Section 3.1: Inertial and Non-inertial Frames of Reference

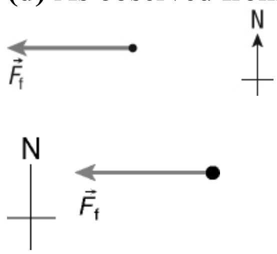
Tutorial 1 Practice, page 110

1. (a) When the car is moving with constant velocity, I see the ball lie still on the floor. I would see the same situation when the car is at rest.

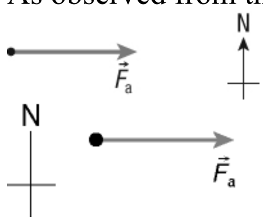
(b) To an observer on the sidewalk, the ball appears to be moving with a constant velocity of 14 m/s [E].

(c) If the car accelerates forward, I see the ball roll backward on the floor.

(d) As observed from the frame of reference of the car:



As observed from the frame of reference of the sidewalk:



The car's frame of reference is non-inertial. I observe a fictitious force (the force pushing the ball backward, west) in the car's frame of reference.

2. (a) Given: m = 22.0 g = 0.0220 kg; θ = 32.5°

Required: a

Analysis: Look at the situation from an Earth (inertial) frame of reference. The horizontal component of the tension F_T balances the acceleration and the vertical component of the tension F_T balances the gravitational force. Express the components of the tension in terms of the horizontal and vertical applied forces. Then calculate the magnitude of the acceleration.

Solution: Vertical component of force:

$$\Sigma F_y = 0$$

$$F_T \cos\theta - mg = 0$$

$$F_T = \frac{mg}{\cos\theta}$$

Horizontal component of force:

$$\Sigma F_x = ma$$
$$F_T \sin\theta = ma$$
$$\left(\frac{\cancel{m}g}{\cos\theta}\right)\sin\theta = \cancel{m}a$$
$$g\left(\frac{\sin\theta}{\cos\theta}\right) = a$$
$$g\tan\theta = a$$
$$a = (9.8\ \text{m/s}^2)\tan 32.5°$$
$$a = 6.2\ \text{m/s}^2$$

Statement: The magnitude of the boat's acceleration is 6.2 m/s^2. I did not need to know the mass of the ball to make the calculation because that value was cancelled out to obtain the forces.

(b) Given: m = 22.0 g = 0.0220 kg; θ = 32.5°

Required: F_T

Analysis: From part (a), $F_T = \dfrac{mg}{\cos\theta}$.

Solution:
$$F_T = \frac{mg}{\cos\theta} = \frac{(0.0220\ \text{kg})(9.8\ \text{m/s}^2)}{\cos 32.5°}$$
$$F_T = 0.26\ \text{N}$$

Statement: The magnitude of the tension in the string is 0.26 N. I need to know the mass to make this calculation because the force is the mass multiplied by the acceleration.

3. (a) When the subway is moving at a constant velocity, there is no tension in the strap.

(b) Given: m = 14 kg; θ = 35°; a = 1.4 m/s^2

Required: F_T

Analysis: Look at the situation from an Earth (inertial) frame of reference. The horizontal component of the tension F_T balances the acceleration, so express the x-components of the tension in terms of the horizontal applied forces. The vertical forces of gravity and the normal force balance each other.

$$\Sigma F_x = ma$$
$$F_T \sin\theta = ma$$
$$F_T = \frac{ma}{\sin\theta}$$

Solution:
$$F_T = \frac{ma}{\sin\theta} = \frac{(14\ \text{kg})(1.4\ \text{m/s}^2)}{\tan 35°}$$
$$F_T = 34\ \text{N}$$

Statement: The tension on the strap during acceleration is 34 N.

4. Given: $\mu_s = 0.42$
Required: maximum a
Analysis: The force due to the acceleration of the train, $F = ma$, must be less than the force of static friction, $F_s = \mu_s F_N$, where $F_N = mg$ $F_N = mg$. So the maximum acceleration occurs when $F = F_s$.

$$\begin{aligned} F &= F_s \\ ma &= \mu_s F_N \\ \cancel{m}a &= \mu_s(\cancel{m}g) \\ a &= \mu_s g \end{aligned}$$

Solution:
$$\begin{aligned} a &= \mu_s g \\ &= (0.42)(9.8 \text{ m/s}^2) \\ a &= 4.1 \text{ m/s}^2 \end{aligned}$$

Statement: The maximum acceleration of the train before the passenger begins to slip along the floor is 4.1 m/s^2.

Tutorial 2 Practice, page 112

1. (a) Given: $m = 55$ kg; $\vec{a} = 2.9$ m/s^2 [up]
Required: F_N
Analysis: Use up as positive and solve for the normal force when $+F_N + (-mg) = ma$.

$$\begin{aligned} +F_N + (-mg) &= ma \\ F_N &= ma + mg \end{aligned}$$

Solution:
$$\begin{aligned} F_N &= ma + mg \\ &= (55 \text{ kg})(2.9 \text{ m/s}^2) + (55 \text{ kg})(9.8 \text{ m/s}^2) \\ F_N &= 7.0 \times 10^2 \text{ N} \end{aligned}$$

Statement: The student's apparent weight is 7.0×10^2 N.

