Section 1.6: Relative Motion

Tutorial 1 Practice, page 47–48

1. (a) Given: $\vec{v}_{\text{SW}} = 2.8$ m/s [forward]; $\vec{v}_{\text{TS}} = 1.1$ m/s [forward]

Required: $\vec{v}_{\text{TS}}$

Analysis: Use $\vec{v}_{\text{TW}} = \vec{v}_{\text{TS}} + \vec{v}_{\text{SW}}$, with forward as the positive direction.

Solution:
$$\begin{aligned}\vec{v}_{\text{TW}} &= \vec{v}_{\text{TS}} + \vec{v}_{\text{SW}}\\ &= 1.1 \text{ m/s [forward]} + 2.8 \text{ m/s [forward]}\\ \vec{v}_{\text{TW}} &= 3.9 \text{ m/s [forward]}\end{aligned}$$

Statement: The velocity of the teenagers with respect to the water is 3.9 m/s [forward].

(b) Given: $\vec{v}_{\text{SW}} = 2.8$ m/s [forward]; $\vec{v}_{\text{TS}} = 1.1$ m/s [backward]

Required: $\vec{v}_{\text{TS}}$

Analysis: Use $\vec{v}_{\text{TW}} = \vec{v}_{\text{TS}} + \vec{v}_{\text{SW}}$, with forward as the positive direction.

Solution:
$$\begin{aligned}\vec{v}_{\text{TW}} &= \vec{v}_{\text{TS}} + \vec{v}_{\text{SW}}\\ &= 1.1 \text{ m/s [backward]} + 2.8 \text{ m/s [forward]}\\ &= -1.1 \text{ m/s [forward]} + 2.8 \text{ m/s [forward]}\\ \vec{v}_{\text{TW}} &= 1.7 \text{ m/s [forward]}\end{aligned}$$

Statement: The velocity of the teenagers with respect to the water is 1.7 m/s [forward].

2. Given: $\vec{v}_{\text{PA}} = 235$ km/h [N]; $\vec{v}_{\text{AG}} = 65$ km/h [E 45° N]

Required: $\vec{v}_{\text{PG}}$

Component Method:

Analysis: Use $(v_{\text{PG}})_x = (v_{\text{PA}})_x + (v_{\text{AG}})_x$ and $(v_{\text{PG}})_y = (v_{\text{PA}})_y + (v_{\text{AG}})_y$, with east and north as positive.

Solution: x-components:
$$\begin{aligned}(v_{\text{PG}})_x &= (v_{\text{PA}})_x + (v_{\text{AG}})_x\\ &= 0 \text{ km/h} + (65 \text{ km/h})(\cos 45°)\\ &= 0 \text{ km/h} + 45.96 \text{ km/h}\\ (v_{\text{PG}})_y &= 45.96 \text{ km/h} \text{ (two extra digits carried)}\end{aligned}$$

y-components:
$$\begin{aligned}(v_{\text{PG}})_y &= (v_{\text{PA}})_y + (v_{\text{AG}})_y\\ &= 235 \text{ km/h} + (65 \text{ km/h})(\sin 45°)\\ &= 235 \text{ km/h} + 45.96 \text{ km/h}\\ (v_{\text{PG}})_y &= 281.0 \text{ km/h} \text{ (two extra digits carried)}\end{aligned}$$

Now use these components to determine $\vec{v}_{PG}$.

$$\left|\vec{v}_{PG}\right| = \sqrt{\left|(v_{PG})_x\right|^2 + \left|(v_{PG})_y\right|^2}$$
$$= \sqrt{(45.96 \text{ km/h})^2 + (281.0 \text{ km/h})^2}$$
$$= 284.7 \text{ km/h} \text{ (two extra digits carried)}$$
$$\left|\vec{v}_{PG}\right| = 280 \text{ km/h}$$

$$\theta_2 = \tan^{-1}\left(\frac{281.0 \cancel{\text{km/h}}}{45.96 \cancel{\text{km/h}}}\right)$$
$$\theta_2 = 81°$$

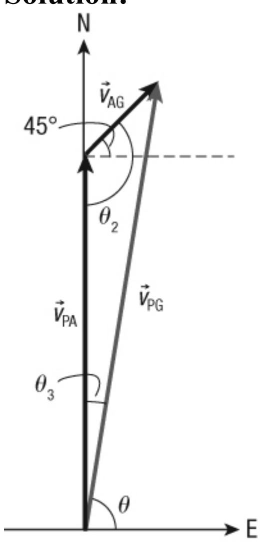
Statement: The speed and direction of the plane with respect to the ground are 280 km/h [E 81° N].

Geometry Method:

Analysis: I know two sides of the vector addition triangle, and the angle that lies in between. Draw a diagram of the situation. Use the cosine and sine laws to determine the speed and direction of the plane.

$$c^2 = a^2 + b^2 - 2ab\cos C \qquad \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Solution:



From the vector addition diagram, $\theta_2 = 90° + 45° = 135°$.

The cosine law gives

$$v_{PG}^2 = v_{PA}^2 + v_{AG}^2 - 2v_{PA}v_{AG}\cos\theta_2$$
$$= (235 \text{ km/h})^2 + (65 \text{ km/h})^2 - 2(235 \text{ km/h})(65 \text{ km/h})(\cos 135°)$$
$$v_{PG}^2 = 81\ 054 \text{ km/h}$$
$$v_{PG} = 284.7 \text{ km/h} \text{ (two extra digits carried)}$$
$$v_{PG} = 280 \text{ km/h}$$

The sine law gives

$$\frac{\sin\theta_3}{v_{\text{AG}}} = \frac{\sin\theta_2}{v_{\text{PG}}}$$

$$\sin\theta_3 = \frac{(65\text{ km/h})(\sin 135°)}{284.7\text{ km/h}}$$

$$\theta_3 = 9.29°$$

$$\theta = 90° - \theta_3$$
$$= 90° - 9.29°$$
$$\theta = 81°$$

Statement: The velocity of the plane with respect to the ground is 280 km/h [E 81°N].

3. Given: $\vec{v}_{\text{HA}} = 175\text{ km/h [S]}$; $\vec{v}_{\text{AG}} = 85\text{ km/h [E]}$

Required: $\vec{v}_{\text{HG}}$

Analysis: The directions of the helicopter and the wind form a right angle with $\vec{v}_{\text{HG}}$ as the hypotenuse of a right-angled triangle. Use the Pythagorean theorem and the tangent ratio to determine $\vec{v}_{\text{HG}}$.

Solution: Determine the magnitude of $\vec{v}_{\text{HG}}$.

$$v_{\text{HG}} = \sqrt{v_{\text{HA}}^2 + v_{\text{AG}}^2}$$
$$= \sqrt{(175\text{ km/h})^2 + (85\text{ km/h})^2}$$
$$v_{\text{HG}} = 190\text{ km/h}$$

Determine the direction of $\vec{v}_{\text{HG}}$

$$\theta = \tan^{-1}\left(\frac{|\vec{v}_{\text{HA}}|}{|\vec{v}_{\text{AG}}|}\right)$$
$$= \tan^{-1}\left(\frac{175\ \cancel{\text{km/h}}}{85\ \cancel{\text{km/h}}}\right)$$
$$\theta = 64°$$

Statement: The velocity of the helicopter with respect to the ground is 190 km/h [E 64°S].

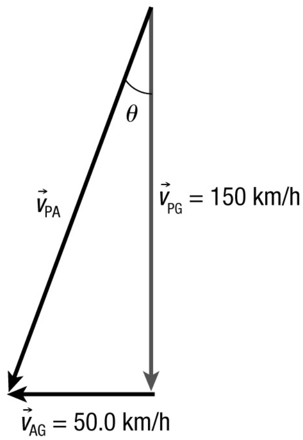
4. Given: $\Delta\vec{d} = 450\text{ km [S]}$; $\Delta t = 3.0\text{ h}$; $\vec{v}_{\text{AG}} = 50.0\text{ km/h [E]}$

Required: $\vec{v}_{\text{PA}}$

Analysis: The displacement of the plane is due south, so the direction of $\vec{v}_{\text{PG}}$ is also south. Determine the magnitude of $\vec{v}_{\text{PG}}$ using the given displacement and time. Then draw the vector addition diagram for $\vec{v}_{\text{PG}} = \vec{v}_{\text{PA}} + \vec{v}_{\text{AG}}$. Since this is a right-angled triangle, use the Pythagorean theorem and the tangent ratio to determine $\vec{v}_{\text{PA}}$.

Solution: The ground speed is

$$v_{PG} = \frac{\Delta d}{\Delta t}$$
$$= \frac{450 \text{ km}}{3.0 \text{ h}}$$
$$v_{PG} = 150 \text{ km/h}$$



Determine the magnitude of $\vec{v}_{PA}$.

$$v_{PA} = \sqrt{(v_{PG})^2 + (v_{AG})^2}$$
$$= \sqrt{(150 \text{ km/h})^2 + (50.0 \text{ km/h})^2}$$
$$v_{PA} = 160 \text{ km/h}$$

Determine the direction of $\vec{v}_{PA}$.

$$\theta = \tan^{-1}\left(\frac{|\vec{v}_{AG}|}{|\vec{v}_{PG}|}\right)$$
$$= \tan^{-1}\left(\frac{50.0 \cancel{\text{ km/h}}}{150 \cancel{\text{ km/h}}}\right)$$
$$\theta = 18°$$

Statement: The plane should head [S 18° W] with an airspeed of 160 km/h.

5. (a) Given: $\vec{v}_{FE} = 4.0$ m/s [N]; $\vec{v}_{CF} = 3.0$ m/s [N]

Required: $\vec{v}_{CE}$

Analysis: Use $\vec{v}_{CE} = \vec{v}_{CF} + \vec{v}_{FE}$, with north as the positive direction.

Solution: $\vec{v}_{CE} = \vec{v}_{CF} + \vec{v}_{FE}$

$$= (+4.0 \text{ m/s}) + (+3.0 \text{ m/s})$$
$$= +7.0 \text{ m/s}$$
$$\vec{v}_{CE} = 7.0 \text{ m/s [N]}$$

Statement: When the child is running north, the velocity of the child with respect to Earth is 7.0 m/s [N].

(b) Given: $\vec{v}_{\text{FE}} = 4.0 \text{ m/s [N]}; \vec{v}_{\text{CF}} = 3.0 \text{ m/s [S]}$

Required: $\vec{v}_{\text{CE}}$

Analysis: Use $\vec{v}_{\text{CE}} = \vec{v}_{\text{CF}} + \vec{v}_{\text{FE}}$, with north as the positive direction.

Solution:
$$\begin{aligned}\vec{v}_{\text{CE}} &= \vec{v}_{\text{CF}} + \vec{v}_{\text{FE}}\\ &= (+4.0 \text{ m/s}) + (-3.0 \text{ m/s})\\ &= +1.0 \text{ m/s}\\ \vec{v}_{\text{CE}} &= 1.0 \text{ m/s [N]}\end{aligned}$$

Statement: When the child is running south, the velocity of the child with respect to Earth is 1.0 m/s [N].

(c) Given: $\vec{v}_{\text{FE}} = 4.0 \text{ m/s [N]}; \vec{v}_{\text{CF}} = 3.0 \text{ m/s [E]}$

Required: $\vec{v}_{\text{CE}}$

Analysis: Use $\vec{v}_{\text{CE}} = \vec{v}_{\text{CF}} + \vec{v}_{\text{FE}}$. This is a right-angled triangle, so use the Pythagorean theorem and the tangent ratio.

Solution: Determine the magnitude of $\vec{v}_{\text{CE}}$.

$$\begin{aligned}v_{\text{CE}} &= \sqrt{(v_{\text{CF}})^2 + (v_{\text{FE}})^2}\\ &= \sqrt{(3.0 \text{ m/s})^2 + (4.0 \text{ m/s})^2}\\ v_{\text{CE}} &= 5.0 \text{ m/s}\end{aligned}$$

Determine the direction of $\vec{v}_{\text{CE}}$.

$$\begin{aligned}\theta &= \tan^{-1}\left(\frac{|\vec{v}_{\text{FE}}|}{|\vec{v}_{\text{CF}}|}\right)\\ &= \tan^{-1}\left(\frac{4.0 \cancel{\text{m/s}}}{3.0 \cancel{\text{m/s}}}\right)\\ \theta &= 53^\circ\end{aligned}$$

Statement: When the child is running east, the child's velocity with respect to Earth is 5.0 m/s [E 53° N], or equivalently 5.0 m/s [N 37° E].

6. (a) Given: $\vec{v}_{\text{PA}} = 3.5 \times 10^2 \text{ km/h [N 35° W]}; \vec{v}_{\text{AG}} = 62 \text{ km/h [S]}$

Required: $\vec{v}_{\text{PG}}$

Analysis: Since one of the given velocities points due south, use the component method of solution. Use $(v_{\text{PG}})_x = (v_{\text{PA}})_x + (v_{\text{AG}})_x$ and $(v_{\text{PG}})_y = (v_{\text{PA}})_y + (v_{\text{AG}})_y$, with east and north as positive.

Solution: *x*-components:

$$\begin{aligned}(v_{\text{PG}})_x &= (v_{\text{PA}})_x + (v_{\text{AG}})_x\\ &= (-350 \text{ km/h})(\sin 35^\circ) + 0 \text{ km/h}\\ (v_{\text{PG}})_x &= -200.8 \text{ km/h} \ \text{(two extra digits carried)}\end{aligned}$$

y-components:

$$(v_{PG})_y = (v_{PA})_y + (v_{AG})_y$$
$$= (350 \text{ km/h})(\cos 35°) + (-62 \text{ km/h})$$
$$(v_{PG})_y = 224.7 \text{ km/h} \text{ (two extra digits carried)}$$

Now use these components to determine $\vec{v}_{PG}$.

$$|\vec{v}_{PG}| = \sqrt{|(v_{PG})_x|^2 + |(v_{PG})_y|^2}$$
$$= \sqrt{(200.75 \text{ km/h})^2 + (224.70 \text{ km/h})^2}$$
$$= 301.3 \text{ km/h} \text{ (two extra digits carried)}$$
$$|\vec{v}_{PG}| = 3.0 \times 10^2 \text{ km/h}$$

$$\theta = \tan^{-1}\left(\frac{200.8 \cancel{\text{ km/h}}}{224.7 \cancel{\text{ km/h}}}\right)$$
$$\theta = 42°$$

Statement: The velocity of plane with respect to the ground is 3.0×10^2 km/h [N 42° W].

(b) Given: $\vec{v}_{PG} = 301.3$ km/h [N 42º W]; $\Delta t = 1.2$ h

Required: $\Delta\vec{d}$

Analysis: Since the plane moves at constant ground velocity, use $\vec{v}_{av} = \dfrac{\Delta\vec{d}}{\Delta t}$; $\Delta\vec{d} = \vec{v}_{av}\Delta t$.

Solution: $\Delta\vec{d} = \vec{v}_{PG}\Delta t$

$$= (301.3 \text{ km/}\cancel{\text{h}} \text{ [N 42° W]})(1.2 \cancel{\text{h}})$$
$$\Delta\vec{d} = 3.6 \times 10^2 \text{ km/h [N 42° W]}$$

Statement: The plane's displacement after 1.2 h is 3.6×10^2 km/h [N 42° W].

7. (a) Given: $\vec{v}_{PW} = 0.70$ m/s [N]; $\vec{v}_{WE} = 0.40$ m/s [E]

Required: $\vec{v}_{PE}$

Analysis: The directions of the current and the swimmer form a right angle with $\vec{v}_{PE}$ as the hypotenuse of a right-angled triangle. Use the Pythagorean theorem and the tangent ratio to determine $\vec{v}_{PE}$.

Solution: Determine the magnitude of $\vec{v}_{PE}$.

$$v_{PE} = \sqrt{(v_{PW})^2 + (v_{WE})^2}$$
$$= \sqrt{(0.70 \text{ m/s})^2 + (0.40 \text{ m/s})^2}$$
$$v_{PE} = 0.81 \text{ m/s}$$

Determine the direction of $\vec{v}_{PE}$.

$$\theta = \tan^{-1}\left(\frac{|\vec{v}_{WE}|}{|\vec{v}_{PW}|}\right)$$
$$= \tan^{-1}\left(\frac{0.40 \cancel{\text{ m/s}}}{0.70 \cancel{\text{ m/s}}}\right)$$
$$\theta = 30°$$

Statement: The velocity of the swimmer with respect to Earth is 0.81 m/s [N 30° E].

(b) Given: $\Delta\vec{d} = 84$ m [N]

Required: time to cross river, Δt

Analysis: The component of $\vec{v}_{\text{PE}}$ pointing north is $\vec{v}_{\text{PW}}$. Use $\vec{v}_{\text{av}} = \dfrac{\Delta\vec{d}}{\Delta t}$; $\Delta t = \dfrac{\Delta d}{v_{\text{av}}}$.

Solution:
$$\begin{aligned}\Delta t &= \frac{\Delta d}{v_{\text{PW}}} \\ &= \frac{84\ \cancel{\text{m}}}{0.70\ \cancel{\text{m}}/\text{s}} \\ \Delta t &= 1.2\times10^2\ \text{s}\end{aligned}$$

Statement: It takes the swimmer 1.2×10^2 s to cross the river.

(c) Given: $\vec{v}_{\text{WE}} = 0.40$ m/s [E]; $\Delta t = 120$ s

Required: distance downstream, Δd

Analysis: The component of $\vec{v}_{\text{PE}}$ pointing east is $\vec{v}_{\text{WE}}$. Use $\vec{v}_{\text{av}} = \dfrac{\Delta\vec{d}}{\Delta t}$; $\Delta\vec{d} = \vec{v}_{\text{av}}\Delta t$.

Solution:
$$\begin{aligned}\Delta d &= v_{\text{WE}}\Delta t \\ &= (0.40\ \text{m}/\cancel{\text{s}})(120\ \cancel{\text{s}}) \\ \Delta d &= 48\ \text{m}\end{aligned}$$

Statement: The swimmer lands 48 m downstream.

(d) Given: $v_{\text{PW}} = 0.70$ m/s; $\vec{v}_{\text{WE}} = 0.40$ m/s [E]; $\vec{v}_{\text{PE}} = ?$ [N]

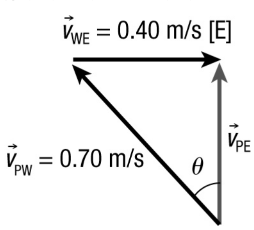
Required: direction of $\vec{v}_{\text{PW}}$

Analysis: The vectors form a right-angled triangle with $\vec{v}_{\text{PW}}$ as the hypotenuse. is a right-angled triangle. Use the sine ratio to determine the direction of $\vec{v}_{\text{PW}}$ with respect to $\vec{v}_{\text{PE}}$ [N]:

$$\sin\theta = \frac{\text{opposite side}}{\text{hypoteuse}}$$

$$\theta = \sin^{-1}\left(\frac{\text{opposite side}}{\text{hypoteuse}}\right)$$

Solution: Determine the direction of $\vec{v}_{\text{PW}}$.



$\theta = \sin^{-1}\left(\frac{|\vec{v}_{\text{WE}}|}{|\vec{v}_{\text{PW}}|}\right)$

$= \sin^{-1}\left(\frac{0.40\ \cancel{\text{m/s}}}{0.70\ \cancel{\text{m/s}}}\right)$

$\theta = 35°$

Statement: The swimmer should head [N 35° W] to land directly north of her starting point.

