3\pi}{2}$	0	1
$\frac{5\pi}{3}$	0.50	0.87
$\frac{7\pi}{4}$	0.71	0.71
$\frac{11\pi}{6}$	0.87	0.50
2π	1	0

b)



c) Yes. The instantaneous rate of change of the cosine function is $y = -\sin x$.



Chapter 5 Section 5 **Question 3 Page 296**

a) **i)**
$$\begin{aligned}\text{Average rate of change} &= \frac{30.9904 - 31.8582}{20 - 15} \\ &= \frac{-0.867812}{5} \\ &\doteq -0.174\end{aligned}$$

ii)
$$\begin{aligned}\text{Average rate of change} &= \frac{30.9904 - 31.1823}{20 - 19} \\ &= \frac{-0.191876}{1} \\ &\doteq -0.192\end{aligned}$$

iii)
$$\begin{aligned}\text{Average rate of change} &= \frac{30.9904 - 31.01}{20 - 19.9} \\ &= \frac{-0.01959}{0.1} \\ &\doteq -0.196\end{aligned}$$

iv)
$$\begin{aligned}\text{Average rate of change} &= \frac{30.9904 - 30.9923}{20 - 19.99} \\ &= \frac{-0.001963}{0.01} \\ &\doteq -0.196\end{aligned}$$

b) The instantaneous rate of change of h at $t = 20$ s is about –0.196 m/s.

c) This instantaneous rate of change represents the vertical speed of the car at $t = 20$ s.

d) No. The graph of the sine function changes its slope continually and would not likely yield the same value at a different value of t.

Chapter 5 Section 5 **Question 4 Page 297**

a)

Daylight in Sarnia, ON	
Month	**Duration (decimal values)**
1	9.08
2	9.95
3	11.20
4	12.73
5	14.10
6	15.13
7	15.32
8	14.52
9	13.18
10	11.75
11	10.30
12	9.25

b) Amplitude, a: $\dfrac{15.32-9.08}{2}=3.12$

Period: 12

$$k=\frac{2\pi}{12}$$
$$=\frac{\pi}{6}$$

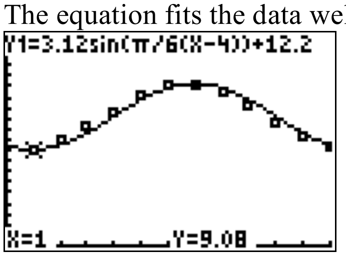
Vertical translation, c: $\dfrac{15.32+9.08}{2}=12.2$

Phase shift: The maximum value of the sine function occurs at $\dfrac{1}{4}$ of a period, or $t = 3$.

However, the maximum occurs at $t = 7$. The phase shift is 7 – 3, or 4 months to the right, $d = 4$.

Equation: $T = 3.12\sin\left[\dfrac{\pi}{6}\left(m-4\right)\right]+12.2$

c) The equation fits the data well.



d) $T \doteq 3.11\sin\left[0.51\left(m-3.63\right)\right]+12.14$

The values for a, k, c, and d compare well with those in the model.

e) I would use a phase shift to model the data using a cosine function.

f) Phase shift: The maximum value of a cosine function occurs at the beginning of the period, or $t = 0$. The phase shift is 7 – 0 = 7 to the right.

Equation: $T = 3.12\sin\left[\frac{\pi}{6}\left(m-7\right)\right]+12.2$ or $T \doteq 3.11\cos\left[0.51\left(m-6.71\right)\right]+12.14$

Chapter 5 Section 5 **Question 5 Page 297**

$$\begin{aligned}\text{Average rate of change} &= \frac{3.12\sin\left[\frac{\pi}{6}(4-4)\right]+12.2-\left[3.12\sin\left[\frac{\pi}{6}(3.9-4)\right]+12.2\right]}{4-3.9}\\ &\doteq 1.63\end{aligned}$$

$$\begin{aligned}\text{Average rate of change} &= \frac{3.12\sin\left[\frac{\pi}{6}(4-4)\right]+12.2-\left[3.12\sin\left[\frac{\pi}{6}(3.99-4)\right]+12.2\right]}{4-3.99}\\ &\doteq 1.63\end{aligned}$$

The rate of change of the number of hours of daylight on April 1 is approximately 1.63 h.

Chapter 5 Section 5 **Question 6 Page 297**

$$\begin{aligned}\text{Amplitude: } a &= \frac{7-1}{2}\\ &= 3\end{aligned}$$

Period is 2

$$\begin{aligned}k &= \frac{2\pi}{2}\\ &= \pi\end{aligned}$$

Vertical translation: 7 – 3 = 4

Equation: $h = 3\cos(\pi t)+4$

Chapter 5 Section 5 **Question 7 Page 297**

a) The instantaneous rate of change seems to be a maximum at (1.5, 4).

b) $\text{Average rate of change} = \frac{3\cos 1.5\pi+4-(3\cos 1.499\pi+4)}{1.5-1.499} \doteq 9.4$

$\text{Average rate of change} = \frac{3\cos 1.5\pi+4-(3\cos 1.4999\pi+4)}{1.5-1.4999} \doteq 9.4$

The instantaneous rate of change at this point is approximately 9.4 m/s.

c) This instantaneous rate of change of the height represents the speed of the spring.

Chapter 5 Section 5 **Question 8 Page 297**

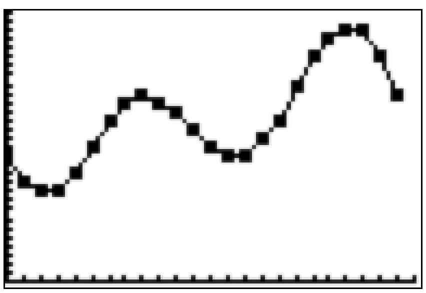
Answers may vary. A sample solution is provided.

Data for Port Alberni, Friday July 11, 2008

a) Times and Heights for High and Low Tides

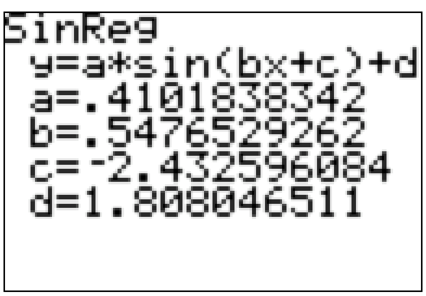
Time (PDT)	Height (m)	Time (PDT)	Height (m)
00:00	1.4	12:00	1.5
01:00	1.1	13:00	1.4
02:00	1.0	14:00	1.4
03:00	1.0	15:00	1.6
04:00	1.2	16:00	1.8
05:00	1.5	17:00	2.2
06:00	1.8	18:00	2.5
07:00	2.0	19:00	2.7
08:00	2.1	20:00	2.8
09:00	2.0	21:00	2.8
10.00	1.9	22:00	2.5
11:00	1.7	23:00	2.1

b)



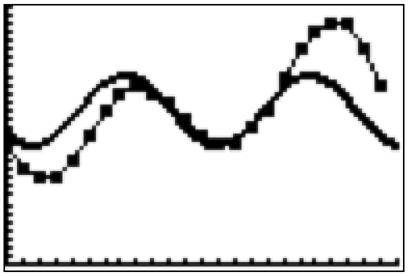
($x \in [0, 23]$, $y \in [0, 3]$, Yscl = 0.1)

c)



$y \doteq 0.41\sin(0.55x - 2.43) + 1.81$

d)



e) The equation obtained by performing a sinusoidal regression on the data is an approximation of a symmetrical sinusoidal function that best models the tide height data.

My equation is a model for only part of the graph. My equation models the average of the height data. My equation is not the best model.

Chapter 5 Section 5 **Question 9 Page 298**

Answers may vary.

a) Point selected from the data in question 8 is (02:00, 1.0).

b) $y \doteq 0.41\sin(0.55x - 2.43) + 1.81$

$$\begin{aligned}\text{Average rate of change} &= \frac{h_2 - h_1}{t_2 - t_1} \\ &= \frac{[0.41\sin(0.55(2) - 2.43) + 1.81] - [0.41\sin(0.55(1.99) - 2.43) + 1.81]}{2 - 1.99} \\ &\doteq \frac{1.42 - 1.41}{0.01} \\ &\doteq 1\end{aligned}$$

The instantaneous rate of change at 02:00 is 1 m/h.

c) The instantaneous rate of change in the water level at 02:00 represents the vertical speed of the tide at t = 02:00 h.