(b) Given: $m = 55$ kg; $\vec{a} = 2.9$ m/s^2 [down]
Required: F_N
Analysis: Use down as positive and solve for the normal force when $-F_N + (mg) = ma$.

$$\begin{aligned} -F_N + (mg) &= ma \\ F_N &= ma - mg \end{aligned}$$

Solution:
$$\begin{aligned} F_N &= mg - ma \\ &= (55 \text{ kg})(9.8 \text{ m/s}^2) - (55 \text{ kg})(2.9 \text{ m/s}^2) \\ F_N &= 3.8 \times 10^2 \text{ N} \end{aligned}$$

Statement: The student's apparent weight is 3.8×10^2 N.

2. (a) Given: m_1 = 9.5 kg; m_2 = 2.5 kg; F_N = 70.0 N
Required: $\vec{a}$
Analysis: Use up as positive and solve for the acceleration when $+F_N + (-mg) = ma$.

$$+F_N + (-mg) = ma$$
$$a = \frac{F_N - mg}{m}$$
$$= \frac{F_N - (m_1 + m_2)g}{m_1 + m_2}$$

Solution: $a = \dfrac{F_N - (m_1 + m_2)g}{m_1 + m_2}$

$$= \frac{70.0\ \cancel{\text{kg}} \cdot \dfrac{\text{m}}{\text{s}^2} - (9.5\ \cancel{\text{kg}} + 2.5\ \cancel{\text{kg}})(9.8\ \text{m/s}^2)}{9.5\ \cancel{\text{kg}} + 2.5\ \cancel{\text{kg}}}$$
$$= -3.967\ \text{m/s}^2\ \text{(two extra digits carried)}$$
$$a = -4.0\ \text{m/s}^2$$

Statement: The acceleration of the elevator is 4.0 m/s^2 [down].
(b) Given: m = 2.5 kg; $\vec{a}$ = 3.967 m/s^2 [down]
Required: $\vec{F}_N$
Analysis: Use down as positive and solve for the normal force when $-F_N + (mg) = ma$.
Solution: $-F_N + (mg) = ma$

$$F_N = mg - ma$$
$$= (2.5\ \text{kg})(9.8\ \text{m/s}^2) - (2.5\ \text{kg})(3.967\ \text{m/s}^2)$$
$$F_N = 15\ \text{N}$$

Statement: The force on the smaller box is 15 N [up].
3. (a) Given: m_1 = 4.2 kg; m_2 = 2.6 kg; a = 0
Required: F_{TA}; F_{TB}
Analysis: Since this is an inertial frame of reference, the tension on rope B balances the force of gravity on block 2. The tension on rope A balances the force of gravity on block 1 and the tension on rope B. $F_g = mg$
Solution: Determine the force on rope B:

$$F_{TB} = m_2 g$$
$$= (2.6\ \text{kg})(9.8\ \text{m/s}^2)$$
$$= 25.48\ \text{N (two extra digits carried)}$$
$$F_{TB} = 25\ \text{N}$$

Determine the force on rope A:

$$F_{TA} = m_1 g + F_{TB}$$
$$= (4.2\ \text{kg})(9.8\ \text{m/s}^2) + 25.48\ \text{N}$$
$$F_{TA} = 67\ \text{N}$$

Statement: The tension on the rope A is 67 N and the tension on the rope B is 25 N.

(b) Given: $m_1 = 4.2$ kg; $m_2 = 2.6$ kg $\vec{a} = 1.2$ m/s^2 [up]
Required: F_{TA}; F_{TB}
Analysis: This is now a non-inertial frame of reference, so instead of just gravity, the acceleration is $g - (-a)$, or $g + a$, where down is positive.
Solution: Determine the force on rope B:

$$\begin{aligned} F_{TB} &= m_2(g+a) \\ &= (2.6\text{ kg})(9.8\text{ m/s}^2 + 1.2\text{ m/s}^2) \\ &= 28.6\text{ N (one extra digit carried)} \\ F_{TB} &= 29\text{ N} \end{aligned}$$

Determine the force on rope A:

$$\begin{aligned} F_{TA} &= m_1(g+a) + F_{TB} \\ &= (4.2\text{ kg})(9.8\text{ m/s}^2 + 1.2\text{ m/s}^2) + 28.6\text{ N} \\ F_{TA} &= 75\text{ N} \end{aligned}$$

Statement: The tension on the rope A is 75 N and the tension on the rope B is 29 N.
4. Given: $\vec{a} = 0.98$ m/s^2 [down]; $m = 61$ kg
Required: F_N
Analysis: Use down as positive and solve for the normal force when $-F_N + (mg) = ma$.
Solution: $-F_N + (mg) = ma$

$$\begin{aligned} F_N &= mg - ma \\ &= (61\text{ kg})(9.8\text{ m/s}^2) - (61\text{ kg})(0.98\text{ m/s}^2) \\ F_N &= 5.4 \times 10^2\text{ N} \end{aligned}$$

Statement: The passenger's apparent weight is 5.4×10^2 m/s^2.

