Section 4.1: Work Done by a Constant Force

Tutorial 1 Practice, page 166

1. Given: $F = 275$ N; $\Delta d = 0.65$ m

Required: W

Analysis: Use the equation for work, $W = F\Delta d \cos\theta$. F and Δd are in the same direction, so the angle between them is zero, stated as $\theta = 0$.

Solution: $W = F\Delta d \cos\theta$

$$= (275 \text{ N})(0.65 \text{ m})\cos 0$$
$$= (180 \text{ N}\cdot\text{m})(1)$$
$$W = 180 \text{ N}\cdot\text{m}$$

Statement: The weightlifter does 1.8×10^2 J of work on the weights.

2. Given: $F = 9.4$ N; $\Delta d = 0$ m

Required: W

Analysis: There is no displacement in the direction of the applied force, so no work is done.

Statement: There is no work (0 J) done on the wall.

3. Given: $F = 0.73$ N; $\Delta d = 0.080$ m(note that the cue stick only does work on the ball when it is in contact with the ball)

Required: W

Analysis: Use the equation for work, $W = F\Delta d \cos\theta$. F and Δd are in the same direction, so the angle between them is zero, $\theta = 0$.

Solution: $W = F\Delta d \cos\theta$

$$= (0.73 \text{ N})(0.080 \text{ m})\cos 0$$
$$= (0.0584 \text{ N}\cdot\text{m})(1)$$
$$W = 0.058 \text{ N}\cdot\text{m}$$

Statement: The cue stick does 0.058 J of work on the ball.

4. Given: $F = 9.9\times10^3$ N; $\Delta d = 4.3$ m; $\theta = 12°$

Required: W

Analysis: The work done on the car by the tow truck depends only on the component of force in the direction of the car's displacement. Use the equation for work, $W = F\Delta d\cos\theta$. F and Δd are at an angle of 12° to each other.

Solution: $W = F\Delta d \cos\theta$

$$= (9.9\times10^3 \text{ N})(4.3 \text{ m})\cos 12°$$
$$W = 4.2\times10^4 \text{ N}\cdot\text{m}$$

Statement: The tow truck does 4.2×10^4 J of work on the car.

Tutorial 2 Practice, page 167

1. (a) Given: $m = 56$ kg; $\Delta d = 78$ m; $g = -9.8 \text{ m/s}^2$

Required: W_r, the work done by the ride on the rider

Analysis: The ride must counteract the force of gravity for it to move at a constant speed. $F_g = mg$, so $F_r = -mg$; $W_r = F_r\Delta d \cos\theta$.

Solution: $W_r = F_r\Delta d\cos\theta$

$= -mg\Delta d\cos\theta$

$= -(56\text{ kg})(-9.8\text{ m/s}^2)(78\text{ m})\cos 0$

$= 4.3\times10^4\ (\text{kg}\cdot\text{m/s}^2)(\text{m})$

$W_r = 4.3\times10^4\ \text{N}\cdot\text{m}$

$= 4.3\times10^4\ \text{J}$

Statement: The work done by the ride on the rider is 4.3×10^4 J.

(b) Given: $m = 56$ kg; $\Delta d = 78$ m; $g = -9.8\text{ m/s}^2$

Required: W_g, the work done by gravity on the rider

Analysis: The force of gravity on the rider is $F_g = mg$; $W_g = F_g\Delta d\cos\theta$.

Solution: $W_g = F_g\Delta d\cos\theta$

$= mg\Delta d\cos\theta$

$= (56\text{ kg})(-9.8\text{ m/s}^2)(78\text{ m})\cos 0$

$= -4.3\times10^4\ (\text{kg}\cdot\text{m/s}^2)(\text{m})$

$= -4.3\times10^4\ \text{N}\cdot\text{m}$

$W_g = -4.3\times10^4\ \text{J}$

Statement: The work done by gravity on the rider is -4.3×10^4 J.

2. (a) Given: $F = -5.21\times10^3\text{N}$; $\Delta d = 355$ m

Required: W

Analysis: $W = F\Delta d\cos\theta$

Solution: $W = F\Delta d\cos\theta$

$= (-5.21\times10^3\ \text{N})(355\text{ m})\cos 0$

$= -1.85\times10^6\ \text{N}\cdot\text{m}$

$W = -1.85\times10^6\ \text{J}$

Statement: The work done by friction on the plane's wheels is -1.85×10^6 J.

(b) Given: $F = -5.21\times10^3\text{N}$; $W = -1.52\times10^6\text{J}$

Required: Δd

Analysis: $W = F\Delta d\cos\theta$ and $W = F\Delta d\cos\theta$; $\Delta d = \dfrac{W}{F\cos\theta}$

Solution: $\Delta d = \dfrac{W}{F\cos\theta}$

$= \dfrac{-1.52\times10^6\ \text{J}}{(-5.21\times10^3\ \text{N})(\cos 0)}$

$= \dfrac{-1.52\times10^6\ \cancel{\text{N}}\cdot\text{m}}{(-5.21\times10^3\ \cancel{\text{N}})(\cos 0)}$

$\Delta d = 292$ m

Statement: The distance travelled by the plane is 292 m.

3. Given: $F = 5.9$ N; $\theta = 150°$; $\Delta d = 3.5$ m

Required: W

Analysis: $W = F\Delta d\cos\theta$

Solution: $W = F\Delta d \cos\theta$

$= (5.9\ \text{N})(3.5\ \text{m})(\cos 150°)$

$= -18\ \text{N} \cdot \text{m}$

$W = -18\ \text{J}$

Statement: The work done on the skier by the snow is −18 J.

