Chapter 1 Polynomial Functions

Chapter 1 Prerequisite Skills

Chapter 1 Prerequisite Skills **Question 1 Page 2**

a) $f(0) = -4(0) + 7$
$= 7$

b) $f(3) = -4(3) + 7$
$= -5$

c) $f(-1) = -4(-1) + 7$
$= 11$

d) $f\left(\frac{1}{2}\right) = -4\left(\frac{1}{2}\right) + 7$
$= 5$

e) $f(-2x) = -4(-2x) + 7$
$= 8x + 7$

f) $f(3x) = -4(3x) + 7$
$= -12x + 7$

Chapter 1 Prerequisite Skills **Question 2 Page 2**

a) $f(0) = 2(0)^2 - 3(0) + 1$
$= 1$

b) $f(3) = 2(3)^2 - 3(3) + 1$
$= 10$

c) $f(-1) = 2(-1)^2 - 3(-1) + 1$
$= 6$

d) $f\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right)^2 - 3\left(\frac{1}{2}\right) + 1$
$= 0$

e) $3f(2x) = 3\left[2(2x)^2 - 3(2x) + 1\right]$
$= 3(8x^2 - 6x + 1)$
$= 24x^2 - 18x + 3$

f) $f(3x) = 2(3x)^2 - 3(3x) + 1$
$= 18x^2 - 9x + 1$

Chapter 1 Prerequisite Skills **Question 3 Page 2**

a) Slope: $m = 3$
y-intercept: $b = 2$

b) put into $y = mx + b$ form first

$$y = -\frac{1}{2}x + \frac{3}{2}$$

Slope: $m = -\frac{1}{2}$

y-intercept: $b = \frac{3}{2}$

c) put into $y = mx + b$ form first
$y = 5x + 7$
Slope: $m = 5$
y-intercept: $b = 7$

d) put into $y = mx + b$ form first
$y = -5x - 11$
Slope: $m = -5$
y-intercept: $b = -11$

e) put into $y = mx + b$ form first

$$y = -\frac{1}{2}x + 1$$

Slope: $m = -\frac{1}{2}$
y-intercept: $b = 1$

Chapter 1 Prerequisite Skills **Question 4 Page 2**

a) Given $m = 3$, $b = 5$
Equation for the line: $y = 3x + 5$

b) Given x-intercept $= -1$, $b = 4$

$$m = \frac{4 - 0}{0 + 1}$$

$m = 4$
$b = 4$
Equation for the line: $y = 4x + 4$

c) Given $m = -4$, point $(7, 3)$
$y = -4x + b$
$3 = -4(7) + b$
$b = 31$
Equation for the line: $y = -4x + 31$

d) Given points (2, –2) and (1, 5)

$$m = \frac{5+2}{1-2}$$
$$m = -7$$
$$-2 = -7(2) + b$$
$$b = 12$$

Equation for the line: $y = -7x + 12$

Chapter 1 Prerequisite Skills **Question 5 Page 2**

a) This function is linear.

x	*y*	First Differences
–2	–7	
–1	–5	2
0	–3	2
1	–1	2
2	1	2
3	3	2
4	5	2

b) This function is neither linear nor quadratic.

x	*y*	First Differences	Second Differences
–1	–8		
0	–2	6	
1	–1	1	–5
2	5	6	5
3	7	2	–4
4	13	6	4
5	20	7	1

c) This function is quadratic.

x	*y*	First Differences	Second Differences
–4	–12		
–3	–5	7	
–2	0	5	–2
–1	3	3	–2
0	4	1	–2
1	3	–1	–2
2	0	–3	–2

Chapter 1 Prerequisite Skills **Question 6 Page 2**

a) Domain: $\{x \in \mathbb{R}\}$
Range: $\{y \in \mathbb{R}, y \geq 1\}$

b) Domain: $\{x \in \mathbb{R}, x \neq -5\}$
Range: $\{y \in \mathbb{R}, y \neq 0\}$

c) Domain: $\{x \in \mathbb{R}, x \leq 0.5\}$
Range: $\{y \in \mathbb{R}, y \geq 0\}$

Chapter 1 Prerequisite Skills **Question 7 Page 2**

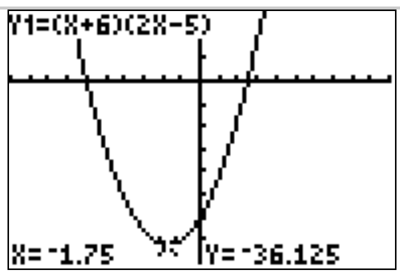
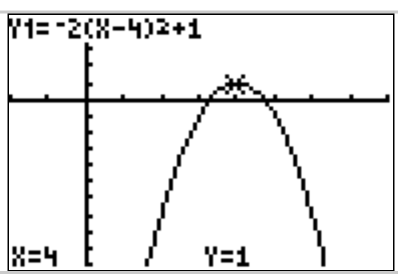
Answers may vary. A sample solution is shown.

a) $y = -3(x + 1)(x - 1)$

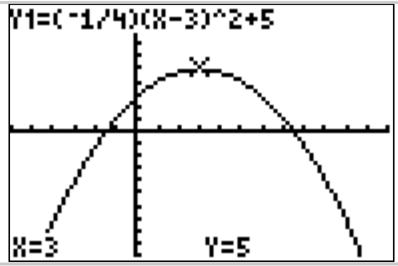
b) $y = -2x^2 - 3x + 3$

c) $y = 4\left(x + \frac{1}{2}\right)(x - 2)$

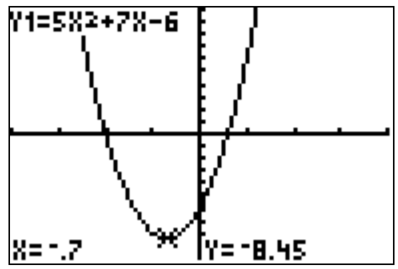
Chapter 1 Prerequisite Skills **Question 8 Page 3**

a) x-intercepts: $-6, \frac{5}{2}$ Vertex: $\left(-\frac{7}{4}, -\frac{289}{8}\right)$ Direction of Opening: opens up Domain: $\{x \in \mathbb{R}\}$ Range: $\{y \in \mathbb{R}, y \geq -\frac{289}{8}\}$	
b) x-intercepts: approximately 3.29, 4.71 Vertex: (4, 1) Direction of Opening: opens down Domain: $\{x \in \mathbb{R}\}$ Range: $\{y \in \mathbb{R}, y \leq 1\}$	

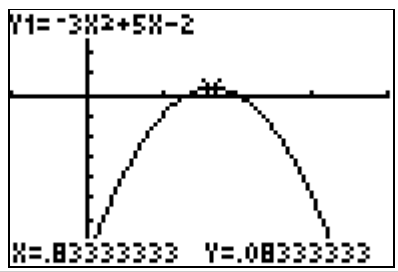
c) x-intercepts: approximately –1.47, 7.47
Vertex: (3, 5)
Direction of Opening: opens down
Domain: $\{x \in \mathbb{R}\}$
Range: $\{y \in \mathbb{R}, y \le 5\}$



d) x-intercepts: –2, $\frac{3}{5}$
Vertex: $\left(-\frac{7}{10}, -\frac{169}{20}\right)$
Direction of Opening: opens up
Domain: $\{x \in \mathbb{R}\}$
Range: $\{y \in \mathbb{R}, y \ge -\frac{169}{20}\}$



e) x-intercepts: 1, $\frac{2}{3}$
Vertex: $\left(-\frac{5}{6}, -\frac{1}{12}\right)$
Direction of Opening: opens down
Domain: $\{x \in \mathbb{R}\}$
Range: $\{y \in \mathbb{R}, y \le \frac{1}{12}\}$



Chapter 1 Prerequisite Skills **Question 9 Page 3**

a) There is a vertical stretch and a reflection in the x-axis.

b) There is a vertical compression.

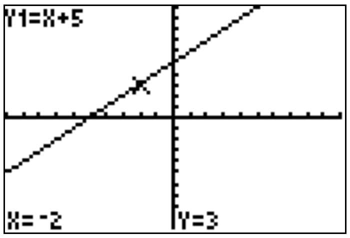
c) There is a horizontal compression.

d) There is a horizontal stretch and a reflection in the y-axis.

e) There is a reflection in the y-axis.

a) i) $f(x) = (x + 2) + 3$
$f(x) = x + 5$

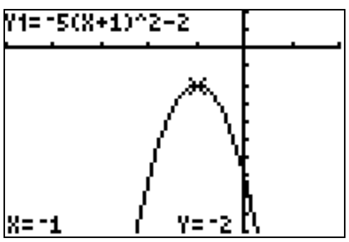
ii)



iii) Domain: $\{x \in \mathbb{R}\}$
Range: $\{y \in \mathbb{R}\}$

b) i) $f(x) = -5(x + 1)^2 - 2$

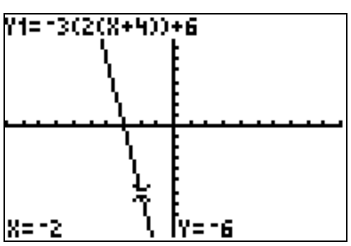
ii)



iii) Domain: $\{x \in \mathbb{R}\}$
Range: $\{y \in \mathbb{R}, y \leq -2\}$

c) i) $f(x) = -3[2(x + 4)] + 6$

ii)



iii) Domain: $\{x \in \mathbb{R}\}$
Range: $\{y \in \mathbb{R}\}$

Chapter 1 Prerequisite Skills **Question 11 Page 3**

a) i) Vertical stretch by a factor of 2
Reflection in the x-axis
Translation of 3 units left
Translation of 1 unit up

ii) $y = -2(x + 3) + 1$

b) i) Vertical compression by a factor of $\frac{1}{3}$
Translation of 2 units down

ii) $y = \frac{1}{3}x^2 - 2$

Chapter 1 Prerequisite Skills **Question 12 Page 3**

To obtain the function, the transformations that must be applied are:
Vertical stretch by a factor of 3
Horizontal stretch by a factor of 2
Reflection in the y-axis
Translation of 1 unit right
Translation of 2 units up

Chapter 1 Section 1 **Power Functions**

Chapter 1 Section 1 **Question 1 Page 11**

a) No. This is a trigonometric function.

b) Yes. This is a polynomial function of degree 1. The leading coefficient is –7.

c) Yes. This is a polynomial function of degree 4. The leading coefficient is 2.

d) Yes This is a polynomial function of degree 5. The leading coefficient is 3.

e) No. This is an exponential function.

f) No. This is a rational function.

Chapter 1 Section 1 **Question 2 Page 11**

a) The degree is 4; the leading coefficient is 5.

b) The degree is 1; the leading coefficient is –1.

c) The degree is 2; the leading coefficient is 8.

d) The degree is 3; the leading coefficient is $-\frac{1}{4}$.

e) The degree is 0; the leading coefficient is –5.

f) The degree is 2; the leading coefficient is 1.

Chapter 1 Section 1 **Question 3 Page 12**

a) **i)** It represents a power function of even degree.
ii) The leading coefficient is negative since the function is facing downward.
iii) Domain: $\{x \in \mathbb{R}\}$; Range: $\{y \in \mathbb{R}, y \leq 0\}$
iv) The function has line symmetry.
v) The function extends from quadrant 3 to 4.

b) **i)** It represents a power function of odd degree.
ii) The leading coefficient is positive since the function is facing up to the right.
iii) Domain: $\{x \in \mathbb{R}\}$; Range: $\{y \in \mathbb{R}\}$
iv) The function has point symmetry.
v) The function extends from quadrant 3 to 1.

c) i) It represents a power function of odd degree.
ii) The leading coefficient is negative since the function is facing down to the left.
iii) Domain: $\{x \in \mathbb{R}\}$; Range: $\{y \in \mathbb{R}\}$
iv) The function has point symmetry.
v) The function extends from quadrant 2 to 4.

d) i) It represents a power function of even degree.
ii) The leading coefficient is positive since the function is facing upwards.
iii) Domain: $\{x \in \mathbb{R}\}$; Range: $\{y \in \mathbb{R}, y \geq 0\}$
iv) The function has line symmetry.
v) The function extends from quadrant 2 to 1.

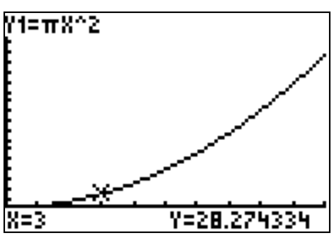
e) i) It represents a power function of odd degree.
ii) The leading coefficient is negative since the function is facing down to the left.
iii) Domain: $\{x \in \mathbb{R}\}$; Range: $\{y \in \mathbb{R}\}$
iv) The function has point symmetry.
v) The function extends from quadrant 2 to 4.

Chapter 1 Section 1 **Question 4 Page 12**

End Behaviour	Function
Extends from quadrant 3 to 1	$y = 5x$, $y = 4x^5$
Extends from quadrant 2 to 4	$y = -x^3$, $y = -0.1x^{11}$
Extends from quadrant 2 to 1	$y = \frac{3}{7}x^2$, $y = 2x^4$
Extends from quadrant 3 to 4	$y = -x^6$, $y = -9x^{10}$

Chapter 1 Section 1 **Question 5 Page 12**

a)

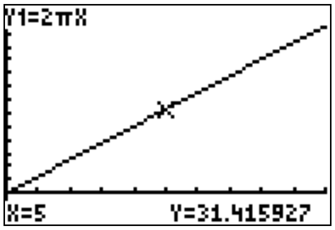


b) Domain: $\{r \in \mathbb{R}, 0 \leq r \leq 10\}$
Range: $\{A(r) \in \mathbb{R}, 0 \leq A(r) \leq 100\pi\}$

c) Answers may vary. A sample solution is shown.
Similarities: vertex (0, 0), x-intercept, y-intercept, end behaviour
Differences: domain, range, overall shape

Chapter 1 Section 1 **Question 6 Page 12**

a)



b) Domain: $\{r \in \mathbb{R}, 0 \le r \le 10\}$
Range: $\{C(r) \in \mathbb{R}, 0 \le C(r) \le 20\pi\}$

c) Answers may vary. A sample solution is shown.
Similarities: end behaviour
Differences: domain, range, overall shape

Chapter 1 Section 1 **Question 7 Page 13**

a) This graph represents a power (cubic) function since it extends from quadrant 2 to 4 and has point symmetry about the origin.

b) This graph represents an exponential function since it has an asymptote at $y = b$, and it is always increasing.

c) This graph represents a periodic function since it repeats at regular intervals.

d) This graph represents a power (constant) function since the degree is 0.

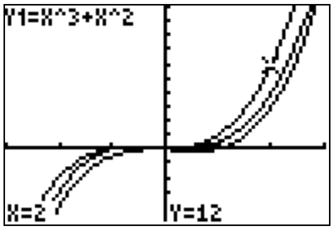
e) This graph represents none of these functions; this is a square root function.

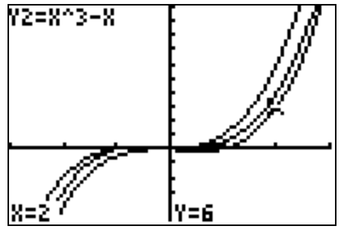
f) This graph represents none of these functions; this is a rational function.

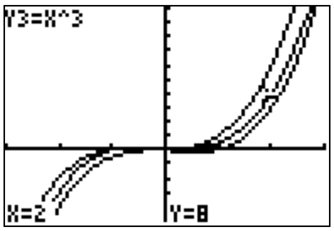
g) This graph represents a power quadratic) function since it extends from quadrant 2 to 1 and it has line symmetry in the y-axis.

Chapter 1 Section 1 **Question 8 Page 13**

a)



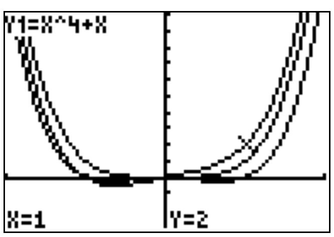


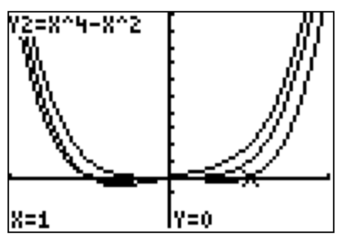


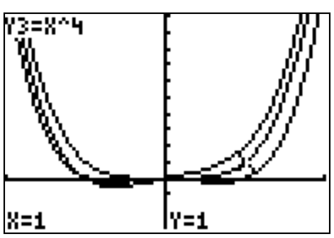
b) All three functions have these key features:
Domain: $\{x \in \mathbb{R}\}$; Range: $\{y \in \mathbb{R}\}$
Extends from quadrant 3 to 1
Point symmetry about (0, 0)
x-intercept = 0, y-intercept = 0

Chapter 1 Section 1 **Question 9 Page 13**

a)





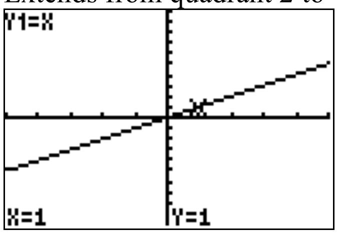


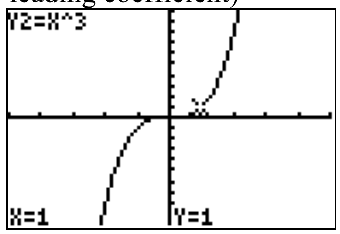
b) All three functions have these key features:
Domain: $\{x \in \mathbb{R}\}$
Extends from quadrant 2 to 1
x-intercept = 0, y-intercept = 0

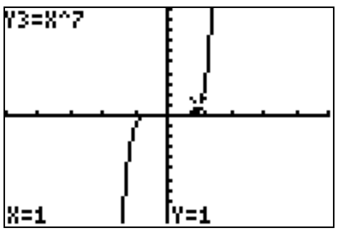
Chapter 1 Section 1 **Question 10 Page 13**

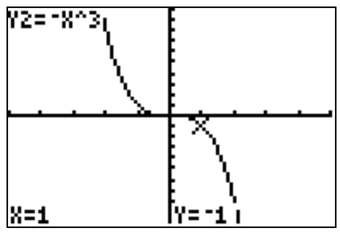
Answers may vary. A sample solution is shown.
Similarities:
- Extends from quadrant 1 to 3 (positive leading coefficient)
- Domain: $\{x \in \mathbb{R}\}$; Range: $\{y \in \mathbb{R}\}$
- Point symmetry about (0, 0)

Differences:
- Overall shape
- Extends from quadrant 2 to 4 (negative leading coefficient)





