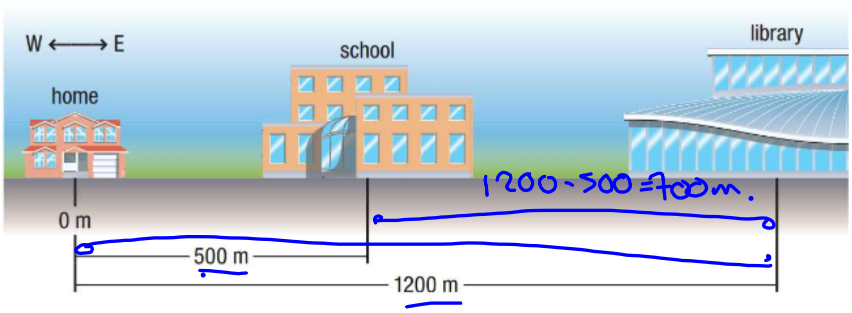
SPH3U: 1.1 Distance, Position, and Displacement

1. What is kinematics?

Kinematics:	the study of motion.
distance d	total length of the path taken by an object. Units: m (metres).



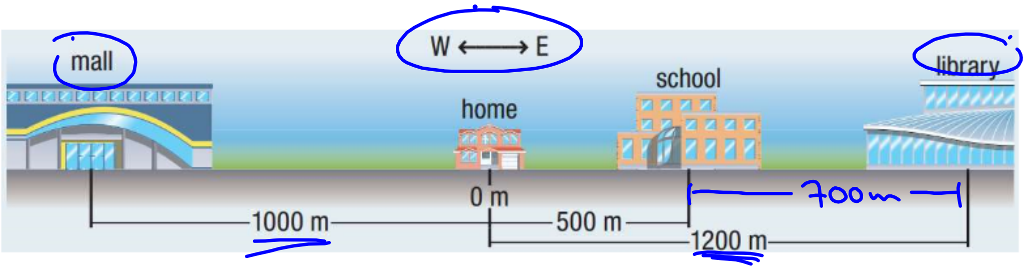
In the figure above, how far will you have traveled if you:

a. walk directly from home to your school? $d = 500\,m$.

b. walk from your school to the library, then return home? $d = 700 + 1200 = 1900\,m$.

2. Position and displacement

Scalar:	a measurement with only magnitude (size).
Vector:	has magnitude and direction. represented by an arrow.
position $\vec{d}$	the distance and direction of an object from a reference point. Vector.
reference point	some location that we choose to be 0.
displacement $\Delta\vec{d}$	change in position. $\Delta\vec{d} = \vec{d}_f - \vec{d}_i$ or $\vec{d}_2 - \vec{d}_1$. Vector.
delta (Δ)	the change in a value ($\Delta x = x_f - x_i$).



In the figure above, what is:

a. the position of the school, with home as a reference point? $\vec{d} = 500\,m\ [E]$.

b. the position of the school, with the library as a reference point? $\vec{d} = 700\,m\ [W]$.

Imagine that you walk from home to school in a straight-line route. What is your displacement? Use your home as a reference point.

$$\Delta\vec{d} = \vec{d}_f - \vec{d}_i = 500\,m\ [E] - 0\,m$$
$$= 500\,m\ [E].$$

What is your displacement if you walk from your school to the library?

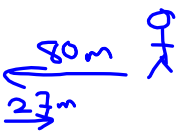
$$\Delta\vec{d} = \vec{d}_f - \vec{d}_i = 1200\,m\ [E] - 500\,m\ [E].$$
$$= 700\,m\ [E].$$

One night after working at the library, you decide to go to the mall. What is your total displacement when walking from the library to the mall?

$$\Delta\vec{d} = \vec{d}_f - \vec{d}_i = 1000\,m\ [W] - 1200\,m\ [E]$$
$$= 1000\,m\ [W] + 1200\,m\ [W]$$
$$= 2200\,m\ [W].$$

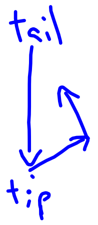
$1200\,m\ [E] = -1200\,m\ [W]$

A dog is practising for her agility competition. She leaves her trainer and runs 80 m due west to pick up a ball. She then carries the ball 27 m due east and drops it into a bucket. What is the dog's total displacement?