8. Given: $|\vec{v}_{\text{C}_1\text{W}}| = |\vec{v}_{\text{C}_2\text{W}}|$; $\vec{v}_{\text{C}_1\text{E}} = 1.2$ m/s [upstream]; $\vec{v}_{\text{C}_2\text{E}} = 2.9$ m/s [downstream]

Required: v_{WE}

Analysis: I know the relative velocity equations, $\vec{v}_{\text{C}_1\text{E}} = \vec{v}_{\text{C}_1\text{W}} + \vec{v}_{\text{WE}}$ and $\vec{v}_{\text{C}_2\text{E}} = \vec{v}_{\text{C}_2\text{W}} + \vec{v}_{\text{WE}}$.

Switching to a simpler notation, $v = |\vec{v}_{\text{C}_1\text{W}}| = |\vec{v}_{\text{C}_2\text{W}}|$ and $w = \vec{v}_{\text{WE}}$. I will rewrite the relative velocity equations, using downstream as the positive direction. Then, I can solve for the required speed.

Solution: The relative velocity equations are

$\vec{v}_{\text{C}_1\text{E}} = \vec{v}_{\text{C}_1\text{W}} + \vec{v}_{\text{WE}}$

$-1.2\ \text{m/s} = (-v) + w \qquad \text{(Equation 1)}$

$\vec{v}_{\text{C}_2\text{E}} = \vec{v}_{\text{C}_2\text{W}} + \vec{v}_{\text{WE}}$

$+2.9\ \text{m/s} = v + w \qquad \text{(Equation 2)}$

Adding Equations 1 and 2,

$(-1.2\ \text{m/s}) + (+2.9\ \text{m/s}) = (-v) + w + v + w$

$1.7\ \text{m/s} = 2w$

$w = 0.85\ \text{m/s}$

Statement: The speed of the water relative to Earth is 0.85 m/s.

(b) Given: $\vec{v}_{\text{C}_2\text{E}} = 2.9$ m/s [downstream]; $v_{\text{WE}} = w = 0.85$ m/s

Required: $v = |\vec{v}_{\text{C}_1\text{W}}| = |\vec{v}_{\text{C}_2\text{W}}|$

Analysis: Substitute the speed of the water from part (a) into either Equation 1 or Equation 2.

Solution: Using Equation 2,

$(+2.9\ \text{m/s}) = v + w$

$v = (+2.9\ \text{m/s}) - w$

$= (+2.9\ \text{m/s}) - (0.85\ \text{m/s})$

$v = 2.0\ \text{m/s}$

Statement: The canoeists paddle at 2.0 m/s with respect to the water.

9. (a) Given: $\vec{v}_{\text{PA}} = 630$ km/h [N]; $\vec{v}_{\text{AG}} = 35$ km/h [S]; $\Delta\vec{d} = 750$ km [N]

Required: Δt

Analysis: Use $\vec{v}_{\text{PG}} = \vec{v}_{\text{PA}} + \vec{v}_{\text{AG}}$, with north as the positive direction, to calculate the ground velocity of the plane. Then use $\vec{v}_{\text{av}} = \frac{\Delta\vec{d}}{\Delta t}$ to determine the time $\Delta t = \frac{\Delta d}{v_{\text{av}}}$.

Solution: The ground velocity is

$$\begin{aligned}\vec{v}_{\text{PG}} &= \vec{v}_{\text{PA}} + \vec{v}_{\text{AG}} \\ &= (+630 \text{ km/h}) + (-35 \text{ km/h}) \\ &= +595 \text{ km/h}\end{aligned}$$

$\vec{v}_{\text{PG}} = 595 \text{ km/h [N]}$ (one extra digit carried)

The required time, Δt, is

$$\begin{aligned}\Delta t &= \frac{\Delta d}{v_{\text{PG}}} \\ &= \frac{750 \cancel{\text{ km}}}{595 \cancel{\text{ km}}\text{/h}}\end{aligned}$$

$\Delta t = 1.3 \text{ h}$

Statement: The flight time is 1.3 h when the wind is blowing south.

(b) Given: $\vec{v}_{\text{PA}} = 630 \text{ km/h [N]}$; $\vec{v}_{\text{AG}} = 35 \text{ km/h [N]}$; $\Delta\vec{d} = 750 \text{ km [N]}$

Required: Δt

Analysis: Use $\vec{v}_{\text{PG}} = \vec{v}_{\text{PA}} + \vec{v}_{\text{AG}}$, with north as the positive direction, to calculate the ground velocity of the plane. Then use $\vec{v}_{\text{av}} = \frac{\Delta\vec{d}}{\Delta t}$ to determine the time Δt.

Solution: The ground velocity is

$$\begin{aligned}\vec{v}_{\text{PG}} &= \vec{v}_{\text{PA}} + \vec{v}_{\text{AG}} \\ &= (+630 \text{ km/h}) + (+35 \text{ km/h}) \\ &= +665 \text{ km/h}\end{aligned}$$

$\vec{v}_{\text{PG}} = 665 \text{ km/h [N]}$ (one extra digit carried)

The required time Δt is

$$v_{\text{av}} = \frac{\Delta d}{\Delta t}$$

$$\begin{aligned}\Delta t &= \frac{\Delta d}{v_{\text{PG}}} \\ &= \frac{750 \cancel{\text{ km}}}{665 \cancel{\text{ km}}\text{/h}}\end{aligned}$$

$\Delta t = 1.1 \text{ h}$

Statement: The flight time is 1.1 h when there is a tail wind. This time is shorter than when there is an opposing wind because the plane moves more quickly with respect to the ground.

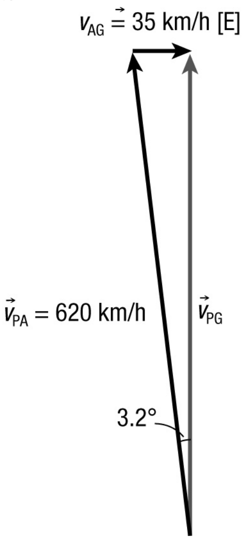
(c) Given: $v_{PA} = 630$ km/h; $\vec{v}_{AG} = 35$ km/h [E]; $\Delta\vec{d} = 750$ km [N]

Required: direction of $\vec{v}_{PA}$, Δt

Analysis: The pilot needs to head somewhat west of north to compensate for the wind that heads east. Sketch the relative velocities, $\vec{v}_{PG} = \vec{v}_{PA} + \vec{v}_{AG}$. Since the vector addition triangle is a right-angled triangle, use the Pythagorean theorem and the sine ratio to determine the direction of the air velocity and the magnitude of the ground velocity. Then determine the flight time using

$\vec{v}_{av} = \dfrac{\Delta\vec{d}}{\Delta t}$; $\Delta t = \dfrac{\Delta d}{v_{av}}$.

Solution:



Determine the direction of $\vec{v}_{PA}$.

$$\theta = \sin^{-1}\left(\frac{|\vec{v}_{AG}|}{|\vec{v}_{PA}|}\right)$$
$$= \sin^{-1}\left(\frac{35\ \cancel{\text{km/h}}}{630\ \cancel{\text{km/h}}}\right)$$
$$\theta = 3.2°$$

Determine the magnitude of $\vec{v}_{PG}$.

$$(v_{PA})^2 = (v_{PG})^2 + (v_{AG})^2$$
$$v_{PG} = \sqrt{(v_{PA})^2 - (v_{AG})^2}$$
$$= \sqrt{(630\text{ km/h})^2 - (35\text{ km/h})^2}$$
$$= 629.0\text{ km/h}\ \ \text{(two extra digits carried)}$$

$$\Delta t = \frac{\Delta d}{v_{PG}}$$
$$= \frac{750\ \cancel{\text{km}}}{629.0\ \cancel{\text{km}}/\text{h}}$$
$$\Delta t = 1.2\ \text{h}$$

Statement: The pilot's heading needs to be [N 3.2° W]. The new flight time is 1.2 h.

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1. (a) Given: $v_{PW} = 1.2$ m/s; $\vec{v}_{WE} = 0.50$ m/s [E]; $\Delta\vec{d}_1 = 1.0$ km [W]; $\Delta\vec{d}_2 = 1.0$ km [E]

Required: $\Delta t = \Delta t_1 + \Delta t_2$

Analysis: Look first at the upstream motion. Determine the speed of the person with respect to Earth using $\vec{v}_{PE} = \vec{v}_{PW} + \vec{v}_{WE}$. Then, rearrange the equation $\vec{v}_{av} = \frac{\Delta\vec{d}}{\Delta t}$ to determine the time required; $\Delta t = \frac{\Delta d}{v_{av}}$. Repeat this procedure for the downstream motion. Finally, use $\Delta t = \Delta t_1 + \Delta t_2$. Throughout, use east as the positive direction.

Solution:

upstream motion:

$$\vec{v}_{PE} = \vec{v}_{PW} + \vec{v}_{WE}$$
$$= (-1.2\ \text{m/s}) + (+0.50\ \text{m/s})$$
$$= -0.70\ \text{m/s}$$
$$\Delta t_1 = \frac{\Delta d}{v_{PW}}$$
$$= \frac{1000\ \cancel{\text{m}}}{0.70\ \cancel{\text{m}}/\text{s}}$$
$$\Delta t_1 = 1429\ \text{s}\ \ \text{(two extra digits carried)}$$

downstream motion:

$$\vec{v}_{PE} = \vec{v}_{PW} + \vec{v}_{WE}$$
$$= (+1.2\ \text{m/s}) + (+0.50\ \text{m/s})$$
$$= +1.70\ \text{m/s}$$
$$\Delta t_2 = \frac{\Delta d}{v_{PW}}$$
$$= \frac{1000\ \cancel{\text{m}}}{1.70\ \cancel{\text{m}}/\text{s}}$$
$$\Delta t_2 = 588.2\ \text{s}\ \ \text{(two extra digits carried)}$$

The total time for the swim is

$$\Delta t = \Delta t_1 + \Delta t_2$$
$$= 1429\ \text{s} + 588.2\ \text{s}$$
$$= 2017\ \cancel{\text{s}} \times \frac{1\ \text{min}}{60\ \cancel{\text{s}}}$$
$$\Delta t = 34\ \text{min}$$

Statement: The swim upstream and back takes 34 min.

(b) Answers may vary. Sample answer: No, the time will not change. The total time downstream and back will also be 34 min. The first leg of the swim will be fast (588 s) and the second leg slow (1429 s), but the whole swim takes the same time.

(c) Given: $v = 1.2$ m/s; $\Delta d = 2.0$ km
Required: Δt
Analysis: $v_{\text{av}} = \dfrac{\Delta d}{\Delta t}$; $\Delta t = \dfrac{\Delta d}{v_{\text{av}}}$

Solution:
$$\begin{aligned}\Delta t &= \frac{\Delta d}{v} \\ &= \frac{2000\ \cancel{\text{m}}}{1.2\ \cancel{\text{m}}/\text{s}} \\ &= 1667\ \cancel{\text{s}} \times \frac{1\ \text{min}}{60\ \cancel{\text{s}}}\ \text{(two extra digits carried)} \\ \Delta t &= 28\ \text{min}\end{aligned}$$

Statement: The total swim would take 28 min in still water. This time is less than for the swims in parts (a) and (b). Swimming against the current is a great disadvantage that is not compensated for fully by swimming with the current for the same distance. The trip takes longer when there is a current because the current slows the swimmer when swimming against it.

2. (a) Given: $\vec{v}_{\text{PA}} = 200$ m/s [W]; $\vec{v}_{\text{AG}} = 60$ m/s [N]
Required: $\vec{v}_{\text{CE}}$
Analysis: $\vec{v}_{\text{PG}} = \vec{v}_{\text{PA}} + \vec{v}_{\text{AG}}$. This is a right-angled triangle, so use the Pythagorean theorem and the tangent ratio.
Solution: Determine the magnitude of $\vec{v}_{\text{PG}}$.

$$\begin{aligned}(v_{\text{PG}})^2 &= (v_{\text{PA}})^2 + (v_{\text{AG}})^2 \\ &= (200\ \text{m/s})^2 + (60\ \text{m/s})^2 \\ (v_{\text{PG}})^2 &= 43\ 600\ \text{m}^2/\text{s}^2 \\ v_{\text{PG}} &= 209\ \text{m/s (two extra digits carried)} \\ v_{\text{PG}} &= 200\ \text{m/s}\end{aligned}$$

Determine the direction of $\vec{v}_{\text{PG}}$.

$$\begin{aligned}\theta &= \tan^{-1}\left(\frac{|\vec{v}_{\text{AG}}|}{|\vec{v}_{\text{PG}}|}\right) \\ &= \tan^{-1}\left(\frac{60\ \cancel{\text{m/s}}}{209\ \cancel{\text{m/s}}}\right) \\ \theta &= 20^\circ\end{aligned}$$

Statement: The ground velocity of the plane is 200 m/s [W 20° N].

(b) Given: $\vec{v}_{PA} = 200$ m/s [W]; $\vec{v}_{AG} = 60$ m/s [E]; $\Delta\vec{d} = 300$ km [W]

Required: Δt

Analysis: First, use $\vec{v}_{PG} = \vec{v}_{PA} + \vec{v}_{AG}$ to determine the ground velocity. Then rearrange the equation $\vec{v}_{av} = \dfrac{\Delta\vec{d}}{\Delta t}$ to calculate the flight time; $\Delta t = \dfrac{\Delta d}{v_{av}}$.

Solution: The ground velocity is

$$\begin{aligned}\vec{v}_{PG} &= \vec{v}_{PA} + \vec{v}_{AG}\\ &= 200 \text{ m/s [W]} + 60 \text{ m/s [E]}\\ &= 200 \text{ m/s [W]} + (-60 \text{ m/s [W]})\\ \vec{v}_{PG} &= 140 \text{ m/s [W]} \text{ (one extra digit carried)}\end{aligned}$$

$$\begin{aligned}\Delta t &= \frac{\Delta d}{v_{PG}}\\ &= \frac{300 \cancel{\text{ km}}}{140 \cancel{\text{ m}}\text{/s}} \times \frac{1000 \cancel{\text{ m}}}{1 \cancel{\text{ km}}}\\ &= 2143 \cancel{\text{ s}} \times \frac{1 \text{ min}}{60 \cancel{\text{ s}}}\\ \Delta t &= 36 \text{ min}\end{aligned}$$

Statement: The flight time is 36 min.

3. (a) Given: $\vec{v}_{HA} = 62$ m/s [N]; $\vec{v}_{AE} = 18$ m/s [N]

Required: $\vec{v}_{HE}$

Analysis: $\vec{v}_{HE} = \vec{v}_{HA} + \vec{v}_{AE}$

Solution:
$$\begin{aligned}\vec{v}_{HE} &= \vec{v}_{HA} + \vec{v}_{AE}\\ &= 62 \text{ m/s [N]} + 18 \text{ m/s [N]}\\ \vec{v}_{HE} &= 8.0 \times 10^1 \text{ m/s [N]}\end{aligned}$$

Statement: The ground velocity of the helicopter is 8.0×10^1 m/s [N].

(b) Given: $\vec{v}_{HA} = 62$ m/s [N]; $\vec{v}_{AE} = 18$ m/s [S]

Required: $\vec{v}_{HE}$

Analysis: $\vec{v}_{HE} = \vec{v}_{HA} + \vec{v}_{AE}$

Solution:
$$\begin{aligned}\vec{v}_{HE} &= \vec{v}_{HA} + \vec{v}_{AE}\\ &= 62 \text{ m/s [N]} + 18 \text{ m/s [S]}\\ &= 62 \text{ m/s [N]} + (-18 \text{ m/s [N]})\\ \vec{v}_{HE} &= 44 \text{ m/s [N]}\end{aligned}$$

Statement: The ground velocity of the helicopter is 44 m/s [N].

(c) Given: $\vec{v}_{\text{HA}} = 62 \text{ m/s [N]}; \vec{v}_{\text{AE}} = 18 \text{ m/s [W]}$

Required: $\vec{v}_{\text{HE}}$

Analysis: The vectors $\vec{v}_{\text{HE}} = \vec{v}_{\text{HA}} + \vec{v}_{\text{AE}}$ form a right-angled triangle with $\vec{v}_{\text{HE}}$ as the hypotenuse. Use the Pythagorean theorem and the tangent ratios to determine $\vec{v}_{\text{HE}}$.

Solution:
$$\begin{aligned} v_{\text{HE}} &= \sqrt{(v_{\text{HA}})^2 + (v_{\text{AE}})^2} \\ &= \sqrt{(62 \text{ m/s})^2 + (18 \text{ m/s})^2} \\ v_{\text{HE}} &= 65 \text{ m/s} \end{aligned}$$

Determine the direction of $\vec{v}_{\text{HE}}$.

$$\begin{aligned} \theta &= \tan^{-1}\left(\frac{|\vec{v}_{\text{AE}}|}{|\vec{v}_{\text{HA}}|}\right) \\ &= \tan^{-1}\left(\frac{18 \cancel{\text{m/s}}}{62 \cancel{\text{m/s}}}\right) \\ \theta &= 16° \end{aligned}$$

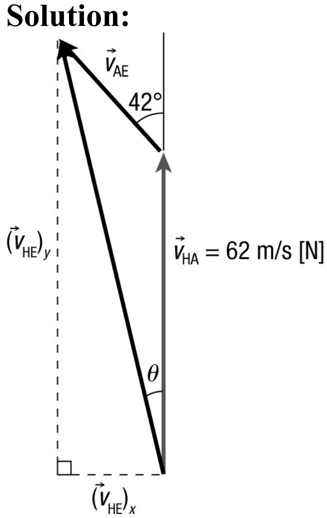
Statement: The velocity of the helicopter with respect to Earth is 65 m/s [N 16° W].

(d) Given: $\vec{v}_{\text{HA}} = 62 \text{ m/s [N]}; \vec{v}_{\text{AE}} = 18 \text{ m/s [N 42° W]}$

Required: $\vec{v}_{\text{HE}}$

Analysis: Draw the vector addition diagram for the situation. Use components to determine $\vec{v}_{\text{HE}}$. Use east and north as positive.

Solution:



x-components:

$$\begin{aligned} (v_{\text{HE}})_x &= (v_{\text{HA}})_x + (v_{\text{AE}})_x \\ &= 0 \text{ m/s} + (-(18 \text{ m/s})\sin 42°) \\ &= 0 \text{ m/s} + (-12.04 \text{ m/s}) \\ (v_{\text{HE}})_x &= -12.04 \text{ m/s} \text{ (two extra digits carried)} \end{aligned}$$

y-components:

$$(v_{\text{HE}})_y = (v_{\text{HA}})_y + (v_{\text{AE}})_y$$
$$= 62 \text{ m/s} + (18 \text{ m/s})(\cos 42°)$$
$$= 62 \text{ m/s} + 13.38 \text{ m/s}$$
$$(v_{\text{HE}})_y = 75.38 \text{ m/s} \text{ (two extra digits carried)}$$

Determine the magnitude of $\vec{v}_{\text{HE}}$.

$$v_{\text{HE}} = \sqrt{\left|(v_{\text{HE}})_x\right|^2 + \left|(v_{\text{HE}})_y\right|^2}$$
$$= \sqrt{(12.04 \text{ m/s})^2 + (75.38 \text{ m/s})^2}$$
$$v_{\text{HE}} = 76 \text{ m/s}$$

Determine the direction of $\vec{v}_{\text{HE}}$.