Section 3.1 Questions, page 113

1. (a) The ball would appear to move straight up and down because I am moving with the same velocity as the other train. From my viewpoint, the other train and passenger are standing still, and the ball is not affected by the train's motion.
(b) If the trains moved in opposite directions, the ball would appear to have horizontal motion, so I would see the path as a parabola.
2. Given: $a = 1.5$ m/s^2
Required: θ
Analysis: The horizontal component of the tension F_T balances the acceleration, and the vertical component of the tension F_T balances the gravitational force. Express the tangent ratio of the angle in terms of the applied force and the gravitational force, then solve for the angle.

$$\begin{aligned} \tan\theta &= \frac{F_a}{F_g} \\ \theta &= \tan^{-1}\left(\frac{\cancel{m}a}{\cancel{m}g}\right) \\ \theta &= \tan^{-1}\left(\frac{a}{g}\right) \end{aligned}$$

Solution: $\theta = \tan^{-1}\left(\dfrac{a}{g}\right)$

$$= \tan^{-1}\left(\frac{1.5\ \frac{\cancel{\text{m}}}{\cancel{\text{s}^2}}}{9.8\ \frac{\cancel{\text{m}}}{\cancel{\text{s}^2}}}\right)$$

$$\theta = 8.7°$$

Statement: The string makes an 8.7° angle with the vertical.

3. Given: v_f = 255 km/h; Δt = 10.0 s

Required: θ

Analysis: Determine the acceleration using $v_\text{f} = a\Delta t$ or $a = \dfrac{v_\text{f}}{\Delta t}$. The horizontal component of the tension F_T balances the acceleration and the vertical component of the tension F_T balances the gravitational force. Express the tangent ratio of the angle in terms of the applied force and the gravitational force, then solve for the angle.

Solution: Determine the acceleration of the plane:

$$a = \frac{v_\text{f}}{\Delta t}$$

$$= \frac{255\ \frac{\cancel{\text{km}}}{\cancel{\text{h}}}}{10.0\ \text{s}} \times \frac{1000\ \text{m}}{1\ \cancel{\text{km}}} \times \frac{1\ \cancel{\text{h}}}{60\ \cancel{\text{min}}} \times \frac{1\ \cancel{\text{min}}}{60\ \text{s}}$$

$$= 7.0833\ \text{m/s}^2 \text{ (two extra digits carried)}$$

$$a = 7.08\ \text{m/s}^2$$

Determine the angle the string makes with the vertical:

$$\tan\theta = \frac{F_\text{a}}{F_\text{g}}$$

$$\theta = \tan^{-1}\left(\frac{\cancel{m}a}{\cancel{m}g}\right)$$

$$= \tan^{-1}\left(\frac{a}{g}\right)$$

$$= \tan^{-1}\left(\frac{7.0833\ \frac{\cancel{\text{m}}}{\cancel{\text{s}^2}}}{9.8\ \frac{\cancel{\text{m}}}{\cancel{\text{s}^2}}}\right)$$

$$\theta = 35.9°$$

Statement: The string makes an 35.9° angle with the vertical.

4. Given: θ = 16°

Required: a

Analysis: Look at the situation from an Earth (inertial) frame of reference. The horizontal component of the tension F_T balances the acceleration and the vertical component of the tension F_T must balance the gravitational force since the cork ball does not move. Express the components of the tension in terms of the horizontal and vertical applied forces. Then calculate the magnitude of the acceleration.

Vertical component of force:

$$\begin{aligned}\Sigma F_y &= 0\\ F_T\cos\theta - mg &= 0\\ F_T &= \frac{mg}{\cos\theta}\end{aligned}$$

Horizontal component of force:

$$\begin{aligned}\Sigma F_x &= ma\\ F_T\sin\theta &= ma\\ \left(\frac{\cancel{m}g}{\cos\theta}\right)\sin\theta &= \cancel{m}a\\ g\left(\frac{\sin\theta}{\cos\theta}\right) &= a\\ g\tan\theta &= a\end{aligned}$$

Solution: $$\begin{aligned}a &= g\tan\theta\\ &= \left(9.8\ \text{m/s}^2\right)\tan 16°\\ a &= 2.8\ \text{m/s}^2\end{aligned}$$

Statement: The magnitude of the car's acceleration is 2.8 m/s^2.

5. Given: v_f = 6.0 m/s; Δt = 10.0 s; m = 64 kg

Required: F_N

Analysis: Determine the upward acceleration using $v_f = a\Delta t$ or $a = \dfrac{v_f}{\Delta t}$. Use up as positive and solve for the normal force when $+F_N + (-mg) = ma$.

$$\begin{aligned}+F_N + \left(-mg\right) &= ma\\ F_N &= ma + mg\end{aligned}$$

Solution: Determine the upward acceleration:

$$\begin{aligned}a &= \frac{v_f}{\Delta t}\\ &= \frac{6.0\ \text{m/s}}{10.0\ \text{s}}\\ a &= 0.60\ \text{m/s}^2\end{aligned}$$

Determine the apparent weight:

$$\begin{aligned}F_N &= ma + mg\\ &= \left(64\ \text{kg}\right)\left(0.6\ \text{m/s}^2\right) + \left(64\ \text{kg}\right)\left(9.8\ \text{m/s}^2\right)\\ F_N &= 6.7\times10^2\ \text{N}\end{aligned}$$

Statement: The passenger's apparent weight is 6.7×10^2 N.

6. Given: F_N = 255 N; m = 52 kg

Required: a

Analysis: Use up as positive and solve for the acceleration when $+F_N + (-mg) = ma$.

$$+F_N + (-mg) = ma$$
$$a = \frac{F_N - mg}{m}$$

Solution: $a = \dfrac{F_N - mg}{m}$

$$= \frac{255\ \cancel{\text{kg}} \cdot \dfrac{\text{m}}{\text{s}^2} - (52\ \cancel{\text{kg}})(9.8\ \text{m/s}^2)}{52\ \cancel{\text{kg}}}$$
$$a = -4.9\ \text{m/s}^2$$

Statement: The acceleration of the ride is 4.9 m/s^2 [down].

