Section 6.1: Newtonian Gravitation

Tutorial 1 Practice, page 293

1. Given: $m_1 = 1.0 \times 10^{20}$ kg; $m_2 = 3.0 \times 10^{20}$ kg; $F_g = 2.2 \times 10^9$ N; $G = 6.67 \times 10^{-11}$ N·m²/kg²
Required: r

Analysis: $F_g = \dfrac{Gm_1m_2}{r^2}$;

$$r^2 = \frac{Gm_1m_2}{F_g}$$

$$r = \sqrt{\frac{Gm_1m_2}{F_g}}$$

Solution: $r = \sqrt{\dfrac{Gm_1m_2}{F_g}}$

$$= \sqrt{\frac{\left(6.67\times10^{-11}\ \frac{\cancel{\text{N}}\cdot\text{m}^2}{\cancel{\text{kg}^2}}\right)\left(1.0\times10^{20}\ \cancel{\text{kg}}\right)\left(3.0\times10^{20}\ \cancel{\text{kg}}\right)}{\left(2.2\times10^9\ \cancel{\text{N}}\right)}}$$

$$r = 3.0\times10^{10}\ \text{m}$$

Statement: The distance between the two asteroids is 3.0×10^{10} m.

2. Given: $m = 1.9 \times 10^{27}$ kg; $r = 7.0 \times 10^7$ m; $G = 6.67 \times 10^{-11}$ N·m²/kg²
Required: g_{Jupiter}

Analysis: Start with the universal law of gravitation, $F_g = \dfrac{Gm_1m_2}{r^2}$, then use $F = ma$ to substitute for F_g with the mass of an object on the surface, m_2, and the acceleration of the object, which will be the magnitude of the gravitational field strength on the surface of Jupiter, g_{Jupiter}.

$$F_g = \frac{Gm_1m_2}{r^2}$$

$$\cancel{m_2}a = \frac{Gm_1\cancel{m_2}}{r^2}$$

$$g_{\text{Jupiter}} = \frac{Gm_1}{r^2}$$

Solution: $g_{\text{Jupiter}} = \dfrac{Gm_1}{r^2}$

$$= \frac{\left(6.67\times10^{-11}\ \frac{\text{N}\cdot\cancel{\text{m}^2}}{\text{kg}^{\cancel{2}}}\right)\left(1.9\times10^{27}\ \cancel{\text{kg}}\right)}{\left(7.0\times10^7\ \cancel{\text{m}}\right)^2}$$

$$g_{\text{Jupiter}} = 26\ \text{m/s}^2$$

Statement: The magnitude of the gravitational field strength on the surface of Jupiter is 26 m/s².

3. (a) Given: m_A = 40.0 kg; m_B = 60.0 kg; m_C = 80.0 kg; r_{AB} = 0.50 m; r_{BC} = 0.75 m; $G = 6.67 \times 10^{-11}$ N·m²/kg²

Required: $\vec{F}_{\text{net}}$

Analysis: $F_g = \dfrac{Gm_1m_2}{r^2}$

Solution: Determine $\vec{F}_{AB}$.

$$F_g = \frac{Gm_1m_2}{r^2}$$

$$F_{AB} = \frac{Gm_Am_B}{r_{AB}^2}$$

$$= \frac{\left(6.67\times10^{-11}\ \dfrac{\text{N}\bullet\cancel{\text{m}^2}}{\cancel{\text{kg}^2}}\right)(40.0\ \cancel{\text{kg}})(60.0\ \cancel{\text{kg}})}{(0.50\ \cancel{\text{m}})^2}$$

$F_{AB} = 6.403\times10^{-7}$ N (two extra digits carried)

Determine $\vec{F}_{BC}$.

$$F_g = \frac{Gm_1m_2}{r^2}$$

$$F_{BC} = \frac{Gm_Bm_C}{r_{BC}^2}$$

$$= \frac{\left(6.67\times10^{-11}\ \dfrac{\text{N}\cdot\cancel{\text{m}^2}}{\cancel{\text{kg}^2}}\right)(60.0\ \cancel{\text{kg}})(80.0\ \cancel{\text{kg}})}{(0.75\ \cancel{\text{m}})^2}$$

$F_{BC} = 5.692\times10^{-7}$ N (two extra digits carried)

$$\vec{F}_{\text{net}} = \vec{F}_{AB} + \vec{F}_{BC}$$
$$= 6.403\times10^{-7}\ \text{N [left]} + 5.692\times10^{-7}\ \text{N [right]}$$
$$= 6.403\times10^{-7}\ \text{N [left]} - 5.692\times10^{-7}\ \text{N [left]}$$
$$\vec{F}_{\text{net}} = 7.1\times10^{-8}\ \text{N [left]}$$

Statement: The net force acting on B is 7.1×10^{-8} N [left].

(b) Given: m_A = 40.0 kg; m_B = 60.0 kg; m_C = 80.0 kg; r_{AB} = 0.50 m; r_{BC} = 0.75 m; $G = 6.67 \times 10^{-11}$ N·m²/kg²

Required: $\vec{F}_{\text{net}}$

Analysis: $F_g = \dfrac{Gm_1m_2}{r^2}$; determine the angle using the inverse tan function.

Solution: Determine $\vec{F}_{\text{AB}}$.

$$F_{\text{g}} = \frac{Gm_1m_2}{r^2}$$

$$F_{\text{AB}} = \frac{Gm_{\text{A}}m_{\text{B}}}{r_{\text{AB}}^2}$$

$$= \frac{\left(6.67\times10^{-11}\ \frac{\text{N}\cdot\cancel{\text{m}^2}}{\cancel{\text{kg}^2}}\right)(40.0\ \cancel{\text{kg}})(60.0\ \cancel{\text{kg}})}{(0.50\ \cancel{\text{m}})^2}$$

$F_{\text{AB}} = 6.403\times10^{-7}$ N (two extra digits carried)

Determine $\vec{F}_{\text{BC}}$.

$$F_{\text{g}} = \frac{Gm_1m_2}{r^2}$$

$$F_{\text{BC}} = \frac{Gm_{\text{B}}m_{\text{C}}}{r_{\text{BC}}^2}$$

$$= \frac{\left(6.67\times10^{-11}\ \frac{\text{N}\bullet\cancel{\text{m}^2}}{\cancel{\text{kg}^2}}\right)(60.0\ \cancel{\text{kg}})(80.0\ \cancel{\text{kg}})}{(0.75\ \cancel{\text{m}})^2}$$

$F_{\text{BC}} = 5.692\times10^{-7}$ N (two extra digits carried)

$$\vec{F}_{\text{net}} = \vec{F}_{\text{AB}} + \vec{F}_{\text{BC}}$$

$$F_{\text{net}} = \sqrt{F_{\text{AB}}^2 + F_{\text{BC}}^2}$$

$$= \sqrt{(6.403\times10^{-7}\ \text{N})^2 + (5.692\times10^{-7}\ \text{N})^2}$$

$$F_{\text{net}} = 8.6\times10^{-8}\ \text{N}$$

$$\theta = \tan^{-1}\left(\frac{F_{\text{BC}}}{F_{\text{AB}}}\right)$$

$$= \tan^{-1}\left(\frac{5.692\times10^{-7}\ \text{N}}{6.403\times10^{-7}\ \text{N}}\right)$$

$$\theta = 42^\circ$$

Statement: The net force acting on B is 8.6×10^{-8} N [W 42° S].

Tutorial 2 Practice, page 295

1. Given: $r = 7.0 \times 10^6$ m; $m_{\text{white dwarf}} = 1.2 \times 10^{30}$ kg; $G = 6.67 \times 10^{-11}$ N·m²/kg²

Required: $g_{\text{white dwarf}}$

Analysis: $g_{\text{white dwarf}} = \dfrac{Gm_{\text{white dwarf}}}{r^2}$

Solution: $g_{\text{white dwarf}} = \dfrac{Gm_{\text{white dwarf}}}{r^2}$

$$= \frac{\left(6.67\times10^{-11}\ \frac{\text{N}\cdot\cancel{\text{m}^2}}{\text{kg}^{\cancel{2}}}\right)\left(1.2\times10^{30}\ \cancel{\text{kg}}\right)}{\left(7.0\times10^{6}\ \cancel{\text{m}}\right)^2}$$

$$g_{\text{white dwarf}} = 1.6\times10^{6}\ \text{N/kg}$$

Statement: The surface gravitational field strength of the white dwarf is 1.6×10^6 N/kg, which is over 100 000 times that of Earth.

2. Given: $r_2 = 2r_{\text{Saturn}}$

Required: g_2

Analysis: $g_2 = \dfrac{Gm}{r_2^2}$

Solution: $g_2 = \dfrac{Gm}{r_2^2}$

$$= \frac{Gm}{\left(2r_{\text{Saturn}}\right)^2}$$

$$= \frac{1}{4}\left(\frac{Gm}{r_{\text{Saturn}}^2}\right)$$

$$g_2 = \frac{1}{4}g_{\text{Saturn}}$$

Statement: The surface gravitational field strength would be one quarter of the old surface gravitational field strength.

Research This: Gravitational Field Maps and Unmanned Underwater Vehicles, page 295

A. Sample answers: A gravitational field map describes the strength of the gravitational field at points across Earth. The map is created by using satellites to detect fine density differences in the crust, which cause increases or decreases in the gravitational force. This information can be used by a UUV to detect where it is on the planet based on the gravitational force.

B. Diagrams may vary depending on the type of UUV chosen. Students should highlight the key feature of the UUV they choose, such as the propulsion system (a propeller is most common), the power source (battery powered), the navigation system, and the sensors, which will vary with purpose of the UUV, but may include depths sensors, sonar, or sensors to measure concentration of compounds in the water.

C. Answers may vary. Students reports should explain the how UUVs use gravitational field maps to compare with measurements collected by the UUV on about the direction, angle, and strength of Earth's magnetic field at its position. Students may also discuss the usefulness of navigation by magnetic fields because UUVs travel to far for remote control and do not have access to satellites for GPS navigation.

Section 6.1 Questions, page 296

1. For your weight to be one half your weight on the surface, the magnitude of the gravitational acceleration must be one half of g.

$$g = \frac{Gm}{r_E^2}$$

$$\frac{1}{2}g = \frac{Gm}{2r_E^2}$$

$$\frac{1}{2}g = \frac{Gm}{\left(\sqrt{2}r_E\right)^2}$$

The altitude from Earth's centre is $\sqrt{2}r_E$, or about $1.41r_E$. Therefore, the altitude above Earth's surface is $1.41r_E - 1r_E = 0.41r_E$.

2. Given: $r = 5.3 \times 10^{-11}$ m; $m_1 = 1.67 \times 10^{-27}$ kg; $m_2 = 9.11 \times 10^{-31}$ kg; $G = 6.67 \times 10^{-11}\ \text{N·m}^2/\text{kg}^2$

Required: F_g

Analysis: $F_g = \frac{Gm_1m_2}{r^2}$

Solution: $F_g = \frac{Gm_1m_2}{r^2}$

$$= \frac{\left(6.67\times10^{-11}\ \frac{\text{N}\cdot\cancel{\text{m}^2}}{\cancel{\text{kg}^2}}\right)\left(1.67\times10^{-27}\ \cancel{\text{kg}}\right)\left(9.11\times10^{-31}\ \cancel{\text{kg}}\right)}{\left(5.3\times10^{-11}\ \cancel{\text{m}}\right)^2}$$

$$F_g = 3.6\times10^{-47}\ \text{N}$$

Statement: The magnitude of the gravitational attraction between the proton and the electron is 3.6×10^{-47} N.

3. (a) The value for r is squared in the denominator, so as r increases, the gravitational force decreases.

(b)

$$F_g = \frac{Gm_1m_2}{r^2}$$

$$= \frac{Gm_1m_2}{(4r_1)^2}$$

$$= \frac{Gm_1m_2}{16r_1^2}$$

$$F_g = \frac{1}{16}\left(\frac{Gm_1m_2}{r_1^2}\right)$$

The gravitational force changes by a factor of $\frac{1}{16}$.

4. (a) Given: $m_1 = 225$ kg; $d = 8.62 \times 10^6$ m; $m_2 = 5.98 \times 10^{24}$ kg; $r_E = 6.38 \times 10^6$ m; $G = 6.67 \times 10^{-11}$ N·m²/kg²
Required: F_g
Analysis:

$$F_g = \frac{Gm_1m_2}{r^2}$$

$$F_g = \frac{Gm_1m_E}{\left(d + r_E\right)^2}$$

Solution: $F_g = \dfrac{Gm_1m_E}{\left(d + r_E\right)^2}$

$$= \frac{\left(6.67 \times 10^{-11}\ \frac{\text{N} \cdot \cancel{\text{m}^2}}{\cancel{\text{kg}^2}}\right)\left(225\ \cancel{\text{kg}}\right)\left(5.98 \times 10^{24}\ \cancel{\text{kg}}\right)}{\left(8.62 \times 10^6\ \cancel{\text{m}} + 6.38 \times 10^6\ \cancel{\text{m}}\right)^2}$$

$$F_g = 399\text{ N}$$

Statement: The gravitational force is 399 N toward Earth's centre.
(b) Given: $m_1 = 225$ kg; $d = 8.62 \times 10^6$ m; $m_E = 5.98 \times 10^{24}$ kg; $r_E = 6.38 \times 10^6$ m; $G = 6.67 \times 10^{-11}$ N·m²/kg²
Required: g
Analysis: $g = \dfrac{Gm}{r^2}$

Solution: $g = \dfrac{Gm}{r^2}$

$$= \frac{Gm_E}{\left(d + r_E\right)^2}$$

$$= \frac{\left(6.67 \times 10^{-11}\ \frac{\text{N} \cdot \cancel{\text{m}^2}}{\text{kg}^{\cancel{2}}}\right)\left(5.98 \times 10^{24}\ \cancel{\text{kg}}\right)}{\left(8.62 \times 10^6\ \cancel{\text{m}} + 6.38 \times 10^6\ \cancel{\text{m}}\right)^2}$$

$$g = 1.77\text{ m/s}^2$$

Statement: The resulting acceleration is 1.77 m/s² toward Earth's centre.
5. Given: $g_{\text{Titan}} = 1.3$ N/kg; $m = 1.3 \times 10^{23}$ kg; $G = 6.67 \times 10^{-11}$ N·m²/kg²
Required: r
Analysis: $g = \dfrac{Gm}{r^2}$

Solution: $g_{\text{Titan}} = \dfrac{Gm}{r^2}$

$$r = \sqrt{\frac{Gm}{g_{\text{Titan}}}}$$

$$= \sqrt{\frac{\left(6.67\times10^{-11}\ \frac{\cancel{\text{N}}\cdot\text{m}^2}{\cancel{\text{kg}^2}}\right)\left(1.3\times10^{23}\ \cancel{\text{kg}}\right)}{\left(1.3\ \cancel{\text{N}}/\cancel{\text{kg}}\right)}}$$

$$r = 2.6\times10^{6}\ \text{m}$$

Statement: The radius of Titan is 2.6×10^6 m.