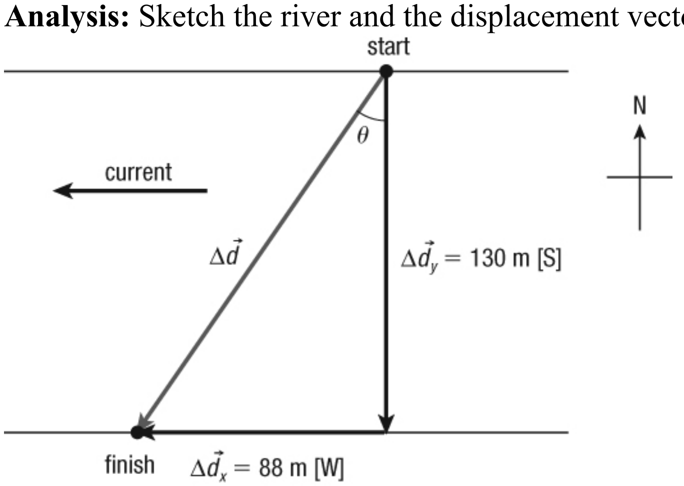
$$\theta = \tan^{-1}\left(\frac{\left|(\vec{v}_{\text{HE}})_x\right|}{\left|(\vec{v}_{\text{HE}})_y\right|}\right)$$
$$= \tan^{-1}\left(\frac{12.04 \cancel{\text{m/s}}}{75.38 \cancel{\text{m/s}}}\right)$$
$$\theta = 9.1°$$

Statement: The velocity of the helicopter with respect to the ground is 76 m/s [N 9.1° W].

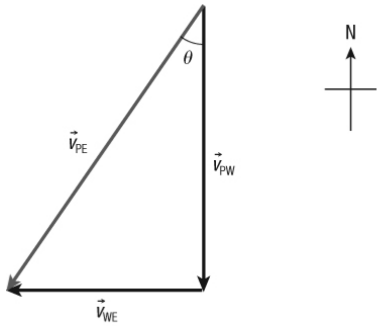
4. (a) Given: $\vec{v}_{\text{PW}} = 0.65$ m/s [S]; $\Delta d_x = 88$ m [W]; $\Delta d_y = 130$ m [S]; river flows [W]

Required: $\vec{v}_{\text{WE}}$

Analysis: Sketch the river and the displacement vectors.



Also draw $\vec{v}_{\text{PE}} = \vec{v}_{\text{PW}} + \vec{v}_{\text{WE}}$.



Vectors $\vec{v}_{\text{PW}}$ and $\vec{v}_{\text{WE}}$ are perpendicular components of $\vec{v}_{\text{PE}}$.
I know the river flows west and that the swimmer drifts 88 m west during her crossing. I can determine the time Δt of the crossing because she swims heading straight across the 130 m at 0.65 m/s. Use $v_{\text{av}} = \dfrac{\Delta d_y}{\Delta t}$ to determine the crossing time and the $v_{\text{av}} = \dfrac{\Delta d_x}{\Delta t}$ to determine the speed of the current. Use south and west as the positive directions.

Solution:

Crossing the river:

$$v_{\text{av}} = \frac{\Delta d_y}{\Delta t}$$

$$\Delta t = \frac{\Delta d_y}{v_{\text{PW}}} = \frac{130\ \cancel{\text{m}}}{0.65\ \cancel{\text{m}}/\text{s}}$$

$$\Delta t = 200\ \text{s}$$

Drifting downstream:

$$v_{\text{av}} = \frac{\Delta d_{\text{x}}}{\Delta t}$$

$$v_{\text{WE}} = \frac{88\ \text{m}}{200\ \text{s}}$$

$$v_{\text{WE}} = 0.44\ \text{m/s}$$

Statement: The water moves at 0.44 m/s [W].

(b) Given: $\Delta d_x = 88$ m [W]; $\Delta d_y = 130$ m [S]

Required: $\vec{v}_{\text{PE}}$

Analysis: The displacement triangle and the velocity triangle are similar, right-angled triangles. Use the displacement triangle to determine the angle θ, and the velocity triangle to calculate $\vec{v}_{\text{PE}}$.

Solution: In the displacement triangle:

$$\theta = \tan^{-1}\left(\frac{|\Delta \vec{d}_x|}{|\Delta \vec{d}_y|}\right) = \tan^{-1}\left(\frac{88\ \cancel{\text{m}}}{130\ \cancel{\text{m}}}\right)$$

$$\theta = 34°$$

In the velocity triangle:

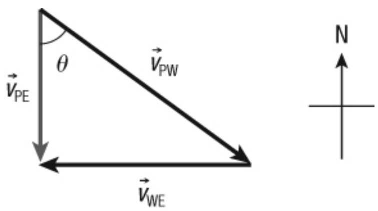
$$(v_{PE})^2 = (v_{PW})^2 + (v_{WE})^2$$
$$= (0.65 \text{ m/s})^2 + (0.44 \text{ m/s})^2$$
$$v_{PE} = 0.78 \text{ m/s}$$

Statement: The velocity of the swimmer with respect to Earth is 0.78 m/s [S 34° W].

(c) Given: $v_{PW} = 0.65$ m/s ; $\vec{v}_{WE} = 0.44$ m/s [W]; $\vec{v}_{PE}$ = ? [S]

Required: direction of $\vec{v}_{PW}$

Analysis: Use $\vec{v}_{PE} = \vec{v}_{PW} + \vec{v}_{WE}$ to draw the relative velocities. This is a right-angled triangle. Use the sine ratio to determine the direction of $\vec{v}_{PW}$.



$$\sin\theta = \frac{|\vec{v}_{WE}|}{|\vec{v}_{PW}|};\ \theta = \sin^{-1}\left(\frac{|\vec{v}_{WE}|}{|\vec{v}_{PW}|}\right)$$

Solution: $\theta = \sin^{-1}\left(\frac{|\vec{v}_{WE}|}{|\vec{v}_{PW}|}\right)$

$$= \sin^{-1}\left(\frac{0.44\ \cancel{\text{m/s}}}{0.65\ \cancel{\text{m/s}}}\right)$$
$$\theta = 43°$$

Statement: The swimmer should head [S 43° E].

5. Given: $\Delta\vec{d} = 1.4 \times 10^3$ km [S 43° E]; $\Delta t = 3.4$ h; $\vec{v}_{AG} = 55$ km/h [S]

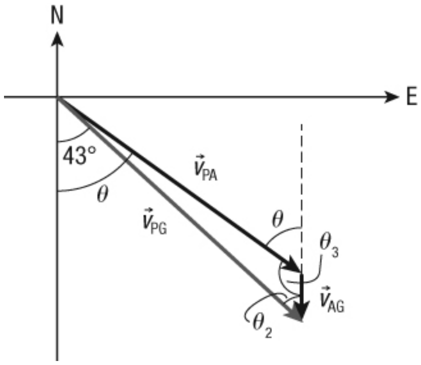
Required: $\vec{v}_{PA}$

Analysis: Draw the relative velocities using $\vec{v}_{PG} = \vec{v}_{PA} + \vec{v}_{AG}$. Solve the triangle using the cosine and sine laws:

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Solution:



$$\vec{v}_{PG} = \frac{\Delta \vec{d}}{\Delta t}$$

$$= \frac{1.4 \times 10^3 \text{ km [S 43° E]}}{3.4 \text{ h}}$$

$$= 411.8 \text{ km/h [S 43° E]} \quad \text{(two extra digits carried)}$$

$$(v_{PA})^2 = (v_{PG})^2 + (v_{AG})^2 - 2(v_{PG})(v_{AG})\cos\theta_2$$

$$= (v_{PG})^2 + (v_{AG})^2 - 2(v_{PG})(v_{AG})\cos\theta_2$$

$$(v_{PA})^2 = (411.8 \text{ km/h})^2 + (55 \text{ km/h})^2 - 2(411.8 \text{ km/h})(55 \text{ km/h})(\cos 43°)$$

$$v_{PA} = 373.4 \text{ km/h} \;\; \text{(two extra digits carried)}$$

$$v_{PA} = 370 \text{ km/h}$$

$$\frac{\sin\theta_3}{v_{PG}} = \frac{\sin\theta_2}{v_{PA}}$$

$$\sin\theta_3 = \frac{v_{PG}\sin\theta_2}{v_{PA}}$$

$$= \frac{(411.8 \; \cancel{\text{km/h}})(\sin 43°)}{373.4 \; \cancel{\text{km/h}}}$$

$$\theta_3 = 131.2°$$

$$\theta = 180° - \theta_3$$

$$= 180° - 131.2°$$

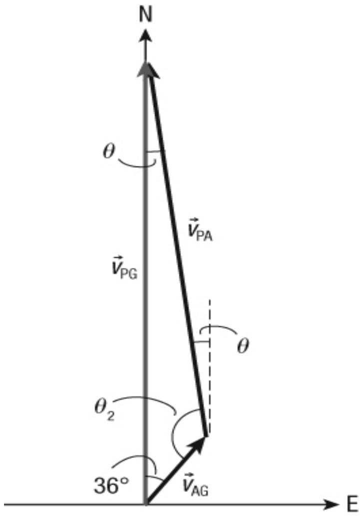
$$\theta = 49°$$

Statement: The plane must maintain an air velocity of 3.7×10^2 km/h [S 49° E].

6. (a) Given: $\Delta\vec{d} = 220$ km [N]; $\vec{v}_{AG} = 42$ km/h [N 36° E]; $v_{PA} = 230$ km/h [?]

Required: direction of $\vec{v}_{PA}$

Analysis: The direction of the ground velocity (the track) of the plane is [N] and I know the wind direction. So I can draw $\vec{v}_{PG} = \vec{v}_{PA} + \vec{v}_{AG}$.



The direction of the air speed (the heading), angle θ in the velocity triangle, can be determined using the sine law.

Solution: $\dfrac{\sin\theta}{v_{AG}} = \dfrac{\sin 36°}{v_{PA}}$

$$\sin\theta = \frac{v_{AG}\sin 36°}{v_{PA}}$$

$$= \frac{(42\text{ km/h})(\sin 36°)}{(230\text{ km/h})}$$

$$\theta = 6.167° \text{ (two extra digits carried)}$$

$$\theta = 6.2°$$

Statement: The heading of the plane should be [N 6.2° W].

(b) Given: $\Delta\vec{d} = 220$ km [N]; $\vec{v}_{AG} = 42$ km/h [N 36° E]; $v_{PA} = 230$ km/h; $\theta = 6.167°$

Required: Δt

Analysis: Use the vector triangle and the sine law to calculate ground speed, v_{PG}.

The calculate Δt using $v_{PG} = \dfrac{\Delta d}{\Delta t}$; $\Delta t = \dfrac{\Delta d}{v_{PG}}$.

Solution: Determine the third angle in the vector triangle, ϕ.

$\phi + \theta + 36° = 180°$

$$\phi = 180° - \theta - 36°$$

$$\phi = 180° - 6.167° - 36°$$

$$\phi = 137.8° \text{ (two extra digits carried)}$$

$$\frac{\sin\phi}{v_{\text{PG}}} = \frac{\sin 36°}{v_{\text{PA}}}$$

$$v_{\text{PG}} = \frac{v_{\text{PA}}\sin\phi}{\sin 36°}$$

$$= \frac{(230\text{ km/h})(\sin 137.8°)}{\sin 36°}$$

$$v_{\text{PG}} = 262.6\text{ km/h (two extra digits carried)}$$

$$\Delta t = \frac{\Delta d}{v_{\text{PG}}}$$

$$= \frac{220\ \cancel{\text{km}}}{262.6\ \cancel{\text{km}}\text{/h}}$$

$$\Delta t = 0.84\text{ h}$$

Statement: The trip will take 0.84 h.

7. (a) Given: $\vec{v}_{\text{PA}} = 250$ m/s [W]; $\vec{v}_{\text{AG}} = 50.0$ m/s [E]

Required: v_{PG}

Analysis: Use $\vec{v}_{\text{PG}} = \vec{v}_{\text{PA}} + \vec{v}_{\text{AG}}$ to determine v_{PG}.

Solution: $\vec{v}_{\text{PG}} = \vec{v}_{\text{PA}} + \vec{v}_{\text{AG}}$

$= 250\text{ m/s [W]} + 50.0\text{ m/s [E]}$

$= 250\text{ m/s [W]} + (-50.0\text{ m/s [W]})$

$\vec{v}_{\text{PG}} = 2.0 \times 10^2\text{ m/s [W]}$

Statement: The ground speed of the plane on its way west is 2.0×10^2 m/s.

(b) Given: $\vec{v}_{\text{PA}} = 250$ m/s [E]; $\vec{v}_{\text{AG}} = 50.0$ m/s [E]

Required: v_{PG}

Analysis: $\vec{v}_{\text{PG}} = \vec{v}_{\text{PA}} + \vec{v}_{\text{AG}}$

Solution: $\vec{v}_{\text{PG}} = \vec{v}_{\text{PA}} + \vec{v}_{\text{AG}}$

$= 250\text{ m/s [E]} + 50.0\text{ m/s [E]}$

$\vec{v}_{\text{PG}} = 3.0 \times 10^2\text{ m/s [E]}$

Statement: The ground speed of the plane on its way east is 3.0×10^2 m/s.

8. (a) Given: $\vec{v}_{\text{PS}} = 2.0$ m/s [up]; $\vec{v}_{\text{SW}} = 3.2$ m/s [E]

Required: $\vec{v}_{\text{PW}}$

Analysis: The relative velocity triangle, $\vec{v}_{\text{PW}} = \vec{v}_{\text{PS}} + \vec{v}_{\text{SW}}$, is right-angled with $\vec{v}_{\text{PW}}$ as the hypotenuse. Use the Pythagorean theorem and the tangent ratio to determine the magnitude and direction of $\vec{v}_{\text{PW}}$.

Solution: $(v_{PW})^2 = (v_{PS})^2 + (v_{SW})^2$

$$v_{PW} = \sqrt{(v_{PS})^2 + (v_{SW})^2}$$
$$= \sqrt{(2.0\text{ m/s})^2 + (3.2\text{ m/s})^2}$$
$$v_{PW} = 3.8\text{ m/s}$$

$$\theta = \tan^{-1}\left(\frac{v_{PS}}{v_{SW}}\right)$$
$$= \tan^{-1}\left(\frac{2.0\ \cancel{\text{m/s}}}{3.2\ \cancel{\text{m/s}}}\right)$$
$$\theta = 32°$$

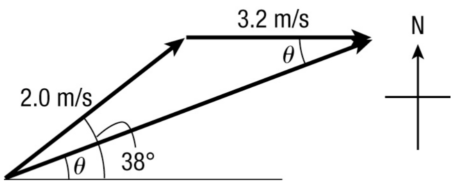
Statement: The people who take the elevator move at 3.8 m/s [E 32° up] relative to the water.

(b) Given: $\vec{v}_{PS} = 2.0$ m/s [E 38° up]; $\vec{v}_{SW} = 3.2$ m/s [E]

Required: $\vec{v}_{PW}$

Analysis: Sketch the relative velocity triangle, $\vec{v}_{PW} = \vec{v}_{PS} + \vec{v}_{SW}$. Use components to determine $\vec{v}_{PW}$, with east and up as positive.

Solution:



x-components:

$$(v_{PW})_x = (v_{PS})_x + (v_{SW})_x$$
$$= +(2.0\text{ m/s})\cos 38° + (+3.2\text{ m/s})$$
$$= 1.576\text{ m/s} + 3.2\text{ m/s}$$
$$(v_{PW})_x = 4.776\text{ m/s} \text{ (two extra digits carried)}$$

y-components:

$$(v_{PW})_y = (v_{PS})_y + (v_{SW})_y$$
$$= (2.0\text{ m/s})\sin 38° + 0\text{ m/s}$$
$$= 1.231\text{ m/s} + 0\text{ m/s}$$
$$(v_{PW})_y = 1.231\text{ m/s} \text{ (two extra digits carried)}$$

Now use these components to determine $\vec{v}_{PW}$.

$$\left|\vec{v}_{PW}\right| = \sqrt{\left|(v_{PW})_x\right|^2 + \left|(v_{PW})_y\right|^2}$$
$$= \sqrt{(4.776\text{ m/s})^2 + (1.231\text{ m/s})^2}$$
$$\left|\vec{v}_{PG}\right| = 4.9\text{ m/s}$$

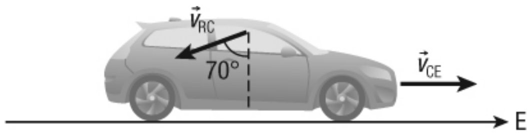
$$\theta = \tan^{-1}\left(\frac{1.231\ \cancel{\text{m/s}}}{4.776\ \cancel{\text{m/s}}}\right)$$
$$\theta = 14°$$

Statement: The people who take the stairs move at 4.9 m/s [E 14° up] relative to the water.

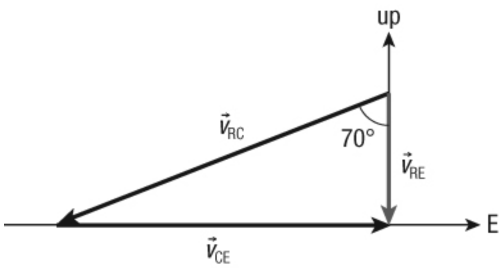
9. (a) Given: $\vec{v}_{\text{CE}} = 60.0$ km/h [E]; $\vec{v}_{\text{RC}}$ = ? [down 70.0° W]

Required: $\vec{v}_{\text{RC}}$

Analysis: Make a sketch of the car with the rain.



Also draw $\vec{v}_{\text{RE}} = \vec{v}_{\text{RC}} + \vec{v}_{\text{CE}}$.



Convert kilometres per hour to metres per second.

$$60\ \frac{\cancel{\text{km}}}{\cancel{\text{h}}} \times \frac{1000\ \text{m}}{1\ \cancel{\text{km}}} \times \frac{1\ \cancel{\text{h}}}{3600\ \text{s}} = 16.667\ \text{m/s (two extra digits carried)}$$

Solve for v_{RC} using the sine ratio.

Solution: $\sin 70° = \dfrac{v_{\text{CE}}}{v_{\text{RC}}}$

$$\begin{aligned} v_{\text{RC}} &= \frac{v_{\text{CE}}}{\sin 70.0°} \\ &= \frac{16.667\ \text{m/s}}{\sin 70.0°} \\ &= 17.737\ \text{m/s (two extra digits carried)} \\ v_{\text{RC}} &= 17.7\ \text{m/s} \end{aligned}$$

Statement: The rain moves at 17.7 m/s [down 70.0° W] with respect to the car.

(b) Given: $\vec{v}_{\text{RC}}$ = 17.737 m/s [down 70.0° W]

Required: $\vec{v}_{\text{RE}}$

Analysis: Look at the triangles in part (a). Solve for v_{RE} using the cosine ratio.