Chapter 5 Section 5 **Question 10 Page 298**

a) Amplitude: $a = \frac{4+4}{2}$
$= 4$

Period is $\frac{2}{5}$

$$k = \frac{2\pi}{\frac{2}{5}}$$
$$= 5\pi$$

b) Equation: $d = 4\sin 5\pi t$

Chapter 5 Section 5 **Question 11 Page 298**

a) It is the graph of $y = x^2$ reflected in the *x*-axis and shifted up 8 units.

Equation: $y = -x^2 + 8$

b) Amplitude: $a = \frac{6-2}{2}$
$= 2$

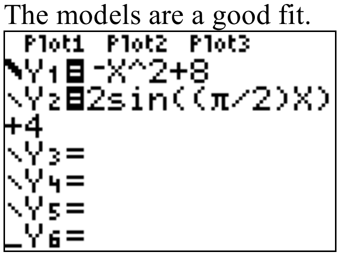
Period is 4

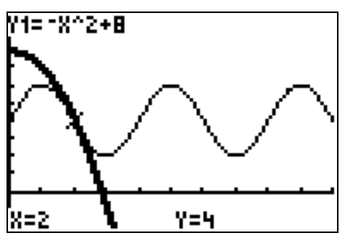
$$k = \frac{2\pi}{4}$$
$$= \frac{\pi}{2}$$

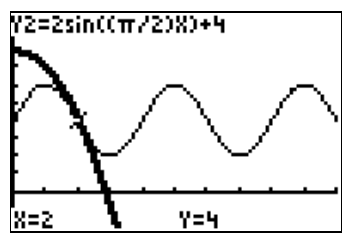
Vertical translation: $c = 6 - 2$
$= 4$

Equation: $y = 2\sin\left(\frac{\pi}{2}x\right) + 4$

c) The models are a good fit.







Chapter 5 Section 5 **Question 12 Page 298**

a) Average rate of change $= \frac{-(2)^2 + 8 - \left[-(1.999)^2 + 8\right]}{2 - 1.999}$
$\doteq -4$

The instantaneous rate of change at $x = 2$ is approximately –4.

b) Average rate of change $= \dfrac{2\sin\left[\dfrac{\pi}{2}(2)\right]+4-\left[2\sin\left[\dfrac{\pi}{2}(1.999)\right]+4\right]}{2-1.999}$

$\doteq -3.14$

The instantaneous rate of change at $x = 2$ is approximately –3.14.

c) The answers in parts a) and b) are different.

d) Answers may vary. A sample solution is shown.
If the instantaneous rate is considerably different, the cars may fall off the track.

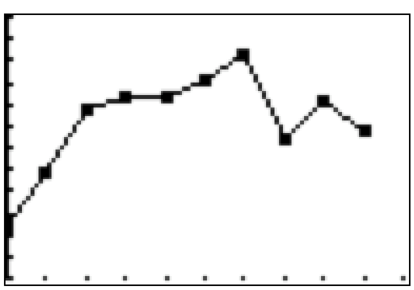
Chapter 5 Section 5 **Question 13 Page 298**

Answers may vary.

b) The data selected is Internal Travellers to Canada, Years: 1999 to 2005, Table 387-0004. Travellers from the United States who stayed overnight in Canada. Unadjusted figures are shown.

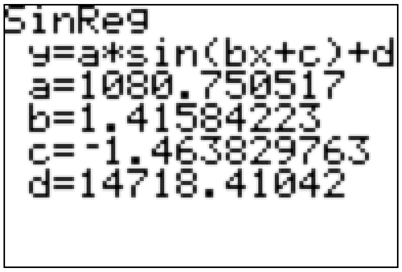
Year	Number of Visitors
1996	12 909 000
1997	13 401 000
1998	14 892 000
1999	15 180 000
2000	15 189 000
2001	15 570 000
2002	16 167 000
2003	14 232 000
2004	15 088 000
2005	14 390 000

c)



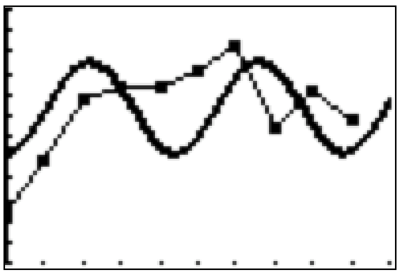
($x \in [0, 9]$, $y \in [10\ 000, 17\ 000]$, Yscl = 500)

d)



$y \doteq 1080.75\sin(1.42x - 1.46) + 14\ 718.41$

e)



f) The equation obtained by performing a sinusoidal regression on the data is an approximation of a symmetrical sinusoidal function that best models the number of travellers data.

My equation models the average of the number of travellers data. My equation is not a very good model.

Chapter 5 Section 5 **Question 14 Page 298**

Answers may vary.

a) Point selected from the data in question 13 is (2000, 15 189 000).

b) $y \doteq 1080.75\sin(1.42x - 1.46) + 14\,718.41$

Average rate of change

$$= \frac{h_2 - h_1}{t_2 - t_1}$$
$$= \frac{[1080.75\sin(1.42(2000) - 1.46) + 14\,718.41] - [1080.75\sin(1.42(1999) - 1.46) + 14\,718.41]}{2000 - 1999}$$
$$\doteq \frac{13\,644.32 - 14\,438.66}{1}$$
$$\doteq -794.34$$

The instantaneous rate of change at year 2000 is –794 340 travellers/year.

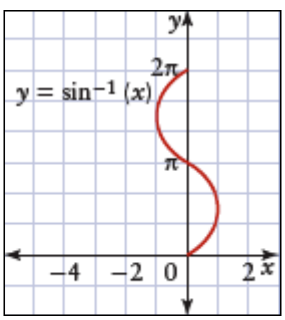
c) The instantaneous rate of change in the number of travellers at year 2000 represents the vertical speed of the change in the number of travellers to Canada. This represents a rate of decline.

Chapter 5 Section 5 **Question 15 Page 299**

Solutions to the Achievement Check can be found in the Teacher's Resource.

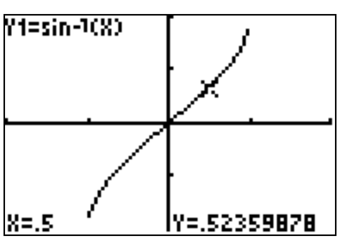
Chapter 5 Section 5 **Question 16 Page 299**

a)



b) No, it is not a function because it does not pass the vertical line test.

In order for it to be a function, restrict the range to the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.



c) When $x = 0$, the instantaneous rate of change appears to be at a maximum.

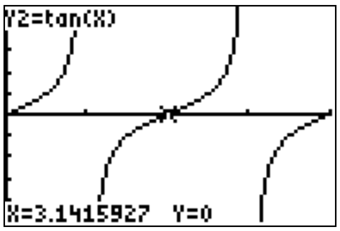
d) Average rate of change $= \dfrac{\sin^{-1} 0 - \sin^{-1} 0.001}{0 - 0.001}$

$= 1$

The instantaneous rate of change at $x = 0$ is 1.

Chapter 5 Section 5 **Question 17 Page 299**

a)

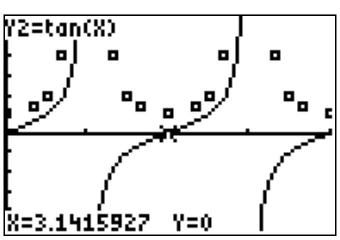


b) The instantaneous rate of change does not appear to equal 0 at any value of x.
There is no maximum or minimum value.

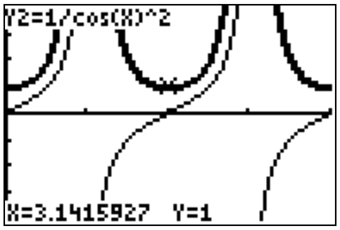
c)

Angle x	$f(x) = \tan x$	Instantaneous Rate of Change
0	0	1
$\frac{\pi}{6}$	0.58	1.33
$\frac{\pi}{4}$	1	2
$\frac{\pi}{3}$	1.73	4
$\frac{\pi}{2}$	undefined	undefined
$\frac{2\pi}{3}$	–1.73	4
$\frac{3\pi}{4}$	–1	2
$\frac{5\pi}{6}$	–0.58	1.33
π	0	1
$\frac{7\pi}{6}$	0.58	1.33
$\frac{5\pi}{4}$	1	2
$\frac{4\pi}{3}$	1.73	4
$\frac{3\pi}{2}$	undefined	undefined
$\frac{5\pi}{3}$	–1.73	4
$\frac{7\pi}{4}$	–1	2
$\frac{11\pi}{6}$	–0.58	1.33
2π	0	1

d)

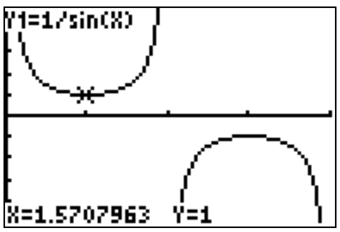


e) Answers may vary. For example, the instantaneous rate of change follows the same pattern as $y = \sec^2 x$.