6. Given: g_E = 9.8 N/kg; g_2 = 3.20 N/kg

Required: r

Analysis: Use the equation $g = \dfrac{Gm}{r^2}$ to determine the change in r given the change in the value of g.

Solution: $\dfrac{g_2}{g_E} = \dfrac{3.20\ \cancel{\text{N/kg}}}{9.8\ \cancel{\text{N/kg}}}$

$$\frac{g_2}{g_E} = \frac{16}{49}$$

$$g_2 = \frac{16}{49} g_E$$

$$= \frac{16}{49}\left(\frac{Gm}{r_E^2}\right)$$

$$= \frac{Gm}{\frac{49}{16} r_E^2}$$

$$g_2 = \frac{Gm}{\left(\frac{7}{4} r_E\right)^2}$$

Statement: The acceleration due to gravity is 3.20 N/kg at $\dfrac{7}{4} r_E$ from Earth's centre, or $0.75 r_E$ above Earth's surface,

7. Given: $m_{\text{Sun}} = 2.0 \times 10^{30}$ kg; $r = 1.5 \times 10^{11}$ m; $G = 6.67 \times 10^{-11}$ N·m²/kg²

Required: g

Analysis: $g = \dfrac{Gm}{r^2}$

Solution: $g = \dfrac{Gm}{r^2}$

$$= \frac{\left(6.67\times10^{-11}\ \dfrac{\text{N}\bullet\cancel{\text{m}^2}}{\text{kg}^{\cancel{2}}}\right)\left(2.0\times10^{30}\ \cancel{\text{kg}}\right)}{\left(1.5\times10^{11}\ \cancel{\text{m}}\right)^2}$$

$$g = 5.9\times10^{-3}\ \text{N/kg}$$

Statement: The gravitational field strength of the Sun at a distance of 1.5×10^{11} m from its centre is 5.9×10^{-3} N/kg.

8. Let m_1 be the larger mass, and let x be the distance from m_1 to the location of zero net force. Set the two gravitational field strengths equal to each other, and develop a quadratic equation. Solve for x.

$$\frac{Gm_1}{x^2} = \frac{Gm_2}{(r-x)^2}$$

$$\cancel{G}(r-x)^2 = \frac{\cancel{G}m_2}{m_1}x^2$$

$$(r-x)^2 = \frac{m_2}{m_1}x^2$$

$$r^2 - 2rx + x^2 = \frac{m_2}{m_1}x^2$$

$$\left(1-\frac{m_2}{m_1}\right)x^2 - 2rx + r^2 = 0$$

Use the quadratic formula:

$$x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$= \frac{-(-2r)\pm\sqrt{(-2r)^2 - 4\left(1-\dfrac{m_2}{m_1}\right)(r^2)}}{2\left(1-\dfrac{m_2}{m_1}\right)}$$

$$= \frac{2r\pm\sqrt{4r^2-4r^2+4\dfrac{m_2}{m_1}r^2}}{2\left(1-\dfrac{m_2}{m_1}\right)}$$

$$= \frac{2r \pm \sqrt{4\frac{m_2}{m_1}r^2}}{2\left(1-\frac{m_2}{m_1}\right)}$$

$$= \frac{2r \pm 2r\sqrt{\frac{m_2}{m_1}}}{2\left(1-\frac{m_2}{m_1}\right)}$$

$$x = r\frac{1 \pm \sqrt{\frac{m_2}{m_1}}}{\left(1-\frac{m_2}{m_1}\right)}$$

Since the greater value will not be between the two masses but will be the other side of m_2 from m_1:

$$x = r\frac{1 - \sqrt{\frac{m_2}{m_1}}}{\left(1-\frac{m_2}{m_1}\right)}$$

$$x = \left(r\frac{1 - \sqrt{\frac{m_2}{m_1}}}{\left(1-\frac{m_2}{m_1}\right)}\right)\left(\frac{1 + \sqrt{\frac{m_2}{m_1}}}{1 + \sqrt{\frac{m_2}{m_1}}}\right)$$

$$x = r\frac{1-\frac{m_2}{m_1}}{\left(1-\frac{m_2}{m_1}\right)\left(1+\sqrt{\frac{m_2}{m_1}}\right)}$$

$$x = \frac{r}{1+\sqrt{\frac{m_2}{m_1}}}$$

The location of zero force is $\frac{r}{1+\sqrt{\frac{m_2}{m_1}}}$ from the larger object, m_1.

9. (a) Given: $m_1 = 537$ kg; $m_E = 5.98 \times 10^{24}$ kg; $r = 2.5 \times 10^7$ m; $G = 6.67 \times 10^{-11}$ N·m²/kg²

Required: g

Analysis: $g = \dfrac{Gm}{r^2}$

Solution:
$$
\begin{aligned}
g &= \frac{Gm}{r^2} \\
&= \frac{Gm_E}{r^2} \\
&= \frac{\left(6.67\times10^{-11}\ \frac{\text{N}\cdot\cancel{\text{m}^2}}{\text{kg}^{\cancel{2}}}\right)\left(5.98\times10^{24}\ \cancel{\text{kg}}\right)}{\left(2.5\times10^{7}\ \cancel{\text{m}}\right)^2}
\end{aligned}
$$

$$g = 0.64\ \text{m/s}^2$$

Statement: The resulting acceleration is 0.64 m/s² toward Earth's centre.

(b) Given: $m_1 = 537$ kg; $m_E = 5.98 \times 10^{24}$ kg; $r = 2.5 \times 10^7$ m; $G = 6.67 \times 10^{-11}$ N·m²/kg²

Required: F_g

Analysis: $F_g = \dfrac{Gm_1m_2}{r^2}$

Solution:
$$
\begin{aligned}
F_g &= \frac{Gm_1m_2}{r^2} \\
&= \frac{Gm_1m_E}{r^2} \\
&= \frac{\left(6.67\times10^{-11}\ \frac{\text{N}\cdot\cancel{\text{m}^2}}{\cancel{\text{kg}^2}}\right)\left(537\ \cancel{\text{kg}}\right)\left(5.98\times10^{24}\ \cancel{\text{kg}}\right)}{\left(2.5\times10^{7}\ \cancel{\text{m}}\right)^2}
\end{aligned}
$$

$$F_g = 340\ \text{N}$$

Statement: The gravitational force is 340 N toward Earth's centre.

10. Given: $r = 2.44 \times 10^6$ m; $m_{\text{Mercury}} = 3.28 \times 10^{23}$ kg; $G = 6.67 \times 10^{-11}$ N·m²/kg²

Required: g_{Mercury}

Analysis: $g_{\text{Mercury}} = \dfrac{Gm_{\text{Mercury}}}{r^2}$

Solution:
$$
\begin{aligned}
g_{\text{Mercury}} &= \frac{Gm_{\text{Mercury}}}{r^2} \\
&= \frac{\left(6.67\times10^{-11}\ \frac{\text{N}\cdot\cancel{\text{m}^2}}{\text{kg}^{\cancel{2}}}\right)\left(3.28\times10^{23}\ \cancel{\text{kg}}\right)}{\left(2.44\times10^{6}\ \cancel{\text{m}}\right)^2}
\end{aligned}
$$

$$g_{\text{Mercury}} = 3.67\ \text{N/kg}$$

Statement: The surface gravitational field strength on Mercury is 3.67 N/kg. The value provided in Table 2 is 3.7 N/kg, which is the same the value that I calculated to two significant digits.

11. (a) Given: g = 5.3 N/kg; $m_E = 5.98 \times 10^{24}$ kg; $r_E = 6.38 \times 10^6$ m; $G = 6.67 \times 10^{-11}$ N·m²/kg²

Required: r

Analysis: $g = \frac{Gm}{r^2}$

Solution: $g = \frac{Gm}{r^2}$

$$r = \sqrt{\frac{Gm}{g}}$$

$$= \sqrt{\frac{\left(6.67\times10^{-11}\ \frac{\cancel{\text{N}}\cdot\text{m}^2}{\cancel{\text{kg}^2}}\right)\left(5.98\times10^{24}\ \cancel{\text{kg}}\right)}{\left(5.3\ \cancel{\text{N}}/\cancel{\text{kg}}\right)}}$$

$$r = 8.675\times10^6\ \text{m (two extra digits carried)}$$

Calculate the altitude above Earth's surface:

8.675×10^6 m – 6.38×10^6 m = 2.3×10^6 m

Statement: The altitude of the satellite is 2.3×10^6 m.

(b) Given: m_1 = 620 kg; $m_E = 5.98 \times 10^{24}$ kg; $r_{satellite} = 8.675 \times 10^6$ m; $G = 6.67 \times 10^{-11}$ N·m²/kg²

Required: F_g

Analysis: $F_g = \frac{Gm_1m_2}{r^2}$

Solution: $F_g = \frac{Gm_1m_2}{r^2}$

$$= \frac{\left(6.67\times10^{-11}\ \frac{\text{N}\cdot\cancel{\text{m}^2}}{\cancel{\text{kg}^2}}\right)\left(620\ \cancel{\text{kg}}\right)\left(5.98\times10^{24}\ \cancel{\text{kg}}\right)}{\left(8.675\times10^6\ \cancel{\text{m}}\right)^2}$$

$$F_g = 3.3\times10^3\ \text{N}$$

Statement: The gravitational force on the satellite is 3.3×10^3 N toward Earth's centre.

12. The motion of the Moon depends on Earth's mass and G through the universal law of gravitation. Using data on the mass and orbital radius of the Moon and G, we can determine Earth's mass using the universal law of gravitation, $F_g = \frac{Gm_1m_2}{r^2}$, and the equation for centripetal acceleration, $F_c = \frac{mv^2}{r}$, since the two forces are equal.

$$F_g = F_c$$

$$\frac{Gm_1m_2}{r^2} = \frac{mv^2}{r}$$

$$\frac{Gm_{\text{Earth}}\cancel{m_{\text{Moon}}}}{r^{\cancel{2}}} = \frac{\cancel{m_{\text{Moon}}}v^2}{\cancel{r}}$$

$$\frac{Gm_{\text{Earth}}}{r} = v^2$$

$$m_{\text{Earth}} = \frac{rv^2}{G}$$

13. From question 8, the location of zero force is $\dfrac{r}{1+\sqrt{\dfrac{m_2}{m_1}}}$ from the larger object, m_1. r is the centre to centre distance between the Moon and Earth.

Since $m_E = 5.98 \times 10^{24}$ kg and $m_{\text{Moon}} = 7.36 \times 10^{22}$ kg:

$$\begin{aligned} r_0 &= \frac{r}{1+\sqrt{\dfrac{m_2}{m_1}}} \\ &= \frac{r}{1+\sqrt{\dfrac{7.36\times10^{22}}{5.98\times10^{24}}}} \\ &= \frac{r}{1.111} \\ r_0 &= 0.9r \end{aligned}$$

The mass should be $0.9r$ from the centre of the Earth, or $\dfrac{r}{10}$ from the centre of the Moon.

Section 6.2: Orbits

Mini Investigation: Exploring Gravity and Orbits, page 298

A. When I increase the size of the Sun, Earth's orbit changes: the orbit is closer to the Sun.
B. The Moon is pulled out of Earth's orbit and orbits the Sun. When you increase the size of Earth, the Moon orbits closer and faster.
C. The orbital radius and orbital period decrease.
D. Answers may vary. Students should describe a system using the satellite and planet simulation that includes a gravity assist by having the satellite use the planet's gravity to change direction. The scale is too small to notice changes in speed.

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1. Given: $r = 5.34 \times 10^{17}$ m; $v = 7.5 \times 10^{5}$ m/s; $G = 6.67 \times 10^{-11}$ N·m²/kg²
Required: m
Analysis: Rearrange the equation for speed to solve for mass:

$$v = \sqrt{\frac{Gm}{r}}$$

$$v^2 = \frac{Gm}{r}$$

$$m = \frac{rv^2}{G}$$

Solution: $m = \dfrac{rv^2}{G}$

$$= \frac{\left(5.34\times10^{17}\ \cancel{\text{m}}\right)\left(7.5\times10^{5}\ \cancel{\text{m}}/\cancel{\text{s}}\right)^2}{6.67\times10^{-11}\ \dfrac{\cancel{\text{kg}}\cdot\dfrac{\cancel{\text{m}}}{\cancel{\text{s}^2}}\cdot\cancel{\text{m}^2}}{\text{kg}^{\cancel{2}}}}$$

$$m = 4.5\times10^{39}\ \text{kg}$$

Statement: The mass of the black hole is 4.5×10^{39} kg.
2. Given: $r = 2.28 \times 10^{11}$ m; $m = 6.42 \times 10^{23}$ kg; $F_g = 1.63 \times 10^{21}$ N; $m_{Sun} = 1.99 \times 10^{30}$ kg; $G = 6.67 \times 10^{-11}$ N·m²/kg²
Required: v; T
Analysis: $v = \sqrt{\dfrac{Gm}{r}}$; $T = \dfrac{2\pi r}{v}$

Solution: Determine the orbital speed of Mars:

$$v = \sqrt{\frac{Gm_{\text{Sun}}}{r}}$$

$$= \sqrt{\frac{\left(6.67\times10^{-11}\ \frac{\cancel{\text{kg}}\cdot\frac{\cancel{\text{m}}}{\text{s}^2}\cdot\text{m}^2}{\text{kg}^{\cancel{2}}}\right)\left(1.99\times10^{30}\ \cancel{\text{kg}}\right)}{2.28\times10^{11}\ \cancel{\text{m}}}}$$

$$= 2.4128\times10^{4}\ \text{m/s (two extra digits carried)}$$

$$v = 2.41\times10^{4}\ \text{m/s}$$

Determine the period:

$$T = \frac{2\pi r}{v}$$

$$= \frac{2\pi\left(2.28\times10^{11}\ \cancel{\text{m}}\right)}{2.4128\times10^{4}\ \frac{\cancel{\text{m}}}{\text{s}}}$$

$$= 5.937\times10^{7}\ \cancel{\text{s}}\times\frac{1\ \cancel{\text{min}}}{60\ \cancel{\text{s}}}\times\frac{1\ \cancel{\text{h}}}{60\ \cancel{\text{min}}}\times\frac{1\ \cancel{\text{d}}}{24\ \cancel{\text{h}}}\times\frac{1\ \text{y}}{365\ \cancel{\text{d}}}$$

$$T = 1.90\ \text{y}$$

Statement: The speed of Mars is 2.41×10^4 m/s, and its period is 1.90 Earth years.

