Section 4.5: The Law of Conservation of Energy

Mini Investigation: Various Energies of a Roller Coaster, page 185

Answers may vary. Sample answers:

A. The total energy graph is a straight line because total energy is conserved and it is constant.

B. At any height, *h*, the sum of the energy values on the potential energy and kinetic energy curves is equal to the value of the total energy at that height.

C. It is necessary to know the height at point A because it represents the potential energy before the roller coaster starts. This is equal to the total mechanical energy. If the height of point A were greater, the change in slopes of the potential and kinetic energy graphs would be greater, and the total mechanical energy graph would be higher, but still horizontal.

D. If the mass of the roller coaster car were greater, the total mechanical energy would be greater, and the change in slopes for the graphs for potential and kinetic energy would be greater.

Tutorial 1 Practice, page 187

1. (a) Given: $m = 0.43\text{ kg};\ \Delta y = 18\text{ m};\ g = 9.8\text{ m/s}^2;\ v_\text{i} = 7.4\text{ m/s}$

Required: v_f

Analysis: The total energy at the top of the hill is equal to the total energy at the bottom of the hill. At the top of the hill, the total energy is the gravitational potential energy, $mg\Delta y$, plus the kinetic energy, $\frac{1}{2}mv_\text{i}^2$. At the bottom of the hill, the total energy is equal to the kinetic energy, $\frac{1}{2}mv_\text{f}^2$, since there is no gravitational potential energy at $\Delta y = 0$.

Solution:
$$\begin{aligned}
\cancel{m}g\Delta y + \frac{1}{2}\cancel{m}v_\text{i}^2 &= \frac{1}{2}\cancel{m}v_\text{f}^2 \\
2g\Delta y + v_\text{i}^2 &= v_\text{f}^2 \\
v_\text{f} &= \sqrt{2g\Delta y + v_\text{i}^2} \\
&= \sqrt{2(9.8\text{ m/s}^2)(18\text{ m}) + (7.4\text{ m/s})^2} \\
v_\text{f} &= 2.0 \times 10^1\text{ m/s}
\end{aligned}$$

Statement: The ball's speed at the bottom of the hill is 2.0×10^1 m/s.

(b) Given: $m = 0.43\text{ kg};\ \Delta y = 18\text{ m};\ v_\text{i} = 4.2\text{ m/s};\ g = 9.8\text{ m/s}^2$

Required: v_f

Analysis: The total energy when the ball is kicked up the hill is equal to the total energy when the ball reaches the bottom of the hill. The energy at any time when the ball is on its way up or down the hill does not matter. When the ball is kicked, $E_\text{T} = mg\Delta y + \frac{1}{2}mv_\text{i}^2$.

At the bottom of the hill, $E_\text{T} = \frac{1}{2}mv_\text{f}^2$.

Solution: $\cancel{m}g\Delta y + \frac{1}{2}\cancel{m}v_\text{i}^2 = \frac{1}{2}\cancel{m}v_\text{f}^2$

$$2g\Delta y + v_\text{i}^2 = v_\text{f}^2$$

$$v_\text{f} = \sqrt{2g\Delta y + v_\text{i}^2}$$

$$= \sqrt{2(9.8\ \text{m/s}^2)(18\ \text{m}) + (4.2\ \text{m/s})^2}$$

$$v_\text{f} = 19\ \text{m/s}$$

Statement: The ball's speed as it reaches the bottom of the hill is 19 m/s.

2. (a) Given: $m = 0.057$ kg; $\Delta y = 1.8$ m; $g = 9.8\ \text{m/s}^2$; $v_\text{f} = 0\ \text{m/s}$

Required: v_i

Analysis: Let the player's hand be the $y = 0$ reference point. The total energy when the ball is released is all kinetic energy, $\frac{1}{2}mv_\text{i}^2$. The total energy at the highest point of the ball is all gravitational potential energy, $mg\Delta y$. Thus, $\frac{1}{2}mv_\text{i}^2 = mg\Delta y$.

Solution: $\frac{1}{2}\cancel{m}v_\text{i}^2 = \cancel{m}g\Delta y$

$$v_\text{i}^2 = 2g\Delta y$$

$$v_\text{i} = \sqrt{2g\Delta y}$$

$$= \sqrt{2(9.8\ \text{m/s}^2)(1.8\ \text{m})}$$

$$v_\text{i} = 5.9\ \text{m/s}$$

Statement: The speed of the ball as it leaves the player's hand is 5.9 m/s.

(b) Given: $m = 0.057$ kg; $v_{\text{i}2} = v_\text{i}$; $g = 9.8\ \text{m/s}^2$

Required: $\Delta y_2 : \Delta y_1$

Analysis: $\frac{1}{2}mv_\text{i}^2 = mg\Delta y$

Solution:

$$\frac{1}{2}\cancel{m}v_{\text{i}1}^2 = \cancel{m}g\Delta y_1 \qquad \frac{1}{2}\cancel{m}v_{\text{i}2}^2 = \cancel{m}g\Delta y_2$$

$$\frac{1}{2}v_{\text{i}1}^2 = g\Delta y_1 \qquad \frac{1}{2}v_{\text{i}2}^2 = g\Delta y_2$$

$$\Delta y_1 = \frac{v_{\text{i}1}^2}{2g} \qquad \Delta y_2 = \frac{v_{\text{i}2}^2}{2g}$$

$$\frac{\Delta y_2}{\Delta y_1} = \frac{\frac{v_{i2}^2}{2g}}{\frac{v_{i1}^2}{2g}}$$

$$= \frac{v_{i2}^2}{\cancel{2g}} \times \frac{\cancel{2g}}{v_{i1}^2}$$

$$= \frac{v_{i2}^2}{v_{i1}^2}$$

$$= \frac{\left(\frac{1}{4} v_{i1}\right)^2}{v_{i1}^2}$$

$$= \frac{\frac{1}{16} \cancel{v_{i1}^2}}{\cancel{v_{i1}^2}}$$

$$\frac{\Delta y_2}{\Delta y_1} = \frac{1}{16}$$

Statement: The ratio of the maximum rise of the ball after leaving the player's hand to the maximum rise in (a) is 1:16.

Tutorial 2 Practice, page 190

1. (a) Given: $v = 1.4 \text{ m/s}; \Delta y = 5.0 \text{ m}; m = 65 \text{ kg}; g = 9.8 \text{ m/s}^2$

Required: P

Analysis: The work done to get to the top of the ladder is equal to the gravitational potential energy at the top of the ladder, $W = mg\Delta y$. The time, t, taken to get to the top of the ladder is the distance, Δy, divided by the speed, v. $P = \frac{W}{t}$

Solution: $t = \frac{\Delta y}{v}$

$$= \frac{5.0 \text{ m}}{1.4 \text{ m/s}}$$

$t = 3.57$ s (one extra digit carried)

$$P = \frac{W}{t}$$
$$= \frac{mg\Delta y}{t}$$
$$= \frac{(65\ \text{kg})(9.8\ \text{m/s}^2)(5.0\ \text{m})}{3.57\ \text{s}}$$
$$P = 890\ \text{W}$$

Statement: The firefighter's power output while climbing the ladder is 890 W.

(b) From (a), it takes the firefighter 3.6 s to climb the ladder.

2. Given: $v_{f2} = 2v_{f1}$; $t_2 = t_1$; $v_i = 0\ \text{m/s}$; $m_2 = m_1$

Required: ratio of power needed, $P_2 : P_1$

Analysis: We are told that the Grand Prix car accelerates to twice the speed of the car in Sample 1, which can be expressed as $v_{f2} = 2v_{f1}$. We are also told that the Grand Prix car accelerates to this speed in the same amount of time as the car in Sample 1, which is stated as 7.7 s. We will assume that the two cars are equal in mass, at $1.1\times10^3\ \text{kg}$.

Solution: $P_1 = \dfrac{mv_{f1}^2}{2t}$ $\quad P_2 = \dfrac{mv_{f2}^2}{2t}$

$$\frac{P_2}{P_1} = \frac{\frac{\cancel{m}v_{f2}^2}{\cancel{2t}}}{\frac{\cancel{m}v_{f1}^2}{\cancel{2t}}}$$
$$= \frac{v_{f2}^2}{v_{f1}^2}$$
$$= \frac{(2v_{f1})^2}{v_{f1}^2}$$
$$= \frac{4\cancel{v_{f1}^2}}{\cancel{v_{f1}^2}}$$
$$\frac{P_2}{P_1} = \frac{4}{1}$$

Statement: The ratio of the power needed by the Grand Prix car to the power needed by the car in Sample Problem 1 is 4:1.

3. Given: $\Delta d = 190\ \text{m}; t = 4\ \text{min}\ 50\ \text{s} = 290\ \text{s}; m = 62\ \text{kg}; g = 9.8\ \text{m/s}^2$

Required: P

Analysis: $P = \dfrac{W}{t}$; $W = mg\Delta y$

Solution: $P = \frac{W}{t}$

$= \frac{mg\Delta y}{t}$

$= \frac{(62\text{ kg})(9.8\text{ m/s}^2)(190\text{ m})}{290\text{ s}}$

$P = 0.40\text{ kW}$

Statement: The racer's average power output during the race is 0.40 kW.

Section 4.5 Questions, page 191

1. (a) Given: $v_{\text{i}} = 11\text{ m/s};\ g = 9.8\text{ m/s}^2$

Required: maximum height that the ball will reach, Δy

Analysis: The kinetic energy when the child tosses the ball is equal to the gravitational potential energy at the ball's maximum height, expressed as $\frac{1}{2}mv_{\text{i}}^2 = mg\Delta y$.

Solution: $\frac{1}{2}\cancel{m}v_{\text{i}}^2 = \cancel{m}g\Delta y$

$\frac{1}{2}v_{\text{i}}^2 = g\Delta y$

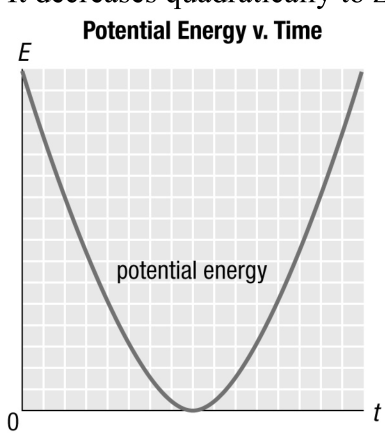
$\Delta y = \frac{v_{\text{i}}^2}{2g}$

$= \frac{(11\text{ m/s})^2}{2(9.8\text{ m/s}^2)}$

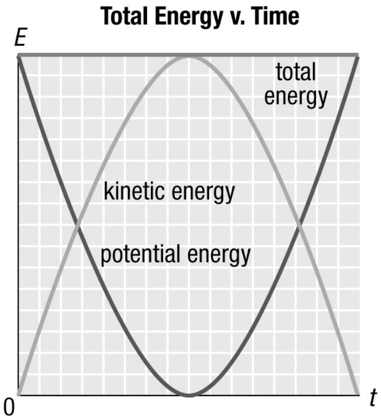
$\Delta y = 6.2\text{ m}$

Statement: The maximum height that the ball will reach is 6.2 m.

(b) As the ball leaves the child's hand, the gravitational potential energy is zero.
It increases quadratically to its maximum when the ball reaches its maximum height.
It decreases quadratically to zero as the ball returns to the level of the child's hand.



(c) The graph has this shape because, as the ball leaves the child's hand, the kinetic energy is at its maximum. It decreases quadratically to zero when the ball reaches its maximum height. It increases quadratically to its maximum as the ball returns to the level of the child's hand. The total energy is conserved, so it is a constant horizontal line equal to the maximum kinetic energy or potential energy.



2. Answers may vary. Sample answers:
(a) The kinetic energy is the greatest just before the apple hits the ground.
(b) The gravitational potential energy is the greatest as the apple leaves the branch.
3. (a) The law of conservation of energy states that energy can neither be created nor destroyed in an isolated system; it can only change form. Assuming the puck and surface form an isolated system, the energy of the hockey puck is conserved. The kinetic energy of the puck is transformed to thermal energy by friction.
(b) The initial kinetic energy is transformed to thermal energy by friction as the puck slows down to a stop.
4. (a) Given: $m = 110$ kg; $\Delta y = 210$ m; $g = 9.8$ m / s^2
Required: W
Analysis: The work done by gravity is equal to the gravitational potential energy at the top of the hill, expressed as $E_g = mg\Delta y$.
Solution:
$$\begin{aligned} W &= E_g \\ &= mg\Delta y \\ &= (110 \text{ kg})(9.8 \text{ m/s}^2)(210 \text{ m}) \\ W &= 2.3 \times 10^5 \text{ J} \end{aligned}$$
Statement: The work done by gravity on the skier is 2.3×10^5 J.
(b) Given: $m = 110$ kg; $\Delta y = 210$ m; $g = 9.8$ m / s^2; $v_i = 0$ m / s
Required: v_f
Analysis: Because the skier has no initial velocity, the total energy at the top of the hill is all potential energy. The total energy at the bottom of the hill is all kinetic energy. The total energy at the top of the hill is equal to the total energy at the bottom of the hill, or $mg\Delta y = \frac{1}{2}mv_f^2$. Solve for v_f and substitute.