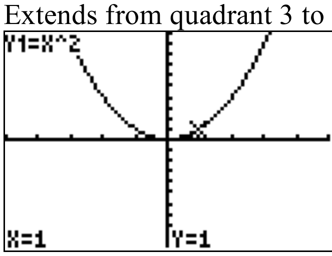


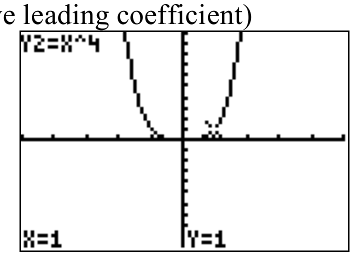


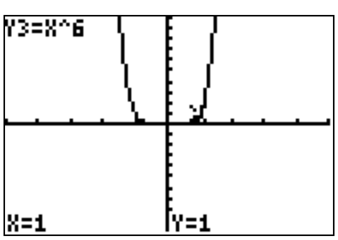
Chapter 1 Section 1 **Question 11 Page 13**

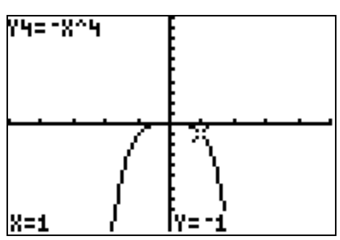
Answers may vary. A sample solution is shown.
Similarities:
- Extends from quadrant 2 to 1 (positive leading coefficient)
- Domain: $\{x \in \mathbb{R}\}$
- Line symmetry

Differences:
- Range
- Extends from quadrant 3 to 4 (negative leading coefficient)



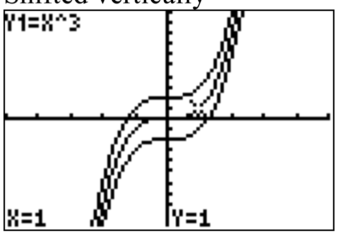


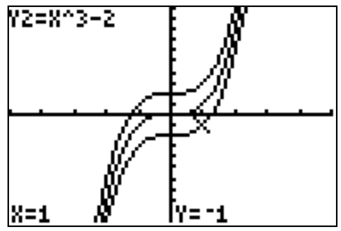


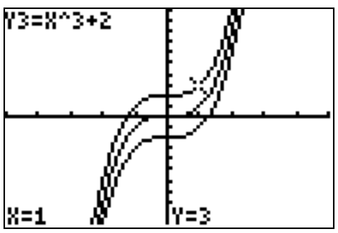


Chapter 1 Section 1 **Question 12 Page 13**

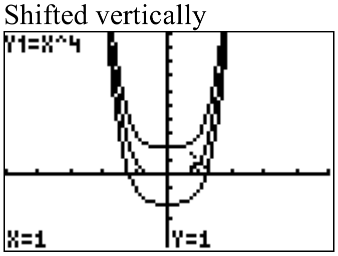
a) Answers may vary. A sample solution is shown.
Similarities:
Extends from quadrant 3 to 1
Domain: $\{x \in \mathbb{R}\}$; Range: $\{y \in \mathbb{R}\}$
Point symmetry
Shape
Difference:
Shifted vertically

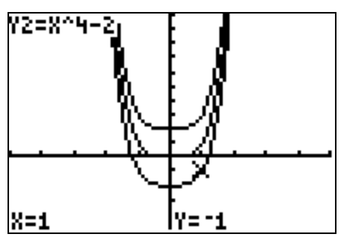


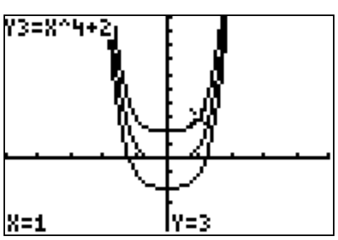




b) Answers may vary. A sample solution is shown.
Similarities:
Extends from quadrant 2 to 1
Domain: $\{x \in \mathbb{R}\}$
Line symmetry
Shape
Differences:
Range
Shifted vertically

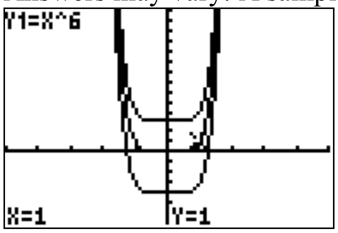


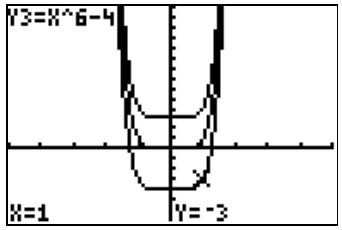


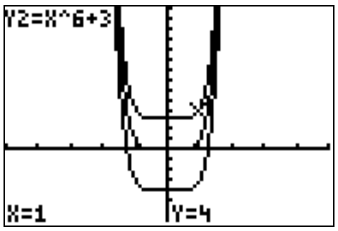


c) c is a vertical shift of x^n, when n is a natural number.

d) Answers may vary. A sample solution is shown.



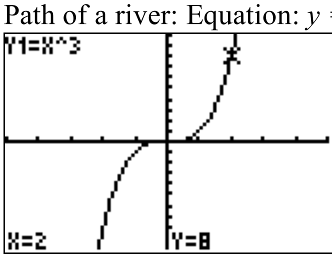




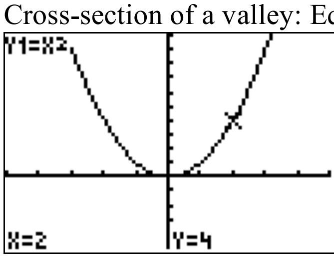
Chapter 1 Section 1 **Question 13 Page 14**

Answers may vary. A sample solution is shown.

Path of a river: Equation: $y = x^3$, Domain: $\{x \in \mathbb{R}\}$, Range: $\{y \in \mathbb{R}\}$



Cross-section of a valley: Equation: $y = x^2$, Domain: $\{x \in \mathbb{R}\}$, Range: $\{y \in \mathbb{R}, y \geq 0\}$

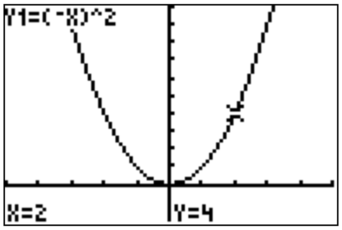


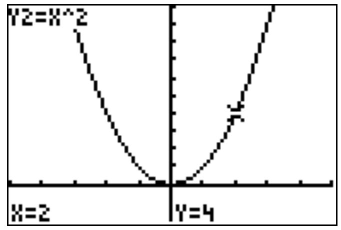
Chapter 1 Section 1 **Question 14 Page 14**

a) $y = (-x)^{2n}$ is the same graph as $y = x^{2n}$, when n is a non-negative integer since,

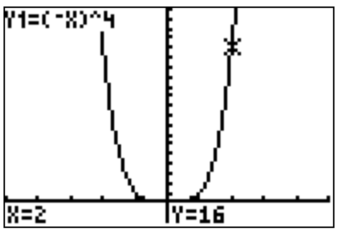
$$\begin{aligned}(-x)^{2n} &= (-1)^{2n}(x)^{2n}\\ &= x^{2n}\end{aligned}$$

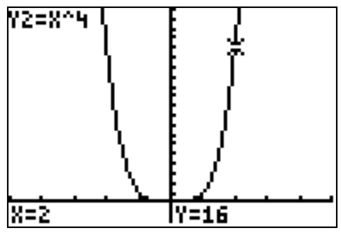
i)





ii)





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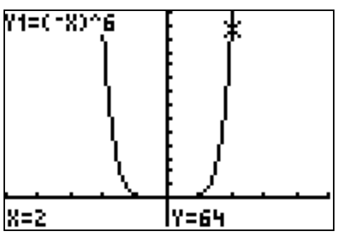
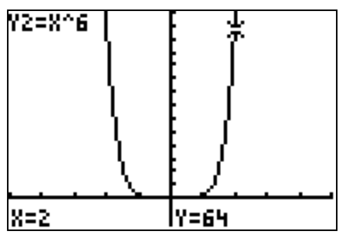
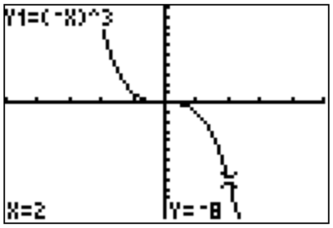


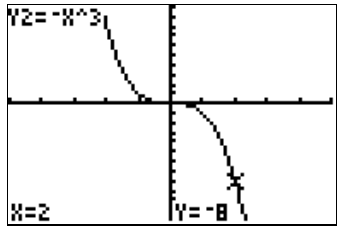


b) $y = (-x)^{2n+1}$ is the same graph as $y = -x^{2n+1}$, when n is a non-negative integer since,

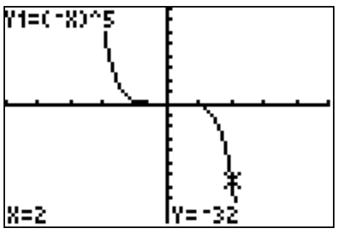
$$\begin{aligned}(-x)^{2n+1} &= (-1)^{2n+1}(x)^{2n+1} \\ &= -x^{2n+1}\end{aligned}$$

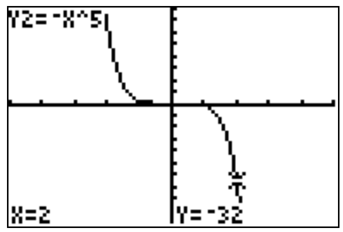
i)



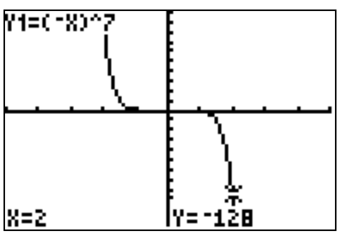


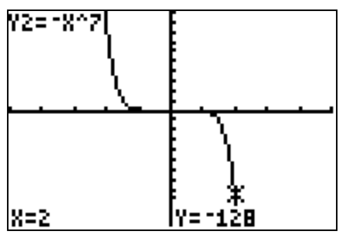
ii)





iii)



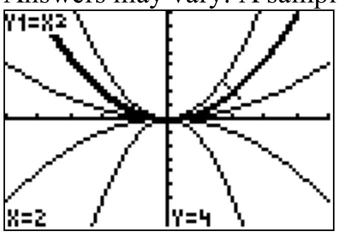


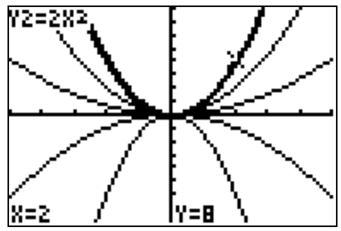
c) Answers may vary. A sample solution is shown.
When n is an even non-negative integer, $(-x)^n = x^n$.
When n is an odd non-negative integer, $(-x)^n = -x^n$.

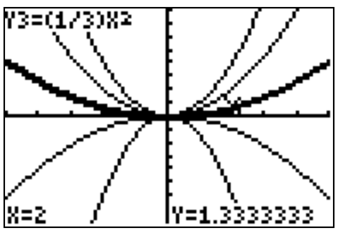
Chapter 1 Section 1 **Question 15 Page 14**

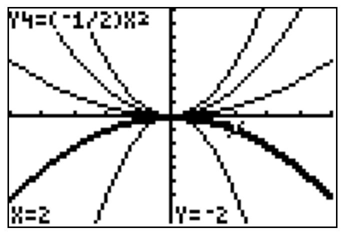
a) Answers may vary. A sample solution is shown.
For the graph of $y = ax^n$
$a > 0$; vertical stretch by a factor of a
$0 < a < 1$; vertical compression by a factor of a
$-1 < a < 0$; vertical compression by a factor of a and a reflection in the x-axis
$a < -1$; vertical stretch by a factor of a and a reflection in the x-axis

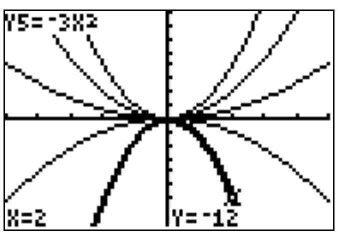
b) Answers may vary. A sample solution is shown.









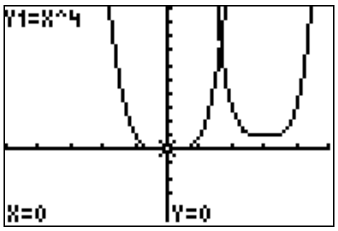


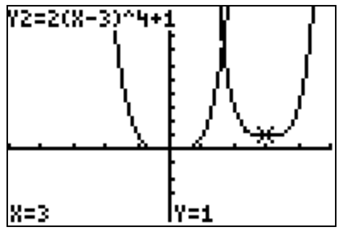
Chapter 1 Section 1 **Question 16 Page 14**

a) vertical stretch by a factor of 2
translation of 3 units right
translation of 1 unit up

b) Prediction:
vertical stretch by a factor of 2
translation of 3 units right
translation of 1 unit up

c)



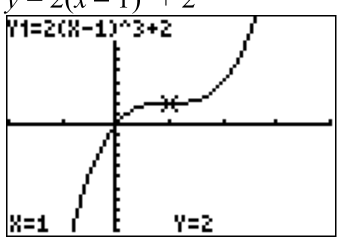


Chapter 1 Section 1 **Question 17 Page 14**

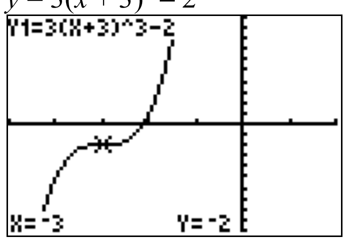
a) Prediction for $y = a(x - h)^3 + k$:
a is a vertical stretch or compression
h is a shift left or right
k is a shift up or down.

b) Answers may vary. A sample solution is shown.

$y = 2(x - 1)^3 + 2$



$y = 3(x + 3)^3 - 2$



Chapter 1 Section 1 **Question 18 Page 14**

$$\left(2^{120}\right)\left(5^{125}\right) = \left(2^{120}\right)\left(5^{120} \times 5^{5}\right)$$
$$= \left(2 \times 5\right)^{120}\left(5^{5}\right)$$
$$= \left(10^{120}\right)\left(3125\right)$$

10^{120} has 120 zeros and the leading 1, which is 121 digits.
Multiplying the leading 1 by 3125 gives me four digits plus the 120 zeros.
So, there are 124 digits in the number.

Chapter 1 Section 1 **Question 19 Page 14**

The distance between the A and B x-coordinates is 6, divided by 3 is 2.
The x-coordinate for the first trisecting point is $2 + 1(2) = 4$.
The x-coordinate for the second trisecting point is $2 + 2(2) = 6$.

The distance between the A and B y-coordinates is 1, divided by 3 is $-\frac{1}{3}$.

The y-coordinate for the first trisecting point is $3 - 1\left(\frac{1}{3}\right) = \frac{8}{3}$.

The y-coordinate for the second trisecting point is $3 - 2\left(\frac{1}{3}\right) = \frac{7}{3}$.

So, the coordinates of the two points that trisect A and B are $\left(4, \frac{8}{3}\right)$ and $\left(6, \frac{7}{3}\right)$.

Chapter 1 Section 2 **Characteristics of Polynomial Functions**

Chapter 1 Section 2 **Question 1 Page 26**

For parts a) to e), the least possible degree equals the number of x-intercepts.

a) The least possible degree of the function is 4.

b) The least possible degree of the function is 5.

c) The least possible degree of the function is 4.

d) The least possible degree of the function is 5.

e) The least possible degree of the function is 3.

Chapter 1 Section 2 **Question 2 Page 26**

	Sign of Leading Coefficient	**End Behaviour (quadrants)**	**Symmetry**	**Number of Maximum Points**	**Number of Minimum Points**	**Number of Local Maximum Points**	**Number of Local Minimum Points**
a)	+	2 to 1	none	0	1	1	2
b)	+	3 to 1	none	0	0	2	2
c)	–	3 to 4	none	1	0	2	1
d)	–	2 to 4	none	0	0	2	2
e)	–	2 to 4	point	0	0	1	1

d) If the function has a minimum and/or maximum point, the degree of the function is even. If the function has no minimum or maximum point, the degree of the function is odd.
The total number of local maximum and local minimum points is equal to or less than the degree of the function minus 1.

Chapter 1 Section 2 **Question 3 Page 26**

	i) End Behaviour (quadrants)	**ii) Constant Finite Differences**	**iii) Value of Constant Finite Differences**
a)	quadrant 2 to 1	2nd	2
b)	quadrant 2 to 4	3rd	–24
c)	quadrant 3 to 4	4th	–168
d)	quadrant 3 to 1	5th	72
e)	quadrant 2 to 4	1st	–1
f)	quadrant 3 to 4	6th	–720

iii)

a) $1(2 \times 1) = 2$

b) $-4(3 \times 2 \times 1) = -24$

c) $-7(4 \times 3 \times 2 \times 1) = -168$

d) $0.6(5 \times 4 \times 3 \times 2 \times 1) = 72$

e) $-1(1) = -1$

f) $-1(6 \times 5 \times 4 \times 3 \times 2 \times 1) = -720$

Chapter 1 Section 2 **Question 4 Page 27**

For parts a) to f), the *n*th difference equals the degree of the polynomial function.

a) The degree of the function is 2.

$$a(2\times 1) = -8$$
$$2a = -8$$
$$a = -4$$

The value of the leading coefficient is –4.

b) The degree of the function is 4.

$$a(4\times 3\times 2\times 1) = -48$$
$$24a = -48$$
$$a = -2$$

The value of the leading coefficient is –2.

c) The degree of the function is 3.

$$a(3\times 2\times 1) = -12$$
$$6a = -12$$
$$a = -2$$

The value of the leading coefficient is –2.

d) The degree of the function is 4.

$$a(4\times 3\times 2\times 1) = 24$$
$$24a = 24$$
$$a = 1$$

The value of the leading coefficient is 1.

e) The degree of the function is 3.

$$a(3\times 2\times 1) = 36$$
$$6a = 36$$
$$a = 6$$

The value of the leading coefficient is 6.

f) The degree of the function is 5.

$$a(5\times 4\times 3\times 2\times 1) = 60$$
$$120a = 60$$
$$a = \frac{1}{2}$$

The value of the leading coefficient is $\frac{1}{2}$.

Chapter 1 Section 2 **Question 5 Page 27**

a) This graph represents an odd-degree function since it has point symmetry about the origin, the graph extends from quadrant 2 to 4, there is no maximum or minimum point, the domain is $\{x \in \mathbb{R}\}$, and the range is $\{y \in \mathbb{R}\}$.

b) This graph represents an even-degree function since it has line symmetry, the graph extends from quadrant 2 to 1, it has a minimum point, and the range is restricted to $\{y \in \mathbb{R}, y \geq 0\}$.

c) This graph represents an odd-degree function since it has point symmetry, the graph extends from quadrant 3 to 1, there is no maximum or minimum point, the domain is $\{x \in \mathbb{R}\}$, and the range is $\{y \in \mathbb{R}\}$.

d) This graph represents an even-degree function since the graph extends from quadrant 3 to 4, there is a maximum point, and the range is restricted to $\{y \in \mathbb{R}, y \leq a\}$, where a is the maximum value of the function.