3. Given: $d = 600.0$ km $= 6.000 \times 10^5$ m; $r_E = 6.38 \times 10^6$ m; $m_E = 5.98 \times 10^{24}$ kg;
$G = 6.67 \times 10^{-11}$ N·m^2/kg^2

Required: v; T

Analysis: Determine the orbital radius, then use the value for r to calculate the speed, $v = \sqrt{\frac{Gm}{r}}$.

Then use the equation for period, $T = \frac{2\pi r}{v}$.

$$r = d + r_E$$

$$= 6.000\times10^{5}\ \text{m} + 6.38\times10^{6}\ \text{m}$$

$$r = 6.98\times10^{6}\ \text{m}$$

Solution:

$$v = \sqrt{\frac{Gm}{r}}$$

$$= \sqrt{\frac{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^{\cancel{2}}/\text{kg}^{\cancel{2}})(5.98\times10^{24}\ \cancel{\text{kg}})}{6.98\times10^{6}\ \cancel{\text{m}}}}$$

$$v = 7.559\times10^{3}\ \text{m/s (one extra digit carried)}$$

$$T = \frac{2\pi r}{v}$$
$$= \frac{2\pi(6.98\times10^{6}\ \cancel{\text{m}})}{7.559\times10^{3}\ \cancel{\text{m}}/\text{s}}$$
$$= 5801.9\ \cancel{\text{s}} \times \frac{1\ \text{min}}{60\ \cancel{\text{s}}}$$
$$T = 97\ \text{min}$$

Statement: The speed of the satellite is 7.56×10^3 m/s, and the period of the satellite is 97 min.

4. Given: d = 25 m; $r_{\text{Moon}} = 1.74 \times 10^6$ m; $m = 7.36 \times 10^{22}$ kg; $G = 6.67 \times 10^{-11}$ N·m^2/kg^2

Required: v

Analysis: $v = \sqrt{\frac{Gm}{r}}$

Solution: Determine the orbital radius:

$$r = d + r_{\text{Moon}}$$
$$= 25\ \text{m} + 1.74\times10^{6}\ \text{m}$$
$$r = 1.740\times10^{6}\ \text{m}$$

Determine the orbital speed:

$$v = \sqrt{\frac{Gm}{r}}$$
$$= \sqrt{\frac{\left(6.67\times10^{-11}\ \frac{\cancel{\text{kg}}\cdot\frac{\cancel{\text{m}}}{\text{s}^2}\cdot\text{m}^2}{\cancel{\text{kg}^2}}\right)\left(7.36\times10^{22}\ \cancel{\text{kg}}\right)}{1.740\times10^{6}\ \cancel{\text{m}}}}$$
$$v = 1.7\times10^{3}\ \text{m/s}$$

Statement: The orbital speed of the satellite is 1.7×10^3 m/s.

Research This: Space Junk, page 302

A. Air resistance will slow a satellite and cause it to slip into a lower orbit.
B. The satellite may hit other satellites or burn up in the atmosphere.
C. Sample answer: Yes. Different styles of rockets and boosters are being considered for space missions. Another way to reduce space junk is to equip satellites with small boosters that would enable them to fall to Earth once they have become obsolete.
D. Sample answer: Space junk is regularly falling to Earth. To speed up the removal of space junk, specific missions to remove space junk can be undertaken. A proposed technology that could help reduce the amount of space junk is a "laser broom." A laser broom is a ground-based laser beam that would heat space junk enough to cause it to break apart into much smaller pieces or change direction and fall to Earth.

E. Answers may vary. Sample answer:
Hi Amelia;

I was researching the Internet and found out that space junk is actually many types of debris. According to the NASA Orbital Debris Space Program Office space junk is

- abandoned spacecraft or spacecraft parts that no longer work—these items float around in space circling Earth until they fall back down or collide with other space junk
- upper stages of launch vehicles—these are parts of space shuttles that are fired off or ejected in stages, usually the upper parts of the rocket which get ejected last and are trapped in Earth's orbit
- solid rocket fuel—some space shuttles use solid rocket fuel for propulsion and some can be left over after launch in the container in which it was sent up
- tiny flecks of paint—when spacecraft enters space, heat or collisions with small particles chip paint from the surface of the spacecraft

I find it interesting that paint flecks are considered space junk, and despite their size they can actually do quite a bit of damage when they strike objects. Space junk can orbit Earth at a speed of more than 3.5×10^4 km/h. If a speck of paint travelling at that speed hits a space station, it can create a 0.6 cm diameter hole in the window of the space station. Hard to believe something that small can cause a lot of damage!

Section 6.2 Questions, page 303

1. Natural satellites are natural objects that revolve around another body due to gravitational attraction, such as the Moon in the Earth–Moon system. Artificial satellites are objects that have been manufactured and intentionally placed in orbit by humans, such as the International Space Station.
2. Microgravity is a more accurate term than "zero gravity" to describe what astronauts experience on the International Space Station. Microgravity is one millionth the value of g.
3. GPS satellites are a network of 24 satellites that coordinate several of their signals at once to locate objects on Earth's surface.
4. (a) A geosynchronous orbit is an orbit at a location above Earth's surface such that the speed of an object in a geosynchronous orbit matches the rate of the Earth's rotation.
(b) A satellite in geosynchronous orbit appears to pass through the same position in the sky at the same time every day to an observer on Earth.
(c) A satellite in a geostationary orbit appears to remain in the same position in the sky to an observer on Earth.
5. Given: $T = 24$ h; $m_E = 5.98 \times 10^{24}$ kg; $G = 6.67 \times 10^{-11}$ N·m^2/kg^2
Required: r

Analysis: Use the equation for period to isolate v, $T = \frac{2\pi r}{v}$. Then set the value for v equal to the equation for v to isolate and solve for r, $v = \sqrt{\frac{Gm}{r}}$.

But first convert 24 h to seconds:

$$T = \frac{24\ \cancel{\text{h}}}{1\ \cancel{\text{d}}} \times \frac{60\ \cancel{\text{min}}}{1\ \cancel{\text{h}}} \times \frac{60\ \text{s}}{1\ \cancel{\text{min}}}$$

$$T = 86\ 400\ \text{s}$$

$$T = \frac{2\pi r}{v}$$

$$v = \frac{2\pi r}{T}$$

$$\sqrt{\frac{Gm}{r}} = \frac{2\pi r}{T}$$

$$\frac{Gm}{r} = \frac{4\pi^2 r^2}{T^2}$$

$$\frac{GmT^2}{4\pi^2} = r^3$$

Solution: $\frac{GmT^2}{4\pi^2} = r^3$

$$r^3 = \frac{\left(6.67 \times 10^{-11}\ \frac{\cancel{\text{kg}} \cdot \frac{\cancel{\text{m}}}{\cancel{\text{s}^2}} \cdot \text{m}^{\cancel{2}}}{\cancel{\text{kg}^2}}\right)\left(5.98 \times 10^{24}\ \cancel{\text{kg}}\right)\left(86\,400\ \cancel{\text{s}}\right)^{\cancel{2}}}{4\pi^2}$$

$$r = 4.2 \times 10^7\ \text{m}$$

Statement: The orbital radius of a satellite in geosynchronous orbit is 4.2×10^7 m.

6. (a) Given: $T = 164.5$ y; $r = 4.5 \times 10^9$ km $= 4.5 \times 10^{12}$ m

Required: v

Analysis: Use the equation for period to isolate and solve for v, $T = \frac{2\pi r}{v}$. But first convert the period to seconds:

$$T = 164.5\ \cancel{\text{y}} \times \frac{365\ \cancel{\text{d}}}{1\ \cancel{\text{y}}} \times \frac{24\ \cancel{\text{h}}}{1\ \cancel{\text{d}}} \times \frac{60\ \cancel{\text{min}}}{1\ \cancel{\text{h}}} \times \frac{60\ \text{s}}{1\ \cancel{\text{min}}}$$

$T = 5.188 \times 10^9$ s (two extra digits carried)

$$T = \frac{2\pi r}{v}$$

$$v = \frac{2\pi r}{T}$$

Solution: Determine the orbital speed of Neptune:

$$\begin{aligned} v &= \frac{2\pi r}{T} \\ &= \frac{2\pi(4.5 \times 10^{12}\ \text{m})}{5.188 \times 10^{9}\ \text{s}} \\ &= 5.450 \times 10^{3}\ \text{m/s (two extra digits carried)} \end{aligned}$$

$$v = 5.5 \times 10^{3}\ \text{m/s}$$

Statement: The orbital speed of Neptune is 5.5×10^3 m/s.

(b) Given: $r = 4.5 \times 10^9$ km $= 4.5 \times 10^{12}$ m; $v = 5.450 \times 10^3$ m/s;
$G = 6.67 \times 10^{-11}$ N·m²/kg²

Required: m

Analysis: Use the equation for speed to isolate and solve for m, $v = \sqrt{\dfrac{Gm}{r}}$:

$$v = \sqrt{\frac{Gm}{r}}$$

$$v^2 = \frac{Gm}{r}$$

$$m = \frac{rv^2}{G}$$

Solution: $m = \dfrac{rv^2}{G}$

$$= \frac{(4.5 \times 10^{12}\ \cancel{\text{m}})(5.450 \times 10^{3}\ \cancel{\text{m}}/\cancel{\text{s}})^2}{6.67 \times 10^{-11}\ \dfrac{\cancel{\text{kg}} \cdot \dfrac{\cancel{\text{m}}}{\cancel{\text{s}^2}} \cdot \cancel{\text{m}^2}}{\text{kg}^{\cancel{2}}}}$$

$$m = 2.0 \times 10^{30}\ \text{kg}$$

Statement: The mass of the Sun is 2.0×10^{30} kg.

7. Given: $T = 29$ y; $v = 9.69$ km/s $= 9.69 \times 10^3$ m/s

Required: r

Analysis: $T = \dfrac{2\pi r}{v}$, but first convert the period to seconds:

$$\begin{aligned} T &= 29\ \cancel{\text{y}} \times \frac{365\ \cancel{\text{d}}}{1\ \cancel{\text{y}}} \times \frac{24\ \cancel{\text{h}}}{1\ \cancel{\text{d}}} \times \frac{60\ \cancel{\text{min}}}{1\ \cancel{\text{h}}} \times \frac{60\ \text{s}}{1\ \cancel{\text{min}}} \\ &= 9.145 \times 10^{8}\ \text{s (two extra digits carried)} \end{aligned}$$

$$T = 9.1 \times 10^{8}\ \text{s}$$

Solution: Determine the orbital radius of Saturn:

$$T = \frac{2\pi r}{v}$$

$$r = \frac{Tv}{2\pi}$$

$$= \frac{(9.145\times10^{8}\ \text{s})(9.69\times10^{3}\ \text{m/s})}{2\pi}$$

$$r = 1.4\times10^{12}\ \text{m}$$

Statement: The orbital radius of Saturn is 1.4×10^{12} m.

8. (a) Given: $r = 5.03 \times 10^{11}$ m; $m_{\text{Sun}} = 1.99 \times 10^{30}$ kg; $G = 6.67 \times 10^{-11}$ N·m²/kg²

Required: v

Analysis: $v = \sqrt{\dfrac{Gm}{r}}$

Solution: $v = \sqrt{\dfrac{Gm}{r}}$

$$= \sqrt{\frac{\left(6.67\times10^{-11}\ \dfrac{\cancel{\text{kg}}\cdot\dfrac{\cancel{\text{m}}}{\text{s}^2}\cdot\text{m}^2}{\text{kg}^{\cancel{2}}}\right)\left(1.99\times10^{30}\ \cancel{\text{kg}}\right)}{5.03\times10^{11}\ \cancel{\text{m}}}}$$

$$= 1.6244\times10^{4}\ \text{m/s}\ \text{(two extra digits carried)}$$

$$v = 1.62\times10^{4}\ \text{m/s}$$

Statement: The speed of the asteroid is 1.62×10^{4} m/s.

(b) Given: $r = 5.03 \times 10^{11}$ m; $v = 1.6244 \times 10^{4}$ m/s

Required: T

Analysis: $T = \dfrac{2\pi r}{v}$

Solution: $T = \dfrac{2\pi r}{v}$

$$= \frac{2\pi\left(5.03\times10^{11}\ \cancel{\text{m}}\right)}{1.6244\times10^{4}\ \dfrac{\cancel{\text{m}}}{\text{s}}}$$

$$= 1.9456\times10^{8}\ \cancel{\text{s}}\times\frac{1\ \cancel{\text{min}}}{60\ \cancel{\text{s}}}\times\frac{1\ \cancel{\text{h}}}{60\ \cancel{\text{min}}}\times\frac{1\ \cancel{\text{d}}}{24\ \cancel{\text{h}}}\times\frac{1\ \text{y}}{365\ \cancel{\text{d}}}$$

$$T = 6.17\ \text{y}$$

Statement: The period of the asteroid is 6.17 y.

