Chapter 3 Rational Functions

Chapter 3 Prerequisite Skills

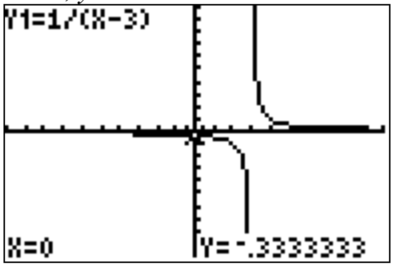
Chapter 3 Prerequisite Skills Question 1 Page 146

Answers may vary. A sample solution is shown.

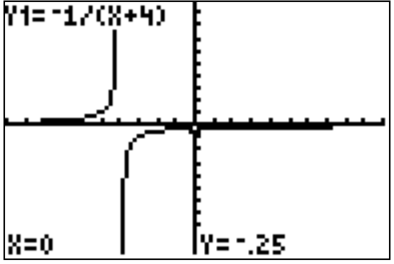
A line or curve that the graph approaches more and more closely. For $f(x) = \frac{1}{x}$, the vertical asymptote is $x = 0$.

Chapter 3 Prerequisite Skills Question 2 Page 146

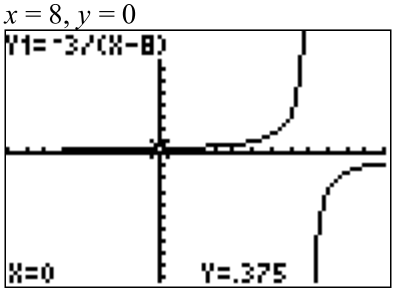
a) $x = 3, y = 0$



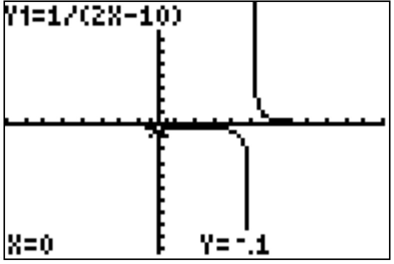
b) $x = –4, y = 0$



c) $x = 8, y = 0$

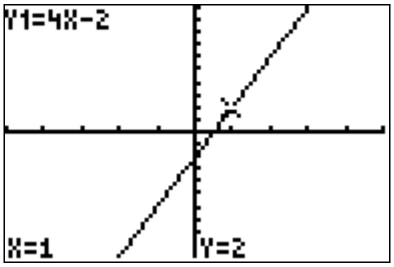


d) $x = 5$, $y = 0$



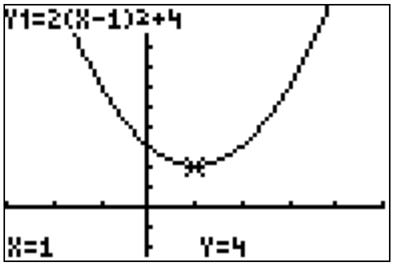
Chapter 3 Prerequisite Skills **Question 3 Page 146**

a)



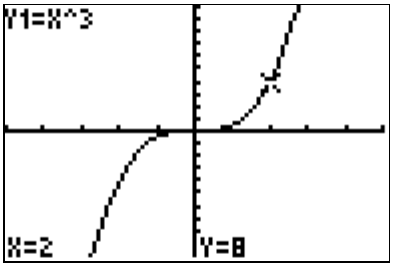
No restrictions on the domain or range.
domain: $\{x \in \mathbb{R}\}$, range: $\{y \in \mathbb{R}\}$

b)



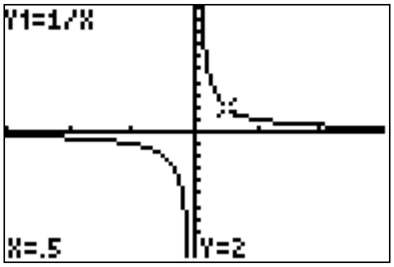
No restrictions on the domain.
domain: $\{x \in \mathbb{R}\}$
From the graph:
range: $\{y \in \mathbb{R}, y \geq 4\}$

c)



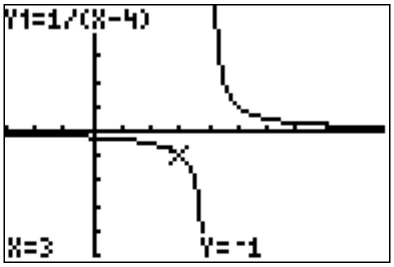
No restrictions on the domain and range.
domain: $\{x \in \mathbb{R}\}$, range: $\{y \in \mathbb{R}\}$

d)



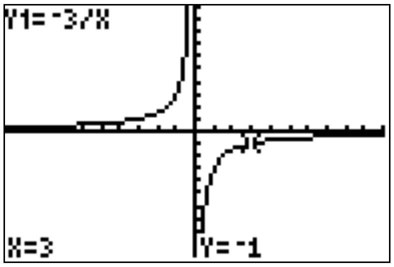
Division by zero is not defined.
$x \neq 0$
domain: $\{x \in \mathbb{R}, x \neq 0\}$
From the graph:
range: $\{y \in \mathbb{R}, y \neq 0\}$

e)



Division by zero is not defined.
$x - 4 \neq 0$
$x \neq 4$
domain: $\{x \in \mathbb{R}, x \neq 4\}$
From the graph:
range: $\{y \in \mathbb{R}, y \neq 0\}$

f)



Division by zero is not defined.
$x \neq 0$
domain: $\{x \in \mathbb{R}, x \neq 0\}$
From the graph:
range: $\{y \in \mathbb{R}, y \neq 0\}$

Chapter 3 Prerequisite Skills **Question 4 Page 146**

a) $\frac{8+5}{2-3} = \frac{13}{-1} = -13$

b) $\frac{3-0}{5+4} = \frac{3}{9} = \frac{1}{3}$

c) $\frac{4-2}{-7-2} = \frac{2}{-9} = -\frac{2}{9}$

d) $\frac{8-9}{1-0} = -1$

e) $\frac{-6-7}{2-1} = -13$

f) $\frac{-9-3}{-7-3} = \frac{-12}{-10} = \frac{6}{5}$

Chapter 3 Prerequisite Skills **Question 5 Page 146**

a) $\frac{7-10}{-1-7} = \frac{-3}{-8}$
$\doteq 0.38$

b) $\frac{11-6}{7-0} = \frac{5}{7}$
$\doteq 0.71$

c) $\frac{4-2}{7+4} = \frac{2}{11}$
$\doteq 0.18$

d) $\frac{4+1}{11+2} = \frac{5}{13}$
$\doteq 0.38$

e) $\frac{-0.9+5.2}{1.5+6.6} = \frac{4.3}{8.1}$
$\doteq 0.53$

f) $\frac{-1.7+3.2}{10.1-5.8} = \frac{1.5}{4.3}$
$\doteq 0.35$

Chapter 3 Prerequisite Skills **Question 6 Page 146**

a) $(x + 4)(x + 3)$ **b)** $(5x - 2)(x - 3)$

c) $(3x + 8)(2x - 1)$ **d)** $(x + 1)(x + 3)(x - 2)$

e) $P\left(-\frac{1}{2}\right) = 0$

$(2x+1)$ is a factor

$$\begin{array}{r}
6x^2 - x - 2 \\
2x+1\overline{\smash{)}12x^3 + 4x^2 - 5x - 2} \\
\underline{12x^3 + 6x^2} \qquad\qquad \\
-2x^2 - 5x \qquad \\
\underline{-2x^2 - x} \qquad \\
-4x - 2 \\
\underline{-4x - 2} \\
0
\end{array}$$

$$\begin{aligned}(2x+1)(6x^2 - x - 2) &= (2x+1)(2x+1)(3x-2) \\ &= (2x+1)^2(3x-2)\end{aligned}$$

f) $(3x-4)(9x^2+12x+16)$

Chapter 3 Prerequisite Skills **Question 7 Page 146**

a) $(x-8)(x+4)=0$
$x = 8$ or $x = -4$

b) $(x+5)(x+1)=0$
$x = -5$ or $x = -1$

c) $(x-3)(2x-3)=0$
$x = 3$ or $x = \frac{3}{2}$

d) $(x+5)(6x+1)=0$
$x = -5$ or $x = -\frac{1}{6}$

e) $(x+7)(2x-1)=0$
$x = -7$ or $x = \frac{1}{2}$

f) $(x-6)(3x+5)=0$
$x = 6$ or $x = -\frac{5}{3}$

a) $x^2 - 4x + 2 = 0$

$$x = \frac{4 \pm \sqrt{(-4)^2 - 4(1)(2)}}{2(1)}$$

$$x = \frac{4 \pm \sqrt{8}}{2}$$

$$x = 2 \pm \sqrt{2}$$

b) $2x^2 + 8x + 1 = 0$

$$x = \frac{-8 \pm \sqrt{8^2 - 4(2)(1)}}{2(2)}$$

$$x = \frac{-8 \pm \sqrt{56}}{4}$$

$$x = \frac{-4 \pm \sqrt{14}}{2}$$

c) $-3x^2 + 5x + 4 = 0$

$$x = \frac{-5 \pm \sqrt{5^2 - 4(-3)(4)}}{2(-3)}$$

$$x = \frac{-5 \pm \sqrt{73}}{-6}$$

$$x = \frac{5 \pm \sqrt{73}}{6}$$

d) no real roots; no *x*-intercepts

e) $3x^2 + 8x + 2 = 0$

$$x = \frac{-8 \pm \sqrt{8^2 - 4(3)(2)}}{2(3)}$$

$$x = \frac{-8 \pm \sqrt{40}}{6}$$

$$x = \frac{-4 \pm \sqrt{10}}{3}$$

f) $-x^2 + 2x + 7 = 0$

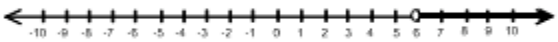
$$x = \frac{-2 \pm \sqrt{2^2 - 4(-1)(7)}}{2(-1)}$$

$$x = \frac{-2 \pm \sqrt{32}}{-2}$$

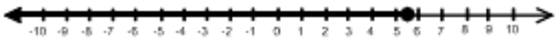
$$x = 1 \pm 2\sqrt{2}$$

Chapter 3 Prerequisite Skills **Question 9 Page 147**

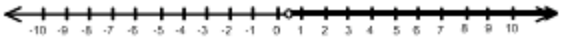
a) $2x > 12$
$x > 6$



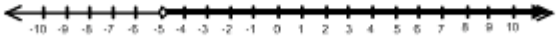
b) $9 + 2 \geq 6x - 4x$
$11 \geq 2x$
$x \leq \frac{11}{2}$



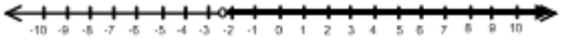
c) $4x - 8x < -2$
$-4x < -2$
$x > \frac{1}{2}$



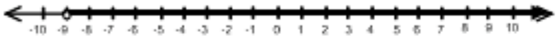
d) $2x - x > -4 - 1$
$x > -5$



e) $3x - x > -1 - 4$
$2x > -5$
$x > -\frac{5}{2}$



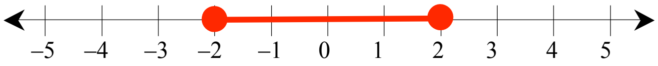
f) $x - 2x < 2 + 7$
$-x < 9$
$x > -9$



Chapter 3 Prerequisite Skills **Question 10 Page 147**

a) $(x - 2)(x + 2) \leq 0$

Case 1
$x \leq 2$ $x \geq -2$
$-2 \leq x \leq 2$ is a solution.

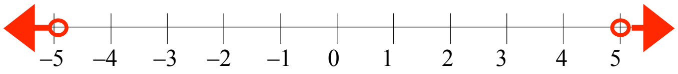


Case 2
$x \geq 2$ $x \leq -2$
No solution.

The solution is $-2 \leq x \leq 2$.

b) $(x - 6)(x + 3) > 0$

Case 1
$x > 6$ $x > -3$
$x > 6$ is a solution.



Case 2
$x < 6$ $x < -3$
$x < -3$ is a solution.

The solution is $x < -3$ or $x > 6$.

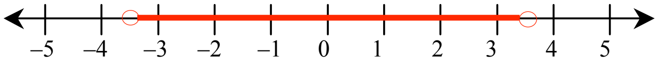
c) $2x^2 - 26 = 0$

$2(x^2 - 13) = 0$

$x^2 = 13$

$x = \sqrt{13}$ or $x = -\sqrt{13}$

$2\left(x - \sqrt{13}\right)\left(x + \sqrt{13}\right) < 0$



Case 1

$x < \sqrt{13}$ $\quad x > -\sqrt{13}$

$-\sqrt{13} < x < \sqrt{13}$ is a solution.

Case 2

$x > \sqrt{13}$ $\quad x < -\sqrt{13}$

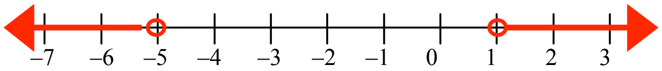
No solution.

The solution is $-\sqrt{13} < x < \sqrt{13}$.

d) $3x^2 - 2x^2 + 5x - 2x - 12 + 2 > 0$

$x^2 + 3x - 10 > 0$

$(x + 5)(x - 2) > 0$



Case 1

$x > –5$ $\quad x > 2$

$x > 2$ is a solution.

Case 2

$x < –5$ $\quad x < 2$

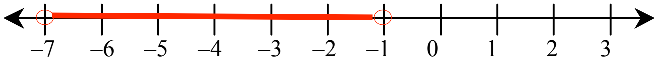
$x < –5$ is a solution.

The solution is $x < –5$ or $x > 2$.

e) $2x^2 - x^2 - x + 9x + 4 + 3 < 0$

$x^2 + 8x + 7 < 0$

$(x + 7)(x + 1) < 0$



Case 1

$x < -7 \qquad x > -1$

No solution.

Case 2

$x > -7 \qquad x < -1$

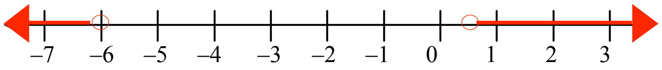
$-7 < x < -1$ is a solution.

The solution is $-7 < x < -1$.

f) $x^2 + x^2 + 2x + 9x + 2 - 8 > 0$

$2x^2 + 11x - 6 > 0$

$(x + 6)(2x - 1) > 0$



Case 1

$x > -6 \qquad x > \frac{1}{2}$

$x > \frac{1}{2}$ is a solution.

Case 2

$x < -6 \qquad x < \frac{1}{2}$

$x < -6$ is a solution.

The solution is $x < -6$ or $x > \frac{1}{2}$.

Chapter 3 Section 1 **Question 1 Page 153**

a)

As $x \to$	**$f(x) \to$**
2^+	$+\infty$
2^-	$-\infty$
$+\infty$	0
$-\infty$	0

b)

As $x \to$	**$f(x) \to$**
-5^+	$+\infty$
-5^-	$-\infty$
$+\infty$	0
$-\infty$	0

c)

As $x \to$	**$f(x) \to$**
8^+	$+\infty$
8^-	$-\infty$
$+\infty$	0
$-\infty$	0

Chapter 3 Section 1 **Question 2 Page 153**

a) i) $x = 2, y = 0$

ii) $x = -3, y = 0$

b) i) $y = \frac{1}{2x}$ shifted 2 to the right

$$y = \frac{1}{2(x-2)}$$

ii) $y = \frac{1}{2x}$ shifted 3 to the left

$$y = \frac{1}{2(x+3)}$$

Chapter 3 Section 1 **Question 3 Page 154**

a) i) $x = 5$ **ii)** $y = 0$ **iii)** let $x = 0$

$$\begin{aligned} f(0) &= \frac{1}{0-5} \\ &= -\frac{1}{5} \end{aligned}$$

b) i) $x = -6$ **ii)** $y = 0$ **iii)** let $x = 0$

$$\begin{aligned} g(0) &= \frac{2}{0+6} \\ &= \frac{1}{3} \end{aligned}$$

c) i) $x = 1$ **ii)** $y = 0$ **iii)** let $x = 0$

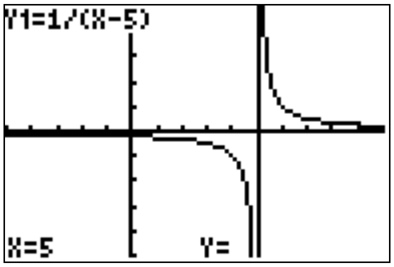
$$\begin{aligned} h(0) &= \frac{5}{1-0} \\ &= 5 \end{aligned}$$

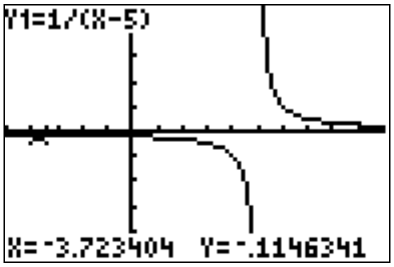
d) i) $x = -7$ **ii)** $y = 0$ **iii)** let $x = 0$

$$\begin{aligned} k(0) &= -\frac{1}{0+7} \\ &= -\frac{1}{7} \end{aligned}$$

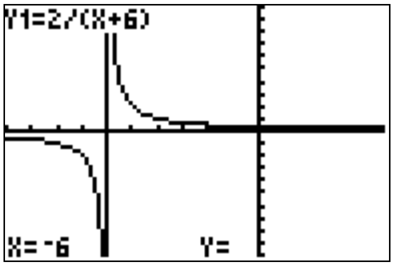
Chapter 3 Section 1 **Question 4 Page 154**

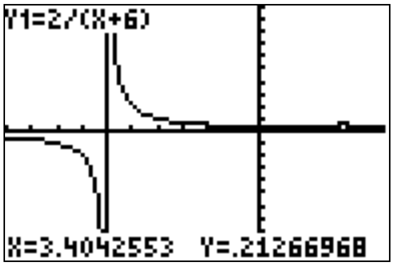
a)



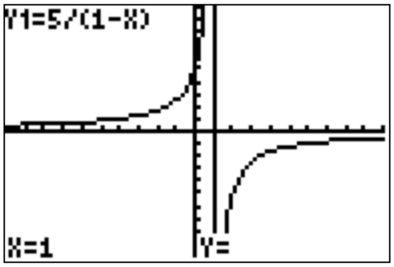


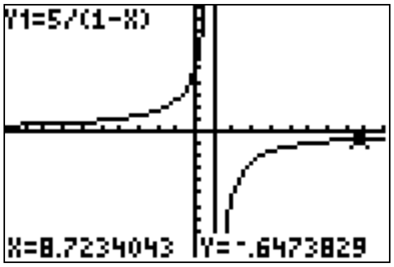
b)



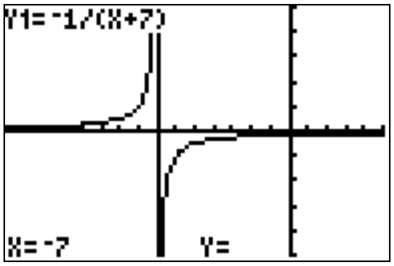


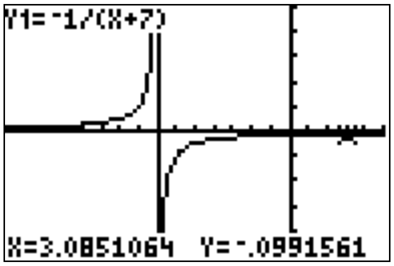
c)





d)





Chapter 3 Section 1 Question 5 Page 154

a) Since the vertical asymptote is $x = 3$:

$$y = \frac{1}{x-3}$$

b) Since the vertical asymptote is $x = –3$:

$$y = \frac{1}{x+3}$$

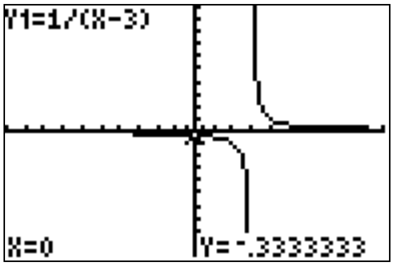
c) Since the vertical asymptote is $x = \frac{1}{2}$:

$$y = \frac{1}{2x-1}$$

d) Since the vertical asymptote is $x = –4$ and it is reflected in the y-axis:

$$y = -\frac{1}{x+4}$$

a)



Select a few points to the left of the asymptote and analyse the slope.

At $x = -1, f(x) = -0.25$
At $x = 0, f(x) \doteq -0.33$
$$\text{Slope} = \frac{-0.33 + 0.25}{0 + 1}$$
$$= -0.08$$

At $x = 1, f(x) = -0.5$
At $x = 2, f(x) = -1$
$$\text{Slope} = \frac{-1 + 0.5}{2 - 1}$$
$$= -0.5$$

Since $-0.5 < -0.08$, the slope is negative and decreasing for the interval $x < 3$.

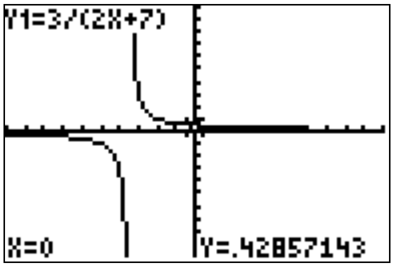
Select a few points to the right of the asymptote and analyse the slope.

At $x = 3.5, f(x) = 2$
At $x = 4, f(x) = 1$
$$\text{Slope} = \frac{1 - 2}{4 - 3.5}$$
$$= -2$$

At $x = 5, f(x) = 0.5$
At $x = 6, f(x) \doteq 0.33$
$$\text{Slope} = \frac{0.33 - 0.5}{6 - 5}$$
$$= -0.17$$

Since $-0.17 > -2$, the slope is negative and increasing for the interval $x > 3$.

b)



Select a few points to the left of the asymptote and analyse the slope.

At $x = –6, f(x) = –0.6$
At $x = –5, f(x) = –1$

$$\text{Slope} = \frac{-1+0.6}{-5+6}$$
$$= –0.4$$

At $x = –4, f(x) = –3$
At $x = –3.8, f(x) = –5$

$$\text{Slope} = \frac{-5+3}{-3+4}$$
$$= –2$$

Since $–2 < –0.4$, the slope is negative and decreasing within the interval $x < -\frac{7}{2}$.

Select a few points to the right of the asymptote and analyse the slope.

At $x = –3, f(x) = 3$
At $x = –2, f(x) = 1$

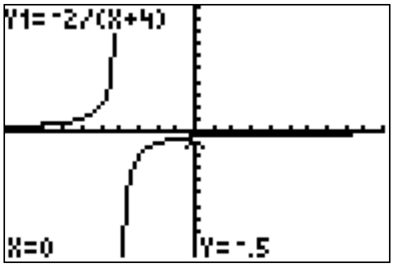
$$\text{Slope} = \frac{1-3}{-2+3}$$
$$= –2$$

At $x = –1, f(x) = 0.6$
At $x = 0, f(x) \doteq 0.43$

$$\text{Slope} = \frac{0.43-0.6}{0+1}$$
$$= –0.17$$

Since $–0.17 > –2$, the slope is negative and increasing for $x > -\frac{7}{2}$.

c)



Select a few points to the left of the asymptote and analyse the slope.

At $x = -5, f(x) = 2$
At $x = -4.5, f(x) = 4$
Slope $= \dfrac{4-2}{-4.5+5}$
$= 4$

Since 4 > 0.33, the slope is positive and increasing for $x < -4$.

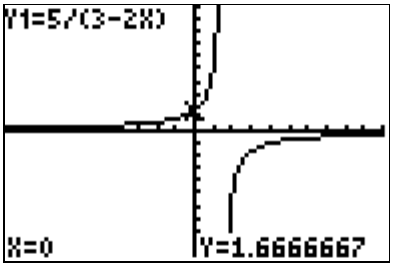
Select a few points to the right of the asymptote and analyse the slope.

At $x = -3, f(x) = -2$
At $x = -2, f(x) = -1$
Slope $= \dfrac{-1+2}{-2+3}$
$= 1$

At $x = 0, f(x) = -0.5$
At $x = 1, f(x) = -0.4$
Slope $= \dfrac{-0.4+0.5}{1-0}$
$= 0.1$

Since 0.1 < 1, the slope is positive and decreasing for $x > -4$.

d)



Select a few points to the left of the asymptote and analyse the slope.

At $x = -2, f(x) \doteq 0.71$
At $x = -1, f(x) = 1$

$$\begin{aligned}\text{Slope} &\doteq \frac{1-0.71}{-1+2}\\ &\doteq 0.29\end{aligned}$$

At $x = 0, f(x) \doteq 1.67$
At $x = 1, f(x) = 5$

$$\begin{aligned}\text{Slope} &\doteq \frac{5-1.67}{1-0}\\ &\doteq 3.33\end{aligned}$$

Since 3.33 > 0.29, the slope is positive and increasing for $x < \frac{3}{2}$.

Select a few points to the right of the asymptote and analyse the slope.

At $x = 2, f(x) = -5$
At $x = 3, f(x) \doteq -1.67$

$$\begin{aligned}\text{Slope} &\doteq \frac{-1.67+5}{3-2}\\ &\doteq 3.33\end{aligned}$$

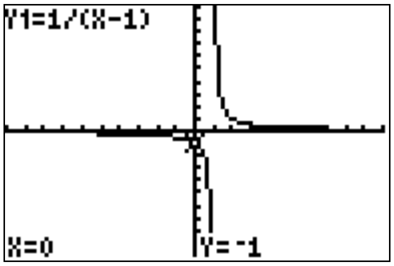
At $x = 4, f(x) = -1$
At $x = 5, f(x) \doteq -0.71$

$$\begin{aligned}\text{Slope} &\doteq \frac{-0.71+1}{5-4}\\ &\doteq 0.83\end{aligned}$$

Since 0.83 < 3.33, the slope is positive and decreasing for $x > \frac{3}{2}$.

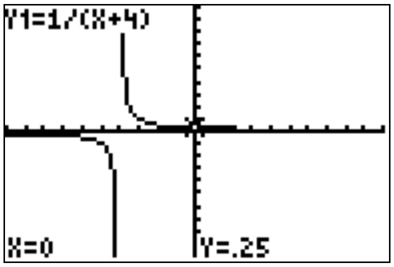
Chapter 3 Section 1 **Question 7 Page 154**

a)



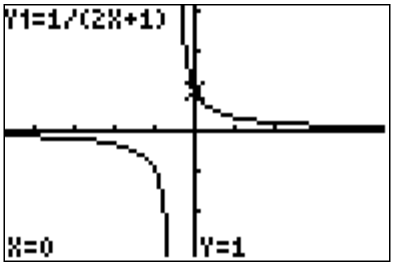
$\{x \in \mathbb{R}, x \neq 1\}$, $\{y \in \mathbb{R}, y \neq 0\}$, $x = 1$, $y = 0$

b)



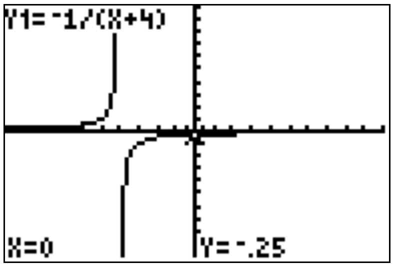
$\{x \in \mathbb{R}, x \neq -4\}$, $\{y \in \mathbb{R}, y \neq 0\}$, $x = -4$, $y = 0$

c)



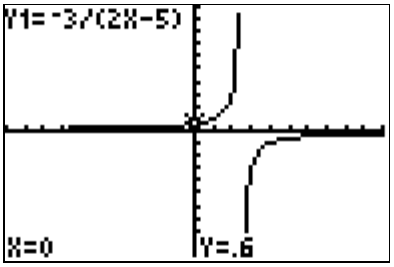
$\{x \in \mathbb{R}, x \neq -\frac{1}{2}\}$, $\{y \in \mathbb{R}, y \neq 0\}$, $x = -\frac{1}{2}$, $y = 0$

d)



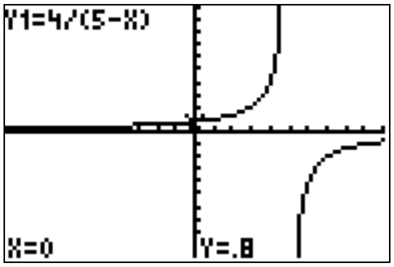
$\{x \in \mathbb{R}, x \neq -4\}$, $\{y \in \mathbb{R}, y \neq 0\}$, $x = -4$, $y = 0$

e)



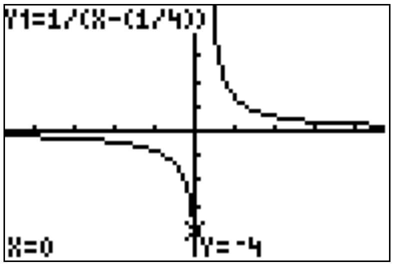
$\{x \in \mathbb{R}, x \neq \frac{5}{2}\}$, $\{y \in \mathbb{R}, y \neq 0\}$, $x = \frac{5}{2}$, $y = 0$

f)



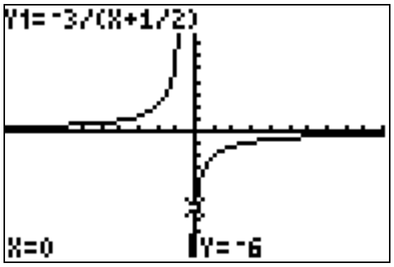
$\{x \in \mathbb{R}, x \neq 5\}$, $\{y \in \mathbb{R}, y \neq 0\}$, $x = 5$, $y = 0$

g)



$\{x \in \mathbb{R}, x \neq \frac{1}{4}\}$, $\{y \in \mathbb{R}, y \neq 0\}$, $x = \frac{1}{4}$, $y = 0$

h)



$\{x \in \mathbb{R}, x \neq -\frac{1}{2}\}$, $\{y \in \mathbb{R}, y \neq 0\}$, $x = -\frac{1}{2}$, $y = 0$

Chapter 3 Section 1 **Question 8 Page 154**

For the *y*-intercept let $x = 0$.

$$f(0) = \frac{1}{0 - c} = -1$$

$$-\frac{1}{c} = -1$$

$$c = 1$$

$$f(x) = \frac{1}{kx - 1}$$

For the asymptote let $x = 1$.

$$kx - 1 = 0$$

$$kx = 1$$

$$k(1) = 1$$

$$k = 1$$

$$f(x) = \frac{1}{x - 1}$$

Chapter 3 Section 1 **Question 9 Page 154**

Let $x = 0$.

$$f(0) = \frac{1}{0 - c}$$

$$= -0.25$$

$$-\frac{1}{c} = -0.25$$

$$c = 4$$

$$f(x) = \frac{1}{kx - 4}$$

Asymptote is at $x = -1$:

$$kx - 4 = 0$$

$$kx = 4$$

$$k(-1) = 4$$

$$k = -4$$

$$f(x) = \frac{1}{-4x - 4}$$

$$f(x) = -\frac{1}{4x + 4}$$

$$y = -\frac{1}{4x + 4}$$

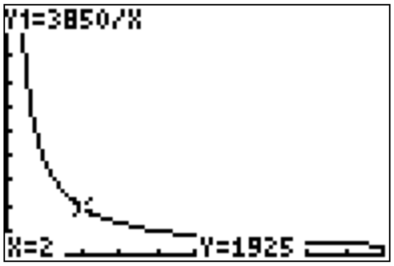
Chapter 3 Section 1 Question 10 Page 155

a) $d = 350 \times 11$

$= 3850$

$t = \frac{3850}{v}$

b)



c) $t = \frac{3850}{500}$

$t = 7.7$

It would take 7.7 h or 7 h and 42 min.

d) As the speed increases the rate of change of time decreases.