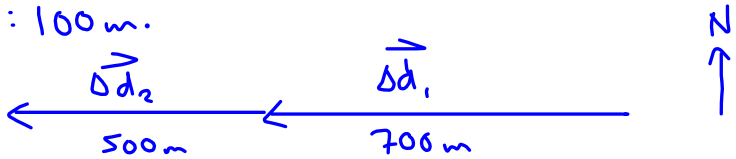
$$\Delta\vec{d}_T = \Delta\vec{d}_1 + \Delta\vec{d}_2$$
$$= 80\,m\ [W] + 27\,m\ [E]$$
$$= 80\,m\ [W] - 27\,m\ [W]$$
$$= 53\,m\ [W].$$

3. Vector scale diagrams



Vector scale diagrams:	draw vectors (arrows) with a certain scale, For instance 1 cm : 1 km.
directed line segment	line with an arrow on it (vector).
tip and tail	tip: end of arrow. tail: start of arrow.
adding vectors	use tip-to-tail: 2nd vector's tail on 1st vector's tip.

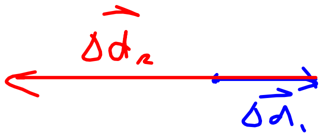
Add the two displacements $\Delta\vec{d}_1$ = 700 m [W] and $\Delta\vec{d}_2$ = 500 m [W] by drawing a vector scale diagram. 1 cm : 100 m.



$\Delta\vec{d}_T$ = 1200 m [W].

Imagine that you are going to visit your friend. Before you get there, you decide to stop at the variety store. If you walk 200 m [N] from your home to the store, and then travel 600 m [S] to your friend's house, what is your total displacement?

1 cm : 100 m → N.



$\Delta\vec{d}_T$ = 400 m

Homework: page 13: #1-6

SPH3U: 1.2 Speed and Velocity

1. Recap

distance	position	displacement
d scalar (m)	$\vec{d}$ vector (m)	$\Delta\vec{d}$ vector. (m).

2. Average speed and velocity

Average speed:	total distance divided by total time. $v_{av} = \frac{\Delta d}{\Delta t}$ Units: m/s.

Your dog runs in a straight line for a distance of 43 m in 28 s. What is your dog's average speed?

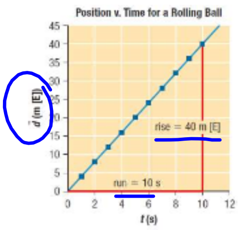
$$v_{av} = \frac{\Delta d}{\Delta t} = \frac{43\text{ m}}{28\text{ s}} = 1.5\text{ m/s}.$$

A baseball rolls along a flat parking lot in a straight line at a constant speed of 3.8 m/s. How far will the baseball roll in 15 s?

$v_{av} = 3.8\text{ m/s}$. $\quad v_{av} = \frac{\Delta d}{\Delta t} \longrightarrow \Delta d = v_{av}\Delta t$

$$\Delta d = v_{av}\Delta t = (3.8\text{ m/s})(15\text{ s}) = 57\text{ m}.$$

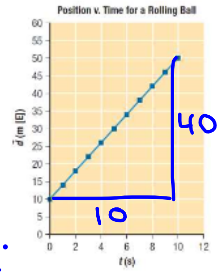
Average velocity: (vector)	total displacement. over total time. $\vec{v}_{av} = \frac{\Delta\vec{d}}{\Delta t}$
position-time graph	a graph with position ($\vec{d}$) on the y-axis and time on the x-axis.
slope	slope of a P-T graph is the average velocity.