Solution: $\not{m}g\Delta y = \frac{1}{2}\not{m}v_{\text{f}}^2$

$$2g\Delta y = v_{\text{f}}^2$$

$$v_{\text{f}} = \sqrt{2g\Delta y}$$

$$= \sqrt{2(9.8\ \text{m/s}^2)(210\ \text{m})}$$

$$v_{\text{f}} = 64\ \text{m/s}$$

Statement: The skier's speed when he reaches the bottom of the hill is 64 m/s.

5. Given: $m = 62$ kg; $v_{\text{i}} = 8.1$ m / s; $g = 9.8$ m / s^2; $\Delta y = 3.7$ m

Required: v_{f}

Analysis: The total energy as the snowboarder leaves the ledge is the sum of her gravitational potential energy, $mg\Delta y$, and her kinetic energy, $\frac{1}{2}mv_{\text{i}}^2$. The total energy when she lands is all kinetic energy, $\frac{1}{2}mv_{\text{f}}^2$, if we take $\Delta y = 0$ at the landing point. Thus, $mg\Delta y + \frac{1}{2}mv_i^2 = \frac{1}{2}mv_{\text{f}}^2$. Solve for v_{f} and substitute the given values.

Solution: $\not{m}g\Delta y + \frac{1}{2}\not{m}v_{\text{i}}^2 = \frac{1}{2}\not{m}v_{\text{f}}^2$

$$2g\Delta y + v_{\text{i}}^2 = v_{\text{f}}^2$$

$$v_{\text{f}} = \sqrt{2g\Delta y + v_{\text{i}}^2}$$

$$= \sqrt{2(9.8\ \text{m/s}^2)(3.7\ \text{m}) + (8.1\ \text{m/s})^2}$$

$$v_{\text{f}} = 12\ \text{m/s}$$

Statement: The snowboarder's speed at the moment she hits the ground is 12 m/s.

6. Given: $\Delta y = 3.5$ m; $\theta = 40°$; $g = 9.8$ m / s^2

Required: the speed in the *y*-direction

Analysis: $mg\Delta y = \frac{1}{2}mv^2$

The energy equations give the speed in the direction along the jump, so we need to use components to solve for the vertical velocity: $v_y = \frac{v}{\sin\theta}$.

Solution: $\not{m}g\Delta y = \frac{1}{2}\not{m}v^2$

$$2g\Delta y = v^2$$

$$v = \sqrt{2g\Delta y}$$

$$= \sqrt{2(9.8\ \text{m/s}^2)(3.5\ \text{m})}$$

$$v = 8.28\ \text{m/s (one extra digit carried)}$$

$$v_y = \frac{v}{\sin\theta}$$
$$= \frac{8.28\text{ m/s}}{\sin 40°}$$
$$v_y = 13\text{ m/s}$$

Statement: The dolphin's minimum speed is 13 m/s.

7. (a) Yes, the mechanical energy of the roller coaster is conserved because there is no friction.

(b) Given: $m = 640$ kg; $v_\text{i} = 0; \Delta y_\text{A} = 30.0$ m; $g = 9.8$ m / s^2

Required: E_T, the total mechanical energy

Analysis: Because the car starts from rest, its total mechanical energy is equal to its potential energy at point A: $E_\text{T} = mg\Delta y_\text{A}$.

Solution: $E_\text{T} = mg\Delta y_\text{A}$
$$= (640\text{ kg})(9.8\text{ m/s}^2)(30.0\text{ m})$$
$$E_\text{T} = 1.9\times10^5\text{ J}$$

Statement: The total mechanical energy at point A is 1.9×10^5 J.

(c) The total mechanical energy is conserved, so it is the same at point B as it is at point A: 1.9×10^5 J.

(d) Given: $m = 640$ kg; $\Delta y_\text{B} = 15.0$ m; $g = 9.8$ m / s^2; $\Delta y_\text{A} = 30.0$ m

Required: v_B; v_C

Analysis for v_B: The total mechanical energy at point B is the sum of the kinetic energy, $\frac{1}{2}mv_\text{B}^2$, and the potential energy, $mg\Delta y_\text{B}$. The total mechanical energy is equal to the potential energy at point A: $mg\Delta y_\text{A}$. Therefore, $\frac{1}{2}mv_\text{B}^2 + mg\Delta y_\text{B} = mg\Delta y_\text{A}$.

Solution for v_B: $\frac{1}{2}\cancel{m}v_\text{B}^2 + \cancel{m}g\Delta y_\text{B} = \cancel{m}g\Delta y_\text{A}$
$$v_\text{B}^2 + 2g\Delta y_\text{B} = 2g\Delta y_\text{A}$$
$$v_\text{B}^2 = 2g\Delta y_\text{A} - 2g\Delta y_\text{B}$$
$$v_\text{B} = \sqrt{2g(\Delta y_\text{A} - \Delta y_\text{B})}$$
$$= \sqrt{2(9.8\text{ m/s}^2)(30.0\text{ m} - 15\text{ m})}$$
$$v_\text{B} = 17\text{ m/s}$$

Statement for v_B: The speed of the car when it reaches point B is 17 m/s.

Analysis for v_C: The total mechanical energy at point C is all kinetic energy, $\frac{1}{2}mv_\text{C}^2$.

The total mechanical energy is equal to the potential energy at point A: $mg\Delta y_\text{A}$.

Thus, $\frac{1}{2}mv_\text{C}^2 = mg\Delta y_\text{A}$.

Solution for v_C: $\frac{1}{2}\cancel{m}v_C^2 = \cancel{m}g\Delta y_A$

$$v_C^2 = 2g\Delta y_A$$
$$v_C = \sqrt{2g(\Delta y_A)}$$
$$= \sqrt{2(9.8\ \text{m/s}^2)(30.0\ \text{m})}$$
$$v_C = 24\ \text{m/s}$$

Statement for v_C: The speed of the car when it reaches point C is 24 m/s.

(e) Given: $\Delta y_A = 30.0$ m; $\Delta y_B = 15.0$ m; $g = 9.8$ m / s^2

Required: v_B; v_C

Analysis: The total energy at A is equal to the total energy at B and at C. At A, the total energy consists of kinetic energy and gravitational potential energy. At B, the total energy also consists of kinetic energy and gravitational potential energy. At C, the total energy consists of kinetic energy only: $E_k = \frac{1}{2}mv^2; E_g = mg\Delta y$

Solution:

$$E_{T\,B} = E_{T\,A}$$
$$\cancel{m}g\Delta y_B + \frac{1}{2}\cancel{m}v_B^2 = \cancel{m}g\Delta y_A + \frac{1}{2}\cancel{m}v_A^2$$
$$\frac{1}{2}v_B^2 = g\Delta y_A + \frac{1}{2}v_A^2 - g\Delta y_B$$
$$v_B^2 = 2g\Delta y_A + v_A^2 - 2g\Delta y_B$$
$$v_B = \sqrt{2g\Delta y_A + v_A^2 - 2g\Delta y_B}$$
$$= \sqrt{2(9.8\ \text{m/s}^2)(30.0\ \text{m}) + (12\ \text{m/s})^2 - 2(9.8\ \text{m/s}^2)(15.0\ \text{m})}$$
$$v_B = 21\ \text{m/s}$$

$$E_{T\,C} = E_{T\,A}$$
$$\frac{1}{2}\cancel{m}v_C^2 = \cancel{m}g\Delta y_A + \frac{1}{2}\cancel{m}v_A^2$$
$$\frac{1}{2}v_C^2 = g\Delta y_A + \frac{1}{2}v_A^2$$
$$v_C^2 = 2g\Delta y_A + v_A^2$$
$$v_C = \sqrt{2g\Delta y_A + v_A^2}$$
$$= \sqrt{2(9.8\ \text{m/s}^2)(30.0\ \text{m}) + (12\ \text{m/s})^2}$$
$$v_C = 27\ \text{m/s}$$

Statement: The speed at B is 21 m/s, and the speed at C is 27 m/s.

8. Given: $m = 52\text{ kg};\ t = 24\text{ s};\ \Delta y = 18\text{ m};\ g = 9.8\text{ m/s}^2$

Required: P

Analysis: $P = \dfrac{W}{t};\ W = mg\Delta y$

Solution:

$$\begin{aligned} P &= \frac{W}{t} \\ &= \frac{mg\Delta y}{t} \\ &= \frac{(52\text{ kg})(9.8\text{ m/s}^2)(18\text{ m})}{24\text{ s}} \\ P &= 380\text{ W} \end{aligned}$$

Statement: The power the woman exerts is 3.8×10^2 W.

Section 4.6: Elastic Potential Energy and Simple Harmonic Motion

Mini Investigation: Spring Force, page 193

Answers may vary. Sample answers:

A. The relationship between F_g and Δx is linear.

B. The slope of the best fit line of my graph is 50. This line represents the relationship between F_g and Δx, where the slope is the spring constant.

C. For the equipment used in this investigation, where *k* is the slope of the line of best fit, the equation is $F_g = 50\Delta x$.

Tutorial 1 Practice, page 195

1. (a) Given: $m = 0.65$ kg; $\Delta x = 0.44$ m; $g = 9.8$ m / s^2

Required: k

Analysis: The force of gravity on the mass points down. The restorative spring force on the mass points up because the spring is stretched down. To calculate the total force, subtract the magnitudes:

$$\vec{F}_g = mg \text{ [down]} = -mg \text{ [up]}; \ \vec{F}_x = -k\Delta\vec{x} = k\Delta x \text{ [up]}$$

Since the mass is not accelerating, $\Sigma\vec{F} = 0$ according to Newton's second law.

Solution:

$$\begin{aligned} \Sigma\vec{F} &= 0 \\ k\Delta x - mg &= 0 \\ k &= \frac{mg}{\Delta x} \\ &= \frac{(0.65 \text{ kg})(9.8 \text{ m/s}^2)}{0.44 \text{ m}} \\ k &= 14.5 \text{ N/m (one extra digit carried)} \end{aligned}$$

Statement: The spring constant is 14 N/m.

(b) Given: $k = 14.5$ N / m; $\Delta x = 0.74$ m; $g = 9.8$ m / s^2

Required: m

Analysis: Use the equation $k\Delta x - mg = 0$ from (a).

Solution:

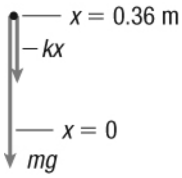
$$\begin{aligned} k\Delta x - mg &= 0 \\ k\Delta x &= mg \\ m &= \frac{k\Delta x}{g} \\ &= \frac{(14.5 \text{ N/m})(0.74 \text{ m})}{9.8 \text{ m/s}^2} \\ m &= 1.1 \text{ kg} \end{aligned}$$

Statement: The new mass is 1.1 kg.

2. Given: $m = 5.3$ kg; $k = 720$ N / m; $\Delta x = 0.36$ m

Required: $\vec{F}_{net}$; $\vec{a}$

Analysis: The free-body diagram for the mass is shown.



The force of gravity on the mass points down. The spring force on the mass points down because the spring is compressed upward:

$$\vec{F}_g = mg \text{ [down]}, \ \vec{F}_x = -k\Delta\vec{x} = k\Delta x \text{ [down]}$$

$$\vec{F}_{net} = \vec{F}_g + \vec{F}_x$$

Use the equation $\vec{F}_{net} = m\vec{a}$ to find the acceleration.

Solution:

$$\begin{aligned}\vec{F}_{net} &= \vec{F}_g + \vec{F}_x \\ &= mg \text{ [down]} + k\Delta x \text{ [down]} \\ &= (5.3 \text{ kg})(9.8 \text{ m/s}^2) \text{ [down]} + (720 \text{ N/m})(0.36 \text{ m}) \text{ [down]} \\ \vec{F}_{net} &= 311 \text{ N (one extra digit carried)}\end{aligned}$$

$$\begin{aligned}\vec{F}_{net} &= m\vec{a} \\ \vec{a} &= \frac{\vec{F}_{net}}{m} \\ &= \frac{311 \text{ N [down]}}{5.3 \text{ kg}} \\ \vec{a} &= 59 \text{ m/s}^2 \text{ [down]}\end{aligned}$$

Statement: The force on the mass is 310 N [down], and the acceleration is 59 m/s^2 [down].

Tutorial 2 Practice, page 196

1. Given: $m_b = 2m$; $k = 2.29 \times 10^3 \text{ N/m}$; $g = 9.8 \text{ m/s}^2$

Required: ratio of $E_{eb} : E_e$

Analysis: $E = \frac{1}{2}k(\Delta x)^2$

To find E_{eb}, we need Δx_b. Use the same equation as in Sample Problem 1, $k\Delta x \text{ [up]} - mg \text{ [down]} = 0$.

Solution: $k\Delta x_b \text{ [up]} - m_b g \text{ [down]} = 0$

$$\begin{aligned}k\Delta x_b &= m_b g \\ \Delta x_b &= \frac{m_b g}{k} \\ \Delta x_b &= \frac{2mg}{k}\end{aligned}$$

$$E_{\text{e b}} = \frac{1}{2}k(\Delta x_{\text{b}})^2 = \frac{1}{2}k\left(\frac{2mg}{k}\right)^2 = \frac{1}{2}k\left(\frac{4m^2g^2}{k^2}\right)$$

$$E_{\text{e b}} = \frac{2m^2g^2}{k}$$

$$E_{\text{e}} = \frac{1}{2}k(\Delta x)^2 = \frac{1}{2}k\left(\frac{mg}{k}\right)^2 = \frac{1}{2}k\left(\frac{m^2g^2}{k^2}\right)$$

$$E_{\text{e}} = \frac{m^2g^2}{2k}$$

$$\frac{E_{\text{e b}}}{E_{\text{e}}} = \frac{\frac{2m^2g^2}{k}}{\frac{m^2g^2}{2k}}$$

$$\frac{E_{\text{e b}}}{E_{\text{e}}} = \frac{4}{1}$$

Statement: The ratio of the elastic potential energy, $E_{\text{eb}} : E_{\text{e}}$ is 4:1.