9. Given: $m = 1.99 \times 10^{30}$ kg; $r = 4.05 \times 10^{12}$ m; $G = 6.67 \times 10^{-11}$ N·m²/kg²

Required: v

Analysis: $v = \sqrt{\dfrac{Gm}{r}}$

Solution: $v = \sqrt{\frac{Gm}{r}}$

$$= \sqrt{\frac{\left(6.67 \times 10^{-11}\ \frac{\cancel{\text{kg}} \cdot \frac{\cancel{\text{m}}}{\text{s}^2} \cdot \text{m}^2}{\cancel{\text{kg}^2}}\right)\left(1.99 \times 10^{30}\ \cancel{\text{kg}}\right)}{4.05 \times 10^{12}\ \cancel{\text{m}}}}$$

$$= 5725.8\ \frac{\cancel{\text{m}}}{\cancel{\text{s}}} \times \frac{60\ \cancel{\text{s}}}{1\ \cancel{\text{min}}} \times \frac{60\ \cancel{\text{min}}}{1\ \text{h}} \times \frac{1\ \text{km}}{1000\ \cancel{\text{m}}}$$

$$v = 2.06 \times 10^4\ \text{km/h}$$

Statement: The orbital speed of the exoplanet is 2.06×10^4 km/h.

10. Given: $r = 4.03 \times 10^{11}$ m; $T = 1100$ d; $G = 6.67 \times 10^{-11}$ N·m^2/kg^2

Required: m

Analysis: Use the equation for T to isolate and then solve for v, $T = \frac{2\pi r}{v}$. Then use the equation for speed to isolate and solve for m, $v = \sqrt{\frac{Gm}{r}}$. But first, convert the period to seconds:

$$T = 1100\ \cancel{\text{d}} \times \frac{24\ \cancel{\text{h}}}{1\ \cancel{\text{d}}} \times \frac{60\ \cancel{\text{min}}}{1\ \cancel{\text{h}}} \times \frac{60\ \text{s}}{1\ \cancel{\text{min}}}$$

$T = 9.504 \times 10^7$ s (two extra digits carried)

$$T = \frac{2\pi r}{v}$$

$$v = \frac{2\pi r}{T}$$

$$v = \sqrt{\frac{Gm}{r}}$$

$$v^2 = \frac{Gm}{r}$$

$$m = \frac{rv^2}{G}$$

Solution: Determine the orbital speed of the exoplanet:

$$v = \frac{2\pi r}{T}$$

$$= \frac{2\pi\left(4.03 \times 10^{11}\ \text{m}\right)}{9.504 \times 10^7\ \text{s}}$$

$v = 2.664 \times 10^4$ m/s (two extra digits carried)

Determine the mass of the star:

$$m = \frac{rv^2}{G}$$

$$= \frac{\left(4.03\times10^{11}\ \cancel{\text{m}}\right)\left(2.664\times10^{4}\ \frac{\cancel{\text{m}}}{\cancel{\text{s}}}\right)^2}{6.67\times10^{-11}\ \frac{\cancel{\text{kg}}\cdot\frac{\cancel{\text{m}}}{\cancel{\text{s}^2}}\cdot\cancel{\text{m}^2}}{\text{kg}^{\cancel{2}}}}$$

$$m = 4.3\times10^{30}\ \text{kg}$$

Statement: The mass of the star is 4.3×10^{30} kg.

11. Given: $r = 9.38 \times 10^6$ m; $m = 6.42 \times 10^{23}$ kg; $G = 6.67 \times 10^{-11}$ N·m^2/kg^2

Required: T

Analysis: Use the equation for period, $T = \frac{2\pi r}{v}$. For v in the equation for period, use $v = \sqrt{\frac{Gm}{r}}$.

$$T = \frac{2\pi r}{v}$$

$$= \frac{2\pi r}{\sqrt{\frac{Gm}{r}}}$$

$$T = \frac{2\pi\sqrt{r^3}}{\sqrt{Gm}}$$

Solution: $T = \frac{2\pi\sqrt{r^3}}{\sqrt{Gm}}$

$$= \frac{2\pi\sqrt{\left(9.38\times10^{6}\ \cancel{\text{m}}\right)^3}}{\sqrt{\left(6.67\times10^{-11}\ \frac{\cancel{\text{kg}}\cdot\frac{\cancel{\text{m}}}{\text{s}^2}\cdot\cancel{\text{m}^2}}{\cancel{\text{kg}^2}}\right)\left(6.42\times10^{23}\ \cancel{\text{kg}}\right)}}$$

$$= 27\,584\ \cancel{\text{s}}\times\frac{1\ \cancel{\text{min}}}{60\ \cancel{\text{s}}}\times\frac{1\ \cancel{\text{h}}}{60\ \cancel{\text{min}}}\times\frac{1\ \text{d}}{24\ \cancel{\text{h}}}$$

$$T = 0.319\ \text{d}$$

Statement: The period of Phobos is 0.319 days.

12. Given: $T = 24$ h; $m_{\text{E}} = 5.98 \times 10^{24}$ kg; $G = 6.67 \times 10^{-11}$ N·m^2/kg^2

Required: v

Analysis: Using the equations for period and speed, isolate r in each: $T = \frac{2\pi r}{v}$, $v = \sqrt{\frac{Gm}{r}}$. Then set the two equations equal to each other and solve for v.

But first convert 24 h to seconds:

$$T = \frac{24\ \cancel{\text{h}}}{1\ \cancel{\text{d}}} \times \frac{60\ \cancel{\text{min}}}{1\ \cancel{\text{h}}} \times \frac{60\ \text{s}}{1\ \cancel{\text{min}}}$$

$$T = 86\ 400\ \text{s}$$

$$T = \frac{2\pi r}{v} \qquad v = \sqrt{\frac{Gm}{r}}$$

$$r = \frac{Tv}{2\pi} \qquad v^2 = \frac{Gm}{r}$$

$$r = \frac{Gm}{v^2}$$

$$\frac{Tv}{2\pi} = \frac{Gm}{v^2}$$

$$v^3 = \frac{2\pi Gm}{T}$$

Solution:

$$\frac{Tv}{2\pi} = \frac{Gm}{v^2}$$

$$v^3 = \frac{2\pi Gm}{T}$$

$$= \frac{2\pi \left(6.67 \times 10^{-11}\ \frac{\cancel{\text{kg}} \cdot \frac{\text{m}}{\text{s}^2} \cdot \text{m}^2}{\cancel{\text{kg}^2}} \right) \left(5.98 \times 10^{24}\ \cancel{\text{kg}} \right)}{86\ 400\ \text{s}}$$

$$= 3073\ \frac{\cancel{\text{m}}}{\cancel{\text{s}}} \times \frac{60\ \cancel{\text{s}}}{1\ \cancel{\text{min}}} \times \frac{60\ \cancel{\text{min}}}{1\ \text{h}} \times \frac{1\ \text{km}}{1000\ \cancel{\text{m}}}$$

$$v = 1.11 \times 10^4\ \text{km/h}$$

Statement: The orbital speed of a satellite in geosynchronous orbit is 1.11×10^4 km/h.

13. (a) Given: $m_{\text{Sun}} = 1.99 \times 10^{30}$ kg; $r_{\text{Mercury}} = 5.79 \times 10^{10}$ m; $r_{\text{Venus}} = 1.08 \times 10^{11}$ m; $r_{\text{Earth}} = 1.49 \times 10^{11}$ m; $r_{\text{Mars}} = 2.28 \times 10^{11}$ m; $G = 6.67 \times 10^{-11}$ N·m²/kg²

Required: v_{Mercury}; v_{Venus}; v_{Earth}; v_{Mars}

Analysis: $v = \sqrt{\frac{Gm}{r}}$

Solution: $v_{\text{Mercury}} = \sqrt{\dfrac{Gm_{\text{Sun}}}{r_{\text{Mercury}}}}$

$$= \sqrt{\frac{\left(6.67\times10^{-11}\ \dfrac{\cancel{\text{kg}}\cdot\dfrac{\cancel{\text{m}}}{\text{s}^2}\cdot\text{m}^2}{\cancel{\text{kg}^2}}\right)\left(1.99\times10^{30}\ \cancel{\text{kg}}\right)}{5.79\times10^{10}\ \cancel{\text{m}}}}$$

$v_{\text{Mercury}} = 4.79\times10^{4}\ \text{m/s}$

$v_{\text{Venus}} = \sqrt{\dfrac{Gm_{\text{Sun}}}{r_{\text{Venus}}}}$

$$= \sqrt{\frac{\left(6.67\times10^{-11}\ \dfrac{\cancel{\text{kg}}\cdot\dfrac{\cancel{\text{m}}}{\text{s}^2}\cdot\text{m}^2}{\cancel{\text{kg}^2}}\right)\left(1.99\times10^{30}\ \cancel{\text{kg}}\right)}{1.08\times10^{11}\ \cancel{\text{m}}}}$$

$v_{\text{Venus}} = 3.51\times10^{4}\ \text{m/s}$

$v_{\text{Earth}} = \sqrt{\dfrac{Gm_{\text{Sun}}}{r_{\text{Earth}}}}$

$$= \sqrt{\frac{\left(6.67\times10^{-11}\ \dfrac{\cancel{\text{kg}}\cdot\dfrac{\cancel{\text{m}}}{\text{s}^2}\cdot\text{m}^2}{\cancel{\text{kg}^2}}\right)\left(1.99\times10^{30}\ \cancel{\text{kg}}\right)}{1.49\times10^{11}\ \cancel{\text{m}}}}$$

$v_{\text{Earth}} = 2.98\times10^{4}\ \text{m/s}$

$v_{\text{Mars}} = \sqrt{\dfrac{Gm_{\text{Sun}}}{r_{\text{Mars}}}}$

$$= \sqrt{\frac{\left(6.67\times10^{-11}\ \dfrac{\cancel{\text{kg}}\cdot\dfrac{\cancel{\text{m}}}{\text{s}^2}\cdot\text{m}^2}{\cancel{\text{kg}^2}}\right)\left(1.99\times10^{30}\ \cancel{\text{kg}}\right)}{2.28\times10^{11}\ \cancel{\text{m}}}}$$

$v_{\text{Mars}} = 2.41\times10^{4}\ \text{m/s}$

Statement: The orbital speed of Mercury is 4.79×10^4 m/s.
The orbital speed of Venus is 3.51×10^4 m/s.
The orbital speed of Earth is 2.98×10^4 m/s.
The orbital speed of Mars is 2.41×10^4 m/s.
(b) The farther a planet is from the Sun, the slower its orbital speed.

14. Given: $d = 410$ km $= 4.1 \times 10^5$ m; $r_{\text{Moon}} = 1.74 \times 10^6$ m; $m = 7.36 \times 10^{22}$ kg; $G = 6.67 \times 10^{-11}$ N·m²/kg²

Required: v; T

Analysis: $v = \sqrt{\dfrac{Gm}{r}}$; $T = \dfrac{2\pi r}{v}$. First, calculate the orbital radius:

$$\begin{aligned} r &= d + r_{\text{Moon}} \\ &= 4.1\times10^5 \text{ m} + 1.74\times10^6 \text{ m} \\ r &= 2.15\times10^6 \text{ m (one extra digit carried)} \end{aligned}$$

Solution:

$$\begin{aligned} v &= \sqrt{\frac{Gm}{r}} \\ &= \sqrt{\frac{\left(6.67\times10^{-11}\ \dfrac{\cancel{\text{kg}}\cdot\dfrac{\cancel{\text{m}}}{\text{s}^2}\cdot\text{m}^2}{\cancel{\text{kg}^2}}\right)\left(7.36\times10^{22}\ \cancel{\text{kg}}\right)}{2.15\times10^6\ \cancel{\text{m}}}} \\ &= 1.511\times10^3 \text{ m/s (two extra digit carried)} \\ v &= 1.5\times10^3 \text{ m/s} \end{aligned}$$

Determine the period:

$$\begin{aligned} T &= \frac{2\pi r}{v} \\ &= \frac{2\pi\left(2.15\times10^6\ \cancel{\text{m}}\right)}{1.511\times10^3\ \dfrac{\cancel{\text{m}}}{\text{s}}} \\ T &= 8.9\times10^3 \text{ s} \end{aligned}$$

Statement: The speed of the satellite is 1.5×10^3 m/s, and its period is 8.9×10^3 s.

Chapter 6 Review, pages 312–317

Knowledge

1. (c)
2. (a)
3. (b)
4. (d)
5. (d)
6. (c)
7. (d)
8. (c)
9. (d)
10. (a)
11. False. The gravitational constant, G, *is the same everywhere.*
12. True
13. True
14. False. Unlike most satellites, a geosynchronous satellite *appears to have a fixed position in the sky because it orbits Earth exactly once every 24 h.*
15. True
16. False. The velocity of a satellite in uniform circular motion depends on the satellite's *orbital radius.*
17. True
18. False. According to the theory of general relativity, *gravitational fields can bend the path of light.*
19. False. Gravitational lensing occurs when the gravitational field changes the direction of motion of *light*.
20. **(a)** (ii)
(b) (iii)
(c) (i)

Understanding

21. The weight of any object on Earth is not infinite. Even though the distance between the object and Earth is zero, the distance r in the proportionality statement $F_{\mathrm{g}} \propto \frac{1}{r^2}$ is measured from Earth's centre, not from Earth's surface.
22. The figure g = 9.806 65 N/kg is not accurate for all places on Earth. The value of g depends on the distance from Earth's centre. The surface of Earth is not perfectly uniform. The surface varies because of ocean basins and mountain peaks, so the value of g varies slightly with altitude on Earth's surface.
23. The gravitational forces that two objects exert on each other have the same magnitude but are in opposite directions, as with action–reaction forces described by Newton's third law of motion.
24. Cavendish made the first precise measurement of the force of gravity between two objects on Earth. He created an apparatus with two dumbbell lead spheres of equal mass at the ends of a support, suspended by a thin wire. He brought another set of suspended smaller spheres near the first set. By measuring the angle between the masses, he calculated the force of gravity between the masses.

25. I cannot feel the gravitational force between me and a car 5 m away because the gravitational force constant, G, is very small. I can only feel the effects of a gravitational force between me and extremely massive objects, such as a planet.
26. (a) The unit of gravitational field strength is newtons per kilogram, N/kg. The unit of force is the newton, N. The unit N/kg is therefore the unit of a force per unit mass.
(b) The value of g decreases as distance increases. The value of g increases proportionately with mass.
(c) The direction of a gravitational field around a spherical object is inward, toward the centre of the object.
27. For spherical objects, the strength of the gravitational field at a distance from the surface is the same whether the mass actually fills its volume or all the mass sits at a point in the centre. The gravitational force equation can be used as though all of the object's mass were located at its centre. This is why we measure centre-to-centre distances.