$m = \frac{rise}{run}$

$\vec{v}_{av} = \frac{40 m [E]}{10 s}$

$= 4 m/s [E].$



40

10

$\vec{v}_{av} = 4 m/s [E].$

On a windy day, the position of a balloon changes as it is blown 82 m [N] away from a child in 15 s. What is the average velocity of the balloon?

$\Delta\vec{d} = 82 m [N]. \quad \Delta t = 15 s.$

$\vec{v}_{av} = \frac{\Delta\vec{d}}{\Delta t} = \frac{82 m [N]}{15 s} = 5.5 m/s [N].$

A subway train travels at an average velocity of 22.3 km/h [W]. How long will it take for the subway train to undergo a displacement of 241 m [W]? m/s.

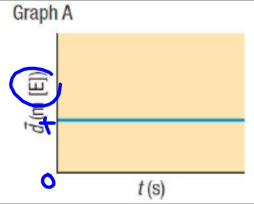
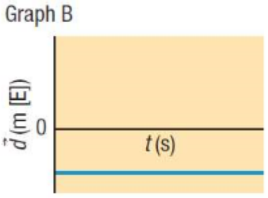
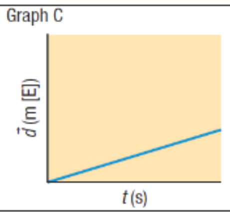
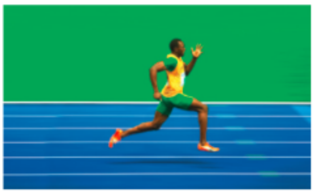
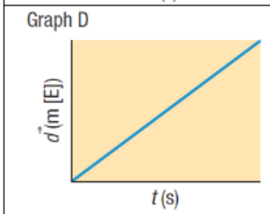
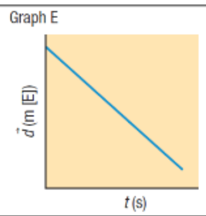
$\vec{v}_{av} = 22.3 \frac{\cancel{km}}{\cancel{h}} \times \frac{1000 m}{1 \cancel{km}} \times \frac{1 h}{60 \cancel{min}} \times \frac{1 \cancel{min}}{60 s} = 22.3 \times \frac{1000}{3600} = 6.194 m/s.$

$\vec{v}_{av} = \frac{\Delta\vec{d}}{\Delta t} \rightarrow \Delta t = \frac{\Delta\vec{d}}{\vec{v}_{av}} = \frac{241 m [W]}{6.194 m/s [W]} = 38.9 s$

3. Motion with uniform and non-uniform velocity

Uniform velocity:	constant velocity in a straight line.
Non-uniform velocity:	velocity that changes or does not travel in a straight line.
accelerated motion	another name for non-uniform velocity.

Example	Uniform?	Why?
A car travels down a straight highway at a steady 100 km/h.	✓	- straight line - constant velocity.
A passenger on an amusement park ride travels in a circle at a constant speed of 1.2 m/s.	✗	- not a straight line.
A parachutist jumps out of an aircraft.	✗	- accelerating towards Earth.

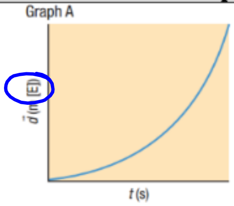
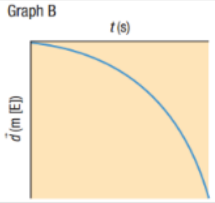
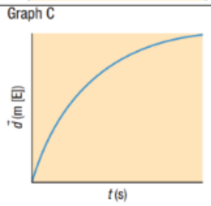
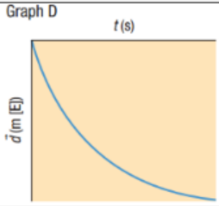
Position-Time Graph	Type of Motion	Example
	-at rest. - $\vec{v}=0$, $\vec{a}=0$. - East of reference.	
	-at rest. -West of reference.	
	-constant velocity. -uniform motion. -moving East.	
	-constant velocity - moving East - faster	
	-constant velocity. -moving West.	