Solution: $\cos 70° = \dfrac{v_{\text{RE}}}{v_{\text{RC}}}$

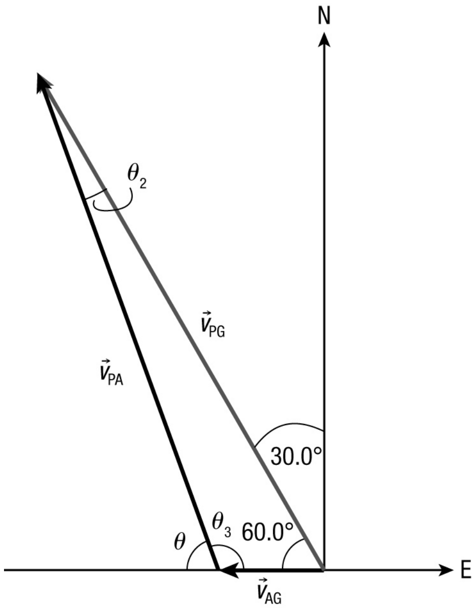
$$\begin{aligned} v_{\text{RE}} &= v_{\text{RC}} \cos 70.0° \\ &= (17.737\ \text{m/s})(\cos 70.0°) \\ v_{\text{RE}} &= 6.07\ \text{m/s} \end{aligned}$$

Statement: The velocity of the rain with respect to Earth is 6.07 m/s [down].

10. (a) Given: $\vec{v}_{\text{PG}}$ = ? [N 30.0° W]; $\vec{v}_{\text{AG}} = 48$ km/h [W]; $v_{\text{PA}} = 260$ km/h

Required: direction of $\vec{v}_{\text{PA}}$

Analysis: The direction of the ground velocity (the track) of the plane is [N 30.0° W], and I know the wind direction, so I can draw $\vec{v}_{\text{PG}} = \vec{v}_{\text{PA}} + \vec{v}_{\text{AG}}$. The angle opposite $\vec{v}_{\text{PA}}$ in the relative velocity triangle is 60.0º (since [N 30.0° W] is equivalent to [W 60.0° N]).



Calculate the direction of the air speed (the heading) from the angles θ_2 and θ_3 in the scalene relative-velocity triangle using the sine law.

Solution: Using the sine law,

$$\frac{\sin\theta_2}{v_{\text{AG}}} = \frac{\sin 60.0°}{v_{\text{PA}}}$$

$$\sin\theta_2 = \frac{v_{\text{AG}} \sin 60.0°}{v_{\text{PA}}}$$

$$= \frac{(48 \text{ km/h})(\sin 60.0°)}{260 \text{ km/h}}$$

$$\theta_2 = 9.2°$$

$$\theta_3 = 180° - 60.0° - \theta_2$$
$$= 180° - 60.0° - 9.2°$$
$$\theta_3 = 110.8°$$

$$\theta = 180° - \theta_3$$
$$= 180° - 110.8°$$
$$\theta = 69°$$

The heading of the plane should be [W 69° N], which is equivalent to [N 21° W].

Statement: The heading of the plane should be [N 21° W].

(b) Given: $\vec{v}_{\text{PG}}$ = ? [W 60.0° N]; $v_{\text{PA}} = 260$ km/h; $\theta_3 = 110.8°$

Required: v_{PG}

Analysis: Use the relative-velocity triangle from part (a) and the sine law to determine v_{PG}.

Solution:
$$\frac{\sin\theta_3}{v_{\text{PG}}} = \frac{\sin 60.0°}{v_{\text{PA}}}$$
$$v_{\text{PG}} = \frac{v_{\text{PA}} \sin\theta_3}{\sin 60.0°}$$
$$= \frac{(260\text{ km/h})(\sin 110.8°)}{\sin 60.0°}$$
$$v_{\text{PG}} = 280\text{ km/h}$$

Statement: The ground speed of the plane is 280 km/h.

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Investigation 1.5.1: Investigating Projectile Motion, page 50

Analyze and Evaluate
(a) The measured accelerations had similar magnitude.
Answers will vary depending on the chosen angles of inclination.
(i) For an object dropping straight down and a change in time of 0.10 s:

Table 1

Change in velocity (cm/s)	Acceleration (cm/s^2)
10.0	100
11.0	110
10.0	100
11.0	110
10.0	100

Average acceleration = 104 cm/s^2 = 1.04 m/s^2
(ii) For a projectile motion speed of 0 m/s and a change in time of 0.10 s:

Table 2

Change in velocity (cm/s)	Acceleration (cm/s^2)
10.0	100
10.0	100
11.0	110
10.0	100
10.0	100

Average acceleration = 102 cm/s^2 = 1.02 m/s^2
(iii) For a projectile motion speed that does not equal 0 m/s and a change in time of 0.10 s:

Table 3

Change in velocity (cm/s)	Acceleration (cm/s^2)
9.0	90
9.0	90
10.0	100
9.0	90
11.0	110

Average acceleration = 96 cm/s^2 = 0.96 m/s^2
(b) Answers may vary. Sample answer:
Given: $\theta = 6.4°$
Required: a
Analysis: $a = g \sin\theta$
Solution:
$$\begin{aligned} a &= g \sin\theta \\ &= (9.8 \text{ m/s}^2)(\sin 6.4°) \\ a &= 1.09 \text{ m/s}^2 \end{aligned}$$
Statement: The magnitude of acceleration down the inclined plane is 1.1 m/s^2.

(c) Answers may vary depending on the measured accelerations. Sample answer:
Table 4 Percent Differences

	Average acceleration	**Percent difference from 1.1 m/s^2**
(i) object dropping straight down	1.04 m/s^2	$\frac{1.1-1.04}{1.1}\times 100 = 5.4\%$
(ii) projectile motion, Δv = 0 m/s	1.02 m/s^2	$\frac{1.1-1.02}{1.1}\times 100 = 7.2\%$
(iii) projectile motion, $\Delta v \neq 0$ m/s	0.96 m/s^2	$\frac{1.1-0.96}{1.1}\times 100 = 13\%$

(d) Percent error is a measure of the difference between a given accepted value and a measured value. The percent difference compares a measured value to a predicted value. We do not have a given accepted value for the acceleration so we must use percent difference to get an indication of the error.
(e) The accelerations were all directed down the plane.
(f) Answers may vary. Sample answer: Some systematic errors are air resistance, friction with surfaces, and the precision of the metre stick. You could minimize the error due to air resistance by making sure to drop the object in a sealed area. You could minimize the error due to friction by making sure the surfaces of the air table and the puck are free of dirt. Some random errors are errors in measurements and in comparing measurements. These errors could be minimized by checking repeating the measurements and determining the average value.

Apply and Extend
(g) Answers may vary. Sample answer: The vertical component of projectile motion is independent from the horizontal component because gravity acts straight down and cannot make the puck accelerate forward parallel to the incline.
(h) Answers may vary. Sample answer: I used a simulator that allowed me to manipulate the initial height, initial speed, angle of inclination, mass, and gravitational acceleration of a projectile. I manipulated the five variables one at a time to observe how changing these variables affected the horizontal distance, the maximum height, and the time of flight.

As the initial height increased, the horizontal distance, the maximum height, and the time of flight all increased.

As the initial speed increased, the horizontal distance, the maximum height, and the time of flight all increased. However, when I changed the angle of inclination to point below the horizontal, increasing the initial speed then caused the time of flight to decrease and the maximum height was always equal to the initial height.

As the angle of inclination increased from 0° to 45°, the horizontal distance, the maximum height, and the time of flight all increased. As the angle of inclination continued to increase to 90°, the horizontal distance began decreasing until it became zero at 90°. The maximum height and time of flight continued to increase. As the angle of inclination decreased from 0° to straight down, the maximum height was always equal to the initial height, while the horizontal distance and time of flight both decreased.

Changing the mass of the projectile has no effect on the motion.

As the gravitational acceleration increased, the horizontal distance, the maximum height, and the time of flight all decreased.

Chapter 1 Review, pages 54–59

Knowledge

1. (a)
2. (b)
3. (a)
4. (a)
5. (c)
6. (b)
7. True
8. False. The average speed is *the slope of the secant drawn between the initial and final times of interest on the position–time curve for the motion.*
9. True
10. False. When a moving objects starts to slow down on a straight track, the average acceleration of the object at any time interval after it starts slowing down is *negative*.
11. True
12. False. The *magnitude* of the average velocity of an object is always *less than* or equal to the average speed.
13. True
14. False. Two students running toward each other with the same speed have *opposite* velocity vectors relative to each other.
15. (a) Answers may vary. Sample answer: No, since the car is moving on a straight track the magnitude of the velocity is the speed. If the velocity is constant then the speed is constant.
(b) Answers may vary. Sample answer: Yes, the ratio of the magnitude of the velocity and the speed is equal to one. On a straight track, the velocity may be positive or negative, but its magnitude is the speed.
16. Answers may vary. Sample answer: Yes, it is possible for an object to have varying velocity and constant speed. An example is a car going around a corner at constant speed.
17. Answers may vary. Sample answer: Yes, two displacement vectors of the same length can have a vector sum of zero. Consider a displacement of 10 m [N] followed by a displacement of 10 m [S]. Their vector sum is zero.
18. Answers may vary. Sample answer: It is not possible for a sprinting football player to stop instantly because this would require a change of velocity (in this case, a slowing down) in a time interval of $\Delta t = 0$. Nothing can occur in an infinitesimally short length of time.
19. When a skier jumps off a ramp and air resistance is not negligible, both the range and the landing speed are reduced. Air resistance causes a slowing down in the horizontal direction, shortening the range. It also reduces the vertical acceleration, so the vertical component of velocity is lower throughout the jump. Together, the two reduced velocity components result in a lower landing speed.
20. Answers may vary. Sample answer: Imagine that you are skiing down a slope and your hat flies off. You see the hat caught in the wind, moving backward—this is the motion of the hat relative to you. A snowboarder at rest on the ski slope sees the hat following behind you and eventually slowing down—this is the motion of the hat relative to the snowboarder.
21. Answers may vary. Sample answer: When two objects are moving at different velocities, each appears to be in motion as seen from the other. This motion is called relative motion because the details of the motion are relative, depending on the point of view.

Understanding

22. Answers may vary. Sample answer: An object can have acceleration because its speed changes or because its direction changes. For example, an object moving around in a circle at a constant speed has acceleration due to its change in direction.

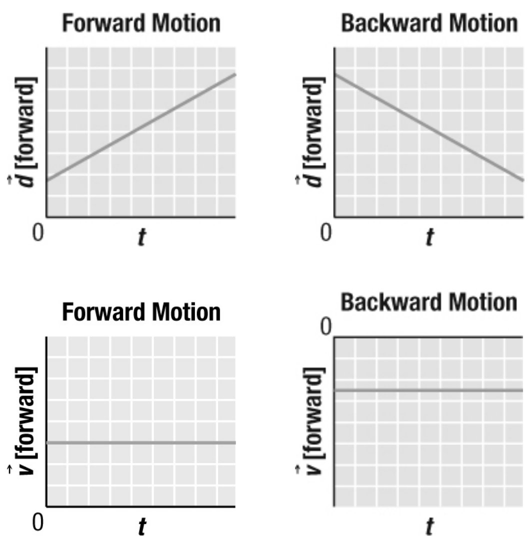
23. Answers may vary. Sample answer:

(a) A car moving to the right and speeding up has both positive velocity and acceleration.

(b) A car moving to the right and slowing down has positive velocity and negative acceleration.

(c) A car moving to the right at constant speed has positive velocity and zero acceleration.

24. Answers may vary. Sample answer: A position–time graph for an object in uniform motion is a straight line. The slope of the graph represents the object's speed. Positive slope is forward motion, and negative slope is backward motion. The intercept on the position axis gives the initial position of the object. The velocity–time graph for the same uniform motion is a horizontal line, intersecting the velocity axis at the constant velocity of the motion.



25. Answers may vary. Sample answer: The odometer records distance travelled since the car was new or the odometer was reset. If you take odometer readings at two times, you can calculate the average speed over the time interval. The speedometer records the instantaneous speed of the car. If you take speedometer readings at two times, you can calculate the average acceleration over the time interval.

26. Answers may vary. Sample answer: A ball tossed into the air and caught at the same height from which it was thrown has symmetric upward and downward motions. Its average velocity is zero. The only time during the flight that its instantaneous velocity is zero is when the ball is at its maximum height.

27. For three displacement vectors to have a vector sum of zero, the vector resulting from the addition of two of the vectors must be the negative of the third vector, that is, it must have the same magnitude as the third vector but point in the opposite direction.

For a sum of 0, the endpoint of the third vector will be the same as starting point of the first vector.

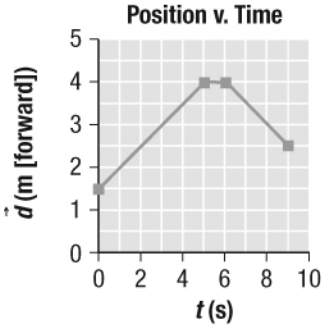
28. Answers may vary. Sample answer: An example in which an object moves in two dimensions but has acceleration in one dimension is a projectile that experiences negligible air resistance. The projectile has motion in both the horizontal and vertical direction but its acceleration is only vertical.

29. Answers may vary. Sample answer: A ball is tossed vertically upward from a roof and lands again on the roof. Displacement is a comparison of two positions. An observer on the roof and an observer on the ground may record the position of the ball differently but they will calculate the same displacement. Because velocity is based on displacement, the two observers see identical velocities.
30. Answers may vary. Sample answer: When you predict the direction of motion of an object you are predicting the direction of the displacement vector of the object over a very short time interval from now. Velocity is based on displacement and time taken, so the velocity vector is parallel to the displacement vector and shows the desired direction. Acceleration is based on a change in velocity. The acceleration vector is not always parallel to the velocity vector. In fact, the acceleration vector predicts which way the motion of an object will curve.
31. Answers may vary. Sample answer: In the long jump, an athlete launches himself into projectile motion. His horizontal speed running up to the jumping pad and the strength and direction of his push off the pad will affect the initial speed and launch angle. A high initial speed and a launch angle close to but below 45° will maximize the range, or horizontal distance, of the jump. Also, any posture or clothing the athlete can use to minimize air resistance will help lengthen the jump.
32. Answers may vary. Sample answer: Assume that air resistance is negligible. Dropping a ball from the window of a moving car rather than stationary car should make no difference to the time it takes for the ball to reach the ground. The motion of the car affects only the horizontal motion of the ball. The time to fall is affected only by gravity.
33. Answers may vary. Sample answer: An object can be at rest as well as in motion at the same time. It depends on who is observing the object. A bag of skis tied to the roof of a car is at rest as seen by someone in the car. If the car is driving along the street, someone standing on the sidewalk would say the skis are in motion. A full description of any object's motion should include mention of a frame of reference, the point of view from which the observations are made.
34. Answers may vary. Sample answer: To navigate a boat so it goes directly across a fast-moving river, I would need to head the boat somewhat upstream. If I have the correct angle, I would observe that the boat is moving along a line directly across the river even though the bow of the boat is not pointing that way. If I am slipping downstream, I need to point more upstream. If I am moving upstream a bit, I need to point more to the opposite shore.

Analysis and Application

35. During interval P, the graph is horizontal, showing that the object is at rest. During interval Q, the graph has a constant positive slope, showing that the object is moving forward at constant speed. During interval R, the graph is horizontal, showing that the object is at rest. During interval S, the graph has a constant negative slope, showing that the object is moving backward at constant speed.

36.



37. Answers may vary. Sample answer: A juggler is able to time the throw and the catch of the balls to ensure they follow certain trajectories. If the juggler throws the balls at the same velocities each time, then they will follow predictable paths. This allows the juggler to determine where the balls will be when they need to be caught.

38. Given: $\vec{v}_i = 0$ m/s; $\Delta t = 2.0$ s; $\vec{a} = 9.8$ m/s^2 [down]

Required: $\Delta\vec{d}$

Analysis: The initial velocity, time taken, and acceleration are known. Use $\Delta\vec{d} = \vec{v}_i\Delta t + \frac{1}{2}\vec{a}\Delta t^2$. Use up as the positive direction.

Solution: $\Delta d = v_i\Delta t + \frac{1}{2}a\Delta t^2$

$$\Delta d = (0\text{ m}/\cancel{\text{s}})(2.0\ \cancel{\text{s}}) + \frac{1}{2}(-9.8\text{ m}/\cancel{\text{s}^2})(2.0\ \cancel{\text{s}})^2$$

$$= -19.6\text{ m}$$

$$\Delta d = -2.0\times10^1\text{ m}$$

Statement: The squirrel was 2.0×10^1 m above the ground.

39. Given: $\vec{v}_i = 9$ m/s [forward]; $\vec{v}_f = 0$ m/s; $\Delta t = 3$ s

Required: $\vec{a}_{av}$

Analysis: The initial and final velocities and the time taken are known. Use $\vec{a} = \frac{\Delta\vec{v}}{\Delta t}$. Use forward as the positive direction.

Solution: $a = \frac{\Delta v}{\Delta t}$

$$= \frac{0\text{ m/s} - 9\text{ m/s}}{3\text{ s}}$$

$$a = -3\text{ m/s}^2$$

Statement: The runner's acceleration is 3 m/s^2 [backward].

40. (a) Given: $\vec{v}_i = 0$ m/s; $\vec{v}_f = 9.0$ m/s [forward]; $\Delta t_1 = 2.0$ s

Required: $\vec{a}$

Analysis: $\vec{a} = \dfrac{\Delta \vec{v}}{\Delta t}$; use forward as positive.

Solution:
$$a = \frac{\Delta v}{\Delta t}$$
$$= \frac{9.0 \text{ m/s} - 0 \text{ m/s}}{2.0 \text{ s}}$$
$$a = 4.5 \text{ m/s}^2$$

Statement: The runner's acceleration is 4.5 m/s^2 [forward].

(b) Given: total displacement: $\vec{v}_i = 0$ m/s; $\vec{v}_f = 9.0$ m/s [forward]; $\Delta t_1 = 2.0$ s; $\Delta \vec{d} = 100.0$ m [forward]; second part of race: $v = 9.0$ m/s [forward]; $\Delta t_1 = 2.0$ s

Required: total time, Δt

Analysis: The total displacement of the runner is 100.0 m but the actual running of the race has two parts. Use $\Delta \vec{d} = \vec{v}_i \Delta t + \dfrac{1}{2}\vec{a}\Delta t^2$ to calculate the displacement $\Delta \vec{d}_1$ during the acceleration of the first part of the race. Then calculate the displacement $\Delta \vec{d}_2$ for the second part of the race and use the constant velocity formula, $\vec{v}_{av} = \dfrac{\Delta \vec{d}}{\Delta t}$, to determine the time Δt_2 for the second part of the race; $\Delta t = \dfrac{\Delta d}{v_{av}}$. Finally, the total time taken is the sum of the two times, $\Delta t = \Delta t_1 + \Delta t_2$.

Solution: The displacement for the first part of the race is
$$\Delta d_1 = v_i \Delta t + \frac{1}{2} a \Delta t^2$$
$$= (0 \text{ m}/\cancel{\text{s}})(2.0 \ \cancel{\text{s}}) + \frac{1}{2}(4.5 \text{ m}/\cancel{\text{s}^2})(2.0 \ \cancel{\text{s}})^2$$
$$\Delta d_1 = 9.0 \text{ m}$$

The displacement for the second part of the race is
$$\Delta d = \Delta d_1 + \Delta d_2$$
$$\Delta d_2 = \Delta d - \Delta d_1$$
$$= 100.0 \text{ m} - 9.0 \text{ m}$$
$$\Delta d_2 = 91.0 \text{ m}$$

The time for the second part of the race is
$$\Delta t_2 = \frac{\Delta d_2}{v_2}$$
$$= \frac{91.0 \ \cancel{\text{m}}}{9.0 \ \cancel{\text{m}}/\text{s}}$$
$$\Delta t_2 = 10.1 \text{ s}$$

The time for the entire race is

$$\Delta t = \Delta t_1 + \Delta t_2$$
$$= 2.0 \text{ s} + 10.1 \text{ s}$$
$$\Delta t = 12 \text{ s}$$

Statement: The runner takes 12 s to run the race.