28. (a) Use the equation $g = \frac{Gm}{r^2}$ to solve for g with mass $4m$ and distance $2d$:

$$\begin{aligned} g &= \frac{Gm}{r^2} \\ &= \frac{G(4m)}{(2d)^2} \\ g &= \frac{Gm}{d^2} \\ g &= 1\frac{Gm}{d^2} \end{aligned}$$

(b) Use the equation $g = \frac{Gm}{r^2}$ to solve for g with mass $6m$ and distance $5d$:

$$\begin{aligned} g &= \frac{Gm}{r^2} \\ &= \frac{G(6m)}{(5d)^2} \\ g &= \frac{6Gm}{25d^2} \\ g &= 0.24\,\frac{Gm}{d^2} \end{aligned}$$

(c) Use the equation $g = \frac{Gm}{r^2}$ to solve for g with mass $2m$ and distance $3d$:

$$\begin{aligned} g &= \frac{Gm}{r^2} \\ &= \frac{G(2m)}{(3d)^2} \\ g &= \frac{2Gm}{9d^2} \\ g &= 0.22\,\frac{Gm}{d^2} \end{aligned}$$

(d) Use the equation $g = \frac{Gm}{r^2}$ to solve for g with mass m and distance $2d$:

$$\begin{aligned} g &= \frac{Gm}{r^2} \\ &= \frac{G(m)}{(2d)^2} \\ g &= \frac{Gm}{4d^2} \\ g &= 0.25\,\frac{Gm}{d^2} \end{aligned}$$

(c) < (b) < (d) < (a)

29. Given: m_1 = 68 kg; r = 1.5 m; m_2 = 27 kg; $G = 6.67 \times 10^{-11}$ N·m^2/kg^2

Required: F_g

Analysis: $F_g = \frac{Gm_1m_2}{r^2}$

Solution: $F_g = \frac{Gm_1m_2}{r^2}$

$$= \frac{\left(6.67 \times 10^{-11}\ \frac{\text{N} \cdot \cancel{\text{m}^2}}{\cancel{\text{kg}^2}}\right)(68\ \cancel{\text{kg}})(27\ \cancel{\text{kg}})}{(1.5\ \cancel{\text{m}})^2}$$

$$F_g = 5.4 \times 10^{-8}\ \text{N}$$

Statement: The magnitude of the gravitational force between the rock and the boulder is 5.4×10^{-8} N.

30. Given: $r = 4.5$ m; $m_1 = 1200$ kg; $F_g = 1.7 \times 10^{-7}$ N; $G = 6.67 \times 10^{-11}$ N·m^2/kg^2
Required: m_2

Analysis: $F_g = \dfrac{Gm_1m_2}{r^2}$

$$m_2 = \frac{F_g r^2}{Gm_1}$$

Solution: $m_2 = \dfrac{F_g r^2}{Gm_1}$

$$= \frac{(1.7\times10^{-7}\ \cancel{\text{N}})(4.5\ \cancel{\text{m}})^2}{\left(6.67\times10^{-11}\ \dfrac{\cancel{\text{N}}\cdot\cancel{\text{m}^2}}{\text{kg}^{\cancel{2}}}\right)(1200\ \cancel{\text{kg}})}$$

$$m_2 = 43\text{ kg}$$

Statement: The mass of the traffic officer is 43 kg.

31. (a) Given: $m_1 = 370$ kg; $m_2 = 9.0 \times 10^{13}$ kg; $F_g = 32$ N; $G = 6.67 \times 10^{-11}$ N·m^2/kg^2
Required: r

Analysis: Rearrange the equation $F_g = \dfrac{Gm_1m_2}{r^2}$ to solve for r.

$$F_g = \frac{Gm_1m_2}{r^2}$$

$$r^2 = \frac{Gm_1m_2}{F_g}$$

$$r = \sqrt{\frac{Gm_1m_2}{F_g}}$$

Solution: $r = \sqrt{\dfrac{Gm_1m_2}{F_g}}$

$$= \sqrt{\frac{\left(6.67\times10^{-11}\ \dfrac{\cancel{\text{N}}\cdot\text{m}^2}{\cancel{\text{kg}^2}}\right)(370\ \cancel{\text{kg}})(9.0\times10^{13}\ \cancel{\text{kg}})}{32\ \cancel{\text{N}}}}$$

$$r = 2.6\times10^2\text{ m}$$

Statement: The gravitational force is 32 N at the distance 2.6×10^2 m from the comet's centre.

(b) Given: $m_1 = 370$ kg; $m_2 = 9.0 \times 10^{13}$ kg; $r = 350$ m; $G = 6.67 \times 10^{-11}$ N·m^2/kg^2
Required: F_g

Analysis: $F_g = \dfrac{Gm_1m_2}{r^2}$

Solution: $F_g = \dfrac{Gm_1m_2}{r^2}$

$$= \frac{\left(6.67\times10^{-11}\ \frac{\text{N}\cdot\cancel{\text{m}^2}}{\cancel{\text{kg}^2}}\right)(370\ \cancel{\text{kg}})(9.0\times10^{13}\ \cancel{\text{kg}})}{(350\ \cancel{\text{m}})^2}$$

$F_g = 18\ \text{N}$

Statement: The magnitude of the gravitational force between the comet and the projectile at a distance of 350 m was 18 N.

(c) Given: $m = 9.0 \times 10^{13}$ kg; $r = 5.0 \times 10^3$ km $= 5.0 \times 10^6$ m; $G = 6.67 \times 10^{-11}\ \text{N·m}^2/\text{kg}^2$

Required: g

Analysis: $g = \dfrac{Gm}{r^2}$

Solution: $g = \dfrac{Gm}{r^2}$

$$= \frac{\left(6.67\times10^{-11}\ \frac{\text{N}\cdot\cancel{\text{m}^2}}{\text{kg}^{\cancel{2}}}\right)(9.0\times10^{13}\ \cancel{\text{kg}})}{(5.0\times10^{6}\ \cancel{\text{m}})^2}$$

$g = 2.4\times10^{-10}\ \text{N/kg}$

Statement: The strength of the comet's gravitational field at the distance of 5.0×10^3 km is 2.4×10^{-10} N/kg.

32. For a satellite to remain in orbit at a certain distance above Earth's surface, speed is more important than mass. The orbital radius is inversely related to the square of the satellite's speed, but it does not depend on the satellite's mass. The mass of Earth is used in the calculation, though.

33. The orbital speed is given by the equation $v = \sqrt{\dfrac{Gm}{r}}$. Examine the orbital speed when the mass doubles, $2m$:

$$v = \sqrt{\frac{G2m}{r}}$$

$$v = \sqrt{2}\sqrt{\frac{Gm}{r}}$$

The speed of the satellite that is orbiting the more massive planet would be greater than the speed of the other satellite by a factor of $\sqrt{2}$.

34. The orbital speed is given by the equation $v = \sqrt{\dfrac{Gm}{r}}$. Examine the mass when the orbital speed doubles, $2v$:

$$2v = 2\sqrt{\frac{Gm}{r}}$$

$$2v = \sqrt{\frac{G(4m)}{r}}$$

The satellite travelling at $2v$ orbits a planet whose mass is four times the mass as the other planet.

35. (a) The Constellation satellites have greater speeds than the RADARSAT satellites because the Constellation satellites have a shorter orbital radius.

(b) The mass of the satellites has no effect on the speeds of the satellites.

36. Scientists have noted that there are significant differences between the measured gravitational field in some regions of the universe and the gravitational fields that known masses can produce. A gravitational field is proportional to the mass of matter that produces the field, so there must be missing mass that scientists cannot detect. This missing mass is called dark matter because scientists know it must be there but they cannot see it.

Analysis and Application

37. All three balls are the same distance from me. The only variation is in mass. Balls A and C have equal mass but different radius. However, they have an equal gravitational force on me because the radius of the ball is not important. The mass of ball B is twice the mass of balls A and C. Therefore, the gravitational force of ball B is twice as great as the gravitational force of balls A and C.

38. Given: $m_1 = 45$ kg; $m_2 = 45$ kg; $r = 1.0$ m; $G = 6.67 \times 10^{-11}$ N·m^2/kg^2

Required: F_g

Analysis: $F_g = \dfrac{Gm_1m_2}{r^2}$

Solution: $F_g = \dfrac{Gm_1m_2}{r^2}$

$$= \frac{\left(6.67\times10^{-11}\ \dfrac{\text{N}\cdot\cancel{\text{m}^2}}{\cancel{\text{kg}^2}}\right)(45\ \cancel{\text{kg}})(45\ \cancel{\text{kg}})}{(1.0\ \cancel{\text{m}})^2}$$

$$F_g = 1.4\times10^{-7}\ \text{N}$$

Statement: The magnitude of the gravitational force between the two people is 1.4×10^{-7} N.

39. (a) Given: $m_1 = 22$ kg; $m_2 = 25$ kg; $r = 1.2$ m; $G = 6.67 \times 10^{-11}$ N·m^2/kg^2

Required: F_g

Analysis: $F_g = \dfrac{Gm_1m_2}{r^2}$

Solution: $F_g = \dfrac{Gm_1m_2}{r^2}$

$$= \frac{\left(6.67\times10^{-11}\ \dfrac{\text{N}\cdot\cancel{\text{m}^2}}{\cancel{\text{kg}^2}}\right)(22\ \cancel{\text{kg}})(25\ \cancel{\text{kg}})}{(1.2\ \cancel{\text{m}})^2}$$

$= 2.548\times10^{-8}$ N (two extra digits carried)

$F_g = 2.5\times10^{-8}$ N

Statement: The magnitude of the gravitational force between the balls is 2.5×10^{-8} N.

(b) Given: m_1 = 16 kg; m_2 = 22 kg; $F_g = 2.548 \times 10^{-8}$ N; $G = 6.67 \times 10^{-11}$ N·m^2/kg^2

Required: r

Analysis: Rearrange the equation $F_g = \dfrac{Gm_1m_2}{r^2}$ to solve for r:

$$F_g = \frac{Gm_1m_2}{r^2}$$

$$r^2 = \frac{Gm_1m_2}{F_g}$$

$$r = \sqrt{\frac{Gm_1m_2}{F_g}}$$

Solution: $r = \sqrt{\dfrac{Gm_1m_2}{F_g}}$

$$= \sqrt{\frac{\left(6.67\times10^{-11}\ \dfrac{\cancel{\text{N}}\cdot\text{m}^2}{\cancel{\text{kg}^2}}\right)(16\ \cancel{\text{kg}})(21\ \cancel{\text{kg}})}{2.548\times10^{-8}\ \cancel{\text{N}}}}$$

$r = 0.94$ m

Statement: The centres of the two balls would have to be 0.94 m apart to have a gravitational force of 2.5×10^{-8} N between them.

40. Given: m_1 = 0.032 kg; m_2 = 5500 kg; r = 0.75 m; $G = 6.67 \times 10^{-11}$ N·m^2/kg^2

Required: F_g

Analysis: $F_g = \dfrac{Gm_1m_2}{r^2}$

Solution: $F_g = \dfrac{Gm_1m_2}{r^2}$

$$= \frac{\left(6.67\times10^{-11}\ \dfrac{\text{N}\cdot\cancel{\text{m}^2}}{\cancel{\text{kg}^2}}\right)(0.032\ \cancel{\text{kg}})(5500\ \cancel{\text{kg}})}{(0.75\ \cancel{\text{m}})^2}$$

$F_g = 2.1\times10^{-8}$ N

Statement: The magnitude of the gravitational force between the two masses is 2.1×10^{-8} N.

41. Given: m = 6520 kg; r = 5.75 m; $G = 6.67 \times 10^{-11}$ N·m²/kg²

Required: g

Analysis: $g = \frac{Gm}{r^2}$

Solution: $g = \frac{Gm}{r^2}$

$$= \frac{\left(6.67 \times 10^{-11}\ \frac{\text{N} \cdot \cancel{\text{m}^2}}{\text{kg}^{\cancel{2}}}\right)(6520\ \cancel{\text{kg}})}{(5.75\ \cancel{\text{m}})^2}$$

$$g = 1.32 \times 10^{-8}\ \text{N/kg}$$

Statement: The strength of the elephant's gravitational field at a distance of 5.75 m is 1.32×10^{-10} N/kg.

42. Given: r = 55.0 m; $g = 1.74 \times 10^{-10}$ N/kg; $G = 6.67 \times 10^{-11}$ N·m²/kg²

Required: m

Analysis: Rearrange the equation $g = \frac{Gm}{r^2}$ to solve for m:

$$g = \frac{Gm}{r^2}$$

$$m = \frac{gr^2}{G}$$

Solution: $m = \frac{gr^2}{G}$

$$= \frac{\left(1.74 \times 10^{-10}\ \frac{\cancel{\text{N}}}{\cancel{\text{kg}}}\right)(55.0\ \cancel{\text{m}})^2}{\left(6.67 \times 10^{-11}\ \frac{\cancel{\text{N}} \cdot \cancel{\text{m}^2}}{\text{kg}^{\cancel{2}}}\right)}$$

$$m = 7.89 \times 10^{3}\ \text{kg}$$

Statement: The mass of the ball of twine is 7.89×10^3 kg.

43. (a) Given: d = 5959 m; $m_E = 5.98 \times 10^{24}$ kg; $r_E = 6.38 \times 10^6$ m; $G = 6.67 \times 10^{-11}$ N·m²/kg²
Required: g

Analysis: $g = \dfrac{Gm}{r^2}$; for r in this equation, add the distance d above the surface and Earth's radius

Solution:
$$\begin{aligned} g &= \frac{Gm}{r^2} \\ &= \frac{Gm_E}{(d + r_E)^2} \\ &= \frac{\left(6.67 \times 10^{-11}\ \dfrac{\text{N} \cdot \cancel{\text{m}^2}}{\text{kg}^{\cancel{2}}}\right)(5.98 \times 10^{24}\ \cancel{\text{kg}})}{(5959\ \cancel{\text{m}} + 6.38 \times 10^6\ \cancel{\text{m}})^2} \\ &= 9.7808 \text{ N/kg (two extra digits carried)} \\ g &= 9.78 \text{ N/kg} \end{aligned}$$

Statement: The strength of Earth's gravitational field at the altitude of Mount Logan is 9.78 N/kg.