Chapter 5 Section 5 **Question 18 Page 299**

a)

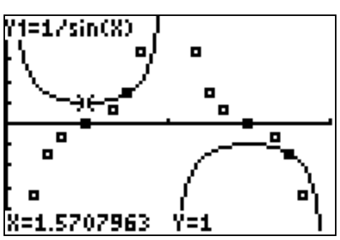


b) The rate of change appears to be zero when $x = \frac{\pi}{2}, \frac{3\pi}{2}$. There are no maximum or minimum values.

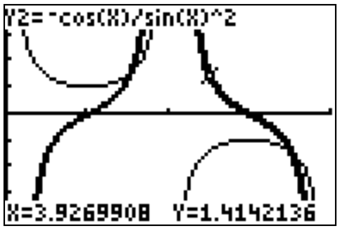
c)

Angle x	$f(x) = \csc x$	**Instantaneous Rate of Change**
0	undefined	undefined
$\frac{\pi}{6}$	2	–3.46
$\frac{\pi}{4}$	1.41	–1.41
$\frac{\pi}{3}$	1.15	–0.67
$\frac{\pi}{2}$	1	0
$\frac{2\pi}{3}$	1.15	0.67
$\frac{3\pi}{4}$	1.41	1.41
$\frac{5\pi}{6}$	2	3.46
π	undefined	undefined
$\frac{7\pi}{6}$	–2	3.46
$\frac{5\pi}{4}$	–1.41	1.41
$\frac{4\pi}{3}$	–1.15	0.67
$\frac{3\pi}{2}$	–1	0
$\frac{5\pi}{3}$	–1.15	–0.67
$\frac{7\pi}{4}$	–1.41	–1.41
$\frac{11\pi}{6}$	–2	–3.46
2π	undefined	undefined

d)



e) Answers may vary. For example, the pattern that is formed is the same as $y = -\dfrac{\cot x}{\sin x}$.



Chapter 5 Section 5 **Question 19 Page 299**

$$\begin{aligned}
\left(\frac{w^2+1}{w}\right)^2 &= 3 \\
\frac{w^4+2w^2+1}{w^2} &= 3 \\
w^2+2+\frac{1}{w^2} &= 3 \\
w^2+\frac{1}{w^2} &= 3-2 \\
w^2+\frac{1}{w^2} &= 1
\end{aligned}$$

The value is 1.

Chapter 5 Section 5 **Question 20 Page 299**

a) sin(P + Q) = sin P cos Q + sin Q + cos P

$$\begin{aligned}
(3\sin \mathrm{P} + 4\cos \mathrm{Q})^2 &= 6^2 \\
9\sin^2 \mathrm{P} + 24\sin \mathrm{P}\cos \mathrm{Q} + 16\cos^2 \mathrm{Q} &= 36 \quad ①
\end{aligned}$$

$$\begin{aligned}
(4\sin \mathrm{Q} + 3\cos \mathrm{P})^2 &= 1^2 \\
16\sin^2 \mathrm{Q} + 24\sin \mathrm{Q}\cos \mathrm{P} + 9\cos^2 \mathrm{P} &= 1 \quad ②
\end{aligned}$$

$$\begin{aligned}
9\sin^2 \mathrm{P} + 24\sin \mathrm{P}\cos \mathrm{Q} + 16\cos^2 \mathrm{Q} + 16\sin^2 \mathrm{Q} + 24\sin \mathrm{Q}\cos \mathrm{P} + 9\cos^2 \mathrm{P} &= 36+1 \quad ① + ② \\
9(\sin^2 \mathrm{P} + \cos^2 \mathrm{P}) + 16(\sin^2 \mathrm{Q} + \cos^2 \mathrm{Q}) + 24(\sin \mathrm{P}\cos \mathrm{Q} + \sin \mathrm{Q}\cos \mathrm{P}) &= 37 \\
9 + 16 + 24\sin(\mathrm{P}+\mathrm{Q}) &= 37 \\
24\sin(\mathrm{P}+\mathrm{Q}) &= 12 \\
\sin(\mathrm{P}+\mathrm{Q}) &= \frac{1}{2}
\end{aligned}$$

b) $\sin(\mathrm{P}+\mathrm{Q}) = \sin(\pi - \mathrm{R})$

$$= \sin \mathrm{R} \qquad \text{(cofunction identity)}$$

$$\sin \mathrm{R} = \frac{1}{2}$$

$$\angle \mathrm{R} = \frac{\pi}{6}$$

Chapter 5 Section 5 **Question 21 Page 299**

$$r\cos\theta = x \qquad r\sin\theta = y$$

$$\cos\theta = \frac{x}{r} \qquad \sin\theta = \frac{y}{r}$$

$$r = a\sin\theta + b\cos\theta$$

$$r = \frac{ax}{r} + \frac{by}{r}$$

$$r^2 = ax + by$$

$$x^2 + y^2 = ax + by \qquad (r^2 = x^2 + y^2)$$

$$x^2 - ax + y^2 - by = 0$$

$$\left(x^2 - ax + \frac{a^2}{4}\right) - \frac{a^2}{4} + \left(y^2 - by + \frac{b^2}{4}\right) - \frac{b^2}{4} = 0 \qquad \text{(complete the square)}$$

$$\left(x - \frac{a}{2}\right)^2 + \left(y - \frac{b}{2}\right)^2 = \frac{a^2 + b^2}{4}$$

$\left(x - \frac{b}{2}\right)^2 + \left(y - \frac{a}{2}\right)^2 = \frac{a^2 + b^2}{4}$ is a circle.

The centre is $\left(\frac{b}{2}, \frac{a}{2}\right)$ and the radius is $\frac{\sqrt{a^2 + b^2}}{2}$.

Therefore, $r = a\sin\theta + b\cos\theta$ is a circle with centre $\left(\frac{b}{2}, \frac{a}{2}\right)$.

Chapter 5 Review

Chapter 5 Review **Question 1 Page 300**

a) Amplitude: $a = \dfrac{6-2}{2}$
$= 2$

b) Vertical translation: $c = \dfrac{6+2}{2}$
$= 4$

Chapter 5 Review **Question 2 Page 300**

$k = \dfrac{2\pi}{3\pi} = \dfrac{2}{3}$

Equation: $y = \cos\left[\dfrac{2}{3}\left(x + \dfrac{\pi}{3}\right)\right]$

Chapter 5 Review **Question 3 Page 300**

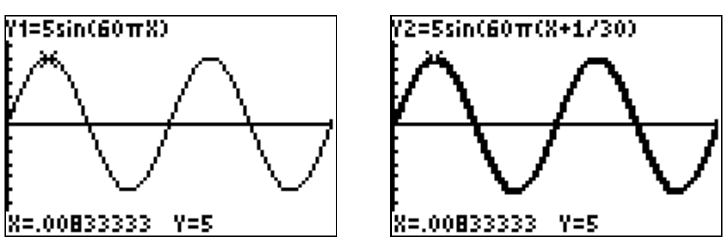
a) Amplitude: $a = \dfrac{10}{2}$
$= 5$

Period is $\dfrac{1}{30}$

$k = \dfrac{2\pi}{\frac{1}{30}}$
$= 60\pi$

Equation: $y = 5\sin 60\pi t$

b) No, a phase shift of the period can generate another possible equation.



Chapter 5 Review **Question 4 Page 300**

$$\csc x = 4$$
$$\frac{1}{\sin x} = 4$$
$$\sin x = \frac{1}{4}$$
$$x \doteq 0.25 \quad \text{or} \quad x \doteq \pi - 0.25$$
$$x \doteq 2.89$$

The solution is $x \doteq 0.25$ or $x \doteq 2.89$.

Chapter 5 Review **Question 5 Page 300**

a) The secant function is a reciprocal of the cosine function and $\cos^{-1}$ is the opposite operation of cosine.