Homework: page 20: #1, 4-8

SPH3U: 1.3 Acceleration

1. Acceleration and graphs

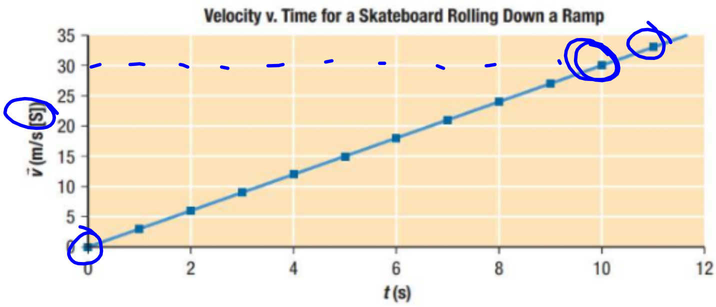
Acceleration:	rate of change of velocity.
velocity-time graph	x-axis: time y-axis: velocity.
position-time graph	is curved if we are accelerating.

Position-Time Graph	Type of Motion	Example
Graph A 	- $\vec{d}$ and $\vec{v}$ are increasing. - speeding up. - moving East.	
Graph B 	- $\vec{d}$ and $\vec{v}$ are decreasing. - speeding up. - moving West.	
Graph C 	- moving East - slowing down.	
Graph D 	- moving West - slowing down.	

2. Determining acceleration from a velocity-time graph

Average acceleration:	$\vec{a}_{av} = \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t}$

- in this course, $\vec{a} = \vec{a}_{av}$ because $\vec{a}$ is always constant.



What is the acceleration of the skateboard in the figure above? Consider the motion between 0 s and 10 s.

$\vec{a} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t} = \frac{30 - 0}{10} = 3.0\,m/s^2\ [S].$

When a rifle is fired, the rifle bullet accelerates from rest to 120 m/s [E] in 1.3 x 10^{-2} s as it travels down the rifle's barrel. What is the bullet's average acceleration?

$\vec{a} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t} = \frac{120 - 0}{1.3 \times 10^{-2}} = 9231\,m/s^2\ [E].$

$= 9.2 \times 10^3\,m/s^2\ [E].$

When a hockey player hits a hockey puck with his stick, the velocity of the puck changes from 8.0 m/s [N] to 10.0 m/s [S] over a time interval of 0.050 s. What is the acceleration of the puck?

$\vec{a} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t} = \frac{10\,[S] - 8\,[N]}{0.05} = \frac{10\,[S] + 8\,[S]}{0.05}$

$= \frac{18}{0.05} = 360\,m/s^2\ [S]$

A racehorse takes 2.70 s to accelerate from a trot to a gallop. If the horse's initial velocity is 3.61 m/s [W] and it experiences an acceleration of 2.77 m/s² [W], what is the racehorse's velocity when it gallops?

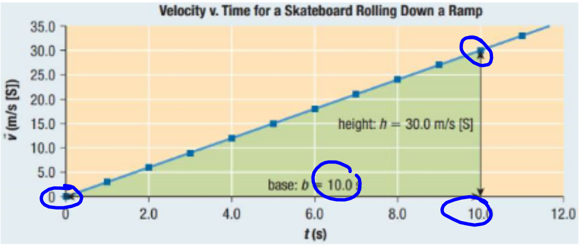
$\vec{a} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t} \rightarrow \vec{v}_f = \vec{a}\Delta t + \vec{v}_i$

$\vec{v}_f = (2.77)(2.70) + 3.61 = 11.089$ m/s [W]

$= 11.1$ m/s [W].