Chapter 3 Section 1 Question 11 Page 155

a) Answers may vary.

b) Answers may vary. A sample solution is shown.

The equation of the asymptote is $x = -\frac{2}{b}$.

When $b = 1$, the asymptote is $x = –2$.

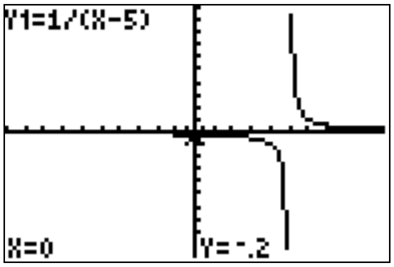
When $b > 1$, $-2 < -\frac{2}{b} < 0$, the vertical asymptote is between –2 and 0.

When $0 < b < 1$, $-\frac{2}{b} < -2$, the vertical asymptote is less than –2.

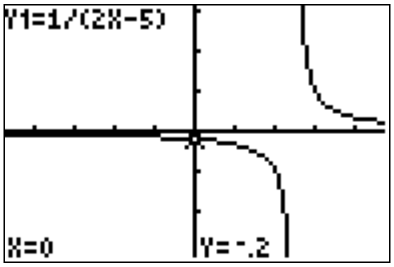
When $b < 0$, the vertical asymptote is bigger than zero.

Chapter 3 Section 1 Question 12 Page 155

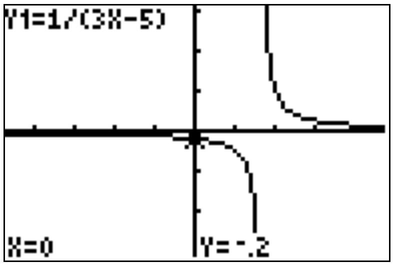
a)



b)



c)

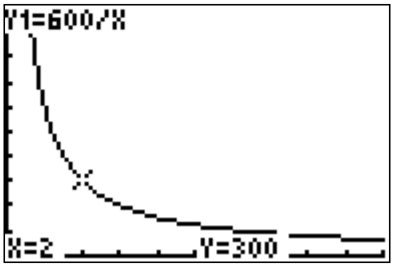


Chapter 3 Section 1 **Question 13 Page 155**

a) $f = 200 \times 3$

$$F = \frac{600}{d}$$

b)



c) $F = \frac{600}{2}$

$F = 300$ N

300 N of force is needed to lift the object 2 m from the fulcrum.

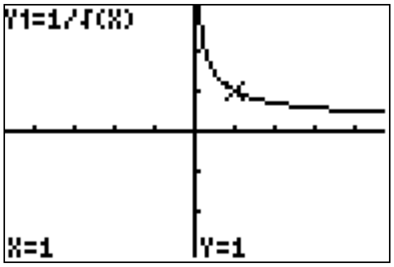
d) $F = \frac{600}{2d}$

$$F = \frac{300}{d}$$

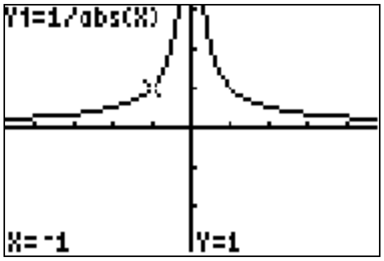
The force is halved.

Chapter 3 Section 1 **Question 14 Page 155**

a) Since division by zero and negative square roots are not defined, $x > 0$ and $y > 0$.
domain: $\{x \in \mathbb{R}, x > 0\}$
range: $\{y \in \mathbb{R}, y > 0\}$
vertical asymptote: $x = 0$
horizontal asymptote: $y = 0$



b) Since division by 0 is not defined, $x \neq 0$ and $y \neq 0$.
Since x is an absolute value, $g(x)$ is positive.
domain: $\{x \in \mathbb{R}, x \neq 0\}$
range: $\{y \in \mathbb{R}, y > 0\}$
vertical asymptote: $x = 0$
horizontal asymptote: $y = 0$



c) Division by zero is not defined.

$$x - 2 \neq 0$$

$$x \neq 2$$

domain: $\{x \in \mathbb{R}, x \neq 2\}$

To find the range, first find the inverse function.

$$x = \frac{3}{y-2} + 4$$

$$x - 4 = \frac{3}{y-2}$$

$$y - 2 = \frac{3}{x-4}$$

$$y = \frac{3}{x-4} + 2$$

Division by zero is not defined.

$$x - 4 \neq 0$$

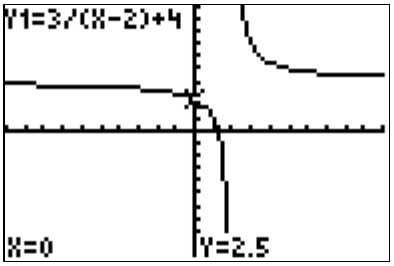
$$x \neq 4$$

The domain of the inverse function is the range of $f(x)$.

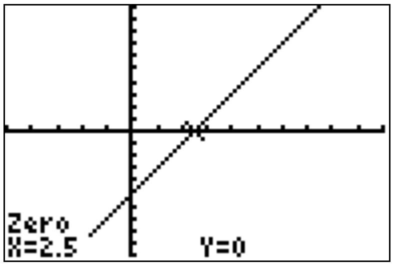
range: $\{y \in \mathbb{R}, y \neq 4\}$

vertical asymptote: $x = 2$

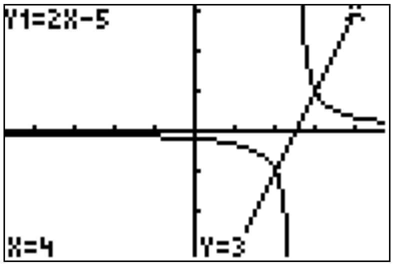
horizontal asymptote: $y = 4$

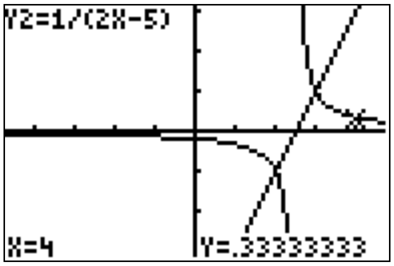


Chapter 3 Section 1 **Question 15 Page 155**



Left side of the x-intercept:	Coordinates for $f(x)=\dfrac{1}{2x-5}$
At $x = 2$, $y = -1$, reciprocal $= -1$	$(2, -1)$
At $x = 1$, $y = -3$, reciprocal $= -\dfrac{1}{3}$	$\left(1, -\dfrac{1}{3}\right)$
At $x = 0$, $y = -5$, reciprocal$= -\dfrac{1}{5}$	$\left(0, -\dfrac{1}{5}\right)$
At $x = -1$, $y = -7$, reciprocal $= -\dfrac{1}{7}$	$\left(-1, -\dfrac{1}{7}\right)$
Right side of the x-intercept:	
At $x = 3$, $y = 1$, reciprocal $= 1$	$(3, 1)$
At $x = 4$, $y = 3$, reciprocal $= \dfrac{1}{3}$	$\left(4, \dfrac{1}{3}\right)$
At $x = 5$, $y = 5$, reciprocal $= \dfrac{1}{5}$	$\left(5, \dfrac{1}{5}\right)$
At $x = 6$, $y = 7$, reciprocal $= \dfrac{1}{7}$	$\left(6, \dfrac{1}{7}\right)$





Answers may vary. A sample solution is shown.
The reciprocal of the y-coordinates on either side of the x-intercept ($y = 2x - 5$) are the y-coordinates of $f(x)=\dfrac{1}{2x-5}$.

Chapter 3 Section 1 **Question 16 Page 155**

$$\frac{1}{x}=\frac{1}{z}-\frac{1}{y}$$

$$\frac{1}{x}=\frac{y}{zy}-\frac{z}{zy}$$

$$\frac{1}{x}=\frac{y-z}{zy}$$

$$x=\frac{zy}{y-z}$$

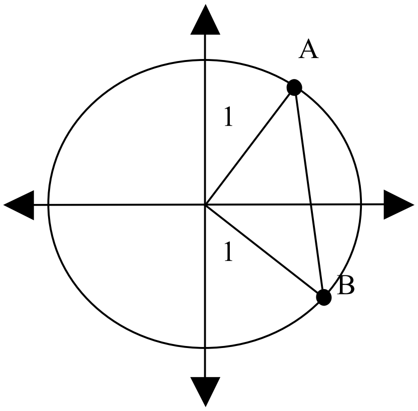
$$x=\frac{yz}{y-z},\ y\neq z,\ x\neq 0,\ z\neq 0$$

Chapter 3 Section 1 **Question 17 Page 155**

$$\frac{75b-5b}{5b}=\frac{70b}{5b}$$
$$=14$$

Chapter 3 Section 1 **Question 18 Page 155**

E $\frac{2}{3}$



If two points are within 1 unit of each other, the angle between them must be less than $\frac{\pi}{3}$ (they form an equilateral triangle).

That means that given any point A, if point B is in the nearest third of the circle to A, the distance will be less than 1 unit. So there is $\frac{2}{3}$ of the circle where the distance is greater than 1 unit.

Chapter 3 Section 2 Reciprocal of a Quadratic Function

Chapter 3 Section 2 Question 1 Page 164

a)

As $x \to$	**$f(x) \to$**
3^-	$-\infty$
3^+	$+\infty$
1^-	$+\infty$
1^+	$-\infty$
$-\infty$	0
$+\infty$	0

b)

As $x \to$	**$f(x) \to$**
-4^-	$+\infty$
-4^+	$-\infty$
5^-	$-\infty$
5^+	$+\infty$
$-\infty$	0
$+\infty$	0

c)

As $x \to$	**$f(x) \to$**
-6^-	$-\infty$
-6^+	$-\infty$
$-\infty$	0
$+\infty$	0

Chapter 3 Section 2 Question 2 Page 165

a) asymptote: $x = 4$ domain: $\{x \in \mathbb{R}, x \neq 4\}$

b) asymptotes: $x = 2, x = -7$ domain: $\{x \in \mathbb{R}, x \neq 2, x \neq -7\}$

c) No asymptotes or restrictions on the domain.
domain: $\{x \in \mathbb{R}\}$

d) $m(x) = \dfrac{3}{(x-5)(x+5)}$

asymptotes: $x = -5, x = 5$ domain: $\{x \in \mathbb{R}, x \neq 5, x \neq -5\}$

e) $h(x)=\frac{1}{(x-3)(x-1)}$

asymptotes: $x = 3$, $x = 1$ domain: $\{x \in \mathbb{R}, x \neq 1, x \neq 3\}$

f) $k(x)=-\frac{2}{(x+4)(x+3)}$

asymptotes: $x = -4$, $x = -3$ domain: $\{x \in \mathbb{R}, x \neq -4, x \neq -3\}$

g) $n(x)=-\frac{2}{(x+2)(3x-4)}$

asymptotes: $x = -2$, $x = \frac{4}{3}$ domain: $\{x \in \mathbb{R}, x \neq -2, x \neq \frac{4}{3}\}$

h) No asymptotes or restrictions on the domain.
domain: $\{x \in \mathbb{R}\}$

Chapter 3 Section 2 **Question 3 Page 165**

a)

Interval	Sign of $f(x)$	Sign of Slope	Change in Slope
$x < 1$	+	+	+
$x > 1$	+	–	–

b)

Interval	Sign of $f(x)$	Sign of Slope	Change in Slope
$x < -2$	+	+	+
$-2 < x < 1$	–	+	–
$x = 1$	–	0	–
$1 < x < 4$	–	–	–
$x > 4$	+	–	+

c)

Interval	Sign of $f(x)$	Sign of Slope	Change in Slope
$x < -3$	–	–	–
$-3 < x < 0$	+	–	+
$x = 0$	+	0	+
$0 < x < 3$	+	+	+
$x > 3$	–	+	–

d)

Interval	Sign of $f(x)$	Sign of Slope	Change in Slope
$x < -4$	–	–	–
$x > -4$	–	+	–

a) asymptote: $x = 1$, y-intercept: 1

$$y = \frac{1}{k(x-1)^2}$$
$$1 = \frac{1}{k(-1)^2}$$
$$k = 1$$
$$y = \frac{1}{(x-1)^2}$$

b) asymptotes: $x = –2$, $x = 4$, point $\left(1, -\frac{1}{9}\right)$

$$y = \frac{1}{k(x+2)(x-4)}$$
$$-\frac{1}{9} = \frac{1}{k(1+2)(1-4)}$$
$$-\frac{1}{9} = \frac{1}{-9k}$$
$$k = 1$$
$$y = \frac{1}{(x+2)(x-4)}$$

c) asymptotes: $x = –3$, $x = 3$, y -intercept: $\frac{1}{9}$, reflected in x-axis

$$y = -\frac{1}{k(x-3)(x+3)}$$
$$\frac{1}{9} = -\frac{1}{k(0-3)(0+3)}$$
$$\frac{1}{9} = \frac{1}{9k}$$
$$k = 1$$
$$y = -\frac{1}{x^2-9}$$

d) asymptote: $x = –4$, point (–3, 1), reflected in the x-axis

$$y = -\frac{1}{k(x+4)^2}$$

$$-1 = -\frac{1}{k(-3+4)^2}$$

$$-1 = -\frac{1}{k}$$

$$k = 1$$

$$y = -\frac{1}{(x+4)^2}$$

Chapter 3 Section 2 **Question 5 Page 165**

a) i) $f(x) = \dfrac{1}{(x-3)(x+3)}$

domain: $\{x \in \mathbb{R}, x \neq –3, x \neq 3\}$

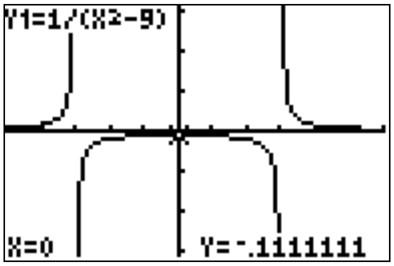
ii) vertical asymptotes: $x = 3$, $x = –3$,
As $x \to \pm\infty$, the denominator approaches $+\infty$, so $f(x)$ approaches 0.
horizontal asymptote: $y = 0$

iii) let $x = 0$

$$f(0) = \frac{1}{(0-3)(0+3)}$$
$$= -\frac{1}{9}$$

y-intercept: $-\dfrac{1}{9}$

iv)



v)

Interval	Sign of *f*(*x*)	Sign of Slope	Change in Slope
$x < -3$	+	+	+
$-3 < x < 0$	–	+	–
$x = 0$	–	0	–
$0 < x < 3$	–	–	–
$x > 3$	+	–	+

vi) $\left\{y \in \mathbb{R},\ y \neq 0\right\}$ $\{y \in \mathbb{R}, y \neq 0\}$

b) i) $t(x) = \dfrac{1}{(x-5)(x+3)}$

domain: $\{x \in \mathbb{R}, x \neq -3, x \neq 5\}$

ii) vertical asymptotes: $x = -3$, $x = 5$

As $x \to \pm\infty$, the denominator approaches $+\infty$, so $f(x)$ approaches 0.

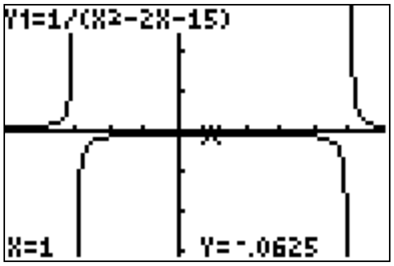
horizontal asymptote: $y = 0$

iii) let $x = 0$

$$t(0) = \frac{1}{(0-5)(0+3)}$$
$$= -\frac{1}{15}$$

y-intercept: $-\dfrac{1}{15}$

iv)



v)

Interval	Sign of *f*(x)	Sign of Slope	Change in Slope
$x < -3$	+	+	+
$-3 < x < 1$	–	+	–
$x = 1$	–	0	–
$1 < x < 5$	–	–	–
$x > 5$	+	–	+

vi) $\{y \in \mathbb{R}, y \neq 0\}$

c) i) The denominator cannot equal zero, there are restrictions at

$$x^2 + 5x - 21 = 0$$

$$x = \frac{-5 \pm \sqrt{5^2 - 4(1)(-21)}}{2(1)}$$

$$x = \frac{-5 + \sqrt{109}}{2} \text{ or } x = \frac{-5 - \sqrt{109}}{2}$$

domain: $\left\{x \in \mathbb{R},\ x \neq \frac{-5 \pm \sqrt{109}}{2}\right\}$

ii) vertical asymptotes at $x = \frac{-5 + \sqrt{109}}{2}$, $x = \frac{-5 - \sqrt{109}}{2}$

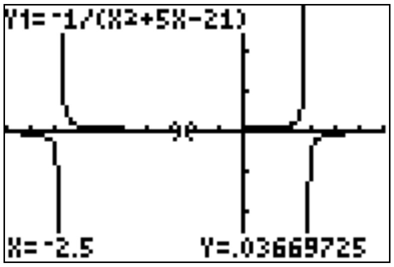
As $x \to \pm\infty$, the denominator approaches $+\infty$, so $f(x)$ approaches 0.
horizontal asymptote: $y = 0$

iii) let $x = 0$

$$p(0) = -\frac{1}{0^2 + 5(0) - 21}$$
$$= \frac{1}{21}$$

y-intercept: $\frac{1}{21}$

iv)



v)

Interval	Sign of $f(x)$	Sign of Slope	Change in Slope
$x < \dfrac{-5-\sqrt{109}}{2}$	$-$	$-$	$-$
$\dfrac{-5-\sqrt{109}}{2} < x < -2.5$	$+$	$-$	$+$
$x = -2.5$	$+$	0	$+$
$-2.5 < x < \dfrac{-5+\sqrt{109}}{2}$	$+$	$+$	$+$
$x > \dfrac{-5+\sqrt{109}}{2}$	$-$	$+$	$-$

vi) $\{y \in \mathbb{R}, y \neq 0\}$

d) i) $w(x) = \dfrac{1}{(x-2)(3x+1)}$

domain: $\left\{x \in \mathbb{R},\ x \neq 2,\ x \neq -\dfrac{1}{3}\right\}$

ii) vertical asymptotes: $x = 2$, $x = -\dfrac{1}{3}$

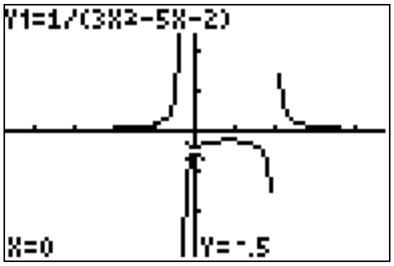
As $x \to \pm\infty$, the denominator approaches $+\infty$, so $f(x)$ approaches 0.
horizontal asymptote: $y = 0$

iii) let $x = 0$

$$w(0) = \frac{1}{(0-2)(3(0)+1)}$$
$$= -\frac{1}{2}$$

y-intercept: $-\dfrac{1}{2}$

iv)



v)

Interval	Sign of $f(x)$	Sign of Slope	Change in Slope
$x < -\frac{1}{3}$	+	+	+
$-\frac{1}{3} < x < \frac{5}{6}$	–	+	–
$x = \frac{5}{6}$	–	0	–
$\frac{5}{6} < x < 2$	–	–	–
$x > 2$	+	–	+

vi) $\{y \in \mathbb{R}, y \neq 0\}$

e) i) No restrictions on the domain.
domain: $\{x \in \mathbb{R}\}$

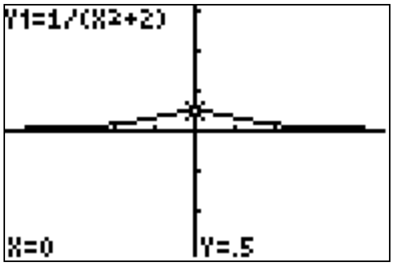
ii) No vertical asymptotes.
As $x \to \pm\infty$, the denominator approaches $+\infty$, so $f(x)$ approaches 0.
horizontal asymptote: $y = 0$

iii) let $x = 0$

$$q(0) = \frac{1}{0^2+2}$$
$$= \frac{1}{2}$$

y-intercept: $\frac{1}{2}$

iv)



v)

Interval	**Sign of $f(x)$**	**Sign of Slope**	**Change in Slope**
$x < 0$	+	+	+
$x = 0$	+	0	–
$x > 0$	+	–	+

vi) $\left\{y \in \mathbb{R}, 0 < y \le \frac{1}{2}\right\}$

Chapter 3 Section 2 **Question 6 Page 166**

a) y-intercept: $-\frac{1}{9}$

The point is approximately (0, –0.1111).

x	***y***	**Slope of Secant with (0, –0.111 11)**
0.1	–0.111 235	–0.001 24
0.01	–0.111 112	–0.0001
0.001	–0.111 111	0

Calculate the slope of the secant with the point (0.001, –0.1111).

$$\begin{aligned}\text{Slope} &= \frac{-0.111\ 111 + 0.111\ 111}{0.001 - 0} \\ &= 0\end{aligned}$$

The slope is approximately 0 at the y-intercept.

b) y-intercept: $-\frac{1}{15}$

The point is approximately (0, –0.066 667).

x	***y***	**Slope of Secant with (0, –0.066 667)**
0.1	–0.065 833	0.008 34
0.01	–0.066 578	0.0089
0.001	–0.066 658	0.009

The slope is approximately 0.009 at the y-intercept.

c) y-intercept: $\frac{1}{21}$

The point is approximately (0, 0.047619).

x	y	**Slope of Secant with (0, 0.047 619)**
0.1	0.048 804	0.011 85
0.01	0.047 733	0.0114
0.001	0.047 63	0.011

The slope is approximately 0.011 at the y-intercept.

d) y-intercept: $-\frac{1}{2}$

The point is (0, –0.5).

x	y	**Slope of Secant with (0, –0.5)**
0.1	–0.404 858	0.95142
0.01	–0.487 876	1.2124
0.001	–0.498 754	1.246
0.0001	–0.499 875	1.25

The slope is approximately 1.25 at the y-intercept.

e) y-intercept: $\frac{1}{2}$

The point is (0, 0.5).

x	y	**Slope of Secant with (0, 0.5)**
0.1	0.497 512	–0.024 88
0.01	0.499 975	–0.0025
0.001	0.5	0

The slope is approximately 0 at the y-intercept.

Chapter 3 Section 2 **Question 7 Page 166**

a) domain and range: $\{x \in \mathbb{R},\ x \neq 0\}$, $\{y \in \mathbb{R},\ y > 0\}$
asymptotes: $x = 0$, $y = 0$
no x- or y-intercepts
$x < 0$: the function is positive and increasing (positive slope)
$x > 0$: the function is positive and decreasing (negative slope)

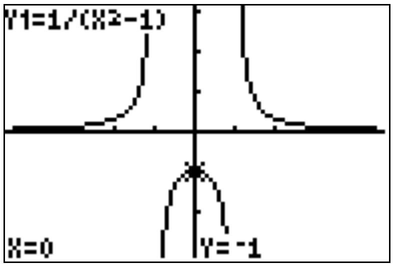
b) domain and range: $\{x \in \mathbb{R},\ x \neq 1\}$, $\{y \in \mathbb{R},\ y > 0\}$
asymptotes: $x = 1$, $y = 0$
y-intercept: 1
$x < 1$: the function is positive and increasing (positive slope)
$x > 1$: the function is positive and decreasing (negative slope)

c) domain and range: $\{x \in \mathbb{R},\ x \neq -2\}$, $\{y \in \mathbb{R},\ y > 0\}$
asymptotes: $x = -2$, $y = 0$
y-intercept: $\frac{1}{4}$
$x < -2$: the function is positive and increasing (positive slope)
$x > -2$: the function is positive and decreasing (negative slope)

Answers may vary. A sample solution is shown.
Key features of the reciprocal of a perfect square function:
There is a vertical asymptote equal to the x value of the vertex of the corresponding quadratic function.
The reciprocal function is positive and increasing (positive slope) when the corresponding quadratic function is positive and decreasing (negative slope).
The reciprocal function is positive and decreasing (negative slope) when the corresponding quadratic function is positive and increasing (positive slope).

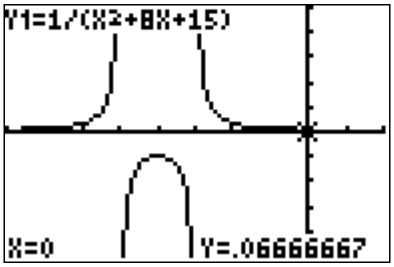
Chapter 3 Section 2 **Question 8 Page 166**

a)



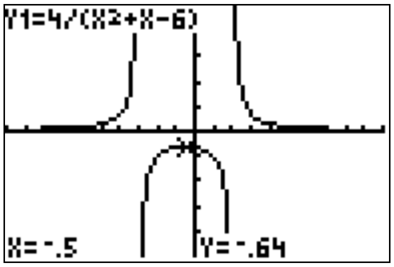
increasing: $x < -1$ and $-1 < x < 0$
decreasing: $0 < x < 1$ and $x > 1$

b)



increasing: $x < -5$ and $-5 < x < -4$
decreasing: $-4 < x < -3$ and $x > -3$

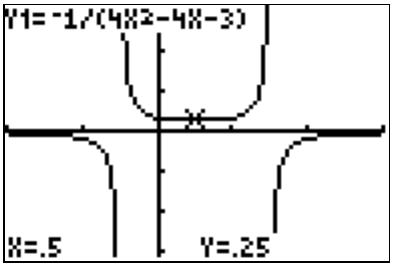
c)



increasing: $x < -3$ and $-3 < x < -\frac{1}{2}$

decreasing: $-\frac{1}{2} < x < 2$ and $x > 2$

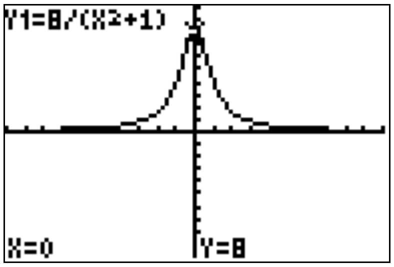
d)



increasing: $\frac{1}{2} < x < \frac{3}{2}$ and $x > \frac{3}{2}$

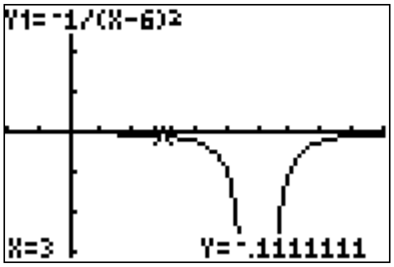
decreasing: $x < -\frac{1}{2}$ and $-\frac{1}{2} < x < \frac{1}{2}$

e)



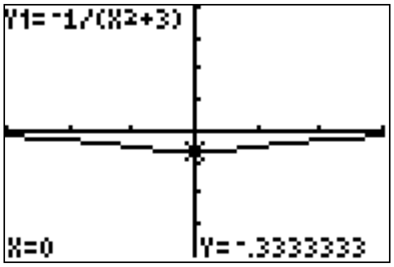
increasing: $x < 0$
decreasing: $x > 0$

f)



increasing: $x > 6$
decreasing: $x < 6$

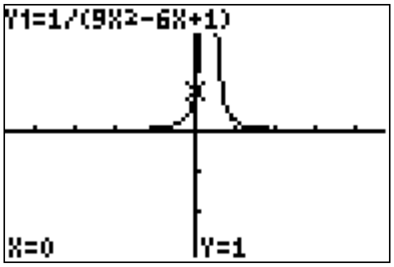
g)



increasing: $x > 0$

decreasing: $x < 0$

h)



increasing: $x < \frac{1}{3}$

decreasing: $x > \frac{1}{3}$

Chapter 3 Section 2 **Question 9 Page 166**

a) Answers may vary. A sample solution is shown.
Complete the square to get it in the form $y = a\left[k(x-d)\right]^2 + c$.
Note that (d, c) is the vertex.

$f(x) = (x^2 + 6x) + 11$ Square half the middle term to get the last term.

$= (x^2 + 6x + 9) + 11 - 9$ Subtract the last term from 11 so the function is not changed.

$= (x+3)^2 + 2$

The vertex is (–3, 2).

b) Answers may vary. A sample solution is shown.
The maximum will be at the vertex of this function.
The vertex of this function will have the same x value as the vertex of the above function.
substitute $x = -3$

$$g(-3) = \frac{1}{(-3)^2 + 6(-3) + 11}$$
$$= \frac{1}{2}$$

maximum point: $\left(-3, \frac{1}{2}\right)$

c) i) $2x^2 - 8x + 9 = 2(x^2 - 4x) + 9$ Divide the first two terms by 2.

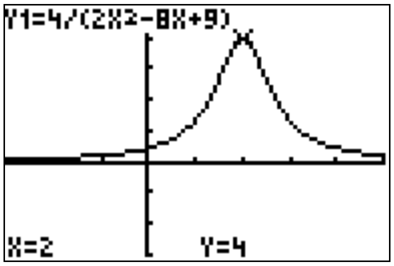
$= 2(x^2 - 4x + 4) + 9 - 8$

$= 2(x-2)^2 + 1$

substitute $x = 2$

$$h(2) = \frac{4}{2(2)^2 - 8(2) + 9}$$
$$= 4$$

maximum point: (2, 4)



ii) $x^2+5x+8=\left(x^2+5x+\frac{25}{4}\right)+8-\frac{25}{4}$

$$=\left(x+\frac{5}{2}\right)^2+\frac{7}{4}$$

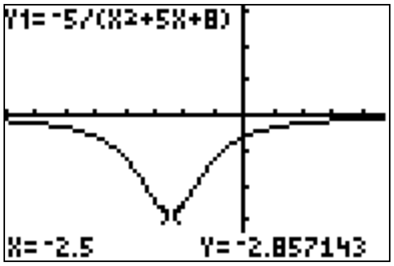
substitute $x=-\frac{5}{2}$

$$k\left(-\frac{5}{2}\right)=-\frac{5}{\left(-\frac{5}{2}\right)^2+5\left(-\frac{5}{2}\right)+8}$$

$$=-\frac{5}{\frac{7}{4}}$$

$$=-\frac{20}{7}$$

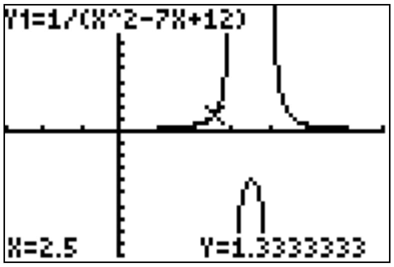
minimum point: $\left(-\frac{5}{2},-\frac{20}{7}\right)$

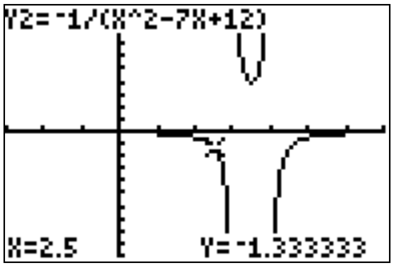


Chapter 3 Section 2 **Question 10 Page 166**

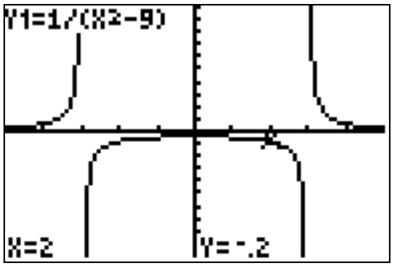
Answers may vary. A sample solution is shown.