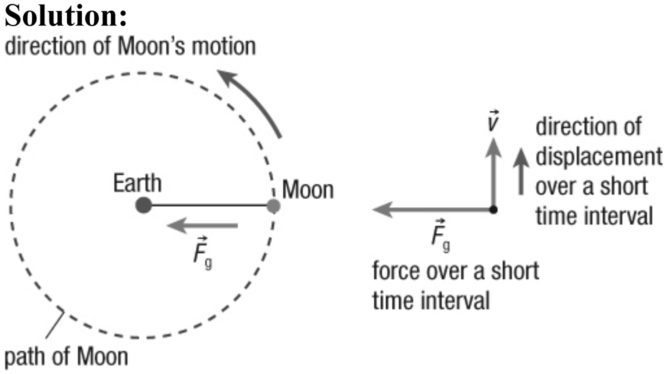
Tutorial 3 Practice, page 168

1. Given: $\theta = 90°$

Required: W

Analysis: The gravitational pull of Earth causes the Moon to experience centripetal acceleration during its orbit. At each moment, the Moon's instantaneous velocity is at an angle of 90° to the centripetal force. During a very short time interval, the very small displacement of the Moon is also at an angle of 90° to the centripetal force. We can break one orbit of the Moon into a series of many small displacements, each occurring during a very short time interval. During each time interval, the centripetal force and the displacement are perpendicular. The total work done during one orbit will equal the sum of the work done during each small displacement. For each small segment, use the work equation, $W = F\Delta d \cos\theta$, with $\theta = 90°$.

Solution:



$$W = F\Delta d \cos\theta$$
$$= F\Delta d \cos 90°$$
$$= F\Delta d(0)$$
$$W = 0\ \text{J}$$

Statement: Summing the work done during all segments of the orbit gives a total of W = 0 J during each revolution. The centripetal force exerted by Earth does zero work on the Moon during the revolution.

Tutorial 4 Practice, page 169

1. Given: $\Delta d = 223\ \text{m}$; $F_h = 122\ \text{N}$; $\theta_h = 37°$; $F_f = 72.3\ \text{N}$; $\theta_f = 180°$

Required: W_h, W_f, W_T

Analysis: $W = F\Delta d \cos\theta$. The total work done is the sum of the work done by the individual forces.

Solution: $W_h = F_h \Delta d \cos\theta$

$= (122\text{ N})(223\text{ m})\cos 37°$

$W_h = 2.17 \times 10^4$ J (one extra digit carried)

$W_f = F_f \Delta d \cos\theta$

$= (72.3\text{ N})(223\text{ m})\cos 180°$

$W_f = -1.61 \times 10^4$ J (one extra digit carried)

$W_T = W_h + W_f$

$= 2.17 \times 10^4\text{ J} + (-1.61 \times 10^4\text{ J})$

$W_T = 5.6 \times 10^3$ J

Statement: The work done by the hiker is 2.2×10^4 J. The work done by friction is -1.6×10^4 J. The total work done is 5.6×10^3 J.

2. Given: $W_T = 2.42 \times 10^4$ J; $F_h = 122$ N; $\theta_h = 37°$; $F_f = 72.3$ N; $\theta_f = 180°$

Required: Δd

Analysis: $W_T = W_h + W_f$

$W_T = F_h \Delta d \cos\theta + F_f \Delta d \cos\theta$

$W_T = \Delta d(F_h \cos\theta + F_f \cos\theta)$

$$\Delta d = \frac{W_T}{F_h \cos\theta + F_f \cos\theta}$$

Solution: $$\Delta d = \frac{W_T}{F_h \cos\theta + F_f \cos\theta}$$

$$= \frac{2.42 \times 10^4\text{ J}}{(122\text{ N})(\cos 37°) + (72.3\text{ N})(\cos 180°)}$$

$\Delta d = 963$ m

Statement: The hiker pulled the sled 963 m.

Section 4.1 Questions, page 170

1. The bottom rope does more work on the box, because it is in the same direction as the displacement. Only the horizontal force component of the top rope does any work on the box.

2. No. There is no work done on an object by a centripetal force, because for each small displacement, the force is perpendicular to the direction of the displacement.

3. Given: $F = 12.6$ N; $\Delta d = 14.2$ m; $\theta = 21.8°$

Required: W

Analysis: $W = F \Delta d \cos\theta$

Solution: $W = F \Delta d \cos\theta$

$= (12.6\text{ N})(14.2\text{ m})\cos 21.8°$

$W = 166$ J

Statement: The shopper does 166 J of work on the cart.

4. Given: $F = 22.8$ N; $\Delta d = 52.6$ m; $W = 9.53\times 10^2$ J

Required: θ

Analysis: $W = F\Delta d\cos\theta$

$$\cos\theta = \frac{W}{F\Delta d}$$

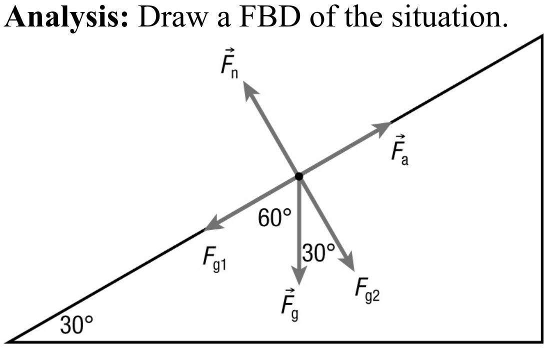
Solution:
$$\begin{aligned}\cos\theta &= \frac{W}{F\Delta d}\\ &= \frac{9.53\times 10^2\ \text{J}}{(22.8\ \text{N})(52.6\ \text{m})}\\ &= \frac{9.53\times 10^2\ \cancel{\text{N}\cdot\text{m}}}{(22.8\ \cancel{\text{N}})(52.6\ \cancel{\text{m}})}\\ &= 0.7946\ \text{(one extra digit carried)}\\ \theta &= 37.4°\end{aligned}$$

Statement: The angle between the rope and the horizontal is 37.4°.

5. (a) Given: $\theta = 30°$; $m = 24$ kg; $g = -9.8\ \text{m/s}^2$

Required: component of gravitational force along the ramp's surface

Analysis: Draw a FBD of the situation.



Let F_{g1} represent the component of gravitational force along the ramp's surface, and F_{g2} represent the component perpendicular to the ramp's surface. Since the angle between F_{g1} and F_g is 60° (from the FBD), $F_{g1} = F_g\cos 60°$.

Solution:
$$\begin{aligned}F_{g1} &= F_g\cos 60°\\ &= mg\cos 60°\\ &= (24\ \text{kg})(-9.8\ \text{m/s}^2)\cos 60°\\ F_{g1} &= -117.6\ \text{N (two extra digits carried)}\end{aligned}$$

Statement: The component of gravitational force along the ramp's surface is 120 N down the ramp.

