## Section 3.4: Rotating Frames of Reference

### Mini Investigation: Foucault Pendulum, page 128

Answers may vary. Sample answers:

**A.** The rotation does not affect the pendulum mass. From our frame of reference, the mass swings back and forth consistently while the globe rotates beneath it.

**B.** From our frame of reference, the period of rotation does not affect the mass. To an observer on the globe, the faster the rotation of Earth, the faster the pendulum appears to move. This implies that the rotation of Earth causes the movement of the Foucault pendulum.

**C.** At the equator the pendulum would not shift at all.

### Tutorial 1 Practice, page 129

**1. (a) Given:** $d$ = 324 m or $r$ = 162 m; $F_c = F_g$

**Required:** $v$

**Analysis:** $F_c = \dfrac{mv^2}{r}$; $F_g = mg$

$$F_c = F_g$$
$$\frac{mv^2}{r} = mg$$
$$\frac{v^2}{r} = g$$
$$v = \sqrt{gr}$$

**Solution:**
$$v = \sqrt{gr}$$
$$= \sqrt{(9.8\ \text{m/s}^2)(162\ \text{m})}$$
$$v = 39.8\ \text{m/s}$$

**Statement:** The relative speed of the astronauts is 39.8 m/s.

**(b) Given:** $a_c = g$; $r$ = 162 m

**Required:** $T$

**Analysis:** $a_c = \dfrac{4\pi^2 r}{T^2}$

$$T = \sqrt{\frac{4\pi^2 r}{a_c}}$$

**Solution:**
$$T = \sqrt{\frac{4\pi^2 r}{a_c}}$$
$$= \sqrt{\frac{4\pi^2(162\ \cancel{\text{m}})}{\left(9.8\ \dfrac{\cancel{\text{m}}}{\text{s}^2}\right)}}$$
$$T = 26\ \text{s}$$

**Statement:** The period of the rotation of the spacecraft is 26 s.

**2.** Yes, both would experience artificial gravity equal to about 30.0 % of Earth's gravity, or 0.300$g$. The mass cancels out in the equation to determine speed, so the effect is independent of mass.

**3. Given:** $g = 10.00\ \text{m/s}^2$; $a_{\text{net}} = 9.70\ \text{m/s}^2$; $r = 6.2 \times 10^6$ m

**Required:** $T$

**Analysis:** $a_{\text{c}} = \dfrac{4\pi^2 r}{T^2}$; the centripetal acceleration is the difference between the acceleration due to gravity and the net acceleration experienced by a falling object.

$$a_{\text{c}} = \frac{4\pi^2 r}{T^2}$$

$$g - a_{\text{net}} = \frac{4\pi^2 r}{T^2}$$

$$T = \sqrt{\frac{4\pi^2 r}{g - a_{\text{net}}}}$$

**Solution:** $T = \sqrt{\dfrac{4\pi^2 r}{g - a_{\text{net}}}}$

$$= \sqrt{\frac{4\pi^2\left(6.2\times10^6\ \cancel{\text{m}}\right)}{\left(10.00\ \dfrac{\cancel{\text{m}}}{\text{s}^2} - 9.70\ \dfrac{\cancel{\text{m}}}{\text{s}^2}\right)}}$$

$$= 2.856\times10^4\ \cancel{\text{s}} \times \frac{1\ \cancel{\text{min}}}{60\ \cancel{\text{s}}} \times \frac{1\ \text{h}}{60\ \cancel{\text{min}}}$$

$$T = 7.9\ \text{h}$$

**Statement:** The length of the day, or period of the planet, is 7.9 h.

**4. Given:** $m_1 = 56$ kg; $r = 250$ m; $m_2 = 42$ kg

**Required:** $v$

**Analysis:** $F_{\text{c}} = \dfrac{mv^2}{r}$; $F_{\text{g}} = mg$; the acceleration on the space station is $\dfrac{42}{56}$ or $\dfrac{3}{4}$ that of Earth because the scale reads the astronaut's weight as 42 kg instead of 56 kg.

$$F_{\text{c}} = F_{\text{N}}$$

$$\frac{mv^2}{r} = \frac{3}{4}mg$$

$$\frac{v^2}{r} = \frac{3}{4}g$$

$$v = \sqrt{\frac{3}{4}gr}$$

**Solution:** $v = \sqrt{\frac{3}{4}gr}$

$= \sqrt{\frac{3}{4}(9.8\text{ m/s}^2)(162\text{ m})}$

$v = 43\text{ m/s}$

**Statement:** The space station floor rotates at a speed of 43 m/s.

**5. Given:** $r = 6.38 \times 10^6$ m

**Required:** $v$

**Analysis:** $F_c = \frac{mv^2}{r}$; $F_g = mg$; the speed of the car would make the centripetal force greater than the gravitational force.

$$F_c = F_g$$
$$\frac{mv^2}{r} = mg$$
$$\frac{v^2}{r} = g$$
$$v = \sqrt{gr}$$

**Solution:** $v = \sqrt{gr}$

$= \sqrt{(9.8\text{ m/s}^2)(6.38\times10^6\text{ m})}$

$v = 7.9\times10^3\text{ m/s}$

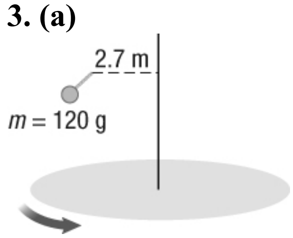
**Statement:** The car would need a speed of $7.9 \times 10^3$ m/s.

## Section 3.4 Questions, page 130

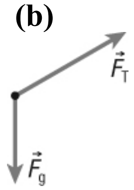
**1.** At just the right speed, the centrifugal acceleration is enough to provide enough force to keep the water in the bucket.

**2.** The spinning washing machine creates a centrifugal acceleration that forces water in the clothes to the outer wall and through pores in the wall, thus removing excess water from the clothes.

**3. (a)**

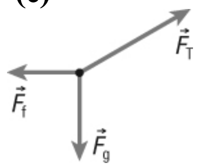



**(b)**



**(c)**




**(d) Given:** $r$ = 2.7 m; $m$ = 120 g or 0.12 kg; $T$ = 2.9 s

**Required:** $\theta$

**Analysis:** The horizontal component of the tension $F_T$ balances the centripetal force and the vertical component of the tension $F_T$ balances the gravitational force. Express the tangent ratio of the angle in terms of the applied force and the gravitational force, then solve for the angle;

$$v = \frac{2\pi r}{T}$$

$$\tan\theta = \frac{F_c}{F_g}$$

$$\theta = \tan^{-1}\left(\frac{\frac{mv^2}{r}}{mg}\right)$$

$$\theta = \tan^{-1}\left(\frac{v^2}{rg}\right)$$

**Solution:** Determine the speed 2.7 m from the centre:

$$\begin{aligned} v &= \frac{2\pi r}{T} \\ &= \frac{2\pi(2.7\text{ m})}{(3.9\text{ s})} \\ &= 4.350\text{ m/s (two extra digits carried)} \end{aligned}$$

$v = 4.3$ m/s

Determine the angle the string makes with the vertical:

$$\begin{aligned} \theta &= \tan^{-1}\left(\frac{v^2}{rg}\right) \\ &= \tan^{-1}\left(\frac{\left(4.350\ \frac{\cancel{\text{m}}}{\cancel{\text{s}}}\right)^2}{(2.7\ \cancel{\text{m}})\left(9.8\ \frac{\cancel{\text{m}}}{\cancel{\text{s}^2}}\right)}\right) \\ &= 35.57^\circ \text{ (two extra digits carried)} \end{aligned}$$

$\theta = 36°$

**Statement:** The string makes a 36° angle with the vertical.

**(e) Given:** $r$ = 2.7 m; $m$ = 120 g = 0.12 kg; $\theta$ = 35.57°

**Required:** $F_T$

**Analysis:** The vertical component of the tension $F_T$ balances the gravitational force. Express the cosine ratio of the angle in terms of the tension and the gravitational force.

$$\cos\theta = \frac{F_g}{F_T}$$
$$F_T = F_g \cos\theta$$
$$F_T = mg\cos\theta$$

**Solution:** $F_T = mg\cos\theta$

$$= (0.12\ \text{kg})\left(9.8\ \frac{\text{m}}{\text{s}^2}\right)\cos 35.57°$$

$$F_T = 0.96\ \text{N}$$

**Statement:** The tension in the string is 0.96 N.

**4. Given:** $r = 6.38 \times 10^6$ m; $T = 24$ h

**Required:** $\frac{a_c}{\text{g}}$ at the equator

**Analysis:** Earth is a non-inertial frame of reference. The acceleration of an object at the equator is the difference between the acceleration due to gravity and the centrifugal acceleration, $g - a_c$.

Use $a_c = \frac{4\pi^2 r}{T^2}$ to determine the centrifugal acceleration, then calculate its ratio with $g$.