2. Given: $\vec{F}_x = 220\text{ N}; \Delta x = 0.14\text{ m}$

Required: E_{e}

Analysis: $\vec{F}_x = -k\Delta\vec{x} = k\Delta x$; $E_{\text{e}} = \frac{1}{2}k(\Delta x)^2$

Solution: $\vec{F}_x = k\Delta x$

$$k = \frac{\vec{F}_x}{\Delta x} = \frac{220\text{ N}}{0.14\text{ m}}$$

$k = 1570$ N/m (one extra digit carried)

$$E_{\text{e}} = \frac{1}{2}k(\Delta x)^2 = \frac{1}{2}(1570\text{ N/m})(0.14\text{ m})^2$$

$$E_{\text{e}} = 15\text{ J}$$

Statement: The elastic potential energy of the toy is 15 J.

Tutorial 3 Practice, page 199

1. Given: $m = 105$ kg; $k = 8.1 \times 10^3$ N / m

Required: *f*, *T*

Analysis: Use the equations for simple harmonic motion period and frequency:

$T = 2\pi\sqrt{\frac{m}{k}}$ and $f = \frac{1}{T}$

Solution: $$\begin{aligned} T &= 2\pi\sqrt{\frac{m}{k}} \\ &= 2\pi\sqrt{\frac{105 \text{ kg}}{7.6 \times 10^3 \text{ N/m}}} \\ T &= 0.74 \text{ s} \end{aligned}$$

$$\begin{aligned} f &= \frac{1}{T} \\ &= \frac{1}{0.74 \text{ s}} \\ f &= 1.4 \text{ Hz} \end{aligned}$$

Statement: The period of the vibrations is 0.74 s, and the frequency is 1.4 Hz.

2. The frequency of oscillations will change if passengers are added to the car because when the mass increases, the period increases. This happens because mass is in the numerator of the equation for the period. If the period increases, the frequency decreases, because frequency is the reciprocal of period.

Section 4.6 Questions, page 200

1. Spring A is more difficult to stretch because it has a greater spring constant.

2. Given: $\vec{F} = 5$ N; $\Delta x = 10$ mm $= 0.01$ m

Required: *k*

Analysis: The spring force opposes the applied force, so $\vec{F}_x = -5$ N . Rearrange the formula $\vec{F}_x = -k\Delta x$ to solve for *k*.

Solution: $$\begin{aligned} \vec{F}_x &= -k\Delta x \\ k &= -\frac{\vec{F}_x}{\Delta x} \\ &= -\frac{(-5 \text{ N})}{0.01 \text{ m}} \\ k &= 500 \text{ N/m} \end{aligned}$$

Statement: The spring constant is 500 N/m.

3. The elastic potential energy stored in a spring is the same whether it is stretched by 1.5 cm or compressed by 1.5 cm. The spring constant is exactly the same whether the spring is stretched or compressed, so the elastic potential energy must also be the same.

4. (a) Given: $k = 5.5\times10^3\text{ N/m}$; $\Delta x = 2.0\text{ cm} = 0.020\text{ m}$
Required: E_e
Analysis: $E_e = \frac{1}{2}k(\Delta x)^2$
Solution: $E_e = \frac{1}{2}k(\Delta x)^2$

$$= \frac{1}{2}(5.5\times10^3\text{ N/m})(0.020\text{ m})^2$$

$$E_e = 1.1\text{ J}$$

Statement: The elastic potential energy of the spring when it stretches 2.0 cm is 1.1 J.
(b) Given: $k = 5.5\times10^3\text{ N/m}$; $\Delta x = -3.0\text{ cm} = -0.030\text{ m}$
Required: E_e
Analysis: $E_e = \frac{1}{2}k(\Delta x)^2$
Solution: $E_e = \frac{1}{2}k(\Delta x)^2$

$$= \frac{1}{2}(5.5\times10^3\text{ N/m})(-0.030\text{ m})^2$$

$$E_e = 2.5\text{ J}$$

Statement: The elastic potential energy of the spring when it compresses 3.0 cm is 2.5 J.
5. (a) Given: $m = 0.63\text{ kg}; k = 65\text{ N/m}; g = 9.8\text{ m/s}^2$
Required: Δx
Analysis: The force of gravity on the mass points down. The restorative force on the mass points up since the spring is compressed.
$\vec{F}_g = mg\text{ [down]} = -mg\text{ [up]}$; $\vec{F}_x = -k\Delta\vec{x} = k\Delta x\text{ [up]}$
Solution: Since the mass is at rest, $\Sigma\vec{F} = 0$.

$$\Sigma\vec{F} = 0$$

$$k\Delta x - mg = 0$$

$$\Delta x = \frac{mg}{k}$$

$$= \frac{(0.63\text{ kg})(9.8\text{ m/s}^2)}{65\text{ N/m}}$$

$$\Delta x = 0.095\text{ m}$$

Statement: The spring is compressed 0.095 m from its equilibrium position.
(b) Given: $m = 0.63\text{ kg}$; $k = 65\text{ N/m}$; $g = 9.8\text{ m/s}^2$
Required: $\vec{a}$
Analysis: The force of gravity on the mass points down. The spring force on the mass points up because the mass is being compressed downward.
$\vec{F}_g = mg\text{ [down]}$; $\vec{F}_x = -k\Delta\vec{x} = k\Delta x\text{ [up]}$

$\vec{F}_{\text{net}} = \vec{F}_{\text{g}} + \vec{F}_x$; $\vec{F}_{\text{net}} = m\vec{a}$

Solution:

$$\begin{aligned}\vec{F}_{\text{net}} &= \vec{F}_{\text{g}} + \vec{F}_x \\ &= mg\ \text{[down]} + k\Delta x\ \text{[up]} \\ &= (0.63\ \text{kg})(9.8\ \text{m/s}^2)\ \text{[down]} + (65\ \text{N/m})(0.041\ \text{cm})\ \text{[up]} \\ \vec{F}_{\text{net}} &= 3.5\ \text{N [down]}\end{aligned}$$

$$\begin{aligned}\vec{F}_{\text{net}} &= m\vec{a} \\ \vec{a} &= \frac{\vec{F}_{\text{net}}}{m} \\ &= \frac{3.5\ \text{N [down]}}{0.63\ \text{kg}} \\ \vec{a} &= 5.6\ \text{m/s}^2\ \text{[down]}\end{aligned}$$

Statement: The acceleration of the mass after it falls 4.1 cm is 5.6 m/s^2 [down].

6. Given: $m = 5.2$ kg; $T = 1.2$ s

Required: k

Analysis: $T = 2\pi\sqrt{\frac{m}{k}}$

Solution:

$$\begin{aligned}T &= 2\pi\sqrt{\frac{m}{k}} \\ \frac{T}{2\pi} &= \sqrt{\frac{m}{k}} \\ \left(\frac{T}{2\pi}\right)^2 &= \frac{m}{k} \\ k &= \frac{m}{\left(\frac{T}{2\pi}\right)^2} \\ &= \frac{5.2\ \text{kg}}{\left(\frac{1.2\ \text{s}}{2\pi}\right)^2} \\ k &= 140\ \text{N/m}\end{aligned}$$

Statement: The spring constant is 140 N/m.

7. Given: $k = 1.5 \times 10^3\ \text{N / m}$; $E_{\text{e}} = 80.0$ J

Required: Δx

Analysis: $E_{\text{e}} = \frac{1}{2}k(\Delta x)^2$

Solution: $E_e = \frac{1}{2}k(\Delta x)^2$

$$\frac{2E_e}{k} = (\Delta x)^2$$

$$\Delta x = \sqrt{\frac{2E_e}{k}}$$

$$= \sqrt{\frac{2(80.0\ \text{J})}{1.5 \times 10^3\ \text{N/m}}}$$

$$\Delta x = 0.33\ \text{m}$$

Statement: The spring should be stretched 0.33 m to store 80.0 J of energy.

8. Given: $\Delta x = 15\ \text{mm} = 0.015\ \text{m};\ k = 400.0\ \text{N / m}$

Required: W

Analysis: The work done is equal to the elastic potential energy:

$$W = E_e;\ E_e = \frac{1}{2}k(\Delta x)^2$$

Solution: $E_e = \frac{1}{2}k(\Delta x)^2$

$$= \frac{1}{2}(400.0\ \text{N/m})(0.015\ \text{m})^2$$

$$E_e = 0.045\ \text{J}$$

Statement: The work done by the spring force acting on the spring is 4.5×10^{-2} J.

9. Given: $E_e = 7.50\ \text{J};\ m = 0.20\ \text{kg};\ k = 240\ \text{N / m}$

Required: f; Δx

Analysis: $T = 2\pi\sqrt{\frac{m}{k}};\ f = \frac{1}{T};\ E_e = \frac{1}{2}k(\Delta x)^2$

Solution: $T = 2\pi\sqrt{\frac{m}{k}}$

$$= 2\pi\sqrt{\frac{0.20\ \text{kg}}{240\ \text{N/m}}}$$

$$T = 0.181\ \text{s (one extra digit carried)}$$

$$f = \frac{1}{T}$$

$$= \frac{1}{0.181\ \text{s}}$$

$$f = 5.5\ \text{Hz}$$

$$E_e = \frac{1}{2}k(\Delta x)^2$$
$$\frac{2E_e}{k} = (\Delta x)^2$$
$$\Delta x = \sqrt{\frac{2E_e}{k}}$$
$$= \sqrt{\frac{2(7.50\text{ J})}{240\text{ N/m}}}$$
$$\Delta x = 0.25\text{ m}$$

Statement: The frequency of oscillation is 5.5 Hz and the amplitude of oscillation is 0.25 m.

10. Given: $m = 5.5 \times 10^2$ kg ; six cycles in 4.4 s

Required: k

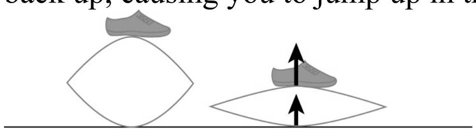
Analysis: Divide 4.4 s by 6 to get T. Use the formula for the period, $T = 2\pi\sqrt{\frac{m}{k}}$, to calculate k.

Solution: $T = \frac{4.4\text{ s}}{6}$

$$T = 2\pi\sqrt{\frac{m}{k}}$$
$$\frac{T}{2\pi} = \sqrt{\frac{m}{k}}$$
$$\frac{T^2}{4\pi^2} = \frac{m}{k}$$
$$k = \frac{4\pi^2 m}{T^2}$$
$$= \frac{4\pi^2(5.5\times 10^2\text{ kg})}{\left(\frac{4.4\text{ s}}{6}\right)^2}$$
$$k = 4.0\times 10^4\text{ N/m}$$

Statement: The spring constant of either spring is 4.0×10^4 N/m.

11. Answers may vary. Sample answer: Pyon pyon shoes strap onto the outside of your regular shoes. They are made of two curved springy pieces of material joined together in a shape similar to that of a football. When you jump, the springy material increases the height you can attain. When your mass presses down on the springs, the springs press back up, causing you to jump up in the air much higher than normal.



Section 4.7: Springs and Conservation of Energy

Tutorial 1 Practice, page 205

1. If the ramp is not frictionless, some of the kinetic energy of the block will be transformed to thermal energy by friction as the block slides down the ramp. Thus, the block will have less kinetic energy to compress the spring, and the amount of compression will be less.

2. Given: $m = 3.5$ kg; $\Delta y = 2.7$ m; $\Delta x = 26$ cm $= 0.26$ m

Required: k

Analysis: $E_\text{g} = mg\Delta y$; $E_\text{e} = \frac{1}{2}k(\Delta x)^2$

Since energy is conserved, the change in potential energy of the mass must equal the change in elastic potential energy when the spring is compressed.

Solution: If we choose the bottom of the ramp to be the $y = 0$ reference point, the mass will have no gravitational potential energy at the bottom of the ramp. The initial gravitational potential energy has transformed into kinetic energy. When the spring is fully compressed, the kinetic energy has transformed into elastic potential energy. Therefore, the spring's initial gravitational potential energy must equal its final elastic potential energy.

$$\begin{aligned} E_\text{e} &= E_\text{g} \\ \frac{1}{2}k(\Delta x)^2 &= mg\Delta y \\ k &= \frac{2mg\Delta y}{(\Delta x)^2} \\ &= \frac{2(3.5\text{ kg})(9.8\text{ m/s}^2)(2.7\text{ m})}{(0.26\text{ m})^2} \\ k &= 2700\text{ N/m} \end{aligned}$$

Statement: The spring constant is 2.7×10^3 N/m.