(b) Analysis: The force up the ramp must exactly balance the component of gravitational force along the ramp's surface for the crate to move up the ramp at a constant speed.

Statement: The force required is 120 N up the ramp.

(c) Given: $F = 117.6$ N; $\Delta d = 23$ m

Required: W

Analysis: $W = F\Delta d\cos\theta$

Solution: $W = F\Delta d\cos\theta$

$= (117.6\text{ N})(23\text{ m})\cos 0°$

$= 2700\text{ J}$

Statement: The work done to push the crate up the ramp is 2.7×10^3 J.

(d) Given: $\mu_k = 0.25$; $m = 24$ kg; $\theta = 30°$; $\Delta d = 16$ m

Required: W_k; W_T

Analysis: The worker has to overcome the force of friction and the force of gravity along the ramp, so the total work done by the worker is the sum of the work done by gravity and the work done by friction.

Solution: The work done by gravity is the force of gravity along the ramp times the distance, which is mg cos 60°(Δd). The work done by friction is the force of friction along the ramp times the distance, which is $\mu_k mg$ cos 30°(Δd). The total work done by the worker is the sum of these two.

$$
\begin{aligned}
W_w &= W_g + W_k \\
&= (mg\cos 60°)(\Delta d) + (\mu_k mg\cos 30°)(\Delta d) \\
&= mg\Delta d(\cos 60° + \mu_k \cos 30°) \\
&= (24\text{ kg})(9.8\text{ m/s}^2)(16\text{ m})(\cos 60° + 0.25\cos 30°) \\
W_w &= 2700\text{ J [up the ramp]}
\end{aligned}
$$

$$
\begin{aligned}
W_k &= F_k\Delta d \\
&= \mu_k F_N \Delta d \\
&= \mu_k mg(\cos 30°)\Delta d \\
&= (0.25)(24\text{ kg})(9.8\text{ m/s}^2)(\cos 30°)(16\text{ m}) \\
W_k &= 810\text{ J [down the ramp]}
\end{aligned}
$$

The total work done by the system is equal to the work done by the worker plus the work done by friction.

$$
\begin{aligned}
W_T &= W + W_k \\
&= 2700\text{ J} + (-810\text{ J}) \\
W_T &= 1900\text{ J}
\end{aligned}
$$

Statement: The work done by the worker is 2.7×10^3 J up the ramp. The work done by kinetic friction is 8.1×10^2 J down the ramp. The total work done is 1.9×10^3 J up the ramp.

6. Given: $F_b = 75$ N; $\theta_b = 32°$; $F_g = 75$ N; $\theta_g = 22°$; $\Delta d = 13$ m

Required: W_T

Analysis: $W_T = W_b + W_g$; $W_b = F_b \Delta d \cos\theta_b$; $W_g = F_g \Delta d \cos\theta_g$

Solution:
$$\begin{aligned} W_T &= W_b + W_g \\ &= F_b \Delta d \cos\theta_b + F_g \Delta d \cos\theta_g \\ &= (75\ \text{N})(13\ \text{m})\cos 32° + (75\ \text{N})(13\ \text{m})\cos 22° \\ W_T &= 1700\ \text{J} \end{aligned}$$

Statement: The total work done by the boy and the girl together is 1.7×10^3 J.

Section 4.2: Kinetic Energy and the Work–Energy Theorem
Tutorial 1 Practice, page 172

1. (a) Given: $v_{\text{i}};\ v_{\text{f}} = 2v_{\text{i}};\ E_{\text{k i}}$

Required: $E_{\text{k f}}$

Analysis:

$$E_{\text{k}} = \frac{1}{2}mv^2$$

Solution:

$$E_{\text{k i}} = \frac{1}{2}mv_{\text{i}}^2$$

$$\begin{aligned} E_{\text{k f}} &= \frac{1}{2}mv_{\text{f}}^2 \\ &= \frac{1}{2}m(2v_{\text{i}})^2 \\ &= \frac{1}{2}m(4v_{\text{i}}^2) \\ &= 2mv_{\text{i}}^2 \\ E_{\text{k f}} &= 4E_{\text{k i}} \end{aligned}$$

Statement: A car's kinetic energy increases by a factor of 4 when the car's speed doubles.

(b) Given: $v_{\text{i}};\ v_{\text{f}} = 3v_{\text{i}}$

Required: $E_{\text{k f}}$

Analysis:

$$E_{\text{k}} = \frac{1}{2}mv^2$$

Solution:

$$E_{\text{k i}} = \frac{1}{2}mv_{\text{i}}^2$$

$$\begin{aligned} E_{\text{k f}} &= \frac{1}{2}mv_{\text{f}}^2 \\ &= \frac{1}{2}m(3v_{\text{i}})^2 \\ &= \frac{1}{2}m(9v_{\text{i}}^2) \\ &= 9\left(\frac{1}{2}mv_{\text{i}}^2\right) \\ E_{\text{k f}} &= 9E_{\text{k i}} \end{aligned}$$

Statement: A car's kinetic energy increases by a factor of 9 when the car's speed triples.

(c) Given: v_i; $v_\text{f} = 1.26v_\text{i}$; $E_\text{k i}$

Required: $E_\text{k f}$

Analysis:

$$E_\text{k} = \frac{1}{2}mv^2$$

Solution:

$$E_\text{k i} = \frac{1}{2}mv_\text{i}^2$$

$$\begin{aligned} E_\text{k f} &= \frac{1}{2}mv_\text{f}^2 \\ &= \frac{1}{2}m(1.26v_\text{i})^2 \\ &= \frac{1}{2}m(1.6v_\text{i}^2) \\ &= 1.6\left(\frac{1}{2}mv_\text{i}^2\right) \\ E_\text{k f} &= 1.6E_\text{k i} \end{aligned}$$

Statement: A car's kinetic energy increases by a factor of 1.6 when the car's speed increases by 26 %.

2. Given: m = 8.0 kg; v = 2.0 m/s

Required: E_k

Analysis:

$$E_\text{k} = \frac{1}{2}mv^2$$

Solution:

$$\begin{aligned} E_\text{k} &= \frac{1}{2}mv^2 \\ &= \frac{1}{2}(8.0\text{ kg})(2.0\text{ m/s})^2 \\ &= 16\text{ kg}\cdot\text{m}^2/\text{s}^2 \\ E_\text{k} &= 16\text{ J} \end{aligned}$$

Statement: The bowling ball's kinetic energy is 16 J.