**Solution:** Determine the centripetal acceleration at the equator:

$$a_c = \frac{4\pi^2 r}{T^2}$$
$$= \frac{4\pi^2 (6.38\times 10^6\ \cancel{\text{m}})}{\left(24\ \cancel{\text{h}} \times \frac{60\ \cancel{\text{min}}}{1\ \cancel{\text{h}}} \times \frac{60\ \text{s}}{1\ \cancel{\text{min}}}\right)^2}$$
$$= \frac{4\pi^2 (6.38\times 10^6\ \cancel{\text{m}})}{(86\ 400\ \text{s})^2}$$
$$a_c = 0.0337\ \text{m/s}^2$$

Therefore, the ratio of the centrifugal acceleration to $g$ is:

$$\frac{a_c}{\text{g}} = \frac{(0.0337\ \text{m/s}^2)}{(9.8\ \text{m/s}^2)}$$
$$\frac{a_c}{\text{g}} = 0.0034$$

**Statement:** The acceleration at the equator is 0.34 % less that $g$.

**5. Given:** $d = 10$ m or $r = 5$ m; $T = 30$ s; $\Delta d = 1.7$ m
**Required:** compare $a_c$ at $r$ and $r - \Delta d$
**Analysis:** $a_c = \dfrac{4\pi^2 r}{T^2}$
**Solution:** Determine the centripetal acceleration at the astronaut's feet:

$$\begin{aligned}a_c &= \frac{4\pi^2 r}{T^2}\\ &= \frac{4\pi^2(5\text{ m})}{(30\text{ s})^2}\\ a_c &= 0.07\text{ m/s}^2\end{aligned}$$

Determine the centripetal acceleration at the astronaut's head:

$$\begin{aligned}a_c &= \frac{4\pi^2(r-\Delta d)}{T^2}\\ &= \frac{4\pi^2(5\text{ m}-1.7\text{ m})}{(30\text{ s})^2}\\ a_c &= 0.05\text{ m/s}^2\end{aligned}$$

**Statement:** The movie did not get the physics right. The acceleration experienced by the astronaut is in the range of 0.05 m/s$^2$ to 0.07 m/s$^2$ instead of 9.8 m/s$^2$.
**6. (a) Given:** $r = 100$ m; $a_c = g$
**Required:** $T$
**Analysis:**

$$\begin{aligned}a_c &= \frac{4\pi^2 r}{T^2}\\ g &= \frac{4\pi^2 r}{T^2}\\ T &= \sqrt{\frac{4\pi^2 r}{g}}\end{aligned}$$

**Solution:**

$$\begin{aligned}T &= \sqrt{\frac{4\pi^2 r}{g}}\\ &= \sqrt{\frac{4\pi^2(100\ \cancel{\text{m}})}{\left(9.8\ \frac{\cancel{\text{m}}}{\text{s}^2}\right)}}\\ &= 20.07\text{ s (two extra digits carried)}\\ T &= 2.0\times10^1\text{ s}\end{aligned}$$

**Statement:** The period of rotation is $2.0 \times 10^1$ s.

**(b) Given:** $r$ = 100 m; $a_c = g$
**Required:** $v$

**Analysis:** $a_c = \frac{v^2}{r}$

$v = \sqrt{a_c r}$

**Solution:** $v = \sqrt{a_c r}$

$= \sqrt{\left(9.8\ \frac{\text{m}}{\text{s}^2}\right)(100\ \text{m})}$

$= 31.30$ m/s (two extra digits carried)

$v = 31$ m/s

**Statement:** The speed of rotation is 31 m/s.
**(c) Given:** $r$ = 100 m; $v_i$ = 31.30 m/s; $v$ = –4.2 m/s
**Required:** $F_N$

**Analysis:** $F_c = \frac{mv}{r^2}$; $F_N = F_c$

**Solution:** $F_N = \frac{mv^2}{r}$

$= \frac{m(31.30\ \text{m/s} - 4.2\ \text{m/s})^2}{(100\ \text{m})}$

$= m(7.34\ \text{m/s}^2)$

$F_N = 7.3m$

**Statement:** The apparent weight is 7.3 times the mass.
**(d) Given:** $r$ = 100 m; $v_i$ = 31.30 m/s; $v$ = +4.2 m/s
**Required:** $F_N$

**Analysis:** $F_c = \frac{mv}{r^2}$; $F_N = F_c$

**Solution:** $F_N = \frac{mv^2}{r}$

$= \frac{m(31.30\ \text{m/s} + 4.2\ \text{m/s})^2}{(100\ \text{m})}$

$= m(12.6\ \text{m/s}^2)$

$F_N = 13m$

**Statement:** The apparent weight is 13 times the mass.
**(e)** Running with the direction of the rotation is a better workout because you experience a greater centrifugal force and it requires more effort or exertion.
**7. (a) Given:** $m$ = 65 kg; $r$ = 150 m; $F_N$ = 540 N
**Required:** $a_c$

**Analysis:** $F_N = F_c$; $F_c = ma_c$; $a_c = \frac{F_N}{m}$

**Solution:** $a_c = \dfrac{F_N}{m}$

$$= \frac{(540\ \text{N})}{(65\ \text{kg})}$$

$$= 8.308\ \text{m/s}^2\ \text{(two extra digits carried)}$$

$$a_c = 8.3\ \text{m/s}^2$$

**Statement:** The acceleration of objects near the floor of the space station is 8.3 m/s$^2$.

**(b) Given:** $r$ = 150 m; $a_c$ = 8.308 m/s$^2$

**Required:** $v$

**Analysis:** $a_c = \dfrac{v^2}{r}$

$$v = \sqrt{a_c r}$$

**Solution:** $v = \sqrt{a_c r}$

$$= \sqrt{\left(8.308\ \frac{\text{m}}{\text{s}^2}\right)(150\ \text{m})}$$

$$= 35.30\ \text{m/s (two extra digits carried)}$$

$$v = 35\ \text{m/s}$$

**Statement:** The speed of rotation of the outer rim is 35 m/s.

**(c) Given:** $r$ = 150 m; $a_c$ = 8.308 m/s$^2$

**Required:** $T$

**Analysis:** $a_c = \dfrac{4\pi^2 r}{T^2}$

$$g = \frac{4\pi^2 r}{T^2}$$

$$T = \sqrt{\frac{4\pi^2 r}{g}}$$

**Solution:** $T = \sqrt{\dfrac{4\pi^2 r}{g}}$

$$= \sqrt{\frac{4\pi^2 (150\ \cancel{\text{m}})}{\left(8.308\ \frac{\cancel{\text{m}}}{\text{s}^2}\right)}}$$

$$T = 27\ \text{s}$$

**Statement:** The period of rotation of the space station is 27 s.

**8. (a) Given:** $r$ = 3.4 cm or 0.034 m; $f = 1.1 \times 10^3$ Hz

**Required:** $a_c$

**Analysis:** $a_c = 4\pi^2 r f^2$

**Solution:** $a_c = 4\pi^2 rf^2$
$$= 4\pi^2(0.034\text{ m})(1.1\times10^3\text{ Hz})^2$$
$$a_c = 1.6\times10^6\text{ m/s}^2$$

**Statement:** From Earth's frame of reference, the magnitude of the centripetal acceleration is $1.6 \times 10^6$ m/s$^2$.

**(b)** Answers may vary. Sample answer: Centrifuges need high frequencies to get the greatest possible acceleration. A high centrifugal force moves the denser particles to the bottom of a test tube, perfectly separating mixed solutions such as plasma and red blood cells.

**(c)** Answers may vary. Sample answer: By separating particles, medical researchers can study the particles in their pure form.

**9.** Answers may vary. Sample answer: A large-scale centrifuge, like all centrifuges, spins to separate a mixture into its components. In a large-scale centrifuge, a wastewater mixture is spun and water is separated from the heavier mixture, often called sludge, which settles on the bottom. The thickened mixture is moved to another facility for treatment while the water is sent on for different treatment before returning to the environment. By separating water from the heavier mixture, these two components of wastewater can receive the appropriate treatment before returning to the environment.

## Section 3.5: Physics Journal: The Physics of Roller Coasters

### Section 3.5 Questions, page 132

**1. (a) Given:** $r$ = 18 m; $F_c = 2.0F_g$

**Required:** $v$

**Analysis:** $F_c = \frac{mv^2}{r}$; $F_g = mg$; The force felt at the top of the loop is the difference between the centrifugal force and the gravitational force.

$$\begin{aligned} F_c - F_g &= 2.0F_g \\ F_c &= 3.0F_g \\ \frac{mv^2}{r} &= 3.0mg \\ \frac{v^2}{r} &= 3.0g \\ v &= \sqrt{3.0gr} \end{aligned}$$

**Solution:** $$\begin{aligned} v &= \sqrt{3.0gr} \\ &= \sqrt{3.0(9.8\text{ m/s}^2)(18\text{ m})} \\ v &= 23\text{ m/s} \end{aligned}$$

**Statement:** The speed at the top of the loop is 23 m/s.

**(b)** The accelerometer experiences the gravitational force and the centripetal force.

**(c)** The accelerometer will read 3$g$, the value of the centripetal acceleration.

**(d)** Answers may vary. Sample answers: You could expect errors due to vibrations of the roller coaster or the difficulty of holding the accelerometer level.