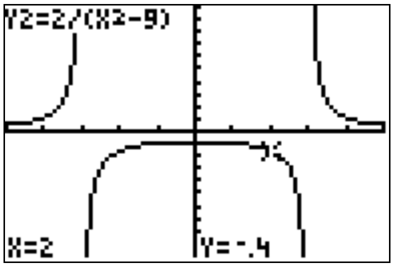
a) *f*(x) and *g*(x) will have the same shape reflected in the x-axis.



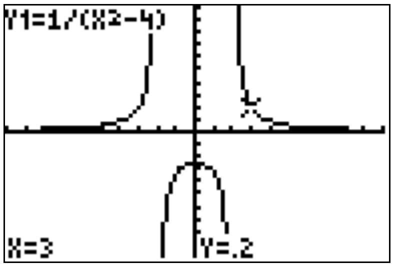


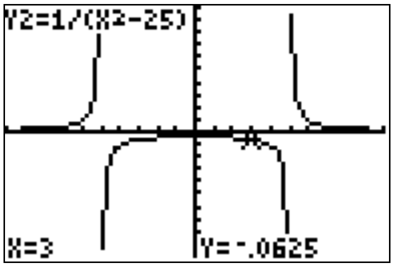
b) $k(x)=\dfrac{2}{x^2-9}$ is a vertical stretch of $h(x)=\dfrac{1}{x^2-9}$ by a factor of 2.





c) $m(x)$ and $n(x)$ will have the same shape but different vertical asymptotes and y-intercept.





Chapter 3 Section 2 Question 11 Page 166

Answers may vary. A sample solution is shown.

a) Since $y = 0$ is the horizontal asymptote, as $x \to \pm\infty$, the denominator approaches $+\infty$, so $f(x)$ approaches 0.
An equation that satisfies the vertical asymptotes:

$$y=\frac{1}{(x-2)(x+3)}$$

For this equation the intervals $x < -3$ and $x > 2$, $y > 0$.
The equation can be written:

$$y=\frac{1}{x^2+x-6}$$

b) Since $y = 0$ is the horizontal asymptote, as $x \to \pm\infty$, the denominator approaches $+\infty$, so approaches 0.
Since there are no vertical asymptotes, the denominator has no real roots.
The maximum point is the vertex.

$$y=\frac{1}{x^2+2}$$

c) Since $y = 0$ is the horizontal asymptote, as $x \to \pm\infty$, the denominator approaches $+\infty$, so $f(x)$ approaches 0.
From the asymptote:

$$y = -\frac{1}{(x+3)^2}$$

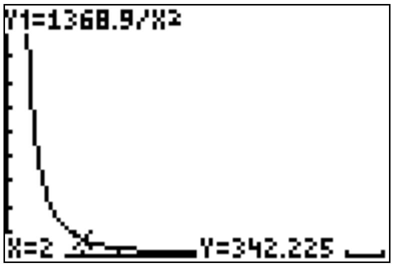
Chapter 3 Section 2 **Question 12 Page 166**

a) $k = 9140 \times 0.387^2$

$k \doteq 1368.89$

$I \doteq \dfrac{1368.89}{d^2}$

b)



c) $I = \dfrac{1368.89}{1^2}$

$= 1368.89$

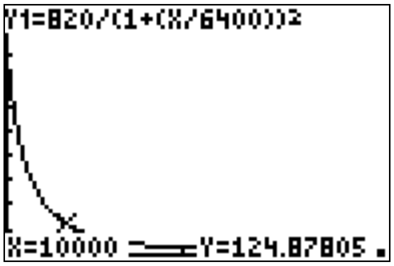
The intensity of radiation on Earth is approximately 1368.89 W/m^2.

x	*y*	**Slope of Secant with (1, 1368.89)**
1.1	1131.31	–2375.8
1.01	1341.92	–2697
1.001	1366.16	–2730

The rate of change is approximately –2730.

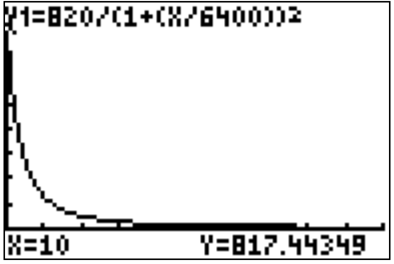
Chapter 3 Section 2 **Question 13 Page 167**

a)



```
Y1=820/(1+(X/6400))²
X=10000          Y=124.87805
```

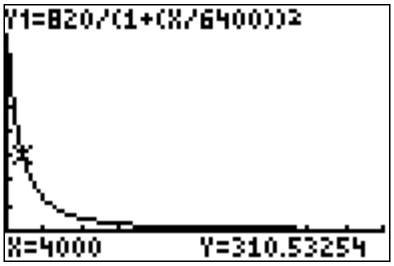
b) i)



```
Y1=820/(1+(X/6400))²
X=10             Y=817.44349
```

$W(h) \doteq 817.4$; approximately 817.4 N

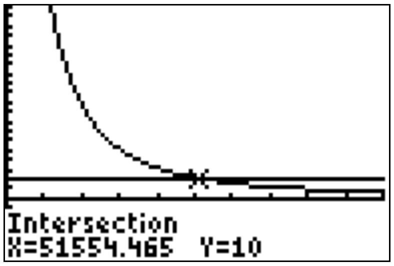
ii)



```
Y1=820/(1+(X/6400))²
X=4000           Y=310.53254
```

$W(h) \doteq 310.5$; approximately 310.5 N

c)



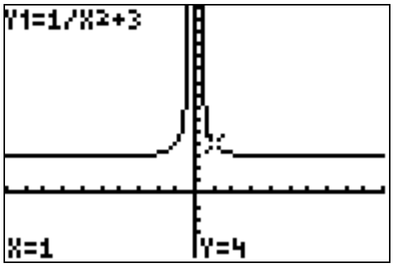
```
Intersection
X=51554.465  Y=10
```

$h \geq 51554.5$

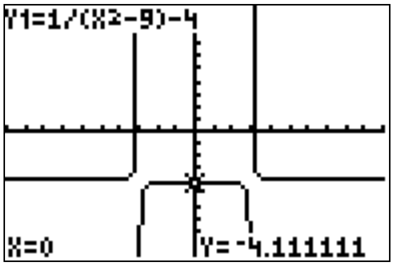
The astronaut will have a weight of less than 10 N at altitudes of at least 51554.5 km.

Chapter 3 Section 2 **Question 14 Page 167**

a) The graph of $y = \frac{1}{x^2}$ shifted up 3.



b) The graph of $y = \frac{1}{x^2 - 9}$ shifted down 4.

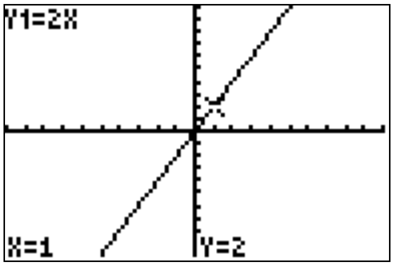


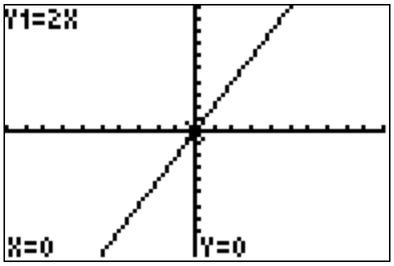
Chapter 3 Section 2 **Question 15 Page 167**

Solutions to Achievement Check questions are provided in the Teacher's Resource.

Chapter 3 Section 2 **Question 16 Page 167**

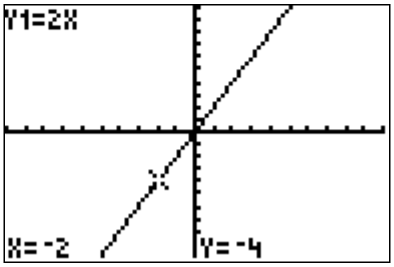
a)

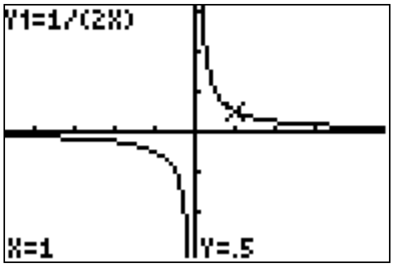




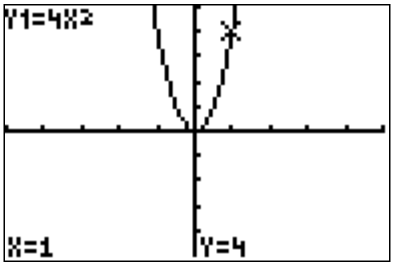
reciprocal: $\frac{1}{2}$

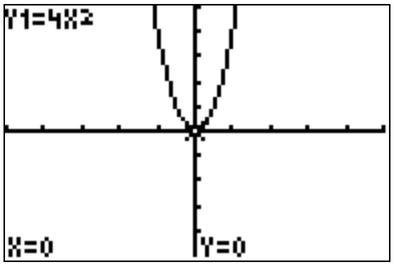
reciprocal: undefined





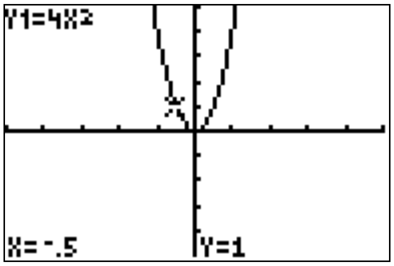
b)

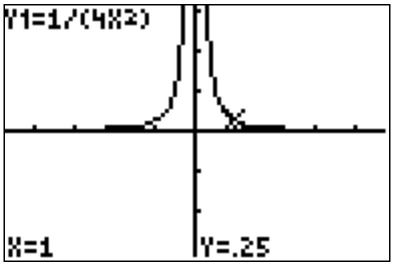




reciprocal: $\frac{1}{4}$

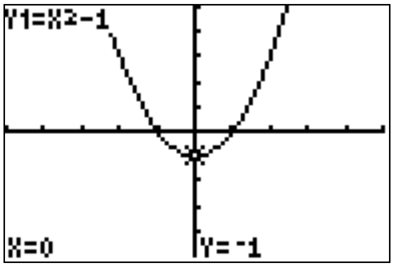
reciprocal: undefined



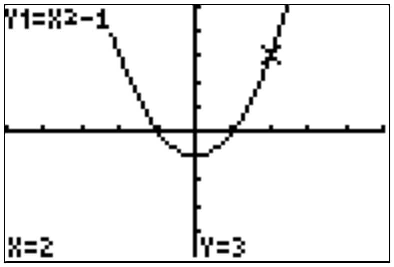


reciprocal: 1

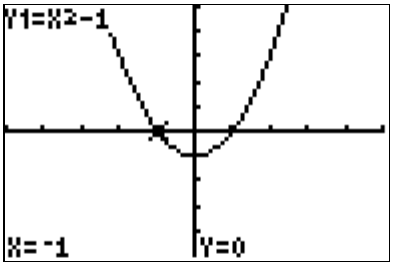
c)



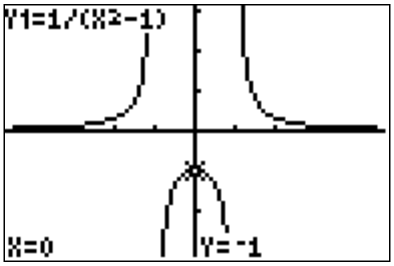
reciprocal: –1



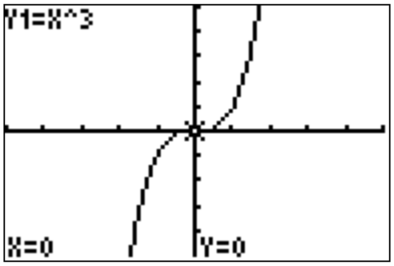
reciprocal: $\frac{1}{3}$



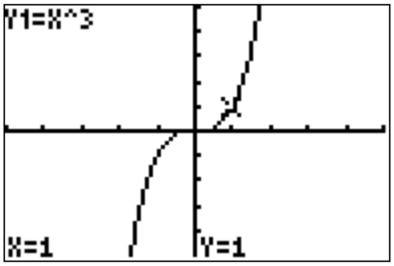
reciprocal: undefined



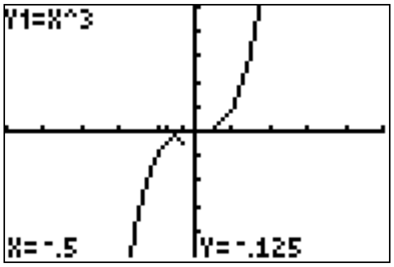
d)



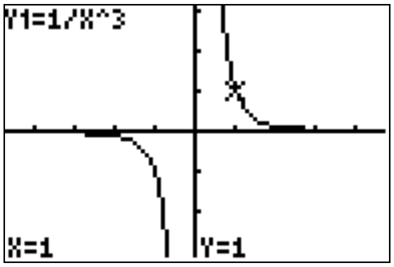
reciprocal: undefined



reciprocal: 1

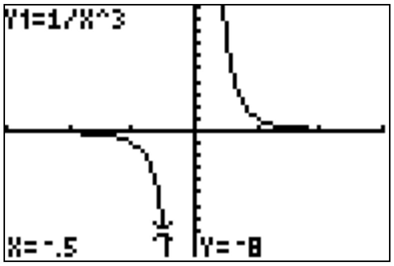


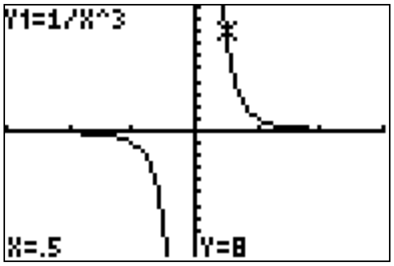
reciprocal: –8



Chapter 3 Section 2 Question 17 Page 167

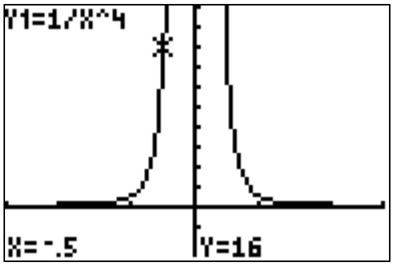
a)

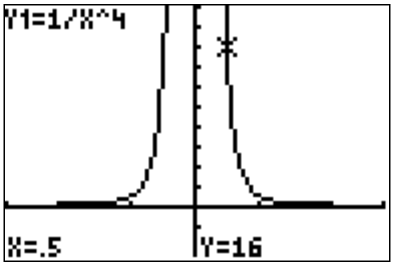




Symmetric about the origin.

b)





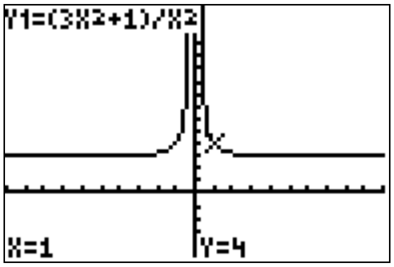
Symmetric about the y-axis.

Chapter 3 Section 2 Question 18 Page 167

Explanations may vary. A sample solution is shown.

a) $f(x)=3+\frac{1}{x^2}$

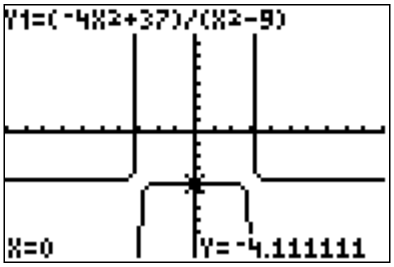
Vertical translation of $y=\frac{1}{x^2}$ up 3.



b)

$$\begin{aligned} g(x) &= \frac{-4x^2+36+1}{x^2-9} \\ &= \frac{-4(x^2-9)+1}{x^2-9} \\ &= \frac{-4(x^2-9)}{x^2-9}+\frac{1}{x^2-9} \\ &= -4+\frac{1}{x^2-9} \end{aligned}$$

Vertical translation of $y=\frac{1}{x^2-9}$ down 4.

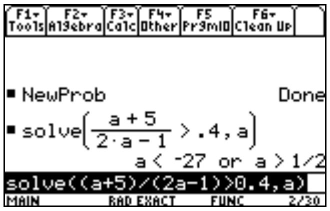


Chapter 3 Section 2 Question 19 Page 167

Answers may vary. A sample solution is shown.

Since $x \neq a, x \neq b, y=\frac{1}{(x-a)(x-b)}$.

Chapter 3 Section 2 Question 20 Page 167



■ NewProb Done

■ solve$\left(\frac{a+5}{2\cdot a-1} > .4, a\right)$

a < -27 or a > 1/2

solve((a+5)/(2a-1)>0.4,a)

$a < -27$ or $a > \frac{1}{2}$

$$\begin{aligned} g(4) &= g(3+1) \\ &= \frac{g(3-2)g(3-1)+1}{g(3)} \\ &= \frac{g(1)g(2)+1}{g(3)} \\ &= \frac{(1)(2)+1}{3} \\ &= 1 \end{aligned}$$

$$\begin{aligned} g(5) &= g(4+1) \\ &= \frac{g(4-2)g(4-1)+1}{g(4)} \\ &= \frac{g(2)g(3)+1}{g(4)} \\ &= \frac{(2)(3)+1}{1} \\ &= 7 \end{aligned}$$

Chapter 3 Section 2 **Question 22 Page 167**

The chord length of both circles will be the same.

chord length = $2R\sin\left(\frac{1}{2}\theta\right)$

chord length of circle O $= 2r\sin(15°)$ chord length of circle P $= 2R\sin(30°)$

$$\begin{aligned} &= 2r\left(\frac{\sqrt{2}\left(\sqrt{3}-1\right)}{4}\right) \\ &= r\left(\frac{\sqrt{2}\left(\sqrt{3}-1\right)}{2}\right) \end{aligned}$$

$$\begin{aligned} &= 2R\times\frac{1}{2} \\ &= R \end{aligned}$$

$$R = r\left(\frac{\sqrt{2}(\sqrt{3}-1)}{2}\right)$$

Ratio of the areas:

$$\pi r^2 : \pi R^2$$

$$\pi r^2 : \pi\left[r(\frac{\sqrt{2}(\sqrt{3}-1)}{2})\right]^2$$

$$\pi r^2 : \pi r^2\left(\frac{2(4-2\sqrt{3})}{4}\right)$$

$$1:1\left(\frac{4-2\sqrt{3}}{2}\right)$$

$$1:2-\sqrt{3}$$

C

Chapter 3 Section 3 Rational Functions of the Form $f(x)=\dfrac{ax+b}{cx+d}$

Chapter 3 Section 3 Question 1 Page 174

a) $x = 7$
domain: $\{x \in \mathbb{R}, x \neq 7\}$

b) $x = -5$
domain: $\{x \in \mathbb{R}, x \neq -5\}$

c) $x = -8$

domain: $\{x \in \mathbb{R}, x \neq -8\}$

d) $x = \dfrac{1}{3}$

domain: $\{x \in \mathbb{R}, x \neq \dfrac{1}{3}\}$

e) $x = -\dfrac{9}{4}$

domain: $\{x \in \mathbb{R}, x \neq -\dfrac{9}{4}\}$

f) $x = 5$

domain: $\{x \in \mathbb{R}, x \neq 5\}$

Chapter 3 Section 3 Question 2 Page 174

a) As $x \to \infty$, the numerator and denominator both approach infinity.
Divide each term by x.

$$p(x) = \frac{\frac{x}{x}}{\frac{x}{x} - \frac{6}{x}}$$

As $x \to \pm\infty$, $\dfrac{6}{x}$ gets very close to 0.

$$p(x) \to \frac{1}{1-0}$$

$$p(x) \to 1$$

The horizontal asymptote is $y = 1$.
range: $\{y \in \mathbb{R}, y \neq 1\}$

b) As x → ∞, the numerator and denominator both approach infinity.
Divide each term by x.

$$q(x)=\frac{\frac{3x}{x}}{\frac{x}{x}+\frac{4}{x}}$$

As $x \to \pm\infty$, $\frac{4}{x}$ gets very close to 0.

$$q(x)\to\frac{3}{1+0}$$

$$q(x)\to 3$$

The horizontal asymptote is $y = 3$.
range: $\{y \in \mathbb{R}, y \neq 3\}$

c) As x → ∞, the numerator and denominator both approach infinity.
Divide each term by x.

$$r(x)=\frac{\frac{x}{x}-\frac{1}{x}}{\frac{x}{x}+\frac{1}{x}}$$

As $x \to \pm\infty$, $\frac{1}{x}$ gets very close to 0.

$$r(x)\to\frac{1-0}{1+0}$$

$$r(x)\to 1$$

The horizontal asymptote is $y = 1$.
range: $\{y \in \mathbb{R}, y \neq 1\}$

d) As $x \to \infty$, the numerator and denominator both approach infinity.
Divide each term by x.

$$s(x)=\frac{\frac{5x}{x}-\frac{2}{x}}{\frac{2x}{x}+\frac{3}{x}}$$

As $x \to \pm\infty$, $\frac{2}{x}$ and $\frac{3}{x}$ get very close to 0.

$$s(x)\to\frac{5-0}{2+0}$$

$$s(x)\to\frac{5}{2}$$

The horizontal asymptote is $y=\frac{5}{2}$.

range: $\{y \in \mathbb{R}, y \neq \frac{5}{2}\}$

e) As $x \to \infty$, the numerator and denominator both approach infinity.
Divide each term by x.

$$t(x)=\frac{\frac{x}{x}-\frac{6}{x}}{\frac{4}{x}-\frac{x}{x}}$$

As $x \to \pm\infty$, $\frac{6}{x}$ and $\frac{4}{x}$ get very close to 0.

$$t(x)\to\frac{1-0}{0-1}$$

$$t(x)\to -1$$

The horizontal asymptote is $y = -1$.

range: $\{y \in \mathbb{R}, y \neq -1\}$

f) As $x \to \infty$, the numerator and denominator both approach infinity.
Divide each term by x.

$$u(x)=\frac{\frac{3}{x}-\frac{4x}{x}}{\frac{1}{x}-\frac{2x}{x}}$$

As $x \to \pm\infty$, $\frac{3}{x}$ and $\frac{1}{x}$ get very close to 0.

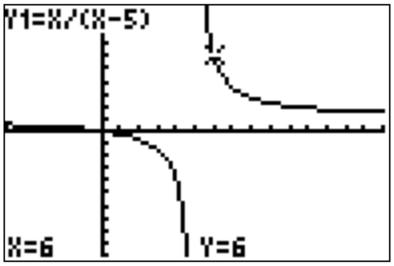
$$u(x)\to\frac{0-4}{0-2}$$

$$u(x)\to 2$$

The horizontal asymptote is $y = 2$.
range: $\{y \in \mathbb{R}, y \neq 2\}$

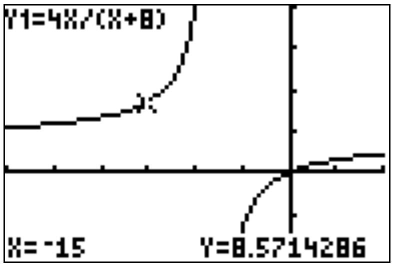
Chapter 3 Section 3 Question 3 Page 174

a)



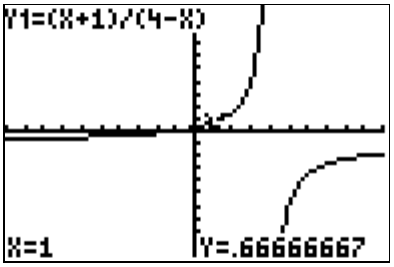
Interval	Sign of *f*(*x*)	Sign of Slope	Change in slope
$x < 0$	+	–	–
$0 < x < 5$	–	–	–
$x > 5$	+	–	+

b)



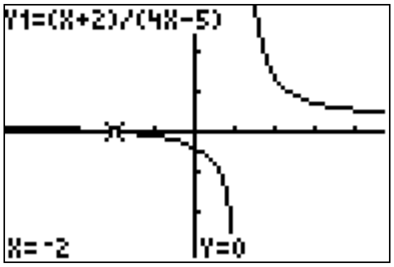
Interval	Sign of *f*(*x*)	Sign of Slope	Change in Slope
$x < -8$	+	+	+
$-8 < x < 0$	–	+	–
$x > 0$	+	+	–

c)



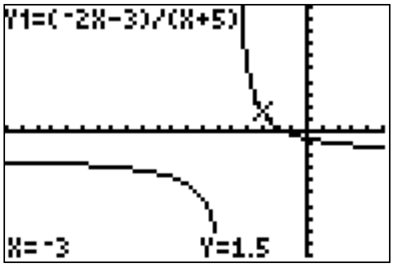
Interval	Sign of $f(x)$	Sign of Slope	Change in Slope
$x < -1$	–	+	+
$-1 < x < 4$	+	+	+
$x > 4$	–	+	–

d)



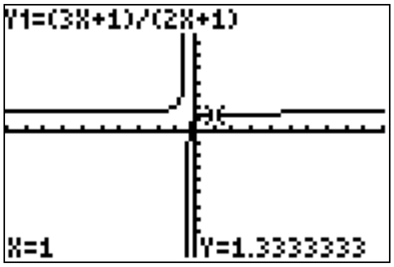
Interval	Sign of $f(x)$	Sign of Slope	Change in Slope
$x < -2$	+	–	–
$-2 < x < \frac{5}{4}$	–	–	–
$x > \frac{5}{4}$	+	–	+

e)



Interval	Sign of $f(x)$	Sign of Slope	Change in Slope
$x < -5$	–	–	–
$-5 < x < -1.5$	+	–	+
$x > -1.5$	–	–	+

f)



Interval	Sign of $f(x)$	Sign of Slope	Change in Slope
$x < -\frac{1}{2}$	+	+	+
$-\frac{1}{2} < x < -\frac{1}{3}$	–	+	–
$x > -\frac{1}{3}$	+	+	–

Chapter 3 Section 3 **Question 4 Page 174**

a) i)

x	***y***	**Slope of Secant with (3.5, 14)**
3.51	13.7647	–23.53
3.501	13.976	–24
3.5001	13.9976	–24

x	***y***	**Slope of Secant with (20, 2.352 94)**
20.1	2.350 88	–0.0206
20.01	2.352 73	–0.021
20.001	2.352 92	–0.02

$m_{3.5} = -24$ $m_{20} = -0.02$

ii)

x	y	**Slope of Secant with (2.5, –10)**
2.51	–10.5449	–54.49
2.501	–10.024	–24
2.5001	–10.0024	–24

x	y	**Slope of Secant with (20, 1.739 13)**
–20.1	1.740 26	–0.0113
–20.01	1.739 24	–0.011
–20.001	1.739 14	–0.01

$m_{2.5} = -24 \qquad m_{-20} = -0.01$

b) The function is decreasing for $x < 3$ and increasing for $x > 3$.

Chapter 3 Section 3 **Question 5 Page 174**

a) i) As $x \to \infty$, the numerator and denominator both approach infinity.
Divide each term by x.

$$f(x) = \frac{\frac{x}{x} - \frac{5}{x}}{\frac{2x}{x} + \frac{1}{x}}$$

As $x \to \pm\infty$, $\frac{5}{x}$ and $\frac{1}{x}$ get very close to 0.

$$f(x) \to \frac{1-0}{2+0}$$

$$f(x) \to \frac{1}{2}$$

The horizontal asymptote is $y = \frac{1}{2}$.

ii) As $x \to \infty$, the numerator and denominator both approach infinity.
Divide each term by x.

$$g(x) = \frac{\frac{3}{x} - \frac{5x}{x}}{\frac{2x}{x} + \frac{1}{x}}$$

As $x \to \pm\infty$, $\frac{3}{x}$ and $\frac{1}{x}$ get very close to 0.

$$g(x) \to \frac{0-5}{2+0}$$

$$g(x) \to -\frac{5}{2}$$

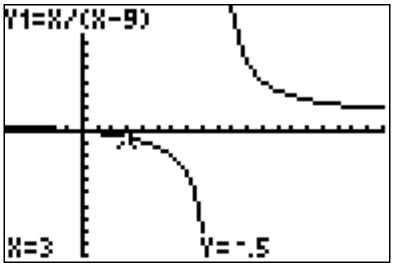
The horizontal asymptote is $y = -\frac{5}{2}$.

b) Answers may vary. A sample solution is shown.
The horizontal asymptote is equal to the coefficient of x in the numerator divided by the coefficient of x in the denominator.