3. Given: $m = 43$ kg; $k = 3.7$ kN / m $= 3700$ N / m; $\Delta x = 37$ cm $= 0.37$ m

Required: Δy

Analysis: $\Delta E_\text{g} = mg\Delta y$; $E_\text{e} = \frac{1}{2}k(\Delta x)^2$

Solution: Choose the lowest point of the bounce as the $y = 0$ reference point. At the maximum height, Δy, all of the elastic potential energy has converted to gravitational potential energy.

$$\Delta E_{\mathrm{g}} = E_{\mathrm{e}}$$

$$mg\Delta y = \frac{1}{2}k(\Delta x)^2$$

$$\Delta y = \frac{k(\Delta x)^2}{2mg}$$

$$= \frac{(3700\ \text{N/m})(0.37\ \text{m})^2}{2(43\ \text{kg})(9.8\ \text{m/s}^2)}$$

$$= \frac{\left(3700\ \cancel{\text{kg}} \cdot \frac{\cancel{\text{m}}}{\cancel{\text{s}^2}}\right)(0.37\ \text{m})}{2(43\ \cancel{\text{kg}})(9.8\ \cancel{\text{m/s}^2})}$$

$$\Delta y = 0.60\ \text{m}$$

Statement: The maximum height he reaches on the following jump is 0.60 m above the compressed point.

4. Given: $m = 0.35$ kg; $h = 2.6$ m; $\Delta x = 0.14$

Required: k

Analysis: $E_{\mathrm{g}} = mg\Delta y$; $E_{\mathrm{e}} = \frac{1}{2}k(\Delta x)^2$

The initial gravitational potential energy of the branch is equal to the final elastic potential energy of the trampoline at its lowest point.

Solution: Choose the lowest point of the trampoline as the $y = 0$ reference point. At this point, all of the gravitational potential energy has transformed to elastic potential energy. Since the lowest point represents $y = 0$, the change in y is the height above the trampoline surface, 2.6 m, plus the maximum compression of the trampoline, 0.14 m. Therefore, $\Delta y = 2.6\ \text{m} + 0.14\ \text{m} = 2.74\ \text{m}$.

$$E_{\mathrm{e}} = E_{\mathrm{g}}$$

$$\frac{1}{2}k(\Delta x)^2 = mg\Delta y$$

$$k = \frac{2mg\Delta y}{(\Delta x)^2}$$

$$= \frac{2(0.35\ \text{kg})(9.8\ \text{m/s}^2)(2.74\ \text{m})}{(0.14\ \text{m})^2}$$

$$k = 960\ \text{N/m}$$

Statement: The spring constant is 960 N/m.

5. (a) Given: m = 4.0 kg; Δy = 0.308 m
Required: v

Analysis: $E_g = mg\Delta y$; $E_k = \frac{1}{2}mv^2$

When the mass is doubled at the top of the ramp, it is at rest, so it has only gravitational potential energy. At the bottom of the ramp, on the horizontal segment, all of the gravitational potential energy has transformed to kinetic energy.
Solution: Let the bottom of the ramp be the $y = 0$ reference point. Therefore, the gravitational potential energy at the top of the ramp is equal to the kinetic energy along the horizontal part of the ramp.

$$E_k = E_g$$

$$\frac{1}{2}\cancel{m}v^2 = \cancel{m}g\Delta y$$

$$v = \sqrt{2g\Delta y}$$

$$= \sqrt{2(9.8\text{ m/s}^2)(0.308\text{ m})}$$

$$v = 2.5\text{ m/s}$$

Statement: The speed of the block as it returns along the horizontal surface is 2.5 m/s.
(b) No, the block does not have the same kinetic energy as before along the horizontal surface. The kinetic energy is equal to the gravitational potential energy, which is greater than before.
(c) Given: $m_d = 2m$

Required: Δx_d

Analysis: The elastic potential energy of the spring at its greatest compression is equal to the gravitational potential energy at the top of the ramp.
Solution: Block in Sample Problem 3: Block with Mass Doubled

$$mg\Delta y = \frac{1}{2}k(\Delta x)^2$$

$$\Delta x = \sqrt{\frac{2mg\Delta y}{k}}$$

$$m_{\mathrm{d}}g\Delta y = \frac{1}{2}k(\Delta x_{\mathrm{d}})^2$$

$$2mg\Delta y = \frac{1}{2}k(\Delta x_{\mathrm{d}})^2$$

$$\begin{aligned}
\Delta x_{\mathrm{d}} &= \sqrt{\frac{4mg\Delta y}{k}} \\
&= \sqrt{\frac{2(2mg\Delta y)}{k}} \\
&= \sqrt{2}\sqrt{\frac{2mg\Delta y}{k}} \\
\Delta x_{\mathrm{d}} &= \sqrt{2}(\Delta x)
\end{aligned}$$

Since the gravitational potential energy is doubled, the compression increases, but it is not doubled. It increases by a factor of $\sqrt{2}$. So the new value of Δx is $\sqrt{2}(0.22\text{ m})$ or 0.31 m.

Statement: The new value of Δx is 31 cm.

(d) Given: $\mu_{\mathrm{k}} = 0.15$; $\Delta y = 0.308$ m; $\Delta d = 0.62$ m; $k = 250$ N / m

Required: Δx

Analysis: $\vec{F}_{\mathrm{f}} = \mu\vec{F}_{\mathrm{N}} = \mu mg$; $W_{\mathrm{f}} = F_{\mathrm{f}}\Delta d$; $E_{\mathrm{k}} = E_{\mathrm{g}} - W_{\mathrm{f}}$; $E_{\mathrm{e}} = E_{\mathrm{k}}$

Solution: Let the bottom of the ramp be the $y = 0$ reference point.

$$\begin{aligned}
\vec{F}_{\mathrm{f}} &= \mu mg \\
&= 0.15(4.0\text{ kg})(9.8\text{ m/s}^2) \\
\vec{F}_{\mathrm{f}} &= 5.88\text{ N (one extra digit carried)}
\end{aligned}$$

$$\begin{aligned}
W_{\mathrm{f}} &= F_{\mathrm{f}}\Delta d \\
&= (5.88\text{ N})(0.62\text{ m}) \\
W_{\mathrm{f}} &= 3.65\text{ J (one extra digit carried)}
\end{aligned}$$

$$\begin{aligned}
E_{\mathrm{k}} &= E_{\mathrm{g}} - W_{\mathrm{f}} \\
&= mg\Delta y - W_{\mathrm{f}} \\
&= (4.0\text{ kg})(9.8\text{ m/s}^2)(0.308\text{ m}) - 3.65\text{ J} \\
E_{\mathrm{k}} &= 8.42\text{ J (one extra digit carried)}
\end{aligned}$$

$$E_e = E_k$$

$$\frac{1}{2}k(\Delta x)^2 = 8.42 \text{ J}$$

$$\Delta x = \sqrt{\frac{2(8.42 \text{ J})}{k}}$$

$$= \sqrt{\frac{2(8.42 \text{ }\cancel{\text{N}} \cdot \text{m})}{250 \text{ }\cancel{\text{N}}/\text{m}}}$$

$$\Delta x = 0.26 \text{ m}$$

Statement: The new value of the compression is 0.26 m.

Research This: Perpetual Motion Machines, page 206

Answers may vary. Sample answers:

A. I chose a metronome. A spring is wound tight, and as it unwinds, the elastic potential energy is converted to kinetic energy, forcing the metronome wand to swing. As the wand swings to its highest point, kinetic energy transforms to potential energy. As the wand swings down again, the gravitational potential energy converts back to kinetic energy. The spring contributes kinetic energy at the bottom of each swing, until the spring is fully unwound. Eventually the metronome stops due to air resistance.

B. The design of the machine has been improved over time by creating quartz metronomes, electronic metronomes, computer metronomes, and even metronome apps for smart phones.

C. The improvements have been the results of all three developments: new materials; new technology; and new scientific discoveries.

Section 4.7 Questions, page 208

1. The total mechanical energy of the system increases. Energy has been added by the person outside the system of the mass and spring.

2. Given: $k = 520 \text{ N / m}$; $m = 4.5 \text{ kg}$; $\Delta x = 0.35 \text{ m}$

Required: v

Analysis: When the spring is compressed, but the mass is at rest, the mass has only elastic potential energy. As the spring is released, the elastic potential energy transforms to kinetic energy. When the mass is no longer touching the spring, all of the energy is kinetic.

$E_e = \frac{1}{2}k(\Delta x)^2$; $E_k = \frac{1}{2}mv^2$

Solution: The kinetic energy when the mass leaves the spring is equal to the elastic potential energy when the spring is at its maximum compression.

$$E_k = E_e$$

$$\frac{1}{2}mv^2 = \frac{1}{2}k(\Delta x)^2$$

$$v = \sqrt{\frac{k(\Delta x)^2}{m}}$$

$$= \sqrt{\frac{(520\ \text{N/m})(0.35\ \text{m})^2}{4.5\ \text{kg}}}$$

$$v = 3.8\ \text{m/s}$$

Statement: The speed of the mass when it leaves the spring is 3.8 m/s [away from the spring].

3. (a) Given: $k = 5.2 \times 10^2\ \text{N/m}; \Delta x = 5.2\ \text{cm} = 0.052\ \text{m}$

Required: E_e

Analysis: $E_e = \frac{1}{2}k(\Delta x)^2$

Solution: $E_e = \frac{1}{2}k(\Delta x)^2$

$$= \frac{1}{2}(5.2 \times 10^2\ \text{N/m})(0.052\ \text{m})^2$$

$$E_e = 0.703\ \text{J (one extra digit carried)}$$

Statement: The elastic potential energy of the compressed spring is 0.70 J.

(b) Given: E_e = 0.703 J; m = 8.4 g = 0.0084 kg; Δx = 5.2 cm = 0.052 m

Required: v

Analysis: $E_g = mg\Delta y$; $E_k = \frac{1}{2}mv^2$

All of the elastic potential energy of the compressed spring has transformed to gravitational potential energy and kinetic energy as the pilot ejects.

Solution: The kinetic energy of the pilot as it ejects and the additional gravitational potential energy is equal to the elastic potential energy of the compressed spring.

$$E_g + E_k = E_e$$

$$mg\Delta x + \frac{1}{2}mv^2 = E_e$$

$$v = \sqrt{\frac{2E_e - 2mg\Delta x}{m}}$$

$$= \sqrt{\frac{2(0.703\ \text{J}) - 2(0.084\ \text{kg})(9.8\ \text{m/s}^2)(0.052\ \text{m})}{0.0084\ \text{kg}}}$$

$$= \sqrt{\frac{2(0.660\ 194\ \cancel{\text{kg}} \cdot \text{m}^2/\text{s}^2)}{0.0084\ \cancel{\text{kg}}}}$$

$$v = 13\ \text{m/s}$$

Statement: The speed of the pilot as it ejects upward from the airplane is 13 m/s above the launch point of the compressed spring.

(c) Given: $m = 8.4\text{ g} = 0.0084\text{ kg};\ k = 5.2\times 10^2\text{ N/m};\ \Delta x = 5.2\text{ cm} = 0.052\text{ m}$

Required: Δy

Analysis: Let the point where the pilot ejects be the $y = 0$ reference point. All of the elastic potential energy of the compressed spring has transformed to gravitational potential energy at the pilot's maximum height.

$E_e = \frac{1}{2}k(\Delta x)^2;\ E_g = mg\Delta y$

Solution: The elastic potential energy of the compressed spring is equal to the gravitational potential energy at the pilot's maximum height.

$$\begin{aligned} E_g &= E_e \\ mg\Delta y &= \frac{1}{2}k(\Delta x)^2 \\ \Delta y &= \frac{k(\Delta x)^2}{2mg} \\ &= \frac{\left(5.2\times 10^2\ \cancel{\text{kg}}\cdot\frac{\cancel{\text{m}}}{\cancel{\text{s}^2}}\right)(0.052\text{ m})^{\cancel{2}}}{2(0.0084\ \cancel{\text{kg}})(9.8\ \cancel{\text{m/s}^2})} \\ \Delta y &= 8.5\text{ m} \end{aligned}$$

Statement: The maximum height that the pilot will reach is 8.5 m above the launch point of the compressed spring.

4. Given: $k = 1.2\times 10^2\text{ N/m};\ m = 82\text{ g} = 0.082\text{ kg};\ \Delta y = 3.4\text{ cm} = 0.034\text{ m}$

Required: Δx

Analysis: The elastic potential energy of the spring transforms to kinetic energy and then to gravitational potential energy as it comes to rest at the top of the ramp.

$E_e = \frac{1}{2}k(\Delta x)^2;\ E_g = mg\Delta y$

Solution: Let the bottom of the ramp represent the $y = 0$ reference point. The elastic potential energy of the spring is equal to the gravitational potential energy at the top of the ramp.

$$\begin{aligned} E_e &= E_g \\ \frac{1}{2}k(\Delta x)^2 &= mg\Delta y \\ \Delta x &= \sqrt{\frac{2mg\Delta y}{k}} \\ &= \sqrt{\frac{2(0.082\text{ kg})(9.8\text{ m/s}^2)(0.034\text{ m})}{1.2\times 10^2\text{ N/m}}} \\ \Delta x &= 0.021\text{ m} \end{aligned}$$

Statement: The distance of the spring's compression is 0.021 m.

5. Given: $m = 75$ kg; $k = 6.5$ N / m

Required: v

Analysis: Let the $y = 0$ reference point be 19 m below the platform. Since the unstretched bungee cord is 11 m long, and the cord is stretched 19 m below the platform, $\Delta x = 19 \text{ m} - 11 \text{ m} = 8 \text{ m}$. The gravitational potential energy is transformed to elastic potential energy and kinetic energy at this point.