3. Given: v = 15 km/h; E_k = 0.83 J

Required: m

Analysis:

$$\begin{aligned} E_\text{k} &= \frac{1}{2}mv^2 \\ 2E_\text{k} &= mv^2 \\ m &= \frac{2E_\text{k}}{v^2} \end{aligned}$$

The speed must be converted to metres per second.

Solution: Convert 15 km/h to metres per second.

$$\left(15\ \frac{\cancel{\text{km}}}{\cancel{\text{h}}}\right)\left(\frac{1\ \cancel{\text{h}}}{3600\ \text{s}}\right)\left(\frac{1000\ \text{m}}{1\ \cancel{\text{km}}}\right) = 4.17\ \text{m/s (one extra digit carried)}$$

$$\begin{aligned} m &= \frac{2E_\text{k}}{v^2} \\ &= \frac{2(0.83\ \text{J})}{(4.17\ \text{m/s})^2} \\ &= 0.095\ \frac{\text{kg}\cdot \cancel{\text{m}^2/\text{s}^2}}{\cancel{\text{m}^2/\text{s}^2}} \\ m &= 0.095\ \text{kg} \end{aligned}$$

Statement: The bird's mass is 0.095 kg.

Tutorial 2 Practice, page 175

1. Given: $m = 22\ \text{g} = 0.022\ \text{kg};\ v_\text{i} = 0;\ v_\text{f} = 220\ \text{km/h}$

Required: W

Analysis: $W = \Delta E_\text{k}$

Solution: Convert the speed to metres per second.

$$v_\text{f} = \left(220\ \frac{\text{km}}{\text{h}}\right)\left(\frac{1000\ \text{m}}{1\ \text{km}}\right)\left(\frac{1\ \text{h}}{3600\ \text{s}}\right)$$

$$v_\text{f} = 61.1\ \text{m/s (one extra digit carried)}$$

$$\begin{aligned} W &= \Delta E_\text{k} \\ &= E_\text{k f} - E_\text{k i} \\ &= \frac{1}{2}mv_\text{f}^2 - \frac{1}{2}mv_\text{i}^2 \\ &= \frac{1}{2}mv_\text{f}^2 \\ &= \frac{1}{2}(0.022\ \text{g})(61.1\ \text{m/s})^2 \\ W &= 41\ \text{J} \end{aligned}$$

Statement: The work done on the arrow by the bowstring is 41 J.

2. Given: $m = 3.8\times10^4\,\text{kg};\ v_\text{i} = 1.5\times10^4\,\text{m/s};\ F = 2.2\times10^5\,\text{N};\ \Delta d = 2.8\times10^6\,\text{m}$

Required: v_f

Analysis: $E_\text{kf} = E_\text{ki} + \Delta E_\text{k};\ E_\text{ki} = \frac{1}{2}mv_\text{i}^2;\ E_\text{kf} = \frac{1}{2}mv_\text{f}^2;\ \Delta E_\text{k} = F\Delta d$

Solution: $E_{kf} = E_{ki} + \Delta E_k$

$$= \frac{1}{2}mv_i^2 + F\Delta d$$

$$= \frac{1}{2}(3.8\times10^4 \text{ kg})(1.5\times10^4 \text{ m/s})^2 + (2.2\times10^5 \text{ N})$$

$$= 4.275\times10^{12} \text{ J} + 6.16\times10^{11} \text{ J}$$

$$E_{kf} = 4.891\times10^{12} \text{ J (two extra digits carried)}$$

$$E_{kf} = \frac{1}{2}mv_f^2$$

$$\frac{2}{m}E_{kf} = v_f^2$$

$$v_f = \sqrt{\frac{2}{m}E_{kf}}$$

$$= \sqrt{\frac{2}{3.8\times10^4 \text{ kg}}(4.891\times10^{12} \text{ J})}$$

$$v_f = 1.6\times10^4 \text{ m/s}$$

Statement: The final speed of the probe is 1.6×10^4 m/s.

3. Given: $v_i = 2.2$ m/s; $v_f = 0$; $F_f = 15$ N

Required: m

Analysis: Friction opposes the motion of the disc, so θ is 180°, and cos θ is –1. The work done by friction is

$$W = F\Delta d\cos\theta$$

$$= (15 \text{ N})(12 \text{ m})\cos180°$$

$$W = -180 \text{ J}$$

Solution: The work–energy theorem tells us that the change in kinetic energy will equal the work done, or –180 J. The final velocity is zero, so the final kinetic energy is zero.

$$W = E_{kf} - E_{ki}$$

$$W = 0 - \frac{1}{2}mv_{ki}^2$$

$$\frac{1}{2}mv_{ki}^2 = -W$$

$$m = -\frac{2W}{v_{ki}^2}$$

$$= -\left(\frac{2(-180 \text{ J})}{(2.2 \text{ m/s})^2}\right)$$

$$m = 74 \text{ kg}$$

Statement: The skater's mass is 74 kg.

Section 4.2 Questions, page 176

1. Answers may vary. Sample answer:
Yes, it is possible. For example, an elephant can have a mass of up to 12 000 kg. Its slow walking speed might be 2 m/s. Thus, its kinetic energy is

$$\begin{aligned} E_k &= \frac{1}{2}mv^2 \\ &= \frac{1}{2}(12\,000\text{ kg})(2\text{ m/s})^2 \\ &= 24\,000\text{ J} \end{aligned}$$

A small cheetah might have a mass of 35 kg, and its top running speed is about 120 km/h, which is about 33 m/s. Its kinetic energy is

$$\begin{aligned} E_k &= \frac{1}{2}mv^2 \\ &= \frac{1}{2}(35\text{ kg})(33\text{ m/s})^2 \\ &= 19\,000\text{ J} \end{aligned}$$

Yes, it is possible that an elephant walking slowly could have greater kinetic energy than the cheetah.

2. (a) Given: $m_c = 5.0$ kg; $m_m = 0.035$ kg; $E_{k\,c} = 100E_{k\,m}$; mouse running at a constant speed, v_m

Required: Will the cat catch up with the mouse?