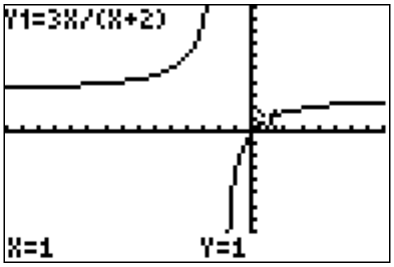
c) $y = \frac{a}{c}$

Chapter 3 Section 3 **Question 6 Page 174**

a) horizontal asymptote: $y = 1$
vertical asymptote: $x = 9$
domain: $\{x \in \mathbb{R}, x \neq 9\}$
range: $\{y \in \mathbb{R}, y \neq 1\}$



b) horizontal asymptote: $y = 3$
vertical asymptote: $x = -2$
domain: $\{x \in \mathbb{R}, x \neq -2\}$
range: $\{y \in \mathbb{R}, y \neq 3\}$

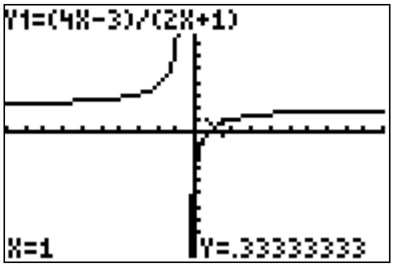


c) horizontal asymptote: $y = 2$

vertical asymptote: $x = -\frac{1}{2}$

domain: $\{x \in \mathbb{R}, x \neq -\frac{1}{2}\}$

range: $\{y \in \mathbb{R}, y \neq 2\}$

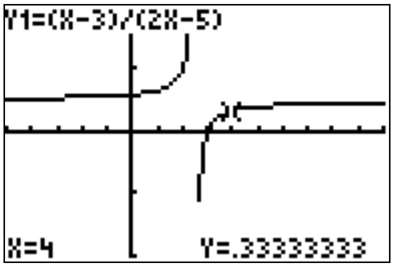


d) horizontal asymptote: $y = \frac{1}{2}$

vertical asymptote: $x = \frac{5}{2}$

domain: $\{x \in \mathbb{R}, x \neq \frac{5}{2}\}$

range: $\{y \in \mathbb{R}, y \neq \frac{1}{2}\}$

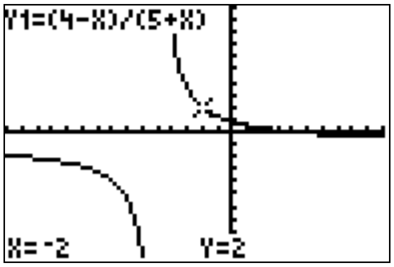


e) horizontal asymptote: $y = -1$

vertical asymptote: $x = -5$

domain: $\{x \in \mathbb{R}, x \neq -5\}$

range: $\{y \in \mathbb{R}, y \neq -1\}$

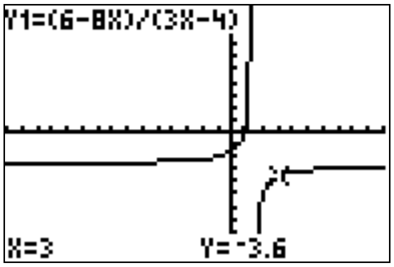


f) horizontal asymptote: $y = -\frac{8}{3}$

vertical asymptote: $x = \frac{4}{3}$

domain: $\{x \in \mathbb{R}, x \neq \frac{4}{3}\}$

range: $\{y \in \mathbb{R}, y \neq -\frac{8}{3}\}$



a) horizontal asymptote: $y = 2$
vertical asymptote: $x = 3$

$$y = \frac{2x+b}{x-3}$$

substitute the point (1.5, 0)

$$0 = \frac{2(1.5)+b}{1.5-3}$$

$$0 = \frac{3+b}{-1.5}$$

$$b = -3$$

$$y = \frac{2x-3}{x-3}$$

b) horizontal asymptote: $y = 1$
vertical asymptote: $x = -1$

$$y = \frac{x+b}{x+1}$$

substitute the point (4, 0)

$$0 = \frac{4+b}{4+1}$$

$$b = -4$$

$$y = \frac{x-4}{x+1}$$

$$y=\frac{ax+b}{cx+d}$$

substitute vertical and horizontal asymptotes

$$y=\frac{x+b}{x-2}$$

substitute x-intercept

$$0=\frac{-4+b}{-4-2}$$

$$0=-4+b$$

$$b=4$$

$$y=\frac{x+4}{x-2}$$

Check y-intercept:

L.S.	**R.S.**
-2	$\frac{0+4}{0-2}$ $=-2$

Since the left side equals the right side, the equation is true for the y-intercept.

$$y=\frac{x+4}{x-2}$$

Chapter 3 Section 3 **Question 9 Page 175**

From the asymptotes:

$$y=\frac{5x+b}{2x+1}$$

substitute y-intercept

$$-3=\frac{0+b}{0+1}$$

$$-3=b$$

$$y=\frac{5x-3}{2x+1}$$

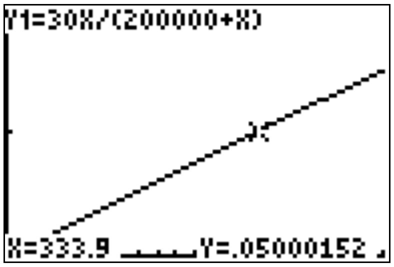
Check the x-intercept:

L.S.	**R.S.**
0	$\dfrac{5\left(\frac{3}{5}\right)-3}{2\left(\frac{3}{5}\right)+1}$
	$=\dfrac{3-3}{\frac{6}{5}+1}$
	$=0$

Since the left side equals the right side, the equation is true for the x-intercept.

Chapter 3 Section 3 **Question 10 Page 175**

a)



Y1=30X/(200000+X)

X=333.9 Y=.05000152

b) As $t \to \infty$, the numerator and denominator both approach infinity.
Divide each term by t.

$$C(t) = \frac{\frac{30t}{t}}{\frac{200\ 000}{t} + \frac{t}{t}}$$

as $t \to \infty$, $\frac{200\ 000}{t}$ gets very close to 0.

$$C(t) \to \frac{30}{0+1}$$

$$C(t) \to 30$$

The amount of pollutant levels off at 30 g/L.

c) From the graph, $t = 333.9$
After approximately 333.9 min.

Chapter 3 Section 3 **Question 11 Page 175**

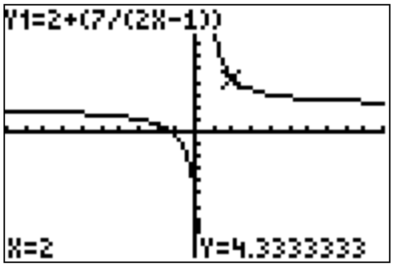
a)

$$\begin{array}{r} 2 \\ 2x-1\overline{)4x+5} \\ \underline{4x-2} \\ 7 \end{array}$$

$$f(x) = 2 + \frac{7}{2x-1}$$

b) Answers may vary. A sample solution is shown.
$f(x) = 2 + \frac{7}{2x-1}$ is the graph of $y = \frac{7}{2x-1}$ vertically translated up 2.

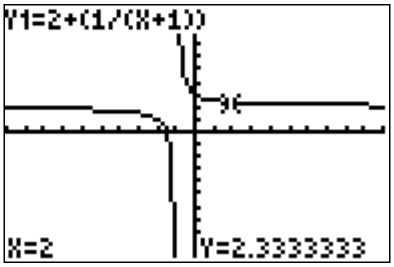
c)



Chapter 3 Section 3 **Question 12 Page 175**

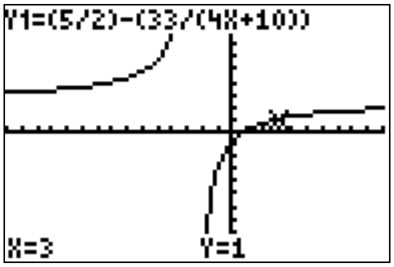
a)
$$\begin{array}{r} 2 \\ x+1\overline{)2x+3} \\ \underline{2x+2} \\ 1 \end{array}$$

$$p(x)=2+\frac{1}{x+1}$$



b)
$$\begin{array}{r} \frac{5}{2} \\ 2x+5\overline{)5x-4} \\ \underline{5x+\frac{25}{2}} \\ -\frac{33}{2} \end{array}$$

$$t(x)=\frac{5}{2}-\frac{33}{2(2x+5)}$$



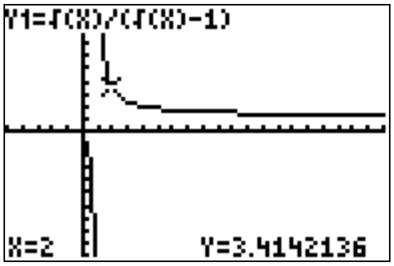
Chapter 3 Section 3 **Question 13 Page 175**

Solutions to Achievement Check questions are provided in the Teacher's Resource.

Chapter 3 Section 3 **Question 14 Page 175**

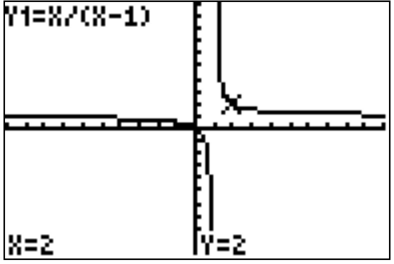
Answers may vary. A sample solution is shown.
As the mass of the club increases, the rate of change of the initial velocity decreases.

Chapter 3 Section 3 Question 15 Page 175



asymptotes: $y = 1$, $x = 1$
domain: $\{x \in \mathbb{R},\ x > 0,\ x \neq 1\}$; range: $\{y \in \mathbb{R},\ y \leq 0,\ y > 1\}$
y-intercept is 0
On $0 < x < 1$, $f(x)$ is negative and decreasing. The slope is negative and decreasing.
On $x > 1$, $f(x)$ is positive and decreasing. The slope is negative and increasing.

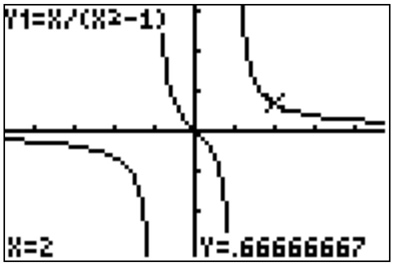
Comparison: Answers may vary. A sample solution is shown.



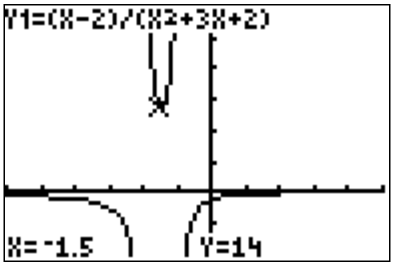
The asymptotes are the same; $y = 1$, $x = 1$
domain and range are different: $\{x \in \mathbb{R},\ x \neq 1\}$; $\{y \in \mathbb{R},\ y \neq 1\}$
The end behaviour is the same for $x > 1$, $f(x)$ is positive and decreasing.
The slope is negative and increasing and for $0 < x < 1$, $f(x)$ is negative and decreasing.
The slope is negative and decreasing.
For $x < 0$, $f(x)$ is positive and decreasing. The slope is negative and decreasing.

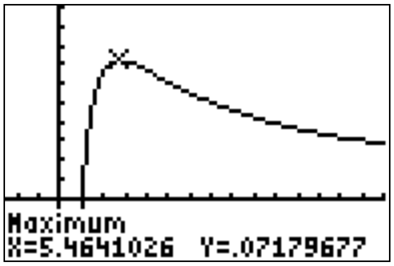
Chapter 3 Section 3 **Question 16 Page 176**

a) x-intercept: 0; y-intercept: 0
vertical asymptotes: $x = 1$, $x = -1$
horizontal asymptote: $y = 0$
domain: $\{x \in \mathbb{R},\ x \neq 1,\ x \neq -1\}$
range: $\{y \in \mathbb{R}\}$
$x < -1$, $f(x)$ is negative and decreasing (negative slope)
$-1 < x < 0$ $f(x)$ is positive and decreasing (negative slope)
$0 < x < 1$, $f(x)$ is negative and decreasing (negative slope)
$x > 1$, $f(x)$ is positive and decreasing (negative slope)



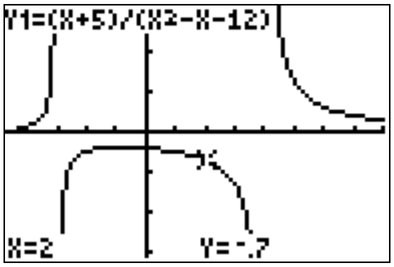
b) $$g(x) = \frac{x-2}{(x+1)(x+2)}$$
x-intercept 2; y-intercept -1
asymptotes: $x = -2$, $x = -1$, $y = 0$
domain: $\{x \in \mathbb{R},\ x \neq -2,\ x \neq -1\}$
range: $\{y \in \mathbb{R},\ y \leq 0.07,\ y \geq 13.93\}$
$x < -2$ $f(x)$ is negative and decreasing (negative slope)
$-2 < x < -1.46$ $f(x)$ is positive and decreasing (negative slope)
$-1.46 < x < -1$, $f(x)$ is positive and increasing (positive slope)
$-1 < x < 2$, $f(x)$ is negative and increasing (positive slope)
$2 < x < 5.46$, $f(x)$ is positive and increasing (positive slope)
$x > 5.46$, $f(x)$ is positive and decreasing (negative slope)



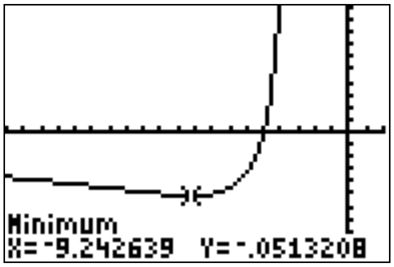


c) $h(x) = \dfrac{x+5}{(x-4)(x+3)}$

x-intercept: -5; y-intercept: $-\dfrac{5}{12}$
asymptotes: $x = -3$, $x = 4$, $y = 0$
domain: $\{x \in \mathbb{R},\ x \neq -3,\ x \neq 4\}$
range: $\{y \in \mathbb{R}\}$
$x < -9.24$, $f(x)$ is negative and decreasing
$-9.24 < x < -5$, $f(x)$ is negative and increasing
$-5 < x < -3$, $f(x)$ is positive and increasing
$-3 < x < -0.76$, $f(x)$ is negative and increasing
$-0.76 < x < 4$, $f(x)$ is negative and decreasing
$x > 4$, $f(x)$ is positive and decreasing



```
Y1=(X+5)/(X²-X-12)
X=2          Y=-.7
```



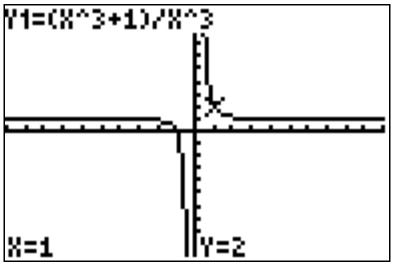
```
Minimum
X=-9.242639  Y=-.0513208
```

Common Features. Answers may vary. A sample solution is shown.
The graphs have the same shape reflected in the x-axis

Chapter 3 Section 3 **Question 17 Page 176**

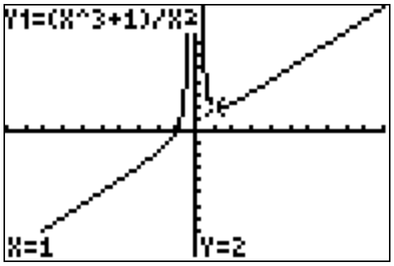
Answers may vary. A sample solution is shown.
When the degree of the polynomial in the numerator is greater than the degree of the polynomial in the denominator you can expect to get an oblique asymptote.

a)



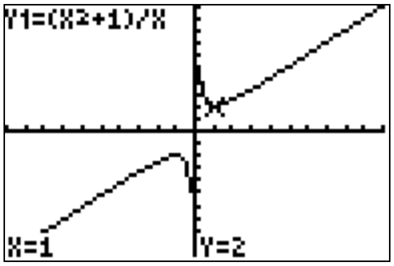
asymptotes: $x = 0$, $y = 1$

b)



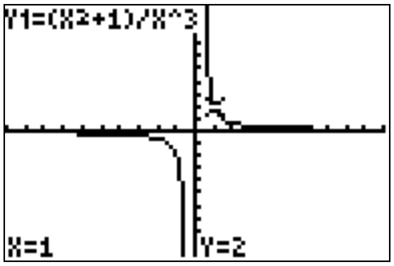
asymptote: $y = x$

c)



asymptote: $y = x$

d)



asymptotes: $x = 0$, $y = 0$

Chapter 3 Section 3 **Question 18 Page 176**

A; $a > b$

Chapter 3 Section 3 **Question 19 Page 176**

a)
$$\begin{array}{r} x \\ x\overline{)x^2-2} \\ \underline{x^2} \\ 0-2 \end{array}$$

quotient: x, remainder: –2

b) $f(x)=x-\dfrac{2}{x}$

c) i)
$$\begin{array}{r} x-1 \\ x-2\overline{)x^2-3x-4} \\ \underline{x^2-2x} \\ -x-4 \\ \underline{-x+2} \\ -6 \end{array}$$

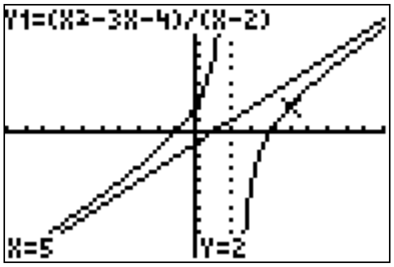
$g(x)=x-1-\dfrac{6}{x-2}$

From $f(x)=q(x)+\dfrac{r(x)}{d(x)}$:

The oblique asymptote is $y = q(x)$.

The vertical asymptote is when $d(x) = 0$.

$y = x – 1$ $x = 2$

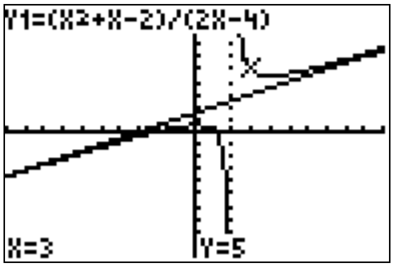


ii) $$\begin{array}{r}
\frac{1}{2}x+\frac{3}{2} \\
2x-4\overline{\smash{)}x^2+x-2} \\
\underline{x^2-2x} \\
3x-2 \\
\underline{3x-6} \\
4
\end{array}$$

$$f(x)=\frac{x+3}{2}+\frac{4x}{2x-4}$$

$$f(x)=\frac{x+3}{2}+\frac{2x}{x-2}$$

asymptotes: $y=\frac{x+3}{2}$; $x=2$



iii) $z(x)=\frac{(x-3)(x+3)}{(x+3)}$

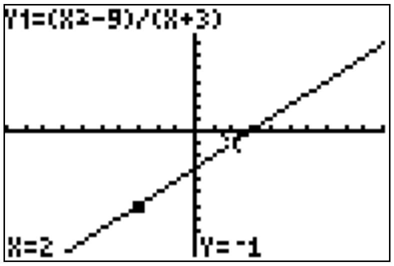
$z(x)=x-3,\ x\neq -3$

There are no asymptotes, but the graph is discontinuous (there's a hole in the graph) at $x=-3$.

To find the y-value where the graph is discontinuous, substitute $x=-3$ into $z(x)$.

$$\begin{aligned} z(-3)&=-3-3 \\ &=-6 \end{aligned}$$

The graph is discontinuous at the point (–3, –6).



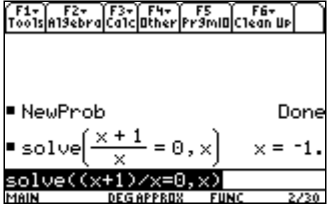
Chapter 3 Section 4 Question 1 Page 183

a) $x+1=0$

$x=-1$

x-intercept is –1

Verify:



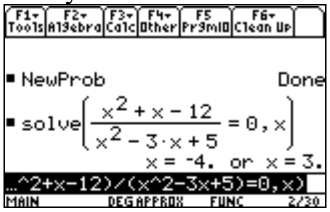
```
■ NewProb                    Done
■ solve((x + 1)/x = 0, x)    x = -1.
solve((x+1)/x=0,x)
```

b) $y=\dfrac{(x+4)(x-3)}{x^2-3x+5}$

$x=-4$ or $x=3$

x-intercepts are –4, 3

Verify:



```
■ NewProb                                Done
■ solve((x^2 + x - 12)/(x^2 - 3·x + 5) = 0, x)
                              x = -4. or x = 3.
…^2+x-12)/(x^2-3x+5)=0,x)
```

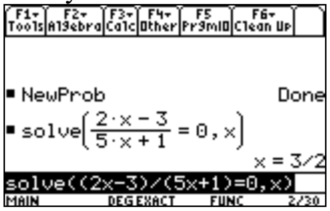
c) $2x-3=0$

$2x=3$

$x=\dfrac{3}{2}$

x-intercept is $\dfrac{3}{2}$

Verify:

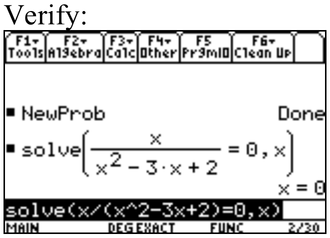


```
■ NewProb                          Done
■ solve((2·x - 3)/(5·x + 1) = 0, x)
                                 x = 3/2
solve((2x-3)/(5x+1)=0,x)
```

d) $x = 0$

x-intercept is 0

Verify:



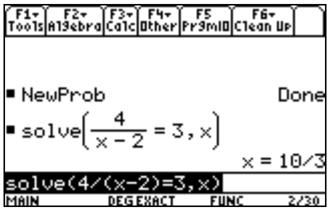
```
■ NewProb                 Done
■ solve(x/(x^2 - 3·x + 2) = 0, x)
                         x = 0
solve(x/(x^2-3x+2)=0,x)
MAIN     DEG EXACT   FUNC  2/30
```

Chapter 3 Section 4 **Question 2 Page 183**

a)

$$\begin{aligned} 4 &= 3(x-2),\ x \neq 2 \\ 4 &= 3x - 6 \\ 3x &= 10 \\ x &= \frac{10}{3} \end{aligned}$$

Check:



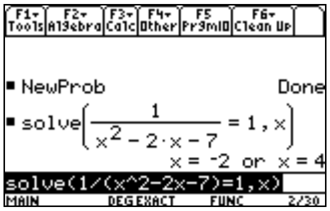
```
■ NewProb                 Done
■ solve(4/(x - 2) = 3, x)
                      x = 10/3
solve(4/(x-2)=3,x)
MAIN     DEG EXACT   FUNC  2/30
```

b)

$$\begin{aligned} 1 &= x^2 - 2x - 7,\ x \neq 1 \pm 2\sqrt{2} \\ x^2 - 2x - 8 &= 0 \\ (x-4)(x+2) &= 0 \\ x &= 4 \text{ or } x = -2 \end{aligned}$$

$$\begin{aligned} x^2 - 2x - 7 &\neq 0 \\ x &\neq \frac{2 \pm \sqrt{(-2)^2 - 4(1)(-7)}}{2(1)} \\ x &\neq \frac{2 \pm \sqrt{32}}{2} \\ x &\neq 1 \pm 2\sqrt{2} \end{aligned}$$

Check:



```
■ NewProb                 Done
■ solve(1/(x^2 - 2·x - 7) = 1, x)
             x = -2 or x = 4
solve(1/(x^2-2x-7)=1,x)
MAIN     DEG EXACT   FUNC  2/30
```

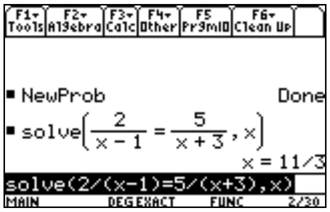
c) $2(x+3)=5(x-1),\ x\neq 1,\ x\neq -3$

$$2x+6=5x-5$$

$$3x=11$$

$$x=\frac{11}{3}$$

Check:



■ NewProb Done

■ $\text{solve}\left(\frac{2}{x-1}=\frac{5}{x+3},x\right)$

x = 11/3

```
solve(2/(x-1)=5/(x+3),x)
MAIN    DEG EXACT    FUNC    2/30
```

d) multiply both sides by x

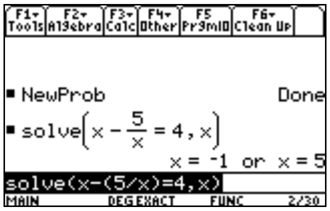
$$x^2-5=4x,\ x\neq 0$$

$$x^2-4x-5=0,$$

$$(x-5)(x+1)=0$$

$$x=5 \text{ or } x=-1$$

Check:



■ NewProb Done

■ $\text{solve}\left(x-\frac{5}{x}=4,x\right)$

x = -1 or x = 5

```
solve(x-(5/x)=4,x)
MAIN    DEG EXACT    FUNC    2/30
```

e) $2x^2=x(x-34),\ x\neq 0$

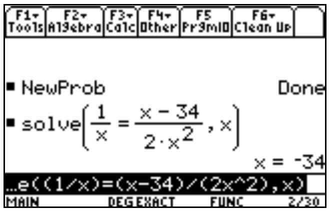
$$2x^2=x^2-34x$$

$$x^2+34x=0$$

$$x(x+34)=0$$

$$\cancel{x=0} \text{ or } x=-34$$

Check:



■ NewProb Done

■ $\text{solve}\left(\frac{1}{x}=\frac{x-34}{2\cdot x^2},x\right)$

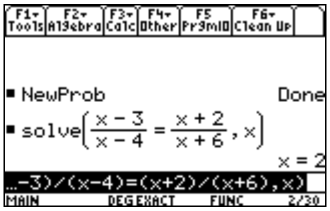
x = -34

```
…e((1/x)=(x-34)/(2x^2),x)
MAIN    DEG EXACT    FUNC    2/30
```

f) $(x-3)(x+6)=(x+2)(x-4),\ x\neq 4,\ x\neq -6$

$$\begin{aligned} x^2+3x-18&=x^2-2x-8\\ 3x+2x&=18-8\\ 5x&=10\\ x&=2 \end{aligned}$$

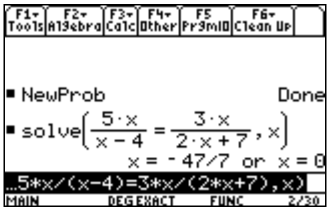
Check:



```
■ NewProb                    Done
■ solve((x - 3)/(x - 4) = (x + 2)/(x + 6), x)
                            x = 2
…-3)/(x-4)=(x+2)/(x+6),x)
```

Chapter 3 Section 4 **Question 3 Page 184**

a)



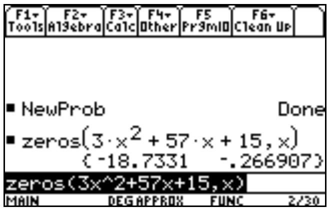
```
■ NewProb                    Done
■ solve(5·x/(x - 4) = 3·x/(2·x + 7), x)
             x = -47/7 or x = 0
…5*x/(x-4)=3*x/(2*x+7),x)
```

$x=0$ or $x\doteq -6.71$

b) $(2x+3)(4x+7)=(5x-1)(x-6),\ x\neq 6,\ x\neq -\dfrac{7}{4}$

$$8x^2+26x+21=5x^2-31x+6$$

$$3x^2+57x+15=0$$



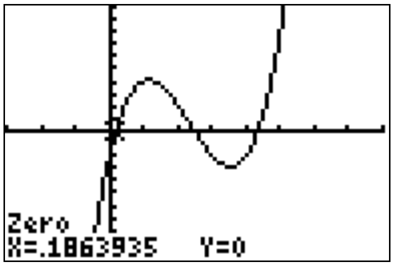
```
■ NewProb                    Done
■ zeros(3·x^2 + 57·x + 15, x)
      {-18.7331   -.266907}
zeros(3x^2+57x+15,x)
```

$x\ \doteq\ -0.27$ or $x\ \doteq\ -18.73$

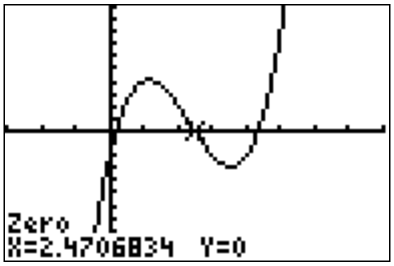
c) $x(x-3)=(x-2)(x^2-4x+1),\ x \neq 2,\ x \neq 3$

$x^2-3x=x^3-4x^2+x-2x^2+8x-2$

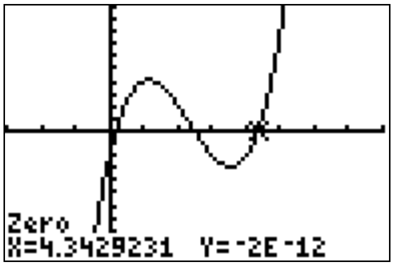
$x^3-7x^2+12x-2=0$



```
Zero
X=.1863935   Y=0
```



```
Zero
X=2.4706834  Y=0
```



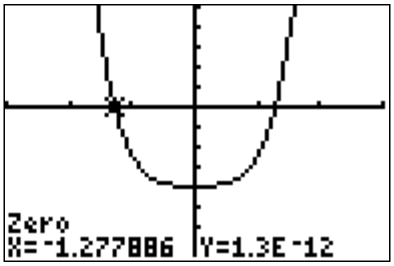
```
Zero
X=4.3429231  Y=-2E-12
```

$x = 0.19$ or $x \doteq 2.47$ or $x \doteq 4.34$

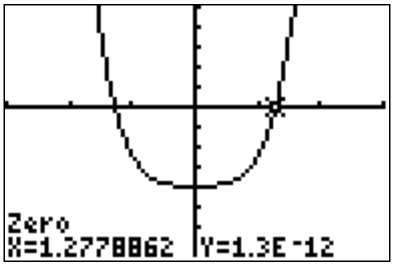
d) $(x^2-1)(x^2+1)=(2x^2+3)(2x^2-3),\ x \neq \pm\sqrt{\frac{3}{2}}$

$x^4-1=4x^4-9$

$3x^4-8=0$



```
Zero
X=-1.277886  Y=1.3E-12
```



```
Zero
X=1.2778862  Y=1.3E-12
```

$x \doteq -1.28$ or $x \doteq 1.28$

a) Because $x - 3 \neq 0$, either $x > 3$ or $x < 3$.

Case 1: If $x > 3$,

$$4 < x - 3$$
$$7 < x$$
$$x > 7$$

$x > 7$ is within the inequality $x > 3$, so the solution is $x > 7$.

Case 2: If $x < 3$:

$4 > x - 3$ Change the inequality when multiplying by a negative.

$$7 > x$$
$$x < 7$$

$x < 3$ is within the inequality $x < 7$, so the solution is $x < 3$.

The solution is $x < 3$ or $x > 7$.

b) Because $x + 1 \neq 0$, either $x > -1$ or $x < -1$.

Case 1: If $x > -1$,

$$7 > 7(x+1)$$
$$7 > 7x + 7$$
$$7x < 0$$
$$x < 0$$

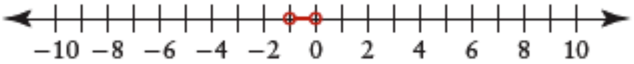
$-1 < x < 0$ is a solution.

Case 2: If $x < -1$:

$$7 < 7(x+1)$$
$$7 < 7x + 7$$
$$7x > 0$$
$$x > 0$$

There is no solution.

The solution is $-1 < x < 0$.



c) Because $x \neq -4,\ x \neq -1,$ either $x > -4$ or $x < -4$ or $x > -1$ or $x < -1$.
$x > -1$ is within $x > -4$, so test $x > -1$
$x < -4$ is within $x < -1$, so test $x < -4$

Case 1: If $x > -1$,

$$\begin{aligned} 5(x+1) &< 2(x+4) \\ 5x+5 &< 2x+8 \\ 3x &< 3 \\ x &< 1 \end{aligned}$$

$-1 < x \leq 1$ is a solution.

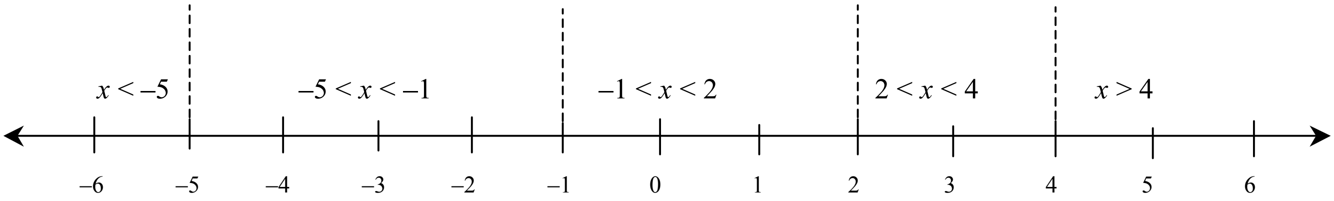
Case 2: If $x < -4$:

$$\begin{aligned} 5(x+1) &< 2(x+4) \\ 5x+5 &< 2x+8 \\ 3x &< 3 \\ x &< 1 \end{aligned}$$

$x < -4$ is within the inequality $x < 1$, so the solution is $x < -4$.

The solution is $x < -4$ or $-1 < x \leq 1$.

d) From the numerator, the zeros occur at $x = 2$ and $x = -1$, so solutions occur at these values of x.
From the denominator, the restrictions occur at $x = 4$ and $x = -5$.
Use a number line to consider the intervals.



Check whether the value of $\frac{(x-2)(x+1)^2}{(x-4)(x+5)} \geq 0$ in each interval is greater or equal to 0.