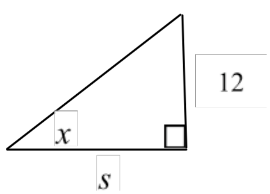
b) $\sec\left(\frac{1}{\sqrt{2}}\right) \doteq 1.32$

$$\cos^{-1}\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}$$

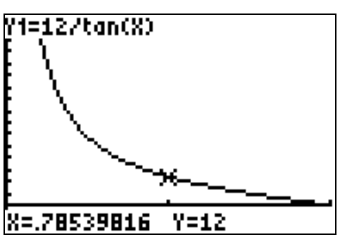
Chapter 5 Review **Question 6 Page 300**

a) $\cot x = \frac{s}{12}$

$$s = 12\cot x$$



b)



c) As x approaches 0, s approaches infinity. This means that the angle of elevation of the Sun approaches the horizon so the length of the shadow approaches infinity. As x approaches $\frac{\pi}{2}$, s approaches 0. This means that the angle of elevation of the Sun approaches an overhead location so the length of the shadow approaches 0.

Chapter 5 Review **Question 7 Page 300**

a) Amplitude: $a = \frac{2+4}{2}$
$= 3$

Vertical translation: $c = 2 - 3$
$= -1$

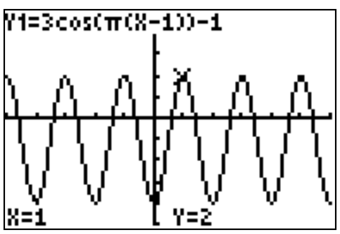
Period is 2

$k = \frac{2\pi}{2}$
$= \pi$

Phase shift: The maximum of a cosine function occurs at the beginning of the period. For this function it occurs at $x = 1$, so the phase shift is 1 unit to the right, $d = 1$.

b) Equation: $y = 3\cos\left[\pi\left(x-1\right)\right] - 1$

c)



d) Answers may vary. For example, they match for all values of x.

Chapter 5 Review **Question 8 Page 300**

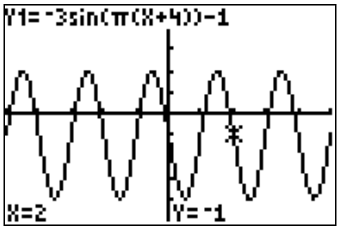
a) The amplitude is 3.

b) The period is $\frac{2\pi}{\pi} = 2$

c) The phase shift is 4 rad left.

d) The vertical translation is 1 unit down.

e) All the characteristics are met.

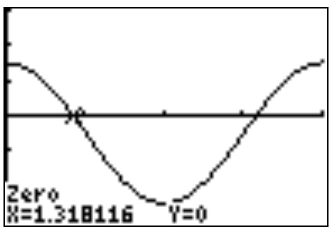


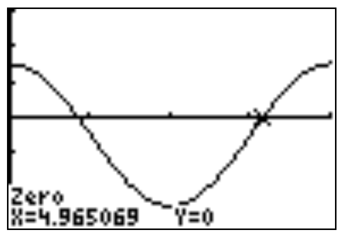
Chapter 5 Review **Question 9 Page 300**

a) $\cos x = \dfrac{1}{4}$

$$x \doteq 1.32 \quad \text{or} \quad x \doteq 2\pi - 1.32$$
$$\doteq 4.97$$

So, $x \doteq 1.32$ or $x \doteq 4.97$.

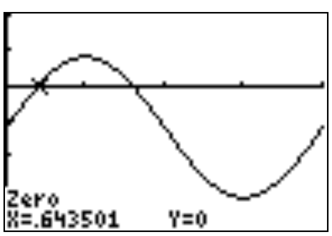


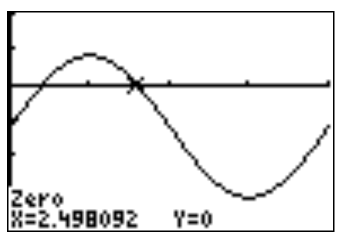


b) $\sin x = 0.6$

$$x \doteq 0.64 \quad \text{or} \quad x \doteq \pi - 0.64$$
$$\doteq 2.50$$

So, $x \doteq 0.64$ or $x \doteq 2.50$.





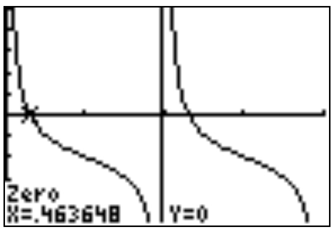
c) $\cot x = 2$

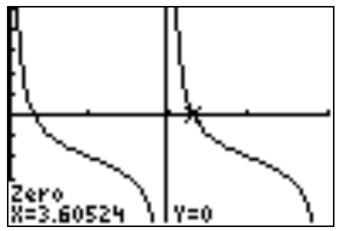
$$\frac{1}{\tan x} = 2$$

$$\tan x = \frac{1}{2}$$

$$x \doteq 0.46 \quad \text{or} \quad x \doteq \pi + 0.46$$
$$\doteq 3.61$$

So, $x \doteq 0.46$ or $x \doteq 3.61$.





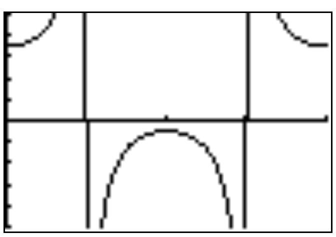
d) $4\sec x = -3$

$$\sec x = -\frac{3}{4}$$

$$\frac{1}{\cos x} = -\frac{3}{4}$$

$$\cos x = -\frac{4}{3}$$

There is no solution.



Chapter 5 Review **Question 10 Page 300**

a) $(2\sin x - 1)(\sin x + 1) = 0$

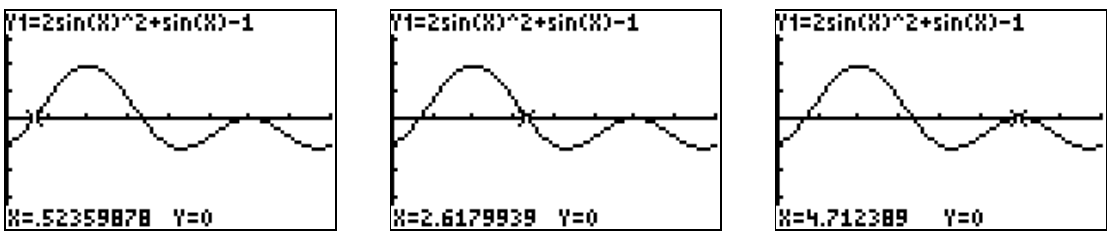
$2\sin x = 1$ or $\sin x = -1$

Case 1:
$$2\sin x = 1$$
$$\sin x = \frac{1}{2}$$
$$x = \frac{\pi}{6} \quad \text{or} \quad x = \frac{5\pi}{6}$$

Case 2:
$$\sin x = -1$$
$$x = \frac{3\pi}{2}$$

$$x = \frac{\pi}{6} \text{ or } x = \frac{5\pi}{6} \text{ or } x = \frac{3\pi}{2}$$

b)



Chapter 5 Review **Question 11 Page 301**

$$\cos^2 x = \frac{3}{4}$$
$$\cos x = \pm\frac{\sqrt{3}}{2}$$

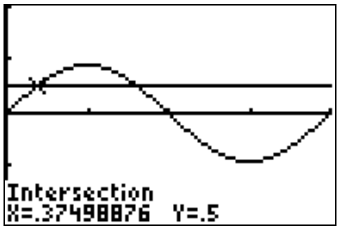
$$x = \frac{\pi}{6} \text{ or } x = \frac{5\pi}{6} \text{ or } x = \frac{7\pi}{6} \text{ or } x = \frac{11\pi}{6}$$

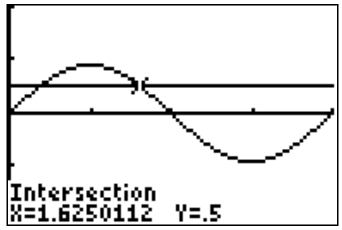
Chapter 5 Review **Question 12 Page 301**

a)
$$a = \frac{1.8}{2} = 0.9$$
$$k = \frac{2\pi}{4} = \frac{\pi}{2}$$

Equation: $y = 0.9\sin\left(\frac{\pi}{2}t\right)$

b)

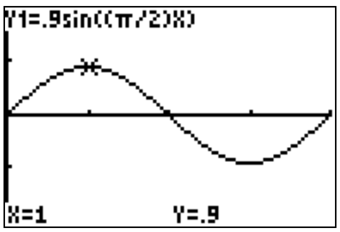


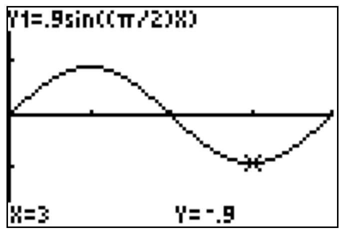


$t \doteq 0.37$ or $t \doteq 1.63$

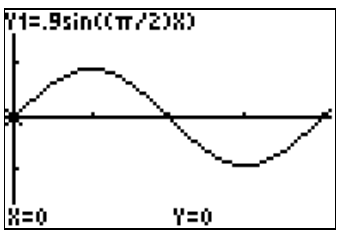
The boat is 0.5 m above ground at approximately 0.37 s and 1.63 s.