Analysis: Determine the cat's speed relative to the mouse's speed using the fact that the cat's kinetic energy is 100 times the mouse's kinetic energy.

Solution:

$$\begin{aligned} E_{k\,c} &= 100E_{k\,m} \\ \frac{1}{2}m_c v_c^2 &= 100\left(\frac{1}{2}m_m v_m^2\right) \\ m_c v_c^2 &= 100 m_m v_m^2 \\ (5.0\ \cancel{\text{kg}})v_c^2 &= 100(0.035\ \cancel{\text{kg}})v_m^2 \\ 5.0v_c^2 &= 3.5v_m^2 \\ v_c^2 &= 0.70v_m^2 \\ v_c &= \sqrt{0.70}v_m \\ v_c &= 0.84v_m \end{aligned}$$

Statement: Since the cat's speed is less than the mouse's speed, the cat will never catch up to the mouse.

(b) Analysis: The cat's speed must be greater than the mouse's speed for the cat to catch up. Let the factor by which the cat's kinetic energy is greater than the mouse's kinetic energy be x. Then $E_{kc} = xE_{km}$.

Solution:

$$E_{\text{k c}} = xE_{\text{k m}}$$

$$\frac{1}{2}m_\text{c}v_\text{c}^2 = x\left(\frac{1}{2}m_\text{m}v_\text{m}^2\right)$$

$$m_\text{c}v_\text{c}^2 = xm_\text{m}v_\text{m}^2$$

$$(5.0\ \cancel{\text{kg}})v_\text{c}^2 = x(0.035\ \cancel{\text{kg}})v_\text{m}^2$$

$$5.0v_\text{c}^2 = 0.035xv_\text{m}^2$$

$$v_\text{c}^2 = 0.0070xv_\text{m}^2$$

$$v_\text{c} = \sqrt{0.0070x}(v_\text{m})$$

For the cat's speed to be greater than the mouse's speed, $\sqrt{0.0070x} > 1$.

$$\sqrt{0.0070x} > 1$$

$$0.0070x > 1$$

$$x > 140$$

Statement: For the cat to catch up with the mouse, its kinetic energy must be greater than 140 times the kinetic energy of the mouse.

3. Given: m = 1.5×10^3 kg; v_i = 11 m/s; v_f = 25 m/s; Δd = 0.20 km

Required: W

Analysis: $W = \Delta E_k$

Solution: $W = \Delta E_\text{k}$

$$= E_{\text{k f}} - E_{\text{k i}}$$

$$= \frac{1}{2}mv_\text{f}^2 - \frac{1}{2}mv_\text{i}^2$$

$$= \frac{1}{2}m(v_\text{f}^2 - v_\text{i}^2)$$

$$= \frac{1}{2}(1.5\times 10^3\ \text{kg})((25\ \text{m/s})^2 - (11\ \text{m/s})^2)$$

$$W = 380\,000\ \text{J}$$

Statement: The work done on the car is 3.8×10^5 J.

4. Given: $m = 9.1\times 10^3$kg; $v_\text{i} = 98$ km / h; $v_\text{f} = 27$ km / h

Required: W

Analysis: $W = \Delta E_\text{k}$

Solution: Convert the speeds to metres per second.

$$v_\text{i} = \left(98\frac{\cancel{\text{km}}}{\cancel{\text{h}}}\right)\left(\frac{1000\ \text{m}}{1\ \cancel{\text{km}}}\right)\left(\frac{1\ \cancel{\text{h}}}{3600\ \text{s}}\right)$$

$v_\text{i} = 27.2$ m/s (one extra digit carried)

$$v_f = \left(27\frac{\cancel{\text{km}}}{\cancel{\text{h}}}\right)\left(\frac{1000\text{ m}}{1\ \cancel{\text{km}}}\right)\left(\frac{1\ \cancel{\text{h}}}{3600\text{ s}}\right)$$

$$v_f = 7.5\text{ m/s}$$

$$\begin{aligned} W &= \Delta E_k \\ &= E_{k\,f} - E_{k\,i} \\ &= \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 \\ &= \frac{1}{2}m(v_f^2 - v_i^2) \\ &= \frac{1}{2}(9.1\times 10^3\text{ kg})((7.5\text{ m/s})^2 - (27.2\text{ m/s})^2) \\ W &= -3\,100\,000\text{ J} \end{aligned}$$

Statement: The work done on the truck is -3.1×10^6 J.

5. Given: $E_{k1} = E_{k2}$; $v_2 = 2.5v_1$

Required: $m_1 : m_2$

Analysis: $E_k = \frac{1}{2}mv^2$;

Solution: Substitute the given values into the equation $E_{k1} = E_{k2}$.

$$\begin{aligned} E_{k\,1} &= E_{k\,2} \\ \cancel{\frac{1}{2}}m_1v_1^2 &= \cancel{\frac{1}{2}}m_2v_2^2 \\ m_1v_1^2 &= m_2(2.5v_1)^2 \\ m_1\cancel{v_1^2} &= 6.25m_2\cancel{v_1^2} \\ m_1 &= 6.2m_2 \\ \frac{m_1}{m_2} &= 6.2 \\ m_1 : m_2 &= 6.2 : 1 \end{aligned}$$

Statement: The ratio of the slower mass to the faster mass is 6.2 : 1.

6. Given: $m_c = 1.2 \times 10^3$ kg; $m_s = 4.1 \times 10^3$ kg; $v_c = 99$ km/h; $E_{k\,c} = E_{k\,s}$

Required: v_s

Analysis: Convert the speed to kilometres per hour. Substitute $E_k = \frac{1}{2}mv^2$ into the equation $E_{kc} = E_{ks}$ and solve for v_s.