For $x < -5$, test $x = -6$:

$$\frac{(-6-2)(-6+1)^2}{(-6-4)(-6+5)} = -20 < 0, \ x < -5 \text{ is not part of the solution.}$$

For $-5 < x < -1$, test $x = -3$:

$$\frac{(-3-2)(-3+1)^2}{(-3-4)(-3+5)} = \frac{10}{7} > 0, \ -5 < x < -1 \text{ is part of the solution.}$$

For $-1 < x < 2$, test $x = 0$:

$$\frac{(0-2)(0+1)^2}{(0-4)(0+5)} = \frac{1}{10} > 0, \ -1 < x < 2 \text{ is part of the solution.}$$

Since $x = 2$ and $x = -1$ are solutions, $-1 \leq x \leq 2$.
Note: $-5 < x < -1$ and $-1 < x < 2$ can be combined so $-5 < x \leq 2$ is a solution.

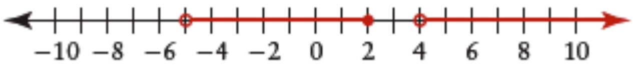
For $2 < x < 4$, test $x = 3$:

$$\frac{(3-2)(3+1)^2}{(3-4)(3+5)} = -2 < 0, \ 2 < x < 4 \text{ is not part of the solution.}$$

For $x > 4$, test $x = 5$:

$$\frac{(5-2)(5+1)^2}{(5-4)(5+5)} = \frac{54}{5} > 0, \ x > 4 \text{ is a solution.}$$

The solution is $-5 < x \leq 2$ or $x > 4$.

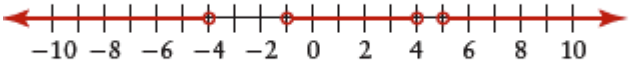


e) $\dfrac{(x-4)(x+4)}{(x-5)(x+1)} > 0$

The zeros occur at $x = 4$ and $x = -4$.
The restrictions occur at $x = 5$ and $x = -1$.

Interval	**Signs of Factors of** $\dfrac{(x-4)(x+4)}{(x-5)(x+1)}$	**Sign of** $\dfrac{(x-4)(x+4)}{(x-5)(x+1)}$
$(-\infty, -4)$	$\dfrac{(-)(-)}{(-)(-)}$	+
-4	$\dfrac{(-)(0)}{(-)(-)}$	0
$(-4, -1)$	$\dfrac{(-)(+)}{(-)(-)}$	–
$(-1, 4)$	$\dfrac{(-)(+)}{(-)(+)}$	+
4	$\dfrac{(0)(+)}{(-)(+)}$	0
$(4, 5)$	$\dfrac{(+)(+)}{(-)(+)}$	–
$(5, \infty)$	$\dfrac{(+)(+)}{(+)(+)}$	+

The solution is $x < -4$ or $-1 < x < 4$ or $x > 5$.



f) $$\frac{(x-2)(x-6)}{x(x-6)}-\frac{x(x-4)}{x(x-6)}<0$$

$$\frac{x^2-8x+12-x^2+4x}{x(x-6)}<0$$

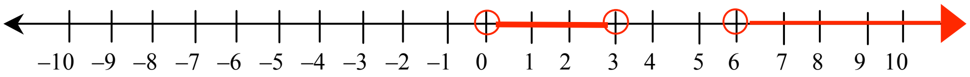
$$\frac{-4x+12}{x(x-6)}<0$$

$$\frac{-4(x-3)}{x(x-6)}<0$$

The zero occurs at $x = 3$.
The restrictions occur at $x = 0$ and $x = 6$.
Critical Values:

x	–1	0	1	3	5	6	7
$\frac{-4(x-3)}{x(x-6)}$	+	∞	–	0	+	∞	–

The solution is $0 < x < 3$ or $x > 6$.



Chapter 3 Section 4 **Question 5 Page 184**

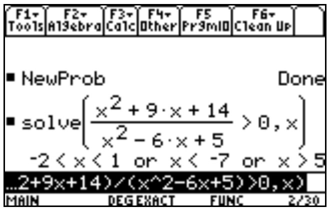
a) $\dfrac{(x+7)(x+2)}{(x-5)(x-1)} > 0$

The zeros are $x = -7$ and $x = -2$.
The restrictions are at $x = 1$ and $x = 5$.

Interval	**Signs of Factors of** $\dfrac{(x+7)(x+2)}{(x-5)(x-1)}$	**Sign of** $\dfrac{(x+7)(x+2)}{(x-5)(x-1)}$
$(-\infty, -7)$	$\dfrac{(-)(-)}{(-)(-)}$	+
-7	$\dfrac{(0)(-)}{(-)(-)}$	0
$(-7, -2)$	$\dfrac{(+)(-)}{(-)(-)}$	–
-2	$\dfrac{(+)(0)}{(-)(-)}$	0
$(-2, 1)$	$\dfrac{(+)(+)}{(-)(-)}$	+
$(1, 5)$	$\dfrac{(+)(+)}{(-)(+)}$	–
$(5, \infty)$	$\dfrac{(+)(+)}{(+)(+)}$	+

The solution is $x < -7$ or $-2 < x < 1$ or $x > 5$.

Check:



```
■ NewProb                              Done
■ solve((x^2 + 9·x + 14)/(x^2 - 6·x + 5) > 0, x)
   -2 < x < 1 or x < -7 or x > 5
…2+9x+14)/(x^2-6x+5)>0,x)
```

b) $\dfrac{(2x-1)(x+3)}{(x+4)^2} < 0$

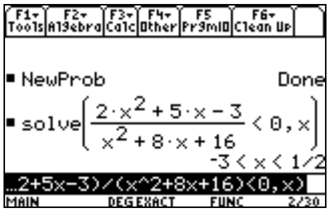
The zeros are $x = –3$ and $x = \dfrac{1}{2}$.

The restriction is at $x = –4$.

Interval	**Signs of Factors of** $\dfrac{(2x-1)(x+3)}{(x+4)^2}$	**Sign of** $\dfrac{(2x-1)(x+3)}{(x+4)^2}$
$(-\infty, -4)$	$\dfrac{(-)(-)}{(+)}$	+
$(-4, -3)$	$\dfrac{(-)(-)}{(+)}$	+
-3	$\dfrac{(-)(0)}{(+)}$	0
$\left(3, \dfrac{1}{2}\right)$	$\dfrac{(-)(+)}{(+)}$	–
$\dfrac{1}{2}$	$\dfrac{(0)(+)}{(+)}$	0
$\left(\dfrac{1}{2}, \infty\right)$	$\dfrac{(+)(+)}{(+)}$	+

The solution is $-3 < x < \dfrac{1}{2}$.

Check:



```
■ NewProb                        Done
■ solve((2·x^2 + 5·x - 3)/(x^2 + 8·x + 16) < 0, x)
                              -3 < x < 1/2
…2+5x-3)/(x^2+8x+16)<0,x)
```

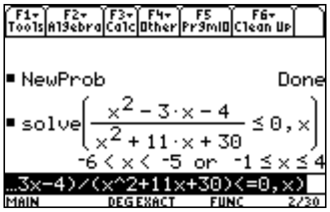
c) $\frac{(x-4)(x+1)}{(x+6)(x+5)} \le 0$

The zeros are $x = -1$ and $x = 4$.
The restrictions are at $x = -6$ and $x = -5$.

Interval	**Signs of Factors of** $\frac{(x-4)(x+1)}{(x+6)(x+5)}$	**Sign of** $\frac{(x-4)(x+1)}{(x+6)(x+5)}$
$(-\infty, -6)$	$\frac{(-)(-)}{(-)(-)}$	+
$(-6, -5)$	$\frac{(-)(-)}{(+)(-)}$	–
$(-5, -1)$	$\frac{(-)(-)}{(+)(+)}$	+
-1	$\frac{(-)(0)}{(+)(+)}$	0
$(-1, 4)$	$\frac{(-)(+)}{(+)(+)}$	–
4	$\frac{(0)(+)}{(+)(+)}$	0
$(4, \infty)$	$\frac{(+)(+)}{(+)(+)}$	+

The solution is $-6 < x < -5$ or $-1 \le x \le 4$.

Check:



```
■ NewProb                          Done
■ solve((x^2 - 3·x - 4)/(x^2 + 11·x + 30) ≤ 0, x)
              -6 < x < -5 or -1 ≤ x ≤ 4
...3x-4)/(x^2+11x+30)<=0,x)
```

d) $\dfrac{(3x-2)(x-2)}{(2x+1)(x-5)} \geq 0$

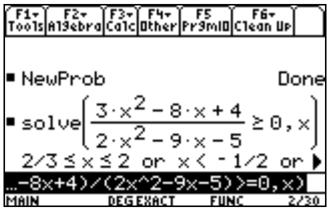
The zeros are $x = \dfrac{2}{3}$ and $x = 2$.

The restrictions are at $x = -\dfrac{1}{2}$ and $x = 5$.

Interval	**Signs of Factors of** $\dfrac{(3x-2)(x-2)}{(2x+1)(x-5)}$	**Sign of** $\dfrac{(3x-2)(x-2)}{(2x+1)(x-5)}$
$\left(-\infty, -\dfrac{1}{2}\right)$	$\dfrac{(-)(-)}{(-)(-)}$	+
$\left(-\dfrac{1}{2}, \dfrac{2}{3}\right)$	$\dfrac{(-)(-)}{(+)(-)}$	−
$\dfrac{2}{3}$	$\dfrac{(0)(-)}{(+)(-)}$	0
$\left(\dfrac{2}{3}, 2\right)$	$\dfrac{(+)(-)}{(+)(-)}$	+
2	$\dfrac{(+)(0)}{(+)(-)}$	0
(2, 5)	$\dfrac{(+)(+)}{(+)(-)}$	−
(5, ∞)	$\dfrac{(+)(+)}{(+)(+)}$	+

The solution is $x < -\dfrac{1}{2}$ or $\dfrac{2}{3} \leq x \leq 2$ or $x > 5$.

Check:

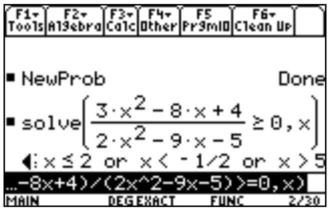


■ NewProb Done

■ solve$\left(\dfrac{3\cdot x^2 - 8\cdot x + 4}{2\cdot x^2 - 9\cdot x - 5} \geq 0, x\right)$

2/3 ≤ x ≤ 2 or x < ⁻1/2 or ▶

…-8x+4)/(2x^2-9x-5))>=0,x)



■ NewProb Done

■ solve$\left(\dfrac{3\cdot x^2 - 8\cdot x + 4}{2\cdot x^2 - 9\cdot x - 5} \geq 0, x\right)$

◀:x ≤ 2 or x < ⁻1/2 or x > 5

…-8x+4)/(2x^2-9x-5))>=0,x)

Chapter 3 Section 4 Question 6 Page 184

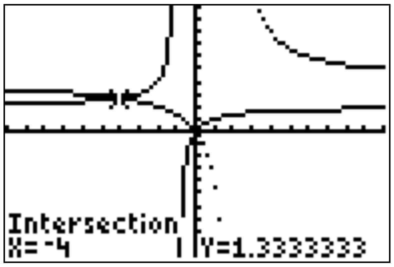
Answers may vary. A sample solution is shown.

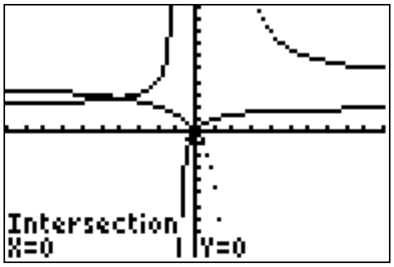
$$\frac{2x-3}{(x-3)(x+5)}=0$$

The restrictions occur at $x = -5$ and $x = 3$.

Chapter 3 Section 4 Question 7 Page 184

f(*x*) is the solid lined graph.
There is an asymptote at $x = -1$ and $x = 2$.





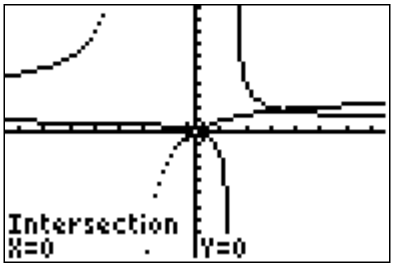
The points of intersection are $\left(-4, \frac{4}{3}\right)$ and (0, 0).
From the graph, the solution is $x < -4$ or $-1 < x < 0$ or $x > 2$.

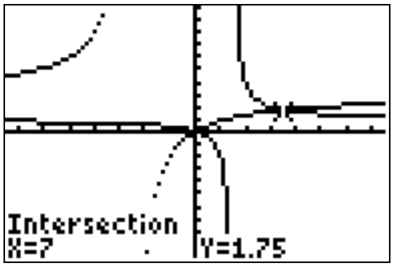
Chapter 3 Section 4 Question 8 Page 184

$f(x)=\frac{x}{x-3}$ is the solid line graph.

$g(x)=\frac{3x}{x+5}$

The asymptotes are at $x = -5$ and $x = 3$.





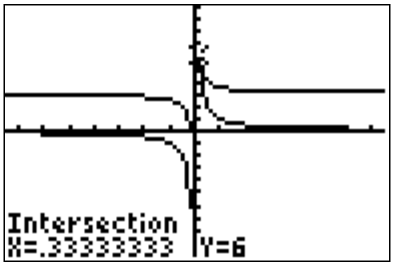
The points of intersection are (0, 0) and (7, 1.75).
From the graph, the solution is $-5 < x < 0$ or $3 < x < 7$.

Chapter 3 Section 4 **Question 9 Page 184**

a) $\frac{1}{x}+3=\frac{2}{x},\ x\neq 0$

$$1+3x=2$$
$$3x=1$$
$$x=\frac{1}{3}$$

Check:



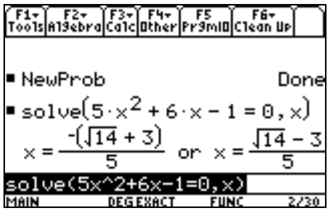
Intersection
X=.33333333 Y=6

The solution is $x=\frac{1}{3}$.

b) $\frac{2}{x+1}+5=\frac{1}{x},\ x\neq -1,\ x\neq 0$

$$\frac{2}{x+1}+\frac{5(x+1)}{x+1}=\frac{1}{x}$$
$$2x+x(5x+5)=x+1$$
$$2x+5x^2+5x-x-1=0$$
$$5x^2+6x-1=0$$
$$x=\frac{-6\pm\sqrt{6^2-4(5)(-1)}}{2(5)}$$
$$x=\frac{-6\pm\sqrt{56}}{10}$$
$$x=\frac{-3\pm\sqrt{14}}{5}$$

Check:



■ NewProb Done

■ solve$(5\cdot x^2+6\cdot x-1=0,x)$

$x=\frac{-(\sqrt{14}+3)}{5}$ or $x=\frac{\sqrt{14}-3}{5}$

solve(5x^2+6x-1=0,x)

The solution is $x=\frac{-3\pm\sqrt{14}}{5}$.

c) $$\frac{12}{x}+x=8,\ x\neq 0$$

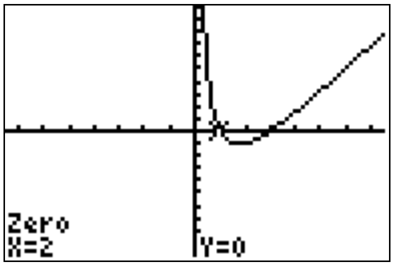
$$12+x^2=8x$$

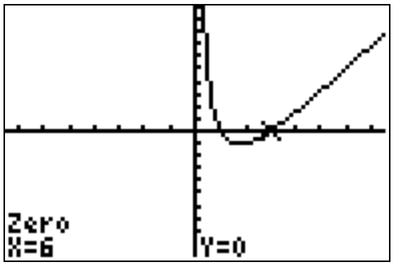
$$x^2-8x+12=0$$

$$(x-6)(x-2)=0$$

$$x=2 \text{ or } x=6$$

Check:





The solution is $x = 2$ or $x = 6$.

d) $$\frac{x}{x-1}=1-\frac{1}{1-x},\ x\neq 1$$

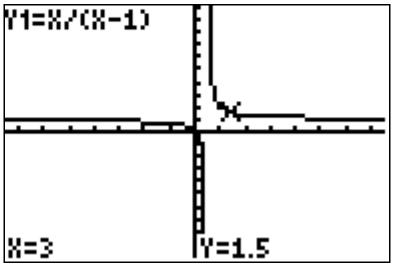
$$\frac{x}{x-1}=1+\frac{1}{x-1}$$

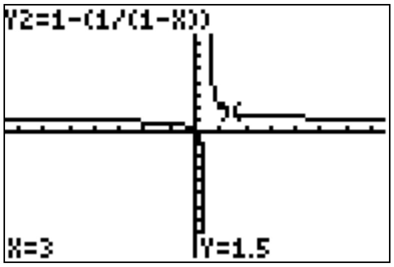
$$x=x-1+1$$

$$0=0$$

True for all values of x except $x = 1$.

Check:





The solution is $\{x \in \mathbb{R}, x \neq 1\}$.

e) $$\frac{2x}{2x+3}-\frac{2x}{2x-3}=1$$

$$2x(2x-3)-2x(2x+3)=(2x-3)(2x+3)$$

$$4x^2-6x-4x^2-6x=4x^2-9$$

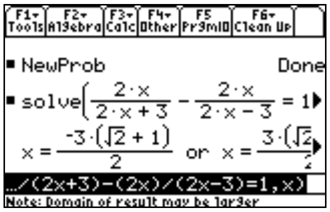
$$4x^2+12x-9=0$$

$$x=\frac{-12\pm\sqrt{12^2-4(4)(-9)}}{2(4)}$$

$$x=\frac{-12\pm\sqrt{288}}{8}$$

$$x=\frac{-3\pm3\sqrt{2}}{2}$$

Check:



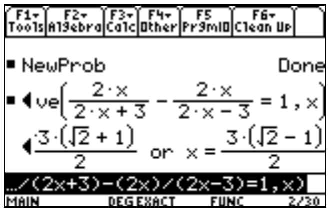
■ NewProb Done

■ solve$\left(\frac{2\cdot x}{2\cdot x+3}-\frac{2\cdot x}{2\cdot x-3}=1\right.$▸

$x=\frac{-3\cdot(\sqrt{2}+1)}{2}$ or $x=\frac{3\cdot(\sqrt{2}}{2}$▸

…/(2x+3)-(2x)/(2x-3)=1,x)

Note: Domain of result may be larger



■ NewProb Done

■ ◂ve$\left(\frac{2\cdot x}{2\cdot x+3}-\frac{2\cdot x}{2\cdot x-3}=1,x\right)$

◂$\frac{3\cdot(\sqrt{2}+1)}{2}$ or $x=\frac{3\cdot(\sqrt{2}-1)}{2}$

…/(2x+3)-(2x)/(2x-3)=1,x)

MAIN DEG EXACT FUNC 2/30

The solution is $x=\frac{-3\pm3\sqrt{2}}{2}$.

f) $$\frac{7}{x-2}-\frac{4}{x-1}+\frac{3}{x+1}=0,\ x\neq-1,\ x\neq1,\ x\neq2$$

$$7(x-1)(x+1)-4(x-2)(x+1)+3(x-2)(x-1)=0$$

$$7(x^2-1)-4(x^2-x-2)+3(x^2-3x+2)=0$$

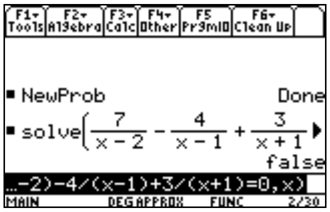
$$7x^2-7-4x^2+4x+8+3x^2-9x+6=0$$

$$6x^2-5x+7=0$$

$$x=\frac{5\pm\sqrt{(-5)^2-4(6)(7)}}{2(6)}$$

$$x=\frac{5\pm\sqrt{-143}}{12}$$

Check:



■ NewProb Done

■ solve$\left(\frac{7}{x-2}-\frac{4}{x-1}+\frac{3}{x+1}\right.$▸

false

…-2)-4/(x-1)+3/(x+1)=0,x)

MAIN DEG APPROX FUNC 2/30

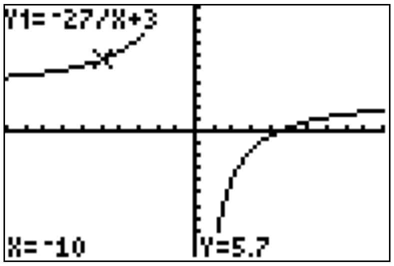
There is no solution.

a) $\frac{2}{x}+3>\frac{29}{x}$

$$\frac{2}{x}-\frac{29}{x}+\frac{3x}{x}>0$$

$$\frac{-27+3x}{x}>0$$

The vertical asymptote is $x = 0$.
The horizontal asymptote is $y = 3$.
The x-intercept is 9.



From the graph, the solution is $x < 0$ or $x > 9$.

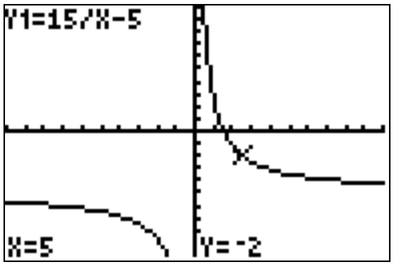
b) $\frac{16}{x}-5<\frac{1}{x}$

$$\frac{16}{x}-\frac{1}{x}-\frac{5x}{x}<0$$

$$\frac{15-5x}{x}<0$$

$$\frac{5(3-x)}{x}<0$$

The vertical asymptote is $x = 0$.
The horizontal asymptote is $y = –5$.
The x-intercept is 3.



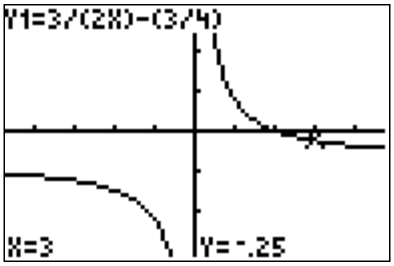
From the graph, the solution is $x < 0$ or $x > 3$.

c)
$$\frac{5}{6x}+\frac{2}{3x}>\frac{3}{4}$$
$$\frac{10}{12x}+\frac{8}{12x}-\frac{9x}{12x}>0$$
$$\frac{18-9x}{12x}>0$$
$$\frac{9(2-x)}{12x}>0$$
$$\frac{3(2-x)}{4x}>0$$

The vertical asymptote is $x = 0$.

The horizontal asymptote is $y=\frac{3}{4}$.

The x-intercept is 2.



Y1=3/(2X)-(3/4)

X=3 Y=-.25

From the graph, the solution is $0 < x < 2$.

d)

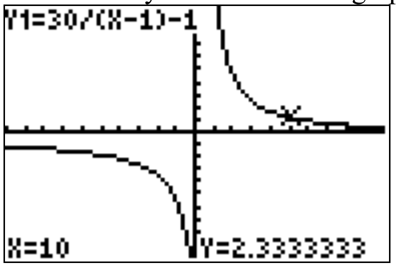
$$6+\frac{30}{x-1}<7$$

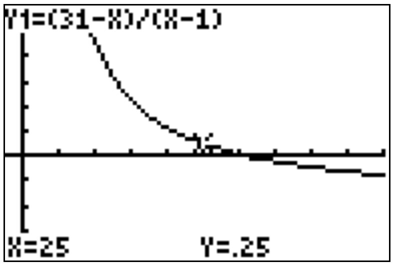
$$6-7+\frac{30}{x-1}<0$$

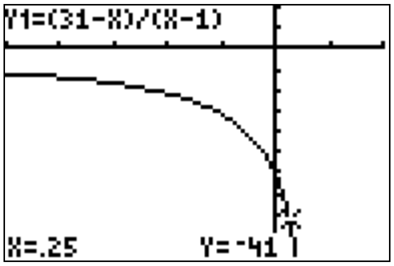
$$\frac{-1(x-1)}{x-1}+\frac{30}{x-1}<0$$

$$\frac{31-x}{x-1}<0$$

The vertical asymptote is $x = 1$.
The horizontal asymptote is $y = -1$.
The x-intercept is 31.
The best way to visualize the graph is to look at it with three different windows.







From the graph, the solution is $x < 1$ or $x > 31$.

$$\begin{aligned}\frac{x+2}{x-5} &> \frac{3}{5}\\ \frac{5(x+2)}{5(x-5)}-\frac{3(x-5)}{5(x-5)} &> 0\\ \frac{5x+10-3x+15}{5(x-5)} &> 0\\ \frac{2x+25}{5(x-5)} &> 0\end{aligned}$$

The zero occurs at $x=-\frac{25}{2}$.

The restriction occurs at $x = 5$.

Interval	**Signs of Factors of** $\frac{2x+25}{5(x-5)}$	**Sign of** $\frac{2x+25}{5(x-5)}$
$\left(-\infty, -\frac{25}{2}\right)$	$\frac{(-)}{(-)}$	+
$-\frac{25}{2}$	$\frac{(0)}{(-)}$	0
$\left(-\frac{25}{2}, 5\right)$	$\frac{(+)}{(-)}$	–
$(5, \infty)$	$\frac{(+)}{(+)}$	+

The solution is $x < -\frac{25}{2}$ or $x > 5$.

$$\begin{aligned}\frac{2x-1}{x+7}&>\frac{x+1}{x+3}\\\frac{(2x-1)(x+3)}{(x+7)(x+3)}-\frac{(x+1)(x+7)}{(x+7)(x+3)}&>0\\\frac{2x^2+5x-3-(x^2+8x+7)}{(x+7)(x+3)}&>0\\\frac{x^2-3x-10}{(x+7)(x+3)}&>0\\\frac{(x-5)(x+2)}{(x+7)(x+3)}&>0\end{aligned}$$

$$\begin{aligned}\frac{2x-1}{x+7}&<\frac{x+1}{x+3}\\\frac{(2x-1)(x+3)}{(x+7)(x+3)}-\frac{(x+1)(x+7)}{(x+7)(x+3)}&<0\\\frac{2x^2+5x-3-(x^2+8x+7)}{(x+7)(x+3)}&<0\\\frac{x^2-3x-10}{(x+7)(x+3)}&<0\\\frac{(x-5)(x+2)}{(x+7)(x+3)}&<0\end{aligned}$$

The zeros occur at $x = -2$ and $x = 5$.
The restrictions occur at $x = -7$ and $x = -3$.

Interval	**Signs of Factors of** $\frac{(x-5)(x+2)}{(x+7)(x+3)}$	**Sign of** $\frac{(x-5)(x+2)}{(x+7)(x+3)}$
$(-\infty, -7)$	$\frac{(-)(-)}{(-)(-)}$	+
$(-7, -3)$	$\frac{(-)(-)}{(+)(-)}$	–
$(-3, -2)$	$\frac{(-)(-)}{(+)(+)}$	+
-2	$\frac{(-)(0)}{(+)(+)}$	0
$(-2, 5)$	$\frac{(-)(+)}{(+)(+)}$	–
5	$\frac{(0)(+)}{(+)(+)}$	0
$(5, \infty)$	$\frac{(+)(+)}{(+)(+)}$	+

For $\frac{(x-5)(x+2)}{(x+7)(x+3)}>0$, the solution is $x < -7$ or $-3 < x < -2$ or $x > 5$.

For $\frac{(x-5)(x+2)}{(x+7)(x+3)}<0$, the solution is $-7 < x < -3$ or $-2 < x < 5$.

Chapter 3 Section 4 Question 13 Page 184

$$\frac{x+1}{x-4} \le \frac{x-3}{x+5}$$

$$\frac{(x+1)(x+5)}{(x-4)(x+5)} - \frac{(x-3)(x-4)}{(x-4)(x+5)} \le 0$$

$$\frac{x^2+6x+5-(x^2-7x+12)}{(x-4)(x+5)} \le 0$$

$$\frac{13x-7}{(x-4)(x+5)} \le 0$$

The zeros occur at $x = \frac{7}{13}$.

The restrictions occur at $x = –5$ and $x = 4$.

$$\frac{x-4}{x+1} \le \frac{x+5}{x-3}$$

$$\frac{(x-4)(x-3)}{(x+1)(x-3)} - \frac{(x+5)(x+1)}{(x+1)(x-3)} \le 0$$

$$\frac{x^2-7x+12-(x^2+6x+5)}{(x+1)(x-3)} \le 0$$

$$\frac{-13x+7}{(x+1)(x-3)} \le 0$$

The zeros occur at $x = \frac{7}{13}$.

The restrictions occur at $x = –1$ and $x = 3$.

First statement:

Interval	**Signs of Factors of** $\frac{13x-7}{(x-4)(x+5)}$	**Sign of** $\frac{13x-7}{(x-4)(x+5)}$
$(-\infty, -5)$	$\frac{(-)}{(-)(-)}$	–
$\left(-5, \frac{7}{13}\right)$	$\frac{(-)}{(-)(+)}$	+
$\frac{7}{13}$	$\frac{(0)}{(-)(+)}$	0
$\left(\frac{7}{13}, 4\right)$	$\frac{(+)}{(-)(+)}$	–
$(4, \infty)$	$\frac{(+)}{(+)(+)}$	+

$\frac{13x-7}{(x-4)(x+5)} \le 0$, the solution is $x < –5$ or $\frac{7}{13} \le x < 4$.

Second statement:

Interval	Signs of Factors of $\frac{-13x+7}{(x+1)(x-3)}$	Sign of $\frac{-13x+7}{(x+1)(x-3)}$
$(-\infty, -1)$	$\frac{(+)}{(-)(-)}$	+
$\left(-1, \frac{7}{13}\right)$	$\frac{(+)}{(+)(-)}$	–
$\frac{7}{13}$	$\frac{(0)}{(+)(-)}$	0
$\left(\frac{7}{13}, 3\right)$	$\frac{(-)}{(+)(-)}$	+
$(3, \infty)$	$\frac{(-)}{(+)(+)}$	–

$\frac{-13x+7}{(x+1)(x-3)} \le 0$, the solution is $-1 < x \le \frac{7}{13}$ or $x > 3$.

Chapter 3 Section 4 **Question 14 Page 184**

a)
$$\left(\frac{1}{a}+\frac{1}{b}\right) \div 2 = \frac{1}{x}$$
$$\frac{1}{2a}+\frac{1}{2b}=\frac{1}{x}$$

b)
$$\begin{aligned}\frac{1}{2(12)}+\frac{1}{2(15)}&=\frac{1}{x}\\ \frac{1}{24}+\frac{1}{30}&=\frac{1}{x}\\ 5x+4x&=120\\ 9x&=120\\ x&=\frac{40}{3}\end{aligned}$$

c) $\frac{1}{2(6)}+\frac{1}{2b}=\frac{1}{1.2}$

$$\frac{1}{12}+\frac{1}{2b}=\frac{5}{6} \quad \text{multiply both sides by } 12b$$

$$b+6=10b$$

$$9b=6$$

$$b=\frac{2}{3}$$

Chapter 3 Section 4 **Question 15 Page 185**

$$k=Id^2$$

$$k=900\times10^2$$

$$k=90\ 000$$

a) i) $I=\frac{90\ 000}{5^2}$

$I=3600$

ii) $I=\frac{90\ 000}{200^2}$

$I=2.25$

b) i) $d^2=\frac{k}{I}$

$d=\sqrt{\frac{k}{I}}$

$d=\sqrt{\frac{90\ 000}{4.5}}$

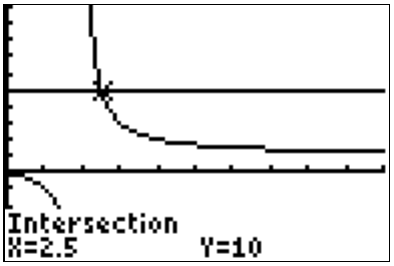
$d \doteq 141.4$

ii) $d\le\sqrt{\frac{90\ 000}{4500}}$

$d\le\sqrt{20}$

$d\le4.5$

Chapter 3 Section 4 **Question 16 Page 185**



```
Intersection
X=2.5         Y=10
```

From the graph:
The point of intersection is (2.5, 10).
The vertical asymptote is $I=2$.
The solution is $2<I<\frac{5}{2}$.

