Unit 4: Quadratics Relations 1
4.1 Modelling With Quadratic Relations

- mathematical models are used to describe real life situations
- The graph of a quadratic relation is a parabola (a symmetrical U shaped curve)
- Every quadratic relation has an x^2 term

Predict which relation is quadratic:

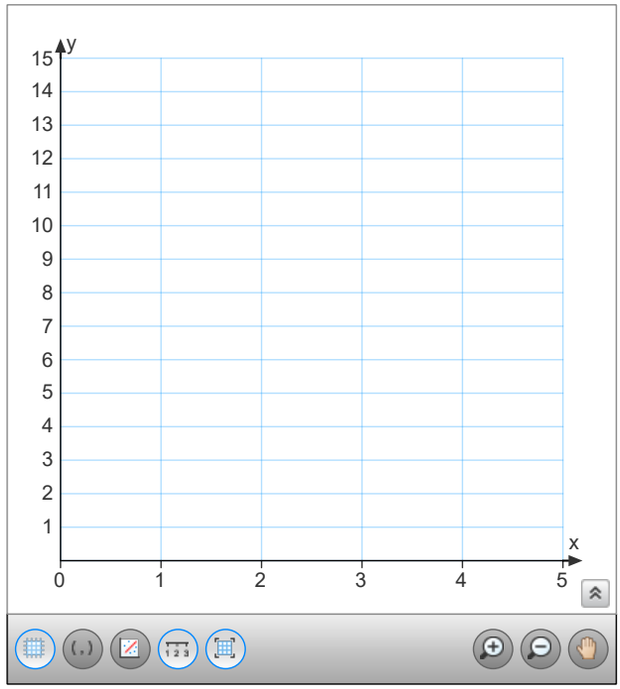
a) $y = x+2$ b) $0.5x^2-5x+5$

Ex.1 The table shows a soccer ball's height above the ground over time after it was kicked in the air.

Time (s)	Height (m)
0	0.10
0.5	7.80
1.0	12.00
1.5	13.80
2.0	13.00
2.5	9.75
3.0	4.00

a) Graph the data
b) Describe the shape of the graph
c) What was the ball's maximum height?
d) For how many seconds was the ball in the air?

X	Y
0	0.1
0.5	7.8
1	12
1.5	13.8
2	13
2.5	9.75
3	4



Ex.2 The table shows how two variable, *x* and *y*, are related.

X	Y
0	1
1	6
2	9
3	10
4	9
5	6
6	1

a) Calculate the first and second differences.

b) Is the relation linear or quadratic? Explain.

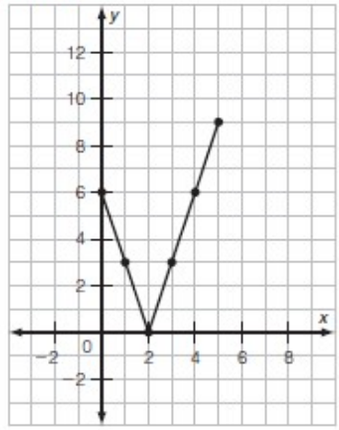
If the first differences are the constant the relation is linear.

If the second differences are constant the relation is quadratic.

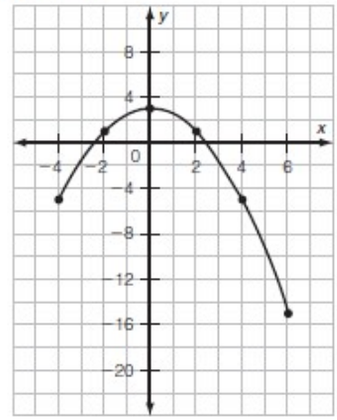
4.1 Modelling with Quadratic Relations, pages 58–60