Solution: $v_c = \left(99\,\frac{\text{km}}{\text{h}}\right)\left(\frac{1000\text{ m}}{1\text{ km}}\right)\left(\frac{1\text{ h}}{3600\text{ s}}\right)$

$v_c = 27.5$ m/s (one extra digit carried)

$$E_{\text{k c}} = E_{\text{k s}}$$

$$\frac{1}{2}m_c v_c^2 = \frac{1}{2}m_s v_s^2$$

$$v_s^2 = \frac{m_c v_c^2}{m_s}$$

$$v_s^2 = \frac{(1.2\times10^3\text{ kg})(27.5\text{ m/s})^2}{(4.1\times10^3\text{ kg})}$$

$$v_s = \sqrt{\frac{(1.2\times10^3\text{ kg})(27.5\text{ m/s})^2}{(4.1\times10^3\text{ kg})}}$$

$= 14.9$ m/s (one extra digit carried)

Convert the speed back to kilometres per hour.

$$v_s = \left(14.9\,\frac{\text{m}}{\text{s}}\right)\left(\frac{1\text{ km}}{1000\text{ m}}\right)\left(\frac{3600\text{ s}}{1\text{ h}}\right)$$

$v_s = 54$ km/h

Statement: The speed of the SUV is 54 km/h.

7. Given: $m_a = 0.020$ kg; $v_a = 250$ km / h; $m_b = 0.14$ kg; $E_{\text{ka}} = E_{\text{kb}}$

Required: v_b

Analysis: $E_k = \frac{1}{2}mv^2$; convert speed to metres per second; substitute into $E_{\text{ka}} = E_{\text{kb}}$

Solution: $v_a = 250\,\frac{\text{km}}{\text{h}}\cdot\frac{1000\text{ m}}{1\text{ km}}\cdot\frac{1\text{ h}}{3600\text{ s}}$

$v_a = 69.4$ m/s (one extra digit carried)

$$E_{\text{k a}} = E_{\text{k b}}$$

$$\cancel{\frac{1}{2}} m_{\text{a}} v_{\text{a}}^2 = \cancel{\frac{1}{2}} m_{\text{b}} v_{\text{b}}^2$$

$$\frac{m_{\text{a}} v_{\text{a}}^2}{m_{\text{b}}} = v_{\text{b}}^2$$

$$v_{\text{b}} = \sqrt{\frac{m_{\text{a}} v_{\text{a}}^2}{m_{\text{b}}}}$$

$$= \sqrt{\frac{(0.020\ \cancel{\text{kg}})(69.4\ \text{m/s})^2}{0.14\ \cancel{\text{kg}}}}$$

$$v_{\text{b}} = 26.2\ \text{m/s}$$

Convert the speed back to kilometres per hour.

$$v_{\text{b}} = \left(26.2 \frac{\cancel{\text{m}}}{\cancel{\text{s}}}\right)\left(\frac{1\ \text{km}}{1000\ \cancel{\text{m}}}\right)\left(\frac{3600\ \cancel{\text{s}}}{1\ \text{h}}\right)$$

$$v_{\text{b}} = 94\ \text{km/h}$$

Statement: The speed of the baseball is 94 km/h.

8. Given: $v = 150\ \text{km / h}$; $m = 0.16\ \text{kg}$; $\Delta d = 0.25\ \text{m}$

Required: F

Analysis: Convert the speed to metres per second; $E_{\text{k}} = \frac{1}{2}mv^2$; $W = F\Delta d$; $W = \Delta E_{\text{k}}$; $\Delta E_{\text{k}} = E_{\text{kf}} - E_{\text{ki}}$. The puck has no initial velocity, so it has no initial kinetic energy, that is, $E_{\text{ki}} = 0$.

Solution: $v = 150\ \frac{\cancel{\text{km}}}{\cancel{\text{h}}} \cdot \frac{1000\ \text{m}}{1\ \cancel{\text{km}}} \cdot \frac{1\ \cancel{\text{h}}}{3600\ \text{s}}$

$v = 41.7$ m/s (one extra digit carried)

$$W = F\Delta d$$

$$F = \frac{W}{\Delta d}$$

$$
\begin{aligned}
W &= \Delta E_{\text{k}} \\
&= E_{\text{k f}} - E_{\text{k i}} \\
&= E_{\text{k f}} \\
&= \frac{1}{2}mv^2 \\
&= \frac{1}{2}(0.16\text{ kg})(41.7\text{ m/s})^2 \\
W &= 139\text{ J (one extra digit carried)}
\end{aligned}
$$

$$
\begin{aligned}
F &= \frac{W}{\Delta d} \\
&= \frac{139\text{ J}}{0.25\text{ m}} \\
F &= 560\text{ N}
\end{aligned}
$$

Statement: The average force exerted on the puck by the player is 560 N.

9. Given: $m = 5.31\times10^{-26}\text{kg};\ E_{\text{k}} = 6.25\times10^{-21}\text{J}$

Required: v

Analysis: $E_{\text{k}} = \frac{1}{2}mv^2$; solve for v and substitute.

Solution:

$$
\begin{aligned}
E_{\text{k}} &= \frac{1}{2}mv^2 \\
\frac{2}{m}E_{\text{k}} &= v^2 \\
v &= \sqrt{\frac{2}{m}E_{\text{k}}} \\
&= \sqrt{\frac{2}{5.31\times10^{-26}\text{ kg}}(6.25\times10^{-21}\text{ J})} \\
v &= 485\text{ m/s}
\end{aligned}
$$

Statement: The speed of the molecule is 485 m/s.

10. Given: $F_{\text{a}} = 15\text{ N}; m = 3.9\text{ kg};\ \mu_{\text{k}} = 0.25;\ v_{\text{i}} = 0.0\text{ m/s};\ \Delta d = 12\text{ m}$

Required: v_{f}

Analysis: $F_{\text{f}} = \mu_{\text{k}}F_{\text{N}};\ F_{\text{N}} = mg;\ F = F_{\text{a}} - F_{\text{f}};\ W = \Delta E_{\text{k}};$ since $v_{\text{i}} = 0.0\text{ m/s};\ \Delta E_{\text{k}} = E_{\text{kf}};$

$E_{\text{kf}} = \frac{1}{2}mv_{\text{f}}^2$

Solution: $F_f = \mu_k F_N$

$$= 0.25mg$$

$$= 0.25(3.9 \text{ kg})(-9.8 \text{ m/s}^2)$$

$$F_f = -9.56 \text{ N (one extra digit carried)}$$

$$F = F_a + F_f$$

$$= 15 \text{ N} + (-9.56 \text{ N})$$

$$F = 5.44 \text{ N (one extra digit carried)}$$

$$W = F\Delta d$$

$$= (5.44 \text{ N})(12 \text{ m})$$

$$W = 65.3 \text{ J}$$

$$W = E_k$$

$$W = \frac{1}{2}mv_f^2$$

$$\frac{2}{m}W = v_f^2$$

$$v_f = \sqrt{\frac{2}{m}W}$$

$$= \sqrt{\frac{2}{3.9 \text{ kg}}(65.3 \text{ J})}$$

$$v_f = 5.8 \text{ m/s}$$

Statement: The final speed of the block is 5.8 m/s.