Chapter 3 Section 4 **Question 17 Page 185**

Solutions to Achievement Check questions are provided in the Teacher's Resource.

Chapter 3 Section 4 **Question 18 Page 185**

a) substitute $l = \frac{23}{w}$ into $l + w = 32$

$$\frac{23}{w} + w = 32$$

$$23 + w^2 = 32w$$

$$w^2 - 32w + 23 = 0$$

$$w = \frac{32 \pm \sqrt{(-32)^2 - 4(1)(23)}}{2(1)}$$

$$w = \frac{32 \pm \sqrt{932}}{2}$$

$$w \doteq 31.26 \text{ or } w \doteq 0.74$$

$$l \doteq 0.74 \text{ or } l \doteq 31.26$$

The rectangle's dimensions are approximately 31.26 cm by 0.74 cm.

b)

$$x^2 + y^2 = 1$$

$$xy = \frac{1}{2}$$

$$y = \frac{1}{2x}$$

$$x^2 + \left(\frac{1}{2x}\right)^2 = 1 \text{ substitute } \frac{1}{2x} \text{ for } y$$

$$x^2 + \frac{1}{4x^2} = 1$$

$$4x^4 + 1 = 4x^2$$

$$4x^4 - 4x^2 + 1 = 0$$

$$(2x^2 - 1)^2 = 0$$

$$2x^2 = 1$$

$$x^2 = \frac{1}{2}$$

$$x = \frac{1}{\sqrt{2}} \text{ or } x = -\frac{1}{\sqrt{2}}$$

$$y = \frac{1}{2\left(\frac{1}{\sqrt{2}}\right)} \text{ or } y = -\frac{1}{2\left(\frac{1}{\sqrt{2}}\right)}$$

$$y = \frac{\sqrt{2}}{2} \text{ or } y = -\frac{\sqrt{2}}{2}$$

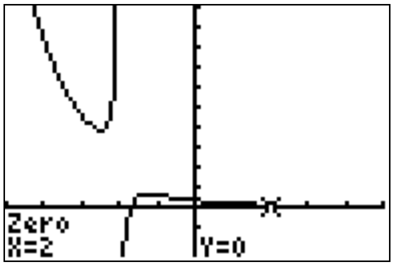
$$y = \frac{\sqrt{2}}{2}, x = \frac{\sqrt{2}}{2} \text{ or } y = \frac{-\sqrt{2}}{2}, x = \frac{-\sqrt{2}}{2}$$

Chapter 3 Section 4 **Question 19 Page 185**

a) $x+2=2^x$

$$10^{x+2}=10^{2^x}$$
$$10^{x+2}=100^x$$
$$10^{x+2}=10^{2x}$$
$$x+2=2x$$
$$x=2$$

Check:

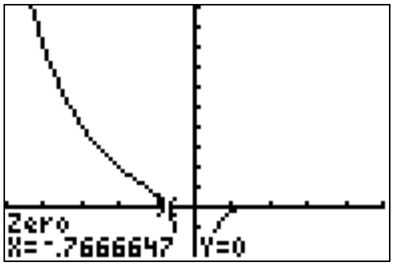


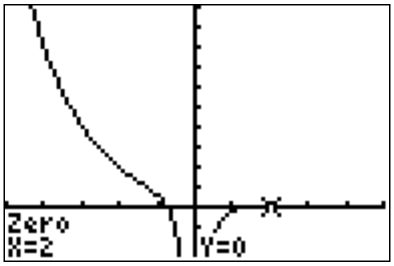
$x=2$

b) $\frac{1}{2^2}=\frac{1}{2^2}$ and $\frac{1}{2^4}=\frac{1}{4^2}$

Look at $\frac{1}{2^x}-\frac{1}{x^2}>0$ to find other points of intersection.

The zeros of the function indicate where it is positive.

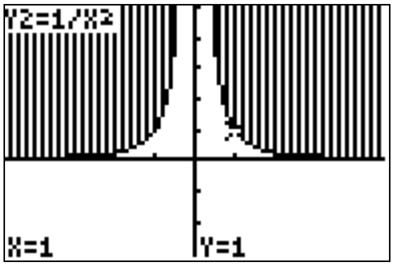




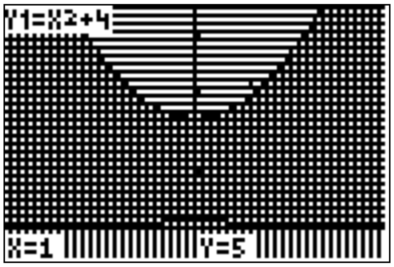
$\frac{1}{2^x}>\frac{1}{x^2}$ for $x<-0.77$, $2<x<4$.

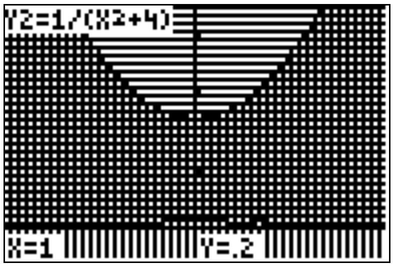
Chapter 3 Section 4 **Question 20 Page 185**

a)



b)





Chapter 3 Section 4 **Question 21 Page 185**

a) $\dfrac{5x+7}{(x+3)(x-1)} = \dfrac{A}{(x+3)} + \dfrac{B}{(x-1)}$

$5x+7 = A(x-1)+B(x+3)$

$5x+7 = Ax-A+Bx+3B$

$5x+7 = (A+B)x+(3B-A)$

$5 = A+B$ and $7 = 3B-A$

$B = 5-A$ substitute into second equation

$7 = 3(5-A)-A$

$7 = 15-3A-A$

$-8 = -4A$

$A = 2 \quad B = 3$

$$\frac{5x+7}{(x+3)(x-1)} = \frac{2}{x+3} + \frac{3}{x-1}$$

b) $\frac{7x+6}{(x-3)(x+2)}=\frac{A}{(x-3)}+\frac{B}{(x+2)}$

$$7x+6=A(x+2)+B(x-3)$$
$$7x+6=Ax+2A+Bx-3B$$
$$7x+6=(A+B)x+(2A-3B)$$
$$7=A+B \text{ and } 6=2A-3B$$

substitute $A=7-B$ into $6=2A-3B$

$$6=2(7-B)-3B$$
$$6=14-2B-3B$$
$$-8=-5B$$
$$B=\frac{8}{5} \qquad A=\frac{27}{5}$$

$$\frac{7x+6}{(x-3)(x+2)}=\frac{27}{5(x-3)}+\frac{8}{5(x+2)}$$

c) $\frac{6x^2-14x-27}{(x+2)(x-3)^2}=\frac{A}{(x+2)}+\frac{B}{(x-3)}+\frac{C}{(x-3)^2}$

$$6x^2-14x-27=A(x-3)^2+B(x+2)(x-3)+C(x+2)$$
$$6x^2-14x-27=A(x^2-6x+9)+B(x^2-x-6)+C(x+2)$$
$$6x^2-14x-27=x^2(A+B)+x(-6A-B+C)+(9A-6B+2C)$$
$$6=A+B \text{ and } -14=-6A-B+C \text{ and } -27=9A-6B+2C$$

Substitute $B=6-A$

$-14=-6A-(6-A)+C\Rightarrow -8=-5A+C \quad (1)$

$-27=9A-6(6-A)+2C\Rightarrow 9=15A+2C \quad (2)$

Subtract (2) from 2(1).

$$-16=-10A+2C$$
$$\underline{9=15A+2C}$$
$$-25=-25A$$

$A=1$ substitute into 2

$$9=15(1)+2C$$
$$C=-3$$
$$B=6-1$$
$$B=5$$

$$\frac{6x^2-14x-27}{(x+2)(x-3)^2}=\frac{1}{x+2}+\frac{5}{x-3}-\frac{3}{(x-3)^2}$$

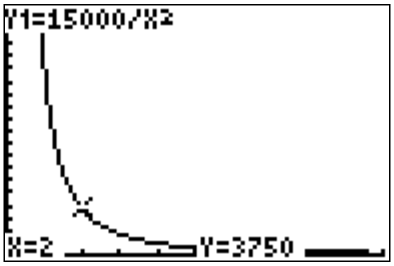
Chapter 3 Section 5 Making Connections With Rational Functions and Equations

Chapter 3 Section 5 Question 1 Page 189

a) $6 = \frac{k}{(50)^2}$

$k = 15\ 000$

$I = \frac{15\ 000}{d^2}$



Parts b) and c) are answered using the graph.

b) The light intensity is less.

c) When d is close to 0, the light intensity is very large.

Chapter 3 Section 5 Question 2 Page 189

a) $V = \frac{k}{P}$

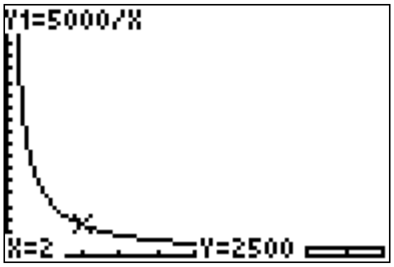
$k = VP$

$k = 10 \times 500$

$k = 5000$

$V = \frac{5000}{P}$

b)



c) From the graph: The volume is halved.

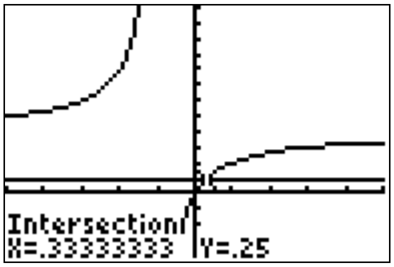
Chapter 3 Section 5 **Question 3 Page 190**

a)
$$
\begin{aligned}
0.5 &= \frac{x}{1+x} \\
0.5(1+x) &= x \\
0.5 + 0.5x - x &= 0 \\
-0.5x &= -0.5 \\
x &= 1
\end{aligned}
$$
The resistance x needs to be 1 Ω, for the total resistance to be 0.5 Ω.

b) $\frac{x}{1+x} < 0.25$

The zero occurs at $x = 0$.

The vertical asymptote occurs at $x = -1$.



Intersection
X=.33333333 Y=.25

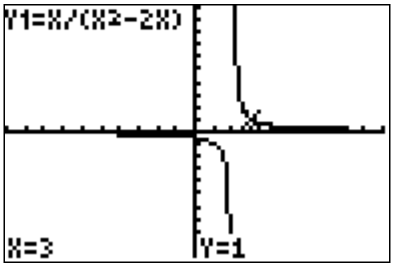
The point of intersection is approximately (0.33, 0.25).

The graph is less than 0.25 between $-1 < x < 0.33$.

The total resistance is less than 0.25 Ω when x is between –1 Ω and approximately 0.33 Ω.

Chapter 3 Section 5 **Question 4 Page 190**

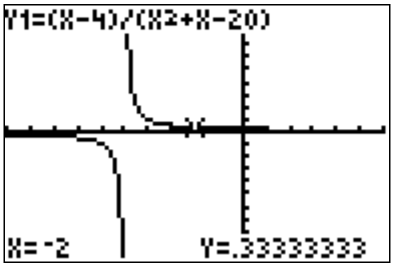
a)



Y1=X/(X²-2X)
X=3 Y=1

discontinuous at $\left(0, -\frac{1}{2}\right)$

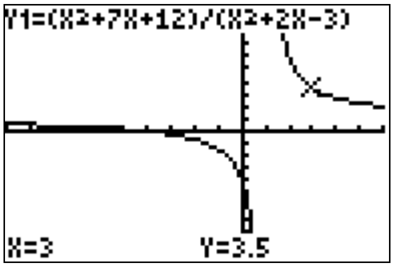
b)



Y1=(X-4)/(X²+X-20)
X=-2 Y=.33333333

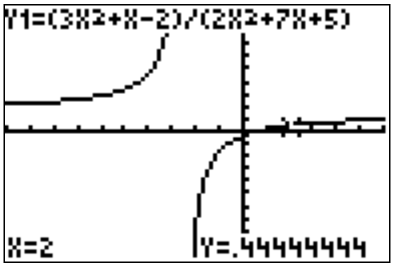
discontinuous at $\left(4, \frac{1}{9}\right)$

c)



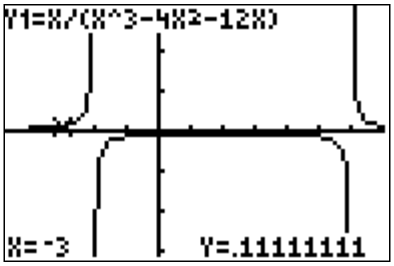
discontinuous at $\left(-3, -\frac{1}{4}\right)$

d)



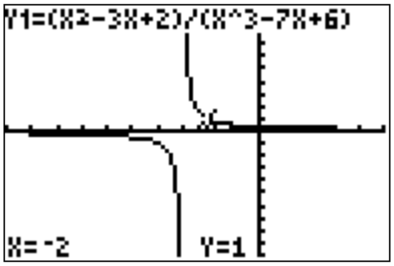
discontinuous at $\left(-1, -\frac{5}{3}\right)$

e)



discontinuous at $\left(0, -\frac{1}{12}\right)$

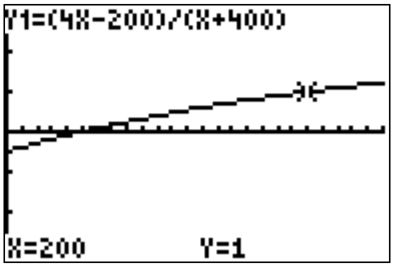
f)



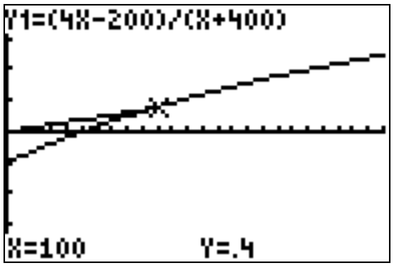
discontinuous at $\left(1, \frac{1}{4}\right)$ and $\left(2, \frac{1}{5}\right)$

Chapter 3 Section 5 **Question 5 Page 190**

a)



b)



b) Answers may vary. A sample solution is shown.

Average profit is modelled by $\frac{P(x)}{x}$ = slope of secant.

See the graph above for an example.

c) slope secant = $\dfrac{P(x)}{x}$

test (100, 0.4); slope = 0.004
test (150, 0.73); slope $\doteq$ 0.0048
test (200, 1); slope = 0.005
test (250, 1.23); slope $\doteq$ 0.0049
The average profit is the greatest when $x = 200$.

d)

x	y	**Slope of Secant with (1000, 2.714 285 7)**
999.9	2.714 193 9	0.000 918

The rate of change of the profit at a sales level of 1000 kg is 9.18×10^{-4}.

Chapter 3 Section 5 Question 6 Page 190

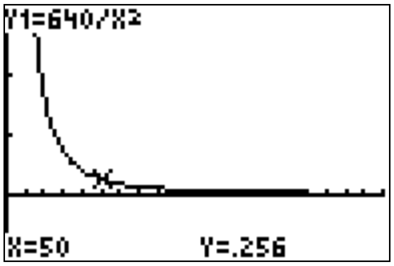
a) $k = \dfrac{Rd^2}{l}$

$k = \dfrac{40(4)^2}{1000}$

$k = 0.64$

$R = \dfrac{0.64l}{d^2}$

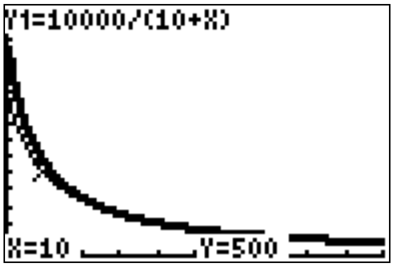
b) $R = \dfrac{640}{d^2}$

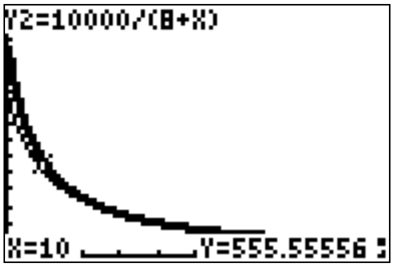


Chapter 3 Section 5 **Question 7 Page 190**

Answers may vary. A sample solution is shown.

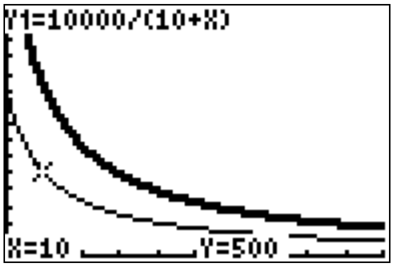
a)

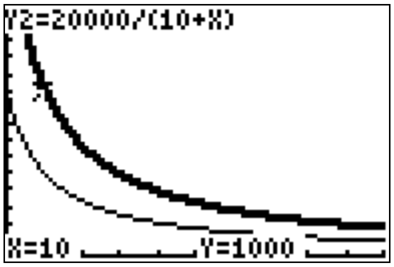




The cost is just slightly greater per person than the original model. The cost decreases at a greater rate at first.

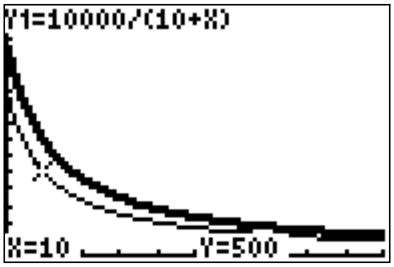
b)

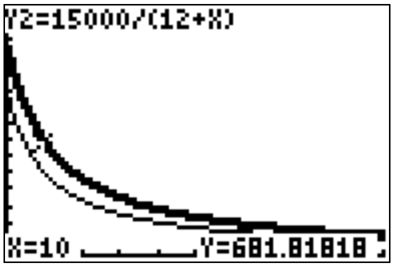




The cost is much greater per person. The gap between the graphs decreases as the number of passengers increases. The cost decreases at a slower rate.

c)

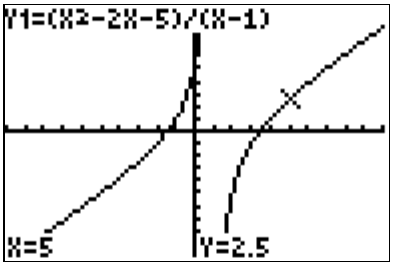




The cost per person is greater. As the number of passengers increase, the cost per person decreases and the graphs get closer. The cost decreases at a slightly slower rate.

Chapter 3 Section 5 **Question 8 Page 190**

a)



b) a slanting asymptote

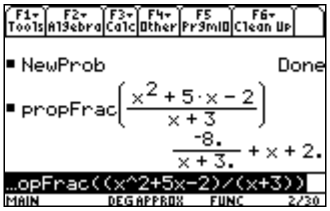
c)
$$\begin{array}{r}
x-1 \\
x-1\overline{\smash{\big)}\,x^2-2x-5} \\
\underline{x^2-x} \\
-x-5 \\
\underline{-x+1} \\
-6
\end{array}$$

$$\frac{x^2-2x-5}{x-1}=x-1-\frac{6}{x-1}$$

d) The equation of the oblique asymptote is the quotient; $y = x - 1$

Chapter 3 Section 5 **Question 9 Page 191**

a)



■ NewProb Done

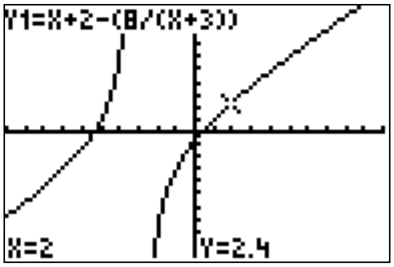
■ propFrac$\left(\frac{x^2+5\cdot x-2}{x+3}\right)$

$\frac{-8.}{x+3.}+x+2.$

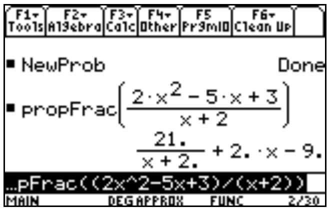
...opFrac((x^2+5x-2)/(x+3))

MAIN DEG APPROX FUNC 2/30

Oblique asymptote occurs at $y = x + 2$.



b)



■ NewProb Done

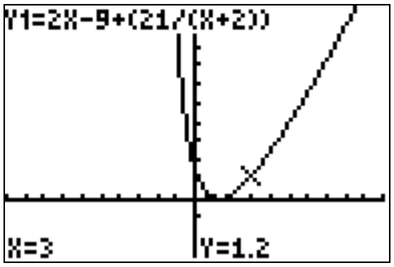
■ propFrac$\left(\frac{2\cdot x^2-5\cdot x+3}{x+2}\right)$

$\frac{21.}{x+2.}+2.\cdot x-9.$

...pFrac((2x^2-5x+3)/(x+2))

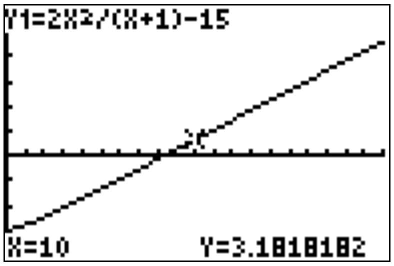
MAIN DEG APPROX FUNC 2/30

Oblique asymptote occurs at $y = 2x - 9$.

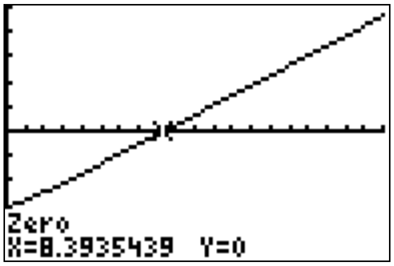


Chapter 3 Section 5 Question 10 Page 191

a)

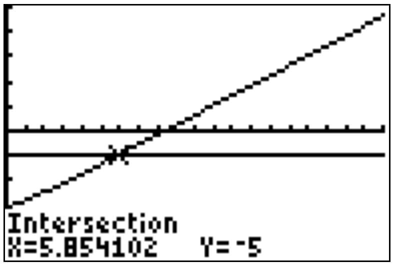


b)



b) approximately 8.39 h

c)



approximately 5.85 hours

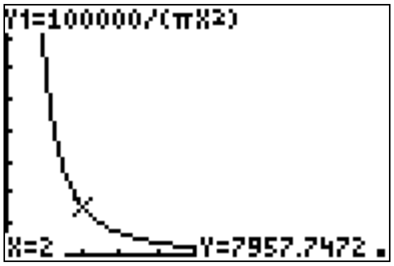
Chapter 3 Section 5 Question 11 Page 191

a) $V = \pi r^2 h$

$$h = \frac{V}{\pi r^2}$$

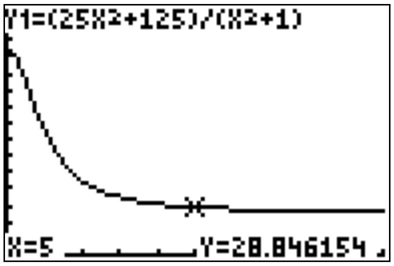
$$h = \frac{100\ 000}{\pi \times r^2}$$

b)



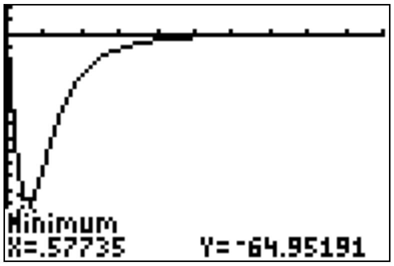
Chapter 3 Section 5 **Question 12 Page 191**

a)



b) The systolic pressure decreases and gets closer to 25.

c)

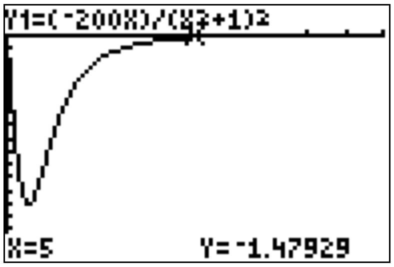


The rate of change decreases until $t \doteq 0.58$ s and then increases gradually, getting closer to 0.

d) Rate of change of $P(t)$.
Approximate slope of the tangent at $t = 5$ s.

$$\frac{28.846154 - 28.847634}{5 - 4.999} = -1.48$$

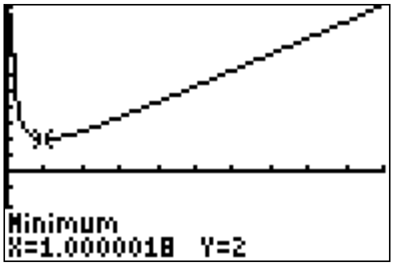
Rate of change of $R(t)$ at $t = 5$ s:



From the graph; approximately –1.48

The rate of change of $R(t)$ and $P(t)$ at $t = 5$ s is –1.48.

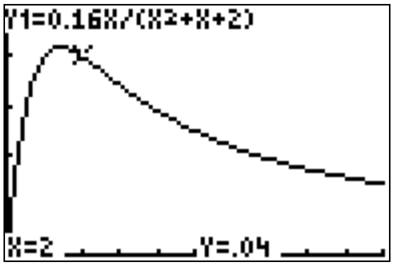
Chapter 3 Section 5 **Question 13 Page 191**



minimum sum at $x = 1$

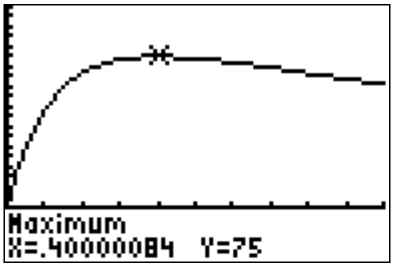
Chapter 3 Section 5 **Question 14 Page 191**

a)



b) The curve increases to reach a maximum concentration of $C = 0.0418$ mg/cm^3 when $t \doteq 1.414$ min and then gradually decreases to C as time increases close to 0.

Chapter 3 Section 5 **Question 15 Page 191**



Increasing for $0 < R < 0.40$.

Chapter 3 Section 5 **Question 16 Page 191**

$$\begin{aligned}\text{For } n = 2;\ g(x) &= \frac{x^2-4}{x-2},\ x \neq 2\\ &= x + 2\end{aligned}$$

False for $n = 2$ because the function is discontinuous at the point (2, 4).

Chapter 3 Section 5 **Question 17 Page 191**

A

One of the zeros of the function is 1.

$$\begin{aligned} f(1) &= 5(1)^4 + 4(1)^3 + 3(1)^2 + P(1) + Q \\ 0 &= 5 + 4 + 3 + P + Q \\ -12 &= P + Q \end{aligned}$$

Chapter 3 Section 5 **Question 18 Page 191**

Answers may vary. A sample solution is shown.

$$\begin{aligned} (\sqrt{3}\sin x)^2 &= (2 - \cos x)^2 \\ 3\sin^2 x &= 4 - 4\cos x + \cos^2 x \\ 3(1 - \cos^2 x) &= 4 - 4\cos x + \cos^2 x \\ 3 - 3\cos^2 x - \cos^2 x + 4\cos x - 4 &= 0 \\ -4\cos^2 x + 4\cos x - 1 &= 0 \\ (2\cos x - 1)^2 &= 0 \\ \cos x &= \frac{1}{2} \end{aligned}$$

$$x = \frac{\pi}{3} + 2k\pi,\ k = 0,\ \pm1,\ \pm2,\ \pm3,\ ...$$

$$\left(\text{i.e., ..., } -\frac{5\pi}{3}, \frac{\pi}{3}, \frac{7\pi}{3}, ...\right)$$

Chapter 3 Review

Chapter 3 Review **Question 1 Page 192**

a) From the denominator, the vertical asymptote occurs at $x = 2$.

$$\frac{\frac{1}{x}}{\frac{x}{x}-\frac{2}{x}}, \text{ as } x \to \pm\infty, \frac{1}{x} \text{ and } \frac{2}{x} \text{ approach 0.}$$

$$= \frac{0}{1-0}$$

$$= 0$$

The horizontal asymptote is $y = 0$.

b) From the denominator, the vertical asymptote occurs at $x = -7$.

$$\frac{\frac{3}{x}}{\frac{x}{x}+\frac{7}{x}} \text{ as } x \to \pm\infty, \frac{3}{x} \text{ and } \frac{7}{x} \text{ approach 0.}$$

$$= \frac{0}{1+7}$$

$$= 0$$

$$= \frac{0}{1+0}$$

$$= 0$$

The horizontal asymptote is $y = 0$.

c) From the denominator, the vertical asymptote occurs at $x = 5$.

$$-\frac{\frac{4}{x}}{\frac{x}{x}-\frac{5}{x}} \text{ as } x \to \pm\infty, \frac{4}{x} \text{ and } \frac{5}{x} \text{ approach 0.}$$

$$= -\frac{0}{1-0}$$

$$= 0$$

The horizontal asymptote is $y = 0$.