Chapter 1 Section 2 **Question 6 Page 27**

Graph	a) Least Possible Degree	b) Sign of the Leading Coefficient	c) End Behaviour (quadrants)	d) Symmetry
5a)	5	–	2 to 4	point
5b)	4	+	2 to 1	line
5c)	3	+	3 to 1	point
5d)	6	–	3 to 4	none

Chapter 1 Section 2 **Question 7 Page 27**

a)

x	y	First Differences	Second Differences	Third Differences
–3	–45			
		29		
–2	–16		–16	
		13		6
–1	–3		–10	
		3		6
0	0		–4	
		–1		6
1	–1		2	
		1		6
2	0		8	
		9		6
3	9		14	
		23		
4	32			

i) Since the third differences are constant, the degree of the polynomial function is 3.
ii) The sign of the leading coefficient is positive since the third differences are positive.
iii)

$$\begin{aligned} a(3\times 2\times 1) &= 6 \\ 6a &= 6 \\ a &= 1 \end{aligned}$$

The value of the leading coefficient is 1.

b)

x	y	First Differences	Second Differences	Third Differences	Fourth Differences
–2	–40				
		52			
–1	12		–44		
		8		42	
0	20		–2		–24
		6		18	
1	26		16		–24
		22		–6	
2	48		10		–24
		32		–30	
3	80		–20		–24
		12		–54	
4	92		–74		
		–62			
5	30				

i) Since the fourth differences are constant, the degree of the polynomial function is 4.
ii) The sign of the leading coefficient is negative since the fourth differences are negative.
iii)

$$\begin{aligned} a(4\times 3\times 2\times 1) &= -24 \\ 24a &= 24 \\ a &= -1 \end{aligned}$$

The value of the leading coefficient is –1.

Chapter 1 Section 2 **Question 8 Page 28**

a) $P(x)$ is a quartic function.

b) The 4th differences are the finite constant differences, since the degree is 4.

$0.001\,25(4 \times 3 \times 2 \times 1) = 0.03$

The value of the constant finite difference is 0.03.

c) End Behaviour: The graph extends from quadrant 2 to 1.

d) Domain: $\{x \in \mathbb{R}, x \geq 0\}$

e) Answers may vary. For example, they represent when the profit is equal to zero.

f) $P(30) = 30 + 0.001\,25(30)^4 - 3$

$= 1039.5$

The profit from the sale of 3000 snowboards is $1 039 500.

Chapter 1 Section 2 **Question 9 Page 28**

a)

x	y	First Differences	Second Differences	Third Differences
0	10			
		24		
1	34		–16	
		8		12
2	42		–4	
		4		12
3	46		8	
		12		12
4	58		20	
		32		12
5	90		32	
		64		12
6	154		44	
		108		
7	262			

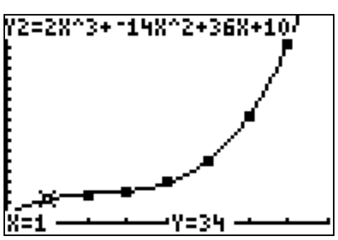
i) The degree of this function is 3.

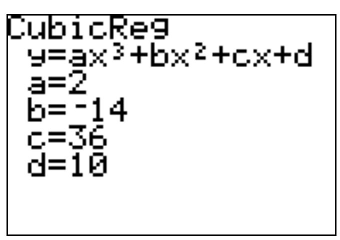
ii)

$$a(3 \times 2 \times 1) = 12$$
$$6a = 12$$
$$a = 2$$

The value of the leading coefficient is 2.

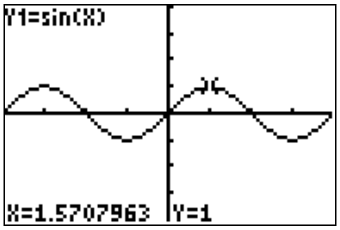
b)

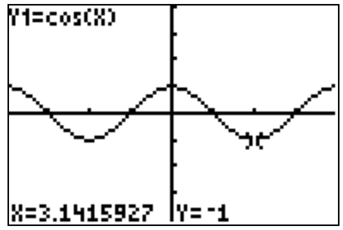




Chapter 1 Section 2 **Question 10 Page 28**

a)

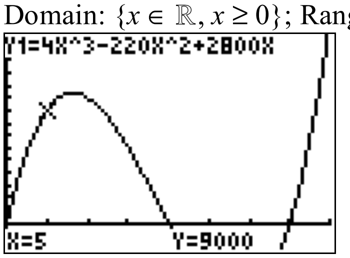




b) Answers may vary. Sample answer: Both have smooth curves (no corners or discontinuities), and for every x-value, there is one and only one y-value, but there can be multiple x-values with the same y-value, and some y-values are never reached by the graphs.
A polynomial function has a finite number of local minima and maxima, while a periodic function such as the sine function has an infinite number.
As a polynomial function goes to infinity (positive or negative) the y-values also go to infinity (either positive or negative). The periodic function repeats the same pattern at either end of the graph.

Chapter 1 Section 2 **Question 11 Page 28**

a) Domain: $\{x \in \mathbb{R}, x \geq 0\}$; Range: $\{y \in \mathbb{R}, y \geq 0\}$



b) $V(x) = 4x(x - 35)(x - 20)$; $x = 35$, $x = 20$, and $x = 0$ are the x-intercepts.

c) $4(3 \times 2 \times 1) = 24$
The constant finite difference for this function is 24.

Chapter 1 Section 2 **Question 12 Page 28**

a) $r(d)$ is a cubic function.

b) The 3rd differences are constant since the degree of the function is 3.
$-0.7(3 \times 2 \times 1) = -4.2$
The value of the constant differences is -4.2.

c) End Behaviour: The graph extends from quadrant 2 to 4.

d) Domain: $\{d \in \mathbb{R}, d \geq 0\}$; Range: $\{r(d) \in \mathbb{R}, r(d) \geq 0\}$

Chapter 1 Section 2 **Question 13 Page 28**

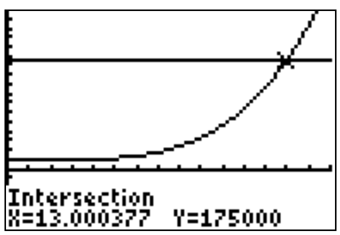
a) Answers may vary. A sample solution is shown.
– End Behaviour: extends from quadrant 2 to 1,
– Domain: $\{t \in \mathbb{R}\}$, Range: $\{P(t) \in \mathbb{R}, P(t) \geq 11\,732\}$
– No x-intercepts

b) The 4th differences are the constant finite differences since the degree of the function is 4.
$6(4 \times 3 \times 2 \times 1) = 144$
The value of the constant differences is 144.

c) $p(0) = 6(0)^4 - 5(0)^3 + 200(0) + 12\,000$
$= 12\,000$
The current population of the town is 12 000.

d) $p(10) = 6(10)^4 - 5(10)^3 + 200(10) + 12\,000$
$= 69\,000$
The population of the town 10 years from now will be 69 000.

e)



The population will be 175 000 in 13 years.

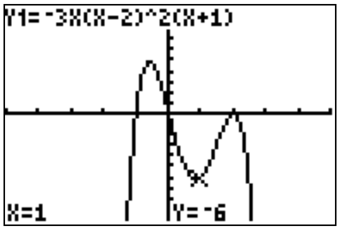
Chapter 1 Section 2 **Question 14 Page 29**

Solutions to Achievement Check questions are provided in the Teacher's Resource.

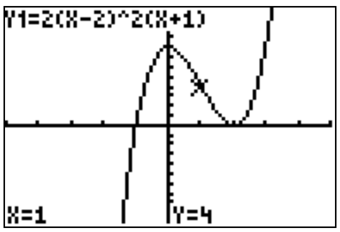
Chapter 1 Section 2 **Question 15 Page 29**

Answers may vary. A sample solution is shown.

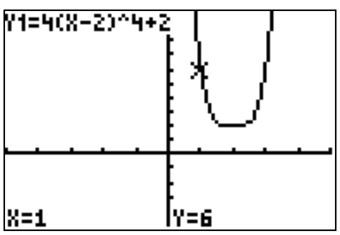
a)



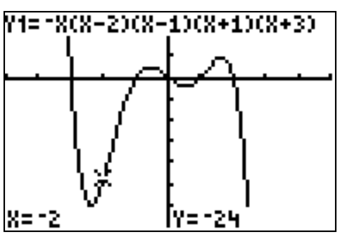
b)



c)



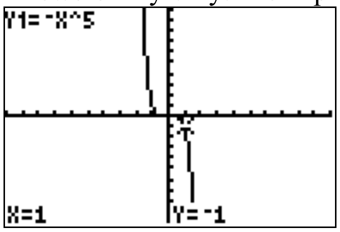
d)

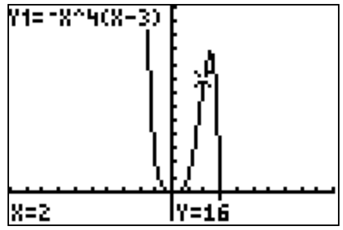


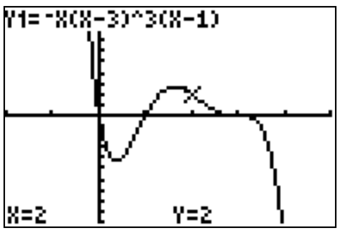
Chapter 1 Section 2 **Question 16 Page 29**

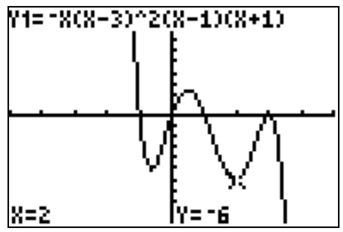
a) A quintic function can have 1 to 5 *x*-intercepts.

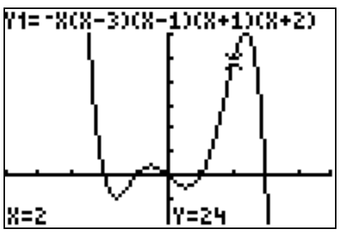
b) Answers may vary. A sample solution is shown.











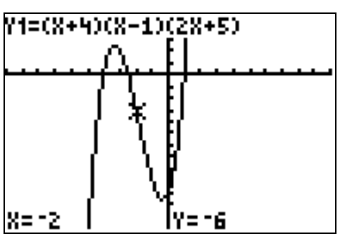
Chapter 1 Section 2 **Question 17 Page 29**

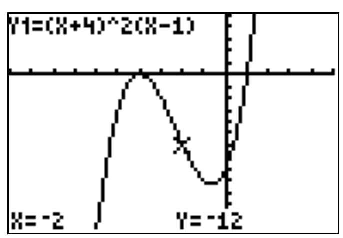
a) i) This is a cubic function since the degree is 3.

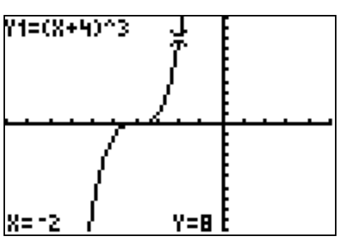
ii) This is a cubic function since the degree is 3.

iii) This is a cubic function since the degree is 3.

b)







c) Answers may vary. A sample solution is shown.
The number of x-intercepts equal the number of roots of the equation.

Chapter 1 Section 2 **Question 18 Page 29**

a) **i)**

$$\begin{aligned} S(r) &= 2\pi r^2 h + 2\pi rh \\ &= 2\pi rh(r+1) \\ &= 2\pi r(3r)(r+1) \\ &= 6\pi r^2(r+1) \end{aligned}$$

ii)

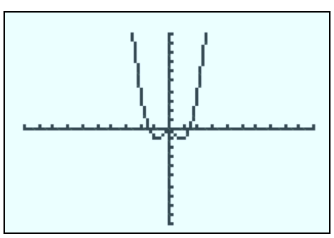
$$\begin{aligned} V(r) &= \pi r^2 h \\ &= 3\pi r^3 \end{aligned}$$

b) Answers may vary. A sample solution is shown.
$S(r)$:
Cubic function
has 2 x-intercepts
Domain: $\{x \in \mathbb{R}\}$, Range: $\{y \in \mathbb{R}\}$
Extends from quadrant 3 to 1

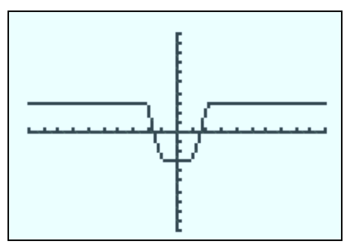
$V(r)$:
Cubic function
has 1 x-intercept
Domain: $\{x \in \mathbb{R}\}$, Range: $\{y \in \mathbb{R}\}$
Extends from quadrant 3 to 1

Chapter 1 Section 2 **Question 19 Page 29**

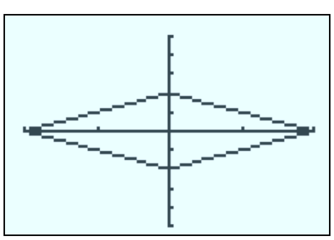
a)



b)



c)



Chapter 1 Section 3 **Equations and Graphs of Polynomial Functions**

Chapter 1 Section 3 **Question 1 Page 39**

	Degree of the Function	**Sign of Leading Coefficient**	**End Behaviour (quadrants)**	***x*-intercepts**
a)	3	+	quadrant 3 to 1	$4, -3, \frac{1}{2}$
b)	4	–	quadrant 3 to 4	–2, 2, 1, –1
c)	5	+	quadrant 3 to 1	$-\frac{2}{3}, \frac{3}{2}, 4, -1$
d)	6	–	quadrant 3 to 4	–5, 5

Chapter 1 Section 3 **Question 2 Page 39**

a) i) The x-intercepts are –4, $-\frac{1}{2}$, and 1.

ii) Positive Intervals: $x > 1$, $-4 < x < -\frac{1}{2}$; Negative Intervals: $x < -4$, $-\frac{1}{2} < x < 1$

iii) The function has no zeros of order 2 or 3.

b) i) The x-intercepts are –1 and 4.

ii) Positive Intervals: none; Negative Intervals: $x < -1$, $-1 < x < 4$, $x > 4$

iii) The function could have zeros of order 2.

c) i) The x-intercepts are –3 and 1.

ii) Positive Intervals: $x < -3$, $x > 1$; Negative Intervals: $-3 < x < 1$

iii) The function could have zeros of order 3.

d) i) The x-intercepts are –5 and 3.

ii) Positive Intervals: $x < -5$, $-5 < x < 3$; Negative Intervals: $x > 3$

iii) The function could have zeros of order 2.

Chapter 1 Section 3 **Question 3 Page 39**

a) **i)** $3, -2, \frac{3}{4}$; all order 1

ii) $1, -1, -3, 3$; all order 1

iii) order 2: $-4, 1$; order 1: $-2, \frac{3}{2}$

iv) order 3: $\frac{2}{3}$; order 2: 5; order 1: -6

b) The graph in **ii)** is even, all others are neither even nor odd.

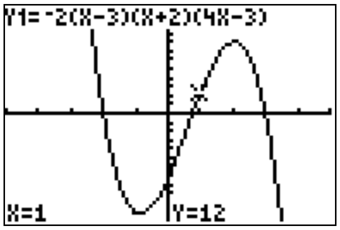
i) $f(x) = -2(x-3)(x+2)(4x-3)$
$f(-x) = -2(-x-3)(-x+2)(-4x-3)$
$= 2(x+3)(x-2)(4x+3)$
$-f(x) = 2(x-3)(x+2)(4x-3)$
$f(-x) \neq f(x)$; this is not an even function.
$f(-x) \neq -f(x)$; this is not an odd function.

ii) $g(x) = (x-1)(x+3)(1+x)(3x-9)$
$= 3(x-1)(x+3)(x+1)(x-3)$
$g(-x) = (-x-1)(-x+3)(1-x)(-3x-9)$
$= 3(x+1)(x-3)(x-1)(x+3)$
$g(-x) = g(x)$; this is an even function.

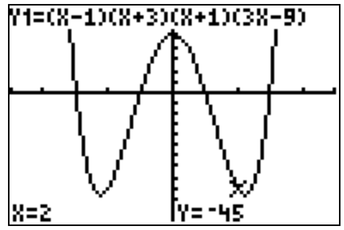
iii) $h(x) = -(x+4)^2(x-1)^2(x+2)(2x-3)$
$h(-x) = -(-x+4)^2(-x-1)^2(-x+2)(-2x-3)$
$= -(x-4)^2(x+1)^2(x-2)(2x+3)$
$-h(x) = (x+4)^2(x-1)^2(x+2)(2x-3)$
$h(-x) \neq h(x)$; this is not an even function.
$h(-x) \neq -h(x)$; this is not an odd function.

iv) $p(x) = 3(x+6)(x-5)^2(3x-2)^3$
$p(-x) = 3(-x+6)(-x-5)^2(-3x-2)^3$
$= 3(x-6)(x+5)^2(3x+2)^3$
$-p(x) = -3(x+6)(x-5)^2(3x-2)^3$
$p(-x) \neq p(x)$; this is not an even function.
$p(-x) \neq -p(x)$; this is not an odd function.

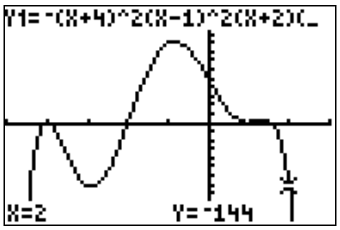
c) **i)**



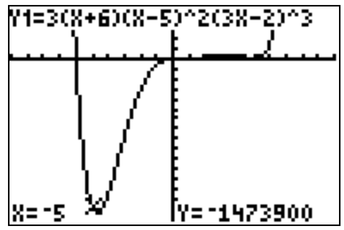
ii)



iii)



iv)



Chapter 1 Section 3 **Question 4 Page 39**

a) Since $f(x)$ is a cubic function, it may be odd and thus have point symmetry about the origin.

$f(-x) = (-x - 4)(-x + 3)(2(-x) - 1)$
$\quad = -(x + 4)(x - 3)(2x + 1)$
$-f(x) = -(x - 4)(x + 3)(2x - 1)$
$f(-x) \neq -f(x)$; the function is not odd and does not have point symmetry about the origin.

b) Since $g(x)$ is a quartic function, it may be even and thus have line symmetry about the y-axis.

$g(x) = -2(x + 2)(x - 2)(x - 1)(x + 1)$
$g(-x) = -2(-x + 2)(-x - 2)(-x - 1)(-x + 1)$
$\quad = -2(x - 2)(x + 2)(x + 1)(x - 1)$
$g(x) = g(-x)$; the function is even and has line symmetry about the y-axis.

c) Since $h(x)$ is a quintic function, it may be odd and thus have point symmetry about the origin.