$E_g = mg\Delta y$; $E_k = \frac{1}{2}mv^2$; $E_e = \frac{1}{2}k(\Delta x)^2$

Solution: The gravitational potential energy at the platform is equal to the sum of the kinetic energy and the elastic potential energy 19 m below the platform.

$$E_g = E_k + E_e$$

$$mg\Delta y = \frac{1}{2}mv^2 + \frac{1}{2}k(\Delta x)^2$$

$$\frac{1}{2}mv^2 = mg\Delta y - \frac{1}{2}k(\Delta x)^2$$

$$v = \sqrt{\frac{2}{m}\left(mg\Delta y - \frac{1}{2}k(\Delta x)^2\right)}$$

$$= \sqrt{\frac{2}{(75 \text{ kg})}\left[(75 \text{ kg})(9.8 \text{ m/s}^2)(19 \text{ m}) - \frac{1}{2}(65.5 \text{ N/m})(8.0 \text{ m})^2\right]}$$

$$v = 18 \text{ m/s}$$

Statement: The speed of the bungee jumper at 19 m below the bridge is 18 m/s.

6. Given: $k = 5.0$ N / m; $m = 0.25$ kg; $\Delta x = 14 \text{ cm} = 0.14 \text{ m}$

Required: h_{max}; v_{max}; a_{max}

Analysis: Let the $y = 0$ reference point be the rest position of the spring. In simple harmonic motion, the maximum height is the opposite of the lowest point.

The maximum velocity occurs as the box passes through the rest position. At this point, there is only kinetic energy. Thus, $\frac{1}{2}k(\Delta x)^2 = \frac{1}{2}mv^2$.

The maximum acceleration occurs at the maximum height. At this point, the spring force is equal to the applied force, so $k\Delta x = ma$.

Solution: The maximum height is 14 cm [above rest position].

For the maximum velocity,

$$\frac{1}{2}k(\Delta x)^2 = \frac{1}{2}mv^2$$

$$v = \sqrt{\frac{k(\Delta x)^2}{m}}$$

$$= \sqrt{\frac{(5.0 \text{ N/m})(0.14 \text{ m})^2}{0.25 \text{ kg}}}$$

$$v = 0.63 \text{ m/s}$$

For the maximum acceleration,

$$k\Delta x = ma$$

$$\begin{aligned} a &= \frac{k\Delta x}{m} \\ &= \frac{(5.0 \text{ N/m})(0.14 \text{ m})}{0.25 \text{ kg}} \\ a &= 2.8 \text{ m/s}^2 \end{aligned}$$

Statement: The maximum height is 14 cm [above rest position]. The maximum speed is 0.63 m/s. The maximum acceleration is 2.8 m/s^2 [toward rest position].

7. Given: $m = 0.22$ kg; $k = 280$ N / m; $\Delta x = 11$ cm $= 0.11$ m

Required: h

Analysis: Let the y = 0 reference point be the fully compressed position of the spring. When the block is dropped, all the energy is in the form of gravitational potential energy. At the full compression of the spring, all the energy has transformed to elastic potential energy.

$$E_\text{g} = mg\Delta y\text{; } E_\text{e} = \frac{1}{2}k(\Delta x)^2$$

Solution: The gravitational potential energy as the block is dropped is equal to the elastic potential energy at full compression of the spring.

$$\begin{aligned} E_\text{g} &= E_\text{e} \\ mg\Delta y &= \frac{1}{2}k(\Delta x)^2 \\ \Delta y &= \frac{k(\Delta x)^2}{2mg} \\ &= \frac{(280 \text{ N/m})(0.11 \text{ m})^2}{2(0.22 \text{ kg})(9.8 \text{ m/s}^2)} \\ \Delta y &= 0.79 \text{ m} \end{aligned}$$

This height represents the total distance from where the block was dropped to the lowest point of the spring, so subtract the maximum compression of the spring, 0.11 m, to get the height from which the block was dropped.

$$\begin{aligned} h &= \Delta y - \Delta x \\ &= 0.79 \text{ m} - 0.11 \text{ m} \\ h &= 0.68 \text{ m} \end{aligned}$$

Statement: The block was dropped from a height of 0.68 m.

8. (a) Given: $m = 1.0$ kg; $v = 1.0$ m / s; $k = 1000.0$ N / m

Required: Δx

Analysis: The kinetic energy of the block transforms fully to elastic potential energy at the maximum compression of the spring, because the block is at rest at this point.

$$E_\text{k} = \frac{1}{2}mv^2\text{; } E_\text{e} = \frac{1}{2}k(\Delta x)^2$$

Solution: The kinetic energy of the block before it hits the spring is equal to the elastic potential energy of the spring at its maximum compression.

$$E_e = E_k$$
$$\frac{1}{2}k(\Delta x)^2 = \frac{1}{2}mv^2$$
$$\Delta x = \sqrt{\frac{mv^2}{k}}$$
$$= \sqrt{\frac{(1.0\text{ kg})(1.0\text{ m/s})^2}{1000.0\text{ N/m}}}$$
$$\Delta x = 0.032\text{ m}$$

Statement: The maximum compression of the spring is 0.032 m.

(b) The block will travel to the maximum compression of the spring, 0.032 m before coming to rest.

9. (a) Given: $m = 6.0$ kg; $v = 3.0$ m / s; $k = 1250$ N / m

Required: Δx

Analysis: The kinetic energy of the block transforms fully to elastic potential energy at the maximum compression of the spring, because the block is at rest at this point.

$E_k = \frac{1}{2}mv^2$; $E_e = \frac{1}{2}k(\Delta x)^2$

Solution:
$$E_e = E_k$$
$$\frac{1}{2}k(\Delta x)^2 = \frac{1}{2}mv^2$$
$$\Delta x = \sqrt{\frac{mv^2}{k}}$$
$$= \sqrt{\frac{(6.0\text{ kg})(3.0\text{ m/s})^2}{1250\text{ N/m}}}$$
$$\Delta x = 0.208\text{ m (one extra digit carried)}$$

Statement: The maximum distance the spring is compressed is 0.21 m.

(b) Given: $\Delta x = 14$ cm $= 0.14$ m; $m = 6.0$ kg; $v_i = 3.0$ m / s; $k = 1250$ N / m

Required: v_f; a

Analysis: Some of the kinetic energy of the block transforms to elastic potential energy as the spring compresses. So the initial kinetic energy transforms to final kinetic energy and elastic potential energy.

$E_k = \frac{1}{2}mv^2$; $E_e = \frac{1}{2}k(\Delta x)^2$

For the acceleration, the only force acting on the block is the spring force, which is equal to $k\Delta x$ away from the spring.

$\vec{F} = m\vec{a}$

Solution: The initial kinetic energy is equal to the sum of the final kinetic energy and the elastic potential energy.

$$E_{\text{k i}} = E_{\text{k f}} + E_{\text{e}}$$

$$\frac{1}{2}mv_{\text{i}}^2 = \frac{1}{2}mv_{\text{f}}^2 + \frac{1}{2}k(\Delta x)^2$$

$$mv_{\text{f}}^2 = mv_{\text{i}}^2 - k(\Delta x)^2$$

$$v_{\text{f}} = \sqrt{\frac{mv_{\text{i}}^2 - k(\Delta x)^2}{m}}$$

$$= \sqrt{\frac{(6.0\text{ kg})(3.0\text{ m/s})^2 - (1250\text{ N/m})(0.14\text{ m})^2}{6.0\text{ kg}}}$$

$$v_{\text{f}} = 2.2\text{ m/s}$$

For the acceleration:

$$\vec{F} = m\vec{a}$$

$$k\Delta x\text{ [away from the spring]} = m\vec{a}$$

$$\vec{a} = \frac{k\Delta x\text{ [away from the spring]}}{m}$$

$$= \frac{(1250\text{ N/m})(0.14\text{ m})\text{ [away from the spring]}}{6.0\text{ kg}}$$

$$\vec{a} = 29\text{ m/s}^2\text{ [away from the spring]}$$

Statement: When the spring is compressed, the speed of the block is 2.2 m/s, and the acceleration is 29 m/s^2 [away from the spring].

10. Given: $k = 440\text{ N / m};\ \Delta x = 45\text{ cm} = 0.45\text{ m};\ m = 57\text{ g} = 0.057\text{ kg};\ d_y = 1.2\text{ m}$

Required: d_x, the horizontal distance

Analysis: Find the speed of the ball as it leaves the machine, and then use projectile motion equations to determine when it will hit the ground.

Let the $y = 0$ reference point be the height at which the ball leaves the machine. Thus, there is no gravitational potential energy at this point. The elastic potential energy of the spring when it is at its maximum compression transforms to kinetic energy when the ball leaves the machine.

$$E_{\text{k}} = \frac{1}{2}mv^2;\ E_{\text{e}} = \frac{1}{2}k(\Delta x)^2$$

Once v has been determined, use projectile motion equations to determine the distance, d_x, the ball travels. Since the ball is projected horizontally, the initial launch angle is 0°, and there is no vertical component of velocity: $v_y = 0$. Use the equation $d_y = v_y t - \frac{1}{2}gt^2$ to determine t, and then use the equation $d_x = v_x t$ to determine d_x.

Solution: The elastic potential energy of the spring when it is at its maximum compression is equal to the kinetic energy when the ball leaves the machine.

$$E_\text{k} = E_\text{e}$$

$$\frac{1}{2} mv^2 = \frac{1}{2} k(\Delta x)^2$$

$$v = \sqrt{\frac{k(\Delta x)^2}{m}}$$

$$= \sqrt{\frac{(440\text{ N/m})(0.45\text{ cm})^2}{0.057\text{ g}}}$$

$$v = 39.5\text{ m/s (one extra digit carried)}$$

Now, use the projectile motion equations.

$$d_y = v_y t - \frac{1}{2} g t^2$$

$$d_y = -\frac{1}{2} g t^2$$

$$t = \sqrt{\frac{-2d_y}{g}}$$

$$= \sqrt{\frac{-2(-1.2\text{ m})}{9.8\text{ m/s}^2}}$$

$$t = 0.495\text{ s (one extra digit carried)}$$

$$d_x = v_x t$$

$$= (39.5\text{ m/s})(0.495\text{ s})$$

$$d_x = 2.0 \times 10^1\text{ m}$$

Statement: The horizontal distance that the tennis ball can travel before hitting the ground is 2.0×10^1 m.

Chapter 4 Review, pages 214–219

Knowledge

1. (b)
2. (c)
3. (b)
4. (c)
5. (a)
6. (d)
7. (b)
8. True
9. True
10. False. The gravitational potential energy of an object 5 m above the ground in Ontario is *greater than that of* an identical object 5 m above the ground on the Moon.
11. False. The joule (J) is the SI unit for *two* quantities: work and energy.
12. True
13. True
14. False. In an oscillating spring, the elastic potential energy when the spring is completely compressed is equal to the *elastic potential* energy when the spring is fully extended.

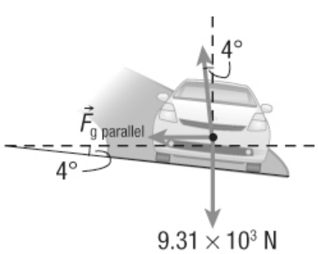
Understanding

15. (a) No work is being performed by the car, because it is not moving.
(b) No work is being performed by the snow bank because it is not moving.
16. Answers may vary. Sample answer: Two examples in which a non-zero force acts on an object, but the total work done by that force is zero, are a person who is pushing an appliance but is unable to move it; a mass is hanging from a string on a frictionless pulley attached to a block on a table, but the block is not moving.
17. No, it is not possible to do work on an object if the object does not move. An object has to move a non-zero distance Δd for work to be done.
18. (a) Given: $\vec{F}_g = 9.31\times10^3$ N [straight downward]; $\theta = 4°$

Required: component of gravitational force parallel to the car's motion, $\vec{F}_{\text{g parallel}}$

Analysis: Determine $\vec{F}_{\text{g parallel}}$ using trigonometry.

Solution:



$$\vec{F}_{\text{g parallel}} = \vec{F}_{\text{g}} \sin\theta$$
$$= (9.31 \times 10^3 \text{ N}) \sin 4°$$
$$\vec{F}_{\text{g parallel}} = 649.4 \text{ N [parallel to the slope] (one extra digit carried)}$$

Statement: The component of the gravitational force that acts parallel to the car's motion is 649 N.

(b) Given: $\vec{F}_{\text{g parallel}} = 649.4$ N [parallel to the slope]; $\Delta d = 30.0$ m

Required: W

Analysis: $W = F\Delta d$

Solution: $W = F\Delta d$
$$= (649.4 \text{ N})(30.0 \text{ m})$$
$$W = 1.95 \times 10^4 \text{ J}$$

Statement: The work done on the car by gravity as the car slides is 1.95×10^4 J.

19. (a) Given: $m = 65$ kg; $\Delta y = 100.0$ m

Required: W

Analysis: $W_{\text{002 on 001}} = -E_{\text{g}}$; $E_{\text{g}} = mg\Delta y$

The work done on 002 by 001 is equal in magnitude but opposite in sign to the work done by gravity, which is equal to the gravitational potential energy.

Solution: $W_{\text{001 on 002}} = -E_{\text{g}}$
$$= -mg\Delta y$$
$$= -(65 \text{ kg})(9.8 \text{ m/s}^2)(100.0 \text{ m})$$
$$W_{\text{001 on 002}} = -6.4 \times 10^4 \text{ J}$$

Statement: The work done by 001 on 002 is -6.4×10^4 J.

(b) Given: $m = 65$ kg; $\Delta y = 100.0$ m

Required: W

Analysis: The work done by gravity is equal to the gravitational potential energy.