11. Given: $m = 5.55 \times 10^3 \text{kg}$; $v_1 = 2.81 \text{ km / s or } 2810 \text{ m / s}$; $v_2 = 3.24 \text{ km / s or } 3240 \text{ m / s}$

Required: W_g

Analysis: $W_g = \Delta E_k$; $\Delta E_k = E_{k2} - E_{k1}$; $E_k = \frac{1}{2}mv^2$

Solution: $W_g = \Delta E_k$

$$= E_{k\,2} - E_{k\,1}$$

$$= \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$$

$$= \frac{1}{2}m(v_2^2 - v_1^2)$$

$$= \frac{1}{2}(5.55\times10^3\ \text{kg})((3240\ \text{m/s})^2 - (2810\ \text{m/s})^2)$$

$$W_g = 7.22\times10^9\ \text{J}$$

Statement: The work done by gravity on the satellite is 7.22×10^9 J.

12. (a) It is a quadratic function.

(b) The graph passes through the origin because when the speed is zero, the kinetic energy is zero.

(c) Given: From the graph, when the speed, v, is 2 m/s, the kinetic energy, E_k, is 4 J.

Required: m

Analysis: $E_k = \frac{1}{2}mv^2$; solve for m.

Solution: $E_k = \frac{1}{2}mv^2$

$$\frac{2}{v^2}E_k = m$$

$$m = \frac{2}{v^2}E_k$$

$$= \frac{2}{(2\ \text{m/s})^2}(4\ \text{J})$$

$$m = 2\ \text{kg}$$

Statement: The mass of the robot is 2 kg.

(d) Substitute the mass of the robot into the equation for kinetic energy, omitting units.

$$E_k = \frac{1}{2}mv^2$$

$$= \frac{1}{2}(2)v^2$$

$$E_k = v^2$$

Section 4.3: Gravitational Potential Energy

Tutorial 1 Practice, page 180

1. Given: m = 0.02 kg; Δd = 8.0 m; g = 9.8 m/s^2

Required: ΔE_g

Analysis: Use the gravitational potential energy equation, $\Delta E_g = mg\Delta y$. Let the y = 0 reference point be the ground.

Solution: $\Delta E_g = mg\Delta y$

$$= (0.02 \text{ kg})(9.8 \text{ m/s}^2)(8.0 \text{ m})$$

$$\Delta E_g = 1.6 \text{ J}$$

Statement: The change in potential energy between the branch and the ground is 1.6 J.

2. Given: $\Delta E_g = 660 \text{ J};\ \Delta y = 2.2 \text{ m}; g = 9.8 \text{ m/s}^2$

Required: m

Analysis: Rearrange the gravitational potential energy equation, $\Delta E_g = mg\Delta y$, to solve for m.

Solution: $\Delta E_g = mg\Delta y$

$$m = \frac{\Delta E_g}{g\Delta y}$$

$$= \frac{660 \text{ J}}{(9.8 \text{ m/s}^2)(2.2 \text{ m})}$$

$$m = 31 \text{ kg}$$

Statement: The mass of the loaded barbell is 31 kg.

3. Given: height of each book, $h = 3.6 \text{ cm} = 0.036 \text{ m}$; number of extra books = 2

Required: W

Analysis: $\Delta E_g = mg\Delta y$

The 11th book is moved 10 × 3.6 cm and the 12th book is moved 11 × 3.6 cm.

Solution: $\Delta E_g = mg\Delta y$

$$= (1.6 \text{ kg})(9.8 \text{ m/s}^2)[10(0.036 \text{ m}) + 11(0.036 \text{ m})]$$

$$\Delta E_g = 12 \text{ J}$$

Statement: The work done by the student to stack the two extra books is 12 J.

Section 4.3 Questions, page 181

1. (a) Given: $m = 2.5 \text{ kg};\ g = 9.8 \text{ m/s}^2;\ \Delta y = 2.0 \text{ m}$

Required: E_k

Analysis: The kinetic energy of the wood when it hits the table is equal to the potential energy of the wood before it falls. $E_k = \Delta E_g = mg\Delta y$

Solution: $E_k = E_g$

$= mg\Delta y$

$= (2.5 \text{ kg})(9.8 \text{ m/s}^2)(2.0 \text{ m})$

$E_k = 49 \text{ J}$

Statement: The kinetic energy of the piece of wood as it hits the table is 49 J.

(b) Given: $m = 2.5 \text{ kg}; E_k = 49 \text{ J}$

Required: v

Analysis: $E_k = \frac{1}{2}mv^2$; solve for v

Solution: $E_k = \frac{1}{2}mv^2$

$\frac{2E_k}{m} = v^2$

$v = \sqrt{\frac{2E_k}{m}}$

$= \sqrt{\frac{2(49 \text{ J})}{2.5 \text{ kg}}}$

$v = 6.3 \text{ m/s}$

Statement: The speed of the wood as it hits the table is 6.3 m/s.

2. Given: $g = 9.8 \text{ m/s}^2; m = 5.0 \text{ kg}; \Delta y = 553 \text{ m}$

Required: E_g

Analysis: $E_g = mg\Delta y$

Solution: $E_g = mg\Delta y$

$= (5.0 \text{ kg})(9.8 \text{ m/s}^2)(553 \text{ m})$

$E_g = 2.7 \times 10^4 \text{ J}$

Statement: The gravitational potential energy of the Canada goose is 2.7×10^4 J.

3. (a) Given: $m = 175 \text{ g} = 0.175 \text{ kg}$; $\Delta y = 1.05 \text{ m}$; $g = -9.8 \text{ m/s}^2$

Required: gravitational potential energy of the puck, E_g

Analysis: $E_g = mg\Delta y$

Solution: $E_g = mg\Delta y$

$= (0.175 \text{ kg})(9.8 \text{ m/s}^2)(1.05 \text{ m})$

$E_g = 1.8 \text{ J}$

Statement: The gravitational potential energy of the puck is 1.8 J.