$h(-x) = (3(-x) + 2)^2(-x - 4)(-x + 1)(2(-x) - 3)$
$\quad = -(3x - 2)^2(x + 4)(x - 1)(2x + 3)$
$-h(x) = -(3x + 2)^2(x - 4)(x + 1)(2x - 3)$
$h(-x) \neq -h(x)$; the function is not odd and does not have point symmetry about the origin.

d) Since $p(x)$ is a function with degree 6, it may be even and thus have line symmetry about the y-axis.

$p(x) = -(x + 5)^3(x - 5)^3$
$p(-x) = -(-x + 5)^3(-x - 5)^3$
$\quad = -(-1)^3(x - 5)^3(-1)^3(x + 5)^3$
$\quad = -(x - 5)^3(x + 5)^3$
$p(x) = p(-x)$; the function is even and has line symmetry about the y-axis.

Chapter 1 Section 3 **Question 5 Page 40**

a) i), ii) Since the exponent of each term is even, $f(x) = x^4 - x^2$ is an even function and has line symmetry about the y-axis.
Verify that $f(-x) = f(x)$.

$$\begin{aligned} f(-x) &= (-x)^4 - (-x)^2 \\ &= x^4 - x^2 \\ &= f(x) \end{aligned}$$

b) i), ii) Since the exponent of each term is odd, $f(x) = -2x^3 + 5x$ is an odd function and has point symmetry about the origin.
Verify that $f(-x) = -f(x)$.

$$\begin{aligned} f(-x) &= -2(-x)^3 + 5(-x) \\ &= 2x^3 - 5x \\ -f(x) &= -(-2x^3 + 5x) \\ &= 2x^3 - 5x \\ f(-x) &= -f(x) \end{aligned}$$

c) i), ii) One of the exponents in $f(x) = -4x^5 + 2x^2$ is even and one is odd, so the function is neither even nor odd and thus does not have either point symmetry about the origin or line symmetry about the y-axis.

d) i), ii) Since $f(x) = x(2x + 1)^2(x - 4)$ is a quartic function, it may be even and thus have line symmetry about the y-axis.

$$\begin{aligned} f(-x) &= (-x)(2(-x) + 1)^2(-x - 4) \\ &= -x(-1)^2(2x - 1)^2(-1)(x + 4) \\ &= x(2x - 1)^2(x + 4) \end{aligned}$$

Since $f(-x) \neq f(x)$, the function is not even and thus does not have line symmetry about the y-axis.

e) i), ii) Since the exponent of each term is even, $f(x) = -2x^6 + x^4 + 8$ is an even function and has line symmetry about the y-axis.
Verify that $f(-x) = f(x)$.

$$\begin{aligned} f(-x) &= -2(-x)^6 + (-x)^4 + 8 \\ &= -2x^6 + x^4 + 8 \\ &= f(x) \end{aligned}$$

Chapter 1 Section 3 **Question 6 Page 40**

a) x-intercepts: –2 (order 3), 3 (order 2)
Equation: $y = (x + 2)^3(x - 3)^2$

b) x-intercepts: –3, –1, 2
$y = a(x + 3)(x + 1)(x - 2)$
Substitute the point (0, 12) from the graph to find a.
$12 = a(3)(1)(-2)$
$a = -2$
Equation: $y = -2(x + 3)(x + 1)(x - 2)$

c) x-intercepts: –2(order 2), 1(order 2)

$y = a(x + 2)^2(x – 1)^2$

Substitute the point (0, –12) from the graph to find a.

$–12 = a(2)^2(–1)^2$

$a = –3$

Equation: $y = –3(x + 2)^2(x – 1)^2$

d) x-intercepts: –2(order 3), 1(order 2)

$y = a(x + 2)^3(x – 1)^2$

Substitute the point (0, 4) from the graph to find a.

$4 = a(2)^3(–1)^2$

$a = 0.5$

Equation: $y = 0.5(x + 2)^3(x – 1)^2$

Chapter 1 Section 3 **Question 7 Page 40**

a) $y = a(x + 2)^2(x – 3)$

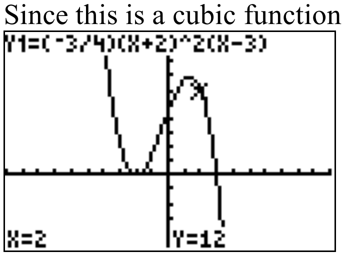
Since y-intercept is 9,

$9 = a(0 + 2)^2(0 – 3)$

$a = -\frac{3}{4}$

Equation: $y = -\frac{3}{4}(x + 2)^2(x – 3)$

Since this is a cubic function it could be odd, but from the graph we can see that is not odd.



b) $y = a(x + 1)^3(x – 1)$

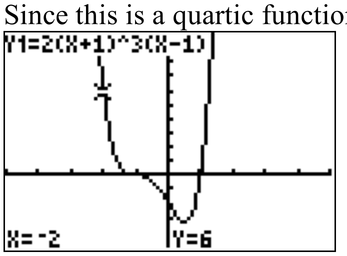
Since the y-intercept is –2,

$–2 = a(0 + 1)^3(0 – 1)$

$a = 2$

Equation: $y = 2(x + 1)^3(x – 1)$

Since this is a quartic function it could be even, but from the graph we can see that is not even.



c) $y = a(x + 1)^3(x - 3)^2$

Substitute the point (–2, 50) to find a.

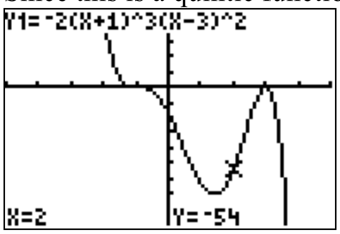
$50 = a((-2) + 1)^3((-2) - 3)^2$

$50 = a(-1)^3(-5)^2$

$a = -2$

Equation: $y = -2(x + 1)^3(x - 3)^2$

Since this is a quintic function it could be odd, but from the graph we can see that is not odd.



d) $y = a(x + 3)(x + 2)^2(x - 2)^2$

Substitute the point (1, –18) to find a.

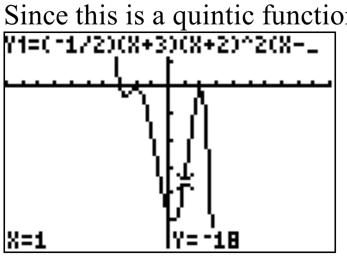
$-18 = a(1 + 3)(1 + 2)^2(1 - 2)^2$

$-18 = a(4)(3)^2(-1)^2$

$a = -\frac{1}{2}$

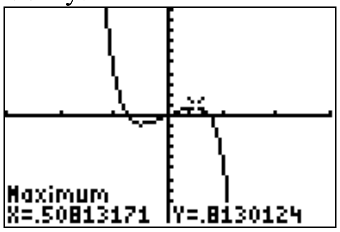
Equation: $y = -\frac{1}{2}(x + 3)(x + 2)^2(x - 2)^2$

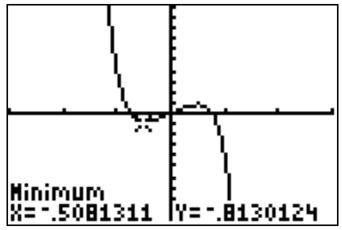
Since this is a quintic function it could be odd, but from the graph we can see that is not odd.



a) Since the exponent of each term is odd, $f(x) = -6x^5 + 2x$ is an odd function and has point symmetry about the origin.

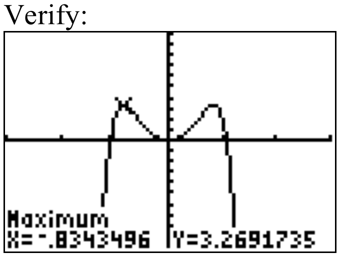
Verify:

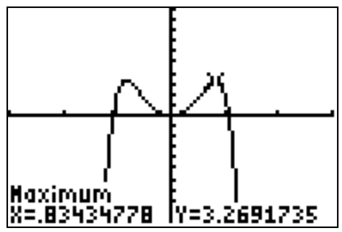




b) Since the exponent of each term is even, $g(x) = -7x^6 + 3x^4 + 6x^2$ is an even function and has line symmetry about the y-axis.

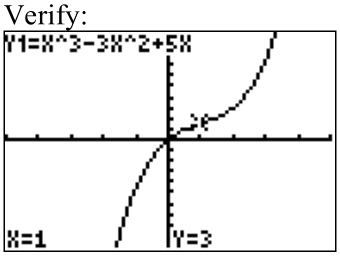
Verify:

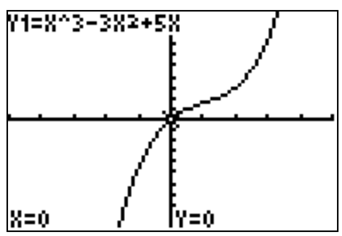


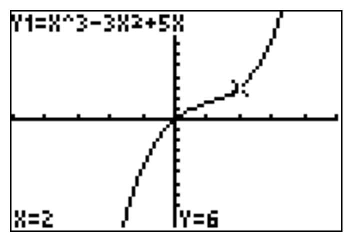


c) Since there are two odd exponents and one even exponent in $h(x) = x^3 - 3x^2 + 5x$, the function is neither odd nor even. The function may have point symmetry since it is cubic.

Verify:



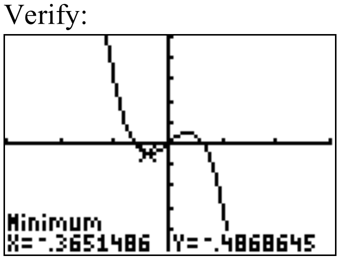


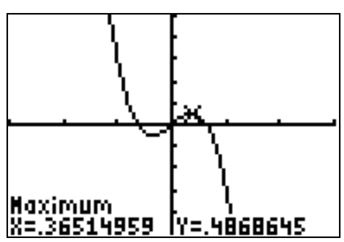


The vertical distance from (1, 3) to (0, 0) and (1, 3) to (2, 6) is 3.
The function has point symmetry.

d) Since the exponent of each term is odd, $p(x) = -5x^3 + 2x$ is an odd function and has point symmetry about the origin.

Verify:

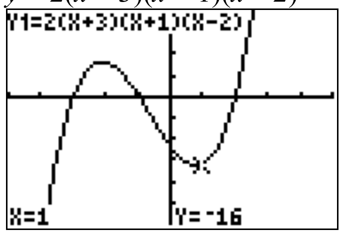




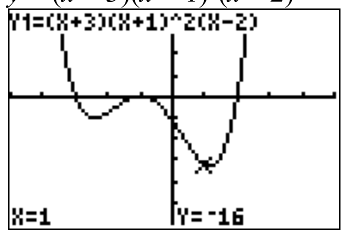
Chapter 1 Section 3 **Question 9 Page 40**

Answers may vary. A sample solution is shown.

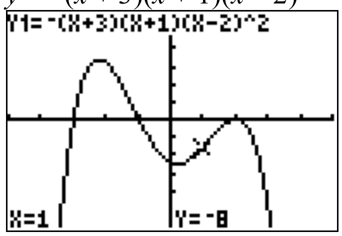
a) $y = 2(x + 3)(x + 1)(x – 2)$



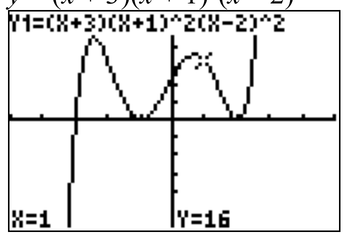
b) $y = (x + 3)(x + 1)^2(x – 2)$



c) $y = –(x + 3)(x + 1)(x – 2)^2$



d) $y = (x + 3)(x + 1)^2(x – 2)^2$

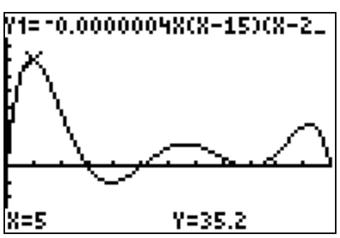


Chapter 1 Section 3 **Question 10 Page 41**

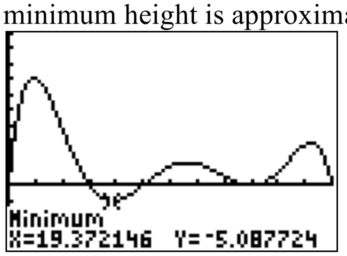
a) Answers may vary. For example, this polynomial function is the fully factored version of the equations learned in Sections 1 and 2.

b) Answers may vary. For example, the equation provides information about the x-intercepts, the degree of the function, the sign of the leading coefficient, and the order of the zeros.

c)



d) The maximum height is approximately 35.3 m above the platform (see graph above). The minimum height is approximately 5.1 m below the platform (see graph below).

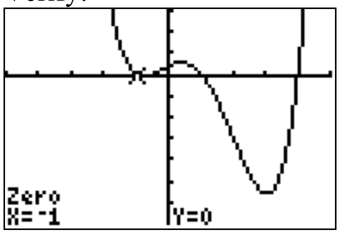


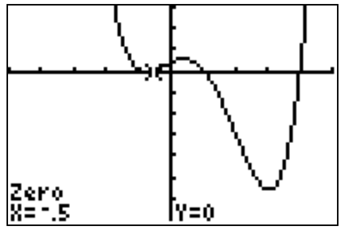
Chapter 1 Section 3 **Question 11 Page 41**

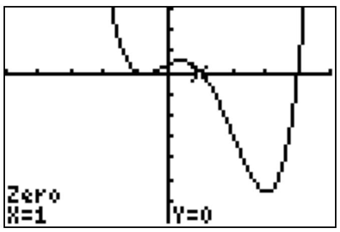
a) $f(x) = (2x + 1)(x - 1)(x - 4)(x + 1)$

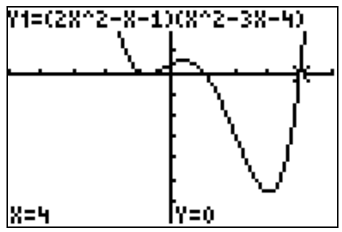
The zeros are 4, 1, –1, and $-\frac{1}{2}$.

b) Verify:









Chapter 1 Section 3 **Question 12 Page 41**

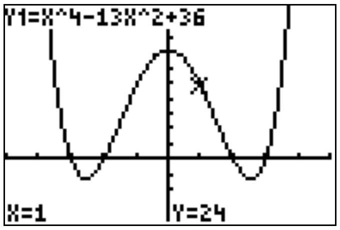
a) i) $f(x) = x^4 - 13x^2 + 36$
$= (x - 3)(x - 2)(x + 2)(x + 3)$
The zeros are 3, 2, –2, and –3.

ii) $g(x) = 6x^5 - 7x^3 - 3x$
$= x\left(\sqrt{2}x + \sqrt{3}\right)\left(\sqrt{2}x - \sqrt{3}\right)\left(3x^2 + 1\right)$
The zeros are 0, $-\frac{\sqrt{6}}{2}$, and $\frac{\sqrt{6}}{2}$.

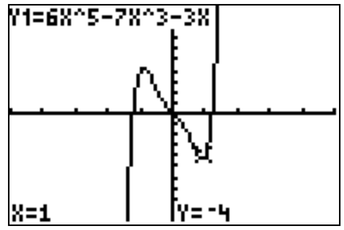
b) i) Since the exponent of each term is even, $f(x)$ is an even function.
Verify that $f(-x) = f(x)$.
$f(-x) = (-x - 3)(-x - 2)(-x + 2)(-x + 3)$
$= (-1)(x + 3)(-1)(x + 2)(-1)(x - 2)(-1)(x - 3)$
$= (x + 3)(x + 2)(x - 2)(x - 3)$
$= f(x)$

ii) Since the exponent of each term is odd, $g(x) = -6x^5 - 7x^3 - 3x$ is an odd function.
Verify that $g(-x) = -g(x)$.
$g(-x) = -6(-x)^5 - 7(-x)^3 - 3(-x)$
$= 6x^5 + 7x^3 + 3x$
$-g(x) = -(-6x^5 - 7x^3 - 3x)$
$= 6x^5 + 7x^3 + 3x$
$g(-x) = -g(x)$

c) i)



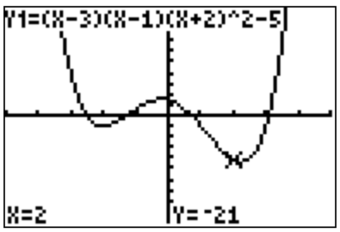
ii)



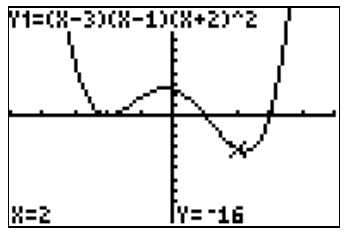
Chapter 1 Section 3 **Question 13 Page 41**

Answers may vary. A sample solution is shown.

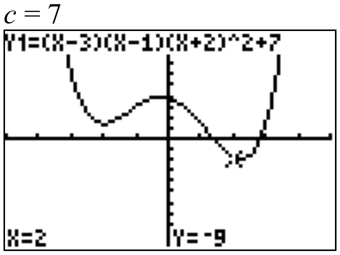
a) $c = -5$



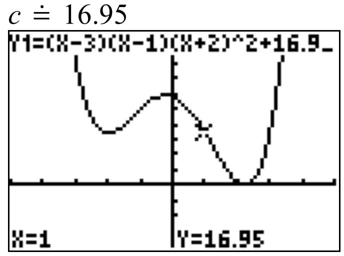
b) $c = 0$



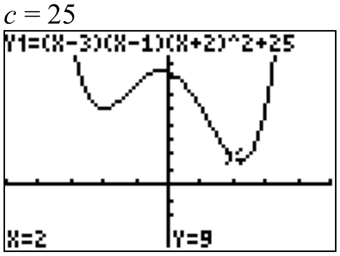
c) $c = 7$



d) $c \doteq 16.95$



e) $c = 25$



Chapter 1 Section 3 **Question 14 Page 41**

a) Answers may vary. A sample solution is shown.
$f(x) = (3x - 2)(3x + 2)(x - 5)(x + 5)$
$g(x) = 2(3x - 2)(3x + 2)(x - 5)(x + 5)$

b) $y = a(3x - 2)(x - 5)$
Substitute the point (–1, –96) to solve for a.
$-96 = a(3(-1) - 2)(-1 - 5)$
$-96 = a(-5)(-6)$
$a = -3.2$
Equation: $y = -3.2(3x - 2)(x - 5)$

c) $y = -[-3.2(3x - 2)(x - 5)]$
$y = 3.2(3x - 2)(x - 5)$

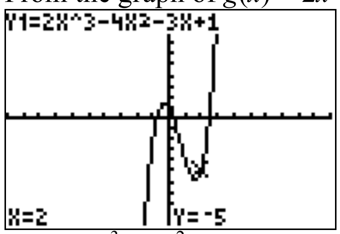
Chapter 1 Section 3 **Question 15 Page 41**

Answers may vary. A sample solution is shown.
An odd function must go through the origin and since a nonzero constant is added, the function shifts away from the origin, either upwards or downwards. Since the function no longer passes through the origin, it is no longer an odd function.
For example, let $f(x) = x^3$, which is an odd function.
$g(x) = f(x) + k$
$= x^3 + k$, where k is a nonzero constant
$g(-x) = (-x)^3 + k$
$= -x^3 + k$
$-g(x) = -(x^3 + k)$
$= -x^3 - \mathrm{k}$
$g(-x) \neq -g(x)$, so the function is not odd.