41. (a) Given: $\vec{v}_1 = 20.0$ m/s [forward]; $\vec{v}_f = 0$ m/s; $\Delta\vec{d} = 50.0$ m

Required: $\vec{a}$

Analysis: Use the initial and final velocities and the displacement. Use $v_f^2 = v_i^2 + 2a\Delta d$ to determine the acceleration: $a = \dfrac{v_f^2 - v_i^2}{2\Delta d}$. Use forward as positive.

Solution: $a = \dfrac{v_f^2 - v_i^2}{2\Delta d}$

$$= \frac{(0 \text{ m/s})^2 - (20.0 \text{ m/s})^2}{2(50.0 \text{ m})}$$
$$a = -4.00 \text{ m/s}^2$$

Statement: The car's acceleration is 4.00 m/s^2 [backward].

(b) Given: $\vec{v}_1 = 20.0$ m/s [forward]; $\vec{v}_f = 0$ m/s; $\vec{a} = 4.00$ m/s^2 [backward]

Required: stopping time, Δt

Analysis: Any of the formulas involving Δt can be used. Use $\vec{a} = \dfrac{\Delta\vec{v}}{\Delta t}$; $\Delta t = \dfrac{\Delta v}{a}$.

Solution: $\Delta t = \dfrac{\Delta v}{a}$

$$= \frac{0 \text{ m/s} - 20.0 \text{ m/s}}{-4.00 \text{ m/s}^2}$$
$$\Delta t = 5.00 \text{ s}$$

Statement: It takes the race car 5.00 s to stop.

42. Given: $\vec{v}_i = 5.0$ m/s [forward]; $\Delta d = 10.0$ m [forward]; $\Delta t = 2.2$ s

Required: $\vec{a}$

Analysis: Use $\Delta\vec{d} = \vec{v}_i\Delta t + \dfrac{1}{2}\vec{a}\Delta t^2$ to calculate the acceleration: $a = \dfrac{\Delta d - v_i\Delta t}{0.5\Delta t^2}$.

Use forward as the positive direction.

Solution: $a = \dfrac{\Delta d - v_i\Delta t}{0.5\Delta t^2}$

$$a = \frac{10.0 \text{ m} - (5 \text{ m}/\cancel{\text{s}})(2.2\ \cancel{\text{s}}) \text{ m}}{0.5(2.2 \text{ s})^2}$$
$$a = -0.41 \text{ m/s}^2$$

Statement: The bowling ball's acceleration is 0.41 m/s^2 [backward].

43. Given: stone 1: $\vec{v}_i = 0$ m/s; $t_i = 0$ s; stone 2: $\vec{v}_i = 10.0$ m/s [down]; $t_i = 0.50$ s

Required: displacement where stone 2 overtakes stone 1, $\Delta\vec{d}$

Analysis: Both stones accelerate down under the influence of gravity. Rewrite the equation $\Delta\vec{d} = \vec{v}_i\Delta t + \frac{1}{2}\vec{a}\Delta t^2$ for each stone. Then compare the equations to determine the time when the stones have the same displacement. Use up as the positive direction.

Solution: Rewrite the equation $\Delta\vec{d} = \vec{v}_i\Delta t + \frac{1}{2}\vec{a}\Delta t^2$ for each stone at time *t* (omitting the units for convenience).

Stone 1:

$$\begin{aligned}\Delta d &= v_i\Delta t + \frac{1}{2}a\Delta t^2 \\ &= (0\ \text{m}/\cancel{\text{s}})(t - 0\ \cancel{\text{s}}) + \frac{1}{2}(-9.8\ \text{m}/\cancel{\text{s}^2})(t - 0\ \cancel{\text{s}})^2 \\ \Delta d &= -4.9t^2 \qquad \text{(Equation 1)}\end{aligned}$$

Stone 2:

$$\begin{aligned}\Delta d &= v_i\Delta t + \frac{1}{2}a\Delta t^2 \\ &= (-10\ \text{m}/\cancel{\text{s}})(t - 0.5\ \cancel{\text{s}}) + \frac{1}{2}(-9.8\ \text{m}/\cancel{\text{s}^2})(t - 0.5\ \cancel{\text{s}})^2 \\ &= (-10t + 5.0) - (4.9)(t^2 - t + 0.25) \\ \Delta d &= -4.9t^2 - 5.1t + 3.775 \qquad \text{(Equation 2)}\end{aligned}$$

Compare Equations 1 and 2 and solve for *t*.

$$\begin{aligned}-4.9t^2 &= -4.9t^2 - 5.1t + 3.775 \\ 5.1t &= 3.775 \\ t &= 0.7402\ \text{s} \quad \text{(two extra digits carried)}\end{aligned}$$

Substitute this value of *t* into either Equation 1 or 2 to determine the common displacement.

$$\begin{aligned}\Delta d &= -(4.9\ \text{m/s}^2)t^2 \qquad \text{(Equation 1)} \\ &= -(4.9\ \text{m}/\cancel{\text{s}^2})(0.7402\ \cancel{\text{s}})^2 \\ \Delta d &= -2.7\ \text{m}\end{aligned}$$

Statement: The second stone overtakes the first stone 2.7 m below the top of the cliff.

44. (a) Answers may vary. Sample answer: To see how the height of a high jumper changes when the acceleration of gravity is reduced, I thought about a projectile problem where you are determining the maximum height attained. For vertical initial velocity, the maximum height is $\Delta d_y = \frac{v_i^2}{2g}$, so a smaller value of *g* would lead to a higher jump. Another way to see this is to realize that it is the effect of gravity that stops you from jumping very high. The weaker the force of gravity is, the higher a jump will be.

(b) Given: $\vec{v}_{\text{i}} = 5$ m/s [up]; $g = 2.0$ m/s^2 [down]

Required: Δd_{y}

Analysis: $\Delta d_y = \dfrac{v_{\text{i}}^2}{2g}$

Solution:

$$\Delta d_y = \frac{(5 \text{ m/}\cancel{\text{s}})^2}{2(2.0 \text{ m/}\cancel{\text{s}^2})}$$

$$\Delta d_y = 6.2 \text{ m}$$

Statement: You could throw a baseball up 6.2 m.

45. Given: $\vec{v} = 60$ m/s [S 60° W]

Required: $\vec{v}_x$

Analysis: Use east and north as positive. The component directed west is in the negative x direction.

Solution:

$$\begin{aligned} v_x &= -v\sin\theta \\ &= -(60 \text{ m/s})(\sin 60°) \\ v_x &= -50 \text{ m/s} \end{aligned}$$

Statement: The required component of velocity is 50 m/s [W].

46. Solutions may vary. Sample answer:

(a) Given: car: $\vec{d}_{1\text{i}} = 0$ m [forward]; $\vec{v}_{1\text{i}} = 0$ m/s; $\vec{a} = 2$ m/s^2 [forward];
truck: $\vec{d}_{2\text{i}} = 0$ m [forward]; $\vec{v}_2 = 10$ m/s [forward]

Required: displacement where the car catches up with the truck, Δd

Analysis: The vehicles start from the same position and travel an equal distance. Set up an equation for each vehicle that relates position and time. The car moves at constant acceleration, $\Delta\vec{d} = \vec{v}_{\text{i}}\Delta t + \dfrac{1}{2}\vec{a}\Delta t^2$, and the truck at constant velocity, $\vec{v} = \dfrac{\Delta\vec{d}}{\Delta t}$ so $\Delta d = v\Delta t$. Compare these equations to solve for time and, later, displacement. Use forward as positive.

Solution: car:

$$\begin{aligned} \Delta d &= v_{\text{i}}\Delta t + \frac{1}{2}a\Delta t^2 \\ \Delta d_1 &= (0 \text{ m/s})\Delta t + \frac{1}{2}(2 \text{ m/s}^2)\Delta t^2 \\ \Delta d_1 &= (1 \text{ m/s}^2)\Delta t^2 \end{aligned}$$

truck:

$$\begin{aligned} v_2 &= \frac{\Delta d_2}{\Delta t} \\ \Delta d_2 &= v_2\Delta t \\ \Delta d_2 &= (10 \text{ m/s})\Delta t \end{aligned}$$

The displacements of the two vehicles are equal,

$$\begin{aligned} \Delta d_1 &= \Delta d_2 \\ (1 \text{ }\cancel{\text{m}}\text{/s}^{\cancel{2}})\Delta t^{\cancel{2}} &= (10 \text{ }\cancel{\text{m}}\text{/}\cancel{\text{s}})\cancel{\Delta t} \\ \Delta t &= 10 \text{ s} \end{aligned}$$

The common displacement can be found using the equation for either the car or the truck.

$$\Delta d_1 = (1\ \text{m/s}^2)\Delta t^2$$
$$= (1\ \text{m}/\cancel{\text{s}^2})(10\ \cancel{\text{s}})^2$$
$$\Delta d_1 = 100\ \text{m}$$

Statement: The car overtakes the truck 100 m beyond the starting point.

(b) Given: car: $\vec{d}_{1i} = 0$ m [forward]; $\vec{v}_{1i} = 0$ m/s; $\vec{a} = 2\ \text{m/s}^2$ [forward]

Required: speed of the car at 10 s, v_f

Analysis: Use $\vec{a} = \dfrac{\Delta\vec{v}}{\Delta t}$ to solve for the final speed of the car; $\vec{a} = \dfrac{\vec{v}_f - \vec{v}_i}{\Delta t}$; $\vec{v}_f = \vec{v}_i + \vec{a}\Delta t$

Solution: $v_f = v_i + a\Delta t$

$$= 0\ \text{m/s} + (2\ \text{m/s}^{\cancel{2}})(10\ \cancel{\text{s}})$$
$$v_f = 20\ \text{m/s}$$

Statement: The car is moving at 20 m/s [forward] when it catches up with the truck.

47. Given: $\vec{v}_i = 100.0$ km/h [forward]; $\vec{v}_f = 0$ m/s; $\Delta t = 5.2$ s

Required: $\Delta\vec{d}$

Analysis: Use $\Delta\vec{d} = \left(\dfrac{\vec{v}_i + \vec{v}_f}{2}\right)\Delta t$ to solve for displacement. Use forward as the positive direction.

First convert kilometres per hour to metres per second.

$$\frac{100.0\ \cancel{\text{km}}}{1\ \cancel{\text{h}}} \times \frac{1000\ \text{m}}{1\ \cancel{\text{km}}} \times \frac{1\ \cancel{\text{h}}}{3600\ \text{s}} = 27.78\ \text{m/s (two extra digits carried)}$$

Solution: $\Delta d = \left(\dfrac{v_i + v_f}{2}\right)\Delta t$

$$= \frac{(27.78\ \text{m}/\cancel{\text{s}} + 0\ \text{m}/\cancel{\text{s}})(5.2\ \cancel{\text{s}})}{2}$$
$$\Delta d = 72\ \text{m}$$

Statement: The car stops in 72 m.

48. (a) Given: $\vec{v}_i = 30.0$ m/s [up]; $\vec{v}_f = 0$ m/s; $\vec{a} = 9.8\ \text{m/s}^2$ [down]

Required: Δt

Analysis: Use $\vec{a} = \dfrac{\Delta\vec{v}}{\Delta t}$ to calculate the time taken; $\Delta t = \dfrac{\Delta v}{a}$. Use up as positive.

Solution: $\Delta t = \dfrac{\Delta v}{a}$

$$= \frac{(0\ \cancel{\text{m}/\text{s}}) - (30.0\ \cancel{\text{m}/\text{s}})}{-9.8\ \cancel{\text{m}}/\text{s}^{\cancel{2}}}$$
$$\Delta t = 3.1\ \text{s}$$

Statement: It takes the ball 3.1 s to rise to its maximum height.

(b) Given: $\vec{v}_{\text{i}} = 30.0 \text{ m/s [up]}$; $\vec{v}_{\text{f}} = 0 \text{ m/s}$; $\vec{a} = 9.8 \text{ m/s}^2 \text{ [down]}$

Required: $\Delta\vec{d}$

Analysis: Use $\Delta d = \dfrac{v_{\text{f}}^2 - v_{\text{i}}^2}{2a}$. (Any of the formulas containing displacement could be used.)

Solution:
$$\Delta d = \frac{v_{\text{f}}^2 - v_{\text{i}}^2}{2a}$$
$$= \frac{(0 \text{ m/}\cancel{\text{s}})^{\cancel{2}} - (30.0 \text{ m/}\cancel{\text{s}})^{\cancel{2}}}{2(-9.8 \cancel{\text{m}}/\cancel{\text{s}^2})}$$
$$\Delta d = 46 \text{ m}$$

Statement: The ball rises 46 m.

(c) Given: $\vec{v}_{\text{i}} = 30.0 \text{ m/s [up]}$; $\vec{v}_{\text{f}} = 10.0 \text{ m/s [up]}$; $\vec{a} = 9.8 \text{ m/s}^2 \text{ [down]}$

Required: Δt

Analysis: Use $\vec{a} = \dfrac{\Delta\vec{v}}{\Delta t}$ to calculate the time taken; $\Delta t = \dfrac{\Delta v}{a}$. Use up as positive.

Solution:
$$\Delta t = \frac{\Delta v}{a}$$
$$= \frac{(10.0 \cancel{\text{m}}/\cancel{\text{s}}) - (30.0 \cancel{\text{m}}/\cancel{\text{s}})}{-9.8 \cancel{\text{m}}/\text{s}^{\cancel{2}}}$$
$$\Delta t = 2.0 \text{ s}$$

Statement: At 2.0 s after the throw, the ball will have a velocity of 10.0 m/s [upward].

(d) Given: $\vec{v}_{\text{i}} = 30.0 \text{ m/s [up]}$; $\vec{v}_{\text{f}} = 10.0 \text{ m/s [down]}$; $\vec{a} = 9.8 \text{ m/s}^2 \text{ [down]}$

Required: Δt

Analysis: Use $\vec{a} = \dfrac{\Delta\vec{v}}{\Delta t}$ to calculate the time taken; $\Delta t = \dfrac{\Delta v}{a}$. Use up as positive.

Solution:
$$\Delta t = \frac{\Delta v}{a}$$
$$= \frac{(-10.0 \cancel{\text{m}}/\cancel{\text{s}}) - (30.0 \cancel{\text{m}}/\cancel{\text{s}})}{-9.8 \cancel{\text{m}}/\text{s}^{\cancel{2}}}$$
$$\Delta t = 4.1 \text{ s}$$

Statement: At 4.1 s after the throw, the ball will have a velocity of 10.0 m/s [downward].

(e) Given: $\vec{v}_{\text{i}} = 30.0 \text{ m/s [up]}$; $\Delta\vec{d} = 0 \text{ m}$; $\vec{a} = 9.8 \text{ m/s}^2 \text{ [down]}$

Required: Δt

Analysis: The displacement is zero when the ball returns to its original height. Its velocity at this time is the opposite of its initial velocity: $\vec{v}_{\text{f}} = 30.0 \text{ m/s [down]}$. Use $\vec{a} = \dfrac{\Delta\vec{v}}{\Delta t}$ to calculate the time taken. Use up as positive.

Solution: $a = \frac{\Delta v}{\Delta t}$

$$\Delta t = \frac{\Delta v}{a}$$
$$= \frac{(-30.0\ \cancel{\text{m}}/\cancel{\text{s}}) - (30.0\ \cancel{\text{m}}/\cancel{\text{s}})}{-9.8\ \cancel{\text{m}}/\text{s}^{\cancel{2}}}$$
$$\Delta t = 6.1\ \text{s}$$

Statement: The displacement is zero after 6.1 s.

49. (a) Given: velocity–time graph Figure 2, page 56

Required: instantaneous acceleration at $t = 3$ s, $t = 10$ s, and $t = 13$ s

Analysis: The velocity–time graph is straight at the requested times. The instantaneous acceleration at these times is equal to the average acceleration close to these times. For each time, read two points from the graph and use $\vec{a}_{\text{av}} = \frac{\Delta \vec{v}}{\Delta t}$. Use south as the positive direction.

Solution: For $t = 3$ s, two points are (0 s, 20 m/s) and (5 s, 20 m/s).

$$a_{\text{av}} = \frac{\Delta v}{\Delta t}$$
$$= \frac{20\ \text{m/s} - 20\ \text{m/s}}{5\ \text{s} - 0\ \text{s}}$$
$$a_{\text{av}} = 0\ \text{m/s}^2$$

For $t = 10$ s, two points are (9 s, 45 m/s) and (12 s, 15 m/s).

$$a_{\text{av}} = \frac{\Delta v}{\Delta t}$$
$$= \frac{15\ \text{m/s} - 45\ \text{m/s}}{12\ \text{s} - 9\ \text{s}}$$
$$a_{\text{av}} = -10\ \text{m/s}^2$$

For $t = 13$ s , two points are (12 s, 15 m/s) and (14 s, 5 m/s).

$$a_{\text{av}} = \frac{\Delta v}{\Delta t}$$
$$= \frac{5\ \text{m/s} - 15\ \text{m/s}}{14\ \text{s} - 12\ \text{s}}$$
$$a_{\text{av}} = -5\ \text{m/s}^2$$

Statement: The instantaneous acceleration at $t = 3$ s is 0 m/s^2; at $t = 10$ s, it is 10 m/s^2 [N]; and at $t = 10$ s, it is 5 m/s^2 [N].

(b) Given: velocity–time graph Figure 2, page 56

Required: average acceleration for the complete motion, $\vec{a}_{\text{av}}$

Analysis: The average acceleration is based only on the change in velocity from the beginning to the end of the motion. Read the initial the final points from the graph and use $\vec{a}_{\text{av}} = \frac{\Delta \vec{v}}{\Delta t}$. Use south as the positive direction.

Solution: The two points are (0 s, 20 m/s) and (16 s, 0 m/s).

$$\begin{aligned} a_{\text{av}} &= \frac{\Delta v}{\Delta t} \\ &= \frac{0 \text{ m/s} - 20 \text{ m/s}}{16 \text{ s} - 0 \text{ s}} \\ a_{\text{av}} &= -1.2 \text{ m/s}^2 \end{aligned}$$

Statement: The average acceleration for the complete motion is 1.2 m/s^2 [N].

50. Given: At $t_1 = 2$ s and $t_2 = 10$ s, $d_y = h$; $g = 9.8 \text{ m/s}^2$

Required: height, h

Analysis: The ball's flight is symmetric on the way up and the way down. The time the ball reaches its maximum is halfway between the two given times. Use this time and $\Delta v_y = -g\Delta t$ to determine the initial vertical velocity. Then use $\Delta d_y = v_{1y}\Delta t - \frac{1}{2}g\Delta t^2$ with t = 2 s, to determine the height h. Use up as the positive direction.