Practise

1. a) neither



b) quadratic



2. a) first and second differences vary

b) first differences vary; second differences are all –4

c) first differences are all –6

3. a) not quadratic; linear **b)** quadratic; x^2 term

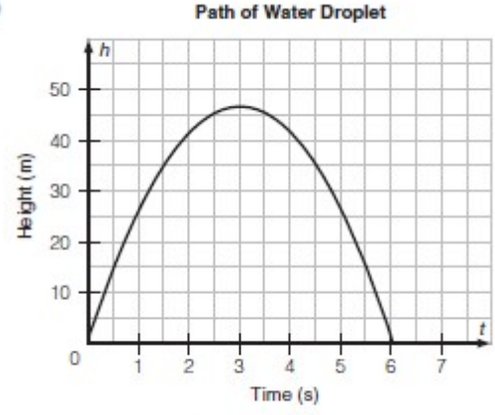
c) not quadratic; linear **d)** quadratic; x^2 term

4. a)

Time (s)	Height (m)
0	**1.0**
1	**26.1**
2	**41.4**
3	**46.9**
4	**42.6**
5	**28.5**
6	**4.6**

b) about 3 s

c)



d) after about 6.2 s, estimates may vary

4.2 The Quadratic Equation

$$y = ax^2 + k$$

Standard quadratic equation
$y = x^2$

Part A: The effect of changing *a*

The value of *a* determines the orientation and shape of the parabola.

- If a > 0 the parabola opens upward
- If a < 0 the parabola is reflected in the x axis if opens downward
- If -1 < a < 1 the parabola is vertically compressed relative to the graph $y=x^2$
- If a > 1 or a < -1 the parabola is vertically stretched relative to the graph $y=x^2$

Part B: The effect of changing *k*

The value of k determines the vertical position of the parabola

- If $k > 0$, the vertex of the parabola is *k* units above the x axis
- If $k < 0$, the vertex of the parabola is *k* units below the x axis

- The coordinates of the vertex are $(0,k)$

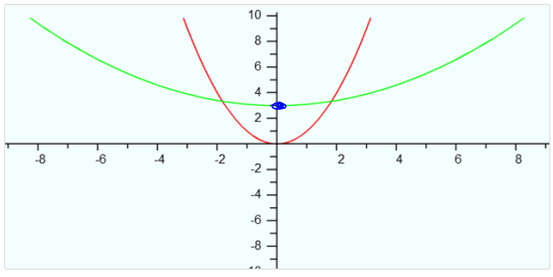
Ex.1 Describe the transformation that would be applied to the graph $y=x^2$. Identify the vertex of the new graph.

a) $y = 3x^2 - 5$

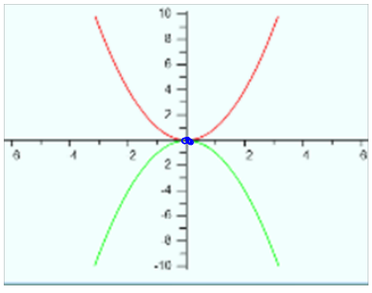
b) $y = 0.4x^2 - 10$

c) $y = -11x^2 + 8$

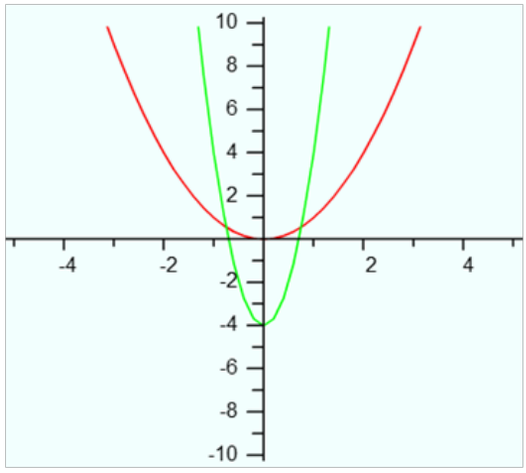
Ex. 2 Describe the shape of the parabola



- The vertex is (0,3)
- the vertex has been translated 3 units up so k=3
- the parabola opens up so a is positive
- the parabola is vertically compressed so $0 < a < 1$



- The vertex is on the x axis so k=0
- the parabola opens downward so a is negative
- the parabola appears to have the same shape as $y=x^2$ so a is -1