Chapter 1 Section 3 **Question 16 Page 41**

a) $f(x) = x^3 - 3x + 1$
$f(0) = 1, f(0.5) = -0.375, f(1) = -1$
The function changes sign between $x = 0$ and $x = 0.5$, so a root lies in this interval.
Since $f(0.25) \doteq 0.266$, there is a root between $x = 0.25$ and $x = 0.5$.
Since $f(0.38) \doteq -0.085$, there is a root between $x = 0.25$ and $x = 0.38$.
Since $f(0.32) \doteq 0.073$, there is a root between $x = 0.32$ and $x = 0.38$.
Since $f(0.35) \doteq -0.007$, this is the approximate root of $f(x)$ that lies between 0 and 1.

b) From the graph of $g(x) = 2x^3 - 4x^2 - 3x + 1$, the greatest root is between 2 and 3.



$g(x) = 2x^3 - 4x^2 - 3x + 1$
$g(2) = -5, g(2.5) = -0.25, g(3) = 10$
The function changes sign between $x = 2.5$ and $x = 3$, so a root lies in this interval.
Since $g(2.75) \doteq 4.094$, there is a root between $x = 2.5$ and $x = 2.75$.
Since $g(2.625) \doteq 1.738$, there is a root between $x = 2.5$ and $x = 2.625$.
Since $g(2.563) \doteq 0.708$, there is a root between $x = 2.5$ and $x = 2.563$.
Since $g(2.532) \doteq 0.225$, there is a root between $x = 2.5$ and $x = 2.532$.
Since $g(2.516) \doteq -0.015$, there is a root between $x = 2.516$ and $x = 2.532$.
Since $g(2.524) \doteq 0.104$, there is a root between $x = 2.516$ and $x = 2.524$.
Since $g(2.52) \doteq 0.044$, there is a root between $x = 2.516$ and $x = 2.52$.
Since $g(2.518) \doteq 0.015$, there is a root between $x = 2.516$ and $x = 2.518$.
Since $g(2.517) \doteq -0.0003$, this is the approximate greatest root of $g(x)$.

Chapter 1 Section 4 **Transformations**

Chapter 1 Section 4 **Question 1 Page 49**

a) $a = 4$; vertical stretch by a factor of 4

$k = 3$; horizontal compression by a factor of $\frac{1}{3}$

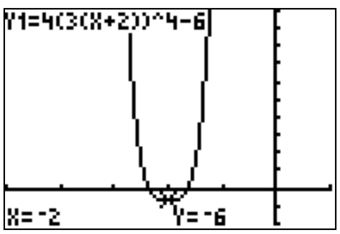
$d = -2$; translation 2 units left

$c = -6$; translation 6 units down

b)

$y = x^4$	$y = (3x)^4$	$y = 4(3x)^4$	$y = 4[3(x + 2)^4] - 6$
(–2, 16)	(–2, 1296)	(–2, 5184)	(–2, –6)
(–1, 1)	(–1, 81)	(–1, 324)	(–1, 318)
(0, 0)	(0, 0)	(0, 0)	(0, 5178)
(1, 1)	(1, 81)	(1, 324)	(1, 26 238)
(2, 16)	(2, 1296)	(2, 5184)	(2, 82 938)

c)



d) Domain: $\{x \in \mathbb{R}\}$; Range: $\{y \in \mathbb{R}, y \geq -6\}$;
Vertex: (–2, –6); Axis of Symmetry: $x = -2$

Chapter 1 Section 4 **Question 2 Page 50**

a) **ii)** reflection in the x-axis

b) **iv)** reflection in the y-axis

c) **iii)** reflection in the x-axis and the y-axis

d) **i)** no reflection

Chapter 1 Section 4 **Question 3 Page 50**

a) iii) vertically stretched by a factor of 2

b) iv) horizontally compressed by a factor of $\frac{1}{2}$

c) ii) vertically compressed by a factor of $\frac{1}{2}$

d) i) horizontally stretched by a factor of 2

Chapter 1 Section 4 **Question 4 Page 50**

a) $k = 3$; horizontal compression by a factor of $\frac{1}{3}$
$c = -1$; vertical translation 1 unit down
$n = 3$; degree of the function

b) $a = 0.4$; vertical compression by a factor of 0.4
$d = -2$; horizontal translation of 2 units left
$n = 2$; degree of the function

c) $c = 5$; vertical translation of 5 units up
$n = 3$; degree of the function

d) $a = \frac{3}{4}$; vertical compression by a factor of $\frac{3}{4}$
$k = -1$; reflection in the y-axis
$d = 4$; horizontal translation 4 units right
$c = 1$; vertical translation 1 unit up
$n = 3$; degree of the function

e) $a = 2$; vertical stretch by a factor of 2
$k = \frac{1}{3}$; horizontal stretch by a factor of 3
$c = -5$; vertical translation 5 units down
$n = 4$; degree of the function

f) Put $y = 8\left[\left(2x\right)^3 + 3\right]$ into the form $y = a\left[k(x-d)\right]^n + c$.

$y = 8(2x)^3 + 24$
$a = 8$; vertical stretch by a factor of 8
$k = 2$; horizontal compression by a factor of $\frac{1}{2}$
$c = 24$; vertical translation 24 units up
$n = 3$; degree of the function

Chapter 1 Section 4 **Question 5 Page 50**

a) **ii)** $y = x^3$ shifted down 1

b) **iv)** $y = x^4$ reflected in the x-axis

c) **i)** $y = x^3$ vertically compressed by a factor of $\frac{1}{4}$ and reflected in the x-axis

d) **iii)** $y = x^5$ horizontally stretched by a factor of 4 and reflected in the y-axis.

Chapter 1 Section 4 **Question 6 Page 51**

a) translation of 2 units left and 1 unit down; $y = (x + 2)^2 - 1$

b) translation of 4 units right and 5 units up; $y = (x - 4)^3 + 5$

Chapter 1 Section 4 **Question 7 Page 51**

a) $a = -3$; $k = \frac{1}{2}$; $d = -4$; $c = 1$

b) a: vertical stretch by a factor of 3 and a reflection in the x-axis
k: horizontal stretch by a factor of 2
d: 4 units left
c: 1 unit up

c) Domain: $\{x \in \mathbb{R}\}$; Range: $\{y \in \mathbb{R}, y \leq 1\}$; Vertex: $(-4, 1)$; Axis of Symmetry: $x = -4$

d) Answers may vary. A sample solution is shown.
1. Vertical stretch by a factor of 3, horizontal stretch by a factor of 2, reflection in the x-axis, left 4 units, and up 1 unit.
2. Horizontal stretch by a factor of 2, vertical stretch by a factor of 3, reflection in the x-axis, up 1 unit, and left 4 units.

Chapter 1 Section 4 **Question 8 Page 51**

a) Vertical compression by a factor of 0.5, a reflection in the x-axis, 4 units right
$f(x) = -0.5(x - 4)^3$

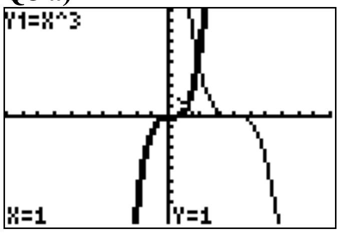
b) Reflection in x-axis, horizontal compression by a factor of $\frac{1}{4}$, 1 unit up
$f(x) = -(4x)^4 + 1$

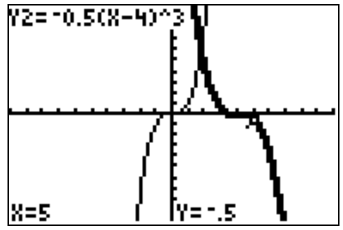
c) Vertical stretch by a factor of 2, horizontal stretch by a factor of 3, 5 units right, 2 units down
$$f(x) = 2\left[\frac{1}{3}(x-5)\right]^3 - 2$$

Chapter 1 Section 4 **Question 9 Page 51**

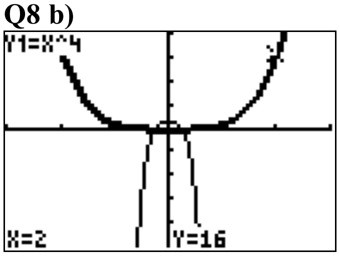
a)

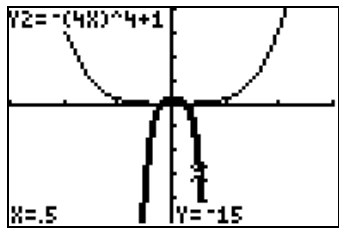
Q8 a)



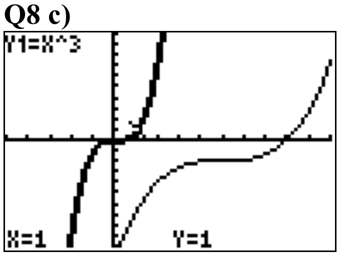


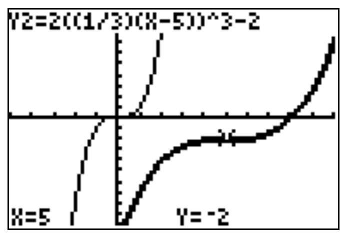
Q8 b)





Q8 c)





b) Q8 a) Domain: $\{x \in \mathbb{R}\}$; Range: $\{y \in \mathbb{R}\}$
Q8 b) Domain: $\{x \in \mathbb{R}\}$; Range: $\{y \in \mathbb{R}, y \le 1\}$; Vertex: (0, 1); Axis of Symmetry: $x = 0$
Q8 c) Domain: $\{x \in \mathbb{R}\}$; Range: $\{y \in \mathbb{R}\}$

Chapter 1 Section 4 **Question 10 Page 51**

a) i) $y = -x^4 + 2$
ii) Domain: $\{x \in \mathbb{R}\}$; Range: $\{y \in \mathbb{R}, y \le 2\}$; Vertex: (0, 2); Axis of Symmetry: $x = 0$

b) i) $y = -(x - 5)^3$
ii) Domain: $\{x \in \mathbb{R}\}$; Range: $\{y \in \mathbb{R}\}$

c) i) $y = (x + 3)^2 - 5$
ii) Domain: $\{x \in \mathbb{R}\}$; Range: $\{y \in \mathbb{R}, y \ge -5\}$; Vertex (–3, –5); Axis of Symmetry: $x = -3$

Chapter 1 Section 4 **Question 11 Page 51**

The equations needed are:
$y = x^3, y = (x-4)^3, y = (x+4)^3,$
$y = x^4 - 6, y = (x-4)^4 - 6, y = (x+4)^4 - 6,$
$y = -x^4 + 6, y = -(x-4)^4 + 6, y = -(x+4)^4 + 6$

Chapter 1 Section 4 **Question 12 Page 52**

a) i) $f(x) = (x + 2)^4 + 3$
ii) Domain: $\{x \in \mathbb{R}\}$; Range: $\{y \in \mathbb{R}, y \geq 3\}$; Vertex: (–2, 3); Axis of Symmetry: $x = -2$

b) i) $f(x) = \left[\frac{1}{5}(x+12)\right]^5$

ii) Domain: $\{x \in \mathbb{R}\}$; Range: $\{y \in \mathbb{R}\}$

c) i) $f(x) = -3(x + 1)^4 - 6$
ii) Domain: $\{x \in \mathbb{R}\}$; Range: $\{y \in \mathbb{R}, y \leq -6\}$; Vertex (–1, –6); Axis of Symmetry: $x = -1$

d) i) $f(x) = -\left[-\frac{1}{5}(x-1)\right]^6 - 3$

ii) Domain: $\{x \in \mathbb{R}\}$; Range: $\{y \in \mathbb{R}, y \leq -3\}$; Vertex (1, –3); Axis of Symmetry: $x = 1$

e) i) $f(x) = -7\left[\frac{5}{4}(x+1)\right]^6 + 9$

ii) Domain: $\{x \in \mathbb{R}\}$; Range: $\{y \in \mathbb{R}, y \leq 9\}$; Vertex (–1, 9); Axis of Symmetry: $x = -1$

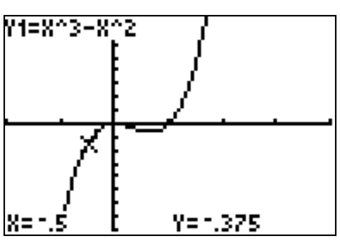
Chapter 1 Section 4 **Question 13 Page 52**

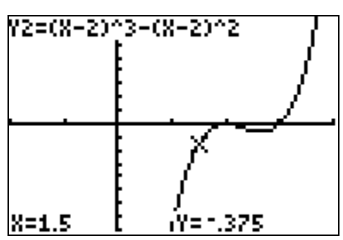
Solutions to Achievement Check questions are provided in the Teacher's Resource.

Chapter 1 Section 4 **Question 14 Page 52**

a) Answers may vary. A sample solution is shown.
I predict there will be a horizontal shift of 2 to the right.

b)





c) $y = x^3 - x^2$
$y = x^2(x - 1)$
$0 = x^2(x - 1)$
$x = 0$ or $x = 1$

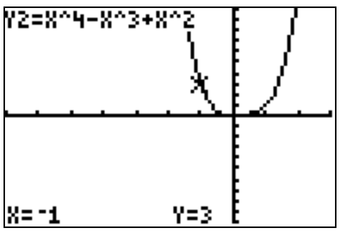
x-intercepts are 0 and 1.

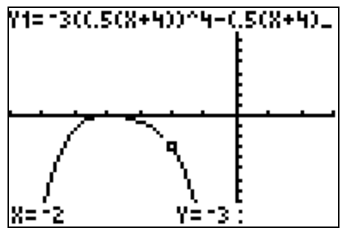
$y = (x - 2)^3 - (x - 2)^2$
$y = (x - 2)^2[(x - 2) - 1]$
$y = (x - 2)^2(x - 3)$
$0 = (x - 2)^2(x - 3)$
$x = 2$ or $x = 3$
x-intercepts are 3 and 2.

Chapter 1 Section 4 **Question 15 Page 52**

a) The translations that must be applied are a vertical stretch by a factor of 3, a reflection in the x-axis, a horizontal stretch by a factor of 2, and a translation of 4 units left.

b)





c) $y = x^4 - x^3 + x^2$
$y = x^2(x^2 - x + 1)$
$0 = x^2(x^2 - x + 1)$
$x = 0$
x-intercept is 0.

$$y = -3\left[\left[\frac{1}{2}(x+4)\right]^4 - \left[\frac{1}{2}(x+4)\right]^3 + \left[\frac{1}{2}(x+4)\right]^2\right]$$

$$y = -3\left[\frac{1}{2}(x+4)\right]^2\left[\left[\frac{1}{2}(x+4)\right]^2 - \left[\frac{1}{2}(x+4)\right] + 1\right]$$

$$0 = -3\left[\frac{1}{2}(x+4)\right]^2\left[\left[\frac{1}{2}(x+4)\right]^2 - \left[\frac{1}{2}(x+4)\right] + 1\right]$$

$x = -4$
x-intercept is -4.

Chapter 1 Section 4 **Question 16 Page 52**

a) $h(x) = 3(x + 1)(x + 6)(x - 1) - 5$

b) $h(x) = -\frac{6}{5}(x + 1)(x + 6)(x - 1) - 5$

Chapter 1 Section 4 **Question 17 Page 52**

The original price of the eggs is 12 × (price per egg) = *y*.
The new price per egg is 13 × (new price per egg) = *y*.

New price per egg = $\frac{y}{13}$.
The new price per egg multiplied by a dozen is 24 cents less.

$$12\left(\frac{y}{13}\right) = (y - 24)$$
$$y = 312$$

The original price of the eggs is \$3.12 per dozen.

Chapter 1 Section 4 **Question 18 Page 52**

a)

$$\begin{aligned} f_1(x) &= \left[f_0(x)\right]^2 \\ &= (x^2)^2 \\ &= x^4 \end{aligned} \qquad \begin{aligned} f_2(x) &= \left[f_1(x)\right]^2 \\ &= (x^4)^2 \\ &= x^8 \end{aligned} \qquad \begin{aligned} f_3(x) &= \left[f_2(x)\right]^2 \\ &= (x^8)^2 \\ &= x^{16} \end{aligned}$$

b) The formula is $f_n(x) = x^{2^{(n+1)}}$.

Chapter 1 Section 5 — Slopes of Secants and Average Rate of Change

Chapter 1 Section 5 — Question 1 Page 62

e) There is only one variable in the situation; for an average rate of change, two variables are needed.

Chapter 1 Section 5 — Question 2 Page 62

a) constant and positive; the graph shows a line with a positive slope.

b) zero; the graph shows a horizontal line with zero slope.

c) constant and negative; the graph shows a line with a negative slope.

Chapter 1 Section 5 — Question 3 Page 62

Solutions may vary, depending on points chosen. Answers will be the same.

a) Two points on the line are (0, 1) and (5, 8).

$$\begin{aligned}\text{Average rate of change} &= \frac{8-1}{5-0} \\ &= \frac{7}{5}\end{aligned}$$

b) Two points on the line are (2, 1) and (6, 1).

$$\begin{aligned}\text{Average rate of change} &= \frac{1-1}{6-2} \\ &= 0\end{aligned}$$

c) Two points on the line are (0, 6) and (7, 2).

$$\begin{aligned}\text{Average rate of change} &= \frac{2-6}{7-0} \\ &= -\frac{4}{7}\end{aligned}$$

Chapter 1 Section 5 — Question 4 Page 62

$$\text{Average rate of change} = \frac{66.8-16.2}{2003-1990} \doteq 3.89$$

The average rate of change of households with a home computer from 1990 to 2003 is 3.89%/year.

Chapter 1 Section 5 **Question 5 Page 62**

a) $$\text{Average rate of change} = \frac{52.1-26.3}{2003-1999}$$
$$\doteq 6.45$$

The average rate of change of households using e-mails from 1999 to 2003 is 6.45%/year.

b) Answers may vary. For example, someone might want to know if it makes sense to use e-mail for their business.

c) 1999-2000 $\frac{37.4-26.3}{2000-1999} \doteq 11.1$

The average rate of change of households using e-mails from 1999 to 2000 is 11.1%/year.