$W = E_{\text{g}}$; $E_{\text{g}} = mg\Delta y$

Solution: $W = E_{\text{g}}$
$$= mg\Delta y$$
$$= (65 \text{ kg})(9.8 \text{ m/s}^2)(100.0 \text{ m})$$
$$W = 6.4 \times 10^4 \text{ J}$$

Statement: The work done by gravity on 002 is 6.4×10^4 J.

(c) Given: $m = 65\text{ kg}; v = 2.5\text{ m/s}$

Required: E_k

Analysis: $E_k = \frac{1}{2}mv^2$

Solution: $E_k = \frac{1}{2}mv^2$

$$= \frac{1}{2}(65\text{ kg})(2.5\text{ m/s})^2$$

$$E_k = 2.0 \times 10^2\text{ J}$$

Statement: Spy 002's kinetic energy is 2.0×10^2 J.

(d) Spy 002's gravitational potential energy when she leaves the top of the building is 6.4×10^4 J, and when she reaches the ground, it is 0. Thus, the change in gravitational potential energy is -6.4×10^4 J.

20. Work is the energy required when a force moves an object a certain distance in the same direction as the force.

21. Given: $m = 68\text{ kg};\ v_1 = 5.8\text{ m/s};\ v_2 = 6.9\text{ m/s}$

Required: W

Analysis: $E_k = \frac{1}{2}mv^2$

The work done is equal to the difference in kinetic energy.

Solution: $W = E_{k\,2} - E_{k\,1}$

$$= \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$$

$$= \frac{1}{2}m(v_2^2 - v_1^2)$$

$$= \frac{1}{2}(68\text{ kg})[(6.9\text{ m/s})^2 - (5.8\text{ m/s})^2]$$

$$W = 470\text{ J}$$

Statement: The work that the sprinter does is 4.7×10^2 J.

22. Given: $m = 20.0\text{ kg};\ v_i = 2.0\text{ m/s};\ v_f = 0\text{ m/s}$

Required: W_f

Analysis: The work done by friction is equal to the opposite of the change in kinetic energy, since friction works against the curling stone.

$W = -\Delta E_k;\ E_k = \frac{1}{2}mv^2$

Solution: $W = -\Delta E_{\text{k}}$

$$= -(E_{\text{k}\,2} - E_{\text{k}\,1})$$

$$= -\left(\frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2\right)$$

$$= -\frac{1}{2}mv_2^2$$

$$= -\frac{1}{2}(20.0\text{ kg})(2.0\text{ m/s})^2$$

$$W = -4.0 \times 10^1\text{ J}$$

Statement: The work done on the stone by friction is -4.0×10^1 J.

23. Given: $m = 60.0$ kg; $\Delta y = 10.0$ m

Required: ΔE_{g}

Analysis: Let the $y = 0$ reference point be the surface of the water. The change in gravitational potential energy is the negative of the gravitational potential energy as the diver leaves the diving board.

$E_{\text{g}} = mg\Delta y$

Solution: $\Delta E_{\text{g}} = -E_{\text{g}}$

$$= -mg\Delta y$$

$$= -(60.0\text{ kg})(9.8\text{ m/s}^2)(10.0\text{ m})$$

$$\Delta E_{\text{g}} = -5.9 \times 10^3\text{ J}$$

Statement: The change in gravitational potential energy is -5.9×10^3 J.

24. Given: $\Delta y = 1.2$ m; $\Delta E_{\text{g}} = 5.8$ J

Required: m

Analysis: Let the $y = 0$ reference point be the floor. The change in gravitational potential energy is equal to the gravitational potential energy at the level of the shelf.

$\Delta E_{\text{g}} = E_{\text{g}} = mg\Delta y$

Solution: $\Delta E_{\text{g}} = mg\Delta y$

$$m = \frac{\Delta E_g}{g\Delta y}$$

$$= \frac{5.8\text{ J}}{(9.8\text{ m/s}^2)(1.2\text{ m})}$$

$$m = 0.49\text{ kg}$$

Statement: The mass of the case of cereal is 0.49 kg.

25. Both mass and height are factors in gravitational energy, and mass is proportional to volume. So volume and height of the waterfall are important for hydroelectric power generation.

26. The energy increases, because height is a factor in gravitational potential energy. So when the height increases, the gravitational potential energy increases.

27. Answers may vary. Sample answer:

	Work	Power
Description	the amount of energy required to move an object a certain distance in the direction of an applied force	the amount of work done over a certain amount of time
Units	joules (J)	watts (W)
Example	A worker pushes a crate up a ramp.	A sled dog pulls a sled to win a 100-m race in 10 s.

28. Given: $\Delta y = 1.2\text{ m};\ m = 55\text{ g} = 0.055\text{ kg}$

Required: E_g

Analysis: Let the $y = 0$ reference point be the ground.

$E_g = mg\Delta y$

Solution:

$$\begin{aligned} E_g &= mg\Delta y \\ &= (0.055\text{ kg})(9.8\text{ m/s}^2)(1.2\text{ m}) \\ E_g &= 0.65\text{ J} \end{aligned}$$

Statement: The gravitational potential energy of the egg before it falls is 0.65 J.

29. Given: $t = 1\text{ s};\ m = 5.7\times10^5\text{kg};\ \Delta y = 21\text{ m}$

Required: P

Analysis: $P = \dfrac{W}{t}$; $W = mg\Delta y$

Solution:

$$\begin{aligned} P &= \frac{W}{t} \\ &= \frac{mg\Delta y}{t} \\ &= \frac{(5.7\times10^5\text{ kg})(9.8\text{ m/s}^2)(21\text{ m})}{1\text{ s}} \\ P &= 1.2\times10^8\text{ W} \end{aligned}$$

Statement: The power generated is 1.2×10^8 W.

30. Answers may vary. Sample answer: A skier going down a hill converts gravitational potential energy to kinetic energy and thermal energy (due to friction with the hill).

31. Answers may vary. Sample answer: A short diving board would be stiffer than a longer diving board. When a person is standing on a short board, the vertical displacement of the board from its equilibrium position will be less than the vertical displacement of a longer board with the same person standing on it. If the force exerted on both boards is the same, but the vertical displacement differs, then the smaller the vertical displacement, the greater the spring constant. The spring constant is a constant of proportionality. Material that is stiff will need a greater force to extend or compress it. Therefore, the smaller diving board will have a larger spring constant.

32. Answers may vary. Sample answer: The elastic potential energy of a spring is proportional to the square of the compression or extension distance of the spring. The proportionality constant is *k*. If *k* is large, the spring is stiff. If *k* is small, the spring is loose.

33. Answers may vary. Sample answer: The mass on the spring with the smaller spring constant will oscillate more quickly than the mass on the spring with the larger spring constant.
34. Answers may vary. Sample answer: The oscillation of the larger mass will have a larger amplitude than that of the smaller mass.
35. Answers may vary. Sample answer: The elastic potential energy is proportional to the distance the spring (in this case the bowstring) is stretched. The kinetic energy when the arrow is released is equal to the elastic potential energy. The kinetic energy is proportional to the square of the speed. Thus, the more the bowstring is stretched, the greater the elastic potential energy, the greater the kinetic energy, and the greater the speed.
36. Answers may vary. Sample answer: In the kinetic energy equation, speed is inversely proportional to the square root of mass. Thus, the lighter the mass, the greater the speed.
37. Given: Δx; k; m
Required: v
Analysis: The kinetic energy of the ball as it leaves the spring is equal to the elastic potential energy when the spring is compressed:

$$E_{\text{k}} = E_{\text{e}};\ E_{\text{k}} = \frac{1}{2}mv^2;\ E_{\text{e}} = \frac{1}{2}k(\Delta x)^2$$

Solution:

$$\begin{aligned} E_{\text{k}} &= E_{\text{e}} \\ \frac{1}{2}mv^2 &= \frac{1}{2}k(\Delta x)^2 \\ v^2 &= \frac{k(\Delta x)^2}{m} \\ v &= \sqrt{\frac{k(\Delta x)^2}{m}} \\ v &= \left(\sqrt{\frac{k}{m}}\right)\Delta x \end{aligned}$$

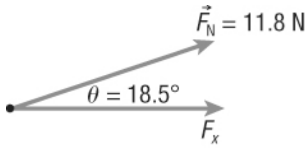
Statement: The ball's initial speed is $v = \left(\sqrt{\frac{k}{m}}\right)\Delta x$.

38. There is no work done on the object by the force. Work requires the object to move in the direction of the applied force.

Analysis and Application

39. Given: $\theta = 18.5°$; $W = 214\ \text{J}$
Required: Δd
Analysis: Draw an FBD of the toboggan. Use trigonometry to determine the applied force in the direction of the movement of the sled.
$W = F\Delta d$

Solution:



$$F_x = F\cos\theta$$

$$W = F_x\Delta d$$

$$W = F\cos\theta(\Delta d)$$

$$\Delta d = \frac{W}{F\cos\theta}$$

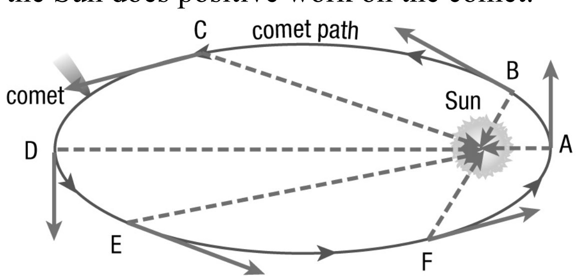
$$= \frac{214\text{ J}}{(11.8\text{ N})\cos 18.5°}$$

$$\Delta d = 19.1\text{ m}$$

Statement: The sled moves 19.1 m.

40. No. You go flying off the merry-go-round in a direction perpendicular to centripetal force. Since the angle is 90°, and $\cos 90° = 0$, the work done is 0.

41. (a) In the diagram, the lettered arrows represent the direction of motion of the comet, and the dashed arrows represent the gravitational force of the Sun. At points A and D, the motion of the comet is perpendicular to the gravitational force of the Sun, so the Sun does no work on the comet. At points B and C, there is a component of the gravitational force of the Sun that is parallel and opposite to the motion of the comet, so the Sun does negative work on the comet. At points E and F, there is a component of the gravitational force of the Sun that is parallel and in the same direction as the motion of the comet, so the Sun does positive work on the comet.

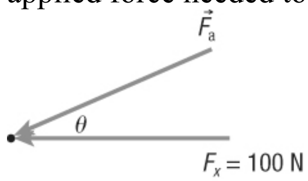


(b) As the comet approaches the Sun, the work done by gravity decreases to zero, in a direction toward the Sun, in the same direction as the comet.

(c) As the comet moves away from the Sun, the work done by gravity increases from zero, but in a direction toward the Sun, opposite the motion of the comet.

42. The units of force are newtons (N), or kilograms times metres per second squared ($kg{\cdot}m/s^2$). The units of distance are metres (m). So the units of work are $\left[\frac{kg \cdot m}{s^2}\right] \cdot [m]$ or $\left[\frac{kg \cdot m^2}{s^2}\right]$. The units of mass are kilograms (kg), and the units of speed are metres per second (m/s). So the units of kinetic energy are $[kg] \cdot \left[\frac{m}{s}\right]^2$ or $\left[\frac{kg \cdot m^2}{s^2}\right]$. The units of work done by a constant force and kinetic energy are the same.

43. We are not given the mass of the motorcycle. A reasonable value for the horizontal applied force needed to move the motorcycle is 100 N. Draw an FBD of the motorcycle.



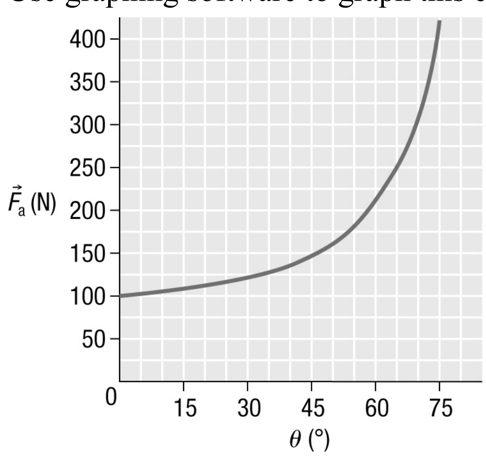
Using trigonometry,

$$\frac{F_x}{F_a} = \cos\theta$$

$$F_a = \frac{F_x}{\cos\theta}$$

$$F_a = \frac{100\ \text{N}}{\cos\theta}$$

Use graphing software to graph this equation.



As the angle increases, the applied force required to move the motorcycle increases.

44. Given: $m_1 = 1000$ kg; $m_2 = 2000$ kg; $m_3 = 3000$ kg; $m_4 = 4000$ kg; $v = 10$ m/s; $\Delta d = 1\ \text{km} = 1000\ \text{m}$

Required: F

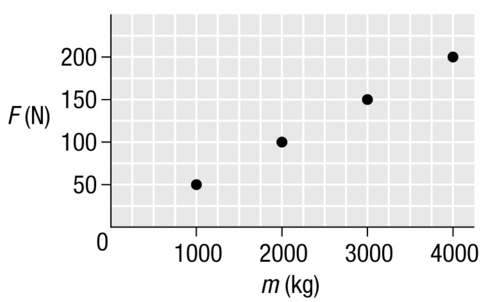
Analysis: The work done by the spacecraft on each load is equal to the kinetic energy of the load.