- the vertex has been translated 4 units down so k=-4
- the vertex is (0,4)
- the parabola opens upward so a is positive
- the graph is verticall stretched so a>1

4.2 The Quadratic Relation $y = ax^2 + k$, pages 61–63

Practise

1. a) **i), ii)** $y = 2x^2 + 7$, parabola is wider and its vertex is farther from x-axis

b) **i), ii)** $y = -0.5x^2 + 2$; parabola is wider and its vertex is farther from x-axis

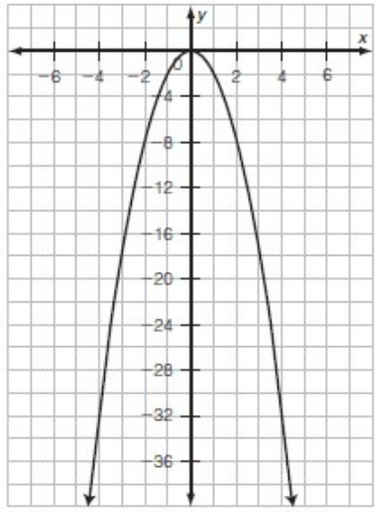
c) **i)** $y = -0.01x^2 - 0.9$, parabola is wider

ii) $y = 0.1x^2 - 9$, vertex is farther from x-axis

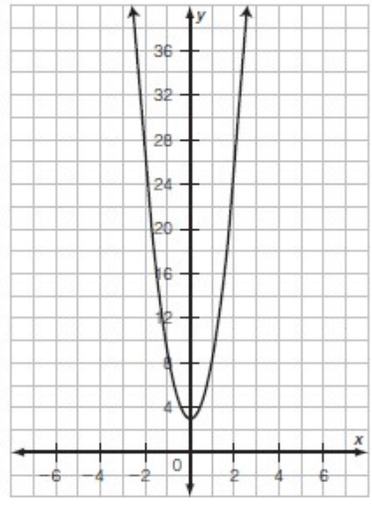
d) **i), ii)** $y = 10x^2 - 10$, parabola is wider and its vertex is farther from x-axis

2. Graphs may vary.

a) reflected in the x-axis, vertically stretched by a factor of 2

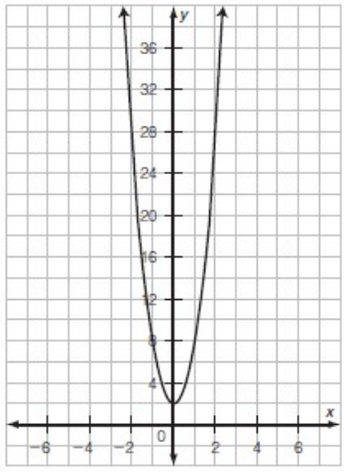


b) translated 3 units up, vertically stretched by a factor of 6

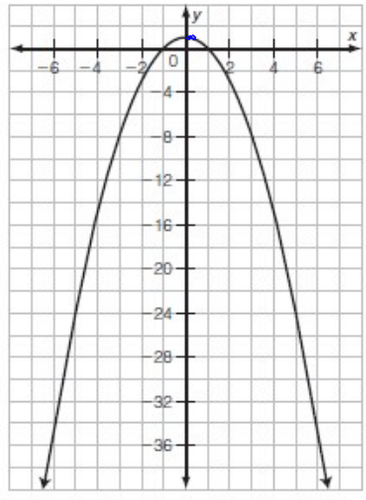


4. Graphs may vary.

a) translate 2 units up, vertically stretch by a factor of 6



b) reflect in the x-axis, translate 1 unit up



5. a) (0, 11) **b)** $y = -4.9x^2 + 11$

4.3 The Quadratic Relation $y = a(x-h)^2$

Graph each relation.

i) $y = (x+2)^2$ ii) $y = (x+4)^2$

a)How does the shape of the graph change compared to $y = x^2$?

b)What is similar about the position of all three graphs?

c)Each relation has a constant term inside the brackets. How does this term relate to the position of the graph?