2000-2001 $\frac{46.1-37.4}{2001-2000} \doteq 8.7$

The average rate of change of households using e-mails from 2000 to 2001 is 8.7%/year.

2001-2002 $\frac{48.9-46.1}{2002-2001} \doteq 2.8$

The average rate of change of households using e-mails from 2001 to 2002 is 2.8%/year.

2002-2003 $\frac{52.1-48.9}{2003-2002} \doteq 3.2$

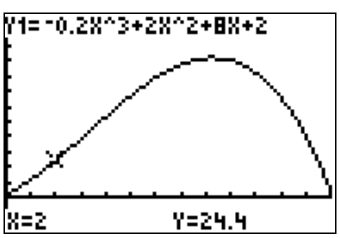
The average rate of change of households using e-mails from 2002 to 2003 is 3.2%/year.

d) Greatest: between 1999 and 2000, the greatest increase of e-mail use
Least: between 2001 and 2002, the smallest increase of e-mail use

e) Answers may vary. For example, the value in a) is the average of the values in c).

Chapter 1 Section 5 **Question 6 Page 63**

a)



b) Positive Interval: $0 \le x < 8.28$; Negative Interval: $8.28 < x \le 13$; Zero: $x = 8.28$

c) **i)** $\dfrac{67-2}{5-0} = 13$

The average rate of change of the purchase price from year 0 to year 5 is \$13/year.

ii) $\dfrac{91.6-67}{8-5} = 8.2$

The average rate of change of the purchase price from year 5 to year 8 is \$8.20/year.

iii) $\dfrac{82-91.6}{10-8} = -4.8$

The average rate of change of the purchase price from year 8 to year 10 is –\$4.80/year.

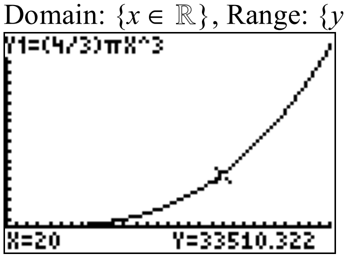
iv) $\dfrac{4.6-91.6}{13-8} = -17.4$

The average rate of change of the purchase price from year 8 to year 13 is –\$17.40/year.

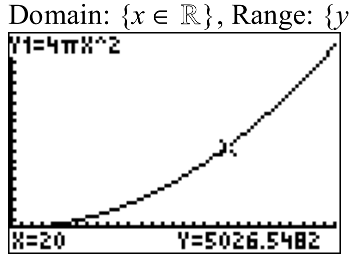
d) Answers may vary. A sample solution is shown.
The best time to buy shares would be from year 8 to year 13 because the purchase price is decreasing at the highest rate. The best time to sell shares is from year 0 to year 5 because the purchase price is increasing at the highest rate.

Chapter 1 Section 5 **Question 7 Page 63**

a) The volume is a cubic function.
Domain: $\{x \in \mathbb{R}\}$, Range: $\{y \in \mathbb{R}\}$



The surface area is a quadratic function.
Domain: $\{x \in \mathbb{R}\}$, Range: $\{y \in \mathbb{R}, y \geq 0\}$



b) From the graphs find the values of V and S for the indicated values of r.

Volume

i) (30, 113 097.34) and (25, 65 449.85)

$$\text{Average rate of change} = \frac{113\,097.34 - 65\,449.85}{30 - 25}$$
$$\doteq 9529.50 \text{ cm}^2$$

ii) (25, 65 449.85) and (20, 33 510.32)

$$\text{Average rate of change} = \frac{65\,449.85 - 33\,510.32}{25 - 20}$$
$$\doteq 6387.91 \text{ cm}^2$$

Surface Area

i) (30, 11 309.73) and (25, 7853.98)

$$\text{Average rate of change} = \frac{11\,309.73 - 7853.98}{30 - 25}$$
$$\doteq 691.15 \text{ cm}$$

ii) (25, 7853.98) and (20, 5026.55)

$$\text{Average rate of change} = \frac{7853.98 - 5026.55}{25 - 20}$$
$$\doteq 565.49 \text{ cm}$$

Answers may vary. For example, as the change in volume decreases, the change in surface area decreases.

c) From the graph, the surface area decreases from 2827.43 cm^2 to 1256.64 cm^2 when the radius decreases from 15 cm to 10 cm.

$$\text{Average rate of change} = \frac{2827.43 - 1256.64}{15 - 10}$$
$$\doteq 314.16 \text{ cm}$$

d) From the graph, the volume decreases from 1675.52 cm^3 to 942.48 cm^3 when the radius decreases from approximately 7.37 cm to 6.08 cm.

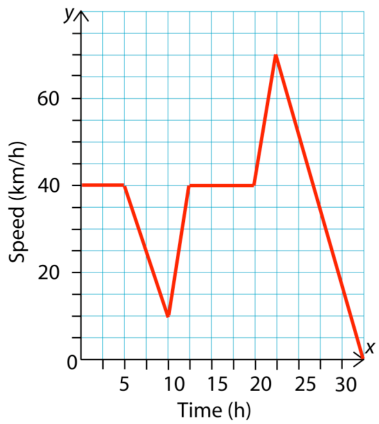
$$\text{Average rate of change} = \frac{1675.52 - 942.48}{7.37 - 6.08}$$
$$\doteq 570.07 \text{ cm}^2$$

e) Answers may vary. A sample solution is shown.
The snowball is decreasing in surface area at an approximate rate of 314.16 cm when the radius decreases from 15 cm to 10 cm. The snowball is decreasing in volume at a rate of approximately 570.07 cm^2 when the radius is approximately between 7.37 cm and 6.08 cm.

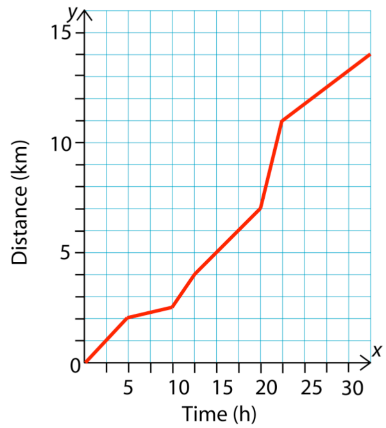
Chapter 1 Section 5 **Question 8 Page 63**

Answers may vary. A sample solution is shown.

Speed vs. Time Graph



Distance vs. Time Graph



Chapter 1 Section 5 **Question 9 Page 63**

a) Solve using the x-intercept form of an equation.
x-intercepts are 0 and 28.

$y = a(x - 0)(x - 28)$

A point on the parabola is (10, 36). Solve for a.

$36 = a(10 - 0)(10 - 28)$

$36 = -180a$

$a = -0.2$

Equation of the parabola: $y = -0.2(x - 0)(x - 28)$
$= -0.2x(x - 28)$

b) Each crossbeam represents the secant lines.

c) Find the endpoints of each beam.

(0, 0) and (16, 38.4) $m = \frac{38.4 - 0}{16 - 0} = 2.4$	(2, 10.4) and (18, 36) $m = \frac{36 - 10.4}{18 - 2} = 1.6$	(6, 26.4) and (20, 32) $m = \frac{32 - 26.4}{20 - 6} = 0.4$
(8, 32) and (20, 32) $m = \frac{32 - 32}{20 - 8} = 0$	(10, 36) and (18, 36) $m = \frac{36 - 36}{18 - 10} = 0$	(12, 38.4) and (16, 38.4) $m = \frac{38.4 - 38.4}{16 - 12} = 0$
(8, 32) and (22, 26.4) $m = \frac{26.4 - 32}{22 - 8} = -0.4$	(10, 36) and (26, 10.4) $m = \frac{10.4 - 36}{26 - 10} = -1.6$	(12, 38.4) and (28, 0) $m = \frac{0 - 38.4}{28 - 12} = -2.4$

d) The slopes of the crossbeams represent the steepness of the crossbeams.

e) The crossbeams have the same steepness in a different direction.

Chapter 1 Section 5 **Question 10 Page 64**

a) Substitute values into the equation to get the endpoints.

i) (0, 2000) and (100, 0)

$$m = \frac{0-2000}{100-0}$$
$$= -20 \text{ L/min}$$

ii) (0, 2000) and (30, 480.2)

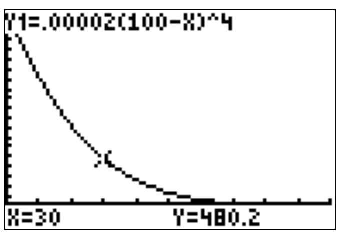
$$m = \frac{480.2-2000}{30-0}$$
$$= -50.66 \text{ L/min}$$

iii) (70, 16.2) and (100, 0)

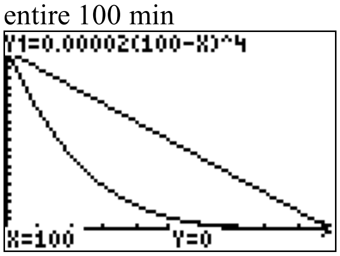
$$m = \frac{0-16.2}{100-70}$$
$$= -0.54 \text{ L/min}$$

b) The rate at which water drains from the tub slows down as the time progresses.

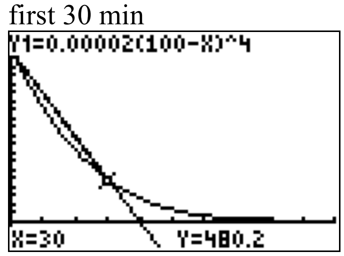
c)



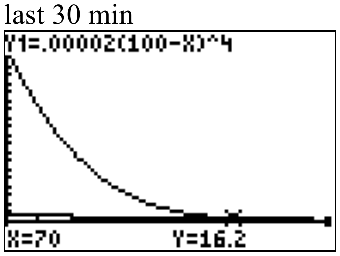
d) i) entire 100 min



ii) first 30 min



iii) last 30 min



a) 0-1 h: Average rate of change $= \frac{17\,280 - 18\,750}{1}$
$= -1470$

1-2 h: Average rate of change $= \frac{15\,870 - 17\,280}{1}$
$= -1410$

2-3 h: Average rate of change $= \frac{14\,520 - 15\,870}{1}$
$= -1350$

3-4 h: Average rate of change $= \frac{13\,230 - 14\,520}{1}$
$= -1290$

4-5 h: Average rate of change $= \frac{12\,000 - 13\,230}{1}$
$= -1230$

5-6 h: Average rate of change $= \frac{10\,830 - 12\,000}{1}$
$= -1170$

6-7 h: Average rate of change $= \frac{9720 - 10\,830}{1}$
$= -1110$

7-8 h: Average rate of change $= \frac{8670 - 9720}{1}$
$= -1050$

8-9 h: Average rate of change $= \frac{7680 - 8670}{1}$
$= -990$

9-10 h: Average rate of change $= \frac{6750 - 7680}{1}$
$= -930$

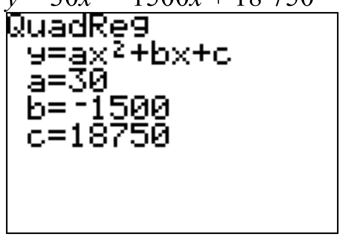
b) Greatest rate of change: between 0-1 h
Least rate of change: between 9-10 h

c) Type of polynomial function: quadratic

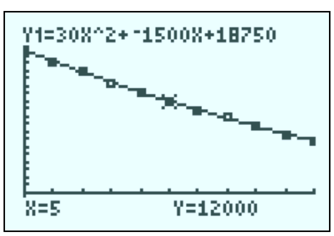
Time (h)	*Amount of Water (L)*	**First Differences**	**Second Differences**
0	18 750	–1470	
1	17 280	–1410	60
2	15 870	–1350	60
3	14 520	–1290	60
4	13 230	–1230	60
5	12 000	–1170	60
6	10 830	–1110	60
7	9720	–1050	60
8	8670	–990	60
9	7680	–930	60
10	6750		

d) The first differences are the same as the average rates of change for the same interval.

e) $y = 30x^2 - 1500x + 18\ 750$



f)



g) When $y = 0$ the pool is fully drained.

$0 = 30x^2 - 1500x + 18\ 750$

$0 = 30(x - 25)^2$

$x = 25$

It takes 25 h for the pool to fully drain.

Chapter 1 Section 5 **Question 12 Page 64**

a) The average rate of change represents the velocity of the ball (m/s).

b) i) Average rate of change $= \dfrac{20.4-15.1}{2-1}$
$= 5.3$

ii) Average rate of change $= \dfrac{18.975-15.1}{1.5-1}$
$= 7.75$

iii) Average rate of change $= \dfrac{16.071-15.1}{1.01-1}$
$= 9.71$

iv) Average rate of change $= \dfrac{15.20151-15.1}{1.01-1}$
$= 10.151$

v) Average rate of change $\doteq \dfrac{15.110195-15.1}{1.001-1}$
$\doteq 10.195$

vi) Average rate of change $\doteq \dfrac{15.10102-15.1}{1.0001-1}$
$\doteq 10.2$

c) Answers may vary. For example, the values are the instantaneous rate at 1 s and as the time interval decreases, the average rate of change gets closer to 10.2.

d) Answers may vary. A sample solution is shown.
To estimate the instantaneous rate of change, estimate the average rate of change at very short intervals.

Chapter 1 Section 5 **Question 13 Page 64**

Given: $A = 25$ cm^2

= base multiplied by height, divided by 2

Let y be the base and x be the height.

Let p be the perimeter and h be the hypotenuse.

$$25 = \frac{1}{2}xy \qquad h^2 = \left(\frac{50}{x}\right)^2 + x^2$$

$$y = \frac{50}{x} \qquad h^2 = \frac{2500}{x^2} + x^2$$

Perimeter:

$$p = x + \frac{50}{x} + h \quad \text{or} \quad p - h = x + \frac{50}{x}$$

$$p^2 = x^2 + 50 + xh + 50 + \frac{2500}{x^2} + \frac{50h}{x} + hx + \frac{50h}{x} + h^2$$

$$p^2 = x^2 + \frac{2500}{x^2} + 100 + 2xh + \frac{100h}{x} + h^2$$

Substitute $h^2 = \frac{2500}{x^2} + x^2$.

$$p^2 = h^2 + 100 + 2xh + \frac{100h}{x} + h^2$$

$$p^2 = 2h^2 + 100 + h\left(2x + \frac{100}{x}\right)$$

$$p^2 = 2h^2 + 100 + h\left(x + x + \frac{50}{x} + \frac{50}{x}\right)$$

$$p^2 = 2h^2 + 100 + h\big(2(p-h)\big)$$

$$p^2 = 2h^2 + 100 + 2ph - 2h^2$$

$$2ph = p^2 - 100$$

$$h(p) = \frac{p^2 - 100}{2p}$$

Chapter 1 Section 6 — Slopes of Tangents and Instantaneous Rate of Change

Chapter 1 Section 6 — Question 1 Page 71

a) The coordinates of the tangent point are (5, 3).

b) Answers may vary. For example, (3, 7) is another point on the tangent line.

c) $m = \frac{7-3}{3-5}$
$= -2$
The slope of the tangent line is –2.

d) The value represents the instantaneous rate of change at $x = 5$.

Chapter 1 Section 6 — Question 2 Page 71

a) At point A, the instantaneous rate of change is positive because the tangent line is positive. At point B, the instantaneous rate of change is zero because the tangent line has a slope of zero. At point C, the instantaneous rate of change is negative because the tangent line is negative.

b) A: tangent point (2, 12) and another point on the tangent line (1, 8)
$m = \frac{8-12}{1-2}$
$= 4$ m/s
C: tangent point (7, 7) and another point on the tangent line (6, 13)
$m = \frac{13-7}{6-7}$
$= -6$ m/s

c) The values in part b) represent the velocity at 2 s and 7 s.

Chapter 1 Section 6 — Question 3 Page 71

a)

Interval	Δh	Δt	$\frac{\Delta h}{\Delta t}$
$3 \le t \le 3.1$	–0.289	0.1	–2.89
$3 \le t \le 3.01$	–0.024 49	0.01	–2.449
$3 \le t \le 3.001$	–0.002 404 9	0.001	–2.4049

b) The velocity after 3 s is decreasing at a rate of approximately 2.4 m/s.

Chapter 1 Section 6 **Question 4 Page 71**

Solutions may vary, but answers are the same. A sample solution is shown.
Tangent point (2.5, 31.25) and another point on the tangent line (4, 68.75)

$$m = \frac{68.75 - 31.25}{4 - 2.5}$$
$$= 25 \text{ m/s}$$

Tangent point (2.5, 31.25) and another point on the tangent line (2, 18.75)

$$m = \frac{18.75 - 31.25}{2 - 2.5}$$
$$= 25 \text{ m/s}$$

Chapter 1 Section 6 **Question 5 Page 72**

a) Average rate of change $= \dfrac{27.9 - 12.3}{2003 - 1999}$
$= 3.9\%/\text{year}$

b) i) Average rate of change from 2000 to 2001 $= \dfrac{24.4 - 18.2}{1}$
$= 6.2$

Average rate of change from 1999 to 2000 $= \dfrac{18.2 - 12.3}{1}$
$= 5.9$

The instantaneous rate of change in 2000 is between 5.9% and 6.2% per year.

ii) Average rate of change from 2003 to 2002 $= \dfrac{27.9 - 25.7}{1}$
$= 2.2$

Average rate of change from 2002 to 2001 $= \dfrac{25.7 - 24.4}{1}$
$= 1.3$

The instantaneous rate of change in 2002 is between 1.3% and 2.2% per year.

c) Answers may vary. For example, the average of the values in b) is equal to the value in a).

Chapter 1 Section 6 **Question 6 Page 72**

a) Average rate of change from 1955 to 2005 = $\frac{127.3-16.8}{2005-1955}$
= 2.21 CPI/year

b) i) Average rate of change from 1965 to 1970 = $\frac{24.2-20}{1970-1965}$
= 0.84

Average rate of change from 1960 to 1965 = $\frac{20-18.5}{1965-1960}$
= 0.3

The instantaneous rate of change in 1965 is between 0.3 and 0.84 CPI/year.

ii) Average rate of change from 1985 to 1990 = $\frac{93.3-75}{1990-1985}$
= 3.66

Average rate of change from 1980 to 1985 = $\frac{75-52.4}{1985-1980}$
= 4.52

The instantaneous rate of change in 1985 is between 3.66 and 4.52 CPI/year.

iii) Average rate of change from 2000 to 2005 = $\frac{127.3-113.5}{2005-2000}$
= 2.76

Average rate of change from 1995 to 2000 = $\frac{113.5-104.2}{2000-1995}$
= 1.86

The instantaneous rate of change in 2000 is between 1.86 and 2.76 CPI/year.

c) Answers may vary. For example, the average of the values in b) is greater than the value in a).