Chapter 3 Review **Question 2 Page 192**

a) From the shape we know that this is a rational function.
The vertical asymptote occurs at $x = 1$.
The horizontal asymptote occurs at $y = 0$
There are no zeros.
A point on the graph is (2, 2).

$$y = \frac{a}{(x-1)}$$

$$2 = \frac{a}{1}$$

$$a = 2$$

$$y = \frac{2}{x-1}$$

b) The vertical asymptote is $x = -4$.
The horizontal asymptote is $y = 0$.
There are no zeros.
A point on the graph is (–3, 1).

$$y = \frac{a}{x+4}$$

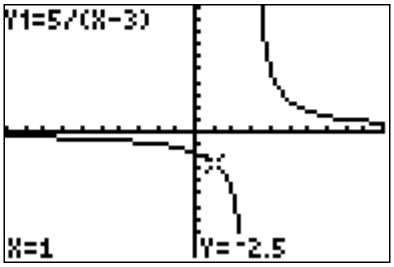
$$1 = \frac{a}{1}$$

$$a = 1$$

$$y = \frac{1}{x+4}$$

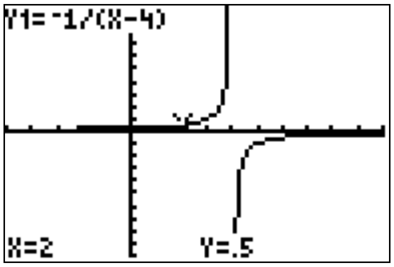
Chapter 3 Review **Question 3 Page 192**

a)



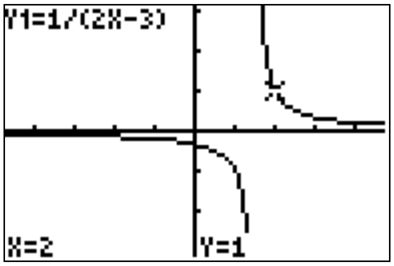
domain: $\{x \in \mathbb{R}, x \neq 3\}$
range: $\{y \in \mathbb{R}, y \neq 0\}$
y-intercept: $-\frac{5}{3}$
asymptotes: $x = 3, y = 0$

b)



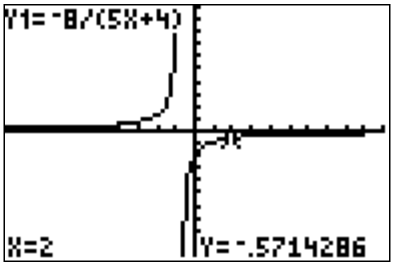
domain: $\{x \in \mathbb{R}, x \neq 4\}$
range: $\{y \in \mathbb{R}, y \neq 0\}$
y-intercept: $\frac{1}{4}$
asymptotes: $x = 4$, $y = 0$

c)



domain: $\{x \in \mathbb{R}, x \neq \frac{3}{2}\}$
range: $\{y \in \mathbb{R}, y \neq 0\}$
y-intercept: $-\frac{1}{3}$
asymptotes: $x = \frac{3}{2}$, $y = 0$

d)



domain: $\{x \in \mathbb{R}, x \neq -\frac{4}{5}\}$
range: $\{y \in \mathbb{R}, y \neq 0\}$
y-intercept: –2
asymptotes: $x = -\frac{4}{5}$, $y = 0$

Chapter 3 Review **Question 4 Page 192**

a) vertical asymptotes: $x = 3, x = –4$ domain: $\{x \in \mathbb{R}, x \neq –4, x \neq 3\}$

b) vertical asymptote: $x = –3$ domain: $\{x \in \mathbb{R}, x \neq –3\}$

c) vertical asymptotes: $x = –6, x = –2$ domain: $\{x \in \mathbb{R}, x \neq –6, x \neq 2\}$

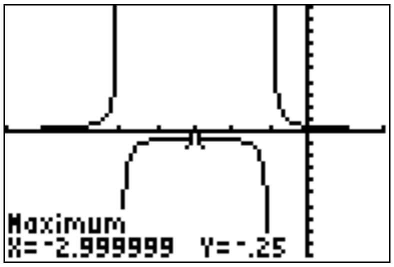
Chapter 3 Review **Question 5 Page 192**

a) **i)** $f(x) = \dfrac{1}{(x+5)(x+1)}$

The asymptotes are $x = –6, x = –2, y = 0$.

ii) y-intercept is $\dfrac{1}{5}$

iii)



```
Maximum
X=-2.999999  Y=-.25
```

iv) function increasing for $x < –5, –5 < x < –3$
function decreasing for $–3 < x < –1, x > –1$

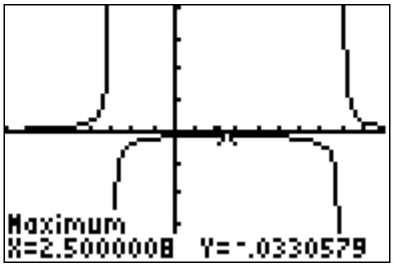
v) domain: $\{x \in \mathbb{R}, x \neq –5, x \neq –1\}$
range: $\{y \in \mathbb{R}, y > 0, y \leq -\dfrac{1}{4}\}$

b) i) $g(x) = \dfrac{1}{(x-8)(x+3)}$

The asymptotes are $x = -3$, $x = 8$, $y = 0$.

ii) y-intercept is $-\dfrac{1}{24}$

iii)



iv) function increasing for $x < -3$, $-3 < x < 2.5$
function decreasing for $2.5 < x < 8$, $x > 8$

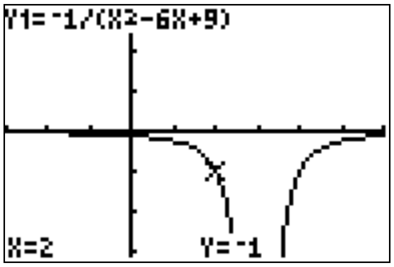
v) domain: $\{x \in \mathbb{R}, x \neq 8, x \neq -3\}$
range: $\{y \in \mathbb{R}, y > 0, y \leq -\dfrac{4}{121}\}$

c) i) $h(x) = -\dfrac{1}{(x-3)^2}$

The asymptotes are $x = 3$, $y = 0$.

ii) y-intercept is $-\dfrac{1}{9}$

iii)



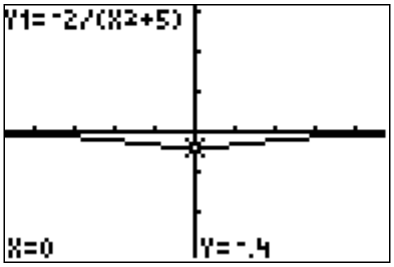
iv) function increasing for $x > 3$
function decreasing for $x < 3$

v) domain: $\{x \in \mathbb{R}, x \neq 3\}$
range: $\{y \in \mathbb{R}, y < 0\}$

d) i) asymptote: $y = 0$

ii) y-intercept is $-\frac{2}{5}$

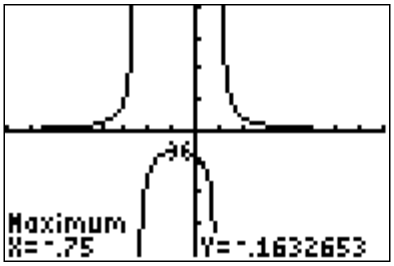
iii)



iv) function increasing for $x > 0$
function decreasing for $x < 0$

v) domain: $\{x \in \mathbb{R}\}$
range: $\{y \in \mathbb{R}, -\frac{2}{5} \le y < 0\}$

Chapter 3 Review **Question 6 Page 192**

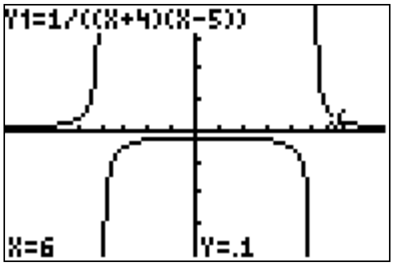


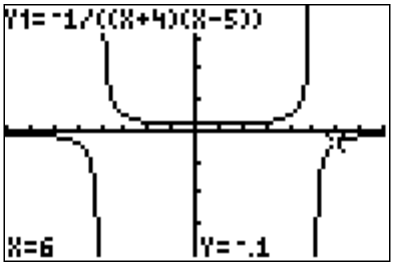
Interval	Sign of Slope	Change in Slope
$x < -\frac{5}{2}$	+	+
$-\frac{5}{2} < x < -\frac{3}{4}$	+	–
$x = -\frac{3}{4}$	0	–
$-\frac{3}{4} < x < 1$	–	–
$x > 1$	–	+

Chapter 3 Review Question 7 Page 192

A reciprocal quadratic function with the given asymptotes is an equation of the form.

$$y = \pm\frac{1}{(x+4)(x-5)}$$





By graphing we can see that the equation that satisfies the interval conditions is:

$$y = -\frac{1}{(x+4)(x-5)}$$

Chapter 3 Review Question 8 Page 192

a) $$a(x) = \frac{\frac{x}{x}}{\frac{x}{x}+\frac{5}{x}}$$

$$a(x) = \frac{1}{1+\frac{5}{x}}$$

As $x \to \pm\infty$, $\frac{5}{x}$ gets very close to 0.

$$a(x) \to \frac{1}{1+0}$$

$$a(x) \to 1$$

The horizontal asymptote has equation $y = 1$.

b) $$b(x) = -\frac{\frac{2x}{x}}{\frac{x}{x} - \frac{3}{x}}$$

$$b(x) = -\frac{2}{1 - \frac{3}{x}}$$

As $x \to \pm\infty$, $\frac{3}{x}$ gets very close to 0.

$$b(x) \to -\frac{2}{1-0}$$
$$b(x) \to -2$$

The horizontal asymptote has equation $y = -2$.

c) $$c(x) = \frac{\frac{x}{x} + \frac{2}{x}}{\frac{x}{x} - \frac{2}{x}}$$

$$c(x) = \frac{1 + \frac{2}{x}}{1 - \frac{2}{x}}$$

As $x \to \pm\infty$, $\frac{2}{x}$ gets very close to 0.

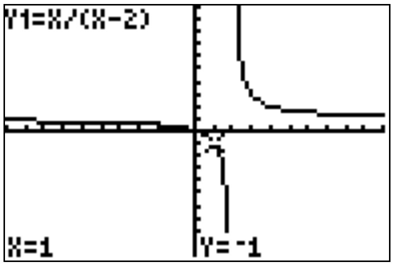
$$c(x) \to \frac{1+0}{1-0}$$
$$c(x) \to 1$$

The horizontal asymptote has equation $y = 1$.

Chapter 3 Review　　　　Question 9 Page 193

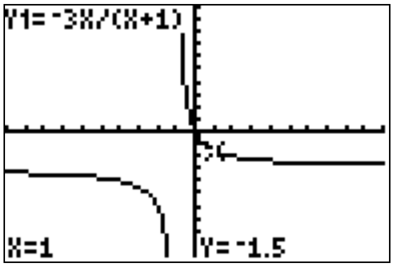
a) asymptotes: $x = 2$, $y = 1$
domain: $\{x \in \mathbb{R}, x \neq 2\}$, range: $\{y \in \mathbb{R}, y \neq 1\}$
y-intercept: 0

$x < 0$	$f(x)$ is positive and decreasing, the slope is negative and decreasing.
$0 < x < 2$	$f(x)$ is negative and decreasing, the slope is negative and decreasing.
$x > 2$	$f(x)$ is positive and decreasing, the slope is negative and increasing.



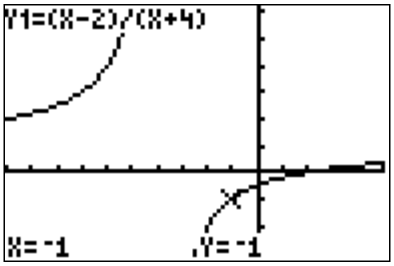
b) asymptotes: $x = -3$, $y = -1$
domain: $\{x \in \mathbb{R}, x \neq -1\}$, range: $\{y \in \mathbb{R}, y \neq -3\}$
y-intercept: 0

$x < -1$	$f(x)$ is negative and decreasing, the slope is negative and decreasing.
$-1 < x < 0$	$f(x)$ is positive and decreasing, the slope is negative and increasing.
$x > 0$	$f(x)$ is negative and decreasing, the slope is negative and increasing.



c) asymptotes: $x = -4$, $y = 1$
domain: $\{x \in \mathbb{R}, x \neq -4\}$, range: $\{y \in \mathbb{R}, y \neq 1\}$
y-intercept: $-\frac{1}{2}$, x-intercept: 2

$x < -4$	$f(x)$ is positive and increasing, the slope is positive and increasing.
$-4 < x < 2$	$f(x)$ is negative and increasing, the slope is positive and decreasing.
$x > 2$	$f(x)$ is positive and increasing, the slope is positive and decreasing.

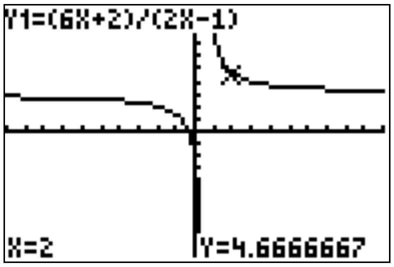


d) asymptotes: $x = \frac{1}{2}$, $y = 3$

domain: $\{x \in \mathbb{R}, x \neq \frac{1}{2}\}$, range: $\{y \in \mathbb{R}, y \neq 3\}$

y-intercept: –2, x-intercept: $-\frac{1}{3}$

$x < -\frac{1}{3}$	$f(x)$ is positive and decreasing, the slope is negative and decreasing.
$-\frac{1}{3} < x < \frac{1}{2}$	$f(x)$ is negative and decreasing, the slope is negative and decreasing.
$x > \frac{1}{2}$	$f(x)$ is positive and decreasing, the slope is negative and increasing.



Chapter 3 Review **Question 10 Page 193**

From the asymptotes:

$$f(x)=\frac{4x+b}{3x+2}$$

Substitute the y-intercept:

$$-\frac{1}{2}=\frac{4(0)+b}{3(0)+2}$$

$$-\frac{1}{2}=\frac{b}{2}$$

$$b=-1$$

$$f(x)=\frac{4x-1}{3x+2}$$

Check the x-intercept:

L.S.	**R.S.**
$=0$	$=\dfrac{4\left(\frac{1}{4}\right)-1}{3\left(\frac{1}{4}\right)+2}$ $=0$

Since **L.S.** = **R.S.**, the equation is true for the x-intercept.

Chapter 3 Review **Question 11 Page 193**

a) $\dfrac{7}{x-4}=2,\ x\neq 4$

$$7=2x-8$$

$$2x=15$$

$$x=\frac{15}{2}$$

b) $\dfrac{3}{x^2+6x-24}=1$

$$3=x^2+6x-24$$

$$x^2+6x-27=0$$

$$(x+9)(x-3)=0$$

$$x=-9 \text{ or } x=3$$

$$x^2+6x-24\neq 0$$

$$x\neq\frac{-6\pm\sqrt{6^2-4(1)(-24)}}{2(1)}$$

$$x\neq -3\pm\sqrt{33}$$

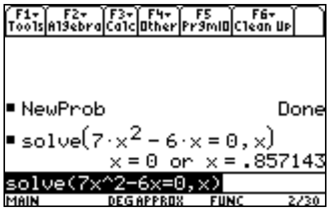
Chapter 3 Review **Question 12 Page 193**

a) $$\frac{4x}{x+2}=\frac{5x}{3x+1},\ x\neq -2,\ x\neq -\frac{1}{3}$$

$$4x(3x+1)=5x(x+2)$$

$$12x^2+4x=5x^2+10x$$

$$7x^2-6x=0$$



```
■ NewProb                    Done
■ solve(7·x^2 - 6·x = 0, x)
          x = 0 or x = .857143
solve(7x^2-6x=0,x)
```

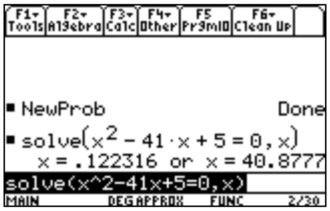
$x=0$ or $x \doteq 0.86$

b) $$\frac{5x+2}{2x-9}=\frac{3x-1}{x+2},\ x\neq \frac{9}{2},\ x\neq -2$$

$$(5x+2)(x+2)=(3x-1)(2x-9)$$

$$5x^2+12x+4=6x^2-29x+9$$

$$x^2-41x+5=0$$



```
■ NewProb                    Done
■ solve(x^2 - 41·x + 5 = 0, x)
   x = .122316 or x = 40.8777
solve(x^2-41x+5=0,x)
```

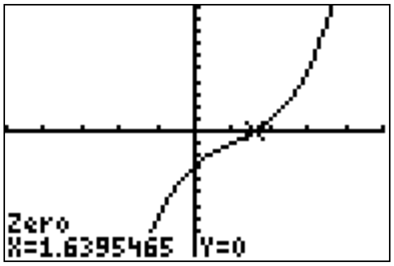
$x \doteq 0.12$ or $x \doteq 40.88$

c) $$\frac{(x^2-3x+1)}{2-x}=\frac{x^2+5x+4}{x-6},\ x\neq 2,\ x\neq 6$$

$$(x^2-3x+1)(x-6)=(x^2+5x+4)(2-x)$$

$$x^3-6x^2-3x^2+18x+x-6=2x^2-x^3+10x-5x^2+8-4x$$

$$2x^3-6x^2+13x-14=0$$



$x \doteq 1.64$

a) $\dfrac{3}{x+5} < 2$

Because $x + 5 \neq 0$, either $x > -5$ or $x < -5$.

Case 1: If $x > -5$,

$3 < 2(x+5)$

$3 < 2x + 10$

$2x > -7$

$x > -\dfrac{7}{2}$

$x > -5$ is within $x > -\dfrac{7}{2}$, so the solution is $x > -\dfrac{7}{2}$.

Case 2: If $x < -5$,

$3 > 2(x+5)$

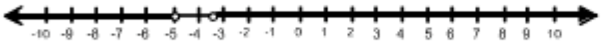
$3 > 2x + 10$

$2x > -7$

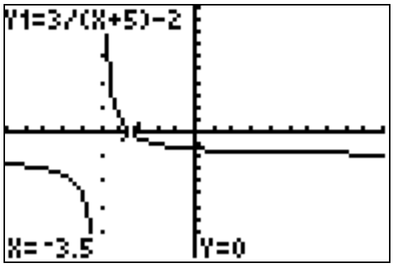
$x < -\dfrac{7}{2}$

$x < -\dfrac{7}{2}$ is within $x < -5$, so the solution is $x < -5$.

The solution is $x < -5$ or $x > -\dfrac{7}{2}$.



Check:



b) $\dfrac{3}{x+2} \le \dfrac{4}{x+3}$

$$\frac{3(x+3)}{(x+2)(x+3)} - \frac{4(x+2)}{(x+2)(x+3)} \le 0$$

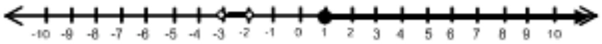
$$\frac{3x+9-4x-8}{(x+2)(x+3)} \le 0$$

$$\frac{-x+1}{(x+2)(x+3)} \le 0$$

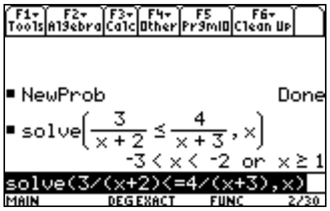
The zero occurs at $x = 1$.
The restrictions occur at $x = -3$, $x = -2$.

Interval	**Signs of Factors of** $\frac{1-x}{(x+2)(x+3)}$	**Sign of** $\frac{1-x}{(x+2)(x+3)}$
$(-\infty, -3)$	$\frac{(+)}{(-)(-)}$	+
$(-3, -2)$	$\frac{+}{(-)(+)}$	–
$(-2, 1)$	$\frac{+}{(+)(+)}$	+
1	$\frac{0}{(+)(+)}$	0
$(1, \infty)$	$\frac{-}{(+)(+)}$	–

The solution is $-3 < x < -2$ or $x \ge 1$.



Check:



■ NewProb Done

■ solve$\left(\frac{3}{x+2} \le \frac{4}{x+3}, x\right)$

$-3 < x < -2$ or $x \ge 1$

```
solve(3/(x+2)<=4/(x+3),x)
MAIN          DEG EXACT     FUNC      2/30
```

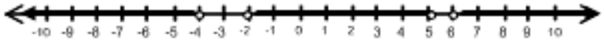
c) $\dfrac{(x-5)(x+4)}{(x-6)(x+2)} > 0$

The zeros are $x = –4$, $x = 5$.
The restrictions occur at $x = –2$, $x = 6$.

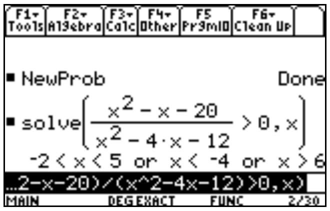
Critical Values in bold:

x	–5	**–4**	–3	**–2**	0	**5**	5.5	**6**	7
$\dfrac{(x-5)(x+4)}{(x-6)(x+2)}$	+	0	–	∞	+	0	–	∞	+

The solution is $x < –4$ or $–2 < x < 5$ or $x > 6$.



Check:



```
■ NewProb                          Done
■ solve((x^2 - x - 20)/(x^2 - 4·x - 12) > 0, x)
  -2 < x < 5 or x < -4 or x > 6
…2-x-20)/(x^2-4x-12)>0,x)
MAIN        DEG EXACT    FUNC    2/30
```

d) $\dfrac{x(x+7)}{(x+5)(x+7)} - \dfrac{(x-1)(x+5)}{(x+5)(x+7)} > 0$

$$\frac{x^2+7x-(x^2+4x-5)}{(x+5)(x+7)} > 0$$

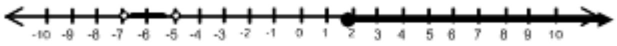
$$\frac{3x+5}{(x+5)(x+7)} > 0$$

The zero occurs at $x = -\frac{5}{3}$.

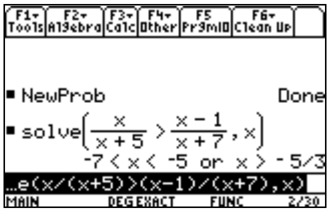
The restrictions occur at $x = -7$, $x = -5$.

Interval	**Signs of Factors of** $\frac{3x+5}{(x+5)(x+7)}$	**Sign of** $\frac{3x+5}{(x+5)(x+7)}$
$-\infty, -7$	$\frac{(-)}{(-)(-)}$	$-$
$(-7, -5)$	$\frac{(-)}{(-)(+)}$	$+$
$\left(-5, -\frac{5}{3}\right)$	$\frac{(-)}{(+)(+)}$	$-$
$-\frac{5}{3}$	$\frac{(0)}{(+)(+)}$	0
$\left(-\frac{5}{3}, \infty\right)$	$\frac{(+)}{(+)(+)}$	$+$

The solution is $-7 < x < -5$ or $x > -\frac{5}{3}$.



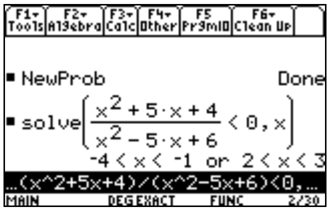
Check:



```
■ NewProb                    Done
■ solve(x/(x+5) > (x-1)/(x+7), x)
        -7 < x < -5 or x > -5/3
…e(x/(x+5))(x-1)/(x+7),x)
```

Chapter 3 Review Question 14 Page 193

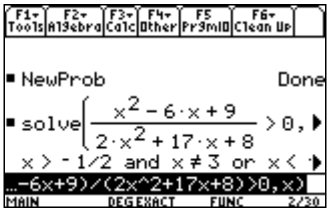
a)



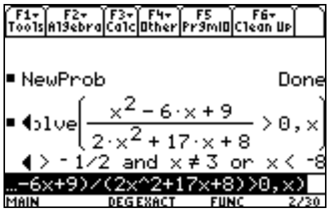
```
▪ NewProb                    Done
▪ solve((x^2 + 5·x + 4)/(x^2 - 5·x + 6) < 0, x)
      -4 < x < -1 or 2 < x < 3
…(x^2+5x+4)/(x^2-5x+6)<0,…
MAIN       DEG EXACT     FUNC     2/30
```

The solution is –4 < x < –1 or 2 < x < 3.

b)



```
▪ NewProb                    Done
▪ solve((x^2 - 6·x + 9)/(2·x^2 + 17·x + 8) > 0, ▶
  x > - 1/2 and x ≠ 3 or x < ▶
…-6x+9)/(2x^2+17x+8)>0,x)
MAIN       DEG EXACT     FUNC     2/30
```

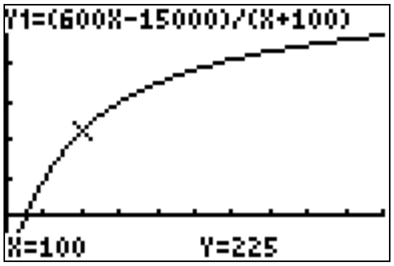


```
▪ NewProb                    Done
▪ ◀ɔlve((x^2 - 6·x + 9)/(2·x^2 + 17·x + 8) > 0, x)
  ◀ > - 1/2 and x ≠ 3 or x < -8
…-6x+9)/(2x^2+17x+8)>0,x)
MAIN       DEG EXACT     FUNC     2/30
```

The solution is $x < -8$ or $x > -\frac{1}{2}$ and $x \neq 3$.

Chapter 3 Review Question 15 Page 193

a)



b) The profits increase as the sales increase.

c) Slope of the secant with (100, 225).

$$\frac{225 - 224.998\ 12}{100 - 99.999} \doteq 1.88$$

Slope of the secant with (500, 475).

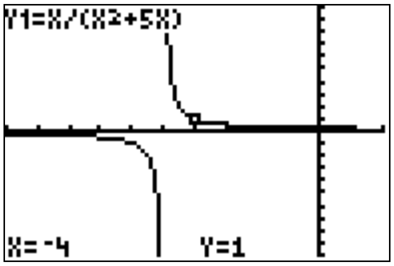
$$\frac{475 - 474.999\ 79}{500 - 499.999} \doteq 0.21$$

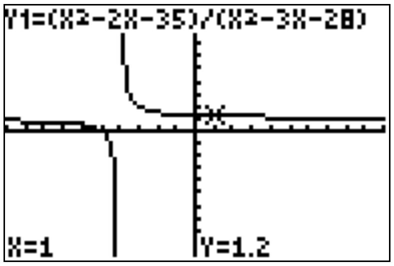
The rate of change of the profit at 100t is approximately 1.88 and approximately 0.21 at 500t, so the rate of change is decreasing.

Chapter 3 Review **Question 16 Page 193**

a) discontinuous at $\left(0, \frac{1}{5}\right)$

b) discontinuous at $\left(7, \frac{12}{11}\right)$





Chapter Problem Wrap-Up

Solutions to the Chapter Problem Wrap-Up are provided in the Teacher's Resource.

Chapter 3 Practice Test

Chapter 3 Practice Test **Question 1 Page 194**

The correct solution is **C**.

The graph has asymptotes at $x = -1$, $x = 1$, $y = 0$.
The graph has a y-intercept at -1.

Chapter 3 Practice Test **Question 2 Page 194**

The correct solution is **B**.

As the denominator approaches infinity, the function approaches 0.

Chapter 3 Practice Test **Question 3 Page 194**

The correct solution is **A**.

The vertical asymptote is $x = 5$.
The horizontal asymptote is $y = 1$.

Chapter 3 Practice Test **Question 4 Page 194**

Answers may vary. A sample solution is shown.

a) $y = \frac{a}{x+2}$

substitute the point (–1, 1)

$$1 = \frac{a}{-1+2}$$

$$1 = \frac{a}{1}$$

$$a = 1$$

$$y = \frac{1}{x+2}$$

b) The asymptotes are $x = -4$, $x = 3$, $y = 0$

$$y = \frac{1}{(x+4)(x-3)}$$

Chapter 3 Practice Test Question 5 Page 195

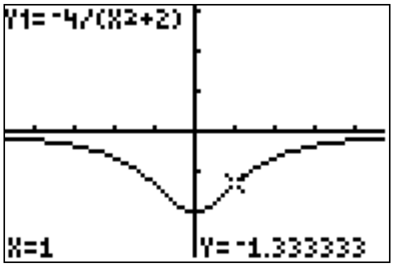
a) i) domain: $\{x \in \mathbb{R}\}$, range: $\{y \in \mathbb{R}, -2 \le y < 3\}$

ii) y-intercept is -2

iii) $y = 0$

iv) $f(x)$ is decreasing for $x < 0$ and increasing for $x > 0$.

b)



Chapter 3 Practice Test Question 6 Page 195

Yes, $\dfrac{1}{f(x)}$ will always yield an asymptote at $y = 0$.

$$g(x) = \frac{1}{f(x)}$$

$$g(x) = \frac{\frac{1}{x}}{\frac{f(x)}{x}}$$

As $x \to \pm\infty$, $\dfrac{1}{x} \to 0$

$$g(x) \to \frac{0}{\frac{f(x)}{x}}$$

$$g(x) \to 0$$

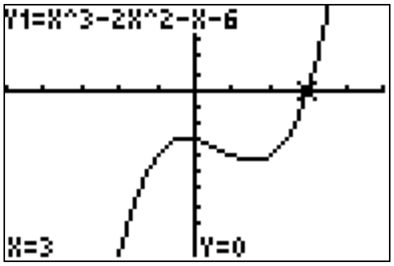
The horizontal asymptote is $y = 0$.

Chapter 3 Practice Test **Question 7 Page 195**

a)
$$\begin{aligned} \frac{3x+5}{x-4} &= \frac{1}{2},\ x \neq 4 \\ 2(3x+5) &= x-4 \\ 6x+10-x+4 &= 0 \\ 5x+14 &= 0 \\ x &= -\frac{14}{5} \end{aligned}$$

b)
$$\begin{aligned} \frac{20}{x^2-4x+7} &= x+2 \\ 20 &= (x+2)(x^2-4x+7) \\ 20 &= x^3-4x^2+7x+2x^2-8x+14 \end{aligned}$$

$$x^3-2x^2-x-6=0$$



$x=3$

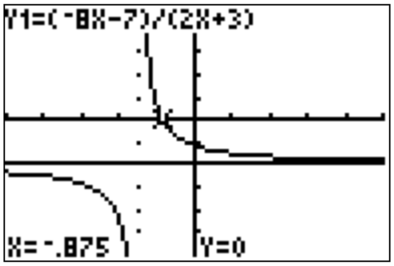
Chapter 3 Practice Test Question 8 Page 195

a) $$\frac{5}{2x+3}-\frac{4(2x+3)}{2x+3}<0$$
$$\frac{5-8x-12}{2x+3}<0$$
$$\frac{-8x-7}{2x+3}<0$$

The vertical asymptote has equation $x = -\frac{3}{2}$.

The horizontal asymptote has equation $y = -4$.

The x-intercept is $-\frac{7}{8}$.

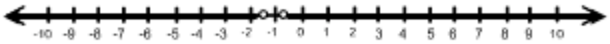


Y1=(-8X-7)/(2X+3)

X=-.875 Y=0

From the graph, the coordinates of all the points below the x-axis would satisfy the inequality $f(x) < 0$.

$x < -\frac{3}{2}, x > -\frac{7}{8}$



b) $$\frac{(x+1)^2}{(x-2)(x+1)}-\frac{(x+7)(x-2)}{(x-2)(x+1)}>0$$

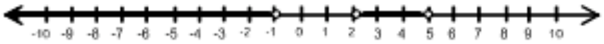
$$\frac{x^2+2x+1-(x^2+5x-14)}{(x-2)(x+1)}>0$$

$$\frac{-3x+15}{(x-2)(x+1)}>0$$

The zero occurs at $x = 5$.
The restrictions occur at $x = -1$, $x = 2$.

Interval	**Signs of Factors of** $\frac{-3x+15}{(x-2)(x+1)}$	**Sign of** $\frac{-3x+15}{(x-2)(x+1)}$
$(-\infty, -1)$	$\frac{(+)}{(-)(-)}$	+
$(-1, 2)$	$\frac{(+)}{(-)(+)}$	–
$(2, 5)$	$\frac{+}{(+)(+)}$	+
5	$\frac{0}{(+)(+)}$	0
$(5, \infty)$	$\frac{-}{(+)(+)}$	–

The solution is $x < -1$ or $2 < x < 5$.



Chapter 3 Practice Test **Question 9 Page 195**

a) Answers may vary. A sample solution is shown.