(b) Given: $m_E = 5.98 \times 10^{24}$ kg; $r_E = 6.38 \times 10^6$ m; $g_{\text{Mount Logan}} = 9.7808$ N/kg; $G = 6.67 \times 10^{-11}$ N·m²/kg²

Required: $\dfrac{g_{\text{Mount Logan}}}{g_{\text{sea level}}}$

Analysis: First, determine the strength of Earth's gravitational field at sea level, $g = \dfrac{Gm}{r^2}$. Then determine the ratio of g for Mount Logan to g at sea level.

Solution:
$$\begin{aligned} g &= \frac{Gm}{r^2} \\ g_{\text{sea level}} &= \frac{Gm_E}{r_E^2} \\ &= \frac{\left(6.67 \times 10^{-11}\ \dfrac{\text{N} \cdot \cancel{\text{m}^2}}{\text{kg}^{\cancel{2}}}\right)(5.98 \times 10^{24}\ \cancel{\text{kg}})}{(6.38 \times 10^6\ \cancel{\text{m}})^2} \\ g_{\text{sea level}} &= 9.7991 \text{ N/kg (two extra digits carried)} \end{aligned}$$

Determine the ratio:

$$\frac{g_{\text{Mount Logan}}}{g_{\text{sea level}}} = \frac{9.7808\ \cancel{\text{N/kg}}}{9.7991\ \cancel{\text{N/kg}}}$$

$$\frac{g_{\text{Mount Logan}}}{g_{\text{sea level}}} = 0.998$$

Statement: The ratio of the strength of Earth's gravitational field at the top of Mount Logan to the strength of Earth's gravitational field at Earth's surface is 0.998.

44. (a) Given: $m_{\text{Sun}} = 1.99 \times 10^{30}$ kg; $m_{\text{Neptune}} = 1.03 \times 10^{26}$ kg; $r = 4.5 \times 10^{9}$ km $= 4.5 \times 10^{12}$ m; $G = 6.67 \times 10^{-11}$ N·m^2/kg^2

Required: g_{Sun}

Analysis: $g = \dfrac{Gm}{r^2}$

Solution:

$$g = \frac{Gm}{r^2}$$

$$g_{\text{Sun}} = \frac{Gm_{\text{Sun}}}{r^2}$$

$$= \frac{\left(6.67\times 10^{-11}\ \frac{\text{N}\cdot \cancel{\text{m}^2}}{\text{kg}^{\cancel{2}}}\right)(1.99\times 10^{30}\ \cancel{\text{kg}})}{(4.5\times 10^{12}\ \cancel{\text{m}})^2}$$

$$g_{\text{Sun}} = 6.6\times 10^{-6}\ \text{N/kg}$$

Statement: The strength of the Sun's gravitational field at Neptune is 6.6×10^{-6} N/kg.

(b) Given: $m_{\text{Neptune}} = 1.03 \times 10^{26}$ kg; $r = 4.5 \times 10^{12}$ m; $G = 6.67 \times 10^{-11}$ N·m^2/kg^2

Required: g_{Neptune}

Analysis: $g = \dfrac{Gm}{r^2}$

Solution:

$$g = \frac{Gm}{r^2}$$

$$g_{\text{Neptune}} = \frac{Gm_{\text{Neptune}}}{r^2}$$

$$= \frac{\left(6.67\times 10^{-11}\ \frac{\text{N}\cdot \cancel{\text{m}^2}}{\text{kg}^{\cancel{2}}}\right)(1.03\times 10^{26}\ \cancel{\text{kg}})}{(4.5\times 10^{12}\ \cancel{\text{m}})^2}$$

$$g_{\text{Neptune}} = 3.4\times 10^{-10}\ \text{N/kg}$$

Statement: The strength of Neptune's gravitational field at the Sun's location is 3.4×10^{-10} N/kg.

(c) Given: $m_{\text{Sun}} = 1.99 \times 10^{30}$ kg; $m_{\text{Neptune}} = 1.03 \times 10^{26}$ kg; $r = 4.5 \times 10^{12}$ m; $G = 6.67 \times 10^{-11}$ N·m^2/kg^2

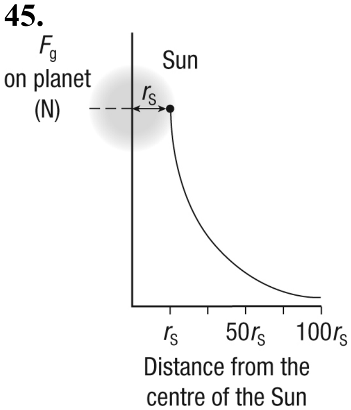
Required: F_g

Analysis: $F_g = \dfrac{Gm_1m_2}{r^2}$

Solution:
$$\begin{aligned} F_g &= \frac{Gm_1m_2}{r^2} \\ &= \frac{\left(6.67\times10^{-11}\ \frac{\text{N}\cdot\cancel{\text{m}^2}}{\cancel{\text{kg}^2}}\right)(1.99\times10^{30}\ \cancel{\text{kg}})(1.03\times10^{26}\ \cancel{\text{kg}})}{(4.5\times10^{12}\ \cancel{\text{m}})^2} \\ F_g &= 6.8\times10^{20}\ \text{N} \end{aligned}$$

Statement: The magnitude of the gravitational force between the Sun and Neptune is 6.8×10^{20} N.

45.



46. Weight is mass times gravitational field strength. The weight of the Mars lander will decrease as its distance from Earth increases. The weight will then eventually increase as the spacecraft gets closer to Mars's surface. The weight will equal zero somewhere in between. The mass of the lander does not change. The mass of the rocket propelling the Mars lander changes as it uses fuel.

47. (a) Given: $\Delta d = 1.25$ m; $\Delta t = 3.0$ s

Required: g

Analysis:
$$\begin{aligned} \Delta d &= \frac{1}{2}a\Delta t^2 \\ \Delta d &= \frac{1}{2}g\Delta t^2 \\ g &= \frac{2\Delta d}{\Delta t^2} \end{aligned}$$

Solution: $g = \dfrac{2\Delta d}{\Delta t^2}$

$$= \frac{2(1.25 \text{ m})}{(3.0 \text{ s})^2}$$

$$= 0.2778 \text{ m/s}^2 \text{(two extra digits carried)}$$

$$g = 0.28 \text{ N/kg}$$

Statement: The strength of the gravitational field of Ceres at the height 1.25 m is 0.28 N/kg.

(b) Given: $r = 4.76 \times 10^5$ m; $g = 0.2778$ N/kg; $G = 6.67 \times 10^{-11}$ N·m^2/kg^2

Required: m

Analysis: Rearrange the equation $g = \dfrac{Gm}{r^2}$ to solve for mass, m:

$$g = \frac{Gm}{r^2}$$

$$m = \frac{gr^2}{G}$$

Solution: $m = \dfrac{gr^2}{G}$

$$= \frac{\left(0.2778 \ \dfrac{\cancel{\text{N}}}{\cancel{\text{kg}}}\right)(4.76 \times 10^5 \ \cancel{\text{m}})^2}{\left(6.67 \times 10^{-11} \ \dfrac{\cancel{\text{N}} \cdot \cancel{\text{m}^2}}{\text{kg}^{\cancel{2}}}\right)}$$

$$= 9.437 \times 10^{20} \text{ kg} \text{ (two extra digits carried)}$$

$$m = 9.4 \times 10^{20} \text{ kg}$$

Statement: The mass of Ceres is 9.4×10^{20} kg.

(c) Given: $r = 4.76 \times 10^5$ m; $d = 150$ km $= 1.5 \times 10^5$ m; $m = 9.437 \times 10^{20}$ kg; $G = 6.67 \times 10^{-11}$ N·m^2/kg^2

Required: g

Analysis: $g = \dfrac{Gm}{r^2}$; for r in this equation, add the distance d above the surface and Ceres's radius

Solution: $g = \dfrac{Gm}{r^2}$

$$= \frac{Gm}{(r+d)^2}$$

$$= \frac{\left(6.67\times10^{-11}\ \dfrac{\mathrm{N}\cdot\cancel{\mathrm{m}^2}}{\mathrm{kg}^{\cancel{2}}}\right)(9.437\times10^{20}\ \cancel{\mathrm{kg}})}{(4.76\times10^{5}\ \cancel{\mathrm{m}}+1.5\times10^{5}\ \cancel{\mathrm{m}})^2}$$

$g = 0.16$ N/kg

Statement: At an altitude of 150 km above Ceres, the strength of the gravitational field is 0.16 N/kg.

48. Given: m_1 = 13 kg; m_2 = 17 kg; m_3 = 12 kg; r_{12} = 6.0 m; r_{13} = 6.0 m; $G = 6.67 \times 10^{-11}$ N·m²/kg²

Required: $\vec{F}_{\text{net}}$

Analysis: Use the equation $F_{\text{g}} = \dfrac{Gm_1m_2}{r^2}$ to calculate the individual forces. Then add them to determine the net force, $\vec{F}_{\text{net}}$.

Solution: Determine $\vec{F}_{12}$.

$$F_{\text{g}} = \frac{Gm_1m_2}{r^2}$$

$$F_{12} = \frac{Gm_1m_2}{r_{12}^2}$$

$$= \frac{\left(6.67\times10^{-11}\ \dfrac{\mathrm{N}\cdot\cancel{\mathrm{m}^2}}{\cancel{\mathrm{kg}^2}}\right)(13\ \cancel{\mathrm{kg}})(17\ \cancel{\mathrm{kg}})}{(6.0\ \cancel{\mathrm{m}})^2}$$

$F_{12} = 4.095\times10^{-10}$ N (two extra digits carried)

The gravitational force acting on m_1 due to m_2 is 4.095×10^{-10} N [toward m_2].

Determine $\vec{F}_{13}$.

$$F_{13} = \frac{Gm_1m_3}{r_{13}^2}$$

$$= \frac{\left(6.67\times10^{-11}\ \dfrac{\mathrm{N}\cdot\cancel{\mathrm{m}^2}}{\cancel{\mathrm{kg}^2}}\right)(13\ \cancel{\mathrm{kg}})(12\ \cancel{\mathrm{kg}})}{(6.0\ \cancel{\mathrm{m}})^2}$$

$F_{13} = 2.890\times10^{-10}$ N (two extra digits carried)

The gravitational force acting on m_1 due to m_3 is 2.890×10^{-10} N [toward m_3].

$$\vec{F}_{net} = \vec{F}_{12} + \vec{F}_{13}$$
$$= 4.095\times10^{-10}\ \text{N [toward } m_2] + 2.890\times10^{-10}\ \text{N [toward } m_3]$$
$$= 4.095\times10^{-10}\ \text{N [toward } m_2] - 2.890\times10^{-10}\ \text{N [toward } m_2]$$
$$\vec{F}_{net} = 1.2\times10^{-10}\ \text{N [toward } m_2]$$

Statement: The net force acting on m_1 is 1.2×10^{-10} N [toward m_2].

49. (a) Given: $r = 2.57 \times 10^6$ m; $m = 1.35 \times 10^{23}$ kg; $G = 6.67 \times 10^{-11}$ N·m²/kg²

Required: g

Analysis: $g = \dfrac{Gm}{r^2}$

Solution: $g = \dfrac{Gm}{r^2}$

$$= \frac{\left(6.67\times10^{-11}\ \frac{\text{N}\cdot\cancel{\text{m}^2}}{\text{kg}^{\cancel{2}}}\right)(1.35\times10^{23}\ \cancel{\text{kg}})}{(2.57\times10^6\ \cancel{\text{m}})^2}$$

$$g = 1.36\ \text{N/kg}$$

Statement: The gravitational field strength on Titan is 1.36 N/kg.

(b) Given: g_{Titan} = 1.36 N/kg; g_{Earth} = 9.8 N/kg

Required: $\dfrac{g_{Titan}}{g_{Earth}}$

Analysis: $\dfrac{g_{Titan}}{g_{Earth}}$

Solution:

$$\frac{g_{Titan}}{g_{Earth}} = \frac{1.36\ \cancel{\text{N/kg}}}{9.8\ \cancel{\text{N/kg}}}$$

$$\frac{g_{Titan}}{g_{Earth}} = 0.139$$

Statement: The ratio of the strength of Titan's gravitational field to the strength of Earth's gravitational field is 0.139:1.

50. (a) Given: d = 375 km = 3.75×10^5 m; $m_E = 5.98 \times 10^{24}$ kg; $r_E = 6.38 \times 10^6$ m; $G = 6.67 \times 10^{-11}$ N·m²/kg²

Required: g

Analysis: $g = \dfrac{Gm}{r^2}$; for r in this equation, add the distance d above the surface and Earth's radius

Solution: $g = \frac{Gm}{r^2}$

$$= \frac{Gm_E}{(d + r_E)^2}$$

$$= \frac{\left(6.67 \times 10^{-11}\ \frac{\text{N} \cdot \cancel{\text{m}^2}}{\text{kg}^{\cancel{2}}}\right)(5.98 \times 10^{24}\ \cancel{\text{kg}})}{(3.75 \times 10^5\ \cancel{\text{m}} + 6.38 \times 10^6\ \cancel{\text{m}})^2}$$

$g = 8.74$ N/kg

Statement: The gravitational field strength on the ISS is 8.74 N/kg.

(b) No, the astronauts are not weightless. They are in free fall as they orbit Earth.

(c) Astronauts and other objects on the ISS appear to float because they are falling at the same rate.

51. Sample answer: The supertankers' force of attraction is very small compared to forces from water waves and from friction, so they are not likely to collide due to gravity. In principle, though, they do feel a force of gravitational attraction between them.