Chapter 1 Section 6 **Question 7 Page 72**

a) Substitute $t = 1$ and $t = 2$ into the equation to get the points (1, 7.6) and (2, 4.9).

$$\text{Average rate of change from 1 s to 2 s} = \frac{4.9 - 7.6}{2 - 1}$$
$$= -2.7 \text{ m/s}$$

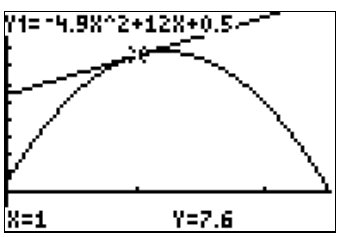
b) $$\text{Average rate of change, } 1 \le t \le 1.01 = \frac{-4.9(1.01)^2 + 12(1.01) + 0.5 - (-4.9(1)^2 + 12(1) + 0.5)}{1.01 - 1}$$
$$= \frac{7.621\,51 - 7.6}{0.01}$$
$$= 2.151$$

$$\text{Average rate of change, } 1 \le t \le 1.001 = \frac{-4.9(1.001)^2 + 12(1.001) + 0.5 - (-4.9(1)^2 + 12(1) + 0.5)}{1.001 - 1}$$
$$= \frac{7.6022 - 7.6}{0.001}$$
$$= 2.1951$$

$$\text{Average rate of change, } 1 \le t \le 1.0001 = \frac{-4.9(1.0001)^2 + 12(1.0001) + 0.5 - (-4.9(1)^2 + 12(1) + 0.5)}{1.0001 - 1}$$
$$= \frac{7.600\,22 - 7.6}{0.0001}$$
$$= 2.199\,51$$

Thus, the approximate instantaneous rate of change of the height of the ball after 1 s is 2.2 m/s.

c)



d) Answers may vary. For example, the average velocity of the ball from 1-3 s is less than then the instantaneous velocity at 1 s.

Chapter 1 Section 6 **Question 8 Page 72**

a) Earth: Average rate of change in the first 3 s = $\dfrac{15.9-60}{3-0}$

$= -14.7$ m/s

Venus: Average rate of change in the first 3 s = $\dfrac{19.95-60}{3-0}$

$= -13.35$ m/s

b) Earth: Average rate of change, $3 \le t \le 3.01$ = $\dfrac{-4.9(3.01)^2+60-(-4.9(3)^2+60)}{3.01-3}$

$\doteq \dfrac{15.6055-15.9}{0.01}$

$\doteq -29.45$

Average rate of change, $3 \le t \le 3.001$ = $\dfrac{-4.9(3.001)^2+60-(-4.9(3)^2+60)}{3.001-3}$

$\doteq \dfrac{15.8706-15.9}{0.001}$

$\doteq -29.4049$

Average rate of change, $3 \le t \le 3.0001$ = $\dfrac{-4.9(3.0001)^2+60-(-4.9(3)^2+60)}{3.0001-3}$

$\doteq \dfrac{15.8971-15.9}{0.0001}$

$\doteq -29.4005$

Thus, the velocity of the rock after 3 s is approximately –29.4m/s on Earth.

Venus: Average rate of change, $3 \le t \le 3.01$ = $\dfrac{-4.45(3.01)^2+60}{3.01-3}$

$\doteq \dfrac{19.6826-19.95}{0.01}$

$\doteq -26.74$

Average rate of change, $3 \le t \le 3.001$ = $\dfrac{-4.45(3.001)^2+60}{3.001-3}$

$\doteq \dfrac{19.9233-19.95}{0.001}$

$\doteq -26.7045$

Average rate of change, $3 \le t \le 3.0001$ = $\dfrac{-4.45(3.0001)^2+60}{3.0001-3}$

$\doteq \dfrac{19.9473-19.95}{0.0001}$

$\doteq -26.7004$

Thus the velocity of the rock after 3 s is approximately –26.7 m/s on Venus.

c) A rock falls faster on Earth than on Venus.

Chapter 1 Section 6 **Question 9 Page 72**

a) Average rate of change $= \dfrac{0.000\,15(200)^3 + 100(200) - \left[0.000\,15(100)^3 + 100(100)\right]}{200 - 100}$

$= \dfrac{21\,200 - 10\,150}{100}$

$= 110.5$

The average rate of change of the cost of producing 100 to 200 MP3 players is \$110.50/MP3 player.

b) $200 \le x \le 201$

Average rate of change $= \dfrac{0.000\,15(201)^3 + 100(201) - \left[0.000\,15(200)^3 + 100(200)\right]}{201 - 200}$

$= \dfrac{21\,318.1 - 21\,200}{1}$

$= 118.1$

$200 \le x \le 200.1$

Average rate of change $= \dfrac{0.000\,15(200.1)^3 + 100(200.1) - \left[0.000\,15(200)^3 + 100(200)\right]}{200.1 - 200}$

$= \dfrac{21\,211.8 - 21\,200}{0.1}$

$= 118.009.$

The instantaneous rate of change of the cost of producing 200 MP3 players is \$118/MP3 player.

c) Answers may vary. For example, the cost of producing 200 MP3 players is higher than the average cost between 100 and 200 MP3 players.

d) No. The cost is always increasing.

Chapter 1 Section 6 **Question 10 Page 73**

a) Average rate of change $= \dfrac{200\left[350 - 0.000\,325(200)^2\right] - 100\left[350 - 0.000\,325(100)^2\right]}{200 - 100}$

$= \dfrac{67\,400 - 34\,675}{100}$

$= 327.25$

The average rate of change of revenue from selling between 100 and 200 MP3 players is \$327.25/MP3 player.

b) $200 \le x \le 201$

$$\text{Average rate of change} = \frac{201\left[350-0.000\,325(201)^2\right]-200\left[350-0.000\,325(200)^2\right]}{201-200}$$
$$= \frac{67\,710.8-67\,400}{1}$$
$$= 310.805$$

$200 \le x \le 200.1$

$$\text{Average rate of change} = \frac{200.1\left[350-0.000\,325(200.1)^2\right]-200\left[350-0.000\,325(200)^2\right]}{200.1-200}$$
$$= \frac{67\,431.1-67\,400}{0.1}$$
$$= 310.98$$

$200 \le x \le 200.01$

$$\text{Average rate of change} = \frac{200.01\left[350-0.000\,325(200.01)^2\right]-200\left[350-0.000\,325(200)^2\right]}{200.01-200}$$
$$= \frac{67\,403.1-67\,400}{0.01}$$
$$= 310.998$$

The instantaneous rate of change of revenue from selling 200 MP3 players is approximately \$311/MP3 player.

c) The average revenue (between 100 and 200) is higher that the revenue for 200 MP3 players.

d) $P(x) = x\left(350-0.000\,325x^2\right)-\left(0.000\,15x^3+100x\right)$

$P(x) = 350x - 0.000\,325x^3 - 0.000\,15x^3 - 100x$

$P(x) = 250x - 0.000\,475x^3$

e) $$\text{Average rate of change} = \frac{250(200)-0.000\,475(200)^3-\left[250(100)-0.000\,475(100)^3\right]}{200-100}$$
$$= \frac{46\,200-24\,525}{100}$$
$$= 216.75$$

The average rate of change of profit from selling between 100 and 200 MP3 players is \$216.75/MP3 player.

f) $200 \le x \le 201$

$$\text{Average rate of change} = \frac{250(201) - 0.000\,475(201)^3 - \left[250(200) - 0.000\,475(200)^3\right]}{201 - 200}$$
$$= \frac{46\,392.7 - 46\,200}{1}$$
$$= 192.715$$

$200 \le x \le 200.1$

$$\text{Average rate of change} = \frac{250(200.1) - 0.000\,475(200.1)^3 - \left[250(200) - 0.000\,475(200)^3\right]}{200.1 - 200}$$
$$= \frac{46\,219.3 - 46\,200}{0.1}$$
$$= 192.971$$

$200 \le x \le 200.01$

$$\text{Average rate of change} = \frac{250(200.01) - 0.000\,475(200.01)^3 - \left[250(200) - 0.000\,475(200)^3\right]}{200.01 - 200}$$
$$= \frac{46\,201.9 - 46\,200}{0.01}$$
$$= 192.997$$

$200 \le x \le 200.001$

$$\text{Average rate of change} = \frac{250(200.001) - 0.000\,475(200.001)^3 - \left[250(200) - 0.000\,475(200)^3\right]}{200.001 - 200}$$
$$= \frac{46\,200.2 - 46\,200}{0.001}$$
$$= 193$$

The instantaneous rate of change of profit from the sale of 200 MP3 players is \$193/MP3 player.

g) Answers may vary. For example, the average profit between 100 and 200 MP3 players is higher than the profit for 200 MP3 players.

Chapter 1 Section 6 **Question 11 Page 73**

a) i) Average rate of change $= \dfrac{-0.09(6)^3 + 1.89(6)^2 + 9(6) - \left[-0.09(2)^3 + 1.89(2)^2 + 9(2)\right]}{6-2}$

$= \dfrac{102.6 - 24.84}{6-2}$

$= 19.44 \times 1000$

The average rate of change of profit earned from selling 2000 to 6000 basketballs is \$19 440.

ii) Average rate of change $= \dfrac{-0.09(20)^3 + 1.89(20)^2 + 9(20) - \left[-0.09(16)^3 + 1.89(16)^2 + 9(16)\right]}{20-16}$

$= \dfrac{216 - 259}{20-16}$

$= -10.8 \times 1000$

The average rate of change of profit earned from selling 16 000 to 20 000 basketballs is –\$10 800.

b) Answers may vary. For example, the profits are increasing between 2000 and 6000 basketballs and decreasing between 16 000 and 20 000 basketballs.

c) i) $5 \le t \le 5.01$

Average rate of change $= \dfrac{-0.09(5.01)^3 + 1.89(5.01)^2 + 9(5.01) - \left[-0.09(5)^3 + 1.89(5)^2 + 9(5)\right]}{5.01-5}$

$= \dfrac{81.2116 - 81}{0.01}$

$= 21.1554 \times 1000$

$5 \le t \le 5.001$

Average rate of change $= \dfrac{-0.09(5.001)^3 + 1.89(5.001)^2 + 9(5.001) - \left[-0.09(5)^3 + 1.89(5)^2 + 9(5)\right]}{5.001-5}$

$= \dfrac{81.0212 - 81}{0.001}$

$= 21.1505 \times 1000$

$5 \le t \le 5.0001$

Average rate of change $= \dfrac{-0.09(5.0001)^3 + 1.89(5.0001)^2 + 9(5.0001) - \left[-0.09(5)^3 + 1.89(5)^2 + 9(5)\right]}{5.0001-5}$

$= \dfrac{81.0021 - 81}{0.0001}$

$= 21.1501 \times 1000$

The instantaneous rate of change of profit earned selling 5000 basketballs is approximately \$21 150.

ii) $18 \le t \le 18.01$, average rate of change:

$$= \frac{-0.09(18.01)^3 + 1.89(18.01)^2 + 9(18.01) - \left[-0.09(18)^3 + 1.89(18)^2 + 9(18)\right]}{18.01 - 18}$$
$$= \frac{249.375 - 249.48}{0.01}$$
$$= -10.4697$$

$18 \le t \le 18.001$, average rate of change:

$$= \frac{-0.09(18.001)^3 + 1.89(18.001)^2 + 9(18.001) - \left[-0.09(18)^3 + 1.89(18)^2 + 9(18)\right]}{18.001 - 18}$$
$$= \frac{249.47 - 249.48}{0.001}$$
$$= -10.443$$

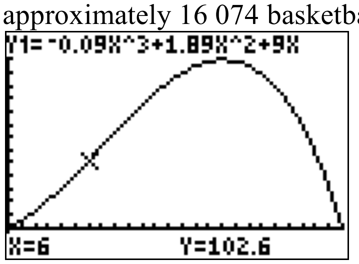
$18 \le t \le 18.0001$, average rate of change

$$= \frac{-0.09(18.0001)^3 + 1.89(18.0001)^2 + 9(18.0001) - \left[-0.09(18)^3 + 1.89(18)^2 + 9(18)\right]}{18.0001 - 18}$$
$$\doteq \frac{249.479 - 249.48}{0.0001}$$
$$\doteq -10.4403$$

The instantaneous rate of change of profit earned selling 18 000 basketballs is approximately –$10 440.

d) The company is losing money when making 18 000 basketballs.

e) Answers may vary. For example, the graph shows that the profit is increasing until approximately 16 074 basketballs are sold, then the profits are decreasing.



Chapter 1 Section 6 **Question 12 Page 73**

a)
$$\begin{aligned}\frac{P(8+h)-P(8)}{h} &= \frac{0.5(8+h)^3+150(8+h)+1200-\left[0.5(8)^3+150(8)+1200\right]}{8+h-8}\\ &= \frac{0.5(h^3+24h^2+192h+512)+1200+150h+1200-256-1200-1200}{h}\\ &= \frac{0.5h^3+12h^2+96h+256+150h-256}{h}\\ &= \frac{0.5h^3+12h^2+246h}{h}\\ &= 0.5h^2+12h+246\end{aligned}$$

b) i) Average rate of change when $h = 2$: $0.5(2)^2 + 12(2) + 246 = 272$

ii) Average rate of change when $h = 4$: $0.5(4)^2 + 12(4) + 246 = 302$

iii) Average rate of change when $h = 5$: $0.5(5)^2 + 12(5) + 246 = 318.5$

c) The values in part b) represent the average rates of change between
i) 8 and 10 years.
ii) 8 and 12 years.
iii) 8 and 13 years.

d) The expression in part a) could be used to find the instantaneous rate of change after 8 years when $h = 0$.

e) Instantaneous rate of change after 8 years $= 0.5(0)^2 + 12(0) + 246$
$= 246$

Chapter 1 Section 6 **Question 13 Page 73**

$$x=\sqrt{10+x}$$
$$x^2=10+x$$
$$x^2-x-10=0$$

Use the Quadratic Formula to find x.

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$x=\frac{-(-1)\pm\sqrt{(-1)^2-4(1)(-10)}}{2(1)}$$
$$x=\frac{1\pm\sqrt{41}}{2}$$

The exact value of $\sqrt{10+\sqrt{10+\sqrt{10+\ldots}}}=\frac{1\pm\sqrt{41}}{2}$.

$\sqrt[3]{m+9} = 3 + \sqrt[3]{m-9}$

$\left(\sqrt[3]{m+9}\right)^3 = \left(3 + \sqrt[3]{m-9}\right)^3$

$m + 9 = 9(m-9)^{\frac{2}{3}} + 27(m-9)^{\frac{1}{3}} + m + 18$

$9(m-9)^{\frac{2}{3}} + 27(m-9)^{\frac{1}{3}} + 9 = 0$

$(m-9)^{\frac{2}{3}} + 3(m-9)^{\frac{1}{3}} + 1 = 0$

Let $x = (m-9)^{\frac{1}{3}}$

So, $x^2 + 3x + 1 = 0$

Use the quadratic formula to find x.

$x = \dfrac{-3+\sqrt{5}}{2}$, or $x = \dfrac{-3-\sqrt{5}}{2}$

Substitute $(m-9)^{\frac{1}{3}}$ back into the equation.

$(m-9)^{\frac{1}{3}} = \dfrac{-3+\sqrt{5}}{2}$ *or* $(m-9)^{\frac{1}{3}} = \dfrac{-3-\sqrt{5}}{2}$

$\left[(m-9)^{\frac{1}{3}}\right]^3 = \left(\dfrac{-3+\sqrt{5}}{2}\right)^3$ $\left[(m-9)^{\frac{1}{3}}\right]^3 = \left(\dfrac{-3-\sqrt{5}}{2}\right)^3$

$m - 9 = 4\sqrt{5} - 9$ $m - 9 = -4\sqrt{5} - 9$

$m = 4\sqrt{5}$ $m = -4\sqrt{5}$

So, $|m| = 4\sqrt{5}$.

Chapter 1 Review

Chapter 1 Review **Question 1 Page 74**

a) Polynomial function with degree 4. The leading coefficient is 3.

b) Polynomial function with degree 2. The leading coefficient is –1.

c) Exponential function since the base is a number and the exponent is x.

d) Rational function since the exponent is negative.

e) Polynomial function with degree 3. The leading coefficient is 5.

Chapter 1 Review **Question 2 Page 74**

a) **i)** The function has an odd degree.
ii) The leading coefficient is negative.
iii) Domain: $\{x \in \mathbb{R}\}$; Range: $\{y \in \mathbb{R}\}$
iv) The graph extends from quadrant 2 to 4.
v) The function has point symmetry.

b) **i)** The function has an even degree.
ii) The leading coefficient is positive.
iii) Domain: $\{x \in \mathbb{R}\}$; Range: $\{y \in \mathbb{R}, y \geq 0\}$
iv) The graph extends from quadrant 2 to 1.
v) The function has line symmetry.

Chapter 1 Review **Question 3 Page 74**

End Behaviour	Function	Reasons
Extends from quadrant 3 to quadrant 1	$y = 4x^3$	positive leading coefficient and odd degree
Extends from quadrant 2 to quadrant 4	$y = -x^5$	negative leading coefficient and odd degree
Extends from quadrant 2 to quadrant 1	$y = \frac{2}{3}x^4$ $y = 0.2x^6$	positive leading coefficient and even degree
Extends from quadrant 3 to quadrant 4	none	

Chapter 1 Review **Question 4 Page 74**

a) **ii)** The function and graph have an odd degree and a positive leading coefficient.

b) **iii)** The function and graph have an even degree and a negative leading coefficient.

c) **i)** The function and graph have an even degree and a positive leading coefficient.

Chapter 1 Review **Question 5 Page 75**

i) $g(x) = 0.5x^4 - 3x^2 + 5x$

a) The 4th finite differences are constant.

b) $(0.5)(4 \times 3 \times 2 \times 1) = 12$; The value of the constant finite differences is 12.

c) There is no symmetry.

ii) $h(x) = x^5 - 7x^3 + 2x - 3$

a) The 5th finite differences are constant.

b) $(1)(5 \times 4 \times 3 \times 2 \times 1) = 120$; The value of the constant finite differences is 120.

c) The function has point symmetry.

iii) $p(x) = -x^6 + 5x^3 + 4$

a) The 6th finite differences are constant.

b) $(-1)(6 \times 5 \times 4 \times 3 \times 2 \times 1) = -720$; The value of the constant finite differences is –720.

c) There is no symmetry.