**2. Given:** $r_1 = 2r_2$; $F_{net} = 1.5g$

**Required:** $\frac{v_1}{v_2}$

**Analysis:** $F_c = \frac{mv^2}{r}$; $F_g = mg$; The force felt at the top of the loop is the difference between the centrifugal force and the gravitational force.

**Solution:** Express the speed at the top of the loop in terms of the radius:

$$\begin{aligned} F_c - F_g &= 1.5F_g \\ F_c &= 2.5F_g \\ \frac{mv^2}{r} &= 2.5mg \\ \frac{v^2}{r} &= 2.5g \\ v &= \sqrt{2.5gr} \end{aligned}$$

Determine the ratio of the speeds:

$$F_c - F_g = 1.5F_g$$

$$F_c = 2.5F_g$$

$$\frac{mv^2}{r} = 2.5mg$$

$$\frac{v^2}{r} = 2.5g$$

$$\frac{v_1}{v_2} = \frac{\sqrt{2.5gr_1}}{\sqrt{2.5gr_2}}$$

$$= \frac{\sqrt{r_1}}{\sqrt{r_2}}$$

$$= \frac{\sqrt{2r_2}}{\sqrt{r_2}}$$

$$\frac{v_1}{v_2} = 1.4$$

**Statement:** The ratio of the speed at the top of the circular loop to the speed at the top of the clothoid loop is 1.4:1.

**3.** Answers may vary. Sample answer: The normal force must be set to zero because that represents the moment when the centrifugal force balances the gravitational force. At greater speeds, there will be a normal force and the rider stays in her seat. At slower speeds, the centrifugal force is insufficient to balance gravity and the riders fall out of their seats.

**4. Given:** $m$ = 62 kg; $v$ = 22 m/s; $r$ = 35 m

**Required:** $F_N$

**Analysis:** $F_c = \frac{mv^2}{r}$; $F_g = mg$

$$F_N = F_g + F_c$$

$$F_N = mg + \frac{mv^2}{r}$$

**Solution:** $F_N = mg + \frac{mv^2}{r}$

$$= (62\text{ kg})(9.8\text{ m/s}^2) + \frac{(62\text{ kg})(22\text{ m/s})^2}{(35\text{ m})}$$

$$F_N = 1500\text{ N}$$

**Statement:** The normal force is 1500 N [up].

# Chapter 3 Review, pages 140–145

### Knowledge

**1.** (a)

**2.** (a)

**3.** (b)

**4.** (b)

**5.** (c)

**6.** (d)

**7.** False. An amusement park ride moving down with a constant velocity is an example of *an inertial* frame of reference.

**8.** True

**9.** False. The direction of centripetal acceleration for a car on a banked curve is always *horizontal into the turn*.

**10.** False. The magnitude of an object's centripetal acceleration increases with the mass and the velocity of the object, *but not the radius of the circular path*.

**11.** True

**12.** False. The Moon *is* an example of an object in uniform circular motion.

**13.** False. Objects moving in a rotating frame of reference experience a force *perpendicular* to the velocity of the object in the rotating frame.

**14.** False. A Foucault pendulum demonstrates that Earth *is* a rotating frame of reference.

**15.** True

**16. (a)** Substitute $v_{\text{new}} = 2v$ in the equation for centripetal acceleration:

$$\begin{aligned} a_c &= \frac{v_{\text{new}}^2}{r} \\ &= \frac{(2v)^2}{r} \\ a_c &= 4\frac{v^2}{r} \end{aligned}$$

The centripetal acceleration is four times its original value.

**(b)** Substitute $r_{\text{new}} = 2r$ in the equation for centripetal acceleration:

$$\begin{aligned} a_c &= \frac{v^2}{r_{\text{new}}} \\ &= \frac{v^2}{2r} \\ a_c &= \frac{1}{2}\frac{v^2}{r} \end{aligned}$$

The centripetal acceleration is half its original value.

**17.** The faster car experiences the greater force because centripetal force increases with speed.

**18.** Centrifuges are used to separate blood into its components parts.

**19. (a)** The source of the centripetal force on the Moon is Earth's gravitational force.
**(b)** The source of the centripetal force on a car turning a corner is static friction.
**(c)** The source of the centripetal force on a rock on a string is tension.

**Understanding**

**20. (a)** You must hold the accelerometer exactly horizontal, so that the angle the bead makes with the vertical is 0° when the car is at rest. If it is tilted slightly, a non-zero angle will introduce an error into all measurements.
**(b)** The bead will be at an angle of 0° with respect to the vertical.
**(c)** The bead will roll to the west side of the accelerometer, or toward you.
**(d)** The bead will be at an angle of 0° with respect to the vertical.
**(e)** The bead will roll to the east side of the accelerometer, or away from you.
**(f) Given:** $\theta = 13°$
**Required:** $a$
**Analysis:** Look at the situation from an Earth (inertial) frame of reference. The horizontal component of the tension $F_T$ balances the acceleration and the vertical component of the tension $F_T$ balances the gravitational force. Express the components of the tension in terms of the horizontal and vertical applied forces. Then calculate the magnitude of the acceleration.
**Solution:** Vertical component of force:

$$\begin{aligned}\Sigma F_y &= 0\\ F_T\cos\theta - mg &= 0\\ F_T &= \frac{mg}{\cos\theta}\end{aligned}$$

Horizontal component of force:

$$\begin{aligned}\Sigma F_x &= ma\\ F_T\sin\theta &= ma\\ \left(\frac{mg}{\cos\theta}\right)\sin\theta &= ma\\ g\left(\frac{\sin\theta}{\cos\theta}\right) &= a\\ g\tan\theta &= a\\ a &= \left(9.8\ \text{m/s}^2\right)\tan 13°\\ a &= 2.3\ \text{m/s}^2\end{aligned}$$

**Statement:** The magnitude of the car's acceleration is 2.3 m/s$^2$.
**21. (a) Given:** $r$ = 13 cm = 0.13 m; $f$ = 33.5 rpm
**Required:** $a_c$
**Analysis:** $a_c = 4\pi^2 rf^2$
**Solution:** Convert the frequency to hertz:

$$\begin{aligned}f &= 33.5\ \text{rpm}\\ &= 33.5\ \frac{1}{1\ \cancel{\text{min}}} \times \frac{1\ \cancel{\text{min}}}{60\ \text{s}}\\ f &= 0.558\ 33\ \text{Hz (two extra digits carried)}\end{aligned}$$

Determine the centripetal acceleration:

$$\begin{aligned} a_c &= 4\pi^2 r f^2 \\ &= 4\pi^2 (0.13 \text{ m})(0.558\ 33 \text{ Hz})^2 \\ a_c &= 1.6 \text{ m/s}^2 \end{aligned}$$

**Statement:** The centripetal acceleration is 1.6 m/s$^2$.
**(b) Given:** $r$ = 4.3 m; $T$ = 1.2 s
**Required:** $a_c$

**Analysis:** $a_c = \dfrac{4\pi^2 r}{T^2}$

**Solution:** $$\begin{aligned} a_c &= \frac{4\pi^2 r}{T^2} \\ &= \frac{4\pi^2 (4.3 \text{ m})}{(1.2 \text{ s})^2} \\ a_c &= 1.2 \times 10^2 \text{ m/s}^2 \end{aligned}$$

**Statement:** The centripetal acceleration of the lasso is $1.2 \times 10^2$ m/s$^2$.
**(c) Given:** $v = 2.18 \times 10^6$ m/s; $d = 1.06 \times 10^{-10}$ m = $r = 5.30 \times 10^{-11}$ m
**Required:** $a_c$

**Analysis:** $a_c = \dfrac{v^2}{r}$

**Solution:** $$\begin{aligned} a_c &= \frac{v^2}{r} \\ &= \frac{(2.18 \times 10^6 \text{ m/s})^2}{(5.30 \times 10^{-11} \text{ m})} \\ a_c &= 8.97 \times 10^{22} \text{ m/s}^2 \end{aligned}$$

**Statement:** The magnitude of the centripetal acceleration is $8.97 \times 10^{22}$ m/s$^2$.
**22. Given:** $r$ = 125 m; $a_c$ = 33.8 m/s$^2$
**Required:** $v$