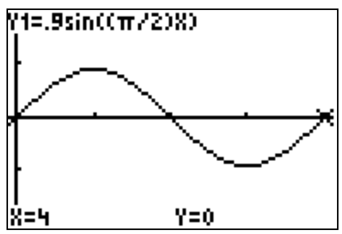
Chapter 5 Review **Question 13 Page 301**





The instantaneous rate of change is equal to 0 at $t = 1$ and $t = 3$.

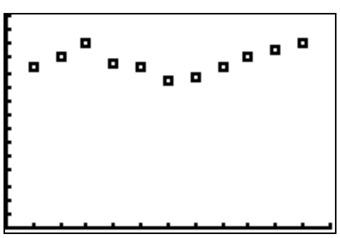




The maximum instantaneous rate of change occurs at $t = 0$ and $t = 4$.

Chapter 5 Review **Question 14 Page 301**

a)



b) Amplitude: $a = \dfrac{261-209}{2}$
$= 26$

Period: 8

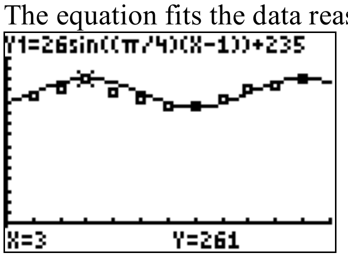
$k = \dfrac{2\pi}{8}$
$= \dfrac{\pi}{4}$

Vertical translation: $c = 261 - 26$
$= 235$

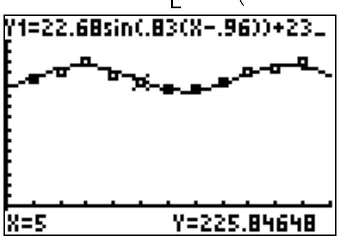
Phase shift: The maximum of the sine function occurs at $\frac{1}{4}$ of the period. $\frac{1}{4}$ of 8 is 2. The maximum for this function occurs at 3, so the phase shift is 1 unit to the right. $d = 1$.

Equation: $y = 26\sin\left[\dfrac{\pi}{4}(x-1)\right] + 235$

c) The equation fits the data reasonably well.



d) Using sinusoidal regression, an equation that better fits the data is
$y \doteq 22.68\ \sin\left[0.83\left(x-0.96\right)\right] + 230.61$.



e) Answers may vary.

Chapter Problem Wrap Up

Solutions for the Chapter Problem Wrap-Up can be found in the Teacher's Resource.

Chapter 5 Practice Test

Chapter 5 Practice Test **Question 1 Page 302**

The correct solution is **B**.

Since the maximum of $y = \cos x$ is 1 and this function is vertically transformed 2 units down, the maximum is $1 - 2 = -1$.

Chapter 5 Practice Test **Question 2 Page 302**

The correct solution is **C**.

$$\begin{aligned} a &= \frac{8+2}{2} \\ &= 5 \end{aligned} \qquad \begin{aligned} c &= 8-5 \\ &= 3 \end{aligned}$$

Chapter 5 Practice Test **Question 3 Page 302**

The correct solution is **C**.

period: $\dfrac{2\pi}{3\pi} = \dfrac{2}{3}$

Chapter 5 Practice Test **Question 4 Page 302**

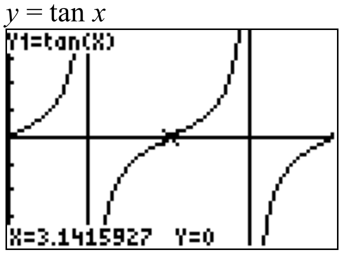
The correct solution is **A**.

$$3\cos\left[2(x - \frac{\pi}{4})\right];\ d = \frac{\pi}{4}$$

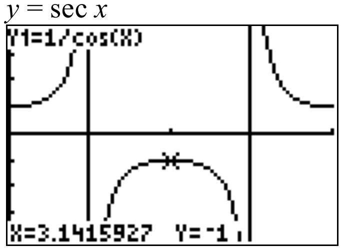
Chapter 5 Practice Test **Question 5 Page 302**

The correct solution is **D**.

$y = \tan x$



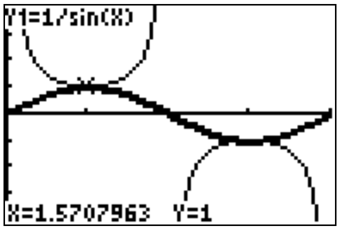
$y = \sec x$

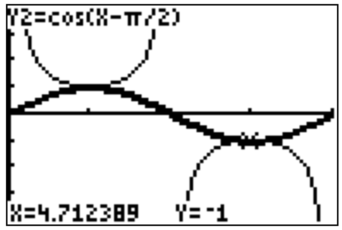


Chapter 5 Practice Test **Question 6 Page 302**

The correct solution is **C**.

$x = \frac{\pi}{2}$ and $x = \frac{3\pi}{2}$

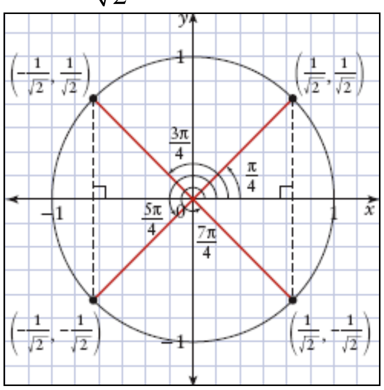




Chapter 5 Practice Test **Question 7 Page 302**

The correct solution is **A**.

$\sin x = \frac{1}{\sqrt{2}}$; $x = \frac{3\pi}{4}$



Chapter 5 Practice Test **Question 8 Page 302**

a) The cosecant function is a reciprocal of the sine function and $\sin^{-1}$ is the opposite operation of sine.

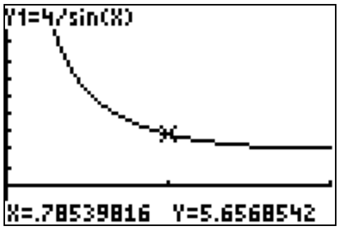
b) $\csc\left(\frac{\sqrt{3}}{2}\right) \doteq 1.31$ $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}$

Chapter 5 Practice Test **Question 9 Page 302**

a) $\csc x = \frac{l}{4}$

$l = 4\csc x$

b)



c) As x approaches 0, t approaches infinity. This means that the angle of inclination of the wire approaches horizontal and so the length of the wire approaches infinity. As x approaches $\frac{\pi}{2}$, t approaches 4. This means that the angle of inclination of the wire approaches vertical and the length of the wire approaches 4 m.

Chapter 5 Practice Test **Question 10 Page 302**

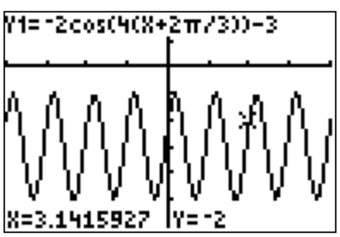
a) Amplitude: $a = 2$

b) Period: $\frac{2\pi}{4} = \frac{\pi}{2}$

c) $d = \frac{-2\pi}{3}$; The phase shift is $\frac{2\pi}{3}$ rad to the left.

d) $c = -3$; The vertical translation is 3 units down.

e) The graph satisfies all the given parameters.

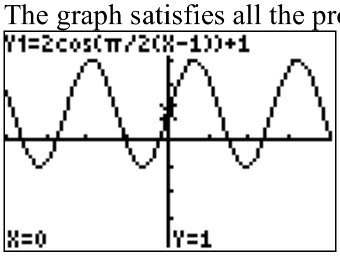


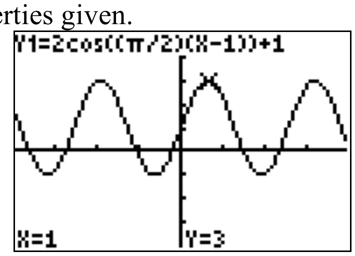
Chapter 5 Practice Test **Question 11 Page 303**

a) $a = \dfrac{3+1}{2}$
$= 2$
$c = 3 - 2$
$= 1$
$k = \dfrac{2\pi}{4}$
$= \dfrac{\pi}{2}$
$d = 1$

Equation: $y = 2\cos\left[\dfrac{\pi}{2}(x-1)\right]+1$

b) The graph satisfies all the properties given.