52. (a) Given: $v = 7.45$ km/s $= 7.45 \times 10^3$ m/s; $r_E = 6.38 \times 10^6$ m; $m_E = 5.98 \times 10^{24}$ kg; $G = 6.67 \times 10^{-11}$ N·m^2/kg^2

Required: d

Analysis: Use the equation for speed to determine the orbital radius, $v = \sqrt{\frac{Gm}{r}}$. Then calculate the altitude of the satellite.

$$v = \sqrt{\frac{Gm}{r}}$$

$$v^2 = \frac{Gm}{r}$$

$$r = \frac{Gm}{v^2}$$

Solution:

$$r = \frac{Gm}{v^2}$$

$$= \frac{\left(6.67 \times 10^{-11}\ \frac{\cancel{\text{kg}} \cdot \frac{\text{m}}{\cancel{\text{s}^2}} \cdot \cancel{\text{m}^2}}{\text{kg}^{\cancel{2}}}\right)(5.98 \times 10^{24}\ \cancel{\text{kg}})}{\left(7.45 \times 10^3\ \frac{\cancel{\text{m}}}{\cancel{\text{s}}}\right)^2}$$

$r = 7.19 \times 10^6$ m

Determine the altitude of the satellite:

$d = r - r_E$

$= 7.19 \times 10^6\text{ m} - 6.38 \times 10^6\text{ m}$

$d = 8.06 \times 10^5\text{ m}$

Statement: The altitude of the satellite is 8.06×10^5 m, or 806 km.

(b) Given: $v = 7.45 \times 10^3$ m/s; $r = 7.19 \times 10^6$ m

Required: T

Analysis: $T = \dfrac{2\pi r}{v}$

Solution:
$$
\begin{aligned}
T &= \frac{2\pi r}{v} \\
&= \frac{2\pi(7.19\times 10^6\ \cancel{\text{m}})}{\left(7.45\times 10^3\ \dfrac{\cancel{\text{m}}}{\text{s}}\right)} \\
&= 6063\ \cancel{\text{s}} \times \frac{1\text{ min}}{60\ \cancel{\text{s}}} \\
T &= 101\text{ min, or 1 h 41 min}
\end{aligned}
$$

Statement: The period of the satellite is 1 h 41 min.

53. (a) Given: $T = 645$ min; $m = 5.69 \times 10^{26}$ m; $G = 6.67 \times 10^{-11}\text{ N·m}^2/\text{kg}^2$

Required: r

Analysis: Rearrange the equation for period, $T = \dfrac{2\pi r}{v}$, to isolate v. Then set the rearranged equation for v equal to the equation for speed, $v = \sqrt{\dfrac{Gm}{r}}$, to solve for r. First, convert 645 min to seconds:

$$645\text{ min} = 645\ \cancel{\text{min}} \times \frac{60\text{ s}}{1\ \cancel{\text{min}}} = 38\ 700\text{ s}$$

$$
\begin{aligned}
T &= \frac{2\pi r}{v} \\
v &= \frac{2\pi r}{T} \\
\sqrt{\frac{Gm}{r}} &= \frac{2\pi r}{T} \\
\frac{Gm}{r} &= \frac{4\pi^2 r^2}{T^2} \\
\frac{GmT^2}{4\pi^2} &= r^3
\end{aligned}
$$

Solution: $\frac{GmT^2}{4\pi^2} = r^3$

$$r^3 = \frac{\left(6.67\times10^{-11}\ \frac{\cancel{\text{kg}}\cdot\frac{\text{m}}{\cancel{\text{s}^2}}\cdot\text{m}^2}{\cancel{\text{kg}^2}}\right)(5.69\times10^{26}\ \cancel{\text{kg}})(38700\ \cancel{\text{s}})^2}{4\pi^2}$$

$$r = 1.13\times10^8\ \text{m}$$

Statement: The radius at which the satellite must orbit is 1.13×10^8 m.

(b) The ratio of the satellite's orbital radius to Saturn's orbital radius is as follows:

$$\frac{r_{\text{satellite}}}{r_{\text{Saturn}}} = \frac{1.13\times10^8\ \cancel{\text{m}}}{6.03\times10^7\ \cancel{\text{m}}}$$

$$\frac{r_{\text{satellite}}}{r_{\text{Saturn}}} = 1.88$$

The ratio is 1.88 to 1, or almost twice the radius of Saturn.

Determine the orbital radius of a geostationary satellite above Earth. A geostationary satellite appears to be "stationary" over the same spot on Earth, so T = 24 h. First, convert 24 h to seconds:

$$24\ \text{h} = 24\ \cancel{\text{h}} \times \frac{60\ \cancel{\text{min}}}{1\ \cancel{\text{h}}} = \frac{60\ \text{s}}{1\ \cancel{\text{min}}} = 86\ 400\ \text{s}$$

$$\frac{GmT^2}{4\pi^2} = r^3$$

$$r^3 = \frac{\left(6.67\times10^{-11}\ \frac{\cancel{\text{kg}}\cdot\frac{\text{m}}{\cancel{\text{s}^2}}\cdot\text{m}^2}{\cancel{\text{kg}^2}}\right)(5.98\times10^{24}\ \cancel{\text{kg}})(86400\ \cancel{\text{s}})^2}{4\pi^2}$$

$$r = 4.22\times10^7\ \text{m}$$

$$\frac{r_{\text{satellite}}}{r_{\text{Earth}}} = \frac{4.22\times10^7\ \cancel{\text{m}}}{6.38\times10^6\ \cancel{\text{m}}}$$

$$\frac{r_{\text{satellite}}}{r_{\text{Earth}}} = 6.62$$

A geostationary satellite above Earth's equator has a ratio of 6.62 : 1, so it is much farther out compared to the radius of Earth than the Saturnian satellite.

54. (a) Given: $r = 4.5 \times 10^9$ km $= 4.5 \times 10^{12}$ m; $m_{\text{Sun}} = 1.99 \times 10^{30}$ kg; $G = 6.67 \times 10^{-11}$ N·m^2/kg^2
Required: v

Analysis: $v = \sqrt{\dfrac{Gm}{r}}$

Solution: $v = \sqrt{\dfrac{Gm}{r}}$

$$= \sqrt{\frac{\left(6.67 \times 10^{-11} \dfrac{\cancel{\text{kg}} \cdot \dfrac{\cancel{\text{m}}}{\text{s}^2} \cdot \text{m}^2}{\cancel{\text{kg}^2}}\right)(1.99 \times 10^{30} \ \cancel{\text{kg}})}{4.5 \times 10^{12} \ \cancel{\text{m}}}}$$

$$= 5.431 \times 10^3 \ \frac{\cancel{\text{m}}}{\cancel{\text{s}}} \times \frac{1 \text{ km}}{1000 \ \cancel{\text{m}}} \times \frac{60 \ \cancel{\text{s}}}{1 \ \cancel{\text{min}}} \times \frac{60 \ \cancel{\text{min}}}{1 \text{ h}}$$

$$v = 2.0 \times 10^4 \text{ km/h}$$

Statement: The orbital speed of Neptune is 5.4×10^3 m/s, or 2.0×10^4 km/h.
(b) Given: $r = 4.5 \times 10^{12}$ m; $m_{\text{Sun}} = 1.99 \times 10^{30}$ kg; $v = 5.431 \times 10^3$ m/s
Required: T

Analysis: $T = \dfrac{2\pi r}{v}$

Solution: $T = \dfrac{2\pi r}{v}$

$$= \frac{2\pi(4.5 \times 10^{12} \ \cancel{\text{m}})}{5.431 \times 10^3 \ \dfrac{\cancel{\text{m}}}{\text{s}}}$$

$$= 5.206 \times 10^9 \ \cancel{\text{s}} \times \frac{1 \ \cancel{\text{min}}}{60 \ \cancel{\text{s}}} \times \frac{1 \ \cancel{\text{h}}}{60 \ \cancel{\text{min}}} \times \frac{1 \ \cancel{\text{d}}}{24 \ \cancel{\text{h}}} \times \frac{1 \text{ y}}{365 \ \cancel{\text{d}}}$$

$$T = 1.7 \times 10^2 \text{ y}$$

Statement: The orbital period of Neptune is 1.7×10^2 y.
55.

Given: $r_{\text{B}} = \dfrac{9}{10} r_{\text{A}}$

Required: $\dfrac{v_{\text{B}}}{v_{\text{A}}}$

Analysis: Solve the equation for orbital velocity, $v = \sqrt{\dfrac{Gm}{r}}$, with $r_{\text{B}} = \dfrac{9}{10} r_{\text{A}}$.

Solution: $v_B = \sqrt{\frac{Gm}{r_B}}$

$= \sqrt{\frac{10Gm}{9r_A}}$

$= \sqrt{\frac{10}{9}}\sqrt{\frac{Gm}{r_A}}$

$\sqrt{\frac{Gm}{r_A}} = v_A$

$v_B = \sqrt{\frac{10}{9}}v_A$

$\frac{v_B}{v_A} = 1.05$

Statement: The orbital velocity of satellite B is 1.05 times the orbital velocity of satellite A.

56. (a) Given: m_1 = 52 kg; d = 820 km = 8.2×10^5 m; $m_2 = 5.98 \times 10^{24}$ kg; $r_E = 6.38 \times 10^6$ m; $G = 6.67 \times 10^{-11}$ N·m²/kg²

Required: F_g

Analysis: $F_g = \frac{Gm_1m_2}{r^2}$; for r in this equation, add the distance d above the surface and Earth's radius

Solution: $F_g = \frac{Gm_1m_2}{r^2}$

$= \frac{Gm_1m_E}{(d + r_E)^2}$

$= \frac{\left(6.67\times10^{-11}\ \frac{\text{N}\cdot\cancel{\text{m}^2}}{\cancel{\text{kg}^2}}\right)(52\ \cancel{\text{kg}})(5.98\times10^{24}\ \cancel{\text{kg}})}{(8.2\times10^5\ \cancel{\text{m}} + 6.38\times10^6\ \cancel{\text{m}})^2}$

$F_g = 4.0\times10^2$ N

Statement: The gravitational force between Earth and the MOST satellite at the altitude of 820 km is 4.0×10^2 N.

(b) Given: $d = 8.2 \times 10^5$ m; $m_E = 5.98 \times 10^{24}$ kg; $r_E = 6.38 \times 10^6$ m; $G = 6.67 \times 10^{-11}$ N·m²/kg²

Required: v

Analysis: $v = \sqrt{\frac{Gm}{r}}$

Solution: $v = \sqrt{\dfrac{Gm}{r}}$

$$v = \sqrt{\frac{Gm_\text{E}}{d + r_\text{E}}}$$

$$= \sqrt{\frac{\left(6.67 \times 10^{-11}\ \frac{\cancel{\text{kg}} \cdot \frac{\cancel{\text{m}}}{\text{s}^2} \cdot \text{m}^2}{\cancel{\text{kg}^2}}\right)(5.98 \times 10^{24}\ \cancel{\text{kg}})}{8.2 \times 10^5\ \cancel{\text{m}} + 6.38 \times 10^6\ \cancel{\text{m}}}}$$

$$= 7.443 \times 10^3\ \frac{\cancel{\text{m}}}{\cancel{\text{s}}} \times \frac{1\ \text{km}}{1000\ \cancel{\text{m}}} \times \frac{60\ \cancel{\text{s}}}{1\ \cancel{\text{min}}} \times \frac{60\ \cancel{\text{min}}}{1\ \text{h}}$$

$$v = 2.7 \times 10^4\ \text{km/h}$$

Statement: The orbital speed of the MOST satellite is 7.4×10^3 m/s, or 2.7×10^4 km/h.

(c) Given: $d = 8.2 \times 10^5$ m; $r_\text{E} = 6.38 \times 10^6$ m; $v = 7.443 \times 10^3$ m/s

Required: T

Analysis: $T = \dfrac{2\pi r}{v}$

Solution: $T = \dfrac{2\pi r}{v}$

$$= \frac{2\pi\left(8.2 \times 10^5\ \cancel{\text{m}} + 6.38 \times 10^6\ \cancel{\text{m}}\right)}{7.443 \times 10^3\ \frac{\cancel{\text{m}}}{\text{s}}}$$

$$= 6078\ \cancel{\text{s}} \times \frac{1\ \cancel{\text{min}}}{60\ \cancel{\text{s}}} \times \frac{1\ \text{h}}{60\ \cancel{\text{min}}}$$

$$T = 1.7\ \text{h}$$

Statement: The orbital period of the MOST satellite is 1.7 h.

57. Given: $m_\text{Moon} = 0.0123 m_\text{E}$; $r_\text{Moon} = r_\text{E}$

Required: $\dfrac{v_\text{E}}{v_\text{Moon}}$

Analysis: Solve the equation for orbital velocity with $m_\text{Moon} = 0.0123 m_\text{E}$ and $r_\text{Moon} = r_\text{E}$, $v = \sqrt{\dfrac{Gm}{r}}$.

Solution:

$$v = \sqrt{\frac{Gm}{r}}$$

$$\begin{aligned} v_{\text{Moon}} &= \sqrt{\frac{Gm_{\text{Moon}}}{r_{\text{Moon}}}} \\ &= \sqrt{\frac{G\left(0.0123 m_{\text{E}}\right)}{r_{\text{E}}}} \\ &= \sqrt{0.0123}\sqrt{\frac{Gm_{\text{E}}}{r}} \end{aligned}$$

$$\sqrt{\frac{Gm_{\text{E}}}{r}} = v_{\text{E}}$$

$$\begin{aligned} v_{\text{Moon}} &= \left(\sqrt{0.0123}\right) v_{\text{E}} \\ v_{\text{Moon}} &= 0.111 v_{\text{E}} \\ \frac{1}{0.111} &= \frac{v_{\text{E}}}{v_{\text{Moon}}} \\ \frac{v_{\text{E}}}{v_{\text{Moon}}} &= 9.0 \end{aligned}$$

Statement: The orbital velocity of the satellite orbiting Earth is 9.0 times the orbital velocity of the satellite orbiting the Moon.

58. Given: d_1 = 390 km = 3.9×10^5 m; d_2 = 390 km – 75 km = 315 km = 3.15×10^5 m; $m = 5.98 \times 10^{24}$ kg; $r_{\text{E}} = 6.38 \times 10^6$ m; $G = 6.67 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2$

Required: Δv

Analysis: Determine the orbital speeds at each altitude using $v = \sqrt{\frac{Gm}{r}}$, then calculate the change in speed. For r in this equation, add the distance d above the surface and Earth's radius.