**Analysis:** $$\begin{aligned} a_c &= \frac{v^2}{r} \\ v &= \sqrt{a_c r} \end{aligned}$$

**Solution:** $$\begin{aligned} v &= \sqrt{a_c r} \\ &= \sqrt{(33.8 \text{ m/s}^2)(125 \text{ m})} \\ v &= 65 \text{ m/s} \end{aligned}$$

**Statement:** The speed of the car is 65 m/s.
**23. Given:** $v$ = 50.0 km/h; $d$ = 33.5 m or $r$ = 16.75 m
**Required:** $a_c$

**Analysis:** $a_c = \dfrac{v^2}{r}$

**Solution:** Convert the speed to metres per second:

$$v = 50.0\ \frac{\cancel{\text{km}}}{\cancel{\text{h}}} \times \frac{1000\ \text{m}}{1\ \cancel{\text{km}}} \times \frac{1\ \cancel{\text{h}}}{60\ \cancel{\text{min}}} \times \frac{1\ \cancel{\text{min}}}{60\ \text{s}}$$

$$= 13.889\ \text{m/s (two extra digits carried)}$$

$$v = 13.9\ \text{m/s}$$

Determine the centripetal acceleration:

$$a_\text{c} = \frac{v^2}{r}$$

$$= \frac{(13.889\ \text{m/s})^2}{(16.75\ \text{m})}$$

$$a_\text{c} = 11.5\ \text{m/s}^2$$

**Statement:** The magnitude of the centripetal acceleration is 11.5 m/s$^2$.

**24. Given:** $d$ = 20 m or $r$ = 10 m; $F_\text{net} = \frac{1}{3}F_\text{g}$

**Required:** $v$

**Analysis:** $F_\text{c} = \frac{mv^2}{r}$; $F_\text{g} = mg$; $F_\text{net} = F_\text{c} + F_\text{g}$

$$F_\text{net} = \frac{1}{3}F_\text{g}$$

$$F_\text{c} + F_\text{g} = \frac{1}{3}F_\text{g}$$

$$F_\text{c} = -\frac{2}{3}F_\text{g}$$

$$\frac{mv^2}{r} = -\frac{2}{3}mg$$

$$v^2 = -\frac{2}{3}\frac{\cancel{m}gr}{\cancel{m}}$$

$$v = \sqrt{-\frac{2}{3}gr}$$

**Solution:** $v = \sqrt{-\frac{2}{3}gr}$

$$= \sqrt{-\frac{2}{3}\left(-9.8\ \text{m/s}^2\right)\left(10\ \text{m}\right)}$$

$$v = 8.1\ \text{m/s}$$

**Statement:** The speed of the roller coaster is 8.1 m/s.

**25.** Substitute $m_{\text{locomotive}} = 3m_{\text{cargo car}}$ in the equation for centripetal force:

$$\begin{aligned} F_{\text{locomotive}} &= \frac{m_{\text{locomotive}} v^2}{r} \\ &= \frac{\left(3m_{\text{cargo car}}\right) v^2}{r} \\ &= 3\frac{m_{\text{cargo car}} v^2}{r} \\ F_{\text{locomotive}} &= 3F_{\text{cargo car}} \end{aligned}$$

The centripetal force on the locomotive is three times the force on the cargo car.

**26. (a) Given:** $m = 2.0$ kg; $v = 20$ m/s; $r = 16$ m

**Required:** $F_c$

**Analysis:** $F_c = \dfrac{mv^2}{r}$

**Solution:** $F_c = \dfrac{mv^2}{r}$

$$\begin{aligned} &= \frac{(90\text{ kg})(20\text{ m/s})^2}{(16\text{ m})} \\ F_c &= 2\times10^3\text{ N} \end{aligned}$$

**Statement:** The centripetal force is $2 \times 10^3$ N.

**(b) Given:** $m = 2.0$ kg; $v = 20$ m/s; $r = 10$ m

**Required:** $F_c$

**Analysis:** $F_c = \dfrac{mv^2}{r}$

**Solution:** $F_c = \dfrac{mv^2}{r}$

$$\begin{aligned} &= \frac{(90\text{ kg})(20\text{ m/s})^2}{(10\text{ m})} \\ F_c &= 4\times10^3\text{ N} \end{aligned}$$

**Statement:** The centripetal force is $4 \times 10^3$ N.

**(c) Given:** $m = 2.0$ kg; $v = 5$ m/s; $r = 10$ m

**Required:** $F_c$

**Analysis:** $F_c = \dfrac{mv^2}{r}$

**Solution:** $F_c = \dfrac{mv^2}{r}$

$$\begin{aligned} &= \frac{(90\text{ kg})(5\text{ m/s})^2}{(10\text{ m})} \\ F_c &= 2\times10^2\text{ N} \end{aligned}$$

**Statement:** The centripetal force is $2 \times 10^2$ N.

**27. Given:** $m$ = 2.0 kg; $F_c = 2.8 \times 10^2$ N; $r$ = 1.00 m

**Required:** $F_c$

**Analysis:** $F_c = \dfrac{mv^2}{r}$

$$v = \sqrt{\frac{F_c r}{m}}$$

**Solution:** $v = \sqrt{\dfrac{F_c r}{m}}$

$$= \sqrt{\frac{\left(2.8\times10^2\ \cancel{\text{kg}}\cdot\frac{\text{m}}{\text{s}^2}\right)(1.00\ \text{m})}{\left(2.0\ \cancel{\text{kg}}\right)}}$$

$$v = 12\ \text{m/s}$$

**Statement:** The speed of the discus is 12 m/s when released.

**28. (a) Given:** $m$ = 2.0 kg; $r$ = 0.5 m; $f$ = 1.0 rpm = $\dfrac{1}{60}$ Hz

**Required:** $F_c$

**Analysis:** $F_c = ma_c$; $a_c = 4\pi^2 rf^2$; $F_c = 4\pi^2 mrf^2$

**Solution:** $F_c = 4\pi^2 mrf^2$

$$= 4\pi^2(2.0\ \text{kg})(0.5\ \text{m})\left(\frac{1}{60}\ \text{Hz}\right)^2$$

$$F_c = 0.011\ \text{N}$$

**Statement:** The centripetal force is 0.011 N.

**(b) Given:** $m$ = 2.0 kg; $r$ = 0.5 m; $f$ = 5.0 rpm = $\dfrac{1}{12}$ Hz

**Required:** $F_c$

**Analysis:** $F_c = ma_c$; $a_c = 4\pi^2 rf^2$; $F_c = 4\pi^2 mrf^2$

**Solution:** $F_c = 4\pi^2 mrf^2$

$$= 4\pi^2(2.0\ \text{kg})(0.5\ \text{m})\left(\frac{1}{12}\ \text{Hz}\right)^2$$

$$F_c = 0.27\ \text{N}$$

**Statement:** The centripetal force is 0.27 N.

**(c) Given:** $m$ = 2.0 kg; $r$ = 0.5 m; $f$ = 1.0 rpm = $\dfrac{1}{120}$ Hz

**Required:** $F_c$

**Analysis:** $F_c = ma_c$; $a_c = 4\pi^2 rf^2$; $F_c = 4\pi^2 mrf^2$

**Solution:** $F_c = 4\pi^2 mrf^2$

$$= 4\pi^2(2.0\text{ kg})(0.5\text{ m})\left(\frac{1}{120}\text{ Hz}\right)^2$$

$F_c = 0.0027\text{ N}$

**Statement:** The centripetal force is 0.0027 N.

**29.** The person's weight equals $mg$, so if the normal force of $1.1 \times 10^3$ N is the result of $2g$, then half the normal, or $5.5 \times 10^2$ N is the person's weight.