Solution: The time of maximum height is

$$\begin{aligned} \Delta t &= \frac{t_1 + t_2}{2} \\ &= \frac{2 \text{ s} + 10 \text{ s}}{2} \\ \Delta t &= 6 \text{ s} \end{aligned}$$

Calculate the initial vertical velocity v_{1y} .

$$\begin{aligned} \Delta v_y &= -g\Delta t \\ v_{yf} - v_{yi} &= -g\Delta t \\ 0 \text{ m/s} - v_{yi} &= (-9.8 \text{ m/s}^{\cancel{2}})(6 \cancel{\text{s}}) \\ v_{yi} &= 58.8 \text{ m/s} \end{aligned}$$

The displacement after 2 s is

$$\begin{aligned} \Delta d_y &= v_{1y}\Delta t - \frac{1}{2}g\Delta t^2 \\ &= (58.8 \text{ m/}\cancel{\text{s}})(2 \cancel{\text{s}}) - (4.9 \text{ m/}\cancel{\text{s}^2})(2 \cancel{\text{s}})^{\cancel{2}} \\ &= 117.6 \text{ m} - 19.6 \text{ m} \\ \Delta d_y &= 98 \text{ m} \end{aligned}$$

Statement: The height at 2 s and at 10 s is 98 m.

51. $\Delta \vec{d} = \vec{v}_i \Delta t + \frac{1}{2}\vec{a}_{\text{av}}\Delta t^2$

$$\begin{aligned} [\text{m}] &= \frac{\text{m}}{\cancel{\text{s}}}[\cancel{\text{s}}] + \frac{\text{m}}{\cancel{\text{s}^2}}[\cancel{\text{s}^2}] \\ [\text{m}] &= [\text{m}] + [\text{m}] \\ [\text{m}] &= [\text{m}] \end{aligned}$$

Since both sides of $\Delta\vec{d} = \vec{v}_{\mathrm{i}}\Delta t + \frac{1}{2}\vec{a}_{\mathrm{av}}\Delta t^2$ have the SI units of distance, the equation is dimensionally correct.

52. (a) To derive $v_{\mathrm{f}}^2 = v_{\mathrm{i}}^2 + 2a\Delta d$ from $\Delta\vec{d} = \left(\frac{\vec{v}_{\mathrm{i}} + \vec{v}_{\mathrm{f}}}{2}\right)\Delta t$ and $\Delta\vec{d} = \vec{v}_{\mathrm{i}}\Delta t + \frac{1}{2}\vec{a}\Delta t^2$, isolate Δt in the first formula and substitute this into the second formula, then solve for v_{f}^2. Since these formulas apply to motion in one dimension, use the convention that the vector $\vec{w}$ is renamed w, with the sign of w indicating direction.

Isolate Δt in $\Delta d = \left(\frac{v_{\mathrm{i}} + v_{\mathrm{f}}}{2}\right)\Delta t$.

$$\Delta d = \left(\frac{v_{\mathrm{i}} + v_{\mathrm{f}}}{2}\right)\Delta t$$

$$\Delta t = \frac{2\Delta d}{v_{\mathrm{i}} + v_{\mathrm{f}}}$$

Substitute this into $\Delta d = v_{\mathrm{i}}\Delta t + \frac{1}{2}a\Delta t^2$.

$$\Delta d = v_{\mathrm{i}}\Delta t + \frac{1}{2}a\Delta t^2$$

$$\Delta d = v_{\mathrm{i}}\left(\frac{2\Delta d}{v_{\mathrm{i}} + v_{\mathrm{f}}}\right) + \frac{1}{2}a\left(\frac{2\Delta d}{v_{\mathrm{i}} + v_{\mathrm{f}}}\right)^2$$

$$\cancel{\Delta d} = 2\cancel{\Delta d}\left(\frac{v_{\mathrm{i}}}{v_{\mathrm{i}} + v_{\mathrm{f}}}\right) + 2a\Delta d^{\cancel{2}}\left(\frac{1}{v_{\mathrm{i}} + v_{\mathrm{f}}}\right)^2$$

$$1 = 2\left(\frac{v_{\mathrm{i}}}{v_{\mathrm{i}} + v_{\mathrm{f}}}\right) + 2a\Delta d\left(\frac{1}{v_{\mathrm{i}} + v_{\mathrm{f}}}\right)^2$$

Multiply both sides by $(v_{\mathrm{i}} + v_{\mathrm{f}})^2$ and simplify.

$$\begin{aligned}(v_{\mathrm{i}} + v_{\mathrm{f}})^2 &= 2v_{\mathrm{i}}(v_{\mathrm{i}} + v_{\mathrm{f}}) + 2a\Delta d \\ v_{\mathrm{i}}^2 + 2v_{\mathrm{i}}v_{\mathrm{f}} + v_{\mathrm{f}}^2 &= 2v_{\mathrm{i}}^2 + 2v_{\mathrm{i}}v_{\mathrm{f}} + 2a\Delta d \\ v_{\mathrm{f}}^2 &= v_{\mathrm{i}}^2 + 2a\Delta d\end{aligned}$$

(b) To derive $\Delta\vec{d} = \vec{v}_{\mathrm{f}}\Delta t - \frac{1}{2}\vec{a}\Delta t^2$ from $\Delta\vec{d} = \left(\frac{\vec{v}_{\mathrm{i}} + \vec{v}_{\mathrm{f}}}{2}\right)\Delta t$ and $\Delta\vec{d} = \vec{v}_{\mathrm{i}}\Delta t + \frac{1}{2}\vec{a}\Delta t^2$, eliminate $\vec{v}_{\mathrm{i}}$.

Isolate $\vec{v}_{\mathrm{i}}$ in the first formula, substitute into the second formula, and then rearrange the result to obtain the required formula.

Isolate v_i.

$$\Delta d = \left(\frac{v_i + v_f}{2}\right)\Delta t$$

$$v_i + v_f = \frac{2\Delta d}{\Delta t}$$

$$v_i = -v_f + \frac{2\Delta d}{\Delta t}$$

Substitute and rearrange.

$$\begin{aligned}\Delta d &= v_i\Delta t + \frac{1}{2}a\Delta t^2\\ &= \left(-v_f + \frac{2\Delta d}{\Delta t}\right)\Delta t + \frac{1}{2}a\Delta t^2\end{aligned}$$

$$\Delta d = -v_f\Delta t + 2\Delta d + \frac{1}{2}a\Delta t^2$$

$$-\Delta d = -v_f\Delta t + \frac{1}{2}a\Delta t^2$$

$$\Delta d = v_f\Delta t - \frac{1}{2}a\Delta t^2$$

53. Answers may vary. Sample answer: I would use a long straight plank and a pile of textbooks arranged on the floor or on a long counter. I would also need a ball and a ranger with its computer or graphing calculator. I would place one end of the plank on some of the books, set the ranger at the top of the plank, and release the ball from in front of the ranger to roll down the plank. Using the software that comes with the ranger, I would draw position–time, velocity–time, and acceleration–time graphs. I would examine the graphs to see if the ball underwent uniform acceleration and, if so, determine the value of the acceleration. It would be interesting to change the number of books under the plank and see how the acceleration changes. Here I would be using the angle of inclination of the plank, as measured from its geometry, as the independent variable and the acceleration of the ball as the dependent variable. For a fixed angle of inclination, I might also compare the acceleration of a slippery sliding object with that of the ball.

54. (a) The sign of the slope of the velocity–time graph gives the sign of the acceleration. The acceleration is positive between 0 s and 10 s, and again between 40 s and 50 s. The acceleration is negative between 20 s and 40 s. The acceleration is 0 m/s^2 between 10 s and 20 s.

(b) The initial velocity and final velocity are the same, so the average acceleration is 0 m/s^2.

(c) The object changes direction when its velocity reverses or changes sign. The object changes direction at 30 s.

(d) Given: velocity–time graph, Figure 3, page 57

Required: displacement between 0 s and 10 s, Δd_1; displacement between 10 s and 15 s, Δd_2

Analysis: Displacement is the area under (or above) the velocity–time curve between the given times. The first displacement is calculated by looking at a triangle; the second is found using a rectangle. Use west as the positive direction.

Solution: From 0 s and 10 s,

$$\Delta d_1 = \frac{1}{2}(\text{base})(\text{height})$$
$$= \frac{1}{2}(10\ \cancel{\text{s}})(60\ \text{m}/\cancel{\text{s}})$$
$$\Delta d_1 = 300\ \text{m}$$

From 10 s and 15 s,

$$\Delta d_2 = (\text{base})(\text{height})$$
$$= (5\ \cancel{\text{s}})(60\ \text{m}/\cancel{\text{s}})$$
$$\Delta d_2 = 300\ \text{m}$$

Statement: Both displacements are 300 m [W].

55. (a) Given: $v_x = 10.0$ m/s; $v_{iy} = 0$ m/s; $\Delta d_y = -0.49$ m

Required: Δd_x

Analysis: In the horizontal direction, I know the constant speed but I do not know the time interval or the displacement. In the vertical direction, I know the displacement, initial velocity, and acceleration. Determine the time interval using $\Delta d_y = v_{1y}\Delta t - \frac{1}{2}g\Delta t^2$;

$$\Delta t = \frac{v_{1y} \pm \sqrt{v_{1y}^2 - 4(\frac{1}{2}g)(\Delta d_y)}}{g}$$

Then go back to the horizontal direction and use $\Delta d_x = v_x \Delta t$ to determine the required horizontal distance.

Solution:

vertical direction:

$$\Delta t = \frac{v_{1y} \pm \sqrt{v_{1y}^2 - 4(\frac{1}{2}g)(\Delta d_y)}}{g}$$

$$\Delta t = \frac{0\ \text{m/s} \pm \sqrt{(0\ \text{m/s})^2 - 4(4.9\ \text{m/s}^2)(-0.49\ \text{m})}}{9.8\ \text{m/s}^2}$$

$$\Delta t = \pm 0.3162\ \text{s}$$

horizontal direction:

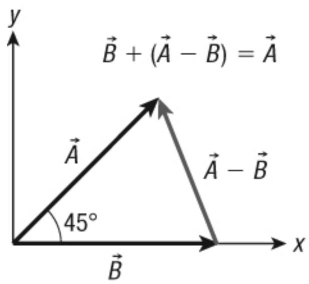
$$\Delta d_x = v_x \Delta t$$
$$= (10.0\ \text{m}/\cancel{\text{s}})(0.3162\ \cancel{\text{s}})$$
$$\Delta d_x = 3.2\ \text{m}$$

Statement: The dart board and player are 3.2 m apart.

56. Given: $\vec{B} = 5.0$ units [along +x-axis]; $\left|\vec{A}\right| = 5.0$ units [at 45° to $\vec{B}$]

Required: components of $\vec{A} - \vec{B}$

Analysis: Make a sketch of the two vectors.



It is not clear whether $\vec{A}$ points above or below $\vec{B}$. Assume that $\vec{A}$ points above $\vec{B}$, into the first quadrant. Determine the *x*- and *y*-components of $\vec{A}$ and $\vec{B}$. Then subtract the components of $\vec{B}$ from those of $\vec{A}$.

Solution:

The components of $\vec{A}$ are

$$\begin{aligned}A_x &= \left|\vec{A}\right|\sin\theta \\ &= (5.0 \text{ units})(\sin 45°) \\ A_x &= 3.535 \text{ units (two extra digits carried)} \\ A_y &= \left|\vec{A}\right|\cos\theta \\ &= (5.0 \text{ units})(\cos 45°) \\ A_y &= 3.535 \text{ units (two extra digits carried)}\end{aligned}$$

The components of $\vec{B}$ are

$$\begin{aligned}B_x &= \left|\vec{B}\right|\sin\theta \\ B_x &= 5.0 \text{ units} \\ B_y &= \left|\vec{B}\right|\cos\theta \\ B_y &= 0 \text{ units}\end{aligned}$$

The components of $\vec{A} - \vec{B}$ are

$$\begin{aligned}(\vec{A} - \vec{B})_x &= \vec{A}_x - \vec{B}_x \\ &= 3.535 \text{ units} - 5.0 \text{ units} \\ (\vec{A} - \vec{B})_x &= -1.5 \text{ units}\end{aligned}$$

$$\begin{aligned}(\vec{A} - \vec{B})_y &= \vec{A}_y - \vec{B}_y \\ &= 3.535 \text{ units} - 0 \text{ units} \\ (\vec{A} - \vec{B})_y &= +3.5 \text{ units}\end{aligned}$$

Looking at the sketch above, this makes sense because $\vec{A} - \vec{B}$ points left and up.

Statement: The *x*- and *y*-components of $\vec{A} - \vec{B}$ are -1.5 units and +3.5 units respectively.

57. Given: $\Delta\vec{d}_1 = 20.0$ km [N]; $\Delta\vec{d}_2 = 25.0$ km [N 60.0° W]

Required: $\Delta\vec{d}$

Analysis: $\Delta\vec{d} = \Delta\vec{d}_1 + \Delta\vec{d}_2$. Determine the *x*- and *y*-components of the given displacement vectors. Add these *x*- and *y*-components to determine the *x*- and *y*-components of the total displacement. Finally, use the Pythagorean theorem and tangent ratio to determine the total displacement vector. Use east and north as positive.

Solution: For the first vector,

$$\begin{aligned}\Delta d_{1x} &= \left|\Delta\vec{d}_1\right|\sin\theta \\ &= +(20.0 \text{ km})(\cos 90.0°) \\ \Delta d_{1x} &= 0 \text{ km}\end{aligned}$$

$$\begin{aligned}\Delta d_{1y} &= +\left|\Delta\vec{d}_1\right|\cos\theta \\ &= +(20.0 \text{ km})(\sin 90.0°) \\ \Delta d_{1y} &= +20.0 \text{ km}\end{aligned}$$

For the second vector,

$$\begin{aligned}\Delta d_{2x} &= -\left|\Delta\vec{d}_2\right|\sin\theta \\ &= -(25.0\text{ km})(\sin 60.0°) \\ \Delta d_{2x} &= -21.651\text{ km (two extra digits carried)}\end{aligned}$$

$$\begin{aligned}\Delta d_{2y} &= +\left|\Delta\vec{d}_2\right|\cos\theta \\ &= +(25.0\text{ km})(\cos 60.0°) \\ \Delta d_{2y} &= +12.5\text{ km}\end{aligned}$$

Add the horizontal components.

$$\begin{aligned}\Delta d_x &= \Delta d_{1x} + \Delta d_{2x} \\ &= 0\text{ km} + (-21.651\text{ km}) \\ \Delta d_x &= -21.651\text{ km (two extra digits carried)}\end{aligned}$$

Add the vertical components.

$$\begin{aligned}\Delta d_y &= \Delta d_{1y} + \Delta d_{2y} \\ &= (+20.0\text{ km}) + (+12.5\text{ km}) \\ \Delta d_y &= +32.5\text{ km}\end{aligned}$$

Combine the total displacement components to determine the total displacement.

$$\begin{aligned}\left|\Delta\vec{d}\right| &= \sqrt{\Delta d_x^2 + \Delta d_y^2} \\ &= \sqrt{(21.651\text{ km})^2 + (32.5\text{ km})^2} \\ \left|\Delta\vec{d}\right| &= 39.1\text{ km}\end{aligned}$$

$$\begin{aligned}\theta &= \tan^{-1}\left(\frac{\left|\Delta\vec{d}_y\right|}{\left|\Delta\vec{d}_x\right|}\right) \\ &= \tan^{-1}\left(\frac{32.5\ \cancel{\text{km}}}{21.651\ \cancel{\text{km}}}\right) \\ \theta &= 56.3°\end{aligned}$$

Statement: The total displacement of the trip is 39.1 km [W 56.3° N] or 39.1 km [N 33.7° W].

58. (a) Given: $\Delta\vec{d}_1 = 6.0$ m [positive y-axis]; $\Delta\vec{d}_2 = 8.0$ m [23° below positive x-axis]

Required: $\Delta\vec{d}$

Analysis: $\Delta\vec{d} = \Delta\vec{d}_1 + \Delta\vec{d}_2$ Determine the x- and y-components of the given displacement vectors. Add these x- and y-components to determine the x- and y-components of the total displacement. Finally, use the Pythagorean theorem and the tangent ratio to determine the total displacement vector.

Solution: For the first vector,

$$\begin{aligned}\Delta d_{1x} &= \left|\Delta\vec{d}_1\right|\cos\theta \\ &= (6.0\text{ m})(\cos 90°) \\ \Delta d_{1x} &= 0\text{ m}\end{aligned}$$

$$\Delta d_{1y} = +\left|\Delta \vec{d}_1\right| \sin\theta$$
$$= +(6.0\text{ m})(\sin 90°)$$
$$\Delta d_{1y} = +6.0\text{ m}$$
For the second vector,
$$\Delta d_{2x} = +\left|\Delta \vec{d}_2\right| \cos\theta$$
$$= +(8.0\text{ m})(\cos 23°)$$
$$\Delta d_{2x} = 7.364\text{ m (two extra digits carried)}$$
$$\Delta d_{2y} = -\left|\Delta \vec{d}_2\right| \sin\theta$$
$$= -(8.0\text{ m})(\sin 23°)$$
$$\Delta d_{2y} = -3.126\text{ m (two extra digits carried)}$$
Add the x-components.
$$\Delta d_x = \Delta d_{1x} + \Delta d_{2x}$$
$$= 0\text{ m} + (+7.364\text{ m})$$
$$= +7.364\text{ m}$$
$$\Delta d_x = +7.4\text{ m}$$
Add the y-components.
$$\Delta d_y = \Delta d_{1y} + \Delta d_{2y}$$
$$= (+6.0\text{ m}) + (-3.126\text{ m})$$
$$= +2.874\text{ m}$$
$$\Delta d_y = +2.9\text{ m}$$
Statement: The x- and y-components of the total displacement vector are +7.4 m and +2.9 m.
(b) Given: $\Delta d_x = +7.364\text{ m}; \Delta d_y = +2.874\text{ m}$

Required: $\Delta \vec{d}$

Analysis: Combine the total displacement components to determine the total displacement.

Solution: $\left|\Delta \vec{d}\right| = \sqrt{\Delta d_x^2 + \Delta d_y^2}$
$$\left|\Delta \vec{d}\right| = \sqrt{(7.364\text{ m})^2 + (2.874\text{ m})^2}$$
$$\left|\Delta \vec{d}\right| = 7.9\text{ m}$$
$$\theta = \tan^{-1}\left(\frac{\left|\Delta \vec{d}_y\right|}{\left|\Delta \vec{d}_x\right|}\right)$$
$$= \tan^{-1}\left(\frac{2.874\ \cancel{\text{m}}}{7.364\ \cancel{\text{m}}}\right)$$
$$\theta = 21°$$
Statement: The total displacement is 7.9 m at 21° above the positive x-axis.

59. Given: $\vec{v}_{\text{i}} = 15$ m/s [E]; $\vec{v}_{\text{f}} = 12$ m/s [E 25° N]; $\Delta t = 5.0$ s

Required: $\vec{a}_{\text{av}}$

Analysis: Calculate the change in velocity using components and $\Delta\vec{v} = \vec{v}_{\text{f}} - \vec{v}_{\text{i}}$. Then, determine the average acceleration using $\vec{a}_{\text{av}} = \dfrac{\Delta\vec{v}}{\Delta t}$. Use east and north as positive.

