9.2 Optimization of Area and Perimeter in Rectangles

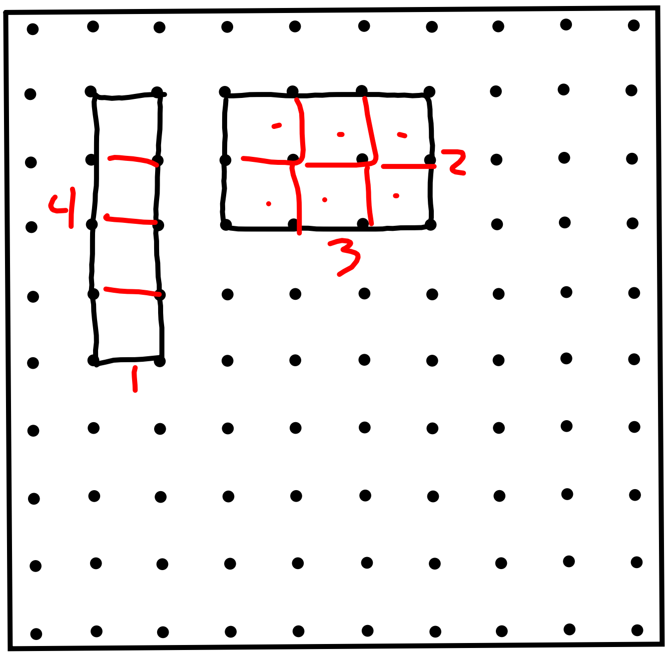
Learning Goal: You will understand that rectangles with the same perimeter can have different dimensions/areas and rectangles with the same area can have different dimensions/perimeter.

9.1 and 9.2 Optimizing Perimeter and Area of a Rectangle

Rectangles with the same perimeter can have different dimensions and contain different areas.

Example 1. If the perimeter of a rectangle is 10, draw examples that would fit this criteria:

$A = 4 \times 1$
$= 4u^2$



$3 \times 2 = 6$
$A = 6$

Rectangles with the same area can have different dimensions and different perimeter.

Example 2. Draw different rectangles with an area of 20 m^2.

Optimization is the process of finding values that make a given quantity the greatest (or least) possible given certain conditions.

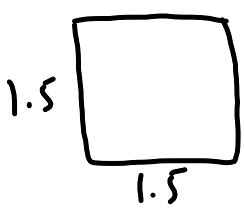
- When we **optimize area**, we find the greatest area for a given perimeter
- When we **optimize perimeter**, we find the least perimeter for a given area

1) Optimizing Area

Need to Know: Rectangles with maximum area (greatest possible) for a given perimeter are a **square**.

Ex. To brighten a room, a rectangular window will be built into a wall. To keep costs as low as possible, the perimeter of the window must be 6.0 m. What window dimensions will allow the maximum amount of light to enter the room?

In this case, we want the biggest area possible for 6 m perimeter. We know that this will be a square, so what must the side lengths be, and what must the area be?



$A = L \times W$

$A = 1.5 \times 1.5$

$= 2.25 \text{ m}^2$

2) Optimizing Perimeter

Need to Know: Rectangles with minimum perimeter for a given area are a **square**.

Example: A farmer wants to enclose a rectangular field with an area of 10 000 m^2 using the minimum amount of fencing. What should the dimensions of the field be?

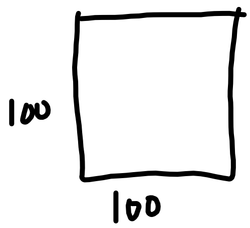
s is side

$$A_{sq} = s^2$$

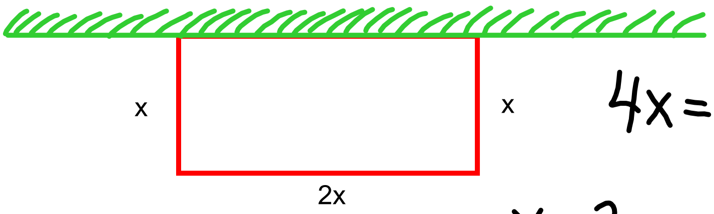
$$10000 = s^2$$

$$\sqrt{10000} = s$$

$$100 = s$$



A rectangle with a border on only three sides has a maximum area for a given border length or a minimum border length for a given area when the side without the border and its opposite side are twice the length of the other two sides.

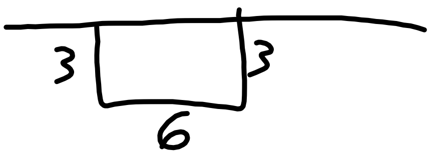


$4x = 12$

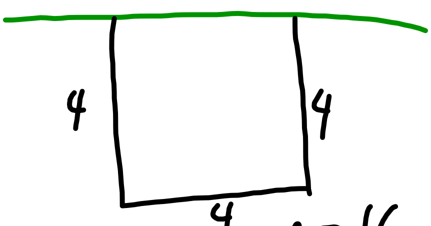
$x + 2x + x = 12$
$4x = 12$

Ex. A rectangular area is to be enclosed with 12 m of fencing. Suppose an existing hedge is used to enclose one side. What is the maximum area that can be enclosed?

$4x = 12$
$x = \frac{12}{4}$
$= 3$

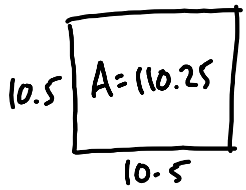


$A = lw$
$= 6 \times 3$
$= 18\ m^2$



$A = 16 m^2$

Example: You have 42 ft of rope to enclose an area. What dimensions will result in the greatest area? What is the area?



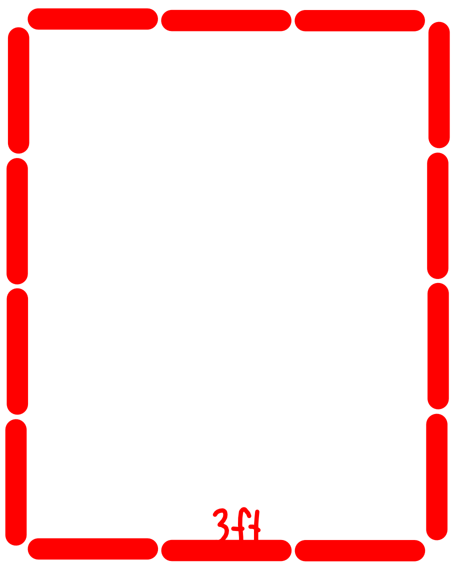
$P = 42\text{ft}$

$\frac{42}{4} = 10.5\text{ ft}$

Suppose that instead of rope you have fourteen 3 ft long metal bars to make the enclosure (total length is still 42 ft). The bars cannot be cut. What are the dimensions of the greatest area you can enclose?

length	width	area
1 bar (3 ft)	6 bar (18 ft)	54 sq ft
2 bar (6 ft)	5 bar (15 ft)	90 sq ft
3 bar (9 ft)	4 bar (12 ft)	108 sq ft
4 bar (12 ft)	3 bar (9 ft)	108 sq ft
5 bar (15 ft)	2 bar (6 ft)	90 sq ft
6 bar (18 ft)	1 bar (3 ft)	54 sq ft

In a situation like this it may not be possible to make a perfect square! In that case, you make the closest thing to a square possible!



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3, 4 (just optimize area, don't "investigate"), 5, 6, 8