**Analysis and Application**

**30. Given:** $r = 0.030$ m; $f = 60$ rpm $= 1.0$ Hz

**Required:** $a_c$

**Analysis:** $a_c = 4\pi^2 rf^2$

**Solution:** $a_c = 4\pi^2 rf^2$

$$= 4\pi^2(0.030\text{ m})(1.0\text{ Hz})^2$$

$a_c = 1.2\text{ m/s}^2$

**Statement:** The centripetal acceleration is 1.2 m/s$^2$.

**31. Given:** $r = 0.50$ m; $m = 1.5$ kg; $F_T \le 25$ N

**Required:** maximum $v$

**Analysis:** The maximum speed of rotation is when the centripetal force equals the maximum tension before the string breaks.

$$F_c = \frac{mv^2}{r}$$

$$v = \sqrt{\frac{F_c r}{m}}$$

**Solution:** $v = \sqrt{\frac{F_c r}{m}}$

$$= \sqrt{\frac{\left(25\ \cancel{\text{kg}} \cdot \frac{\text{m}}{\text{s}^2}\right)(0.50\text{ m})}{(1.5\ \cancel{\text{kg}})}}$$

$v = 2.9\text{ m/s}$

**Statement:** The maximum speed of rotation before breaking the string is 2.9 m/s.

**32. Given:** $F_N = 3.5F_g$; $v = 26$ m/s

**Required:** $r$

**Analysis:** $F_c = \frac{mv^2}{r}$; $F_g = mg$; $F_N = F_c + F_g$

**Solution:**
$$\begin{aligned} F_N &= F_c + F_g \\ 3.5F_g &= F_c + F \\ 2.5F_g &= F_c \\ 2.5mg &= \frac{mv^2}{r} \\ r &= \frac{mv^2}{2.5mg} \\ &= \frac{v^2}{2.5g} \\ &= \frac{(26 \text{ m/s})^2}{2.5(9.8 \text{ m/s}^2)} \\ r &= 28 \text{ m} \end{aligned}$$

**Statement:** The radius of the track's curvature is 28 m.

**33. (a) Given:** $m$ = 35 kg; $d$ = 22 m or $r$ = 11 m; $f$ = 3.5 rpm = $\frac{3.5}{60}$ Hz

**Required:** $F_N$ at the top
**Analysis:** $F_c = 4\pi^2 mrf^2$; $F_N = F_g - F_c$
**Solution:**
$$\begin{aligned} F_N &= F_g - F_c \\ &= mg - 4\pi^2 mrf^2 \\ &= (35 \text{ kg})(9.8 \text{ m/s}^2) - 4\pi^2(35 \text{ kg})(11 \text{ m})\left(\frac{3.5}{60} \text{ Hz}\right)^2 \\ F_N &= 2.9 \times 10^2 \text{ N} \end{aligned}$$

**Statement:** The force of the seat is $2.9 \times 10^2$ N.

**(b) Given:** $m$ = 35 kg; $d$ = 22 m or $r$ = 11 m; $f$ = 3.5 rpm = $\frac{3.5}{60}$ Hz

**Required:** $F_N$ at the bottom
**Analysis:** $F_c = 4\pi^2 mrf^2$; $F_N = F_g + F_c$
**Solution:**
$$\begin{aligned} F_N &= F_g + F_c \\ &= mg + 4\pi^2 mrf^2 \\ &= (35 \text{ kg})(9.8 \text{ m/s}^2) + 4\pi^2(35 \text{ kg})(11 \text{ m})\left(\frac{3.5}{60} \text{ Hz}\right)^2 \\ F_N &= 3.9 \times 10^2 \text{ N} \end{aligned}$$

**Statement:** The force of the seat is $3.9 \times 10^2$ N.
**34. Given:** $m$ = 1.5 kg; $r$ = 2.0 m
**Required:** $F_T$ at the bottom
**Analysis:** Since the tension at the top of the loop equals the gravitational force, but the speed constant, the centripetal force must equal the gravitational force at all points along the circular path: $F_c = mg$; $F_T = F_g + F_c$

**Solution:** $F_T = F_g + F_c$

$= mg + mg$

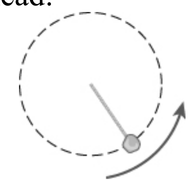
$= 2mg$

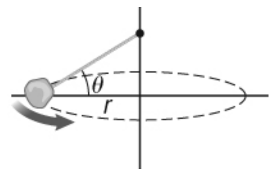
$= 2(1.5\text{ kg})(9.8\text{ m/s}^2)$

$F_T = 29\text{ N}$

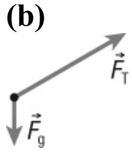
**Statement:** The tension when the rock is at the bottom is 29 N.

**35. (a)** Overhead: Side view:






**(b)**




**(c)** The centripetal force is the sum of the tension and the gravitational force. Given the angle, the centripetal force is equal to $F_T \cos\theta$.

**(d) Given:** $r$ = 1.5 m; $v$ = 10.0 m/s

**Required:** $\theta$

**Analysis:** $F_c = \dfrac{mv^2}{r}$; $F_g = mg$

$$\tan\theta = \frac{F_g}{F_c}$$

$$= \frac{mg}{\frac{mv^2}{r}}$$

$$= \frac{gr}{v^2}$$

$$\theta = \tan^{-1}\left(\frac{gr}{v^2}\right)$$

**Solution:** $\theta = \tan^{-1}\left(\dfrac{gr}{v^2}\right)$

$$= \tan^{-1}\left(\frac{\left(9.8\text{ kg}\cdot\frac{\cancel{\text{m}}}{\cancel{\text{s}^2}}\right)(1.5\ \cancel{\text{m}})}{\left(10.0\ \frac{\cancel{\text{m}}}{\cancel{\text{s}}}\right)^2}\right)$$

$\theta = 8.4°$

**Statement:** The string makes an 8.4° angle with the horizontal.

**36. Given:** $m = 1.7 \times 10^3$ kg; $r = 35$ m; $v = 12$ m/s
**Required:** $F_s$
**Analysis:** $F_c = \frac{mv^2}{r}$; $F_s = F_c$
**Solution:** $F_c = \frac{mv^2}{r}$

$$F_s = \frac{mv^2}{r}$$
$$= \frac{(1.7 \times 10^3 \text{ kg})(12 \text{ m/s})^2}{(35 \text{ m})}$$
$$F_s = 7.0 \times 10^3 \text{ N}$$

**Statement:** The friction force is $7.0 \times 10^3$ N.
**37. Given:** $m = 50.0$ kg; $r = 0.75$ m; $F_T \le 50.0$ N
**Required:** maximum $v$
**Analysis:** The maximum speed of rotation is when the centripetal force equals the maximum tension before the string breaks.

$$F_c = \frac{mv^2}{r}$$
$$v = \sqrt{\frac{F_c r}{m}}$$

**Solution:** $v = \sqrt{\frac{F_c r}{m}}$

$$= \sqrt{\frac{\left(50.0 \ \cancel{\text{kg}} \cdot \frac{\text{m}}{\text{s}^2}\right)(0.75 \text{ m})}{(0.30 \ \cancel{\text{kg}})}}$$
$$v = 11 \text{ m/s}$$

**Statement:** The maximum speed of rotation before breaking the string is 11 m/s.
**38. Given:** $m = 70.0$ kg; $v = 2.0$ m/s; $r = 1.0$ m
**Required:** $F_c$
**Analysis:** $F_c = \frac{mv^2}{r}$
**Solution:** $F_c = \frac{mv^2}{r}$

$$= \frac{(70.0 \text{ kg})(2.0 \text{ m/s})^2}{(1.0 \text{ m})}$$
$$F_c = 2.8 \times 10^2 \text{ N}$$

**Statement:** The centripetal force is $2.8 \times 10^2$ N.

**39. Given:** $m$ = 30.0 kg; $d$ = 20.0 m or $r$ = 10.0 m; $F_c$ = 32 N
**Required:** $v$

**Analysis:** $F_c = \dfrac{mv^2}{r}$

$$v = \sqrt{\frac{F_c r}{m}}$$

**Solution:** $v = \sqrt{\dfrac{F_c r}{m}}$

$$= \sqrt{\frac{\left(32\ \cancel{\text{kg}} \cdot \dfrac{\text{m}}{\text{s}^2}\right)(10.0\ \text{m})}{(30.0\ \cancel{\text{kg}})}}$$

$$v = 3.3\ \text{m/s}$$

**Statement:** The speed of the child cycling is 3.3 m/s.

**40. Given:** $v$ = 55 m/s; $m$ = 125 kg; $r$ = 25 m
**Required:** $F_c$

**Analysis:** $F_c = \dfrac{mv^2}{r}$

**Solution:** $F_c = \dfrac{mv^2}{r}$

$$= \frac{(125\ \text{kg})(55\ \text{m/s})^2}{(25\ \text{m})}$$

$$F_c = 1.5 \times 10^4\ \text{N}$$

**Statement:** The centripetal force is $1.5 \times 10^4$ N.

**41. (a) Given:** $r$ = 2 m; $v$ = 2 m/s; $F_c$ = 16 N
**Required:** $m$

**Analysis:** $F_c = \dfrac{mv^2}{r}$

$$m = \frac{F_c r}{v^2}$$

**Solution:** $m = \dfrac{F_c r}{v^2}$

$$= \frac{\left(16\ \text{kg} \cdot \dfrac{\cancel{\text{m}}}{\cancel{\text{s}^2}}\right)(2\ \cancel{\text{m}})}{\left(2\ \dfrac{\cancel{\text{m}}}{\cancel{\text{s}}}\right)^2}$$

$$m = 8\ \text{kg}$$

**Statement:** The mass of the boat is 8 kg.

**(b) Given:** $r$ = 2 m; $m$ = 8 kg; $F_c$ = 4 N
**Required:** $v$

**Analysis:** $F_c = \dfrac{mv^2}{r}$

$$v = \sqrt{\frac{F_c r}{m}}$$

**Solution:** $v = \sqrt{\dfrac{F_c r}{m}}$

$$= \sqrt{\frac{\left(4\ \cancel{\text{kg}} \cdot \frac{\text{m}}{\text{s}^2}\right)(2\ \text{m})}{\left(8\ \cancel{\text{kg}}\right)}}$$

$$v = 1\ \text{m/s}$$

**Statement:** The speed should be decreased to 1 m/s.

**42. (a)** Substitute $v_2 = 2v_1$ in the equation for centripetal force:

$$F_2 = \frac{mv_2{}^2}{r}$$

$$= \frac{m(2v_1)^2}{r}$$

$$= 4\frac{mv_1{}^2}{r}$$

$$F_2 = 4F_1$$

The centripetal force on the car is four times the force at half the speed.