(b) Given: $E_g = 1.8$ J

Required: change in gravitational potential energy of puck, ΔE_g

Analysis: Since the gravitational potential energy of the puck when it hits the ice is equal to 0, it is expressed as $\Delta E_g = -E_g$.

Solution: $\Delta E_g = -E_g$

$\Delta E_g = -1.8\ \text{J}$

Statement: The change in gravitational potential energy of the puck is −1.8 J.

(c) Given: $\Delta E_g = -1.8\ \text{J}$

Required: work done by the puck, *W*

Analysis: Since work and energy use the same units, *W* is equal to the change in gravitational potential energy of the puck.

Solution: $W = \Delta E_g$

$W = -1.8\ \text{J}$

Statement: The work done on the puck by gravity is 1.8 J.

4. The total work done is 0 J. The work done by gravity while you lift the cat is exactly balanced by the work done by gravity while you lower the cat.

5. Given: $\Delta y = -5.4\ \text{m};\ \Delta E_g = -3.1 \times 10^3\ \text{J};\ g = 9.8\ \text{m/s}^2$

Required: *m*

Analysis: $E_g = mg\Delta y$

Solution: At the mat, the pole vaulter's gravitational potential energy is 0 J.
Thus, $\Delta E_g = -E_g$.

$$\Delta E_g = -E_g$$

$$\Delta E_g = -mg\Delta y$$

$$m = -\frac{\Delta E_g}{g\Delta y}$$

$$= \frac{-3.1 \times 10^3\ \text{J}}{(9.8\ \text{m/s}^2)(-5.4\ \text{m})}$$

$$m = 59\ \text{kg}$$

Statement: The pole vaulter's mass is 59 kg.

6. Given: $m = 0.46\ \text{kg}; \Delta E_g = 155\ \text{J}; g = 9.8\ \text{m/s}^2$

Required: Δy

Analysis: $\Delta E_g = mg\Delta y$

Solution: $\Delta E_g = mg\Delta y$

$$\Delta y = \frac{\Delta E_g}{mg}$$

$$= \frac{155\ \text{J}}{(0.46\ \text{kg})(9.8\ \text{m/s}^2)}$$

$$\Delta y = 34\ \text{m}$$

Statement: The maximum height of the ball above the tee is 34 m.

7. Given: $m = 59$ kg; $\Delta y = 1.3$ km $= 1300$ m; $\theta = 14°$; $g = 9.8$ m/s^2

Required: E_g

Analysis: $\sin\theta = \dfrac{\Delta y}{d}$; $E_g = mg\Delta y$

$$\Delta y = d \sin\theta$$

Solution: $\Delta y = d \sin\theta$

$$= (1300 \text{ m}) \sin 14°$$

$$\Delta y = 314.498 \text{ m (four extra digits carried)}$$

$$E_g = mg\Delta y$$

$$= (59 \text{ kg})(9.8 \text{ m/s}^2)(314.498 \text{ m})$$

$$E_g = 1.8 \times 10^5 \text{ J}$$

Statement: The snowboarder's gravitational potential energy is 1.8×10^5 J.

8. (a) The work done on the first box is zero, because it doesn't move. The second box is lifted a height of Δy, the third is lifted a height of $2\Delta y$, the fourth is lifted a height of $3\Delta y$, and so on until the Nth box, which is lifted a height of $(N - 1)\Delta y$. Therefore, the work done to raise the last box to the top of the pile is expressed as $mg(N - 1)\Delta y$.

(b) As in Sample Problem 3 of Tutorial 1 on page 180, the gravitational potential energy of the stack of boxes is the sum of the gravitational potential energies of the individual boxes.

$$\Delta E_g = mg[0 \times \Delta y + 1 \times \Delta y + 2 \times \Delta y + 3 \times \Delta y + \cdots + (N-1)\Delta y]$$

$$\Delta E_g = mg\Delta y[0 + 1 + 2 + 3 + \cdots + (N-1)]$$

The sum of an arithmetic sequence is given by the formula $S_n = \dfrac{n}{2}[2a_1 + (n-1)d]$. To find the sum of the sequence $0 + 1 + 2 + 3 + ... + (N-1)$, substitute $n = N$, $a_1 = 0$, and $d = 1$.

$$S_n = \frac{n}{2}[2a_1 + (n-1)d]$$

$$S_N = \frac{N}{2}[2(0) + (N-1)(1)]$$

$$S_N = \frac{N(N-1)}{2}$$

Therefore, the gravitational potential energy that is stored in the entire pile is expressed as:

$$\Delta E_g = \frac{mg\Delta y N(N-1)}{2}$$

$$\Delta E_g = mgN(N-1)\frac{\Delta y}{2}$$

9. Answers may vary. Sample answer:

Given: $E_c = 1.3\times10^8$ J; 3.79 L = 1 gal; $g = 9.8$ m/s^2; 30 students in class; average mass of each student = 70 kg

Required: Δy

Analysis: Find the chemical potential energy, $E_{c\,1}$, in 1 L of gas by dividing E_c by 3.79. Find the total mass of the class of students. Solve the equation $E_g = mg\Delta y$ for Δy.

Solution:

$$E_{c\,1} = \frac{E_c}{3.79} = \frac{1.3\times10^8 \text{ J}}{3.79}$$

$$E_{c\,1} = 3.43\times10^7 \text{ J (one extra digit carried)}$$

There are 3.43×10^7 J of chemical potential energy in 1 L of gas. Assuming that there are 30 students in the class, each with an average mass of 70 kg, this equals a total mass of $30\times70 \text{ kg} = 2100 \text{ kg}$.

Therefore,

$$E_g = mg\Delta y$$

$$\Delta y = \frac{E_g}{mg} = \frac{3.43\times10^7 \text{ J}}{(2100 \text{ kg})(9.8 \text{ m/s}^2)}$$

$$\Delta y = 1700 \text{ m}$$

Statement: The chemical potential energy of the gas could lift the students 1700 m if it could all be converted to gravitational potential energy.