3. Motion with uniform and non-uniform velocity

Area under a v-t graph:	$\vec{\Delta d} = \frac{1}{2}(\vec{v}_f + \vec{v}_i)\Delta t$ - area is equal to displacement.

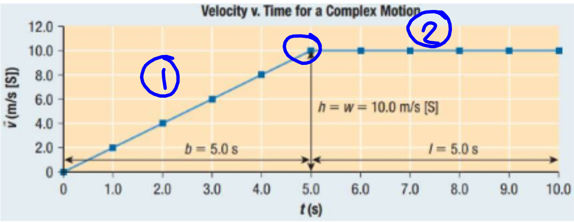


What is the displacement represented by the graph above?

$\Delta d = \frac{1}{2}(v_f + v_i)\Delta t = \frac{1}{2}(30 + 0)(10)$

$= 150$ m [S].

What is the displacement represented by the graph below over the time interval from 0 s to 10.0 s?



1 $\vec{\Delta d} = \frac{1}{2}(v_f + v_i)\Delta t$

$= \frac{1}{2}(10 + 0)(5)$

$= 25$ m [S].

2 $\vec{\Delta d} = \frac{1}{2}(v_f + v_i)\Delta t$

$= \frac{1}{2}(10 + 10)(5)$

$= 50$ m [S].

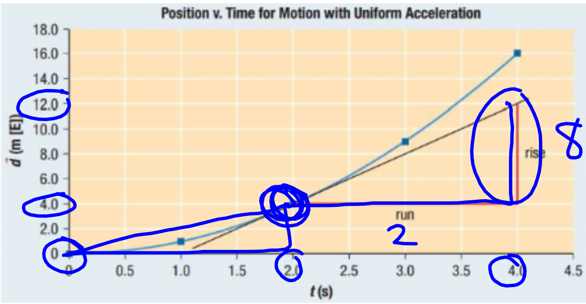
Total: $\vec{\Delta d}_T = \vec{\Delta d}_1 + \vec{\Delta d}_2$

$= 25 + 50$

$= 75$ m [S].

4. Instantaneous velocity and average velocity

Uniform acceleration:	motion where the velocity changes at a constant rate (straight line).
average velocity	average over a time interval.
instantaneous velocity	velocity at a specific instant in time. slope of a tangent on a position-time graph.
tangent	a line that touches a graph at one point.



Consider the point on the curve in the figure above at 2.0 s on the *x*-axis. What is the instantaneous velocity of the object at this time?

$\vec{v}$ = slope of the tangent

$= \frac{\text{rise}}{\text{run}}$

$= \frac{8.0 \text{ m [E]}}{2.0 \text{ s}} = \underline{4.0 \text{ m/s [E]}}.$

What is the average velocity of the object in the figure above over the time interval from 0.0 s to 2.0 s?

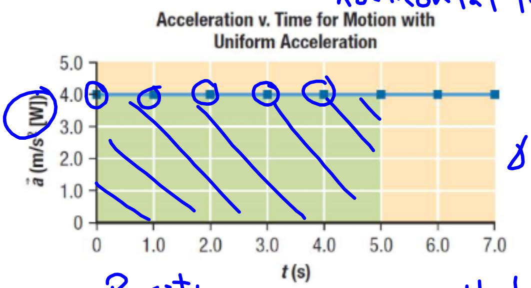
$\vec{v}_{av} = \frac{\Delta \vec{d}}{\Delta t} = \frac{4.0 - 0}{2.0 \text{ s}} = \underline{2.0 \text{ m/s [E]}}$