7. (a) At rest: Accelerating downhill:

When the car is at rest, the dice are aligned with the vertical (with respect to level ground), and make an angle of 17° with respect to the normal (perpendicular to the roof). The only forces acting on the dice are gravity and tension. When the car is accelerating, a horizontal fictitious force pointing to the rear of the car deflects the dice so that they are aligned with the normal, and make an angle of 17° with respect to the vertical, as viewed from the level ground.

(b) Given: θ = 17°

Required: a

Analysis: Look at the situation from an inertial frame of reference on the same angle as the hill. The "vertical" component of the gravitational force F_g balances the tension and the "horizontal" component of the gravitational force F_g balances the force applied by the acceleration. Express the horizontal component of F_g in terms of the applied forces. Then calculate the magnitude of the acceleration.

$$\Sigma F_x = 0$$
$$F_g \sin\theta - ma = 0$$
$$mg \sin\theta - ma = 0$$
$$g \sin\theta - a = 0$$
$$a = g \sin\theta$$

Solution: $a = g\sin\theta$

$$= (9.8\ \text{m/s}^2)\sin 17°$$
$$a = 2.9\ \text{m/s}^2$$

Statement: The magnitude of the car's acceleration is 2.9 m/s^2.

8. (a) Given: m_1 = 1.8 kg; m_2 = 1.2 kg; m_2 = 1.2 kg; F_N = 70.0 N
Required: F_a

Analysis: Since mass 1 does not slide, the acceleration due to the horizontal force must balance the acceleration due to the tension, which equals m_2g. Use $F = ma$ to determine the acceleration.

$$\frac{F_a}{m_1 + m_2 + m_3} = \frac{F_T}{m_1}$$

$$\frac{F_a}{m_1 + m_2 + m_3} = \frac{m_2 g}{m_1}$$

$$F_a = \frac{m_2 g}{m_1}(m_1 + m_2 + m_3)$$

Solution: $F_a = \dfrac{m_2 g}{m_1}(m_1 + m_2 + m_3)$

$$\frac{(1.2\text{ kg})(9.8\text{ m/s}^2)}{(1.8\text{ kg})}(1.8\text{ kg} + 1.2\text{ kg} + 3.0\text{ kg})$$

$$F_a = 39\text{ N}$$

Statement: The applied force is 39 N.
(b) Given: m_1 = 1.2 kg; m_2 = 2.8 kg; θ = 25°
Required: F_a
Analysis: Since mass 1 does not slide, the applied force on mass 1, the gravitational force, and the normal force must balance each other: $\Sigma F = 0$. Use $F = ma$ to determine the acceleration. Since the applied force is entirely horizontal and the gravitational force is entirely vertical, use the tangent ratio to relate them.
Determine the applied acceleration:

$$F = ma$$

$$F_a = (m_1 + m_2)a$$

$$a = \frac{F_a}{m_1 + m_2}$$

Determine the horizontal acceleration of mass 1:

$$\tan\theta = \frac{\cancel{m_1} a}{\cancel{m_1} g} = \frac{a}{g}$$

$$a = g\tan\theta$$

Determine the applied force:

$$\frac{F_a}{m_1 + m_2} = g\tan\theta$$

$$F_a = g\tan\theta(m_1 + m_2)$$

Solution: $F_{\text{a}} = g\tan\theta\left(m_1 + m_2\right)$

$$\left(9.8\text{ m/s}^2\right)\left(\tan 25°\right)\left(1.2\text{ kg} + 2.8\text{ kg}\right)$$

$$F_{\text{a}} = 17\text{ N}$$

Statement: The applied force is 17 N.

Section 3.2: Centripetal Acceleration

Tutorial 1 Practice, page 118

1. Given: r = 25 km = 2.5 × 10^4 m; v = 50.0 m/s

Required: a_c

Analysis: $a_c = \dfrac{v^2}{r}$

Solution:
$$\begin{aligned} a_c &= \frac{v^2}{r} \\ &= \frac{(50.0\ \text{m/s})^2}{(2.5\times10^4\ \text{m})} \\ a_c &= 0.10\ \text{m/s}^2 \end{aligned}$$

Statement: The magnitude of the centripetal acceleration is 0.10 m/s^2.

2. Given: r = 1.2 m; v = 4.24 m/s

Required: $\vec{a}_c$

Analysis: $a_c = \dfrac{v^2}{r}$; Centripetal acceleration is always directed toward the centre of rotation. Since the hammer's velocity is directed south and it is spinning clockwise, the centre of rotation is west of the hammer.

Solution:
$$\begin{aligned} a_c &= \frac{v^2}{r} \\ &= \frac{(4.24\ \text{m/s})^2}{(1.2\ \text{m})} \\ a_c &= 15\ \text{m/s}^2 \end{aligned}$$

Statement: The centripetal acceleration is 15 m/s^2 [W].

3. Given: r = 1.4 m; a_c = 12 m/s^2

Required: v

Analysis:
$$\begin{aligned} a_c &= \frac{v^2}{r} \\ v &= \sqrt{a_c r} \end{aligned}$$

Solution:
$$\begin{aligned} v &= \sqrt{a_c r} \\ &= \sqrt{(12\ \text{m/s}^2)(1.4\ \text{m})} \\ v &= 4.1\ \text{m/s} \end{aligned}$$

Statement: The speed of the ball is 4.1 m/s.