Solution: The *x*- component of $\Delta\vec{v}$ is

$$\begin{aligned}
\Delta v_x &= v_{\text{f}x} - v_{\text{i}x} \\
&= (12 \text{ m/s})(\cos 25°) - 15 \text{ m/s} \\
&= 10.875 \text{ m/s} - 15 \text{ m/s} \\
\Delta v_x &= -4.124 \text{ m/s}
\end{aligned}$$

The *y*-component of $\Delta\vec{v}$ is

$$\begin{aligned}
\Delta v_y &= v_{\text{f}y} - v_{\text{i}y} \\
&= (12 \text{ m/s})(\sin 25°) - 0 \text{ m/s} \\
\Delta v_y &= +5.071 \text{ m/s}
\end{aligned}$$

Determine the change in velocity from its components.

$$\begin{aligned}
|\Delta\vec{v}| &= \sqrt{\Delta v_x^2 + \Delta v_y^2} \\
&= \sqrt{(4.124 \text{ m/s})^2 + (5.071 \text{ m/s})^2} \\
|\Delta\vec{v}| &= 6.5 \text{ m/s}
\end{aligned}$$

$$\begin{aligned}
\theta &= \tan^{-1}\left(\frac{|\Delta\vec{v}_y|}{|\Delta\vec{v}_x|}\right) \\
&= \tan^{-1}\left(\frac{5.071 \ \cancel{\text{m/s}}}{4.124 \ \cancel{\text{m/s}}}\right) \\
&= 50.88° \\
\theta &= 51°
\end{aligned}$$

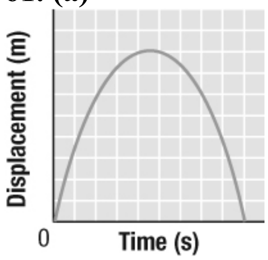
Calculate the average acceleration.

$$\begin{aligned}
\vec{a}_{\text{av}} &= \frac{\Delta\vec{v}}{\Delta t} \\
&= \frac{6.536 \text{ m/s [W 51° N]}}{5.0 \text{ s}} \\
\vec{a}_{\text{av}} &= 1.3 \text{ m/s}^2 \text{ [W 51° N]}
\end{aligned}$$

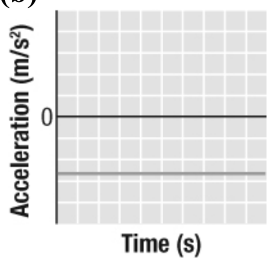
Statement: The average acceleration of the car is 1.3 m/s^2 [W 51° N].

60. Answers may vary. Sample answer: Assume that air resistance is negligible. The golf ball is in projectile motion so its horizontal motion is at constant speed. The average acceleration in the *x*-direction is 0 m/s^2.

61. (a)



(b)



62. Given: $\Delta t = 6.0$ s; $\Delta\vec{d} = 60.0$ m [forward]; $\vec{v}_\mathrm{f} = 15$ m/s [forward]

Required: a; v_i

Analysis: Use $\Delta d = v_\mathrm{f}\Delta t - \frac{1}{2}a\Delta t^2$ to determine the acceleration: $a = \dfrac{2(v_\mathrm{f}\Delta t - \Delta d)}{\Delta t^2}$.

Any of the other motion formulas can be used to calculate the initial velocity. Use $v_\mathrm{f} = v_\mathrm{i} + a\Delta t$.
Use forward as the positive direction.

Solution:

Solve for acceleration.

$$a = \frac{2(v_\mathrm{f}\Delta t - \Delta d)}{\Delta t^2}$$

$$a = \frac{2((15\ \text{m/}\cancel{\text{s}})(6.0\ \cancel{\text{s}}) - 60.0\ \text{m})}{(6.0\ \text{s})^2}$$

$$= 1.667\ \text{m/s}^2 \text{ (two extra digits carried)}$$

$$a = 1.7\ \text{m/s}^2$$

Solve for initial velocity.

$$\begin{aligned} v_\mathrm{f} &= v_\mathrm{i} + a\Delta t \\ v_\mathrm{i} &= v_\mathrm{f} - a\Delta t \\ &= 15\ \text{m/s} - (1.667\ \text{m/s}^2)(6.0\ \text{s}) \\ &= 15\ \text{m/s} - 10\ \text{m/s} \\ v_\mathrm{i} &= 5\ \text{m/s} \end{aligned}$$

Statement: The acceleration is 1.7 m/s^2 [forward] and the initial speed is 5 m/s.

63. Solutions may vary. Sample solution:

Given: maximum height: $\Delta d_y = 3.7$ m; $\theta = 45°$; $g = 9.8$ m/s^2

Required: v_i

Analysis: Imagine the puma completing its jump and returning to the ground. The puma's jump is a projectile motion problem. The maximum height of a projectile is $\Delta d_y = \dfrac{(v_\mathrm{i}\sin\theta)^2}{2g}$.

Solve for v_i; $v_\mathrm{i}^2 = \dfrac{2g\Delta d_y}{\sin^2\theta}$.

Solution: $v_{\rm i}^2 = \dfrac{2g\Delta d_y}{\sin^2\theta}$

$$= \frac{2(9.8\ \text{m/s}^2)(3.7\ \text{m})}{\sin^2 45°}$$

$$v_{\rm i}^2 = 145.04\ \text{m}^2/\text{s}^2$$

$$v_{\rm i} = 12\ \text{m/s}$$

Statement: The initial speed of the puma is 12 m/s.

64. Answers may vary. Sample answer:

(a) Imagine the footballs returning to the ground. There are three formulas developed for such symmetric projectile motion. The one for maximum height is $\Delta d_y = \dfrac{(v_{\rm i}\sin\theta)^2}{2g}$. If the two balls are kicked with the same initial speed, the one with the greater angle (A) goes higher.

(b) The formula for time of flight is $\Delta t = \dfrac{2v_{\rm i}\sin\theta}{g}$. The football with the greater angle (A) stays in the air longer.

(c) No because neither angle is known. The formula for the horizontal range is $\Delta d_x = \dfrac{v_{\rm i}^2\sin 2\theta}{g}$. Footballs kicked at complementary angles have the same range. Otherwise the football with the angle closer to 45° would have the longer range.

65. Given: $\Delta d_x = 200.0$ m; $\theta = 45°$; $d_{\rm iy} = d_{\rm fy}$; $g = 9.8\ \text{m/s}^2$

Required: $v_{\rm i}$

Analysis: Use the range formula, $\Delta d_x = \dfrac{v_{\rm i}^2\sin 2\theta}{g}$.

Solution: $\Delta d_x = \dfrac{v_{\rm i}^2\sin 2\theta}{g}$

$$v_{\rm i}^2 = \frac{\Delta d_x g}{\sin 2\theta}$$

$$= \frac{(200.0\ \text{m})(9.8\ \text{m/s}^2)}{\sin 90°}$$

$$v_{\rm i}^2 = 1960\ \text{m}^2/\text{s}^2$$

$$v_{\rm i} = 44\ \text{m/s}$$

Statement: The baseball leaves the bat at 44 m/s.

66. Given: $\Delta d_x = 9.5$ m; $d_{yi} = 2.0$ m; $d_{yf} = 3.1$ m; $\theta = 35°$; $g = 9.8$ m/s^2

Required: v_i

Analysis: I do not know the time interval for the shot or the initial speed of the ball. I do know that $v_x = v_i \cos\theta$ and $v_y = v_i \sin\theta$. Write an equation for the horizontal motion using $\Delta d_x = v_x \Delta t$ and for the vertical motion using $\Delta d_y = v_{1y}\Delta t - \frac{1}{2}g\Delta t^2$. Use these two equations to write a quadratic equation for Δt.

Solution: Horizontal motion:

$$\Delta d_x = v_x \Delta t$$

$$9.5\text{ m} = v_i \cos 35°\Delta t$$

$$v_i = \frac{9.5\text{ m}}{(\cos 35°)\Delta t} \quad \text{(Equation 1)}$$

Vertical motion:

$$\Delta d_y = v_{1y}\Delta t - \frac{1}{2}g\Delta t^2$$

$$(3.1\text{ m} - 2.0\text{ m}) = v_1(\sin 35°)\Delta t - (4.9\text{ m/s}^2)\Delta t^2$$

$$1.1\text{ m} = v_1(\sin 35°)\Delta t - (4.9\text{ m/s}^2)\Delta t^2 \quad \text{(Equation 2)}$$

Substitute Equation 1 into Equation 2 and simplify.

$$1.1\text{ m} = \left(\frac{9.5\text{ m}}{\cos 35° \cancel{\Delta t}}\right)(\sin 35° \cancel{\Delta t}) - (4.9\text{ m/s}^2)\Delta t^2$$

$$1.1\text{ m} = 6.652\text{ m} - (4.9\text{ m/s}^2)\Delta t^2$$

$$\Delta t^2 = \frac{6.652\ \cancel{\text{m}} - 1.1\ \cancel{\text{m}}}{4.9\ \cancel{\text{m}}/\text{s}^2}$$

$$\Delta t = 1.064\text{ s} \quad \text{(two extra digits carried)}$$

Finally, substitute this time interval into Equation 1.

$$v_i = \frac{9.5\text{ m}}{\cos 35°\Delta t}$$

$$= \frac{9.5\text{ m}}{(\cos 35°)(1.064\text{ s})}$$

$$v_i = 11\text{ m/s}$$

Statement: The initial speed of the basketball is 11 m/s.

67. Given: $\vec{v}_i = 30.0$ m/s [45° above horizontal]; $d_{iy} = d_{fy}$; $g = 9.8$ m/s^2

Required: Δt

Analysis: Since the ball lands at the same height from which it was launched, the time taken is given by $\Delta t = \dfrac{2v_i \sin\theta}{g}$.

Solution: $\Delta t = \dfrac{2v_i \sin\theta}{g}$

$= \dfrac{2(30.0\ \cancel{\text{m}}/\cancel{\text{s}})(\sin 45°)}{9.8\ \cancel{\text{m}}/\text{s}^{\cancel{2}}}$

$\Delta t = 4.3\ \text{s}$

Statement: The time the ball stays in the air is 4.3 s.

68. (a) Given: $\vec{v}_i = 60.0\ \text{m/s}$ [60.0° above horizontal]; $d_{iy} = d_{fy}$; $g = 9.8\ \text{m/s}^2$

Required: maximum height, d_y

Analysis: The maximum height occurs when $v_y = 0\ \text{m/s}$. Use the acceleration formula $v_{fy}^2 = v_{iy}^2 - 2g\Delta d_y$ to determine d_y. Since $v_{iy} = v_i \sin\theta$ and $v_{fy} = 0\ \text{m/s}$, d_y can be calculated.

$v_{fy}^2 = v_{iy}^2 - 2g\Delta d_y$

$\Delta d_y = \dfrac{v_{fy}^2}{2g}$

$\Delta d_y = \dfrac{(v_i \sin\theta)^2}{2g}$

Solution: $\Delta d_y = \dfrac{(v_i \sin\theta)^2}{2g}$

$= \dfrac{((60.0\ \text{m/s})\sin 60.0°)^2}{2(9.8\ \text{m/s}^2)}$

$\Delta d_y = 138\ \text{m}$

Statement: The maximum height of the stream of water is 138 m.

(b) Given: $\vec{v}_i = 60.0\ \text{m/s}$ [60.0° above horizontal]; $g = 9.8\ \text{m/s}^2$

Required: horizontal range, Δd_x

Analysis: Use the range formula, $\Delta d_x = \dfrac{v_i^2 \sin 2\theta}{g}$.

Solution: $\Delta d_x = \dfrac{v_i^2 \sin 2\theta}{g}$

$= \dfrac{(60.0\ \text{m/s})^2(\sin 120.0°)}{9.8\ \text{m/s}^2}$

$\Delta d_x = 318\ \text{m}$

Statement: The water hits the ground 318 m away.

69. Given: $v_x = 8.0\ \text{m/s}$; $\theta = 60.0°$

Required: v_y

Analysis: From basic trigonometry, $v_y = v_x \tan\theta$.

Solution:
$v_y = v_x \tan\theta$
$= (8.0 \text{ m/s})(\tan 60.0°)$
$v_y = 14 \text{ m/s}$

Statement: The vertical component of the dolphin's velocity is 14 m/s.

70. Answers may vary. Sample answer: The tennis player could have either hit the ball such that it had a horizontal speed greater than 28.0 m/s or hit the ball at a different angle so the vertical component of its velocity was greater.

71. Answers may vary. Sample answer: The long jumper can increase the length of his jump by jumping at an angle closer to 45°.

72. (a) Given: $\vec{v}_i = 26 \text{ m/s [75° above horizontal]}$; $d_{iy} = d_{fy}$; $g = 9.8 \text{ m/s}^2$

Required: Δd_x

Analysis: Since the snowball lands at the height it was thrown from, use the range formula,
$\Delta d_x = \dfrac{v_i^2 \sin 2\theta}{g}$.

Solution: $\Delta d_x = \dfrac{v_i^2 \sin 2\theta}{g}$

$$= \frac{(26 \text{ m/s})^2 (\sin 150°)}{9.8 \text{ m/s}^2}$$

$\Delta d_x = 34 \text{ m}$

Statement: The horizontal range of the snowball is 34 m.

(b) Since the two snowballs have the same initial speed and the same range, their angles of inclination must be complementary. The second snowball is thrown at 15° to the horizontal.

(c) Given: $\vec{v}_{i1} = 26 \text{ m/s [75° above horizontal]}$; $\vec{v}_{i2} = 26 \text{ m/s [15° above horizontal]}$; $g = 9.8 \text{ m/s}^2$

Required: time delay before throwing second snowball

Analysis: Calculate the time of flight for each snowball using $\Delta t = \dfrac{2v_i \sin\theta}{g}$. The required time delay between the throws is the difference in these times of flight.

Solution: snowball 1:

$$\Delta t = \frac{2v_{i1} \sin\theta}{g}$$

$$= \frac{2(26 \cancel{\text{m}}/\cancel{\text{s}})(\sin 75°)}{9.8 \cancel{\text{m}}/\text{s}^{\cancel{2}}}$$

$\Delta t_1 = 5.125 \text{ s}$ (two extra digits carried)

snowball 2:

$$\Delta t = \frac{2v_{i2} \sin\theta}{g}$$

$$= \frac{2(26 \cancel{\text{m}}/\cancel{\text{s}})(\sin 15°)}{9.8 \cancel{\text{m}}/\text{s}^{\cancel{2}}}$$

$\Delta t_2 = 1.373 \text{ s}$ (two extra digits carried)

The time delay should be
$\Delta t = \Delta t_1 - \Delta t_2$
$= 5.125 \text{ s} - 1.373 \text{ s}$
$\Delta t = 3.8 \text{ s}$

Statement: He should throw the second snowball 3.8 s after throwing the first.

73. (a) Given: $\vec{v}_i = 27$ m/s [20.0° above horizontal]; $d_{iy} = d_{fy}$; $g = 9.8$ m/s^2

Required: maximum height, d_y

Analysis: The maximum height occurs when $v_y = 0$ m/s . Use the formula $\Delta d_y = \frac{(v_i \sin\theta)^2}{2g}$ to determine d_y.

Solution: $\Delta d_y = \frac{(v_i \sin\theta)^2}{2g}$

$= \frac{((27 \text{ m/s})\sin 20.0°)^2}{2(9.8 \text{ m/s}^2)}$

$\Delta d_y = 4.4$ m

Statement: The ball's maximum height is 4.4 m above the ground.

(b) When air resistance is negligible, the flight is symmetric going up and coming down. The speed of the ball as it hits the ground is 27 m/s.

74. (a) Given: $v_x = 27$ m/s; $v_{iy} = 0$ m/s; $\Delta d_y = -2.4$ m; $g = 9.8$ m/s^2

Required: Δt

Analysis: Use $\Delta d_y = v_{1y}\Delta t - \frac{1}{2}g\Delta t^2$ to determine Δt: $\Delta t = \frac{v_{1y} \pm \sqrt{v_{1y}^2 - 4\left(\frac{1}{2}g\right)(\Delta d_y)}}{g}$

Solution: $\Delta t = \frac{v_{1y} \pm \sqrt{v_{1y}^2 - 4\left(\frac{1}{2}g\right)(\Delta d_y)}}{g}$

$\Delta t = \frac{0 \text{ m/s} \pm \sqrt{(0 \text{ m/s})^2 - 4(4.9 \text{ m/s}^2)(-2.4 \text{ m})}}{9.8 \text{ m/s}^2}$

$\Delta t = \pm 0.70$ s

Statement: The volleyball hits the floor after 0.70 s.

(b) Given: $v_x = 27$ m/s; $\Delta t = 0.6999$ s

Required: Δd_x

Analysis: $\Delta d_x = v_x \Delta t$

Solution: $\Delta d_x = v_x \Delta t$

$= (27 \text{ m}/\cancel{\text{s}})(0.6999 \; \cancel{\text{s}})$

$\Delta d_x = 19$ m

Statement: The volleyball travels a horizontal distance of 19 m.

(c) Given: $v_{iy} = 0$ m/s; $\Delta t = 0.6999$ s; $g = 9.8$ m/s^2

Required: v_f

Analysis: The horizontal component of the velocity is constant throughout. The vertical component changes with constant acceleration: $\Delta v_y = -g\Delta t$. Determine the vertical component of the final velocity and then construct the final speed

Solution:

Determine the vertical component of the final velocity.

$$\begin{aligned}\Delta v_y &= -g\Delta t \\ v_{fy} &= v_{iy} - g\Delta t \\ &= 0\text{ m/s} - (9.8\text{ m/s}^{\cancel{2}})(0.6999\ \cancel{\text{s}}) \\ v_{fy} &= -6.585\text{ m/s}\ \text{ (two extra digits carried)}\end{aligned}$$

Since $v_{fx} = v_x = 1.93$ m/s, combine the components to determine the final speed.

$$\begin{aligned}|\vec{v}_f| &= \sqrt{(v_{fx})^2 + (v_{fy})^2} \\ &= \sqrt{(27\text{ m/s})^2 + (-6.585\text{ m/s})^2} \\ |\vec{v}_f| &= 28\text{ m/s}\end{aligned}$$

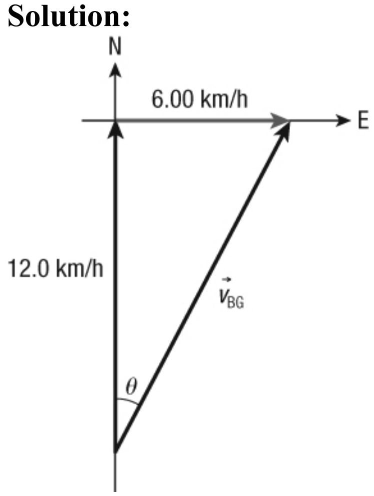
Statement: The final speed of the volleyball is 28 m/s.

75. Given: $\vec{v}_{BW} = 12.0$ km/h [N]; $\vec{v}_{WG} = 6.00$ km/h [E]

Required: $\vec{v}_{BG}$

Analysis: The vector addition diagram for $\vec{v}_{BG} = \vec{v}_{BW} + \vec{v}_{WG}$ is a right-angled triangle. Use the Pythagorean theorem and the tangent ratio to determine $\vec{v}_{BG}$.