Homework: page 30: #4-8, 11

SPH3U: 1.4 Comparing Graphs of Linear Motion

1. Acceleration-time graphs

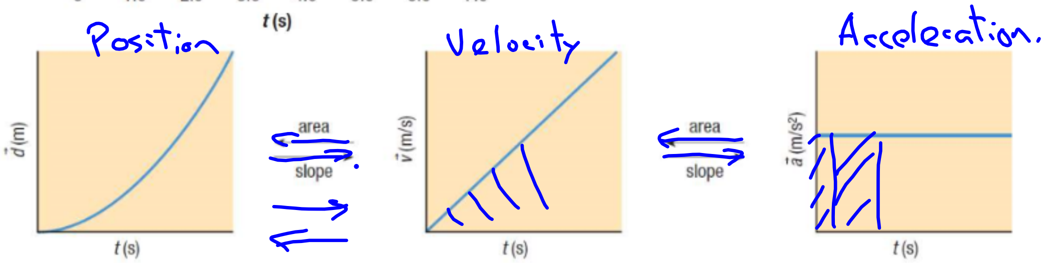
Acceleration-time graph:	accel on the y axis, time on the x-axis -in constant accel, accel will be a straight horizontal line.



Area = bh

$= (5.0\,s)(4.0\,m/s^2)$

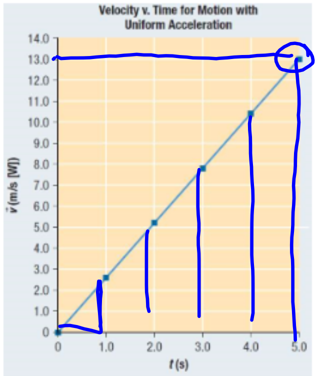
$\Delta\vec{v} = 20\,m/s\ [W]$.



Use the acceleration-time graph above to generate velocity and time data for the object. Then use these data to plot a velocity-time graph.

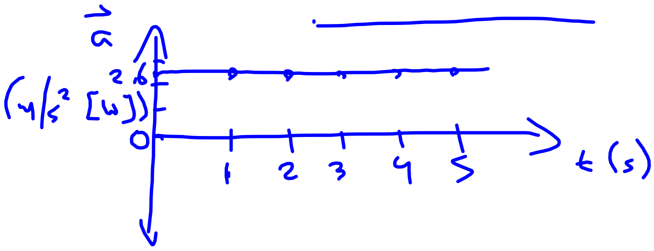
Time t (s)	Acceleration $\vec{a}$ (m/s² [W])	Equation $\Delta\vec{v} = \vec{a}\Delta t$	Velocity $\vec{v}$ (m/s [W])
0	4.0	$\Delta\vec{v} = (4.0\,m/s^2)(0\,s)$	0
1.0	4.0	$\Delta\vec{v} = (4.0\,m/s^2)(1\,s)$	4.0
2.0	4.0	$\Delta\vec{v} = (4.0\,m/s^2)(2\,s)$	8.0
3.0	4.0	(3 s)	12.0
4.0	4.0	(4 s)	16.0
5.0	4.0	(5 s)	20.

Use the velocity-time graph shown to the right to plot the corresponding acceleration-time graph.



$$\vec{a}_{av} = \frac{\Delta \vec{v}}{\Delta t} = \frac{13.0\ m/s\ [W] - 0\ m/s}{5.0\ s - 0\ s.}$$

$$= 2.6\ m/s^2\ [W].$$



2. Summary

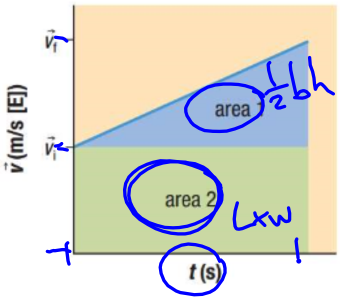
Homework: page 35: #1-4

SPH3U: 1.5 Five Key Equations for Motion with Uniform Acceleration

1. A displacement equation

Area under a V-T graph:	displacement.

Find the area under the graph to the right. This is Equation 1!