4. (a) Given: r = 1.08 × 10^{11} m; a_c = 1.12 × 10^{-2} m/s^2

Required: v

Analysis: $a_c = \dfrac{4\pi^2 r}{T^2}$

$$T = \sqrt{\frac{4\pi^2 r}{a_c}}$$

Solution: $T = \sqrt{\dfrac{4\pi^2 r}{a_c}}$

$$= \sqrt{\frac{4\pi^2 (1.08\times10^{11}\ \cancel{\text{m}})}{\left(1.12\times10^{-2}\ \dfrac{\cancel{\text{m}}}{\text{s}^2}\right)}}$$

$$T = 1.95\times10^7\ \text{s}$$

Statement: The period of Venus is 1.95×10^7 s.

(b) Convert the period in seconds to days:

$$T = 1.95\times10^7\ \text{s}\times\frac{1\ \cancel{\text{min}}}{60\ \cancel{\text{s}}}\times\frac{1\ \cancel{\text{h}}}{60\ \cancel{\text{min}}}\times\frac{1\ \text{d}}{24\ \cancel{\text{h}}}$$

$$T = 226\ \text{days}$$

The period of Venus is 226 days.

5. Given: $v = 7.27 \times 10^3$ m.s; $r = 7.54 \times 10^6$ m

Required: a_c

Analysis: $a_c = \dfrac{v^2}{r}$

Solution: $a_c = \dfrac{v^2}{r}$

$$= \frac{(7.27\times10^3\ \text{m/s})^2}{(7.54\times10^6\ \text{m})}$$

$$a_c = 7.01\ \text{m/s}^2$$

Statement: The magnitude of the centripetal acceleration is $7.01\ \text{m/s}^2$.

6. (a) Given: $a_c = 3.3 \times 10^6\ \text{m/s}^2$; $r = 8.4$ cm $= 8.4 \times 10^{-2}$ m

Required: f

Analysis: $a_c = 4\pi^2 r f^2$

$$f = \sqrt{\frac{a_c}{4\pi^2 r}}$$

Solution: $f = \sqrt{\dfrac{a_c}{4\pi^2 r}}$

$= \sqrt{\dfrac{\left(3.3\times10^6\ \dfrac{\cancel{m}}{s^2}\right)}{4\pi^2\left(8.4\times10^{-2}\ \cancel{m}\right)}}$

$f = 1.0\times10^4\ \text{Hz}$

Statement: The frequency of the centrifuge is 1.0×10^4 Hz.

(b) Convert the frequency in hertz to revolutions per minute:

$f = 1.0\times10^4\ \dfrac{1}{1\ \cancel{s}}\times\dfrac{60\ \cancel{s}}{1\ \text{min}}$

$f = 6.0\times10^5\ \text{rpm}$

The frequency of the centrifuge is 6.0×10^5 rpm.

Section 3.2 Questions, page 119

1. (a) The tension in the string provides the force to keep the puck in its circular path at constant speed, and so provides the acceleration of the puck.

(b) The centripetal acceleration is half as large because centripetal acceleration depends on the inverse of the radius: $\dfrac{1}{2}a_c = \dfrac{v^2}{2r}$.

(c) The centripetal acceleration is four times as great because centripetal acceleration depends on the square of the speed: $4a_c = \dfrac{(2v)^2}{r}$.

2. The centripetal acceleration for the first athlete's hammer is four times greater than that of the second athlete. Centripetal acceleration depends on the square of the speed: $a_c = \dfrac{v^2}{r}$. So if the hammer spins two times as fast, the centripetal acceleration is 2^2, or 4, times larger: $4a$.

3. Given: r = 0.42 m; T = 1.5 s

Required: a_c

Analysis: $a_c = \dfrac{4\pi^2 r}{T^2}$

Solution: $a_c = \dfrac{4\pi^2 r}{T^2}$

$= \dfrac{4\pi^2(0.42\ \text{m})}{(1.5\ \text{s})^2}$

$a_c = 7.4\ \text{m/s}^2$

Statement: The magnitude of the centripetal acceleration of the lasso is 7.4 m/s^2.

4. Given: v = 28 m/s; r = 135 m

Required: a_c

Analysis: $a_c = \dfrac{v^2}{r}$

Solution: $a_c = \dfrac{v^2}{r}$

$$= \frac{(28\text{ m/s})^2}{(135\text{ m})}$$

$$a_c = 5.8\text{ m/s}^2$$

Statement: The magnitude of the centripetal acceleration is 5.8 m/s^2.

5. Given: $r = 6.38 \times 10^6$ m; T = 1 day or 86 400 s

Required: a_c

Analysis: $a_c = \dfrac{4\pi^2 r}{T^2}$

Solution: $a_c = \dfrac{4\pi^2 r}{T^2}$

$$= \frac{4\pi^2 (6.38\times10^6\text{ m})}{(86\,400\text{ s})^2}$$

$$a_c = 3.37\times10^{-2}\text{ m/s}^2$$

Statement: The centripetal acceleration at Earth's equator is 3.37×10^{-2} m/s^2.