**(b)** If there were no friction, the car would not be able to turn and would continue in the same direction.

**43. Given:** $m$ = 105 kg; $v$ = 7.0 m/s; $r$ = 15 m
**Required:** $F_c$

**Analysis:** $F_c = \dfrac{mv^2}{r}$

**Solution:** $F_c = \dfrac{mv^2}{r}$

$$= \frac{(105\ \text{kg})(7.0\ \text{m/s})^2}{(15\ \text{m})}$$

$$F_c = 3.4 \times 10^2\ \text{N}$$

**Statement:** The centripetal force is $3.4 \times 10^2$ N.

**44. Given:** $T$ = 2.00 s; $m_1$ = 3.00 kg; $m_2$ = 5.00 kg; $r_1$ = 4.00 m; $r_2$ = 6.00 m
**Required:** $F_c$

**Analysis:** $F_c = \dfrac{4\pi^2 mr}{T^2}$

**Solution:** Determine the tension in string B:

$$F_B = \frac{4\pi^2 m_2 r_2}{T^2}$$
$$= \frac{4\pi^2 (5.00\text{ kg})(6.00\text{ m})}{(2.00\text{ s})^2}$$
$$F_B = 296\text{ N}$$

Determine the tension in string A:

$$F_A = \frac{4\pi^2 m_1 r_1}{T^2} + F_B$$
$$= \frac{4\pi^2 (3.00\text{ kg})(4.00\text{ m})}{(2.00\text{ s})^2} + 296\text{ N}$$
$$F_A = 414\text{ N}$$

**Statement:** The tension in string A is 414 N and the tension in string B is 296 N.

**45. (a) Given:** $m_1 = 2.0$ kg; $m_2 = 5.0$ kg; $\mu_s = 0.30$; $r = 5.0$ m

**Required:** maximum $v$

**Analysis:** $F_c = \frac{mv^2}{r}$; $F_s = \mu_s mg$; The maximum speed is the point at which the centripetal acceleration on mass 1 equals the force of static friction keeping mass 1 on top of mass 2.

**Solution:**
$$F_c = F_s$$
$$\frac{mv^2}{r} = \mu_s mg$$
$$v^2 = \frac{\mu_s mgr}{m}$$
$$v = \sqrt{\mu_s gr}$$
$$= \sqrt{(0.30)(9.8\text{ m/s}^2)(5.0\text{ m})}$$
$$v = 3.8\text{ m/s}$$

**Statement:** The maximum speed is 3.8 m/s.

**(b) Given:** $m_1 = 2.0$ kg; $m_2 = 5.0$ kg; $\mu_s = 0.30$; $r = 5.0$ m

**Required:** $F_T$

**Analysis:** $F_c = \frac{mv^2}{r}$

**Solution:**
$$F_c = \frac{mv^2}{r}$$
$$F_T = \frac{(m_1 + m_2)v^2}{r}$$
$$= \frac{(2.0\text{ kg} + 5.0\text{ kg})(3.834\text{ m/s})^2}{(5.0\text{ m})}$$
$$F_T = 21\text{ N}$$

**Statement:** The tension in the string at the maximum speed is 21 N.

**46. Given:** $T$ = 4.0 s; $m_1$ = 1200 kg; $m_2$ = 1800 kg; $r_1$ = 4.0 m; $r_2$ = 4.0 m + 3.0 m = 7.0 m

**Required:** $F_T$ at the bottom for each support

**Analysis:** $F_c = \dfrac{4\pi^2 mr}{T^2}$ ; $F_T = F_g + F_c$

**Solution:** Determine the tension in support B:

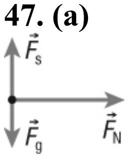
$$\begin{aligned} F_{T_B} &= F_g + F_c \\ &= mg + \frac{4\pi^2 mr}{T^2} \\ &= (1800\text{ kg})(9.8\text{ m/s}^2) + \frac{4\pi^2(1800\text{ kg})(7.0\text{ m})}{(4.0\text{ s})^2} \\ &= 4.873\times10^4\text{ N (two extra digits carried)} \\ F_{T_B} &= 4.9\times10^4\text{ N} \end{aligned}$$

Determine the tension in support A:

$$\begin{aligned} F_{T_A} &= F_g + F_c + F_{T_B} \\ &= mg + \frac{4\pi^2 mr}{T^2} \\ &= (1200\text{ kg})(9.8\text{ m/s}^2) + \frac{4\pi^2(1200\text{ kg})(4.0\text{ m})}{(4.0\text{ s})^2} + 4.873\times10^4\text{ N} \\ F_{T_A} &= 7.2\times10^4\text{ N} \end{aligned}$$

**Statement:** The tension in support A is $7.2 \times 10^4$ N and the tension in support B is $4.9 \times 10^4$ N.

**47. (a)**




The rider feels the gravitational force, centripetal force, and the force due to static friction.

**(b) Given:** $r$ = 3.0 m; $\mu_s$ = 0.40

**Required:** minimum $v$

**Analysis:** In this scenario the normal is horizontal and opposite the centripetal force instead of the gravitational force. Determine the speed at which the force due to static friction equals the gravitational force; $F_c = \dfrac{mv^2}{r}$ ; $F_s = \mu_s F_c$

**Solution:** $F_g = F_s$

$$mg = \mu_s F_c$$

$$mg = \mu_s \left(\frac{mv^2}{r}\right)$$

$$g = \frac{\mu_s v^2}{r}$$

$$v^2 = \frac{gr}{\mu_s}$$

$$v = \sqrt{\frac{gr}{\mu_s}}$$

$$= \sqrt{\frac{(9.8 \text{ m/s}^2)(3.0 \text{ m})}{(0.40)}}$$

$$v = 8.6 \text{ m/s}$$

**Statement:** The minimum speed of rotation to keep the person stuck to the wall 8.6 m/s.

**48. Given:** $m$ = 6.0 kg; $r_A$ = 5.0 m; $r_B$ = 5.0 m; $h$ = 8.0 m; $v$ = 12 m/s

**Required:** $F_T$ in each string

**Analysis:** In this scenario the tensions will not be equal because of the gravitational force. The *x*-components of the tensions will balance the centripetal force while the *x*-components of the tensions balance the gravitational force; $F_c = \dfrac{mv^2}{r}$; $F_s = \mu_s F_c$. The radius of the circular path is one side of a right triangle with string A as the hypotenuse and half the height cylinder as the other side.

**Solution:** Determine the radius of the circular path using the Pythagorean theorem:

$$r^2 + \left(\frac{1}{2}h\right)^2 = r_A^2$$

$$r^2 = r_A^2 - \left(\frac{1}{2}h\right)^2$$

$$r = \sqrt{r_A^2 - \left(\frac{1}{2}h\right)^2}$$

$$= \sqrt{(5.0 \text{ m})^2 - \left(\frac{1}{2}(8.0 \text{ m})\right)^2}$$

$$r = 3.0 \text{ m}$$

Determine the angle the strings make with the horizontal:

$$\cos\theta = \frac{4.0 \text{ m}}{5.0 \text{ m}}$$

$$\theta = 36.87°$$

Also, we can use cos $\theta$ = 0.8 (and therefore sin $\theta$ = 0.6) later in the solution.