$W = F\Delta d$; $E_k = \frac{1}{2}mv^2$

Solution: Solve the equation $W = E_k$ for F, and complete a table of values for the four values of m. Then, draw a scatter plot graph of the data.

$$\begin{aligned} W &= E_k \\ F\Delta d &= \frac{1}{2}mv^2 \\ F &= \frac{mv^2}{2\Delta d} \\ &= \frac{m(10 \text{ m/s})^2}{2(1000 \text{ m})} \\ F &= \frac{m}{20} \text{ m/s}^2 \end{aligned}$$

Mass, *m*	**Force, $F = \frac{m}{20}$ m / s²**
1000 kg	50 N
2000 kg	100 N
3000 kg	150 N
4000 kg	200 N



Statement: The force, in newtons, is one twentieth the mass, in kilograms. The force and the mass are linearly related.

45. Given: $m = 2000$ kg; $v_1 = 10$ m / s; $v_2 = 15$ m / s; $v_3 = 20$ m / s; $v_4 = 25$ m / s; $\Delta d = 1$ km $= 1000$ m

Required: F

Analysis: The work done by the spacecraft on each load is equal to the kinetic energy of the load.

$W = F\Delta d$; $E_k = \frac{1}{2}mv^2$

Solution: Solve the equation $W = E_k$ for F, and complete a table of values for the four values of v. Then, draw a scatter plot graph of the data.

$$W = E_k$$

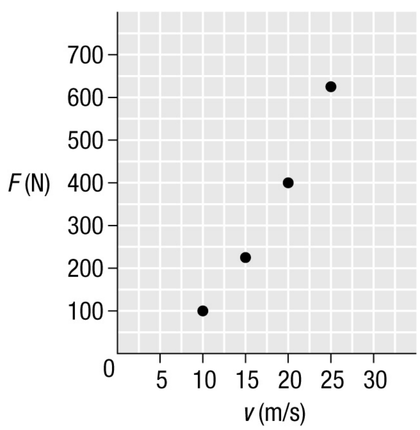
$$F\Delta d = \frac{1}{2}mv^2$$

$$F = \frac{mv^2}{2\Delta d}$$

$$= \frac{(2000\text{ kg})v^2}{2(1000\text{ m})}$$

$$F = v^2\text{ kg/m}$$

Speed, *v*	**Force, $F = v^2$ kg / m**
10 m/s	100 N
15 m/s	225 N
20 m/s	400 N
25 m/s	625 N



Statement: The force, in newtons, is the square of the speed, in metres per second. The force and the speed are quadratically related.

46. (a) The student's velocity is changing, because the direction is changing.

(b) The student's speed is constant. There is no friction, so the initial speed does not change.

(c) The student's kinetic energy is constant, since the student's speed is constant, and kinetic energy is proportional to the square of speed.

47. The value of g decreases as we move farther from Earth's surface.

48. (a) Use the value of g for the Moon instead of that for Earth.

(b) Answers may vary. Sample answer:

You could use a motion sensor to measure the speed an object attains when dropped from a certain height. If the mass is m, the speed is v, and the change in height is Δy, then

$$E_g = E_k$$

$$\cancel{m}g\Delta y = \frac{1}{2}\cancel{m}v^2$$

$$g = \frac{v^2}{2\Delta y}$$

Use the height and the value of v from the motion detector to determine g on the Moon. Do this experiment several times with different height and speed values, and determine the average value of g.

49. (a) Since gravitational potential energy is proportional to height, if the height doubles, the energy doubles. So plant X generates twice as much power as plant Y.

(b) Gravitational potential energy is also proportional to mass, which is proportional to volume, so if the volume doubles, the energy doubles. So plants X and Y now generate the same amount of power.

50. Given: $m = 1.3\text{ kg};\ v_\text{i} = 1.8\text{ m/s};\ v_\text{f} = 0.9\text{ m/s};\ \Delta y = 4.0\text{ m}$

Required: energy lost through air resistance and friction

Analysis: The energy at the top of the chute is a combination of kinetic energy and gravitational potential energy. The energy when the package reaches the floor is all kinetic energy. The difference in energy between that at the top of the chute and that at the floor is the energy lost through air resistance and friction.

$E_\text{g} = mg\Delta y;\ E_\text{k} = \frac{1}{2}mv^2$

Solution: Let E_lost represent the energy lost.

$$\begin{aligned}
E_\text{lost} &= E_\text{g top} + E_\text{k top} - E_\text{k floor} \\
&= mg\Delta y + \frac{1}{2}mv_\text{i}^2 - \frac{1}{2}mv_\text{f}^2 \\
&= m\left(g\Delta y + \frac{1}{2}v_\text{i}^2 - \frac{1}{2}v_\text{f}^2\right) \\
&= (1.3\text{ kg})\left[(9.8\text{ m/s}^2)(4.0\text{ m}) + \frac{1}{2}(1.8\text{ m/s})^2 - \frac{1}{2}(0.9\text{ m/s})^2\right] \\
E_\text{lost} &= 53\text{ J}
\end{aligned}$$

Statement: The energy lost through air resistance and friction is 53 J.

51. Given: $m_1 = m_2;\ \Delta y_2 = \frac{1}{4}\Delta y_1$

Required: comparison of v_1 to v_2

Analysis: The gravitational potential energy when the balloons are dropped is equal to the kinetic energy when they hit the ground.

$E_\text{g} = mg\Delta y;\ E_\text{k} = \frac{1}{2}mv^2$

Solution:

$$\begin{aligned}
E_\text{g 1} &= E_\text{k 1} \\
\cancel{m_1} g\Delta y_1 &= \frac{1}{2}\cancel{m_1} v_1^2 \\
v_1 &= \sqrt{2g\Delta y_1}
\end{aligned}$$

$$E_{g\,2} = E_{k\,2}$$
$$\cancel{m_2} g\Delta y_2 = \frac{1}{2}\cancel{m_2} v_2^2$$
$$v_2 = \sqrt{2g\Delta y_2}$$
$$= \sqrt{2g\left(\frac{1}{4}\Delta y_1\right)}$$
$$= \sqrt{\frac{1}{4}(2g\Delta y_1)}$$
$$v_2 = \frac{1}{2}\sqrt{2g\Delta y_1}$$

Statement: The speed of the blue balloon is half the speed of the red balloon.

52. The pole vaulter's kinetic energy, which is proportional to the square of his speed, as he approaches the jump transforms into the gravitational potential energy that will help him to clear the bar. Thus, approach speed is important to the pole vaulter.

53. Given: $m = 57$ kg; $\Delta y_1 = 45$ m; $\Delta y_2 = 25$ m

Required: v

Analysis: The skier's total energy is the same at the top of the first peak as at the top of the second peak. At the top of the first peak, her energy is all gravitational potential energy, because she starts from rest. At the top of the second peak, her energy is a combination of gravitational potential energy and kinetic energy.

$E_g = mg\Delta y$; $E_k = \frac{1}{2}mv^2$

Solution: Solve for v in the equation $E_{g\,1} = E_{g\,1} + E_k$.

$$E_{g\,1} = E_{g\,2} + E_k$$
$$\cancel{m}g\Delta y_1 = \cancel{m}g\Delta y_2 + \frac{1}{2}\cancel{m}v^2$$
$$\frac{1}{2}v^2 = g(\Delta y_1 - \Delta y_2)$$
$$v = \sqrt{2g(\Delta y_1 - \Delta y_2)}$$
$$= \sqrt{2(9.8\text{ m/s}^2)(45\text{ m} - 25\text{ m})}$$
$$v = 2.0 \times 10^1\text{ m/s}$$

Statement: The skier's speed at the second peak is 2.0×10^1 m/s.

54. Given: $m = 450$ kg; $v = 3.5$ m / s; $\Delta x = 2.0$ m

Required: k

Analysis: The kinetic energy at the moment the car hits the spring is transformed completely to elastic potential energy when the spring is fully compressed.

$E_k = \frac{1}{2}mv^2$; $E_e = \frac{1}{2}k(\Delta x)^2$

Solution: The kinetic energy at the moment the car hits the spring is equal to the elastic potential energy when the spring is fully compressed.

$$E_e = E_k$$

$$\frac{1}{2}k(\Delta x)^2 = \frac{1}{2}mv^2$$

$$k = \frac{mv^2}{(\Delta x)^2}$$

$$= \frac{(450 \text{ kg})(3.5 \text{ m/s})^2}{(2.0 \text{ m})^2}$$

$$k = 1400 \text{ N/m}$$

Statement: The spring constant is 1.4×10^3 N/m.

55. (a) Given: $v_i = 27 \text{ m/s}; \ \theta = 20°; m = 0.43 \text{ kg}; \ g = 9.8 \text{ m/s}^2$

Required: Δy, the maximum height of the ball

Analysis: Determine the *y*-component of the initial velocity using trigonometry.
At the maximum height of the ball, the *y*-component of the velocity is zero, so the kinetic energy is zero. The kinetic energy when the ball is kicked is equal to the potential energy at the maximum height.

$E_g = mg\Delta y$; $E_k = \frac{1}{2}mv^2$

Solution: The *y*-component of the initial velocity is $v_i \sin\theta$.

$$E_g = E_k$$

$$\cancel{m}g\Delta y = \frac{1}{2}\cancel{m}v^2$$

$$\Delta y = \frac{v^2}{2g}$$

$$= \frac{[(27 \text{ m/s})\sin 20°]^2}{2(9.8 \text{ m/s}^2)}$$

$$\Delta y = 4.4 \text{ m}$$

Statement: The maximum height of the ball is 4.4 m.

(b) Since there is no air resistance, the speed of the ball when it lands is equal to the speed of the ball when it was kicked. Thus, the speed of the ball when it lands is 27 m/s.

56. (a) Given: $m = 55 \text{ kg}; \ \Delta y = 1.3 \text{ m}$

Required: W_g

Analysis: The work done by gravity is the negative of the gravitational potential energy.

$W_g = -E_g$; $E_g = mg\Delta y$

Solution: $W_g = -E_g$

$$= -mg\Delta y$$

$$= -(55 \text{ kg})(9.8 \text{ m/s}^2)(1.3 \text{ m})$$

$$W_g = -701 \text{ J (one extra digit carried)}$$

Statement: The work done by gravity is -7.0×10^2 J.

(b) Given: $m = 55\text{ kg};\ v_\text{i} = 5.4\text{ m/s};\ \Delta y = 1.3\text{ m}$

Required: v_f

Analysis: The student's total energy when she leaves the trampoline's surface is all kinetic energy. As she rises in the air, the kinetic energy transforms into gravitational potential energy. If she has not reached her maximum height, she will still have some kinetic energy.

$E_\text{k} = \frac{1}{2}mv^2;\ E_\text{g} = mg\Delta y$

Solution: $E_\text{k i} = E_\text{g} + E_\text{k f}$

$$\frac{1}{2}\cancel{m}v_\text{i}^2 = \cancel{m}g\Delta y + \frac{1}{2}\cancel{m}v_\text{f}^2$$

$$\frac{1}{2}v_\text{f}^2 = \frac{1}{2}v_\text{i}^2 - g\Delta y$$

$$v_\text{f}^2 = v_\text{i}^2 - 2g\Delta y$$

$$v_\text{f} = \sqrt{v_\text{i}^2 - 2g\Delta y}$$

$$= \sqrt{(5.4\text{ m/s})^2 - 2(9.8\text{ m/s}^2)(1.3\text{ m})}$$

$$v_\text{f} = 1.9\text{ m/s}$$

Statement: The student's speed at 1.3 m above the trampoline is 1.9 m/s.

(c) No, she has not reached her maximum height because she still has kinetic energy. At the maximum height, there is no kinetic energy.

57. (a) Given: $k = 4.5\text{ N/m};\ \Delta x_1 = 0.75\text{ m};\ \Delta x_2 = 0.5\text{ m}$

Required: E_lost

Analysis: The amount of energy lost is equal to the change in elastic potential energy.

$E_\text{e} = \frac{1}{2}k(\Delta x)^2$

Solution: $E_\text{lost} = E_\text{e 1} - E_\text{e 2}$

$$= \frac{1}{2}k(\Delta x_1)^2 - \frac{1}{2}k(\Delta x_2)^2$$

$$= \frac{1}{2}k[(\Delta x_1)^2 - (\Delta x_2)^2]$$

$$= \frac{1}{2}(4.5\text{ N/m})[(0.75\text{ m})^2 - (0.50\text{ m})^2]$$

$$= 0.703\text{ J (two extra digits carried)}$$

$$E_\text{lost} = 0.7\text{ J}$$

Statement: The system has lost 0.7 J of energy.

(b) Given: $E_{\text{lost}} = 0.703\text{ J};\ t = 15\text{ min} = 900\text{ s}$
Required: P
Analysis: The work done by the damping is equal to the energy lost.

$$P = \frac{W}{t}$$

Solution:
$$\begin{aligned} P &= \frac{W}{t} \\ &= \frac{E_{\text{lost}}}{t} \\ &= \frac{0.703\text{ J}}{900\text{ s}} \\ P &= 7.8\times 10^{-4}\text{ W} \end{aligned}$$

Statement: The power loss of the system is 7.8×10^{-4} W.