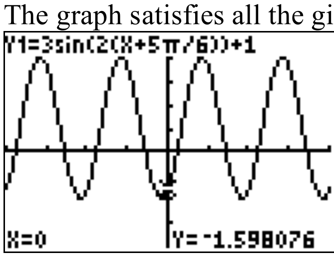


Chapter 5 Practice Test **Question 12 Page 303**

a) $a = \dfrac{4+2}{2}$
$= 3$
$c = 4 - 3$
$= 1$
$k = \dfrac{2\pi}{\pi}$
$= 2$
$d = -\dfrac{5\pi}{6}$

Equation: $y = 3\sin\left[2\left(x+\dfrac{5\pi}{6}\right)\right]+1$

b) The graph satisfies all the given properties.



Chapter 5 Practice Test **Question 13 Page 303**

$$\cos^2 x = 0.49$$
$$\cos x = \pm 0.7$$

Case 1:

$\cos x = 0.7$

$x \doteq 0.8$ or $x \doteq 2\pi - 0.8$

$\doteq 5.49$

Case 2:

$\cos x = -0.7$

$x \doteq 2.35$ or $x \doteq 2\pi - 2.35$

$\doteq 3.94$

$x \doteq 0.8$ or $x \doteq 2.35$ or $x \doteq 3.94$ or $x \doteq 5.49$

Chapter 5 Practice Test **Question 14 Page 303**

$$(2\sin x - 1)(\sin x - 1) = 0$$

$2\sin x = 1$ or $\sin x = 1$

Case 1:

$2\sin x = 1$

$\sin x = \frac{1}{2}$

$x = \frac{\pi}{6}$ or $x = \frac{5\pi}{6}$

Case 2:

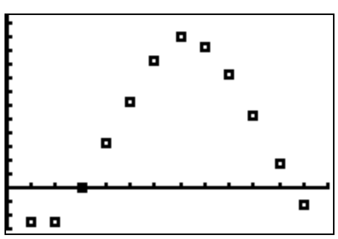
$\sin x = 1$

$x = \frac{\pi}{2}$

$x = \frac{\pi}{6}$ or $x = \frac{\pi}{2}$ or $x = \frac{5\pi}{6}$

Chapter 5 Practice Test **Question 15 Page 303**

a)



b) Amplitude, a: $\dfrac{21.9+4.8}{2}=13.35$

Period: 12

$$k=\frac{2\pi}{12}$$
$$=\frac{\pi}{6}$$

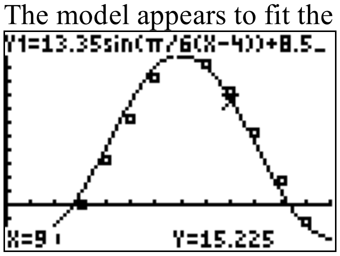
Vertical translation, c: 21.9 – 13.35 = 8.55

Phase shift: The maximum value of a sine function occurs at $\frac{1}{4}$ of a period, $\frac{1}{4}$ of 12 is 3.

The maximum of this function occurs at 7. The phase shift is 7 – 3 = 4 to the right; $d = 4$.

Equation: $y = 13.35\sin\left[\frac{\pi}{6}(x-4)\right]+8.55$

c) The model appears to fit the data well.



Chapter 5 Practice Test **Question 16 Page 303**

a), b)

Phases of the Moon 2007	
Date (days from beginning of year)	**Phase (percent illumination)**
3	100
11	50
19	0
25	50
33	100
41	50
48	0
55	50
62	100
71	50
78	0
84	50
92	100
100	50
107	0
114	50

c) Amplitude, a: $\dfrac{100-0}{2}=50$

Period: 30

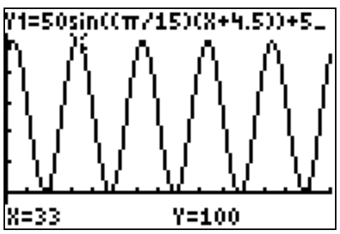
$$k=\frac{2\pi}{30}$$
$$=\frac{\pi}{15}$$

Vertical translation, c: 100 – 50 = 50

Phase shift: the maximum value of a sine function occurs at $\frac{1}{4}$ of a period, $\frac{1}{4}$ of 30 is 7.5. The maximum for this function occurs at 3, so the phase shift is 7.5 – 3 = 4.5; d = 4.5.

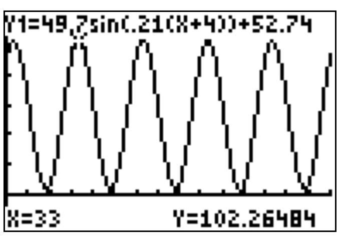
Equation: $y = 50\sin\left[\frac{\pi}{15}\left(x+4.5\right)\right]+50$

d)



e) Using sinusoidal regression, the equation is $y \doteq 49.75\sin\left[0.21(x+4)\right]+52.74$. The values of a, k, c, and d compare well with the model.

f)



Chapter 5 Practice Test **Question 17 Page 303**

a) Answers may vary. Sample Answer: Using your model, first find the average rate of change of the percent of illumination, then estimate the instantaneous rate of change.

b) $$\text{Average rate of change} = \frac{50\sin\left[\frac{\pi}{15}(25+4.5)\right]-50\sin\left[\frac{\pi}{15}(24.999+4.5\right]}{25-24.999}$$
$$\doteq 10.4$$

The instantaneous rate of change on January 25 is approximately 10.4% per day.

c) This instantaneous rate of change represents the percent change in illumination of the moon on January 25.

Chapters 4 and 5 Review

Chapters 4 and 5 Review **Question 1 Page 304**

a) $100° \times \dfrac{\pi}{180°} = \dfrac{5\pi}{9}$

b) $\dfrac{7\pi}{12} \times \dfrac{180°}{\pi} = 105°$

Chapters 4 and 5 Review **Question 2 Page 304**

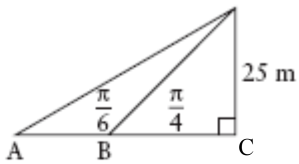
$$\begin{aligned}\theta &= \frac{a}{r}\\ &= \frac{60}{20}\\ &= 3\end{aligned}$$

The sector angle is 3 radians.

Chapters 4 and 5 Review **Question 3 Page 304**

$$\frac{1}{\sqrt{2}} - \left(-\frac{1}{\sqrt{3}}\right) = \frac{\sqrt{3}+\sqrt{2}}{\sqrt{6}}$$

Chapters 4 and 5 Review **Question 4 Page 304**



$$\tan\frac{\pi}{6}=\frac{25}{AC}$$
$$AC=\frac{25}{\tan\frac{\pi}{6}}$$
$$AC=\frac{25}{\frac{1}{\sqrt{3}}}$$
$$AC=25\sqrt{3}$$

$$\tan\frac{\pi}{4}=\frac{25}{BC}$$
$$BC=\frac{25}{\tan\frac{\pi}{4}}$$
$$BC=\frac{25}{1}$$
$$BC=25$$

$$\begin{aligned}AB&=AC-BC\\&=25\sqrt{3}-25\\&=25(\sqrt{3}-1)\end{aligned}$$

Chapters 4 and 5 Review **Question 5 Page 304**

Since $\frac{29\pi}{30}$ lies in the second quadrant, it can be expressed as a sum of $\frac{\pi}{2}$ and an angle a.

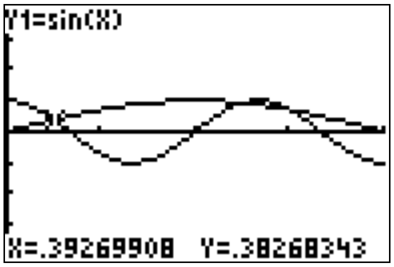
$$\begin{aligned}a+\frac{\pi}{2}&=\frac{29\pi}{30}\\a&=\frac{29\pi}{30}-\frac{\pi}{2}\\a&=\frac{29\pi}{30}-\frac{15\pi}{30}\\a&=\frac{14\pi}{30}\\a&=\frac{7\pi}{15}\end{aligned}$$

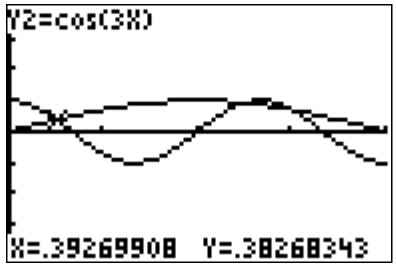
Now apply a trigonometric identity.

$$\begin{aligned}\sin\frac{29\pi}{30}&=\sin\left(\frac{7\pi}{15}+\frac{\pi}{2}\right)\\&=\cos\frac{7\pi}{15}\\&\doteq 0.1045\end{aligned}$$

Chapters 4 and 5 Review **Question 6 Page 304**

Answers may vary. A sample solution is shown.