Determine the centripetal force on the mass:

$$F_c = \frac{mv^2}{r}$$
$$= \frac{(6.0\text{ kg})(12\text{ m/s})^2}{(3.0\text{ m})}$$
$$F_c = 288\text{ N}$$

Express the balancing of the horizontal forces and isolate one of the tensions:

$$F_{\text{TA}_x} + F_{\text{TB}_x} = F_c$$
$$F_{\text{TA}}\sin\theta + F_{\text{TB}}\sin\theta = 288\text{ N}$$
$$F_{\text{TA}} = \frac{288\text{ N} - F_{\text{TB}}\sin\theta}{\sin\theta}$$

Express the balancing of the vertical forces and solve for the tension on string B:

$$F_{\text{TA}_y} - F_{\text{TB}_y} = F_g$$
$$F_{\text{TA}}\cos\theta - F_{\text{TB}}\cos\theta = mg$$
$$\left(\frac{288\text{ N} - F_{\text{TB}}\sin\theta}{\sin\theta}\right)\cos\theta - F_{\text{TB}}\cos\theta = (6.0\text{ kg})(9.8\text{ m/s}^2)$$
$$\frac{(288\text{ N})\cos\theta}{\sin\theta} - \frac{F_{\text{TB}}\cancel{\sin\theta}\cos\theta}{\cancel{\sin\theta}} - F_{\text{TB}}\cos\theta = 58.8\text{ N}$$
$$\frac{(288\text{ N})\cos\theta}{\sin\theta} - 2F_{\text{TB}}\cos\theta = 58.8\text{ N}$$
$$\frac{(288\text{ N})(0.8)}{(0.6)} - 2F_{\text{TB}}(0.8) = 58.8\text{ N}$$
$$384\text{ N} - 1.6F_{\text{TB}} = 58.8\text{ N}$$
$$F_{\text{TB}} = \frac{58.8\text{ N} - 384\text{ N}}{-1.6}$$
$$= 203.2\text{ N}$$
$$F_{\text{TB}} = 2.0 \times 10^2\text{ N}$$

Solve for the tension on string A:

$$F_{\text{TA}} = \frac{288\text{ N} - F_{\text{TB}}\sin\theta}{\sin\theta}$$
$$= \frac{288\text{ N} - (203.2\text{ N})(0.6)}{(0.6)}$$
$$F_{\text{TA}} = 2.8 \times 10^2\text{ N}$$

**Statement:** The tension in string A is $2.8 \times 10^2$ N and the tension in string B is $2.0 \times 10^2$ N.

**49. Given:** $f = 2\text{ rpm} = \dfrac{2}{60}\text{ Hz}$; $r = 30.0$ m; $m = 9.8 \times 10^2$ kg

**Required:** $F_c$

**Analysis:** $F_c = ma_c$; $a_c = 4\pi^2 rf^2$

**Solution:**
$$\begin{aligned} F_c &= ma_c \\ &= m\left(4\pi^2 rf^2\right) \\ &= 4\pi^2\left(9.8\times10^2\text{ kg}\right)\left(30.0\text{ m}\right)\left(\frac{2}{60}\text{ Hz}\right)^2 \\ F_c &= 1.3\times10^3\text{ N} \end{aligned}$$

**Statement:** The driver will feel a force of $1.3 \times 10^3$ N.

**50. Given:** $m = 2.0$ kg; $r = 0.35$ m; $f = 50.0\text{ rpm} = \dfrac{50.0}{60}\text{ Hz}$

**Required:** $F_c$

**Analysis:** $F_c = ma_c$; $a_c = 4\pi^2 rf^2$

**Solution:**
$$\begin{aligned} F_c &= ma_c \\ &= m\left(4\pi^2 rf^2\right) \\ &= 4\pi^2\left(2.0\text{ kg}\right)\left(0.35\text{ m}\right)\left(\frac{50.0}{60}\text{ Hz}\right)^2 \\ F_c &= 19\text{ N} \end{aligned}$$

**Statement:** The force acting on the clothes is 19 N.

**51. Given:** $f = 0.30\text{ rps} = 0.30$ Hz; $r = 15\text{ cm} = 0.15$ m

**Required:** $\mu_s$

**Analysis:** $F_c = \dfrac{mv^2}{r}$; $F_s = \mu_s mg$; The speed is the point at which the centripetal acceleration on the coin equals the force of static friction.

**Solution:**
$$\begin{aligned} F_c &= F_s \\ 4\pi^2 mrf^2 &= \mu_s mg \\ 4\pi^2 rf^2 &= \mu_s g \\ \mu_s &= \frac{4\pi^2 rf^2}{g} \\ &= \frac{4\pi^2\left(0.15\text{ m}\right)\left(0.30\text{ Hz}\right)^2}{\left(9.8\text{ m/s}^2\right)} \\ \mu_s &= 0.053 \end{aligned}$$

**Statement:** The coefficient of static friction is 0.053.

**52. Given:** $r$ = 20.0 m; $F_N = 3.00F_g$
**Required:** $v$

**Analysis:** $F_c = \frac{mv^2}{r}$; $F_g = mg$; The normal force at the bottom of the loop is the sum of the centrifugal force and the gravitational force.

**Solution:**
$$\begin{aligned}
F_c + F_g &= 3.00F_g \\
F_c &= 2.00F_g \\
\frac{mv^2}{r} &= 2.00mg \\
\frac{v^2}{r} &= 2.00g \\
v &= \sqrt{2.00gr} \\
&= \sqrt{2.00(9.8\text{ m/s}^2)(20.0\text{ m})} \\
v &= 19.8\text{ m/s}
\end{aligned}$$

**Statement:** The speed at the bottom of the loop is 19.8 m/s.

**53. Given:** $v$ = 12 m/s; $a_c = 0.500g$
**Required:** $r$

**Analysis:** $F_c = \frac{mv^2}{r}$; $F_c = ma_c$

**Solution:**
$$\begin{aligned}
F_c &= ma_c \\
\frac{\cancel{m}v^2}{r} &= \cancel{m}a_c \\
\frac{v^2}{r} &= 0.500g \\
r &= \frac{v^2}{0.500g} \\
&= \frac{(12\text{ m/s})^2}{0.500(9.8\text{ m/s}^2)} \\
r &= 29\text{ m}
\end{aligned}$$

**Statement:** The radius of the station is 29 m.

**54. (a)** The bucket moves in a circle because of the fictitious centrifugal force.
**(b)** The source of the centrifugal force is the tension in the rope.
**(c) Given:** $m$ = 15 kg; $v$ = 2 m/s; $r$ = 2 m
**Required:** $F_c$

**Analysis:** $F_c = \frac{mv^2}{r}$

**Solution:** $F_c = \dfrac{mv^2}{r}$

$= \dfrac{(15\text{ kg})(2\text{ m/s})^2}{(2\text{ m})}$

$F_c = 30\text{ N}$

**Statement:** The magnitude of the force is 30 N.

**55. (a) Given:** $d$ = 4 m or $r$ = 2 m; $m$ = 65 kg + 95 kg = 160 kg; $f$ = 22 rpm = $\dfrac{22}{60}$ Hz

**Required:** $\mu_s$

**Analysis:** $F_c = \dfrac{mv^2}{r}$; $F_s = \mu_s F_c$; The force of static friction needs to offset the gravitation force.

**Solution:**

$$\begin{aligned}
F_g &= F_s \\
mg &= \mu_s F_c \\
mg &= \mu_s 4\pi^2 mrf^2 \\
g &= \mu_s 4\pi^2 rf^2 \\
\mu_s &= \frac{g}{4\pi^2 rf^2} \\
&= \frac{(9.8\text{ m/s}^2)}{4\pi^2(2\text{ m})\left(\dfrac{22}{60}\text{ Hz}\right)^2} \\
\mu_s &= 0.92
\end{aligned}$$

**Statement:** The coefficient of static friction is 0.92.

**(b)** The frequency of the rider is $\dfrac{22}{60}$ Hz or 0.37 loops/s.

**(c) Given:** $r$ = 2 m; $m$ = 65 kg + 95 kg = 160 kg; $v$ = 6 m/s

**Required:** $F_N$ at top and bottom

**Analysis:** $F_c = \dfrac{mv^2}{r}$; The force at the top of the sphere is the difference between the centrifugal force and the gravitational force: $F_N = F_c - F_g$. The force at the bottom of the sphere is the sum of the centrifugal force and the gravitational force: $F_N = F_c + F_g$.

**Solution:** Determine the force at the top of the sphere:

$$\begin{aligned}
F_N &= F_s - F_g \\
&= \frac{mv^2}{r} - mg \\
&= \frac{(160\text{ kg})(6.0\text{ m/s})^2}{(2\text{ m})} - (160\text{ kg})(9.8\text{ m/s}^2) \\
F_N &= 1.3\times10^3\text{ N}
\end{aligned}$$

Determine the force at the bottom of the sphere:

$$\begin{aligned}F_\text{N} &= F_\text{s} + F_\text{g}\\ &= \frac{mv^2}{r} + mg\\ &= \frac{(160\text{ kg})(6.0\text{ m/s})^2}{(2\text{ m})} + (160\text{ kg})(9.8\text{ m/s}^2)\\ F_\text{N} &= 4.4\times10^3\text{ N}\end{aligned}$$

**Statement:** The normal force at the top of the sphere $1.3 \times 10^3$ N. The normal force at the bottom of the sphere $4.4 \times 10^3$ N.