Chapter 1 Review **Question 6 Page 75**

a) **i)** The degree of the function is 1.

ii) The degree of the function is 5.

iii) The degree of the function is 4.

iv) The degree of the function is 2.

v) The degree of the function is 3.

vi) The degree of the function is 3.

b) **i)** The value of the leading coefficient is –5.

ii) $a(5 \times 4 \times 3 \times 2 \times 1) = -60$

$a = -\frac{1}{2}$; The value of the leading coefficient is $-\frac{1}{2}$.

iii) $a(4 \times 3 \times 2 \times 1) = 36$

$a = \frac{3}{2}$; The value of the leading coefficient is $\frac{3}{2}$.

iv) $a(2 \times 1) = 18$

$a = 9$; The value of the leading coefficient is 9.

v) $a(3 \times 2 \times 1) = 42$

$a = 7$; The value of the leading coefficient is 7.

vi) $a(3 \times 2 \times 1) = -18$

$a = -3$; The value of the leading coefficient is –3.

Chapter 1 Review **Question 7 Page 75**

a)

x	y	First Differences	Second Differences	Third Differences
–3	124	–83	50	–24
–2	41	–33	26	–24
–1	8	–7	2	–24
0	1	–5	–22	–24
1	–4	–27	–46	–24
2	–31	–73	–70	
3	–104	–143		
4	–247			

i) The degree of the function is 3.
ii) The sign of the leading coefficient is negative.
iii) $a(3 \times 2 \times 1) = -24$
$a = -4$; The value of the leading coefficient is –4.

b)

x	y	First Differences	Second Differences	Third Differences	Fourth Differences	Fifth Differences
–2	–229	224	–216	198	–216	240
–1	–5	8	–18	–18	24	240
0	3	–10	–36	6	264	240
1	–7	-46	–30	270	504	
2	–53	–76	240	774		
3	–129	164	1014			
4	35	1178				
5	1213					

i) The degree of the function is 5.
ii) The sign of the leading coefficient is positive.
iii) $a(5 \times 4 \times 3 \times 2 \times 1) = 240$
$a = 2$; The value of the leading coefficient is 2.

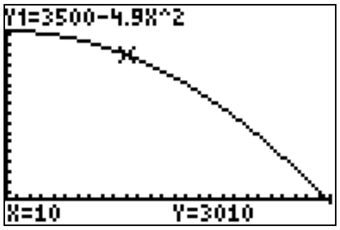
Chapter 1 Review **Question 8 Page 75**

a) $h(t)$ is a quadratic function.

b) i) The 2nd finite differences are constant.
ii) $-4.9(2 \times 1) = -9.8$; The value of the constant finite differences is –9.8.

c) The graph extends from quadrant 3 to 4.

d) Domain: $0 \le t \le 26.73$



e) The t-intercepts represent when the parachutist gets to the ground.

Chapter 1 Review **Question 9 Page 75**

a) i) The least possible degree is 3 since there are 3 x-intercepts of order 1. The sign of the leading coefficient is positive.

ii) The x-intercepts and factors of the function are 0, –2, and 4.

iii) Positive Intervals: $-2 < x < 0$, $x > 4$

Negative Intervals: $x < -2$, $0 < x < 4$

b) i) The least possible degree is 4 since there are 2 x-intercepts of order 1 and 1 x-intercept order 2. The sign of the leading coefficient is negative.

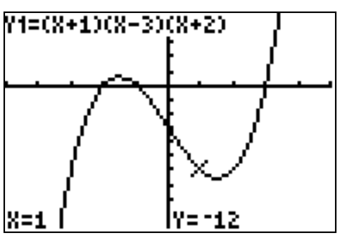
ii) The x-intercepts and factors of the function are –2, $\frac{1}{2}$, and 3.

iii) Positive Intervals: $-2 < x < \frac{1}{2}$

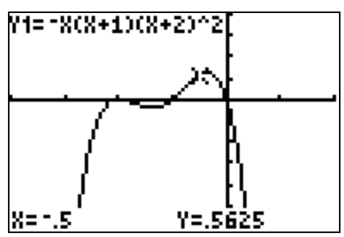
Negative Intervals: $x < -2$, $\frac{1}{2} < x < 3$, $x > 3$

Chapter 1 Review **Question 10 Page 76**

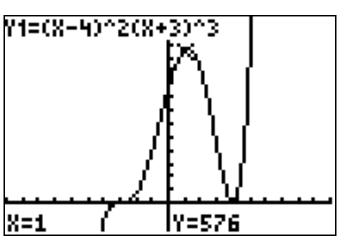
a)



b)



c)



Chapter 1 Review **Question 11 Page 76**

a) Answers may vary. A sample solution is shown.
Equation 1: $y = (x + 3)(x + 1)(x - 2)^2$
Equation 2: $y = (x + 3)(x + 1)(x - 2)^2$

b) Solve for a and substitute the point (1, 4).

$4 = a(1 + 3)(1 + 1)(1 - 2)^2$

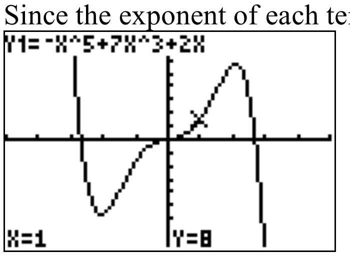
$4 = 8a$

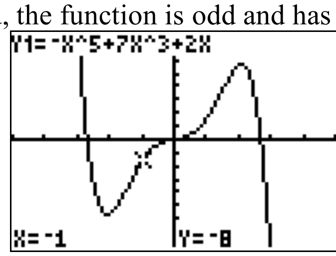
$a = \frac{1}{2}$

Equation: $y = \frac{1}{2}(x + 3)(x + 1)(x - 2)^2$

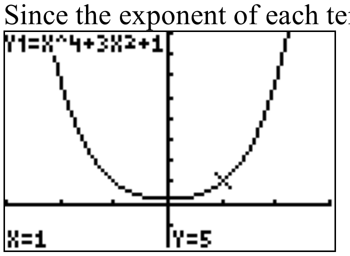
Chapter 1 Review **Question 12 Page 76**

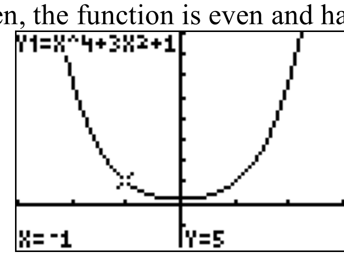
a) Since the exponent of each term is odd, the function is odd and has point symmetry.



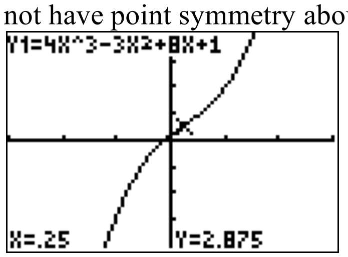


b) Since the exponent of each term is even, the function is even and has line symmetry.





c) Some exponents are even and some are odd, so the function is neither even nor odd and does not have point symmetry about the origin or line symmetry about the y-axis.



Chapter 1 Review **Question 13 Page 76**

a) x-intercepts: –2, 4; point: (2, –16)
Solve for a and substitute in the point.
$-16 = a(2 + 2)(2 - 4)^2$
$a = -1$
Equation: $y = -(x + 2)(x - 4)^2$

b) x-intercepts: –2, $\frac{1}{2}$, 4; point: (1, 5)
Solve for a and substitute in the point.
$5 = a(1 + 2)^2(1 - 4)\left(1 - \frac{1}{2}\right)$

$$a = \frac{5}{-13.5}$$
$$= -\frac{10}{27}$$

Equation: $y = -\frac{10}{27}(x + 2)^2(x - 4)\left(x - \frac{1}{2}\right)$

Chapter 1 Review **Question 14 Page 76**

a) i) Vertical compression by a factor of $\frac{1}{4}$; reflection in the x-axis; vertical translation 2 units down
Equation: $y = -\frac{1}{4}x^3 - 2$

ii) Domain: $\{x \in \mathbb{R}\}$; Range: $\{y \in \mathbb{R}\}$

b) i) Vertical stretch by a factor of 5; horizontal stretch by a factor of $\frac{5}{2}$; horizontal translation 3 units right; vertical translation 1 unit up
Equation: $y = 5\left[\frac{2}{5}(x - 3)\right]^4 + 1$

ii) Domain: $\{x \in \mathbb{R}\}$; Range: $\{y \in \mathbb{R}, y \geq 1\}$; Vertex: (3, 1); Axis of Symmetry: $x = 3$

Chapter 1 Review **Question 15 Page 76**

a) i) Equation: $y = \frac{3}{5}\left[-\frac{1}{2}\left(x+4\right)\right]^4 + 1$

ii) Domain: $\{x \in \mathbb{R}\}$; Range: $\{y \in \mathbb{R}, y \geq 1\}$; Vertex: (–4, 1); Axis of Symmetry: $x = -4$

b) i) Equation: $y = -5\left[4\left(x+2\right)\right]^3 + 7$

ii) Domain: $\{x \in \mathbb{R}\}$; Range: $\{y \in \mathbb{R}\}$

Chapter 1 Review **Question 16 Page 76**

Answers may vary. A sample solution is shown.

a) This is not an example of an average rate of change because it is just the average height.

c) This is not an example of an average rate of change because it is the instantaneous rate of change.

d) This is not an example of an average rate of change because it is just the class average.

Chapter 1 Review **Question 17 Page 77**

a) At the beginning of the year the stock was \$14; at the end of the year the stock was \$10.

b) i) From 0-6 months, the average rate of change is negative.
ii) From 6-9 months, the average rate of change is positive.
iii) From 9-12 months, the average rate of change is zero.

c) i) Average rate of change $= \frac{4-14}{6-0}$
$= -\$1.6\text{/month}$

ii) Average rate of change $= \frac{10-4}{9-6}$
$= \$2\text{/month}$

iii) Average rate of change $= \frac{10-10}{12-9}$
$= \$0\text{/month}$

Chapter 1 Review **Question 18 Page 77**

a) Average rate of change $= \dfrac{30.8 - 8.0}{2003 - 1999}$

$= 5.7\%/\text{year}$

The average rate of change from 1999 to 2003 is 5.7% per year.

b) 2000:

Average rate of change from 1999 to 2000 $= \dfrac{14.7 - 8}{1}$

$= 6.7$

Average rate of change from 2000 to 2001 $= \dfrac{21.6 - 14.7}{1}$

$= 6.9$

The instantaneous rate of change is between 6.7% and 6.9% in 2000.

2002:

Average rate of change from 2001 to 2002 $= \dfrac{26.2 - 21.6}{1}$

$= 4.6$

Average rate of change from 2002 to 2003 $= \dfrac{30.8 - 26.2}{1}$

$= 4.6$

The instantaneous rate of change is 4.6% in 2002.

c) Answers may vary. For example, part a) is the average of the values in part b).

Chapter Problem Wrap Up

Solutions to Chapter Problem Wrap-Up are provided in the Teacher's Resource.

Chapter 1 Practice Test

Chapter 1 Practice Test **Question 1 Page 78**

The correct solution is **C**.

A False; $y = x^3 + x^2$ is degree 3 (odd), but is not an odd function since there is a term with an even exponent.

B False; $y = x^2$ has one x-intercept of order 2.

D False; $y = x^4 + x$ is degree 4 (even), but not all the terms have even exponents, so the function is not even.

Chapter 1 Practice Test **Question 2 Page 78**

The correct solution is **B**.

A False; n degree equals constant nth differences, so a function with constant third differences has degree 3.

C False; $y = x^6 + x^4$ is a power function with even degree and it has line symmetry about the y-axis since all the exponents of its terms are even.

D False; it may have a higher degree, $y = x^6 - 19x^4 + 99x^2 - 81$ has four x-intercepts and has degree 6.

Chapter 1 Practice Test **Question 3 Page 78**

The correct solution is **E**.

A False; $y = \frac{1}{3}x^2$ does not equal $y = \left(\frac{1}{3}x\right)^2$.

B. False; stretches and compressions are applied first.

C False; it does not matter, it can be either way.

D False; it results in a reflection in the y-axis.

Chapter 1 Practice Test **Question 4 Page 78**

a) i) It is an odd degree function extending from quadrant 2 to 4 with point symmetry. There are 3 x-intercepts of order 1, so this is a cubic function with a negative leading coefficient.

b) iii) It is an even function with line symmetry about the y-axis. It extends from quadrant 3 to 4, so it has a negative leading coefficient.

c) ii) It is an odd degree function with point symmetry. It extends from quadrant 3 to 1, so it has a positive leading coefficient.

Chapter 1 Practice Test **Question 5 Page 78**

i) $f(x) = -2x^3 + 7x + 1$

a) The 3rd finite differences are constant.
b) $-2(3 \times 2 \times 1) = -12$; the value of the constant finite differences is -12.
c) The function has point symmetry about (0, 1). $f(x)$ is the graph of $g(x) = -2x^3 + 7x$ shifted up one. Since $g(x)$ has point symmetry about (0, 0) (because all the terms have odd exponents) then $f(x)$ has point symmetry about (0, 1).

ii) $h(x) = x^5 - 7x^3 + 2x + 1$

a) The 5th finite differences are constant.
b) $1(5 \times 4 \times 3 \times 2 \times 1) = 120$; the value of the constant finite differences is 120.
c) The function has point symmetry about (0, 1). $h(x)$ is the graph of $r(x) = x^5 - 7x^3 + 2x$ shifted up one. Since $r(x)$ has point symmetry about (0, 0) (because all the terms have odd exponents) then $h(x)$ has point symmetry about (0, 1).

iii) $p(x) = -x^6 + 5x^2 + 1$

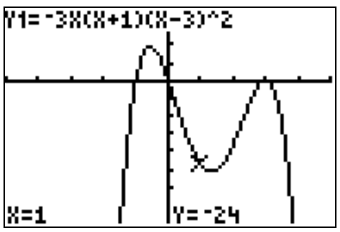
a) The 6th finite differences are constant.
b) $-1(6 \times 5 \times 4 \times 3 \times 2 \times 1) = -720$; the value of the constant finite differences is -720.
c) The function has line symmetry since all the exponents in the function have even degrees.

Chapter 1 Practice Test **Question 6 Page 78**

a) Answers may vary. A sample solution is shown.
Equation 1: $y = 2x(x + 1)(x - 3)^2$
Equation 2: $y = -x(x + 1)(x - 3)^2$

b) Solve for a and substitute the point.
$-18 = a(2)(3)(-1)^2$
$-18 = 6a$
$a = -3$
Equation: $y = -3x(x + 1)(x - 3)^2$

c)



Positive Intervals: $-1 < x < 0$
Negative Intervals: $x < -1$, $0 < x < 3$, $x > 3$

Chapter 1 Practice Test **Question 7 Page 79**

The x-intercepts (roots) are –1 (order 2), 0 (order 1), 1 (order 1), and 2 (order 3). The graph extends from quadrant 2 to 4 so the leading coefficient is negative.
The equation is $y = -x(x + 1)^2(x - 1)(x - 2)^3$.

Chapter 1 Practice Test **Question 8 Page 79**

a) $a = \frac{1}{3}$; vertical compression by a factor of $\frac{1}{3}$

$k = -2$; horizontal compression by a factor of $\frac{1}{2}$, reflection in the y-axis

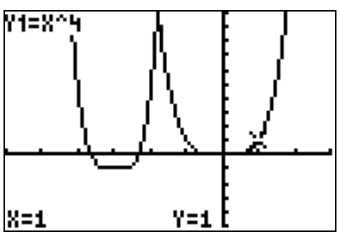
$d = -3$; horizontal translation 3 units left
$c = -1$; vertical translation 1 unit down

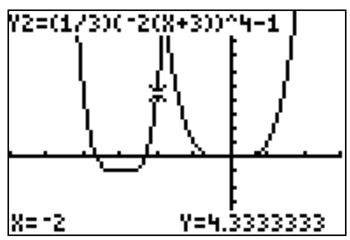
b) Domain: $\{x \in \mathbb{R}\}$; Range: $\{y \in \mathbb{R}, y \leq -1\}$; Vertex: $(-3, -1)$; Axis of Symmetry: $x = -3$

c) Answers may vary. A sample solution is shown.

1. horizontal compression by a factor of $\frac{1}{2}$, vertical compression by a factor of $\frac{1}{3}$, reflection in the y-axis, horizontal translation 3 units left, vertical translation 1 unit down
2. horizontal compression by a factor of $\frac{1}{2}$,vertical compression by a factor of $\frac{1}{3}$, reflection in the y-axis, vertical translation 1 unit down, horizontal translation 3 units left

d)





Chapter 1 Practice Test **Question 9 Page 79**

The equation is $y = -2(x - 3)^3 - 5$.

Chapter 1 Practice Test **Question 10 Page 79**

Answers may vary. A sample solution is shown.
a) An example could be distance versus time at a constant speed.

b) An example could be the drop in temperature versus time at a constant rate.

c) An example could be acceleration.

d) An example could be no change in revenue over a period of time.

Chapter 1 Practice Test **Question 11 Page 79**

$$\text{Average rate of change from 1990 to 2003} = \frac{76.1-15.5}{2003-1990} \doteq 4.66$$

The average rate of change of the percent of households that had a CD player from 1990 to 2003 is 4.66%/year.

Chapter 1 Practice Test **Question 12 Page 79**

a) $V(0) = 0.2(25-0)^3 = 3125$

There is 3125 L of oil in the tank initially.

b) i) Average rate of change in the first 10 min $= \dfrac{0.2(25-10)^3 - \left[0.2(25-0)^3\right]}{10-0} = \dfrac{675-3125}{10} = -245$

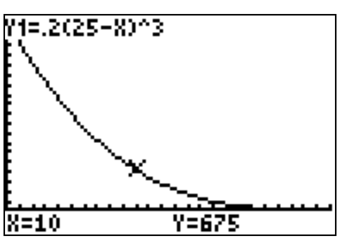
The average rate of change of volume in the first 10 min is –245 L/min.

ii) Average rate of change in the last 10 min $= \dfrac{0.2(25-25)^3 - \left[0.2(25-15)^3\right]}{25-15} = \dfrac{0-200}{10} = -20$

The average rate of change of volume in the last 10 min is –20 L/min.

c) Answers may vary. For example, the average rate of change of volume over time is negative and increasing.

d)



e) The values in part b) represent the slopes of the secants.

a) Average rate of change $= \dfrac{0.002(10)^3 + 0.05(10)^2 + 0.3(10) - \left[0.002(0)^3 + 0.05(0)^2 + 0.3(0)\right]}{10 - 0}$

$$= \frac{10 - 0}{10}$$
$$= 1$$

The average rate of change of the distance travelled in the first 10 s is 1 m/s.

b) $10 \le t \le 10.001$

Average rate of change $= \dfrac{0.002(10.001)^3 + 0.05(10.001)^2 + 0.3(10.001) - \left[0.002(10)^3 + 0.05(10)^2 + 0.3(10)\right]}{10.001 - 10}$

$$= \frac{10.0019 - 10}{0.001}$$
$$= 1.9$$

The instantaneous rate of change of the distance travelled 10 s after leaving the shore is approximately 1.9 m/s.

c) Answers may vary. For example, the speed is increasing since the average in part a) is lower than the instantaneous rate in part b).