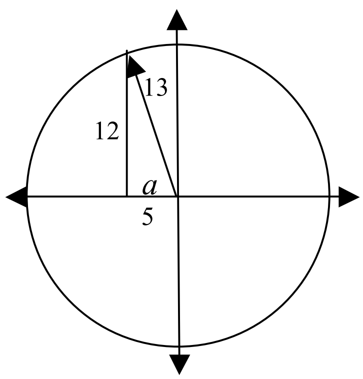


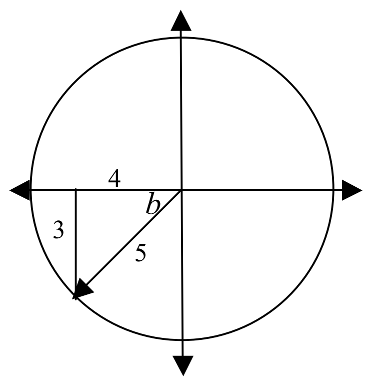


$y = \frac{\pi}{8}$

Chapters 4 and 5 Review **Question 7 Page 304**

$$\begin{aligned} x^2 &= 13^2 - 12^2 \\ x^2 &= 169 - 144 \\ x^2 &= 25 \\ x &= 5 \end{aligned} \qquad \begin{aligned} y^2 &= 5^2 - 4^2 \\ y^2 &= 25 - 16 \\ y^2 &= 9 \\ y &= 3 \end{aligned}$$

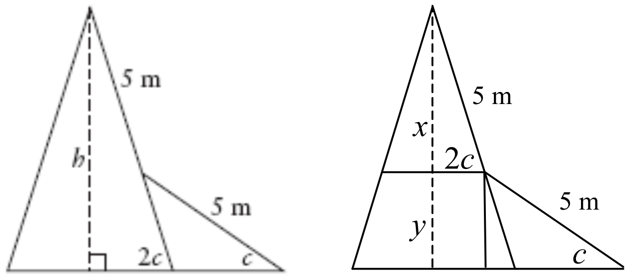




$$\cos a = -\frac{5}{13} \qquad \sin b = -\frac{3}{5}$$

$$\begin{aligned} \sin(a+b) &= \sin a \cos b + \sin b \cos a \\ &= \left(\frac{12}{13}\right)\left(-\frac{4}{5}\right) + \left(-\frac{3}{5}\right)\left(-\frac{5}{13}\right) \\ &= -\frac{48}{65} + \frac{15}{65} \\ &= -\frac{33}{65} \end{aligned}$$

Chapters 4 and 5 Review **Question 8 Page 304**



$$\sin 2c = \frac{x}{5} \qquad\qquad \sin c = \frac{y}{5}$$

$$x = 5\sin 2c \qquad\qquad y = 5\sin c$$

$$x = 5(2\sin c\cos c) \quad \text{double angle formula}$$

$$\begin{aligned} x + y &= 5(2\sin c\cos c) + 5\sin c \\ &= 5\sin c(2\cos c + 1) \end{aligned}$$

Chapters 4 and 5 Review **Question 9 Page 304**

a) $\dfrac{\sin^2 x}{\cot^2 x} + \sin^2 x = \tan^2 x$

b) $$\begin{aligned} \textbf{L.S.} &= \frac{\sin^2 x}{\cot^2 x} + \sin^2 x \\ &= \sin^2 x\tan^2 x + \sin^2 x && \text{reciprocal identity} \\ &= \sin^2 x\left(\frac{\sin^2 x}{\cos^2 x}\right) + \sin^2 x\left(\frac{\cos^2 x}{\cos^2 x}\right) && \text{quotient identity} \\ &= \frac{\sin^2 x(\sin^2 x + \cos^2 x)}{\cos^2 x} \\ &= \frac{\sin^2 x(1)}{\cos^2 x} && \text{Pythagorean identity} \\ &= \tan^2 x && \text{quotient identity} \end{aligned}$$

$\textbf{R.S.} = \tan^2 x$

Since **L.S.** = **R.S.**, $\dfrac{\sin^2 x}{\cot^2 x} + \sin^2 x = \tan^2 x$ is an identity.

Chapters 4 and 5 Review **Question 10 Page 304**

$$\textbf{L.S.} = \frac{\sin 2x}{\sec x}$$
$$= \frac{2\sin x \cos x}{\frac{1}{\cos x}} \quad \text{double angle formula and reciprocal identity}$$
$$= 2\sin x \cos^2 x$$

$$\textbf{R.S.} = \frac{2\cos^2 x}{\csc x}$$
$$= \frac{2\cos^2 x}{\frac{1}{\sin x}} \quad \text{reciprocal identity}$$
$$= 2\cos^2 x \sin x$$

Since **L.S.** = **R.S.**, $\frac{\sin 2x}{\sec x} = \frac{2\cos^2 x}{\csc x}$ is an identity.

Chapters 4 and 5 Review **Question 11 Page 304**

$$\textbf{L.S.} = \sin(x+y)\cos(x-y)$$
$$= [\sin x \cos y + \sin y \cos x][\cos x \cos y + \sin x \sin y]$$
$$= \sin x \cos x \cos^2 y + \sin y \cos y \sin^2 x + \sin y \cos y \cos^2 x + \sin x \cos x \sin^2 y$$
$$= \sin x \cos x(\cos^2 y + \sin^2 y) + \sin y \cos y(\sin^2 x + \cos^2 x)$$
$$= \sin x \cos x + \sin y \cos y \quad \text{Pythagorean identity}$$

$$\textbf{R.S.} = \frac{\sin x}{\sec x} + \frac{\cos y}{\csc y}$$
$$= \frac{\sin x}{\frac{1}{\cos x}} + \frac{\cos y}{\frac{1}{\sin y}} \quad \text{reciprocal identities}$$
$$= \sin x \cos x + \cos y \sin y$$

Since **L.S.** = **R.S.**, $\sin(x+y)\cos(x-y) = \frac{\sin x}{\sec x} + \frac{\cos y}{\csc y}$ is an identity.

Chapters 4 and 5 Review **Question 12 Page 304**

a)
$$\begin{aligned}\text{period} &= \frac{\pi}{3}-\left(-\frac{\pi}{4}\right)\\ &= \frac{\pi}{3}+\frac{\pi}{4}\\ &= \frac{4\pi}{12}+\frac{3\pi}{12}\\ &= \frac{7\pi}{12}\end{aligned}$$

b) The function $y = \sin x$ begins the period at $x = 0$. This sine function starts at $-\frac{\pi}{4}$, so the phase shift is $\frac{\pi}{4}$ to the left. $d = -\frac{\pi}{4}$

Chapters 4 and 5 Review **Question 13 Page 304**

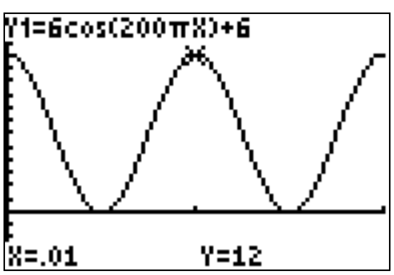
a)
$$\begin{aligned}a &= \frac{12}{2}\\ &= 6\end{aligned}$$

b) period is $\frac{1}{100}$ s

c)
$$\begin{aligned}k &= \frac{2\pi}{\frac{1}{100}}\\ &= 200\pi\end{aligned}$$

d) $c = 6 - 0$; $y = 6 \cos 200\pi x + 6$

e)



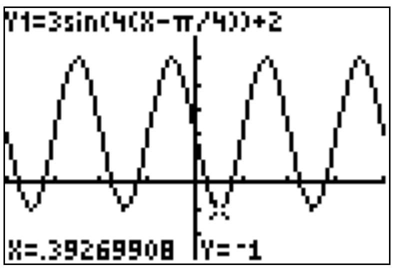
a) $a = 3$

b) The period is $\frac{2\pi}{4} = \frac{\pi}{2}$.

c) $d = \frac{\pi}{4}$; $\frac{\pi}{4}$ radians to the right

d) $c = 2$; vertical shift of 2 upwards

e)



The graph satisfies the characteristics expected.

a) $\csc x = \dfrac{d}{500}$

$d = 500\csc x$

b) $0 \le d \le 250\sqrt{13}$; Contestants have a choice not to go cross country so the lower limit is 0. The upper limit is when d = AC.