Graph each relation.

i) $y = (x-2)^2$ ii) $y = (x-4)x^2$

a)How does the shape of the graph change compared to $y = x^2$?

b)What is similar about the position of all three graphs?

c)Each relation has a constant term inside the brackets. How does this term relate to the position of the graph?

4.3 The Quadratic Relation $y = a(x-h)^2$

Notice the negative sign in the equation $y = (x-h)^2$

When h is positive the number after the subtraction symbol is positive.

ex. $y=3(x-8)^2$ $y=a(x-h)^2$ so h=8

When h is negative, the number after the subtraction symbol is negative.

$y=4(x+6)^2$ becomes $y=4(x-(-6))^2$ $y=a(x-h)^2$

h=-6

The value of h determines the horizontal position of the parabola

- if $h > 0$ the vertex of the parabola is h units to the right of the y axis
- if $h < 0$ the vertex of the parabola is h units to the left of the y axis

Ex. 1.Describe the transformation that would be applied to the graph $y = x^2$. Identify the vertex of the new graph. Sketch the graph.

a) $y = -0.1(x-6)^2$

b) $y = 4(x+7)^2$

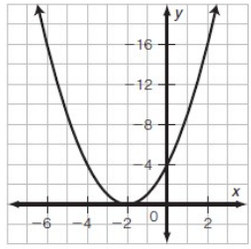
c) $y = 0.9(x+3)^2$

4.3 The Quadratic Relation $y = a(x - h)^2$, pages 64–66

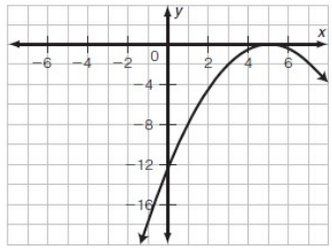
Practise

1. Graphs may vary.

a) $a = 1$, not stretched or compressed; $h = -2$, translated 2 units left



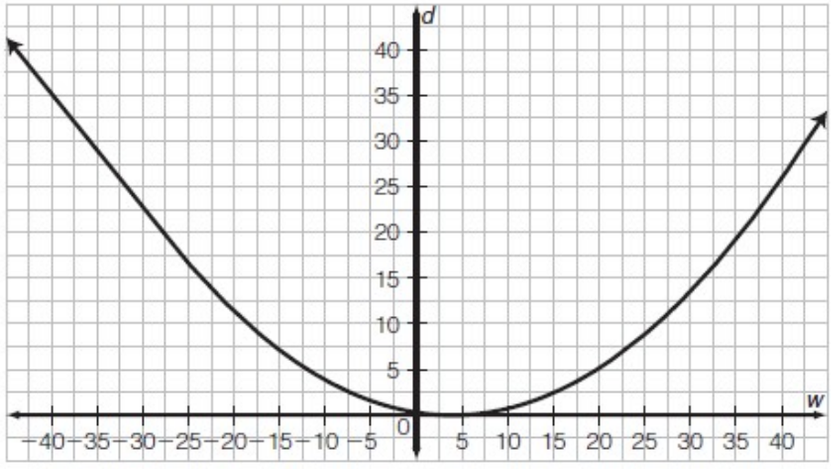
b) $a = -0.5$, vertically compressed by a factor of 0.5, reflected in the x-axis; $h = 5$, translated 5 units right



3. a) $y = 10(x + 7)^2$; $7 > 3$ **b)** $y = 0.375(x - 10)^2$, $10 > 9$

4. $a = \frac{5}{3}$, vertically stretched by a factor of $\frac{5}{3}$; $h = -2$, translated 2 units left

5. a) Graphs may vary.



b) about 70 cm; about 6.5 cm wider

4.4 The Quadratic Relation $y = a(x-h)^2 + k$

Sketch each graph and describe each relation relative to $y = x^2$.