Solution:



Determine the magnitude of $\vec{v}_{\text{BG}}$.

$$v_{\text{BG}} = \sqrt{(v_{\text{BW}})^2 + (v_{\text{WG}})^2}$$
$$= \sqrt{(12.0\text{ km/h})^2 + (6.00\text{ km/h})^2}$$
$$v_{\text{BG}} = 13.4\text{ km/h}$$

Determine the direction of $\vec{v}_{\text{BG}}$

$$\theta = \tan^{-1}\left(\frac{|\vec{v}_{\text{WG}}|}{|\vec{v}_{\text{BW}}|}\right)$$
$$= \tan^{-1}\left(\frac{6.00\ \cancel{\text{km/h}}}{12.0\ \cancel{\text{km/h}}}\right)$$
$$\theta = 26.6°$$

Statement: The velocity of boat with respect to the ground is 13.4 km/h [N 26.6° E].

76. (a) Given: swimmer leg 1: $\Delta t_1 = 0.25$ h; swims upstream ;
swimmer leg 2: swims downstream, meets raft; Raft: drifts downstream $\Delta d_{\text{R}} = 1.0$ km

Required: speed of current, *w*

Analysis: There are three different motions that interconnect: the speed of the current, *w*; the speed of the swimmer with respect to the water, *v*; and the swimmer's speed with respect to the shore, v_1. For the swimmer going upstream, I know the time interval Δt_1. The swimmer's speed with respect to the shore is $v_1 = v - w$. Call the distance covered Δd_1.

For the swimmer going downstream, his speed with respect to the water is $v_2 = v + w$. Call the time taken Δt_2 and the distance covered Δd_2.

For the raft, the distance covered is $\Delta d_{\text{R}} = 1.0\text{ km}$ and its speed is v_{R}. Call the time taken $\Delta t_{\text{R}} = t$. All of the motions occur at constant speed, so rewrite the relation $\Delta d = v\Delta t$ for each motion. Units will be omitted for clarity and convenience. At this point some algebraic manipulation should yield the value of *w*.

Solution:

swimmer leg 1:

$$\Delta d_1 = v_1 \Delta t_1$$
$$\Delta d_1 = \frac{(v - w)}{4}$$

swimmer leg 2:

$$\Delta d_2 = v_2 \Delta t_2$$
$$\Delta d_2 = (v + w)\left(t - \frac{1}{4}\right)$$

raft:

$$\Delta d_{\text{R}} = v_{\text{R}} \Delta t_{\text{R}}$$
$$1 = wt$$
$$t = \frac{1}{w}$$

The swimmer swims to meet the raft, so

$$\Delta d_2 = \Delta d_1 + \Delta d_{\text{R}}$$

Substitute into this last equation and simplify.

$$\Delta d_2 = \Delta d_1 + \Delta d_R$$
$$(v+w)\left(t-\frac{1}{4}\right) = \frac{(v-w)}{4}+1$$
$$(v+w)\left(\frac{1}{w}-\frac{1}{4}\right) = \frac{(v-w)}{4}+1$$
$$(v+w)\left(\frac{4-w}{\cancel{4w}}\right) = \frac{(v-w)w}{\cancel{4w}}+\frac{4w}{\cancel{4w}}$$
$$(v+w)(4-w) = (v-w)w+4w$$
$$4v+\cancel{4w}-vw-\cancel{w^2} = vw-\cancel{w^2}+\cancel{4w}$$
$$4\cancel{v} = 2\cancel{v}w$$
$$w = 2$$

Statement: The speed of the current is 2.0 km/h.

77. Given: $\vec{v}_{\text{SW}} = 0.45$ m/s [N]; $\vec{v}_{\text{WG}} = 2.5$ m/s [W]; $\Delta t = 200.0$ s

Required: width of river, Δd

Analysis: Since the swimmer heads directly across the river, her velocity, $\vec{v}_{\text{SW}}$, is actually the [N]-component of $\vec{v}_{\text{SG}}$. Just use $\Delta d = v\Delta t$ and calculate Δd. Note that the size of the current is irrelevant in this problem.

Solution:
$$\begin{aligned}\Delta d &= v\Delta t\\ &= (0.45\text{ m/s})(200.0\text{ s})\\ \Delta d &= 9.0\times 10^1\text{ m}\end{aligned}$$

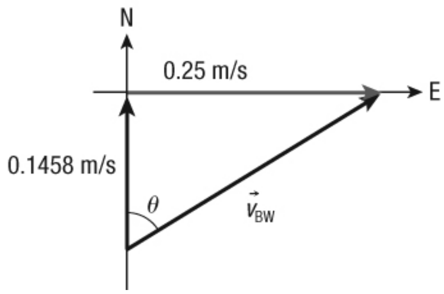
Statement: The river is 9.0×10^1 m wide.

78. Given: $\Delta d = 35$ m; $\Delta t = 4.0$ min $= 240$ s; $\vec{v}_{\text{WG}} = 0.25$ m/s [parallel to shore];
$\vec{v}_{\text{BG}}$ is [perpendicular to shore]

Required: $\vec{v}_{\text{BW}}$

Analysis: Use the given time interval and displacement to determine the speed of the boat with respect to the ground. The vector addition diagram for $\vec{v}_{\text{BG}} = \vec{v}_{\text{BW}} + \vec{v}_{\text{WG}}$ is a right-angled triangle. Use the Pythagorean theorem and the tangent ratio to determine $\vec{v}_{\text{BW}}$.

Solution: Determine the magnitude of $\vec{v}_{\text{BG}}$.
$$\begin{aligned}v_{\text{BG}} &= \frac{\Delta d}{\Delta t}\\ &= \frac{35\text{ m}}{240\text{ s}}\\ v_{\text{BG}} &= 0.1458\text{ m/s}\end{aligned}$$



Determine the magnitude of $\vec{v}_{BW}$.

$$v_{BW} = \sqrt{(v_{WG})^2 + (v_{BG})^2}$$
$$= \sqrt{(0.25\text{ m/s})^2 + (0.1458\text{ m/s})^2}$$
$$v_{BW} = 0.29\text{ m/s}$$

Determine the direction of $\vec{v}_{BW}$

$$\theta = \tan^{-1}\left(\frac{|\vec{v}_{WG}|}{|\vec{v}_{BE}|}\right)$$
$$= \tan^{-1}\left(\frac{0.25\ \cancel{\text{m/s}}}{0.1458\ \cancel{\text{m/s}}}\right)$$
$$\theta = 60°$$

Statement: The velocity of the boat with respect to the ground is 0.29 m/s at 60° upstream from straight across.

79. Given: $\vec{v}_{PA} = 290$ km/h [E 42° S]; $\vec{v}_{AG} = 65$ km/h [E 25° N]

Required: $\vec{v}_{PG}$

Analysis: Use the component method of solution. Use $(v_{PG})_x = (v_{PA})_x + (v_{AG})_x$ and $(v_{PG})_y = (v_{PA})_y + (v_{AG})_y$, with east and north as positive.

Solution: x-components:

$$(v_{PG})_x = (v_{PA})_x + (v_{AG})_x$$
$$= (290\text{ km/h})(\cos 42°) + (65\text{ km/h})(\cos 25°)$$
$$= 215.51\text{ km/h} + 58.91\text{ km/h}$$
$$(v_{PG})_x = 274.42\text{ km/h}$$

y-components:

$$(v_{PG})_y = (v_{PA})_y + (v_{AG})_y$$
$$= (-290\text{ km/h})(\sin 42°) + (65\text{ km/h})(\sin 25°)$$
$$= -194.05\text{ km/h} + 27.47\text{ km/h}$$
$$(v_{PG})_y = -166.58\text{ km/h}$$

Now use these components to determine $\vec{v}_{\text{PG}}$:

$$\left|\vec{v}_{\text{PG}}\right| = \sqrt{(v_{\text{PG}x})^2 + (v_{\text{PG}y})^2}$$
$$= \sqrt{(274.42 \text{ km/h})^2 + (166.58 \text{ km/h})^2}$$
$$\left|\vec{v}_{\text{PG}}\right| = 320 \text{ km/h}$$

$$\theta = \tan^{-1}\left(\frac{166.58 \ \cancel{\text{km/h}}}{274.42 \ \cancel{\text{km/h}}}\right)$$
$$\theta = 31^\circ$$

Statement: The velocity of the plane relative to the ground is 320 km/h [E 31° S].

Evaluation

80. Answers may vary. Sample answer: The time for both balls to reach the ground is equal to $\sqrt{2h/g}$, where h is the height they are dropped from. This equation does not depend on the mass of the balls, so the two balls will take the same amount of time to reach the ground.

81. Answers may vary. Sample answer: Navigation of space probes requires using displacement, velocity, and acceleration vectors. Total displacement and relative-velocity calculations involve the addition and subtraction of vectors.

82. Answers may vary. Sample answer: The argument is not correct because the condition of the javelin being in the air longer will not *always* result in it travelling a greater distance. This is because the time of flight depends on the vertical component of the initial velocity. If the javelin stays in the air for a longer duration of time, it will not necessarily cover a greater horizontal range. To be able to throw the javelin a greater distance, the thrower should also consider the speed and angle at which the javelin is thrown.

83. Answers may vary. Sample answer: If there were no gravity, the arrow would follow a straight-line trajectory and the archer should aim directly at the bull's-eye. Including the effect of gravity means the arrow follows a parabolic trajectory, not a straight line. To hit the bull's-eye, the archer should give the arrow's speed a small vertical component, ensuring that by the time it reaches the target, it will be at the right height to hit the bull's-eye. This small vertical component is achieved by aiming higher than the bull's-eye.

84. Answers may vary. Sample answer: The golfer can increase the range by increasing the angle of flight of the ball. This ensures that the ball stays in the air for longer. Since the wind is blowing from behind, this will carry the ball a greater distance.

85. Answers may vary. Sample answer: The equations will remain the same, but the values for time of flight, horizontal range, and maximum height will be smaller for planet A; for planet B, the equations will remain the same and the values will all be larger.

86. Answers may vary. Sample answer: The range of the snowball will be larger than 81 m. The friend's answer comes from using the range formula for an object landing at the same height as it was launched. In this case, the snowball was launched from a height of 45 m, returned to 45 m and landed on the ground. To determine the range of the snowball, you would have to solve this more general problem.

Reflect on Your Learning

87. Answers may vary. Sample answer: Even though I studied falling objects in grade 11, I keep thinking about a falling object speeding up for a bit and then falling at constant speed. I have realized this comes from my daily experience where many falling things experience lots of air resistance. It takes a while to automatically picture the idealized situation as a starting point and then add more realistic details as needed. I sometimes find it difficult to see the difference between similar scalar and vector quantities such as speed and velocity or distance and displacement. Drawing diagrams, especially in two dimensions, and actually following the difference between distance and displacement was useful.

88. Answers may vary. Sample answer: Our work discussed a lot of projectile motion and relative velocities. I realize that what we did is like a skeleton for solving more complex problems. Certainly air resistance must come into projectile motion somehow. Launching a rocket is a projectile problem, but the wind, the round and spinning Earth, decreasing air density, and all sorts of complications need to be included. Airplanes navigating across the continent will need to compensate for wind speeds and directions that change continuously and for how their own airspeed depends on the load and type of fuel they are using. It feels good to begin to separate the simple basic picture from the more realistic situations that need much more experience.

89. Answers may vary. Sample answer: The strobe images like Figure 2 on page 37 and the ones we made on the air table really showed me that falling objects don't immediately move quickly. Their speed really does build up in time. These images also convinced me that the horizontal motion has no effect on the vertical acceleration.

90. Answers may vary. Sample answer: Looking at projectile motion helped me understand the factors that affect the range of thrown ball. Knowing to throw a basketball at an angle close to 45° will help me make a better long pass.

Research

91. Answers may vary. Students' answers should include some of the following information.
370: Aristotle claims that free falling bodies accelerate but heavier bodies fall faster.
1589: Galileo demonstrates that objects fall at the same rate independent of mass.
1604: Galileo discovers that the distance for falling objects increases as square of time.
1613: Galileo discovers the principle of inertia.
1637: Descartes formalizes the principle of inertia.
1684: Newton discovers the inverse square law and mass dependence of gravity.
1687: Newton publishes the laws of motion and gravitation.

92. Answers may vary. Students should describe how the ramps and turns on the track are designed and what factors are taken into consideration. They can create a schematic diagram of the dirt track model, conduct experiments and prepare data tables and validate various theories they have learned in the chapter. Sample answer: On dirt bike tacks, the turns are banked so that the bikers can go as fast as possible through the turns. Changing the angle of the bank will change the velocity of the bikers as they pass through the turn. If turns are not banked, bikers must decrease their acceleration in order to pass through them. For the jumps, the trajectory depends on the angle of the jump and the velocity of the bikers. If the jump is angled at 45°, the bikers will jump the farthest.

93. Answers may vary. The details of the report depend on the sport chosen. Students should describe the motion of the objects that are involved in the sport. If possible, graphs may be drawn to represent critical ideas. Sample answer: In the sport of football, accuracy is very important in throwing and kicking the ball. Quarterbacks and punters try to throw and kick the ball so that it spirals, which reduces the air drag on the ball allowing them to throw and kick the ball farther and also more accurately. When quarterbacks and punters throw or kick the ball, the ball follows a parabolic path, otherwise known as projectile motion. Since accuracy is very important, the quarterback can change the trajectory of the ball by increasing or decreasing the force that is applied to the ball or by changing the angle that the ball is thrown. To throw the ball as far as possible, the quarterback should apply the maximum possible force and throw the ball at a 45° angle, while throwing a perfect spiral. For punters, accuracy is not always important. Most of the time punters want to kick the ball as far as possible and so they should apply the maximum possible force and kick the ball at a 45° angle, while kicking the ball so it travels in a perfect spiral. When accuracy does matter, such as when they want to determine their opponent's field position, like quarterbacks, the punter can change the trajectory of the ball by increasing or decreasing the force that is applied to the ball or by changing the angle that the ball is kicked.
94. Answers may vary. Sample answer:

Distance (m)	Cumulative Time (s)	Time for Interval (s)	Speed (m/s)
10	1.85	1.85	5.41
20	2.87	1.02	9.80
30	3.78	0.91	11.0
40	4.65	0.87	11.5
50	5.50	0.85	11.8
60	6.32	0.82	12.2
70	7.14	0.82	12.2
80	7.96	0.82	12.2
90	8.79	0.83	12.0
100	9.69	0.90	11.1

This table shows the split times for Usain Bolt's record-breaking run in the 2008 Olympics. Bolt accelerated to a speed around 12 m/s, which he maintained for about 40 m, before decreasing his acceleration at the end of the race. The acceleration was largest early in the race because this is when the *time for interval* changed the most. From 50 m to 80 m, Bolt's speed was exactly the same at 12.2 m/s. Near the end of the race, Bolt started to slow down. Due to fatigue, any runner would begin to slow down near the end; however, because Bolt had a sizable lead, he began to celebrate before crossing the finish line, which slowed him down, so it is difficult to determine how much of this decrease in speed was due to fatigue.
95. Answers may vary. Sample answer:
(a) Given: Reaction time v. Number of beers graph; possible initial speeds
Required: corresponding reaction distances
Analysis: A driver continues to drive at constant speed during the reaction time. Then braking begins. The reaction distance is the distance travelled before the brakes are applied. For each situation below, read the reaction time from the graph and then calculate the reaction distance using $\Delta d = v\Delta t$.

Solution: The reaction time for "No alcohol" is 0.8 s. At 17 m/s, the reaction distance is

$$\begin{aligned}\Delta d &= v\Delta t\\ &= (17\text{ m}/\cancel{\text{s}})(0.8\ \cancel{\text{s}})\\ \Delta d &= 14\text{ m}\end{aligned}$$

At 25 m/s, the reaction distance is

$$\begin{aligned}\Delta d &= v\Delta t\\ &= (15\text{ m}/\cancel{\text{s}})(0.8\ \cancel{\text{s}})\\ \Delta d &= 20\text{ m}\end{aligned}$$

At 33 m/s, the reaction distance is

$$\begin{aligned}\Delta d &= v\Delta t\\ &= (33\text{ m}/\cancel{\text{s}})(0.8\ \cancel{\text{s}})\\ \Delta d &= 26\text{ m}\end{aligned}$$

The reaction time for 4 bottles of beer is 2.0 s. At 17 m/s, the reaction distance is

$$\begin{aligned}\Delta d &= v\Delta t\\ &= (17\text{ m}/\cancel{\text{s}})(2.0\ \cancel{\text{s}})\\ \Delta d &= 34\text{ m}\end{aligned}$$

At 25 m/s, the reaction distance is

$$\begin{aligned}\Delta d &= v\Delta t\\ &= (25\text{ m}/\cancel{\text{s}})(2.0\ \cancel{\text{s}})\\ \Delta d &= 50\text{ m}\end{aligned}$$

At 33 m/s, the reaction distance is

$$\begin{aligned}\Delta d &= v\Delta t\\ &= (33\text{ m}/\cancel{\text{s}})(2.0\ \cancel{\text{s}})\\ \Delta d &= 66\text{ m}\end{aligned}$$

The reaction time for 5 bottles of beer is 3.0 s. At 17 m/s, the reaction distance is

$$\begin{aligned}\Delta d &= v\Delta t\\ &= (17\text{ m}/\cancel{\text{s}})(3.0\ \cancel{\text{s}})\\ \Delta d &= 51\text{ m}\end{aligned}$$

At 25 m/s, the reaction distance is

$$\begin{aligned}\Delta d &= v\Delta t\\ &= (25\text{ m}/\cancel{\text{s}})(3.0\ \cancel{\text{s}})\\ \Delta d &= 75\text{ m}\end{aligned}$$

At 33 m/s, the reaction distance is

$$\begin{aligned}\Delta d &= v\Delta t\\ &= (33\text{ m}/\cancel{\text{s}})(0.8\ \cancel{\text{s}})\\ \Delta d &= 99\text{ m}\end{aligned}$$

Table 2

Speed	**Reaction Distance (m)**		
	No alcohol	4 bottles of beer	5 bottles of beer
17 m/s (60 km/h)	14	34	51
25 m/s (90 km/h)	20	50	75
33 m/s (120 km/h)	26	66	99

(b) Answers may vary. Sample answer: Alcohol interferes with the transmission of the signals from nerve cells to the brain. This results in slower reaction times because alcohol impairs comprehension and coordination, in particular. The reaction time increases when the amount of alcohol in the blood stream increases.
(c) Answers may vary. Sample answer: There is no safe level of alcohol that can be consumed by drivers because alcohol affects people differently. People metabolize alcohol at different rates, and many factors, such as one's metabolic rate and body mass, affect the rate of metabolism. The data in Table 2 show that at faster speeds, the distance travelled before brakes are applied increases. Alcohol affects many other abilities, such as concentration, judgment, and vision, which are very important for driving safely and other activities.

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1. (b)
2. (d)
3. (b)
4. (c)
5. (a)
6. (d)
7. (d)
8. False. The instantaneous velocity at a particular time is the slope of the *position*–time curve at that *time*.
9. True
10. False. The addition of two displacement vectors *does not depend* on the order in which they are added.
11. False. If the velocity vector of an object changes only in direction, the average acceleration is *not* zero.
12. True
13. False. A stone projected horizontally from a cliff will reach the ground *at the same time as* a stone dropped vertically down from the same cliff.
14. True
15. False. If $\vec{v}_{\mathrm{AB}} = 18.3\ \mathrm{m/s\ [S]}$, then $\vec{v}_{AB} = -18.3\ \mathrm{m/s\ [N]}$.