$\vec{\Delta d} = Lw + \frac{1}{2}bh = \Delta t \vec{v_i} + \frac{1}{2}\Delta t(\vec{v_f} - \vec{v_i})$

$= \Delta t(\vec{v_i} + \frac{1}{2}\vec{v_f} - \frac{1}{2}\vec{v_i})$

$= \frac{1}{2}\Delta t(\vec{v_f} + \vec{v_i})$

$\boxed{① \ \vec{\Delta d} = \frac{v_f + v_i}{2}\Delta t.}$

Solve the average acceleration equation for v_f. This is Equation 2!

$\vec{a}_{av} = \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v_f} - \vec{v_i}}{\Delta t}$

$\boxed{② \ \vec{v_f} = \vec{a}\Delta t + \vec{v_i}}$

Substitute v_f into the first equation. This is Equation 3!

$\vec{\Delta d} = \frac{\vec{a}\Delta t + \vec{v_i} + \vec{v_i}}{2}\Delta t$

$\boxed{③ \ \vec{\Delta d} = \vec{v_i}\Delta t + \frac{1}{2}\vec{a}\Delta t^2}$

2. The five key equations of accelerated motion

	Equation	Variables in the equation	Variables not in the equation
Equation 1	$\Delta\vec{d} = \left(\frac{\vec{v_f} + \vec{v_i}}{2}\right)\Delta t$	$\vec{\Delta d}, \vec{v_f}, \vec{v_i}, \Delta t$	$\vec{a}$
Equation 2	$\vec{v_f} = \vec{v_i} + \vec{a}\Delta t$	$\vec{v_f}, \vec{v_i}, \vec{a}, \Delta t$	$\vec{\Delta d}$
Equation 3	$\Delta\vec{d} = \vec{v_i}\Delta t + \frac{1}{2}\vec{a}\Delta t^2$	. . .	$\vec{v_f}$
Equation 4	$\Delta\vec{d} = \vec{v_f}\Delta t - \frac{1}{2}\vec{a}\Delta t^2$	. . .	$\vec{v_i}$
Equation 5	$\vec{v_f}^2 = \vec{v_i}^2 + 2\vec{a}\Delta d$	. . .	Δt

A sports car approaches a highway on-ramp at a velocity of 20.0 m/s [E]. If the car accelerates at a rate of 3.2 m/s² [E] for 5.0 s, what is the displacement of the car?

G iven
R equired
E quations
S olution
S tatement

G: $\vec{v}_i = 20.0$ m/s [E], $\vec{a} = 3.2$ m/s² [E], $\Delta t = 5.0$ s.

R: $\Delta \vec{d}$

E: $\Delta \vec{d} = v_i \Delta t + \frac{1}{2} a \Delta t^2$

S: $\Delta \vec{d} = (20)(5) + \frac{1}{2}(3.2)(5.0)^2$
$= 140$ m [E].

S: ∴ the displacement is 140 m [E].

$\vec{a}$, ~~$\vec{v_f}$~~, $\vec{v_i}$, $\Delta\vec{d}$, Δt

A sailboat accelerates uniformly from 6.0 m/s [N] to 8.0 m/s [N] at a rate of 0.50 m/s² [N]. What distance does the boat travel?

G: $\vec{v}_i = 6.0$ m/s [N], $\vec{v}_f = 8.0$ m/s [N], $\vec{a} = 0.50$ m/s² [N]

R: Δd

E: $v_f^2 = v_i^2 + 2a\Delta d$

S: $\Delta d = \frac{v_f^2 - v_i^2}{2a} = \frac{8^2 - 6^2}{2(0.5)} = \frac{64-36}{1} = 28$ m.

S: ∴ the displacement is 28 m.

$\vec{a}$, $\vec{v_f}$, $\vec{v_i}$, $\Delta\vec{d}$, ~~Δt~~

A dart is thrown at a target that is supported by a wooden backstop. It strikes the backstop with an initial velocity of 350 m/s [E]. The dart comes to rest in 0.0050 s.

a. What is the acceleration of the dart?