a) $y = 15(x + 5)^2 + 7$

b) $y = 0.5(x - 1)^2 + 9$

c) $y = -4(x + 6)^2 - 8$

4.4 The Quadratic Relation $y = a(x-h)^2 + k$

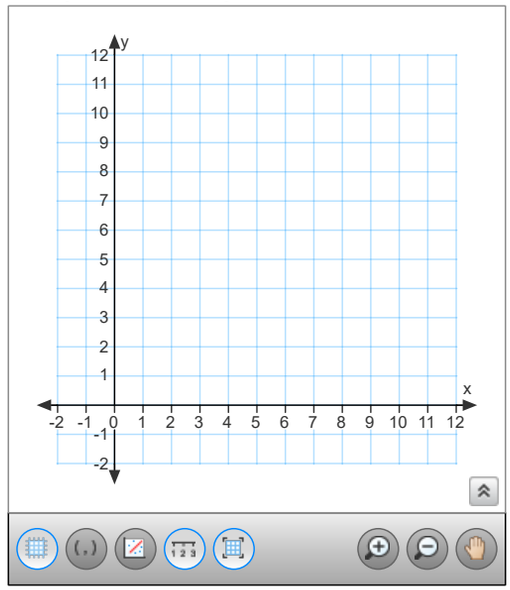
The equation **$y = a(x-h)^2 + k$** is the vertex form of a quadratic relation.

The coordinates of the vertex are (h,k).

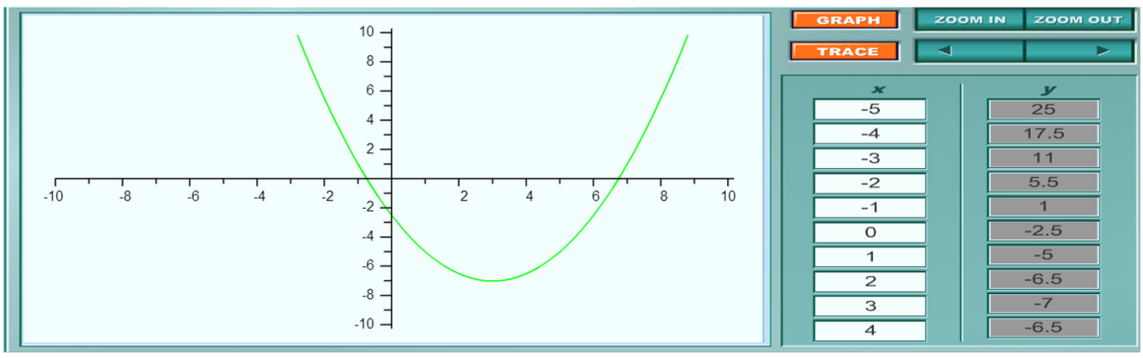
- To sketch a given quadratic relation, plot the vertex and two other points, one of either side of the vertex. Draw a smooth curve through the points.
- To determine the equation use the vertex and another point to solve for a.

Ex. 1. for $y = 2(x - 6)^2 + 3$

- identify the coordinates of the vertex
- determine the x coordinate of a point
  - > 2 units to the left of the vertex
  - > 2 units to the right of the vertex
- use the x value to find 2 points on the parabola, plot
- sketch the graph



Ex. 2
a) find the coordinates of the vertex
b) identify the coordinates of two other points
find the value of a by substituting the vertex and another point
c) write an equation



4.4 The Quadratic Relation $y = a(x - h)^2 + k$, pages 67–69

Practise

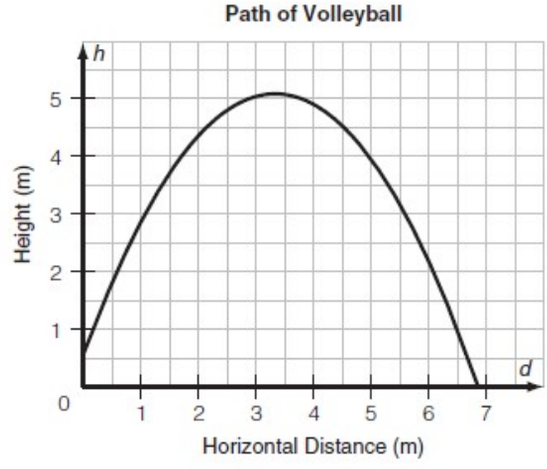
1. a) **i)** (4, −2) **ii)** upward **iii)** vertically stretched
b) **i)** (−1, −5) **ii)** downward **iii)** neither