From the vertical asymptote and the x-intercept.

$$y=\frac{a(x-2)}{c(x+1)}$$

From the horizontal asymptote:

$$y=\frac{a(x-2)}{c(x+1)}$$

$$y=\frac{\frac{ax}{x}-\frac{2a}{x}}{\frac{cx}{x}+\frac{c}{x}}$$

As $x \to \pm\infty$, $\frac{2a}{x} \to 0$ and $\frac{c}{x} \to 0$.

$$y \to \frac{a-0}{c+0}$$

$$y=\frac{a}{c}$$

$$-\frac{1}{2}=\frac{a}{c}$$

$$y=\frac{-(x-2)}{2(x+1)}$$

$$y=\frac{-x+2}{2(x+1)}$$

b) Yes. A sample solution is shown.

$$y=\frac{-2x+4}{4(x+1)}$$

Chapter 3 Practice Test **Question 10 Page 195**

a) $g=\frac{k}{d^2}$

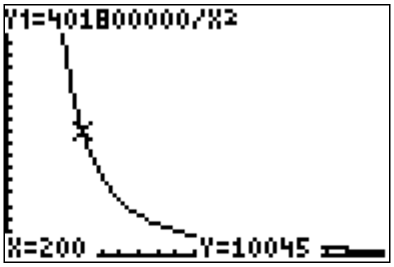
$$k=gd^2$$

$$k=8.2\times 7000^2$$

$$k=401\ 800\ 000$$

$$g=\frac{401\ 800\ 000}{d^2}$$

b)



c) $g = \dfrac{401\ 800\ 000}{d^2}$

$$d^2 = \frac{401\ 800\ 000}{g}$$

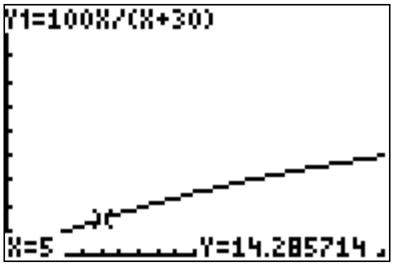
$$d = \sqrt{\frac{401\ 800\ 000}{g}}$$

$$d = \sqrt{\frac{401\ 800\ 000}{6}}$$

$$d \doteq 8183.3$$

Chapter 3 Practice Test **Question 11 Page 195**

a)

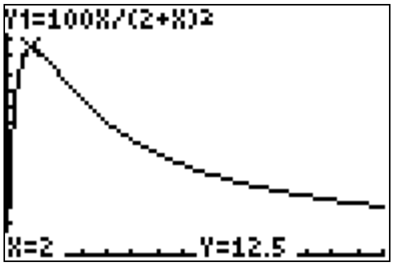


b) domain: $\{t \in \mathbb{R}, t \geq 0\}$, range: $\{P \in \mathbb{R}, 0 \leq P < 100\}$

c) The percentage lost can get close to 100% but not equal to 100%.

Chapter 3 Practice Test Question 12 Page 195

a)



b) The power output increases from 0 Ω to 2 Ω. The power decreases from 2 Ω to 20 Ω.

c) Rate of change at $R = 2$:

$$\frac{12.5 - 12.5}{2 - 1.999} \doteq 0$$

The power is constant at $R = 2$ (not changing).

Chapter 3 Practice Test Question 13 Page 195

Answers may vary. A sample solution is shown.

$x = 0, y = 0$
Slopes increasing and decreasing faster as n increases.

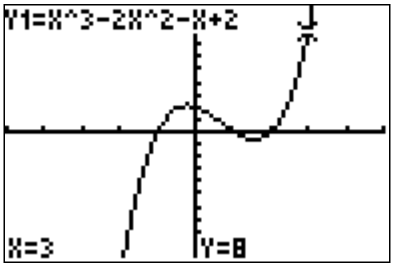
n even:
For $x < 0, f(x)$ is positive and the slope is positive and increasing.
For $x > 0, f(x)$ is positive and the slope is negative and increasing.

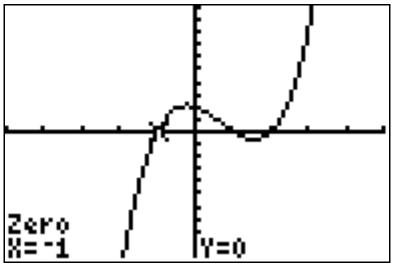
n odd:
For $x < 0, f(x)$ is negative and the slope is negative and decreasing.
For $x > 0, f(x)$ is positive and the slope is negative and increasing.

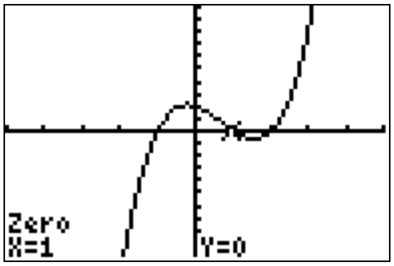
Chapters 1 to 3 Review

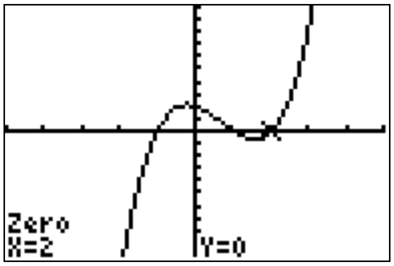
Chapters 1 to 3 Review **Question 1 Page 196**

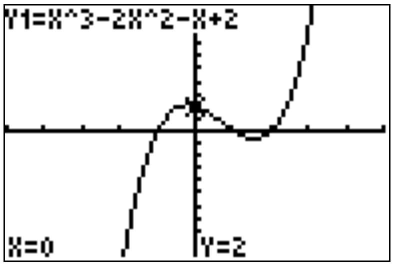
a)





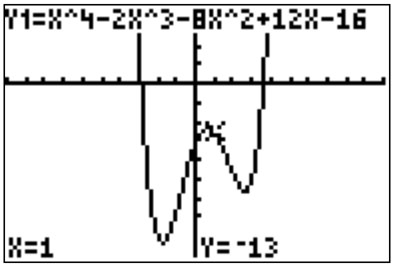




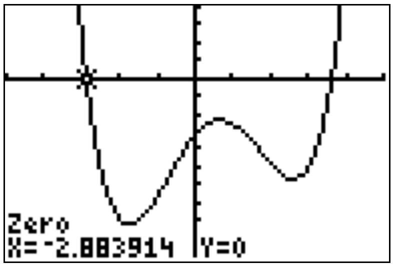


The x-intercepts are –1, 1, and 2.
The y-intercept is 2.

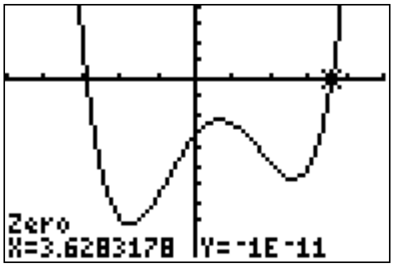
b)



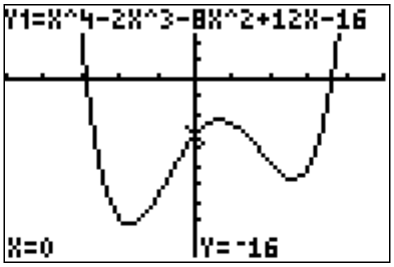
```
Y1=X^4-2X^3-8X^2+12X-16
X=1          Y=-13
```



```
Zero
X=-2.883914  Y=0
```



```
Zero
X=3.6283178  Y=-1E-11
```

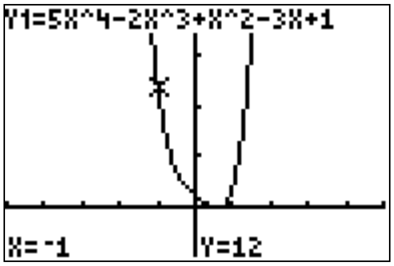


```
Y1=X^4-2X^3-8X^2+12X-16
X=0          Y=-16
```

The x-intercepts are approximately –2.88 and 3.63.
The y-intercept is –16.

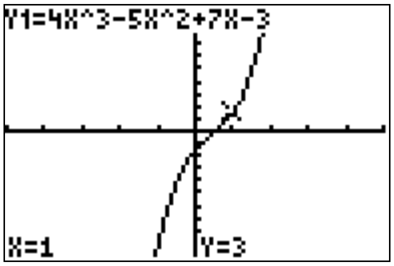
Chapters 1 to 3 Review **Question 2 Page 196**

a) Since the function is even degree and has a positive leading coefficient, the graph extends from quadrant 2 to 1. Therefore, as $x \to -\infty$, $y \to \infty$ and as $x \to \infty$, $y \to \infty$.
The graph does not have symmetry.



```
Y1=5X^4-2X^3+X^2-3X+1
X=-1         Y=12
```

b) Since the function is odd degree and has a positive leading coefficient, the graph extends from quadrant 3 to 1. Therefore, as $x \to -\infty$, $y \to -\infty$ and as $x \to \infty$, $y \to \infty$.
The graph does not have symmetry.



```
Y1=4X^3-5X^2+7X-3
X=1          Y=3
```

Chapters 1 to 3 Review Question 3 Page 196

a) **i)** Average rate: $$\begin{aligned}\text{Slope} &= \frac{-32+62.5}{2.0-2.5}\\ &= \frac{30.5}{-0.5}\\ &= -61\end{aligned}$$

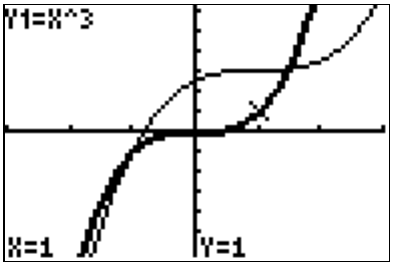
ii) Average rate: $$\begin{aligned}\text{Slope} &= \frac{-13.5+32}{1.5-2.0}\\ &= \frac{18.5}{-0.5}\\ &= -37\end{aligned}$$

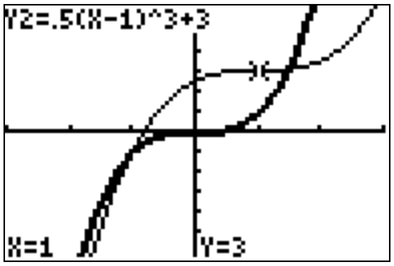
b) $\frac{-61+(-37)}{2} = -49$

The average of the two rates of change approximates the instantaneous rate at $x = 2$.

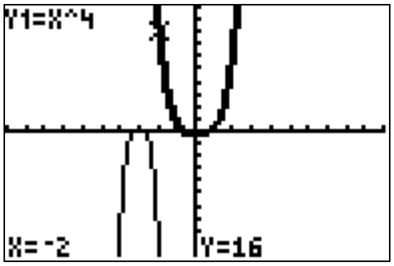
Chapters 1 to 3 Review Question 4 Page 196

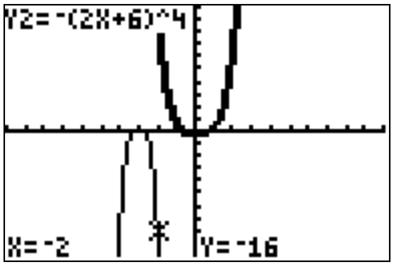
a)





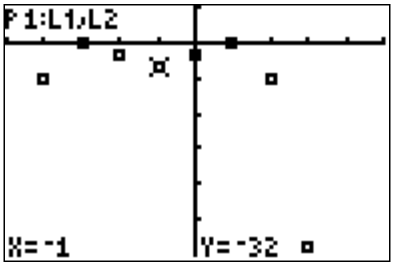
b)

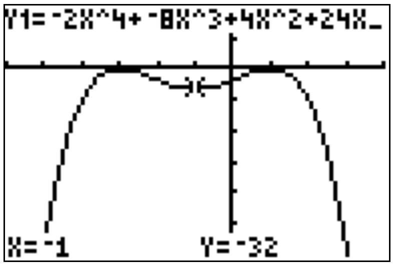




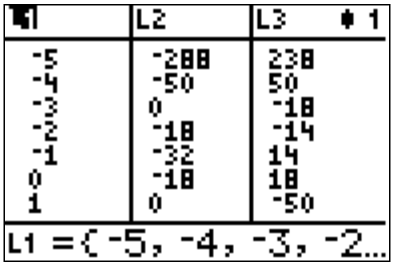
Chapters 1 to 3 Review **Question 5 Page 196**

a)



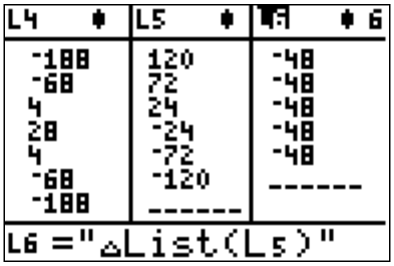


b) The differences are shown in the screen shots.
L3, the first differences; L4, the second differences; L5, the third differences; and L6, the fourth differences.



L1	L2	L3 ◆ 1
-5	-288	238
-4	-50	50
-3	0	-18
-2	-18	-14
-1	-32	14
0	-18	18
1	0	-50

L1 ={-5, -4, -3, -2…



L4 ◆	L5 ◆	L6 ◆ 6
-188	120	-48
-68	72	-48
4	24	-48
28	-24	-48
4	-72	-48
-68	-120	------
-188	------	

L6 ="ΔList(L5)"

The degree is 4.

c) From the table and information given in part a):

$$y = a(x-1)^2(x+3)^2$$
$$-48 = a(4\times 3\times 2\times 1)$$
$$-48 = 24a$$
$$a = -2$$

$$y = -2(x-1)^2(x+3)^2$$

d) Answers may vary. A sample solution is shown.
Reflects and stretches the graph. Also, since the function has even degree, a negative leading coefficient means the graph extends from quadrant 3 to quadrant 4 and has at least one maximum point.

Chapters 1 to 3 Review Question 6 Page 196

a)

x	y	**Secant to the point (2, –9)**
1.9	–8.591	–4.09
1.99	–8.9599	–4.01
1.999	–8.996	–4.00

The instantaneous rate of change at $x = 2$ is –4.

b)

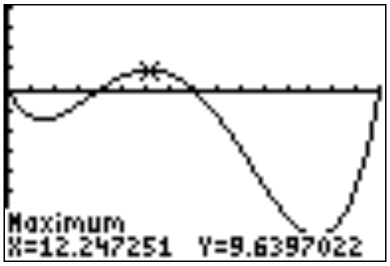
x	y	**Secant to the point (4, –5)**
3.9	–6.131	11.31
3.99	–5.119 3	11.93
3.999	–5.011 99	11.99
3.9999	–5.0012	12.00

The instantaneous rate of change at $x = 4$ is 12.

c) local minimum; changes from negative to positive slope

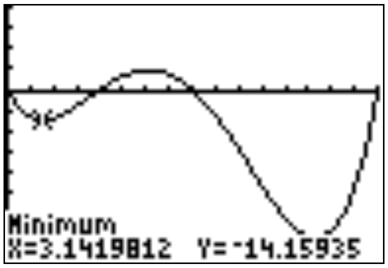
Chapters 1 to 3 Review Question 7 Page 196

a)

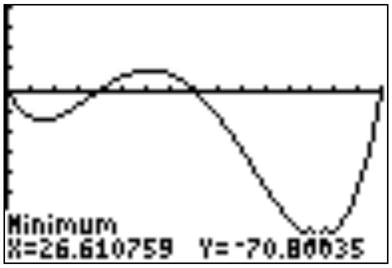


```
Maximum
X=12.247251  Y=9.6397022
```

Maximum (12.25, 9.64)



```
Minimum
X=3.1419812  Y=⁻14.15935
```



```
Minimum
X=26.610759  Y=⁻70.80035
```

Minima (3.14, –14.16) and (26.61, –70.80)

b) Slope: $\frac{9.64+14.16}{12.25-3.14} \doteq 2.61$

The slope from (3.14, –14.16) to (12.25, 9.64) is approximately 2.61.

Slope: $\frac{-70.80-9.64}{26.61-12.25} \doteq -5.60$

The slope from (12.25, 9.64) to (26.61, –70.80) is approximately –5.60.

c) It looks like the graph is the steepest when x is between 27 and 32.

Test $x = 29$:

Secant: $\frac{-59.3775+59.387\ 85}{29-28.999} \doteq 10.35$

Test $x = 32$:

Secant: $\frac{0+0.0307\ 158}{32-31.999} \doteq 30.7158$

The instantaneous rate of change would be the greatest at $x = 32$.

Chapters 1 to 3 Review **Question 8 Page 196**

Answers may vary. A sample solution is shown.

$y = 2x(x + 7)(x - 3)^2$

$y = -\frac{1}{3}x(x+7)(x-3)^2$

Chapters 1 to 3 Review **Question 9 Page 196**

$y = k(x - 2)^2(x + 5)$

Answers may vary. A sample solution is shown.

$y = 2(x - 2)^2(x + 5)$

Let $x = 0$:

$y = 2(-2)^2(5)$

$= 40$

The y-intercept is 40.

$y = -3(x - 2)^2(x + 5)$

Let $x = 0$:

$y = -3(-2)^2(5)$

$= -60$

The y-intercept is –60.

Chapters 1 to 3 Review **Question 10 Page 196**

a)

$$\begin{array}{rl}
 & 2x^2-\frac{7}{2}x+\frac{19}{4} \\
2x+1\big) & \overline{4x^3-5x^2+6x+2} \\
 & \underline{4x^3+2x^2} \\
 & \quad -7x^2+6x \\
 & \quad \underline{-7x^2-\frac{7}{2}x} \\
 & \qquad \frac{19}{2}x+2 \\
 & \qquad \underline{\frac{19}{2}x+\frac{19}{4}} \\
 & \qquad\qquad -\frac{11}{4}
\end{array}$$

$$4x^3-5x^2+6x+2=(2x+1)\left(2x^2-\frac{7}{2}x+\frac{19}{4}\right)-\frac{11}{4},\ x\neq-\frac{1}{2}$$

b)

$$\begin{array}{rl}
 & 3x^3+6x^2+7x+14 \\
x-2\big) & \overline{3x^4+0x^3-5x^2+0x-28} \\
 & \underline{3x^4-6x^3} \\
 & \quad 6x^3-5x^2 \\
 & \quad \underline{6x^3-12x^2} \\
 & \qquad 7x^2+0x \\
 & \qquad \underline{7x^2-14x} \\
 & \qquad\quad 14x-28 \\
 & \qquad\quad \underline{14x-28} \\
 & \qquad\qquad 0
\end{array}$$

$$3x^4-5x^2-28=(x-2)(3x^3+6x^2+7x+14)\,,\ x\neq 2$$

Chapters 1 to 3 Review **Question 11 Page 197**

a)

$$\begin{aligned} P(2)&=6(2)^3-7(2)^2+5(2)+8 \\ &=38 \end{aligned}$$

b)

$$\begin{aligned} P\left(-\frac{4}{3}\right)&=3\left(-\frac{4}{3}\right)^4+\left(-\frac{4}{3}\right)^3-2\left(-\frac{4}{3}\right)+1 \\ &=\frac{97}{9} \end{aligned}$$

Chapters 1 to 3 Review Question 12 Page 197

a)
$$\begin{aligned} P(-5) &= 4(-5)^5 - 3(-5)^3 - 2(-5)^2 + 5 \\ &= -12\ 500 + 375 - 50 + 5 \\ &= -12\ 170 \end{aligned}$$

Since $P(-5) \neq 0$, $x + 5$ is not a factor.

b)
$$\begin{aligned} P(4) &= 3(4)^3 - 15(4)^2 + 10(4) + 8 \\ &= 192 - 240 + 40 + 8 \\ &= 0 \end{aligned}$$

Since $P(4) = 0$, $x - 4$ is a factor.

Chapters 1 to 3 Review Question 13 Page 197

$$\begin{aligned} P(2) &= 3(2)^4 - 4(2)^3 + k(2) - 3(2) + 6 \\ 3 &= 48 - 32 + 2k - 6 + 6 \\ 3 &= 16 + 2k \\ -13 &= 2k \\ k &= -\frac{13}{2} \end{aligned}$$

Chapters 1 to 3 Review Question 14 Page 197

a) Difference of cubes:
$(x - 3)(x^2 + 3x + 9)$

b)
$$\begin{aligned} P(2) &= 2(2)^3 + 4(2)^2 - 13(2) - 6 \\ &= 16 + 16 - 26 - 6 \\ &= 0 \end{aligned}$$

$x - 2$ is a factor.

−2	2	4	−13	−6
−		−4	−16	−6
×	2	8	3	0

$(x - 2)(2x^2 + 8x + 3)$

Chapters 1 to 3 Review **Question 15 Page 197**

a)
$$\begin{aligned}P(-4) &= (-4)^3 - 2(-4)^2 - 19(-4) + 20\\ &= -64 - 32 + 76 + 20\\ &= 0\\ P(1) &= (1)^3 - 2(1)^2 - 19(1) + 20\\ &= 1 - 2 - 19 + 20\\ &= 0\\ P(5) &= (5)^3 - 2(5)^2 - 19(5) + 20\\ &= 125 - 50 - 95 + 20\\ &= 0\end{aligned}$$

$x = -4$ or $x = 1$ or $x = 5$

b)
$$\begin{aligned}P(-3) &= 5(-3)^3 + 23(-3)^2 - 9 + 21(-3)\\ &= -135 + 207 - 9 - 63\\ &= 0\end{aligned}$$

$x + 3$ is a factor.

3	5	23	21	–9
–		15	24	–9
×	5	8	–3	0

$$\begin{aligned}(x+3)(5x^2 + 8x - 3) &= 0\\ x &= -3\\ &\text{or}\\ x &= \frac{-8 \pm \sqrt{8^2 - 4(5)(-3)}}{2(5)}\\ x &= \frac{-8 \pm \sqrt{124}}{10}\\ x &= \frac{-4 \pm \sqrt{31}}{5}\end{aligned}$$

$x = -3$ or $x = \dfrac{-4 - \sqrt{31}}{5}$ or $x = \dfrac{-4 + \sqrt{31}}{5}$

a) Factor first.

$(x-6)(x-1)\geq 0$

Case 1:

$x-6\geq 0 \qquad x-1\geq 0$

$x\geq 6 \qquad x\geq 1$

$x\geq 6$ is included in the inequality $x\geq 1$. So the solution is $x\geq 6$.

Case 2:

$x-6\leq 0 \qquad x-1\leq 0$

$x\leq 6 \qquad x\leq 1$

$x\leq 1$ is included in the inequality $x\leq 6$. So the solution is $x\leq 1$.

The solution is $x\leq 1$ or $x\geq 6$.

b) Factor first.

$$x^2(x+3)-4(x+3)<0$$
$$(x^2-4)(x+3)<0$$
$$(x-2)(x+2)(x+3)<0$$

Case 1:

$x-2<0 \qquad x+2<0 \qquad x+3<0$

$x<2 \qquad x<-2 \qquad x<-3$

The solution is $x<-3$.

Case 2:

$x-2<0 \qquad x+2>0 \qquad x+3>0$

$x<2 \qquad x>-2 \qquad x>-3$

The solution is $-2<x<2$.

Case 3:

$x-2>0 \qquad x+2<0 \qquad x+3>0$

$x>2 \qquad x<-2 \qquad x>-3$

No solution.

Case 4:

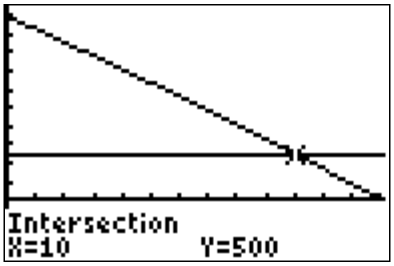
$x-2>0 \qquad x+2>0 \qquad x+3<0$

$x>2 \qquad x>-2 \qquad x<-3$

No solution.

The solution is $x<-3$ or $-2<x<2$.

Chapters 1 to 3 Review Question 17 Page 197

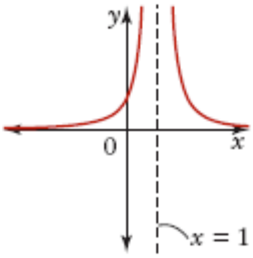


```
Intersection
X=10          Y=500
```

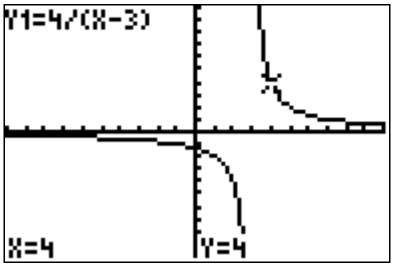
From 0 min to 10 min the mass of the fuel is greater than 500 t.

Chapters 1 to 3 Review Question 18 Page 197

asymptotes: $x = 1, y = 0$
No x-intercept.
The y-intercept is 1.
A



Chapters 1 to 3 Review Question 19 Page 197



```
Y1=4/(X-3)
X=4          Y=4
```

a) $f(x) \to 0$

b) $f(x) \to 0$

c) $f(x) \to \infty$

d) $f(x) \to -\infty$

Chapters 1 to 3 Review Question 20 Page 197

asymptotes: $x = -1$, $y = 1$
Domain: $\{x \in \mathbb{R},\ x \neq -1\}$, Range: $\{y \in \mathbb{R},\ y \neq 1\}$

Chapters 1 to 3 Review Question 21 Page 197

a) $f(x) = \dfrac{6x+1}{2(x-2)}$

i) Domain: $\{x \in \mathbb{R},\ x \neq 2\}$, Range: $\{y \in \mathbb{R},\ y \neq 3\}$

The x-intercept is $-\dfrac{1}{6}$.

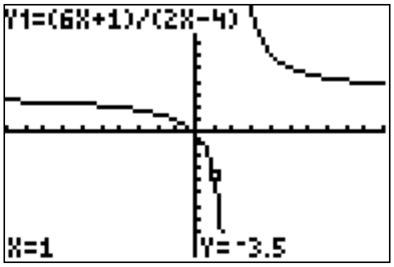
Let $x = 0$:

$$\begin{aligned} y &= \frac{6(0)+1}{2(0)-4} \\ &= -\frac{1}{4} \end{aligned}$$

The y-intercept is $-\dfrac{1}{4}$.

asymptotes: $x = 2$, $y = 3$
negative slope: $x < 2$, $x > 2$
slope decreasing: $x < 2$; slope increasing: $x > 2$

ii)



b) $f(x)=\dfrac{1}{(x-3)(x+3)}$

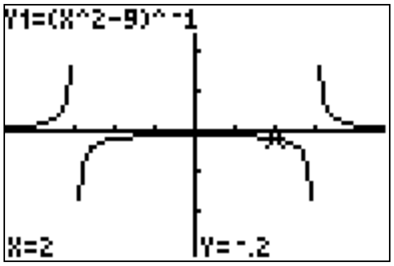
i) Domain: $\{x \in \mathbb{R},\ x \neq -3,\ x \neq 3\}$, Range: $\left\{y \in \mathbb{R},\ y \leq -\dfrac{1}{9},\ y > 0\right\}$

No x-intercept.
Let $x = 0$:

$$y=\frac{1}{0^2-9}$$
$$=-\frac{1}{9}$$

The y-intercept is $-\dfrac{1}{9}$.
asymptotes: $x = -3$, $x = 3$, $y = 0$
positive slope: $x < -3$, $-3 < x < 0$; negative slope: $0 < x < 3$, $x > 3$
slope decreasing: $-3 < x < 0$, $0 < x < 3$; slope increasing: $x < -3$, $x > 3$

ii)



Chapters 1 to 3 Review **Question 22 Page 197**

Answers may vary. A sample solution is shown.
From the x-intercept: $ax + b = k(x + 2)$
From the vertical asymptote: $cx + d = m(x - 1)$
From the horizontal asymptote: $\dfrac{k(x+2)}{m(x-1)}=\dfrac{3(x+2)}{(x-1)}$

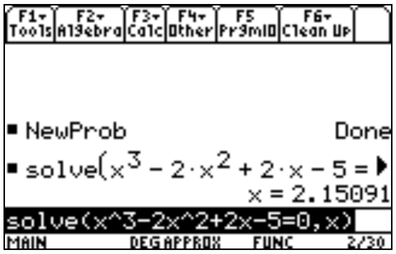
$$f(x)=\frac{3x+6}{x-1}$$

Chapters 1 to 3 Review **Question 23 Page 197**

a)
$$\begin{aligned} 3(2x+3) &= (x-2) \\ 6x-x &= -2-9 \\ 5x &= -11 \\ x &= -\frac{11}{5} \text{ or } -2.2 \end{aligned}$$

b)
$$\begin{aligned} (x+1)(x^2-3x+5) &= 10 \\ x^3-3x^2+5x+x^2-3x+5 &= 10 \\ x^3-2x^2+2x-5 &= 0 \end{aligned}$$

No exact solution.



```
■ NewProb                Done
■ solve(x^3 - 2·x^2 + 2·x - 5 =▶
                    x = 2.15091
solve(x^3-2x^2+2x-5=0,x)
MAIN      DEG APPROX   FUNC   2/30
```

$x \doteq 2.15$

a) $\dfrac{3}{2x+4}+\dfrac{2(2x+4)}{2x+4}>0$

$$\frac{3+4x+8}{2x+4}>0$$

$$\frac{4x+11}{2x+4}>0$$

The zero occurs at $x=-\dfrac{11}{4}$.

The restriction occurs at $x = –2$.

Interval	**Signs of Factors of** $\dfrac{4x+11}{2x+4}$	**Sign of** $\dfrac{4x+11}{2x+4}$
$\left(-\infty,-\dfrac{11}{4}\right)$	$\dfrac{(-)}{(-)}$	+
$-\dfrac{11}{4}$	$\dfrac{(0)}{(-)}$	0
$\left(-\dfrac{11}{4},-2\right)$	$\dfrac{(+)}{(-)}$	–
(–2, ∞)	$\dfrac{(+)}{(+)}$	+

The solution is $x<-\dfrac{11}{4}$ or $x>-2$.

b) $\dfrac{(x+3)(x-2)}{(x-1)(x-2)}-\dfrac{(x+4)(x-1)}{(x-1)(x-2)}\le 0$

$$\frac{x^2+x-6-(x^2+3x-4)}{(x-1)(x-2)}\le 0$$

$$\frac{-2x-2}{(x-1)(x-2)}\le 0$$

$$\frac{-2(x+1)}{(x-1)(x-2)}\le 0$$

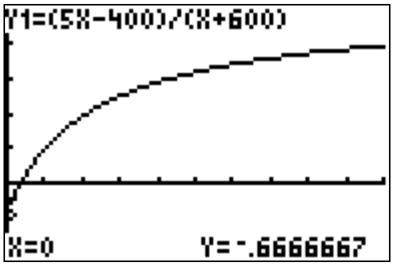
The zero occurs at $x=-1$.
The restrictions occur at $x = 1$ and $x = 2$.

Interval	**Signs of Factors of** $\frac{-2(x+1)}{(x-1)(x-2)}$	**Sign of** $\frac{-2(x+1)}{(x-1)(x-2)}$
$(-\infty,-1)$	$\frac{(+)}{(-)(-)}$	+
-1	$\frac{(0)}{(-)(-)}$	0
$(-1, 1)$	$\frac{(-)}{(-)(-)}$	–
$(1, 2)$	$\frac{(-)}{(+)(-)}$	+
$(2, \infty)$	$\frac{(-)}{(+)(+)}$	–

The solution is $-1 \le x < 1$ or $x > 2$.

Chapters 1 to 3 Review **Question 25 Page 197**

a)



b) The zero is at $x = 80$.

The y-intercept is $-\frac{2}{3}$ (approximately –0.67).

The horizontal asymptote occurs at $y = 5$.

Domain: $\{x \in \mathbb{R},\ x \geq 0\}$, Range: $\left\{P(x) \in \mathbb{R},\ -\frac{2}{3} \leq P(x) < 5\right\}$

c) The profit is always less than \$5000

Chapters 1 to 3 Review **Question 26 Page 197**

Domain: $\{t \in \mathbb{R},\ t \geq 0,\ t \neq 10\}$

Since t represents time, $t \geq 0$; $t \neq 10$ because the denominator cannot be zero.