**56. Given:** $r$ = 3.0 m; $f$ = 2.0 rpm = $\frac{2}{60}$ Hz; $m$ = 54 kg

**Required:** $F_\text{c}$

**Analysis:** $F_\text{c} = 4\pi^2 mrf^2$

**Solution:** $F_\text{c} = 4\pi^2 mrf^2$

$$\begin{aligned}&= 4\pi^2(54\text{ kg})(3.0\text{ m})\left(\frac{2}{60}\text{ Hz}\right)^2\\ F_\text{c} &= 7.1\text{ N}\end{aligned}$$

**Statement:** The magnitude of the centripetal force is 7.1 N.

**57. Given:** $m$ = 450 kg; $f$ = 1 rpm = $\frac{1}{60}$ Hz; $F_\text{c}$ = 48 N

**Required:** $r$

**Analysis:** $F_\text{c} = 4\pi^2 mrf^2$

**Solution:** $F_\text{c} = 4\pi^2 mrf^2$

$$\begin{aligned}r &= \frac{F_\text{c}}{4\pi^2 mf^2}\\ &= \frac{48\text{ N}}{4\pi^2(450\text{ kg})\left(\frac{1}{60}\text{ Hz}\right)^2}\\ r &= 9.7\text{ m}\end{aligned}$$

**Statement:** The length of the lead rope is 9.7 m.

**58. Given:** $r$ = 25 m; $a_\text{c}$ = 1.0$g$

**Required:** $v$

**Analysis:** $a_\text{c} = \frac{v^2}{r}$

**Solution:** $a_c = \dfrac{v^2}{r}$

$$\begin{aligned}
v &= \sqrt{a_c r} \\
&= \sqrt{1.0gr} \\
&= \sqrt{1.0(9.8\ \text{m/s}^2)(25\ \text{m})} \\
&= 15.65\ \frac{\cancel{\text{m}}}{\text{s}} \times \frac{60\ \cancel{\text{s}}}{1\ \cancel{\text{min}}} \times \frac{60\ \cancel{\text{min}}}{1\ \text{h}} \times \frac{1\ \text{km}}{1000\ \cancel{\text{m}}} \\
v &= 56\ \text{km/h}
\end{aligned}$$

**Statement:** The speed of the horse is 56 km/h.

**Evaluation**

**59. (a)** The centrifugal force is the fictitious force that occurs in a rotating frame of reference that is directed away from the centre of rotation.

**(b)** The Coriolis force is the fictitious force in a rotating frame of reference that acts perpendicular to an object's velocity.

**(c)** Fictitious forces are not actual forces; they only appear when the natural frame of reference for a given situation is itself accelerating and they are always proportional to the mass of the object on which they act.

**60. (a)** As a person rides an elevator to the top of the CN Tower, the frame of reference accelerates upward. The person will experience an increase in the normal force and their apparent weight increases.

**(b)** As a person is in free fall while skydiving, the normal force acting on the person is zero and the person also has an apparent weight of zero.

**61. (a)** Equations for Centripetal Acceleration

| Equation | Variables in equation | Variables not in equation |
|---|---|---|
| $a_c = \dfrac{v^2}{r}$ | $v$, $r$ | $T$, $f$ |
| $a_c = \dfrac{4\pi^2 r}{T^2}$ | $r$, $T$ | $v$, $f$ |
| $a_c = 4\pi^2 r f^2$ | $r$, $f$ | $v$, $T$ |

**(b)** The first equation follows from the definition of average acceleration, similar triangles, and looking at very small periods of time. The second equation follows from the first equation by inserting the circumference of the circular path for distance and using period of revolution for the time interval in the basic equation for speed. The third equation follows from the second equation by simply substituting $\dfrac{1}{f}$ for $T$.

**62. (a)** Twirling the rope allows her to increase its speed while it travels in a circular path.

**(b)** By spinning the rope faster, she can ensure it has a greater speed when it is released. This will help the rope go farther.

**(c)** The rope will travel away from the rodeo performer in a straight-line path in the direction it was moving when she let go.

**63.** Banked off- and on-ramps on highways use centripetal forces to allow vehicles to stay on curved paths. The banked curve provides a component of the normal force in the direction of the centripetal acceleration towards the centre, making it more difficult for a car driving on a curve to leave the road. The angles are calculated to ensure safe driving conditions when the coefficient of static friction between tires and the road is high (dry conditions) and low (wet or icy conditions).
**64.** Answers may vary. Sample answer: I would use the analogy of a bucket of water being swung around in a circle. Just as the water stays in the bucket, a rotating space station forces astronauts to the outer walls with the centrifugal force acting to simulate gravity.

**Reflect on Your Learning**
**65.** Answers may vary. Sample answer: The thing I found most surprising was the speeds required for artificial gravity on a spacecraft and that a small radius on the spacecraft means that the artificial gravity will feel different at one's feet compared to one's head.
**66.** Answers may vary. Sample answer: I was not comfortable working with frequency and periods. Also I did not fully understand the difference between a centripetal force and a centrifugal force.
**67.** Answers may vary. Sample answer:

| Know | What I Learned | What I Would Like to Learn |
|---|---|---|
| • artificial gravity is not the gravity I experience with the pull that Earth exerts on me | • artificial gravity is a man-made situation where the value of gravity is changed artificially<br>• rotating a spacecraft at an appropriate frequency can simulate the effects of Earth's gravity | • Does the current space station use rotation to create artificial gravity? |

**68.** Answers may vary. Sample answer: I learned that centrifugal forces are fictitious forces. They help to better explain artificial gravity and objects moving as a frame of reference accelerates.
**69.** Answers may vary. Students' should report on topic from the chapter that has an impact on them. For example, drivers will be interested in the importance of banked curves for driving and how the angle must be carefully selected so that it is safe for dry and slippery conditions when the coefficient of static friction decreases.

**Research**
**70.** Answers may vary. Students' answers should include historic moments such as the earliest roller coasters, the first circular loops, and the introduction of clothoid loops. Students may wish to go into recent developments in roller coasters and research maximum speeds and forces of acceleration.
**71.** Answers may vary. Students should address that storms such as hurricanes tend to rotate counterclockwise in the Northern Hemisphere and clockwise in the Southern Hemisphere as a result of the Coriolis force.

**72.** Answers may vary. Students' answers should include the concept of geotropism (sometimes called gravitropism), whereby seedlings grow upward in response to gravity. In a rotating frame of reference, plants would therefore grow in response to the fictitious centrifugal force, into the centre of rotation.
**73.** Answers may vary. Students should explain how the centrifuge separates the heavier isotopes from the lighter ones while the uranium is in gaseous form as uranium hexafluoride, helping to get higher concentrations of the desired uranium-235 isotope. With higher concentrations of uranium-235, nuclear power plants are more efficient because energy is not wasted working with other uranium isotopes.
**74.** Answers may vary. Students may explore changes in training programs, which requires astronauts to be capable of enduring the intense acceleration of take-off and the disorientation of working in the free fall conditions of the space station. Early astronauts were trained test pilots while modern astronauts are more likely to be academics. Thanks to the introduction of shuttles, astronauts are no longer returning to Earth at sea, so there has been a de-emphasis on survival skills. Students should also discuss the knowledge requirements in addition to the physical testing.
Students should discuss health risks such as muscle loss and decreased bone density, and how astronauts must exercise in space to avoid excessive muscle loss. To combat the health risks associated with extended space travel, research into artificial gravity, better exercise equipment, and improved life-support systems is being conducted.
**75. (a)** Answers may vary. Students' should present information on the historical use of centrifuges in an industry. For example, simple, hand-cranked centrifuges have been used to separate the cream in fresh milk since the 19th century. The technology is still used today, however the centrifuges are automated and more efficient.
**(b)** Answers may vary. Centrifuges have been useful for separating and purifying for over a century. Students may list medical research, cancer treatment, fuel purification, sewage treatment, or something as simple as a home washing machine.
**76.** Answers may vary. Students' concept maps should include that although the roller coaster does not include any loops, it does have multiple tight turns (with banked tracks) where riders will experience a centripetal force that is the sum of the gravitational force and the normal force. Riders will also experience a centrifugal force since the roller coaster is a non-inertial frame of reference.
**77. (a)** For optimal performance, a wind turbine or windmill is pointed directly at the direction the wind is coming from. The kinetic energy of wind is transferred into the curved blades of the wind turbine. The blades are attached to a rotor that rotates with them. The rotation of the rotor causes a shaft into the body of the wind turbine to rotate. A generator in the head of the wind turbine transfers the circular motion of the shaft into alternating current. Students' diagrams must include the blades, attached to the rotor, pointing into the wind, the rotating shaft, and the generator.
**(b)** Using wind power reduces the environmental impact of power generation by reducing pollutants associated with nuclear or fossil fuel plants. Wind power does not require coal mining or oil drilling. One negative aspect is the effect on migratory birds that can be injured or killed by flying into wind turbines.
**78.** Answers may vary. Students' designs should take advantage of material they have learned in this chapter, such as the advantage of clothoid loops and the benefits of banked curves for maintaining greater speeds.