2. a) **i)** (3, −2) **ii)** positive **iii)** $y = 2(x - 3)^2 - 2$
b) **i)** (−2, 5) **ii)** negative **iii)** $y = -0.5(x + 2)^2 + 5$

3. a) $y = 1.5(x - 4)^2 - 1$ **b)** $y = -2(x + 3)^2 + 4$

4. a) (3.3, 5.1) **b)** 0.5 m

c) Graphs may vary.



d) the maximum height of the ball and the ball's horizontal distance

e) no; the ball will clear the net

4.5 Interpret Graphs of Quadratic Relations

Ex. 1 A construction worker drops his wrench. Its fall is modelled by the relation $h = -4.9t^2 + 342$ where h is the height above the ground in meters and t is the time after the wrench was dropped in seconds.

a) graph the relation.
b) how far above the ground was the wrench when it was dropped?
c) How far did the wrench fall after 5 seconds?
d) how far has the wrench fallen after 10 seconds?
e) when will the wrench hit the ground?

Ex.2 A football player kicks a football held 0.5 m above the grouind. The football reaches a maximum height of 30 m at a horizontal distance of 18m from the player.

a) Determine a quadratic relation that models the path of the football.
b) At what horizontal distance from the player does the football hit the ground?

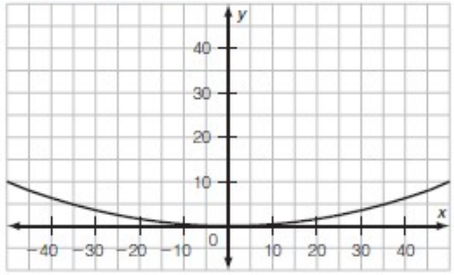
4.5 Interpret Graphs of Quadratic Relations
pages 70–72

Practise

1. **a)** $y = 1$ **b)** $y = -75$

2. **a)** $x = -1$, $x = 5$; $y = 5$; maximum 9; (2, 9)
b) none; $y = -4$; maximum -4; (0, -4)

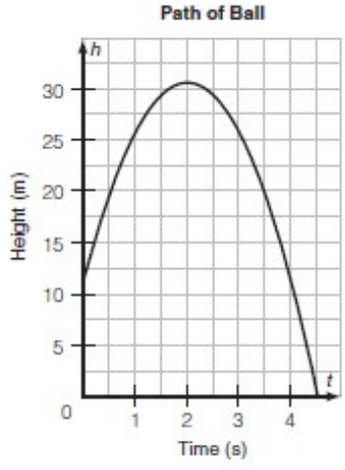
3. **a)** Graphs may vary.



b) $y = 0.004x^2$

4. **a)**

t	h
0	**11**
0.5	**19.575**
1.0	**25.7**
1.5	**29.375**
2.0	**30.6**
2.5	**29.375**
3.0	**25.7**
3.5	**19.575**
4.0	**11**
4.5	**−0.025**

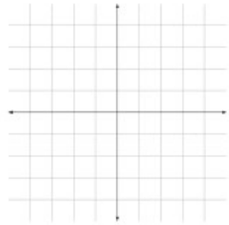
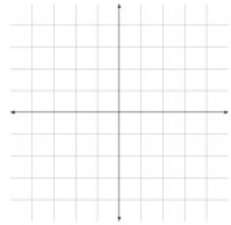
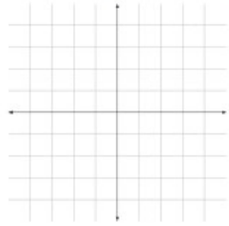
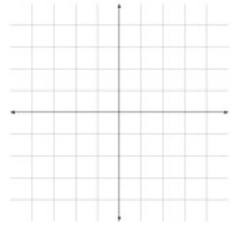