G: $\vec{v}_i = 350$ m/s [E], $\Delta t = 0.0050$ s, $\vec{v}_f = 0$ m/s

R: $\vec{a}$

E: $v_f = v_i + a\Delta t$

S: $\vec{a} = \frac{v_f - v_i}{\Delta t} = \frac{0 - 350}{0.005} = -70\ 000$ m/s² [E]
$= 70\ 000$ m/s² [W].

S: ∴ the acceleration is 70 000 m/s² [W].

$\vec{a}$, $\vec{v_f}$, $\vec{v_i}$, ~~$\Delta\vec{d}$~~, Δt

b. How far does the dart penetrate into the backstop?

G: $\vec{v}_i = 350$ m/s [E], $\Delta t = 0.0050$ s, $\vec{v}_f = 0$ m/s.

R: $\Delta \vec{d}$

E: $\Delta \vec{d} = \left(\frac{v_f + v_i}{2}\right)\Delta t$

S: $\Delta \vec{d} = \left(\frac{0 + 350}{2}\right)(0.0050) = 0.875$ m
$= 0.88$ m.

~~$\vec{a}$~~, $\vec{v_f}$, $\vec{v_i}$, $\Delta\vec{d}$, Δt

Homework: page 39: #1-4

S: ∴ the displacement is 0.88 m.

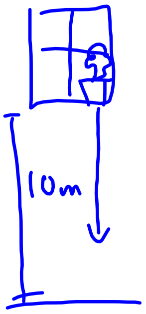
SPH3U: 1.6 Acceleration Near Earth's Surface

1. Acceleration due to gravity

Acceleration due to gravity:	acceleration when an object is allowed to fall freely. $g = 9.8\,m/s^2$.
free fall	when there is no air resistance. only actually happens in a vacuum (close enough).

2. Falling straight down

A flowerpot is knocked off a window ledge and accelerates uniformly to the ground. If the window ledge is 10.0 m above the ground and there is no air resistance, how long does it take the flowerpot to reach the ground?



G: $a = 9.8\,m/s^2$, $\Delta d = 10.0\,m$, $v_i = 0\,m/s$.

R: Δt

F: $\Delta d = \cancel{v_i \Delta t}^{\,0} + \frac{1}{2} a \Delta t^2$

S: $\Delta d = \frac{1}{2}(9.8)\Delta t^2$

$$\Delta t = \sqrt{\frac{2\Delta d}{a}}$$

$$= \sqrt{\frac{2(10)}{9.8}} = 1.43\,s.$$

What is the final velocity of the flowerpot just before it hits the ground?

R: v_f

F: $v_f^2 = v_i^2 + 2a\Delta d$

S: $v_f = \sqrt{0^2 + 2(9.8)(10)}$

$= 14.0\,m/s.$

3. Thrown straight up

A tennis ball is thrown straight up in the air, leaving the person's hand with an initial velocity of 3.0 m/s, as shown to the right. How high, from where it was thrown, does the ball go?

G: $\vec{v_i} = 3.0\ m/s$ [up], $\vec{a} = 9.8\ m/s^2$ [down], $\vec{v_f} = 0\ m/s$.

R: $\vec{\Delta d}$

E: $\vec{v_f}^2 = \vec{v_i}^2 + 2\vec{a}\vec{\Delta d}$

S: $\vec{\Delta d} = \frac{v_f^2 - v_i^2}{2\vec{a}} = \frac{0^2 - 3^2}{2(-9.8)}$

$= \frac{-9}{-19.6} = 0.459\ m$

$= 0.46\ m$.

How long will it take the ball above to reach its maximum height?

R: Δt

E: $\vec{v_f} = \vec{v_i} + \vec{a}\Delta t$

S: $\Delta t = \frac{\vec{v_f} - \vec{v_i}}{\vec{a}} = \frac{0 - 3}{-9.8}$

$= 0.306\ s$

$= 0.31\ s$.

Homework: page 43: #3-7