58. Given: $k = 6.0\text{ N / m};\ \Delta x = 0.40\text{ m}$
Required: m
Analysis: Let the $y = 0$ reference point be the point where the ball begins its bounce back up. Therefore, $\Delta y = \Delta x$. The gravitational potential energy when the ball is released transforms to elastic potential energy at the point where the ball bounces.

$$E_{\text{g}} = mg\Delta y;\ E_{\text{e}} = \frac{1}{2}k(\Delta x)^2$$

Solution:
$$\begin{aligned} E_{\text{g}} &= E_{\text{e}} \\ mg\Delta y &= \frac{1}{2}k(\Delta x)^2 \\ m &= \frac{k(\Delta x)^2}{2g\Delta y} \\ &= \frac{(6.0\text{ N/m})(0.40\text{ m})^2}{2(9.8\text{ m/s}^2)(0.40\text{ m})} \\ &= \frac{(6.0\text{ kg/}\cancel{\text{s}^2})(0.40\ \cancel{\text{m}})^{\cancel{2}}}{2(9.8\ \cancel{\text{m/s}^2})(0.40\ \cancel{\text{m}})} \\ m &= 0.12\text{ kg} \end{aligned}$$

Statement: The mass of the ball is 0.12 kg.

59. (a) Given: $m = 0.50\text{ kg};\ \Delta x_1 = 0.25\text{ m};\ v = 1.5\text{ m/s}$
Required: k
Analysis: The elastic potential energy when the ball is released transforms to kinetic energy at the maximum speed of the ball.

$$E_{\text{e}} = \frac{1}{2}k(\Delta x)^2;\ E_{\text{k}} = \frac{1}{2}mv^2$$

Solution: $E_e = E_k$

$$\frac{1}{2}k(\Delta x_1)^2 = \frac{1}{2}mv^2$$

$$k = \frac{mv^2}{(\Delta x_1)^2}$$

$$= \frac{(0.50\text{ kg})(1.5\text{ m/s})^2}{(0.25\text{ m})^2}$$

$$k = 18\text{ N/m}$$

Statement: The spring constant is 18 N/m.

(b) Given: $\Delta x_1 = 0.25$ m; $\Delta x_2 = 0.125$ m; $k = 18$ N / m; $m = 0.50$ kg

Required: v

Analysis: When the ball is halfway to its equilibrium point, its total energy consists of kinetic energy and elastic potential energy. At its equilibrium point, the total energy consists only of elastic potential energy.

$E_e = \frac{1}{2}k(\Delta x)^2$; $E_k = \frac{1}{2}mv^2$

Solution: $E_{e\,1} = E_{e\,2} + E_k$

$$\frac{1}{2}k(\Delta x_1)^2 = \frac{1}{2}k(\Delta x_2)^2 + \frac{1}{2}mv^2$$

$$mv^2 = k(\Delta x_1)^2 - k(\Delta x_2)^2$$

$$v = \sqrt{\frac{k(\Delta x_1)^2 - k(\Delta x_2)^2}{m}}$$

$$= \sqrt{\frac{(18\text{ N/m})(0.25\text{ m})^2 - (18\text{ N/m})(0.125\text{ m})^2}{0.50\text{ kg}}}$$

$$v = 1.3\text{ m/s}$$

Statement: The speed of the ball halfway to its equilibrium point is 1.3 m/s.

(c) Given: $v = 1.3$ m / s; $m = 0.50$ kg; $k = 18$ N / m; $\Delta x = 0.25$ m

Required: fraction of energy converted from elastic potential energy to kinetic energy

Analysis: Determine the fraction of E_k over $E_{e\,1}$ from (b).

$E_k = \frac{1}{2}mv^2$; $E_e = \frac{1}{2}k(\Delta x)^2$

Solution: $\dfrac{E_\text{k}}{E_\text{e}} = \dfrac{\frac{1}{2}mv^2}{\frac{1}{2}k(\Delta x_1)^2}$

$$= \frac{mv^2}{k(\Delta x_1)^2}$$

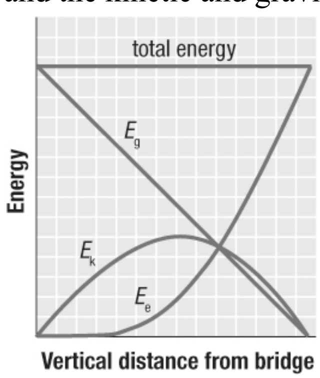
$$= \frac{(0.50\text{ kg})(1.3\text{ m/s})^2}{(18\text{ N/m})(0.25\text{ m})^2}$$

$$\frac{E_\text{k}}{E_\text{e}} = 0.75$$

Statement: The fraction of energy converted from elastic potential energy to kinetic energy is $\frac{3}{4}$.

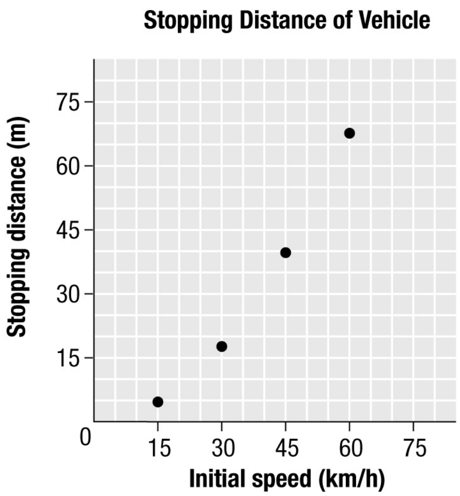
60. The gravitational potential energy of the heavier object at the equilibrium point is greater than that of the lighter object, so the elastic potential energy at the maximum stretch of the heavier object will be greater, since the gravitational potential energy equals the elastic potential energy. Since elastic potential energy is proportional to the square of the maximum stretch amount, if the elastic potential energy is greater, the maximum stretch amount will be greater.

61. At the beginning of the jump, the jumper's gravitational potential energy is at a maximum. As the jumper falls, the gravitational potential energy decreases and the kinetic energy increases. At the full length of the bungee cord before it starts stretching, the kinetic energy is at a maximum and the gravitational potential energy continues to increase. As the bungee cord stretches, the elastic potential energy increases from zero up to its maximum when the cord is fully stretched. When the cord is fully stretched, the elastic potential energy is at its maximum, and gravitational potential energy and kinetic energy are zero. As the jumper bounces back up, the elastic potential energy decreases and the kinetic and gravitational potential energy increase.



62. Answers may vary. Sample answer:
thermal energy: When a skater stops on the ice, kinetic energy transforms to thermal energy through friction between the skate blades and the ice.

63. (a)



(b) The kinetic energy of the car is equal to the work done by the road on the car to stop the car. Therefore,

$$E_k = W$$

$$\frac{1}{2}mv^2 = F\Delta d$$

$$\Delta d = \frac{mv^2}{2F}$$

Since the force, F, and the mass, m, are constant, the stopping distance, Δd, is proportional to the square of the speed, v. The graph is quadratic.

(c) Answers may vary. Sample answer: The data shows that the stopping distance is proportional to vehicle speed, so the faster you are driving, the greater the distance required to stop the vehicle. Therefore, it is important to make sure that there is enough distance between your car and the car in front of you so that you can stop safely. This data also proves that tailgating is dangerous and should be avoided.

Evaluation

64. Answers may vary. Sample answer: The gravitational force is the force exerted by gravity on the crate due to its mass, and it is directed downwards. The normal force is the force between two objects that are in contact, and its direction is perpendicular to the surface. The applied force is the force the worker applies to push the crate, and its direction is up the ramp. Friction opposes the applied force, so its direction is down the ramp. The gravitational force, the applied force, and friction do work. The normal force does zero work. This is because the normal force is perpendicular to the direction of the displacement. The applied force does work because it is in the direction of the displacement, and friction does negative work because it opposes the direction of the displacement. The gravitational force does work on the crate as well because when it is separated into component vectors, one of the components does negative work.

65. Answers may vary. Sample answers:

(a) If Earth were less massive, the force of gravity would be less, so the Moon would not be attracted as strongly. Tides would be different. Birds would fly more easily. It would not take as much fuel to fly an airplane.

(b) If Earth were more massive, the force of gravity would be greater, so the Moon would be attracted more strongly. This would affect the same things as in (a), but in the opposite way.

66. Answers may vary. Sample answer: The gravitational potential energy of the spring toy at the top of the stairs is transformed to kinetic energy when the toy is started down the stairs. The energy is passed from coil to coil in the spring as it moves down the stairs.

67. Answers may vary. Sample answer: Roller coasters take advantage of gravitational potential energy to start moving down hills. The higher the hill, the faster the roller coaster car will move at the bottom of the hill, and the more easily it will make it up the next hill, and so on.

68. Answers may vary. Sample answer: As the frame of the car crumples, it absorbs the energy from the collision, so less of the impact is felt by the people in the car.

Reflect on Your Learning

69. Answers may vary. Sample answer: I found the concept of work difficult to understand since work is done only if a displacement occurs. If I push on a stationary object like a wall, I exert a force or energy pushing against the wall, but no work is done because the wall does not move. To understand this concept, I would have to search for many examples of work being done versus no work being done to keep this concept clear. Another concept that I found difficult to understand is that the direction of the spring force is opposite to the displacement of the spring from its equilibrium position. If you stretch as a spring upwards so that the displacement from the equilibrium position is upwards, the spring force is actually downwards. To understand this concept fully, it would help if I could compress and stretch some springs to see how the forces work in real life.

70. After having read the chapter, I have a better understanding of the different sources of commercial energy, especially hydroelectric power. While hydroelectric power is cleaner than burning fossil fuels for electricity generation, it still has a huge environmental impact. Water resources are diminished, land use is diverted for power dam construction, wildlife habitats are destroyed, and the natural path of water sources are artificially diverted, changing ecosystems. While I still prefer this over fossil fuel–burning plants, alternative sources for electricity generation, such as solar and wind power, should be investigated further.

71. Answers may vary. Sample answer: One topic that I am still unsure of is the use of components in determining work. What confuses me is which angle and trigonometric ratio I should use to calculate the component vectors and when I should be separating the components. One way I can improve my understanding is to do more questions involving these components such as questions involving ramps. Another way for me to improve my understanding is to draw vector diagrams, including right angle triangles to help me visualize these problems.

72. Answers may vary. Sample answer: One instance where I can apply what I learned in this chapter is when I am shovelling snow. Since I know that work is done only when there is a displacement, I should decrease the angle at which I apply the force on my shovel so that the work is done more efficiently. Another instance is when I am at the swimming pool. To make little to no splash when I am diving, I need to pick a diving board that is long and loose so that its spring constant is small. This means that I will to use less force to go up a greater distance. This will give me more time in the air to execute my dive.

Research

73. Answers may vary. Sample answer: One famous scientist who used pendulums to study gravity and the orbital motions of the plant was Galileo Galilei. He formulated his Law of Falling Bodies by studying the motions of pendulums as well as objects rolling and sliding down inclines. He theorized that all bodies would fall with the same acceleration due to gravity, which he calculated to be 32 ft/s or 9.75 m/s. This is very close to the value 9.81 m/s, the accepted value of *g* in today's society.

Leon Foucault proved that Earth rotates by using a pendulum tied to a building. A pendulum works because it is affected by gravity and inertia. As a pendulum is pulled back and then released, it swings down because of gravity and swings up because of inertia. As the pendulum swings, it seems to be rotating. However, since the way it is suspended prevents it from rotating, it must be the building that is rotating. Since the building is attached to Earth, it must be Earth that is rotating.

A simple experiment that can be done to determine the value of *g* using a pendulum.

1. Set up a pendulum so that it swings from a fixed end. For example, a ring stand and a pendulum clamp can be used.
2. Pull back the pendulum and release it making sure that there is very little sideways motion involved.
3. Determine the period of the pendulum, T, by recording the time it takes for the pendulum to swing a set number of times and dividing the time by that number.
4. Repeat this several times with pendulums of different lengths, L.
5. Graph the T^2 v. L using the data collected.
6. From the rearrangement of the equation below, *g* can be determined.

$$T = 2\pi\sqrt{\frac{L}{g}}$$

$$T^2 = 4\pi^2\left(\frac{L}{g}\right)$$

$$\frac{T^2}{L} = \frac{4\pi^2}{g}$$

$$\text{slope} = \frac{4\pi^2}{g}$$

$$g = \frac{4\pi^2}{\text{slope}}$$

74. Answers may vary. Sample answer:

Hydroelectric Plant	Advantages	Disadvantages
Churchill, New Brunswick	• natural basin so no need to construct dams • brings economic opportunities to New Brunswick and Labrador • provides enough electricity to power three cities the size of Montreal	• long distance transmissions for electricity to be used • remote location and harsh climate • alters natural beauty of Churchill Falls and surrounding area
Niagara Falls, Ontario	• supplies one quarter of power used in Ontario and New York State • hydroelectric power may be sold between generating stations in Ontario and generating stations in New York state when needed • constraints made to limit the amount of water used for industry because of tourism • regulating flow of water slows down erosion so prolongs life of Niagara Falls	• not all of the power goes directly to Ontario; Canadians are able to draw 56 500 cubic feet of water per second and Americans are allowed 32 500 cubic feet of water per second through international agreement • issues with the environment such as ice flows and flooding

75. Answers may vary. Sample answer: The physics of carousels is based on centripetal force. Centripetal force is the force that keeps the horses and people moving in a circular motion. In carousels, this centripetal force is supplied by the platform that supports the carousel. The carousel has to be at a low speed so that centrifugal force does not get strong enough to take over the centripetal force that is acting on the people riding the carousel. The centrifugal force is not an actual force. It is when centripetal force stops working and a body's inertia takes over. Since inertia is the resistance of a body to its change in motion, the rider would move in a straight line that is tangent to the centripetal force.