Solution: Determine the orbital speeds at each altitude:

$$v = \sqrt{\frac{Gm}{r}}$$

$$v_1 = \sqrt{\frac{Gm_E}{d_1 + r_E}}$$

$$= \sqrt{\frac{\left(6.67 \times 10^{-11} \, \frac{\cancel{\text{kg}} \cdot \frac{\cancel{\text{m}}}{\text{s}^2} \cdot \text{m}^2}{\cancel{\text{kg}^2}}\right)(5.98 \times 10^{24} \, \cancel{\text{kg}})}{3.9 \times 10^5 \, \cancel{\text{m}} + 6.38 \times 10^6 \, \cancel{\text{m}}}}$$

$v_1 = 7.676 \times 10^3$ m/s (two extra digits carried)

$$v_2 = \sqrt{\frac{Gm_E}{d_2 + r_E}}$$

$$= \sqrt{\frac{\left(6.67 \times 10^{-11} \, \frac{\cancel{\text{kg}} \cdot \frac{\cancel{\text{m}}}{\text{s}^2} \cdot \text{m}^2}{\cancel{\text{kg}^2}}\right)(5.98 \times 10^{24} \, \cancel{\text{kg}})}{3.15 \times 10^5 \, \cancel{\text{m}} + 6.38 \times 10^6 \, \cancel{\text{m}}}}$$

$v_2 = 7.719 \times 10^3$ m/s (two extra digits carried)

Determine the change in orbital speed:

$$\Delta v = v_2 - v_1$$

$$= 7.719 \times 10^3 \text{ m/s} - 7.676 \times 10^3 \text{ m/s}$$

$$\Delta v = 43 \text{ m/s}$$

Statement: The orbital speed of the vehicle would have to increase by 43 m/s in order for its altitude to decrease by 75 km.

59. (a) Given: d = 35 000 km = 3.5×10^7 m; $m_E = 5.98 \times 10^{24}$ kg; $r_E = 6.38 \times 10^6$ m; $G = 6.67 \times 10^{-11}$ N·m²/kg²

Required: g

Analysis: $g = \frac{Gm}{r^2}$; for r in this equation, add the distance d above the surface and Earth's radius.

Solution: $g = \frac{Gm}{r^2}$

$$= \frac{Gm_E}{(d + r_E)^2}$$

$$= \frac{\left(6.67 \times 10^{-11}\ \frac{\text{N} \cdot \cancel{\text{m}^2}}{\cancel{\text{kg}^2}}\right)(5.98 \times 10^{24}\ \cancel{\text{kg}})}{(3.5 \times 10^7\ \cancel{\text{m}} + 6.38 \times 10^6\ \cancel{\text{m}})^2}$$

$g = 0.23$ N/kg

Statement: The gravitational field of Earth at the altitude of the Anik F2 satellite is 0.23 N/kg.

(b) Given: $m_1 = 5900$ kg; $d = 3.5 \times 10^7$ m; $m_E = 5.98 \times 10^{24}$ kg; $r_E = 6.38 \times 10^6$ m; $G = 6.67 \times 10^{-11}\ \text{N·m}^2/\text{kg}^2$

Required: F_g

Analysis: $F_g = \frac{Gm_1m_2}{r^2}$

Solution: $F_g = \frac{Gm_1m_2}{r^2}$

$$= \frac{Gm_1m_E}{(d + r_E)^2}$$

$$= \frac{\left(6.67 \times 10^{-11}\ \frac{\text{N} \cdot \cancel{\text{m}^2}}{\cancel{\text{kg}^2}}\right)(5900\ \cancel{\text{kg}})(5.98 \times 10^{24}\ \cancel{\text{kg}})}{(3.5 \times 10^7\ \cancel{\text{m}} + 6.38 \times 10^6\ \cancel{\text{m}})^2}$$

$F_g = 1.4 \times 10^3$ N

Statement: The gravitational force between Earth and the Anik F2 satellite is 1.4×10^3 N.

(c) Given: $d = 3.5 \times 10^7$ m; $m_E = 5.98 \times 10^{24}$ kg; $r_E = 6.38 \times 10^6$ m; $G = 6.67 \times 10^{-11}\ \text{N·m}^2/\text{kg}^2$

Required: v

Analysis: $v = \sqrt{\frac{Gm}{r}}$; for r in this equation, add the distance d above the surface and Earth's radius

Solution: $v = \sqrt{\dfrac{Gm}{r}}$

$$v = \sqrt{\dfrac{Gm_E}{d + r_E}}$$

$$= \sqrt{\dfrac{\left(6.67 \times 10^{-11} \ \dfrac{\cancel{\text{kg}} \cdot \dfrac{\cancel{\text{m}}}{\text{s}^2} \cdot \text{m}^2}{\cancel{\text{kg}^2}}\right)(5.98 \times 10^{24} \ \cancel{\text{kg}})}{3.5 \times 10^7 \ \cancel{\text{m}} + 6.38 \times 10^6 \ \cancel{\text{m}}}}$$

$$= 3.105 \times 10^3 \ \dfrac{\cancel{\text{m}}}{\cancel{\text{s}}} \times \dfrac{1 \text{ km}}{1000 \ \cancel{\text{m}}} \times \dfrac{60 \ \cancel{\text{s}}}{1 \ \cancel{\text{min}}} \times \dfrac{60 \ \cancel{\text{min}}}{1 \text{ h}}$$

$$v = 1.1 \times 10^4 \text{ km/h}$$

Statement: The orbital speed of the Anik F2 satellite is 3.1×10^3 m/s, or 1.1×10^4 km/h.
(d) Given: $d = 3.5 \times 10^7$ m; $r_E = 6.38 \times 10^6$ m; $v = 3.105 \times 10^3$ m/s
Required: T

Analysis: $T = \dfrac{2\pi r}{v}$

Solution: $T = \dfrac{2\pi r}{v}$

$$= \dfrac{2\pi\left(3.5 \times 10^7 \ \cancel{\text{m}} + 6.38 \times 10^6 \ \cancel{\text{m}}\right)}{3.105 \times 10^3 \ \dfrac{\cancel{\text{m}}}{\text{s}}}$$

$$= 8.374 \times 10^4 \ \cancel{\text{s}} \times \dfrac{1 \ \cancel{\text{min}}}{60 \ \cancel{\text{s}}} \times \dfrac{1 \text{ h}}{60 \ \cancel{\text{min}}}$$

$$T = 23 \text{ h}$$

Statement: The orbital period of the Anik F2 satellite is 23 h.

60. (a) When comparing the equation for gravitational force, $F_g = \dfrac{Gm_1m_2}{r^2}$, exerted by the black hole on a 1 kg object and the gravitational force exerted by the Sun on a 1 kg object, there is only one difference in the equations: the mass of the black hole is 4.3×10^6 times greater than the mass of the Sun. Therefore, the gravitational force of the black hole would be 4.3×10^6 times greater on the object than the Sun's gravitational force on the same object.
(b) Given: $m_1 = 8.5$ kg; $m_2 = (4.3 \times 10^6)m_{Sun}$; $m_{Sun} = 1.99 \times 10^{30}$ kg; $r = 4.5 \times 10^{12}$ m; $G = 6.67 \times 10^{-11}$ N·m^2/kg^2
Required: F_g

Analysis: $F_g = \dfrac{Gm_1m_2}{r^2}$

Solution: $F_g = \frac{Gm_1m_2}{r^2}$

$$= \frac{\left(6.67\times10^{-11}\ \frac{\text{N}\cdot\cancel{\text{m}^2}}{\cancel{\text{kg}^2}}\right)(8.5\ \cancel{\text{kg}})(4.3\times10^{6})(1.99\times10^{30}\ \cancel{\text{kg}})}{(4.5\times10^{12}\ \cancel{\text{m}})^2}$$

$F_g = 2.4\times10^{2}$ N

Statement: The gravitational force exerted by the black hole on the probe is 2.4×10^2 N.

Evaluation

61. (a) Given: m_1 = 85 kg; $m_2 = 4.85 \times 10^{24}$ kg; $r = 1.5 \times 10^{10}$ m

Required: F_{Venus}

Analysis: $F_g = \frac{Gm_1m_2}{r^2}$

Solution: $F_{\text{Venus}} = \frac{Gm_1m_2}{r^2}$

$$= \frac{\left(6.67\times10^{-11}\ \frac{\text{N}\cdot\cancel{\text{m}^2}}{\cancel{\text{kg}^2}}\right)(85\ \cancel{\text{kg}})(4.85\times10^{24}\ \cancel{\text{kg}})}{(1.5\times10^{10}\ \cancel{\text{m}})^2}$$

$F_{\text{Venus}} = 1.2\times10^{-4}$ N

Statement: The gravitational force of Venus on an 85 kg person is 1.2×10^{-4} N.

(b) Given: m_1 = 85 kg; m_2 = 10 000 kg; r = 0.5 m

Required: F_{bus}

Analysis: $F_g = \frac{Gm_1m_2}{r^2}$

Solution: $F_{\text{bus}} = \frac{Gm_1m_2}{r^2}$

$$= \frac{\left(6.67\times10^{-11}\ \frac{\text{N}\cdot\cancel{\text{m}^2}}{\cancel{\text{kg}^2}}\right)(85\ \cancel{\text{kg}})(10\ 000\ \cancel{\text{kg}})}{(0.5\ \cancel{\text{m}})^2}$$

$F_{\text{bus}} = 2.3\times10^{-4}$ N

Statement: The gravitational force of the school bus on an 85 kg person is 2.3×10^{-4} N.

(c) The ratio of the gravitational pull of the bus to the gravitational pull of Venus is

$\frac{2.3\times10^{-4}\ \text{N}}{1.2\times10^{-4}\ \text{N}} = 1.9$.

(d) The gravitational force exerted by the school bus is almost two stronger than the gravitational force exerted by Venus. The gravitational forces exerted by distant objects such as planets are less influential than the gravitational forces of large objects near us on Earth.

62. An increase in the value of G would affect calculations of gravitational force, gravitational field strength, and orbital velocity. The values would all increase, but only slightly.
63. Answers may vary. Sample answer: NASA instructed the astronauts on *Apollo 13* to use the Moon's gravitational field because then the rockets would use less fuel. Going behind the Moon gives the spacecraft a slingshot to get them home. Using the rockets would have gotten the astronauts back to Earth faster, but this strategy uses more fuel, leaving no room for other emergency use of fuel.

64. (a) The satellite is a distance d above Earth's surface, so in the equation $v = \sqrt{\frac{Gm}{r}}$, we have to add d and r because r represents the distance from Earth's centre to the satellite's centre: $v = \sqrt{\frac{Gm}{r+d}}$. When the mass changes to $2m$ and the orbital radius changes to $2r$, we have $v = \sqrt{\frac{G(2m)}{2r+d}}$. The two velocities are not the same. The student is incorrect. The student treated d as the distance from the planet's centre, rather than from the planet's surface.

(b) Compare $v = \sqrt{\frac{Gm}{r+d}}$ to $v = \sqrt{\frac{G(2m)}{2r+d}}$. The actual velocity around the more massive planet would be slightly higher because of the doubling of the mass; also, r in the equation is the radius of the planet plus d.
65. (a) If Earth's orbital speed is greater during the winter, then according to the equation $v = \sqrt{\frac{Gm}{r}}$, the orbital radius must be less in winter than in summer. Therefore, Earth is closest to the Sun in winter.
(b) No, my answer to (a) does not explain why summer in the northern hemisphere is so much warmer than winter. Earth's distance from the Sun is not responsible for seasonal temperatures. Otherwise, it would be warmer in the winter than in the summer.
66. Answers may vary. Sample answer: Weather satellites can track hurricanes and allow us to prepare for them. This type of information can help reduce some damage to property, and it can definitely save lives.
67. Posters may vary. Students should include information about how satellites affect their daily lives, such as weather forecasting, cellphone telephone usage, television signals, and GPS technology in cars and phones. The poster should also include an evaluation of the effect on their daily lives.
68. Answers may vary. Sample answer: As more and more satellites end up in orbit, there will be more space junk. Routes out of the atmosphere will have to take space junk into account. Spacecraft will have to be strong enough to withstand impact with unexpected space junk.

Reflect on Your Learning

69. Answers may vary. Sample answer: The universal law of gravitation describes the relationship between the masses of two objects, the distance between the objects, and the gravitational force they exert on each other. It is useful to understand this relationship first because it can then be related to Newton's second law to derive g, the gravitational field strength.

70. Answers may vary. Students should describe how specific diagrams or images were helpful. For example, they might explain how arrows in images were helpful in understanding the way gravitational field decreases from the centre of a mass. They might also describe how a diagram of a satellite orbiting a planet helped emphasize the point that the distance used in the universal law of gravitation equation is measured from the centre of a planet, not the surface.
71. Answers may vary. Students should choose a topic and describe its importance in their lives. For example, a student might describe how satellites enable them to receive television transmission from news reports around the world. This enables a person to better understand different cultures. Students should also identify one or more aspects of the topic that they would like to learn more about and identify ways they could learn about it.

Research
72. Answers may vary. Students should provide a description of the satellite radar images they viewed and report on the content and use of the information. For example, students could research precipitation and cloud-cover data, sea-state conditions at various heights, ice-sheet characteristics and dynamics, natural hazard predicting and tracking, and responding to natural hazards, and surface-deformation monitoring.
73. Answers may vary. Students should provide an overview about how gravitational concepts have enabled advancements in astronomical knowledge. For example, students could explain how an understanding of gravitational concepts led to the understanding of gravitational lensing.
74. Students should describe the five Lagrange points and explain why only L4 and L5 are relatively stable. They should also identify and explain why some satellites are currently at certain Lagrange points. For example, the placement of the SOHO satellite at the L1 point between Earth and the Sun provides a continuous view of the Sun.
75. A geostationary satellite orbits Earth at the same rate that Earth rotates, but the satellite only sits over one surface location if it orbits in the same direction that Earth rotates. A location above the equator is the only spot where this is possible.
76. Answers may vary. Students should provide a description of a gravity survey and how gravitational fields are used to search for mineral deposits. In their results, they should include the relation to an equation from this chapter and perhaps provide an example from Ontario, if possible.