$$\begin{aligned}
\text{AC}^2 &= 750^2 + 500^2 \\
\text{AC}^2 &= 562\ 500 + 250\ 000 \\
\text{AC}^2 &= 812\ 500 \\
\text{AC} &= \sqrt{812\ 500} \\
\text{AC} &= 250\sqrt{13}
\end{aligned}$$

c) Total time on pavement:

$$\begin{aligned}
t &= \frac{d}{v} \\
&= \frac{(750 + 500)}{10} \\
&= \frac{1250}{10} \\
&= 125
\end{aligned}$$

Total time cross country:

$$\begin{aligned}
t &= \frac{d}{v} \\
&= \frac{(250\sqrt{13})}{6} \\
&\doteq 150 \text{ s}
\end{aligned}$$

749 m on pavement the rest cross country:

$$\begin{aligned}
d^2 &= 500^2 + 1^2 \\
d^2 &= 250\ 000 + 1 \\
d^2 &= 250\ 001 \\
d &= 500.001
\end{aligned}$$

$$\frac{749}{10} + \frac{500.001}{6} \doteq 158$$

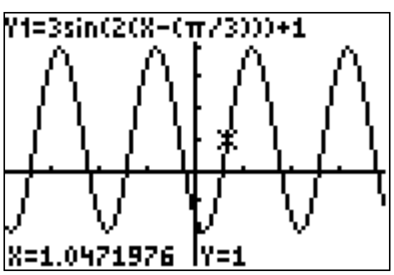
The total time will be a minimum when the contestant stays on the pavement.

a)

$$
\begin{aligned}
k &= \frac{2\pi}{\pi} \\
&= 2 \\
y &= 3\sin\left[2(x-d)\right]+1 \\
1 &= 3\sin\left[2\left(\frac{\pi}{3}-d\right)\right]+1 \\
1-1 &= 3\sin\left[2\left(\frac{\pi}{3}-d\right)\right] \\
0 &= \sin\left[2\left(\frac{\pi}{3}-d\right)\right] \\
\sin^{-1} 0 &= \left[2\left(\frac{\pi}{3}-d\right)\right] \\
0 &= \left[2(\frac{\pi}{3}-d)\right] \\
0 &= \left(\frac{\pi}{3}-d\right) \\
d &= \frac{\pi}{3}
\end{aligned}
$$

The phase shift is $\frac{\pi}{3}$ radians to the right.

b)



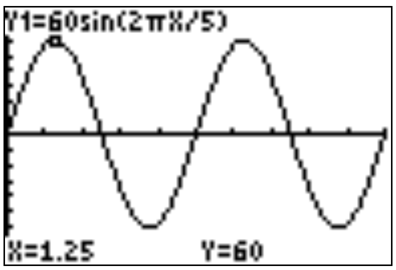
a) $a = \frac{120}{2}$

$= 60$

period is 5s; $k = \frac{2\pi}{5}$

$h(t) = 60 \sin \frac{2\pi t}{5}$

b)



c) Increasing the speed to a cycle in 3 s changes the value of k from $\frac{2\pi}{5}$ to $\frac{2\pi}{3}$, making the equation $h(t) = 60 \sin \frac{2\pi t}{3}$.

a)
$$\sec x = 5$$
$$\frac{1}{\cos x} = 5$$
$$\cos x = \frac{1}{5}$$
$$x = \cos^{-1}\left(\frac{1}{5}\right)$$
$$x \doteq 1.37$$
or
$$x \doteq 2\pi - 1.37$$
$$x \doteq 4.91$$
$$x \doteq 1.37 \text{ or } x \doteq 4.91$$

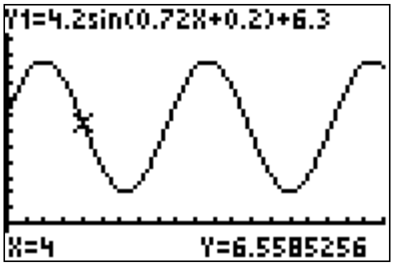
b) $(3 \sin x - 1)(4 \sin x + 1) = 0$
$3 \sin x = 1$ or $4 \sin x = -1$

Case 1:
$$3\sin x = 1$$
$$\sin x = \frac{1}{3}$$
$$x \doteq 0.34$$
or
$$x \doteq \pi - 0.34$$
$$x \doteq 2.80$$

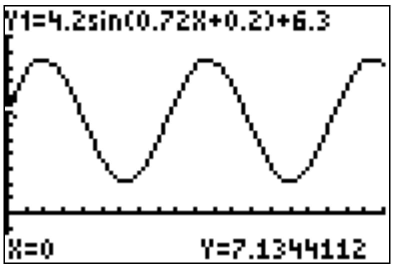
Case 2:
$$4\sin x = -1$$
$$\sin x = -\frac{1}{4}$$
$$x \doteq 2\pi - 0.25$$
$$x \doteq 6.03$$
or
$$x \doteq \pi + 0.25$$
$$x \doteq 3.39$$

$x \doteq 0.34$ or $x \doteq 2.80$ or $x \doteq 3.39$ or $x \doteq 6.03$

a)

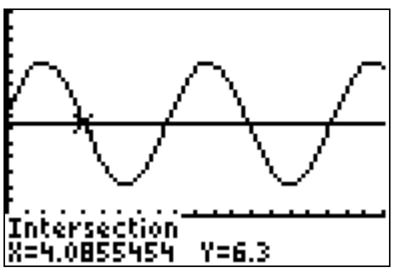


b)



The demand in 1990 was approximately 7134 cones.

c)

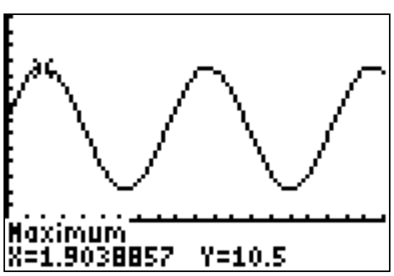


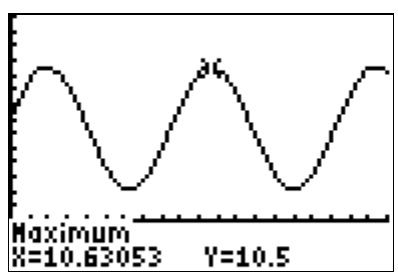
The average demand for ice-cream cones is 6300 cones.

d) period = $\dfrac{2\pi}{0.72}$, which is approximately 8.7

The period of the model is approximately 8.7 years.

e) Answers may vary. A sample solution is shown.





The model forecasts a peak demand in approximately Oct 1991 and June 2000.

f) Answers may vary. A sample solution is shown.

The temperature follows the same trend.

Chapters 4 and 5 Review **Question 20 Page 305**

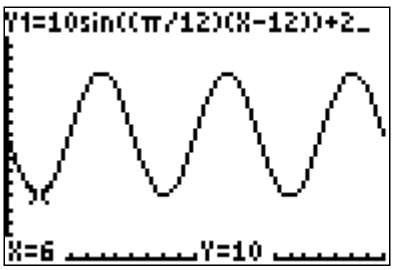
a) $a = \dfrac{30-10}{2}$ or 10; $c = 30 - 10$ or 20

period is 24 hours; $k = \dfrac{2\pi}{24}$ or $\dfrac{\pi}{12}$

The maximum of $y = \sin x$ occurs at $\dfrac{1}{4}$ of a period, $\dfrac{1}{4}$ of 24 is 6. The maximum of this function occurs at 18, $d = 12$ (since $d = 18 - 6$), so the phase shift is 12 units to the right.

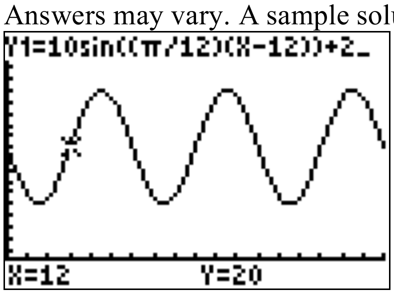
$C(t) = 10\sin\left[\dfrac{\pi}{12}(t-12)\right] + 20$

b)



Chapters 4 and 5 Review **Question 21 Page 305**

a) Answers may vary. A sample solution is shown.



The first point on the graph where the instantaneous rate of change is a maximum is at $t = 12$.

b)

$$\text{Average rate of change} = \frac{10\sin\left[\dfrac{\pi}{12}(12-12)\right] + 20 - \left[10\sin\left[\dfrac{\pi}{12}(11.999-12)\right] + 20\right]}{12 - 11.999}$$
$$\doteq 2.6$$

The instantaneous rate of change at this point is approximately 2.6 ppm/h.