6. Given: a_c = 25 m/s^2; r = 2.0 m

Required: f

Analysis: $a_c = 4\pi^2 r f^2$

$$f = \sqrt{\frac{a_c}{4\pi^2 r}}$$

Solution: $f = \sqrt{\dfrac{a_c}{4\pi^2 r}}$

$$= \sqrt{\frac{\left(25\ \dfrac{\cancel{\text{m}}}{\text{s}^2}\right)}{4\pi^2 (2.0\ \cancel{\text{m}})}}$$

$$f = 0.56\text{ Hz}$$

Statement: The minimum frequency of the cylinder is 0.56 Hz.

7. Given: v = 22 m/s; a_c = 7.8 m/s^2

Required: r

Analysis: $a_c = \dfrac{v^2}{r}$

$$r = \frac{v^2}{a_c}$$

Solution: $r = \dfrac{v^2}{a_c}$

$$= \frac{(22\text{ m/s})^2}{(7.8\text{ m/s}^2)}$$

$$r = 62\text{ m}$$

Statement: The radius of the curve is 62 m.

8. Given: C = 478 m; a_c = 0.146 m/s^2

Required: v

Analysis: $C = 2\pi r$ or $r = \dfrac{C}{2\pi}$;

$$a_c = \frac{v^2}{r}$$

$$v = \sqrt{a_c r}$$

$$v = \sqrt{a_c\left(\frac{C}{2\pi}\right)}$$

Solution: $v = \sqrt{a_c\left(\dfrac{C}{2\pi}\right)}$

$$= \sqrt{\frac{(0.146\text{ m/s}^2)(478\text{ m})}{2\pi}}$$

$$= 3.333\ \frac{\cancel{\text{m}}}{\text{s}} \times \frac{60\ \cancel{\text{s}}}{1\ \cancel{\text{min}}} \times \frac{60\ \cancel{\text{min}}}{1\text{ h}} \times \frac{1\text{ km}}{1000\ \cancel{\text{m}}}$$

$$v = 12.0\text{ km/h}$$

Statement: The speed of the jogger is 12.0 km/h.

9. (a) Given: r = 0.300 m; f = 60.0 rpm

Required: T

Analysis: $T = \dfrac{1}{f}$

Solution: Convert the frequency to hertz:

$$f = 60.0\text{ rpm}$$

$$= 60.0\ \frac{1}{1\ \cancel{\text{min}}} \times \frac{1\ \cancel{\text{min}}}{60\text{ s}}$$

$$f = 1.00\text{ Hz}$$

Determine the period:

$$T = \frac{1}{f}$$
$$= \frac{1}{1.00 \text{ Hz}}$$
$$T = 1.00 \text{ s}$$

Statement: The period of the bicycle wheel is 1.00 s.

(b) Given: r = 0.300 m; f = 1.00 Hz

Required: $\vec{a}_c$

Analysis: $a_c = 4\pi^2 r f^2$; Centripetal acceleration is always directed toward the centre of rotation. Since the wheel's velocity is directed west and it is spinning clockwise, the centre of rotation is north of the point.

Solution: $a_c = 4\pi^2 r f^2$

$$= 4\pi^2 (0.300 \text{ m})(1.00 \text{ Hz})^2$$
$$a_c = 11.8 \text{ m/s}^2$$

Statement: The centripetal acceleration of a point on the edge of that wheel is 11.8 m/s^2 [N] if it is moving westward at that instant.

10. (a) Given: T = 27.3 days; a_c = 2.7 × 10^{-3} m/s^2

Required: r

Analysis: $a_c = \dfrac{4\pi^2 r}{T^2}$

Solution: Convert the period to seconds:

$$T = 27.3 \text{ days} \times \frac{24 \cancel{\text{h}}}{1 \cancel{\text{day}}} \times \frac{60 \cancel{\text{min}}}{1 \cancel{\text{h}}} \times \frac{60 \text{ s}}{1 \cancel{\text{min}}}$$
$$= 2.3587 \times 10^6 \text{ s (two extra digits carried)}$$
$$T = 2.36 \times 10^6 \text{ s}$$

Determine the radius:

$$a_c = \frac{4\pi^2 r}{T^2}$$
$$r = \frac{a_c T^2}{4\pi^2}$$
$$= \frac{\left(2.7 \times 10^{-3} \ \frac{\text{m}}{\cancel{\text{s}^2}}\right)(2.3587 \times 10^6 \ \cancel{\text{s}})^2}{4\pi^2}$$
$$r = 3.8 \times 10^8 \text{ m}$$

Statement: The radius of the curve is 3.8 × 10^8 m.

(b) The values are the same to two significant digits. Any difference beyond that may be because the orbit is not perfectly circular or the speed is not constant.

11. (a) Given: a_c = 711 m/s^2; r = 1.21 m
Required: v

Analysis: $a_c = \frac{v^2}{r}$; $v = \sqrt{a_c r}$

Solution: $v = \sqrt{a_c r}$

$$= \sqrt{(711\text{ m/s}^2)(1.21\text{ m})}$$

$$v = 29.3\text{ m/s}$$

Statement: The speed of the hammer is 29.3 m/s.
(b) Given: Δd = –2.0 m; v_i = 29.3 m/s; θ = 42°
Required: Δd_x
Analysis: Use $v_f^2 = v_i^2 + 2a\Delta d$ to calculate the y-component of the final speed, then calculate the time of flight $v_f = v_i + a\Delta t$. Finally, calculate the range using $\Delta d = v\Delta t$.
Solution: Determine the y-component of the final speed:

$$v_f^2 = v_i^2 + 2a\Delta d$$

$$v_{fy}^2 = v_{iy}^2 + 2g\Delta d$$

$$v_{fy} = \sqrt{v_{iy}^2 + 2g\Delta d}$$

$$= \sqrt{((29.3\text{ m/s})(\sin 42°))^2 + 2(9.8\text{ m/s}^2)(-2.0\text{ m})}$$

$$= 18.58\text{ m/s (two extra digits carried)}$$

$$v_{fy} = 19\text{ m/s}$$

Determine the time of flight:

$$v_{fy} = v_{iy} + a\Delta t$$

$$\Delta t = \frac{v_{fy} - v_{iy}}{a}$$

$$= \frac{18.58\ \frac{\cancel{\text{m}}}{\text{s}} - \left(-29.3\ \frac{\cancel{\text{m}}}{\text{s}}\right)(\sin 42°)}{9.8\ \frac{\cancel{\text{m}}}{\text{s}^2}}$$

$$\Delta t = 3.896\text{ s (two extra digits carried)}$$

Determine the range of the ball:

$$\Delta d_x = v_x \Delta t$$

$$= v_i \Delta t \cos\theta$$

$$= \left(29.3\ \frac{\text{m}}{\cancel{\text{s}}}\right)(3.896\ \cancel{\text{s}})(\cos 42°)$$

$$\Delta d_x = 85\text{ m}$$

Statement: The range of the ball is 85 m.

Section 3.3: Centripetal Force

Tutorial 1 Practice, page 123

1. Given: m = 0.211 kg; r = 25.6 m; v = 21.7 m/s

Required: F_c

Analysis: Lift is equivalent to the normal force, so it is the sum of the centripetal force and the gravitational force; $F_c = \frac{mv^2}{r}$; $F_g = mg$

$$F_N = F_c + F_g$$

$$F_N = \frac{mv^2}{r} + mg$$

Solution: $F_N = \frac{mv^2}{r} + mg$

$$= \frac{(0.211\text{ kg})(21.7\text{ m/s})^2}{(25.6\text{ m})} + (0.211\text{ kg})(9.8\text{ m/s}^2)$$

$$F_c = 5.9\text{ N}$$

Statement: The lift on the plane at the bottom of the arc is 5.9 N.

2. Given: r = 450 m; v = 97 km/h

Required: θ

Analysis: The vertical component of the normal F_N balances the gravitational force and the horizontal component of the normal F_N represents the centripetal force, $F_c = \frac{mv^2}{r}$. Use the tangent ratio to determine the angle the normal makes with the vertical.

$$\tan\theta = \frac{F_{Nx}}{F_{Ny}}$$

$$= \frac{F_c}{F_g}$$

$$= \frac{\left(\frac{mv^2}{r}\right)}{mg}$$

$$= \frac{v^2}{rg}$$

$$\theta = \tan^{-1}\left(\frac{v^2}{rg}\right)$$

Solution: Convert the speed to metres per second:

$$v = 97\ \frac{\cancel{\text{km}}}{\cancel{\text{h}}} \times \frac{1000\text{ m}}{1\ \cancel{\text{km}}} \times \frac{1\ \cancel{\text{h}}}{60\ \cancel{\text{min}}} \times \frac{1\ \cancel{\text{min}}}{60\text{ s}}$$

$= 26.94$ m/s (two extra digits carried)

$v = 27$ m/s

Determine the angle:

$$\theta = \tan^{-1}\left(\frac{v^2}{rg}\right)$$
$$= \tan^{-1}\left(\frac{(26.94\text{ m/s})^2}{(450\text{ m})(9.8\text{ m/s}^2)}\right)$$
$$\theta = 9.3°$$

Statement: The banking angle is 9.3°.

3. Given: m = 2.0 kg; $f = \dfrac{5.00}{2.00\text{ s}}$ = 2.50 Hz; r = 4.00 m

Required: F_T

Analysis: $F_T = ma_c$; $a_c = 4\pi^2 rf^2$; $F_c = 4\pi^2 mrf^2$

Solution: $F_c = 4\pi^2 mrf^2$

$$= 4\pi^2(2.00\text{ kg})(4.00\text{ m})(2.50\text{ Hz})^2$$
$$F_T = 2.0\times10^3\text{ N}$$

Statement: The magnitude of the tension in the string is 2.0×10^3 N.

4. Given: r = 150 m; $F_c = F_g$

Required: v

Analysis: $F_g = mg$; $F_c = \dfrac{mv^2}{r}$

$$F_c = F_g$$
$$\frac{mv^2}{r} = mg$$
$$\frac{v^2}{r} = g$$
$$v = \sqrt{gr}$$

Solution: $v = \sqrt{gr}$

$$= \sqrt{(9.8\text{ m/s}^2)(150\text{ m})}$$
$$v = 38\text{ m/s}$$

Statement: The speed of the barn swallow is 38 m/s.

5. I predict that the maximum speed will decrease because the road conditions are slippery.

Given: μ_s = 0.25; $r = 2.0 \times 10^2$ m; θ = 20.0°

Required: maximum v

Analysis: From Sample Problem 3: $v = \sqrt{gr\left(\dfrac{\sin\theta + \mu_s\cos\theta}{\cos\theta - \mu_s\sin\theta}\right)}$

Solution: $v = \sqrt{gr\left(\dfrac{\sin\theta + \mu_s \cos\theta}{\cos\theta - \mu_s \sin\theta}\right)}$

$$= \sqrt{\left(9.8\text{ m/s}^2\right)\left(2.0\times 10^2\text{ m}\right)\left(\frac{\sin 20.0° + (0.25)\cos 20.0°}{\cos 20.0° - (0.25)\sin 20.0°}\right)}$$

$v = 36$ m/s

Statement: The maximum speed in slippery conditions is 36 m/s.