b) $h = -4.9(t - 2)^2 + 30.6$

c) about 1.3 s and 2.7 s

Unit 4: Quadratic Relations I Review

Vertex form of a quadratic relation	
The coordinates of the vertex	

<table>
<tr><td rowspan="2">The value of a determines the _____ of the parabola.</td><td>If a > 0 the parabola opens _____.
  </td><td>If a < 0 the parabola opens _____.
  </td></tr>
<tr><td>If -1 < a <1 the parabola is _____ _____.
  </td><td>If a > 1 or a < -1 the parabola is _____ _____.
  </td></tr>
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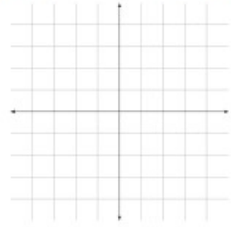
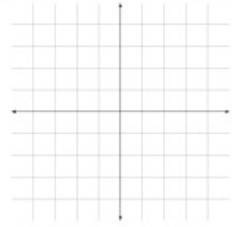
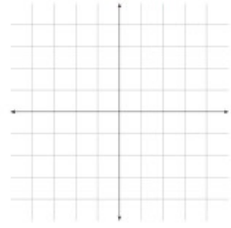
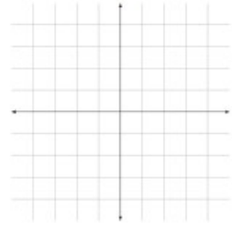
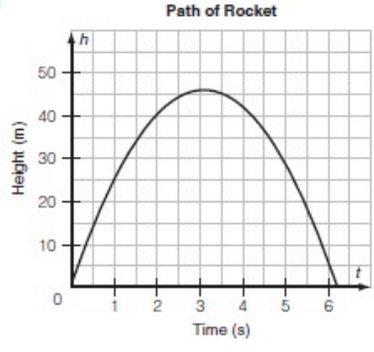
The value of *h* determines the _____ _____ of the parabola.	If h > 0 the vertex of the parabola is h units to the _____ of the y axis. 	If h < 0 the vertex of the parabola is h units to the _____ of the y axis. 
The value of *k* determines the _____ _____ of the parabola.	If k > 0 the vertex of the parabola is k units _____ the y axis. 	If k < 0 the vertex of the parabola is k units _____ the y axis. 

Chapter 4 Review, pages 73–74

1. a)

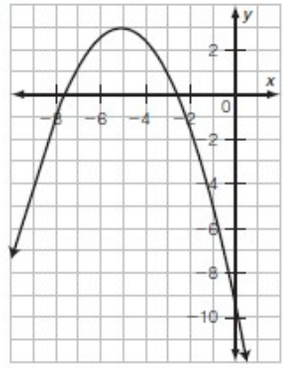


b) about 6.1 s **c)** B

2. a) $y = 1.5x^2 + 1.5$ **b)** $y = 0.5x^2 - 8$

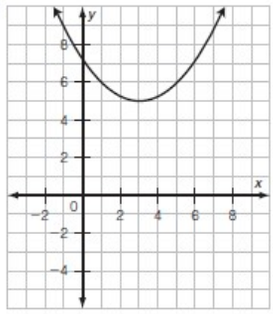
3. Graphs may vary.

a)



$y = -0.5(x + 5)^2 + 3$

b)



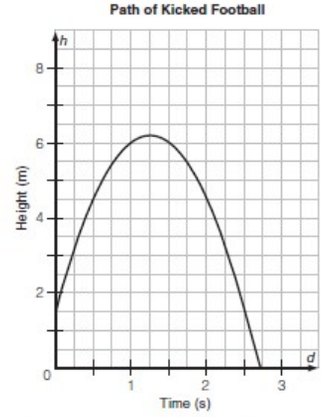
$y = 0.25(x - 3)^2 - 5$

4. a) translated 2 units down, 3 units right, reflected in the x-axis, vertically stretched by a factor of 9

b) translated 1 unit up, 1 unit right, vertically compressed by a factor of 0.9

5. a)

d	0	0.5	1.0	1.5	2.0	2.5
h	1.5	4.5	6	6	4.5	1.5



b) 1.5 m

c) $h = -3(d - 1.25)^2 + 6.1875$