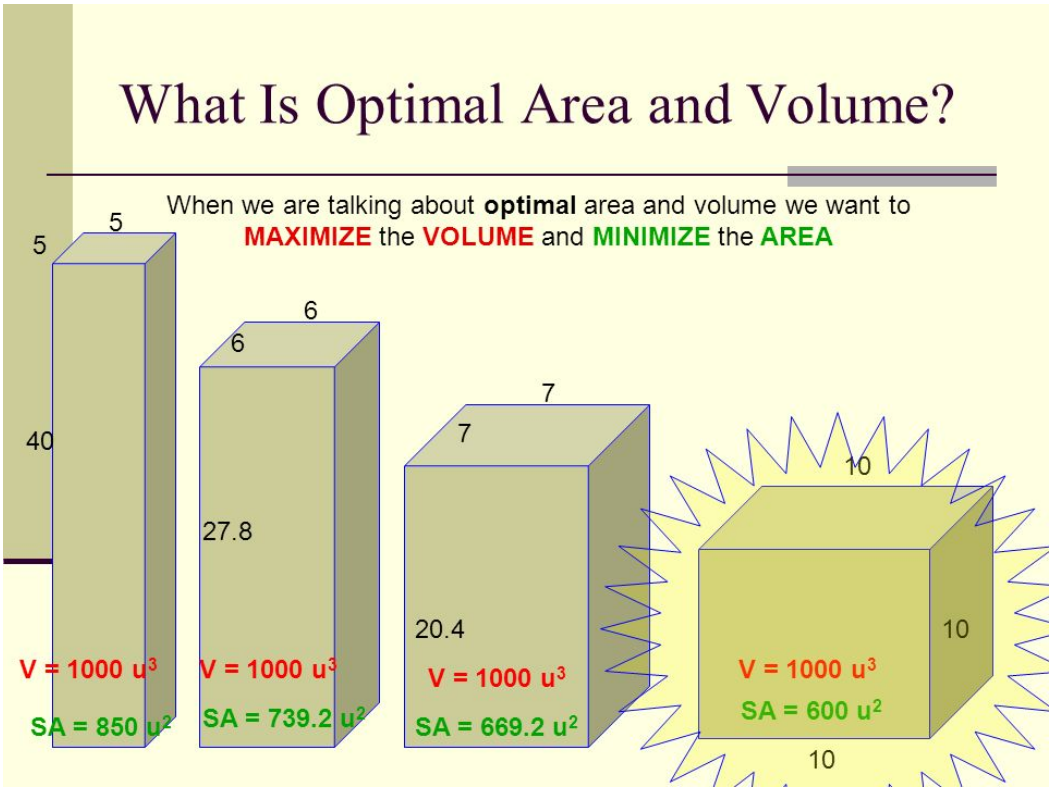
9.3 Minimizing the Surface Area of a Square- Based Prism for a given Volume

- Minimizing surface area for a given volume is important when designing packages and containers to save on materials and reduce heat loss.

- For a square-based prism with a given volume, the minimum surface area occurs when the prism is a cube



- Given a volume , you can find the dimensions of a square-based prism with minimum surface area by solving for s in the formula $V = s^3$, where V is the given volume and s is the length of a side of the cube.

Example one: The Pop-a-Lot popcorn company ships kernels of popcorn to movie theatres in large cardboard boxes with a volume of 500 000 cm^3.

a) Determine the dimensions of the square-based prism box, to the nearest tenth of a centimeter, that will require the least amount of cardboard.

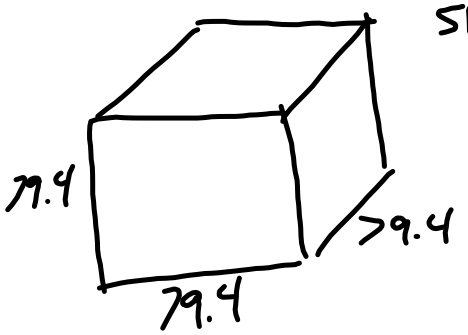
since it will be a cube, each dimension is s units.

$V = s^3$

$500000 = s^3$

$\sqrt[3]{500000} = 79.4\text{ cm}$

b) Find the amount of cardboard required to make this box, to the nearest tenth of a square meter. Describe any assumptions you have made.



since each face is s^2, $SA = 6s^2$

$= 6(79.4)^2$

$= 6(6304.36)$

$= 37826.2\text{ cm}^2$

Example 2. Tyler has been asked to design an insulated square-based prism container to transport hot food. To keep heat loss to a minimum, the total surface area must be minimized. Find the dimensions of the container with volume 145 000 cm^3 that has the minimum heat loss. Round the dimensions to the nearest tenth of a centimeter.

$V = 145000\ cm^3$

$V = s^3$

$145000 = s^3$

$\sqrt[3]{145000} = s$

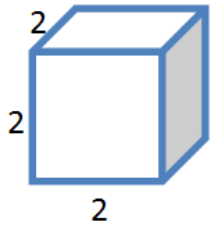
$52.5 = s$

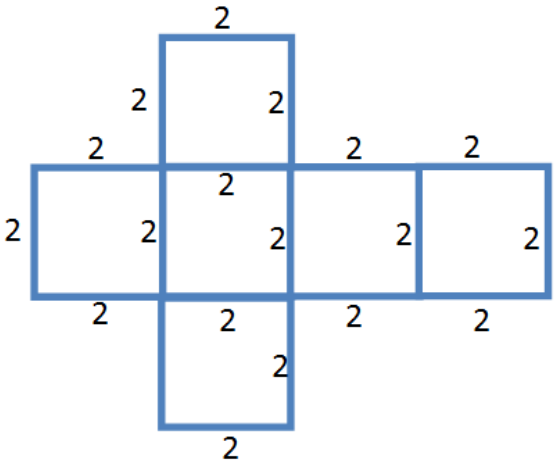
$\therefore$ each side length is 52.5 cm

9.4 Maximize the Volume of a Square-Based Prism

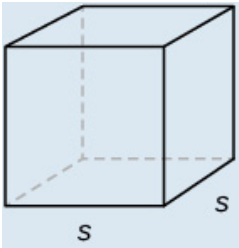
The maximum volume for a given surface area of a square-based prism always occurs when the prism is a cube

The surface area of a cube is given by the formula $SA = 6s^2$, where s is the side length of a cube. When you are given the surface area, solve for s to find the dimensions of the square-based prism with maximum volume.





Know this!



Volume: $V = s^3$

Surface Area: $SA = 6s^2$

Example 3:

need S

given SA

a) Determine the dimensions of the square based prism with maximum volume that can be formed using $5400cm^2$ of cardboard.

$SA = 5400cm^2$

$SA = 6s^2$

$\frac{5400}{6} = \frac{6s^2}{6}$

$900 = s^2$

$\sqrt{900} = s$

$30 = s$

solving is SAMDEB!

b) What is the volume of the prism?

$V = s^3$

$= 30^3$

$= 27000\ cm^3$

Practice: pg 495 #2, 3, 5, 6, 7, 11

pg 502 #5, 7, 8