Section 3.3 Questions, page 124

1. Given: $d = 24$ m or $r = 12$ m; $F_{net} = \dfrac{1}{3}F_g$

Required: v

Analysis: $F_c = \dfrac{mv^2}{r}$; $F_g = mg$; $F_{net} = F_c + F_g$

$$F_{net} = \frac{1}{3}F_g$$
$$F_c + F_g = \frac{1}{3}F_g$$
$$F_c = -\frac{2}{3}F_g$$
$$\frac{mv^2}{r} = -\frac{2}{3}mg$$
$$v^2 = -\frac{2}{3}\frac{\cancel{m}gr}{\cancel{m}}$$
$$v = \sqrt{-\frac{2}{3}gr}$$

Solution: $v = \sqrt{-\dfrac{2}{3}gr}$

$$= \sqrt{-\frac{2}{3}\left(-9.8\text{ m/s}^2\right)(12\text{ m})}$$

$v = 8.9$ m/s

Statement: The speed of the roller coaster is 8.9 m/s.

2. (a)

(b) Given: m = 1000.0 kg; r = 40.0 m; v = 15 m/s
Required: F_N

Analysis: $F_c = \dfrac{mv^2}{r}$; $F_g = mg$;

$F_N = F_g + F_c$

$F_N = mg + \dfrac{mv^2}{r}$

Solution:
$$\begin{aligned} F_N &= mg + \frac{mv^2}{r} \\ &= (1000.0 \text{ kg})(9.8 \text{ m/s}^2) + \frac{(1000.0 \text{ kg})(15 \text{ m/s})^2}{(40.0 \text{ m})} \\ F_N &= 1.5 \times 10^4 \text{ N} \end{aligned}$$

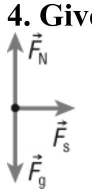
Statement: The magnitude of the normal force is 1.5×10^4 N.
(c) Given: m = 1000.0 kg; r = 40.0 m; v = 15 m/s; $F_{net} = 0$
Required: v

Analysis: $F_c = \dfrac{mv^2}{r}$; $F_g = mg$

$$\begin{aligned} F_{net} &= F_g + F_c \\ 0 &= mg + \frac{mv^2}{r} \\ 0 &= g + \frac{v^2}{r} \\ v &= \sqrt{-gr} \end{aligned}$$

Solution:
$$\begin{aligned} v &= \sqrt{-gr} \\ &= \sqrt{-(-9.8 \text{ m/s}^2)(40.0 \text{ m})} \\ v &= 20 \text{ m/s} \end{aligned}$$

Statement: The speed required to make the driver feel weightless is 20 m/s.
3. (a) When the banking angle increases, the maximum speed of a car also increases because the horizontal component of the normal force has increased.
(b) When the coefficient of friction increases, the maximum speed of a car also increases because the force due to friction, which points into the turn, has increased.
(c) When the mass of the car increases, the maximum speed of a car also increases because the normal force and the force due to friction both increase.
4. Given: $r = 1.2 \times 10^2$ m; $\mu_s = 0.72$; $F_{net} = 0$



Required: maximum v

Analysis: $F_c = \dfrac{mv^2}{r}$; $F_g = mg$; $F_s = \mu_s F_N$

$$F_c = F_s$$
$$\frac{mv^2}{r} = \mu_s F_N$$
$$\frac{\cancel{m}v^2}{r} = \mu_s \cancel{m}g$$
$$\frac{v^2}{r} = \mu_s g$$
$$v = \sqrt{\mu_s g r}$$

Solution: $v = \sqrt{\mu_s g r}$

$$= \sqrt{(0.72)(9.8\ \text{m/s}^2)(1.2 \times 10^2\ \text{m})}$$
$$v = 29\ \text{m/s}$$

Statement: The maximum speed of the car is 29 m/s.

5. (a) The banking angle creates a horizontal component of the normal force, which is only vertical on a horizontal round. This horizontal component increases the net force pushing into the curve. Thanks to this force into the curve, cars can navigate the turn at higher speeds without losing friction.

(b) Drivers must go much more slowly because the coefficient of static friction is dramatically reduced and the net force pushing into the curve is much less.

(c) Answers may vary. Sample answer: The banking angle must work for conditions where the coefficient of static friction is high and when it is low. Making the angles significantly larger would allow for greater speeds but would be much more dangerous in slippery conditions.

6. Given: $m_1 = 0.26$ kg; $m_2 = 0.68$ kg; $r = 1.2$ m

Required: v

Analysis: $F_c = \dfrac{mv^2}{r}$; $F_g = mg$; The tension in the string equals the gravitational force on m_2 and the centripetal force on m_1.

$$F_c = F_g$$
$$\frac{m_1 v^2}{r} = m_2 g$$
$$v = \sqrt{\frac{m_2 g r}{m_1}}$$

Solution: $v = \sqrt{\dfrac{m_2 g r}{m_1}}$

$$= \sqrt{\frac{(0.68\ \text{kg})(9.8\ \text{m/s}^2)(1.2\ \text{m})}{(0.26\ \text{kg})}}$$
$$v = 5.5\ \text{m/s}$$

Statement: The speed of the air puck is 5.5 m/s.